Understanding Non-perturbative Gauge Theories Using Anomaly Mediation of Supersymmetry Breaking

Hitoshi Murayama (Berkeley, Kavli IPMU)

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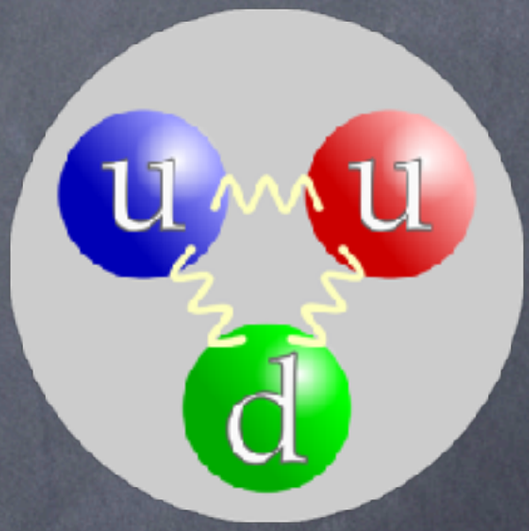
Csaba Csaki, Ofri Telem, Bea Noether, Digvijay Varier,
Bethany Suter, Jacob Leedom, Dan Kondo, Jack Gu,
Zhiyao Lu, Jason Wong, Andrew Goh, Gurpreet Singh

Can we solve QCD?



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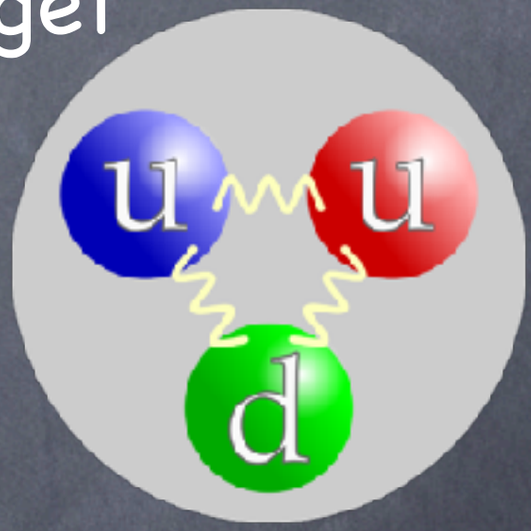

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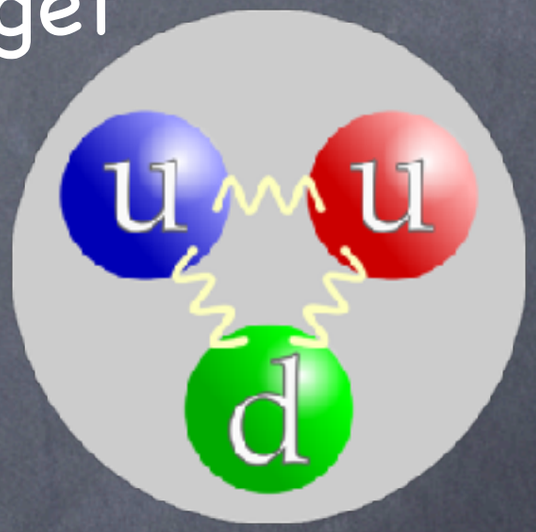

Dear friend,
I am Andre Ouedraogo, a banker by profession from Burkina Faso in West Africa and currently holding the post of Director Auditing and Accounting unit of the bank. It's my urgent need for a foreign partner that made me to contact you for this business. I have the opportunity of transferring the left over funds ($11.5 million) of one of my bank clients who died along with his entire family on 31 July 2000 in a plane crash. You can confirm the genuineness of the deceased death by clicking on this website.
http://news.bbc.co.uk/1/hi/world/europe/859479.stm
I need a foreign partner who will support me because i can not claim this money alone without a foreign partner since the deceased client (the owner of the fund) was a foreigner.

This fund ($11.5 million) will be shared between us in the ratio of 60/40. I agreed that 40% of this money will be for you as a respect to the provision of a foreign account while 60% will be for me and I want to assure you that this transaction is absolutely legal and risk free since i work in this bank and i have all the necessary information that might be needed. Before we proceed, i would like to know your ability to handle this over there in your country.
Please tell me more about the political/economic stability/monetary policy of your country. I need to know all these because i don't want to have problem with the Government of your country.

Kindly update me with the
following information because i want to know you more before we proceed on this transaction. Hope you will understand the importance of this request.

1. Your full name.....
2. Your age/sex .....
3. your occupation .....
4. Your residential address .....
5. Your nationality .....
6. Your private phone number .....
7. Your fax number .....

I will be waiting for your response.
Thanks for your understanding.
Have a great day.
Yours.
Andre Ouedraogo

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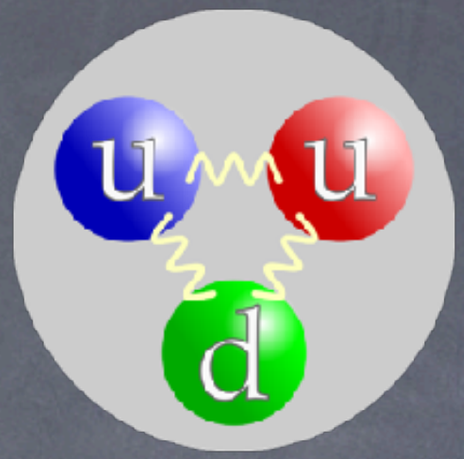

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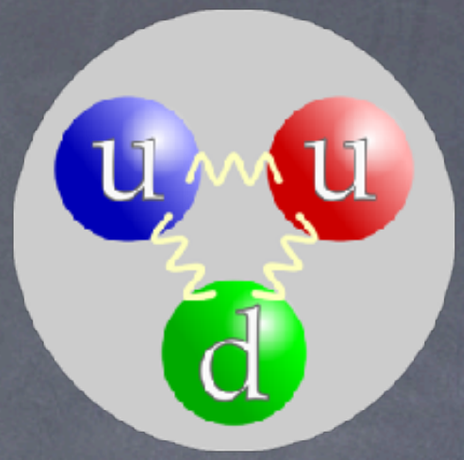

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  - Confinement!

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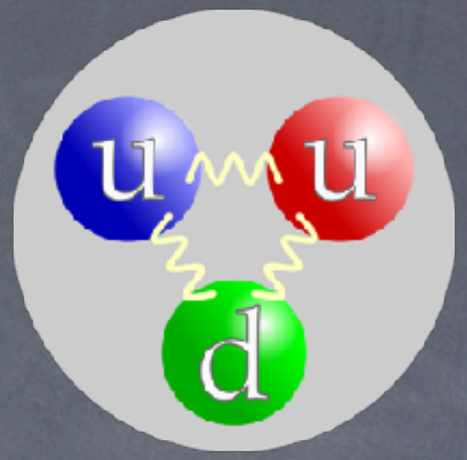

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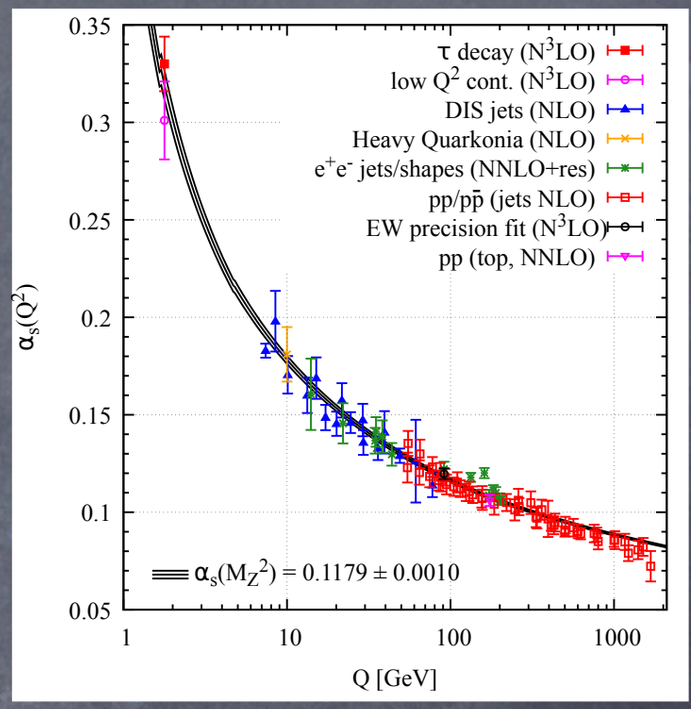

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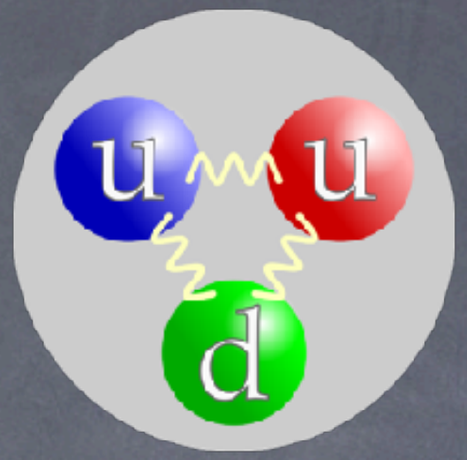

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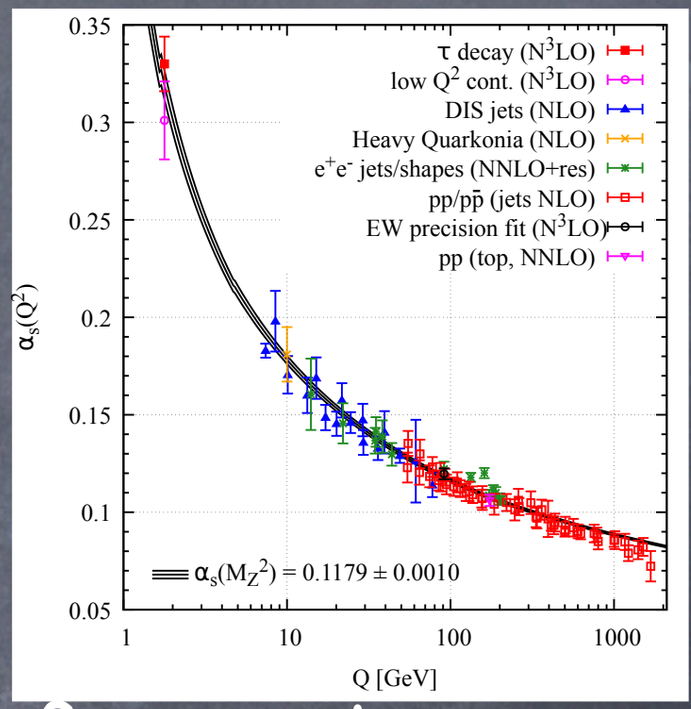

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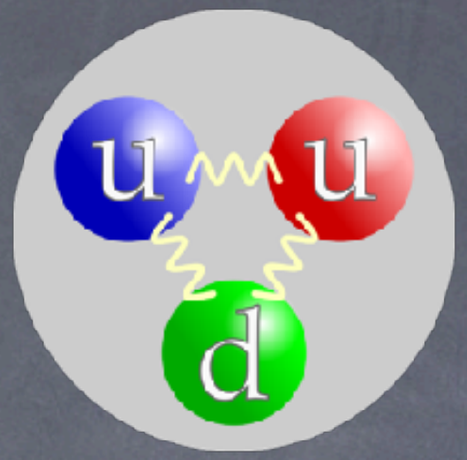

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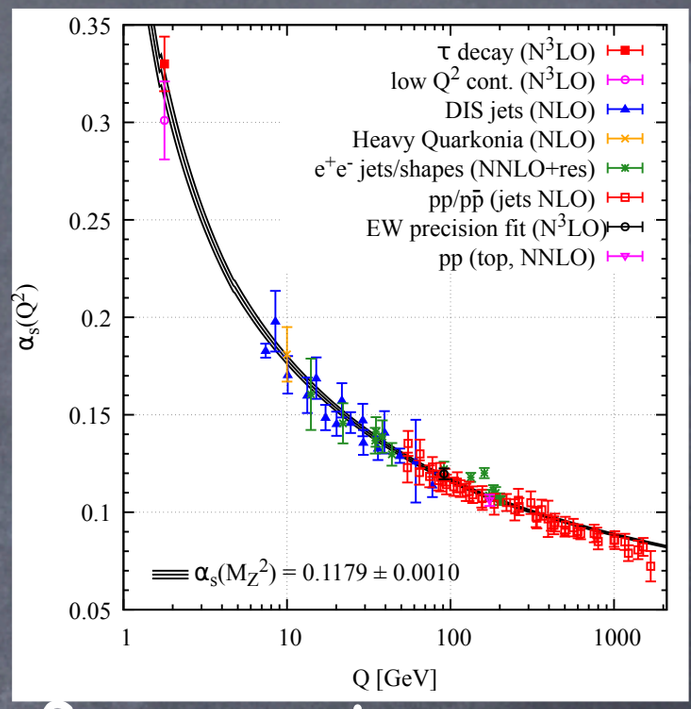

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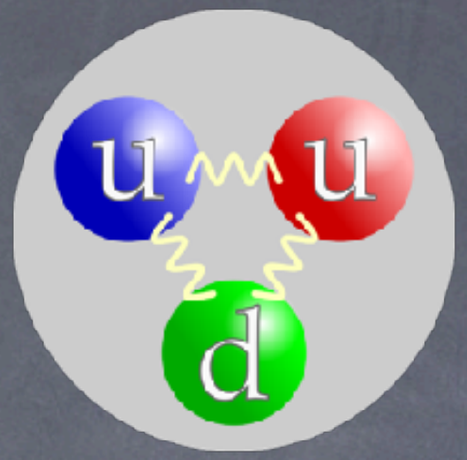

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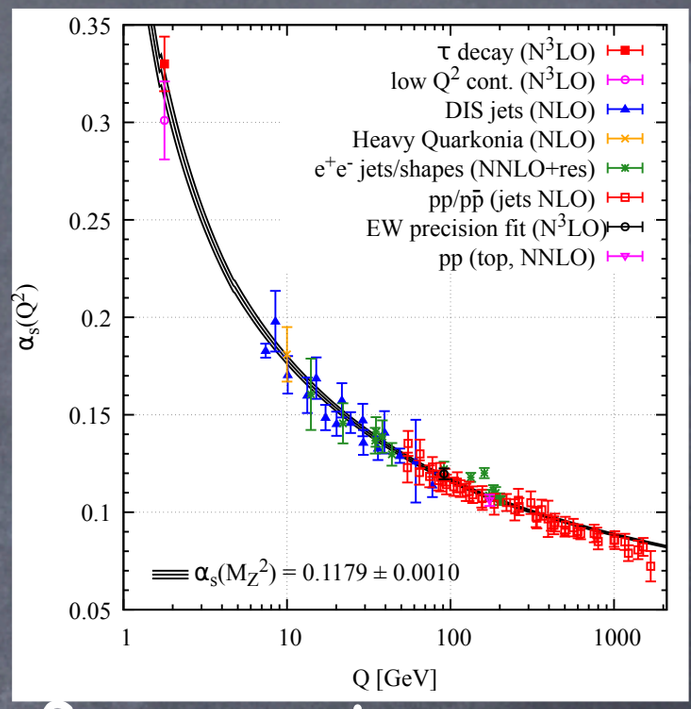

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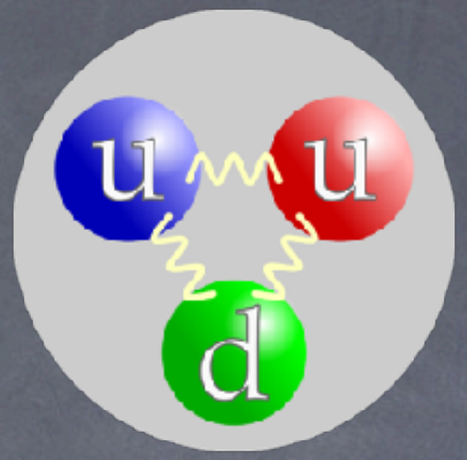

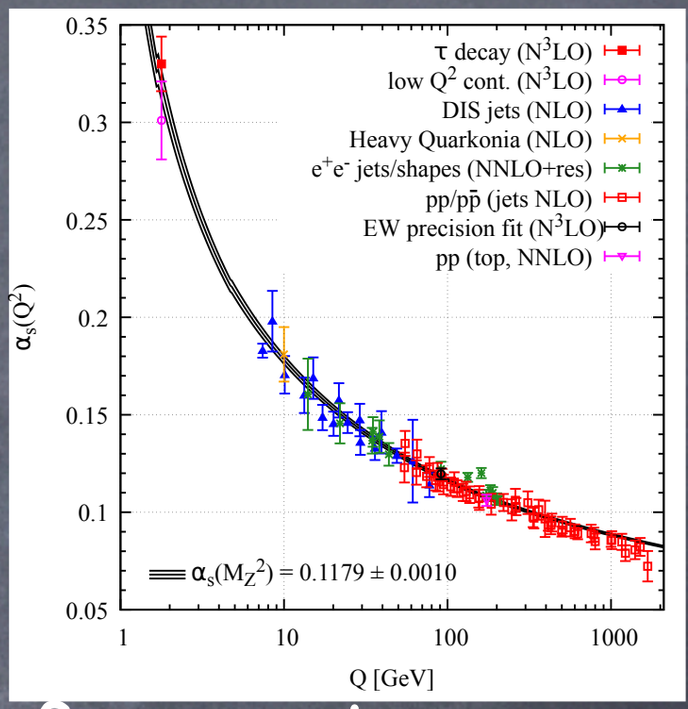

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  - why pion ≈ massless ≪ proton?
- very mysterious!

Feeling better

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  - assume monopole condensation
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  - massless QCD invariant under $SU(N_f)_L \times SU(N_f)_R \times U(1)_B$
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- but still not derived from QCD!

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  - N=2 SU(2) SYM a triplet field = vacua
  - SU(2) → U(1): magnetic monopoles!
  - special points = massless monopole/dyon
  - N=1 perturbation: monopole condensation!

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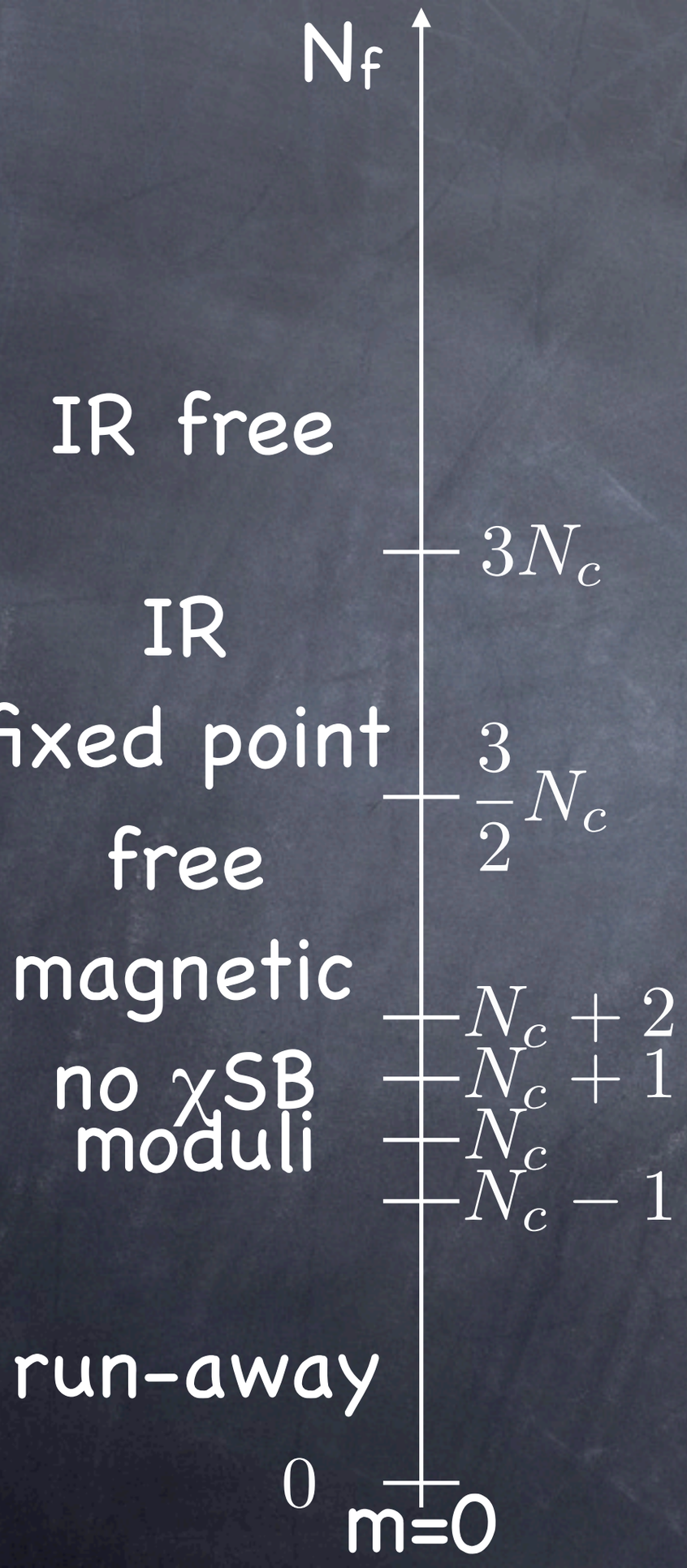
- Chiral symmetry breaking
  - N=2 doesn't have the chiral symmetry
  - N=1 (Seiberg) has too unusual phases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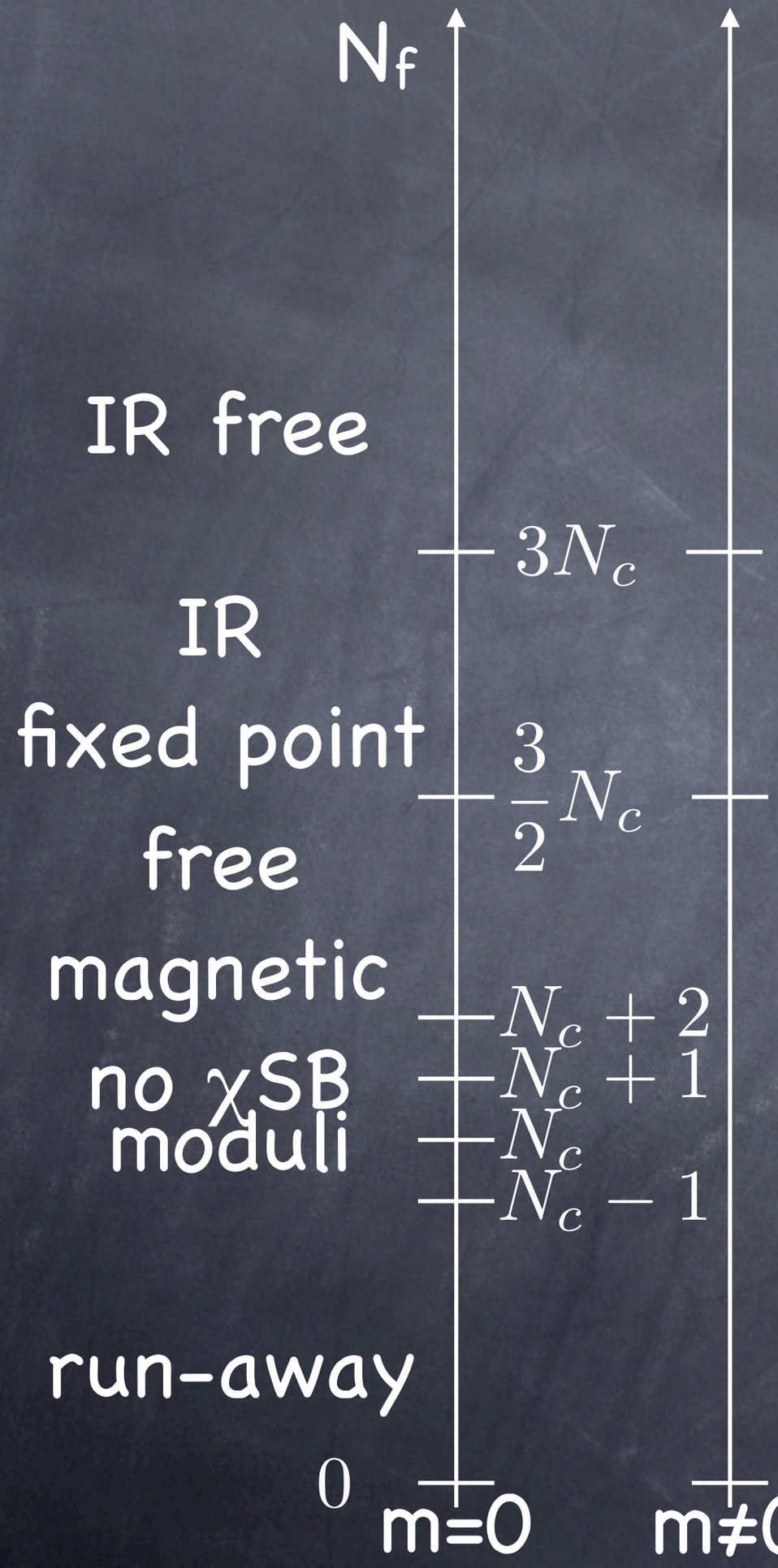
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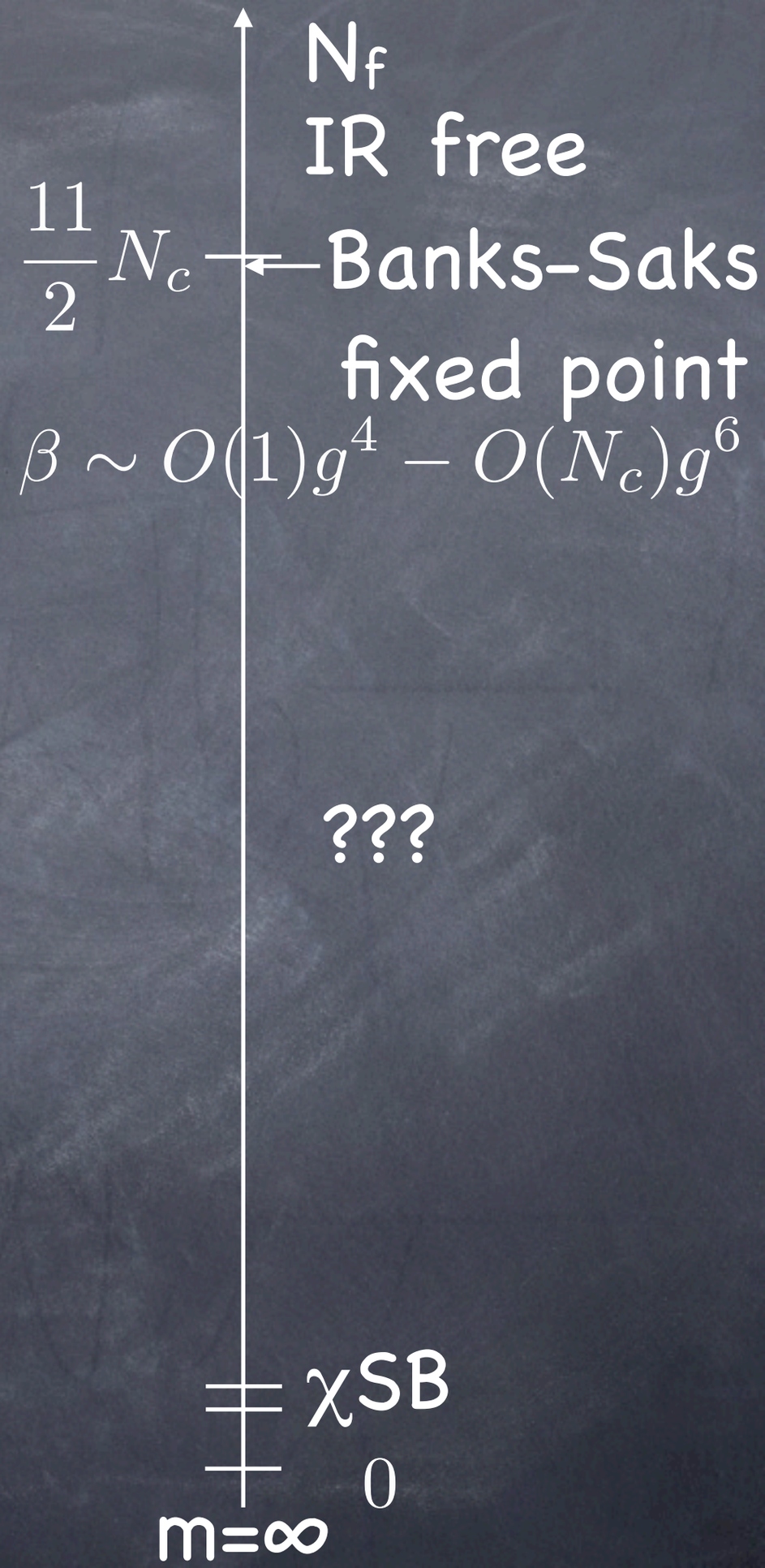
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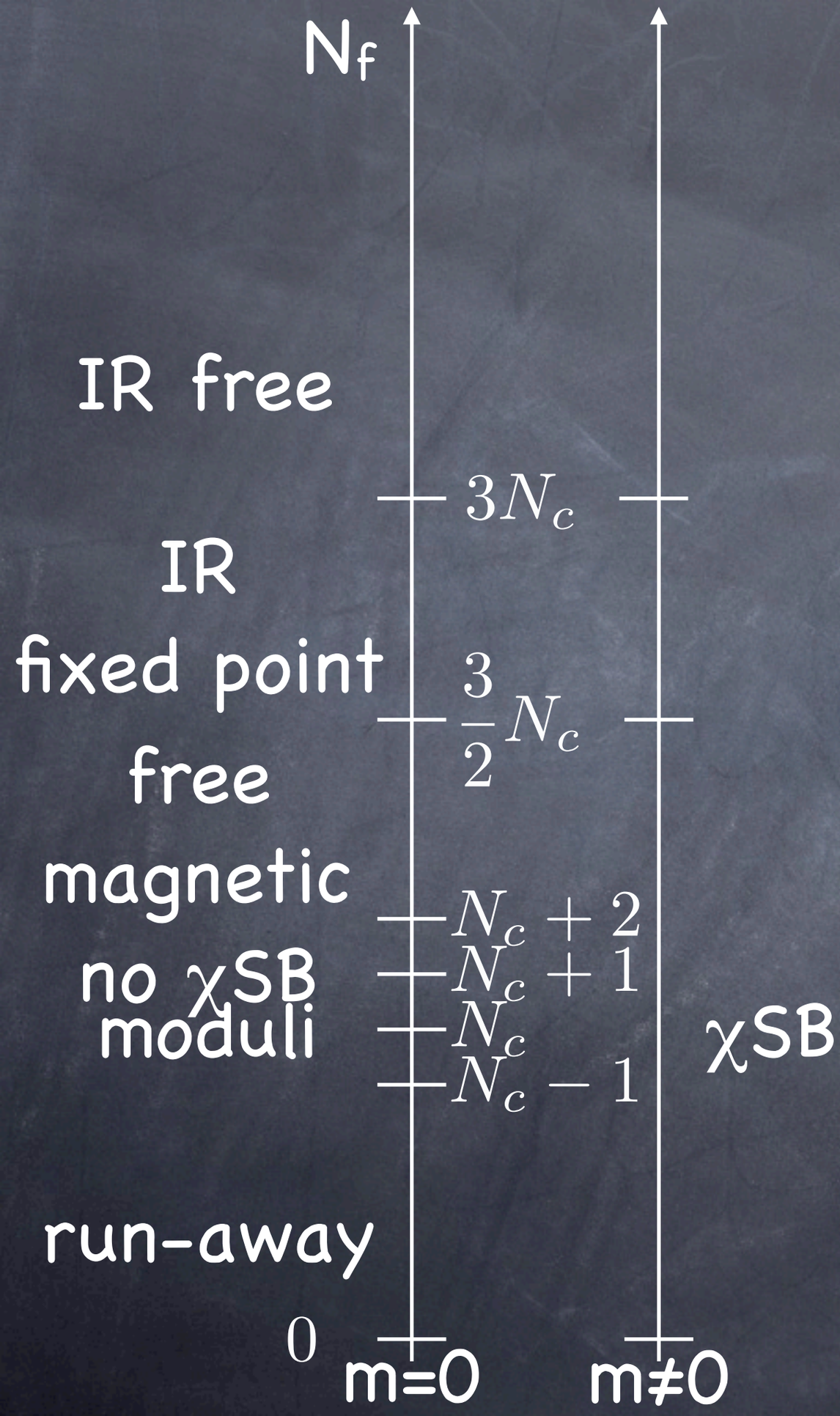
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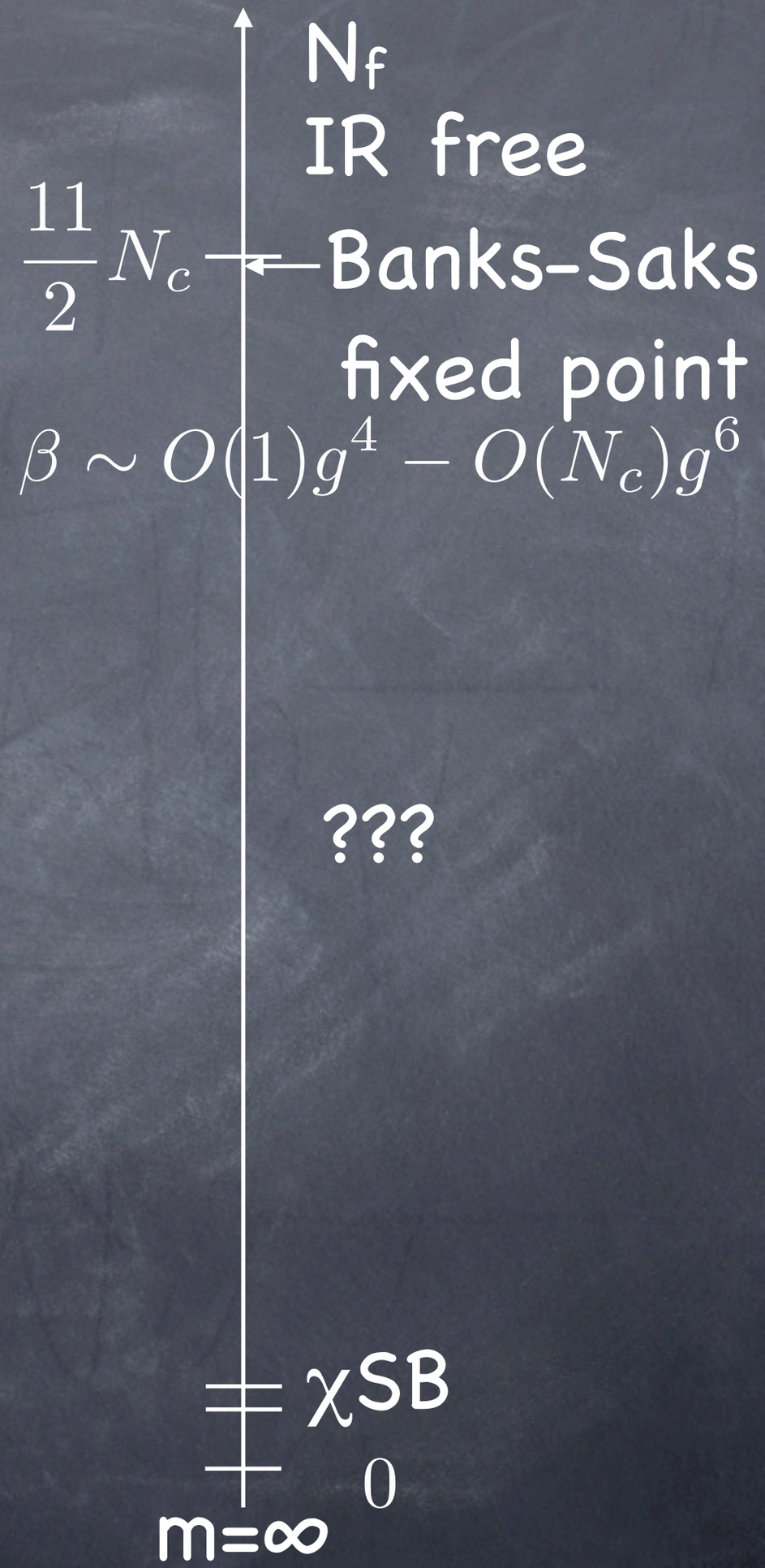
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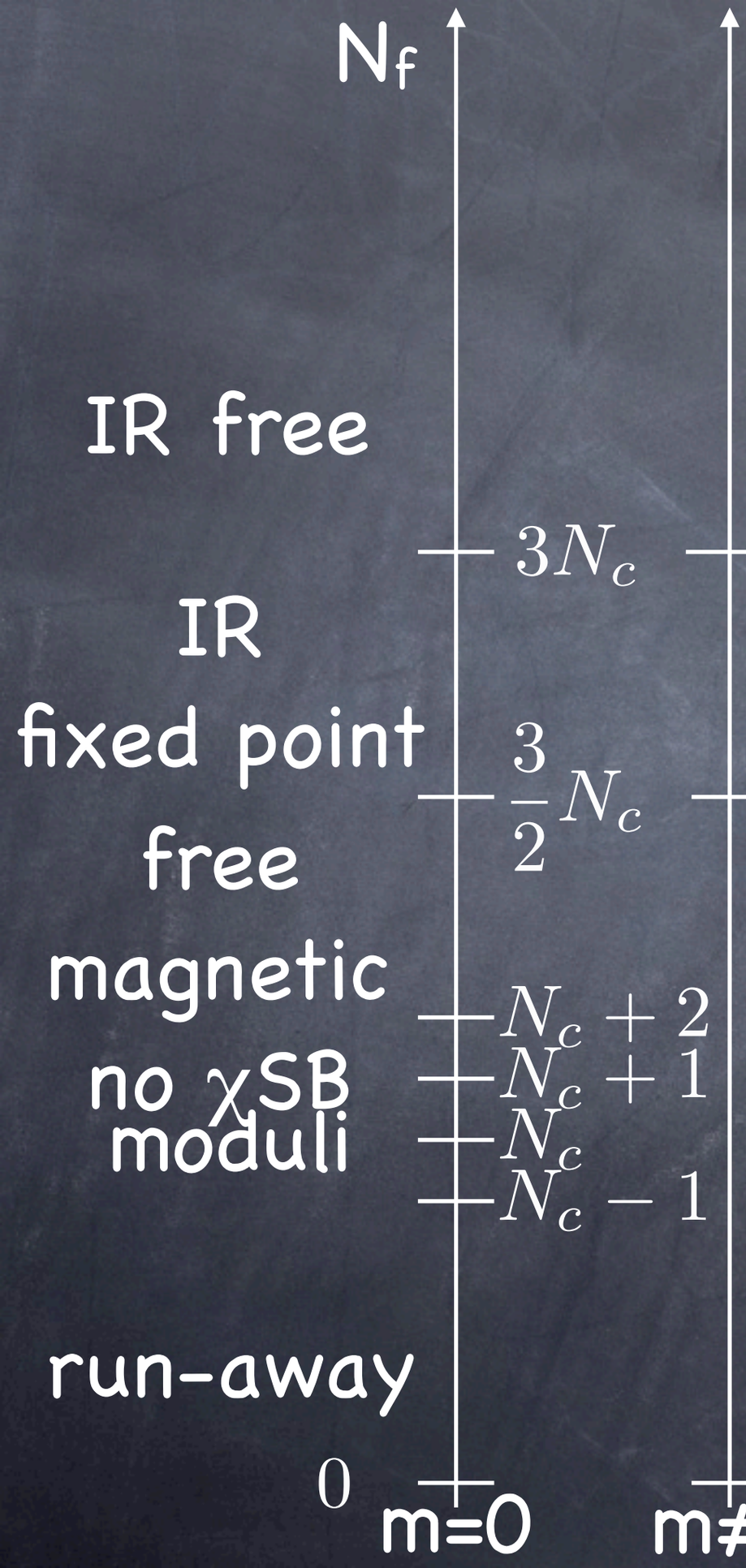
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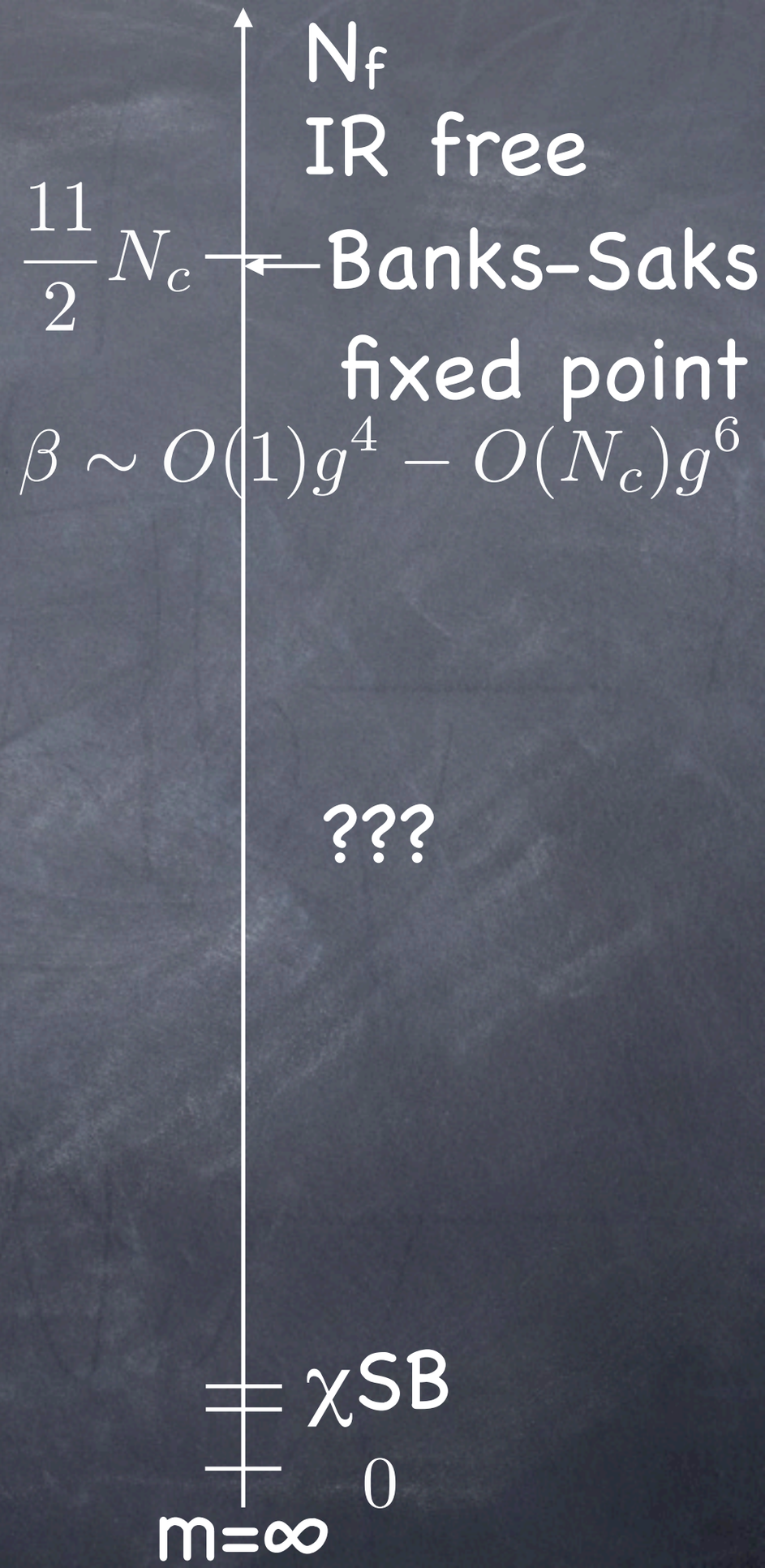
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Anomaly Mediation of SUSY Breaking

supergravity

$m_{3/2} = e^{K/2}|W|$

SUSY breaking

$V = e^K \left(|F|^2 - 3\,|W|^2 \right)$

no interaction

gauge theory

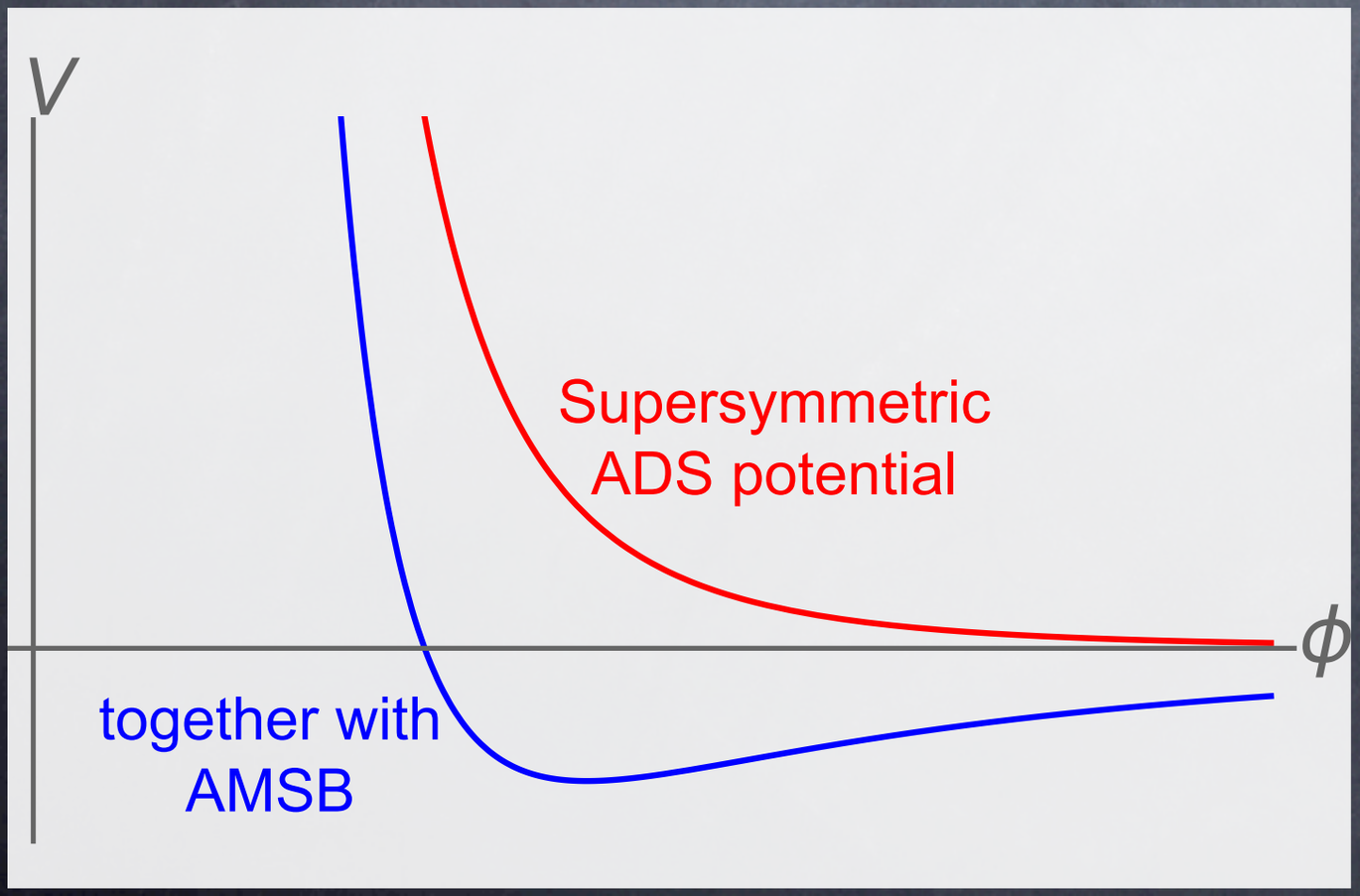
$m = \dfrac{W}{M_{Pl}^2}$

$N_f < N_c$

- run-away superpotential for $M^{ij} = \tilde{Q}^i Q^j$

$$W = (N_c - N_f)\left(\frac{\Lambda^{3N_c - N_f}}{\det M}\right)^{1/(N_c - N_f)} \qquad M^{ij} = \delta^{ij}\phi^2$$

$$V = 2N_f \left|\frac{\Lambda^{3N_c - N_f}}{\phi^{N_c + N_f}}\right|^{\frac{2}{N_c - N_f}} - m(3N_c - N_f)\left(\frac{\Lambda^{3N_c - N_f}}{\phi^{2N_f}}\right)^{\frac{1}{N_c - N_f}} + c.c.$$



$$M_{ij} = \delta_{ij}\left(\left(\frac{N_c + N_f}{3N_c - N_f}\right)^{N_c - N_f}\frac{\Lambda^{3N_c - N_f}}{m^{N_c - N_f}}\right)^{1/N_c}$$

$SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V$

χSB! Proving Nambu

mesino/gaugino loop

$\rightarrow$WZW term

N_f=1 special

no NGB, gapped

Deriving Chiral Lagrangian

$$Q = \begin{pmatrix} v\xi^T \\ 0 \end{pmatrix}, \quad \tilde{Q} = \begin{pmatrix} v\xi \\ 0 \end{pmatrix} \qquad \xi \to g_L \xi h^T, \quad \xi \to h^* \xi g_R^T$$

$$U = \xi\xi \to g_L U g_R^T$$

$$|D_\mu Q|^2 + \left|D_\mu \tilde{Q}\right|^2 = \frac{v^2}{2} \mathrm{Tr} \partial_\mu U^\dagger \partial^\mu U$$

$$f_\pi^2 = 2v^2 = 2\left(\left(\frac{N_c + N_f}{3N_c - N_f}\right) \frac{\Lambda^{3N_c - N_f}}{m^{N_c - N_f}}\right)^{N_c - N_f \; 1/N_c}$$

$\Lambda^{3N_c - N_f} = M^{3N_c - N_f} e^{-8\pi^2 / g_h^2(M)}$ holomorphic coupling

$= M^{3N_c - N_f} e^{-8\pi^2 / g_c^2(M)} Z^{-N_f} (g_c^2)^{-N_c}$ 1PI coupling

$= M^{3N_c - N_f} e^{-8\pi^2 N_c / g_{\mathrm{tH}}^2(M)} Z^{-N_f} N_c^{N_c} (g_{\mathrm{tH}}^2)^{-N_c}$

$f_\pi^2 \propto N_c$ 't Hooft coupling

Non-perturbative Condensates

$$M_{ij} = \delta_{ij} \left(\left(\frac{N_c + N_f}{3N_c - N_f} \right)^{N_c - N_f} \frac{\Lambda^{3N_c - N_f}}{m^{N_c - N_f}} \right)^{1/N_c}$$

$$\langle \bar{q}_i q_j \rangle = 4m\delta_{ij} \left(\left(\frac{N_c + N_f}{3N_c - N_f} \right)^{-N_f} \frac{\Lambda^{3N_c - N_f}}{m^{N_c - N_f}} \right)^{1/N_c}$$

$$\langle \mathrm{Tr}\lambda\lambda \rangle = 32\pi^2 \left(\left(\frac{N_c + N_f}{3N_c - N_f} \right)^{-N_f} \Lambda^{3N_c - N_f} m^{N_f} \right)^{1/N_c}$$

$$\langle \mathrm{Tr}G_{\mu\nu}G^{\mu\nu} \rangle = 32\pi^2 m \left(\left(\frac{N_c + N_f}{3N_c - N_f} \right)^{-N_c - N_f} \Lambda^{3N_c - N_f} m^{N_f} \right)^{1/N_c}$$

$$m_{\eta'}^2 = 2m^2 \frac{N_f (3N_c - N_f)^2}{(N_c - N_f)^2 (N_c + N_f)} \propto N_c^{-1}$$

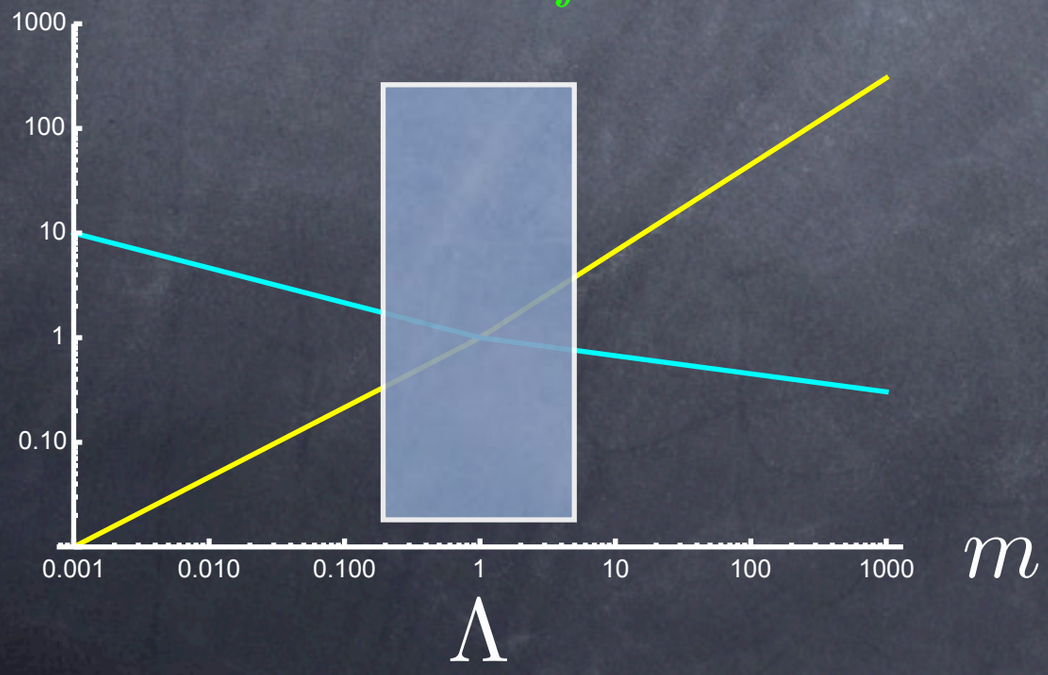
fermion bilinear

$$M^{ij} = \tilde{q}_L^{i*} \tilde{q}_R^j + \theta^2 \bar{q}_L^i q_R^j$$

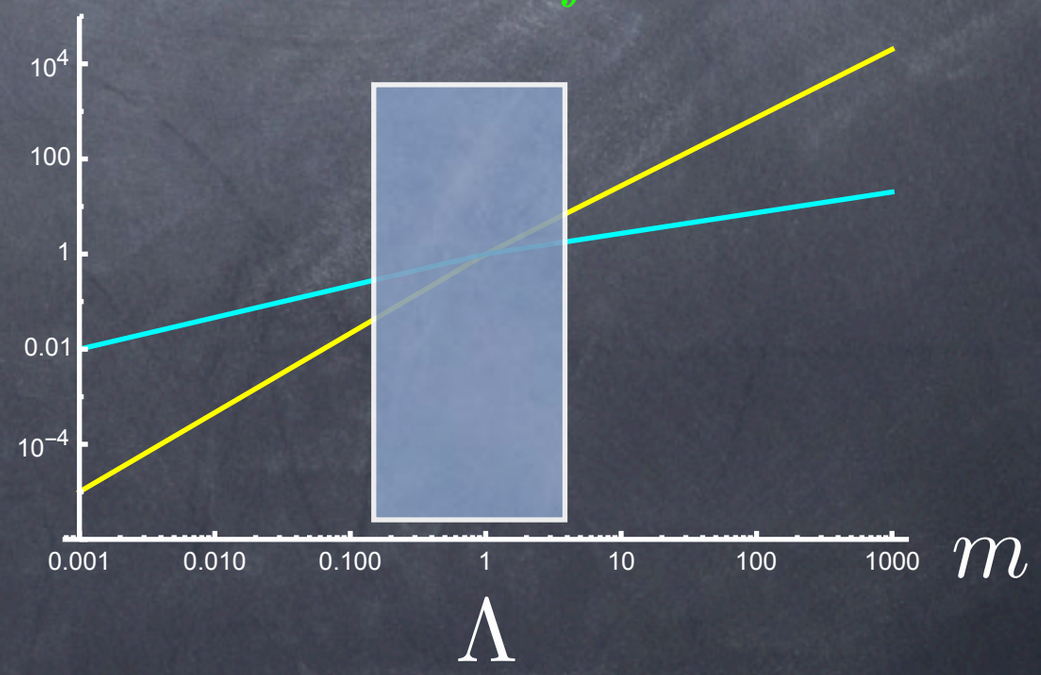
$$\tilde{q}_R^{i*} \tilde{q}_L^j \sim \left(m^{N_f - N_c} \Lambda^{3N_c - N_f} \right)^{1/N_c}$$

$$\bar{q}_R^i q_L^j \sim m \left(m^{N_f - N_c} \Lambda^{3N_c - N_f} \right)^{1/N_c}$$

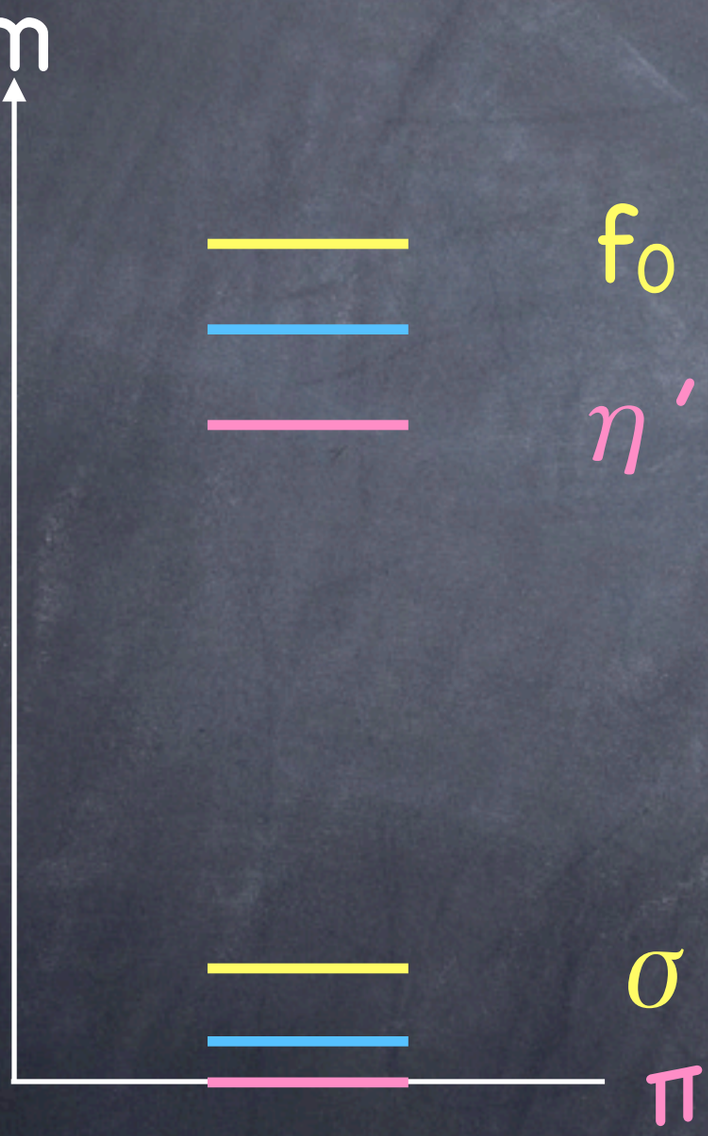
$N_c = 3, N_f = 2$

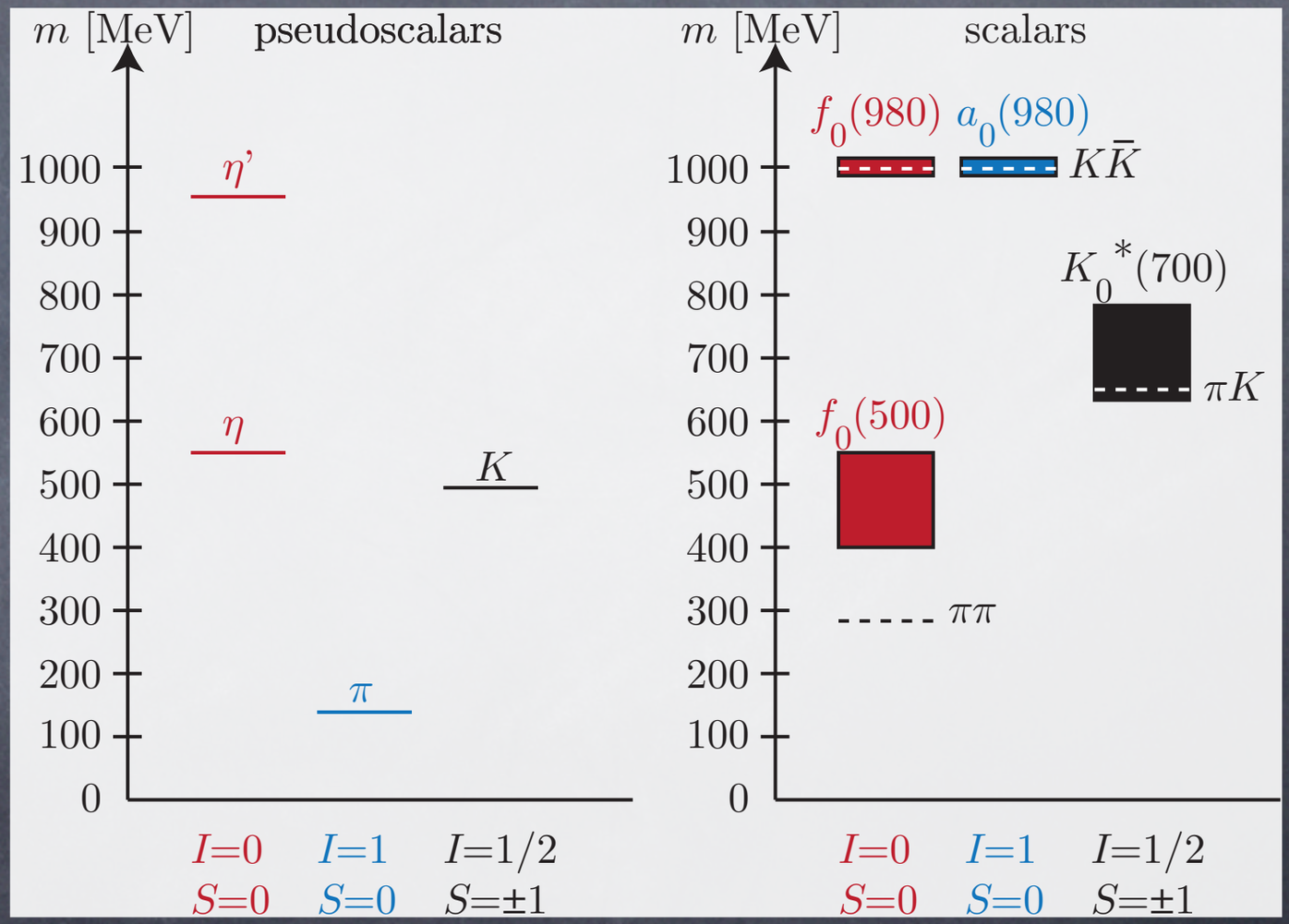


$N_c = 3, N_f = 5$



Light spectrum





— fermions

N_f

IR free

$3(N_c-2)$

IR fixed point

?

$\frac{3}{2}(N_c-2)$

free magnetic

N_c

no χSB

N_c-1

Coulomb

N_c-2

χSB

2 branches

N_c-3

2 branches

N_c-4

N_c-5

run-away

0

m=0

m≠0

Λ

N_f

IR free

$\frac{11}{2}N_c$ ← Banks-Saks fixed point

$3(N_c-2)$

?

$\frac{3}{2}(N_c-2)$

???

χSB

χSB

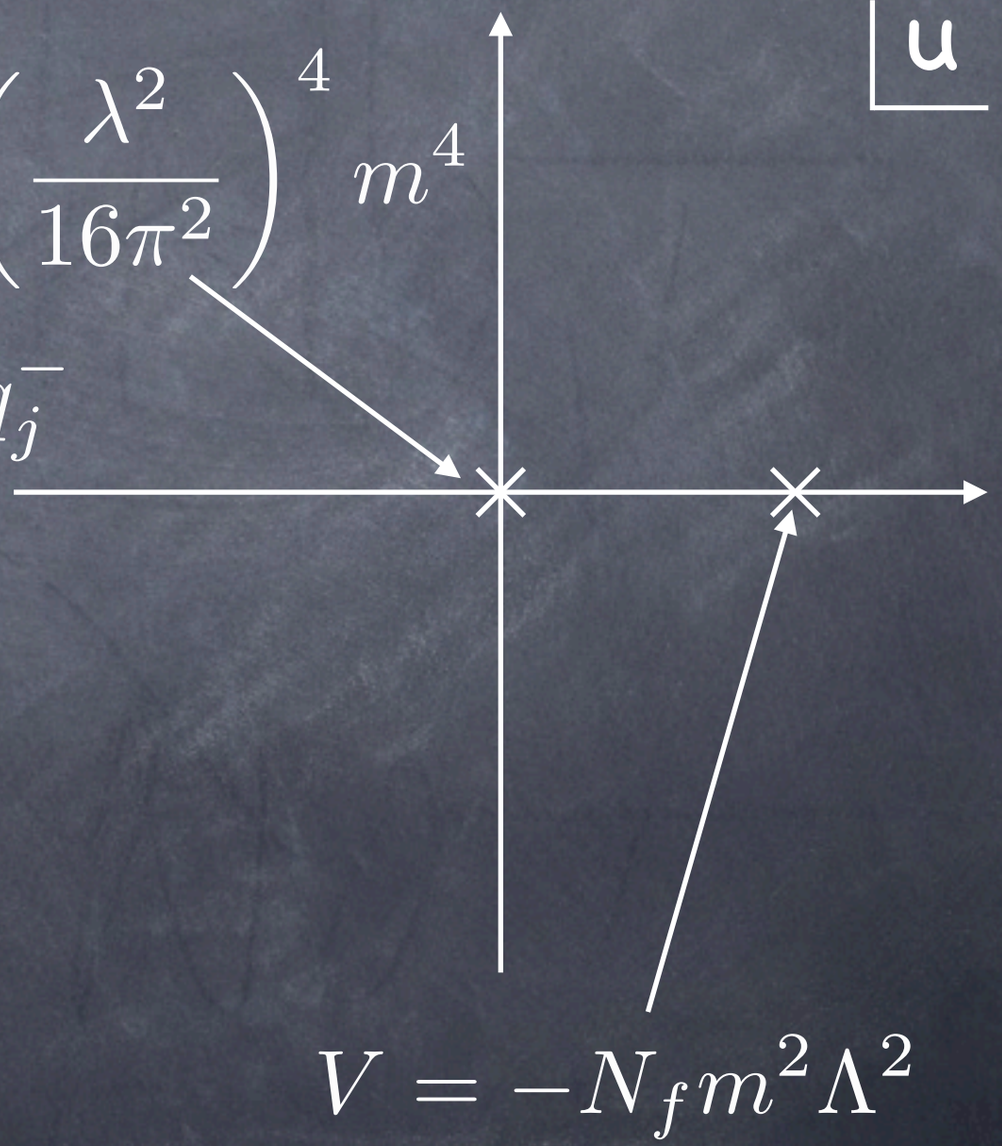
0

m=∞

$N_f = N_c - 2$

- for $M^{ij}=Q^iQ^j\neq 0$ with rank $M=N_f$, $SO(N_c)$ is broken to $SO(2)$
- Coulomb branch $u = \det M$
- two singularities
- $u = \det M = 0$
  - dyons: $q_i^{\pm}$ $W = \frac{1}{\mu} M^{ij} q_i^+ q_j^-$
- $u = \det M = \Lambda^{2N_f}$
  - monopoles: $W = (u - \Lambda^{2N_f}) E^+ E^-$
    $|E^{\pm}| = (m\Lambda)^{1/2}$
  - both monopoles and meson condense!

$$V \approx -\left(\frac{\lambda^2}{16\pi^2}\right)^4 m^4$$

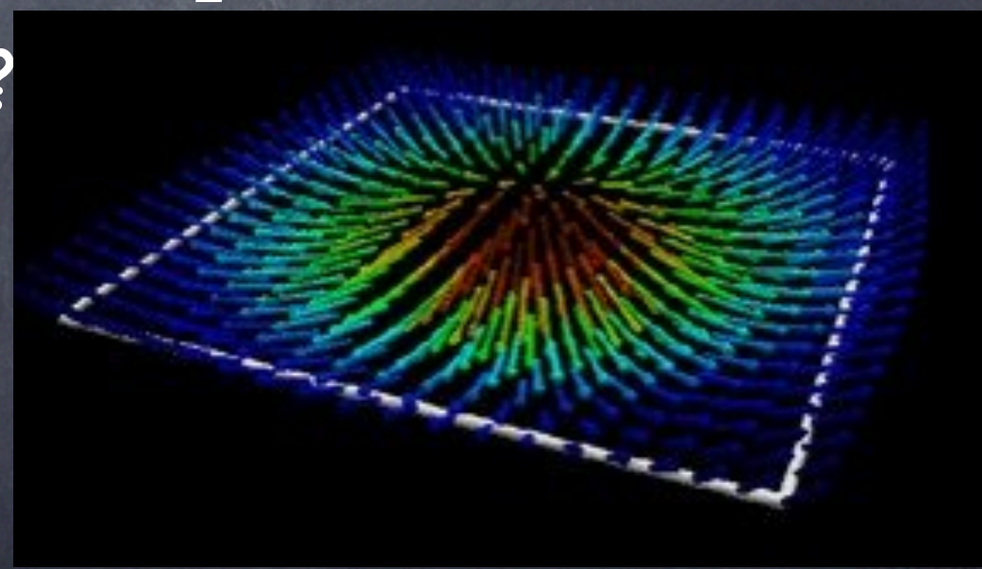


Color-Flavor Flux Tube

- Witten's famous paper on Skyrmions=baryons
- SU(N_c): SU(N_f)xSU(N_f)/SU(N_f) $\langle \bar{q}_i q_j \rangle \propto \delta_{ij}$
- Sp(N_c): SU($2N_f$)/Sp($2N_f$) $\langle q_i q_j \rangle \propto J_{ij}$
- SO(N_c): SU(N_f)/SO(N_f) $\langle q_i q_j \rangle \propto \delta_{ij}$
- He also conjectured the color flux tube solution in the chiral Lagrangian

$$\pi_2(SU(N_f)/SO(N_f)) = \mathbb{Z}_2$$

- Color and flavor intertwined?



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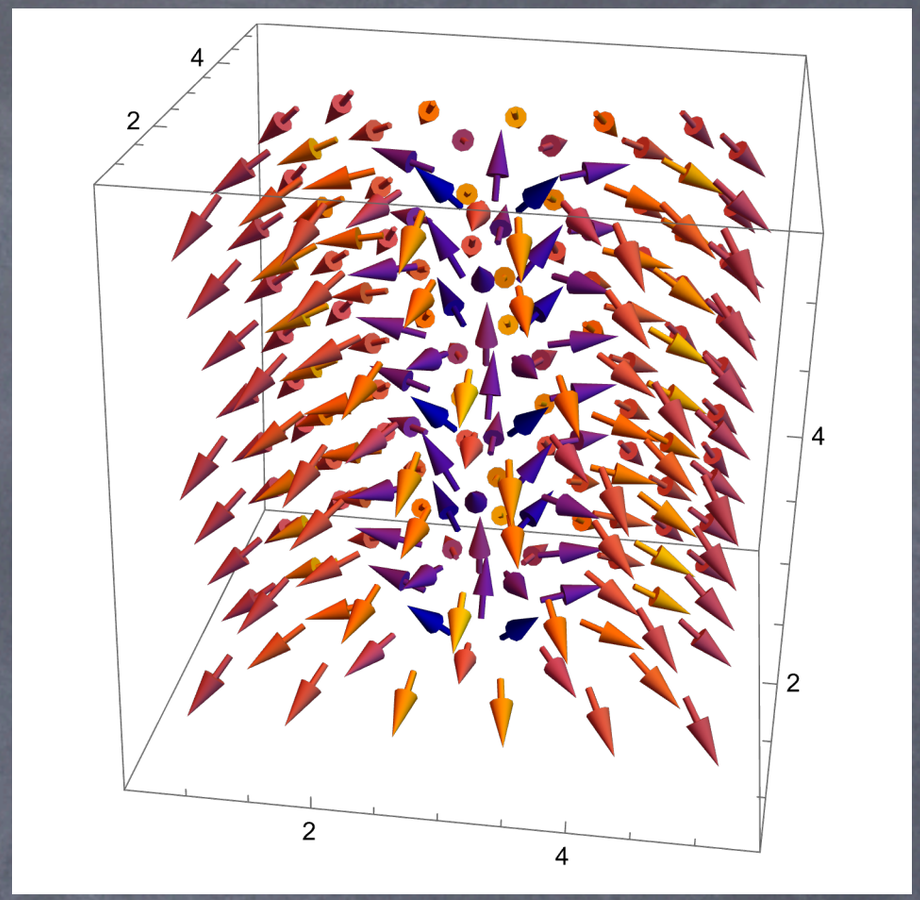

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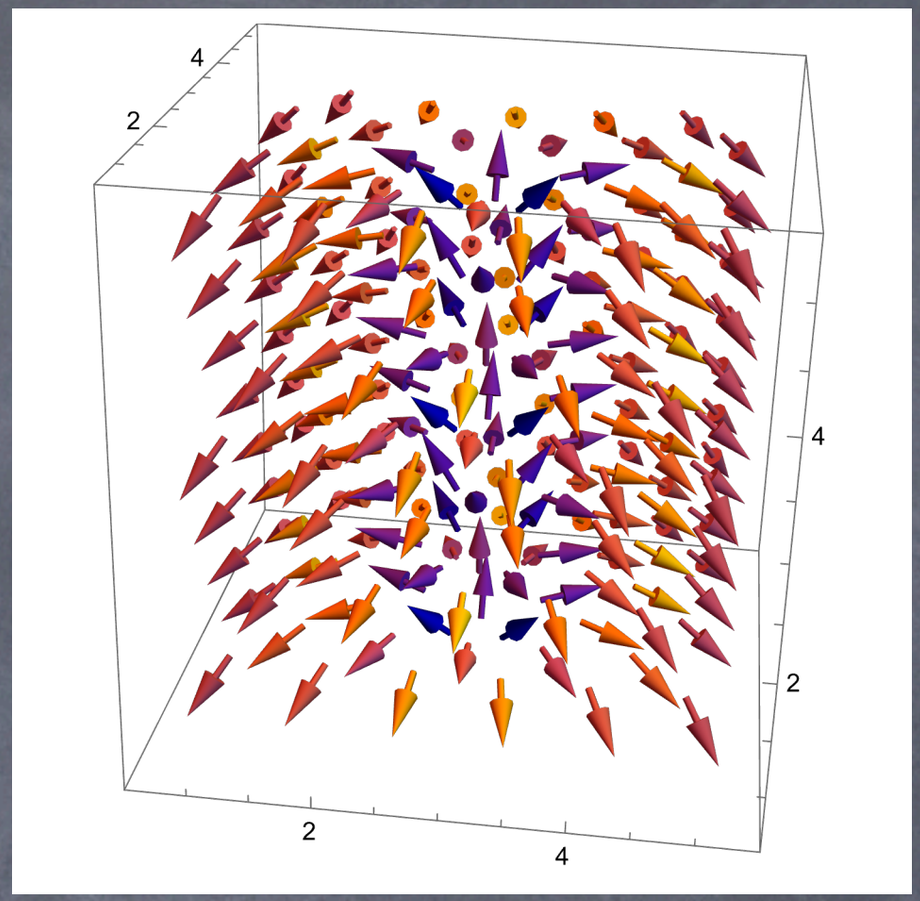


$$E^{\pm}(r,\theta) = \sqrt{\frac{mv}{2\lambda}} f(r) e^{\pm i\theta}$$

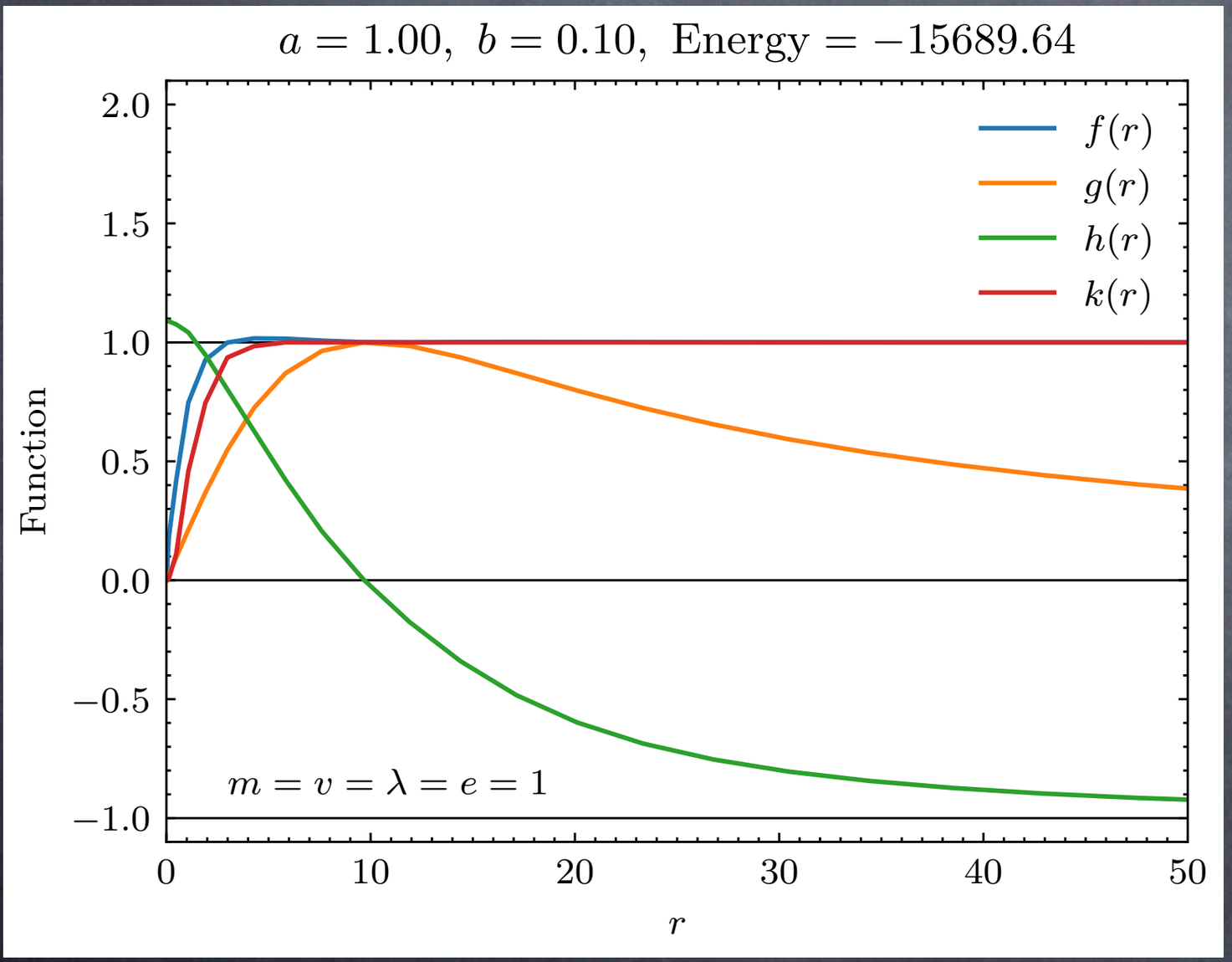
$$eA_\theta = k(r)$$

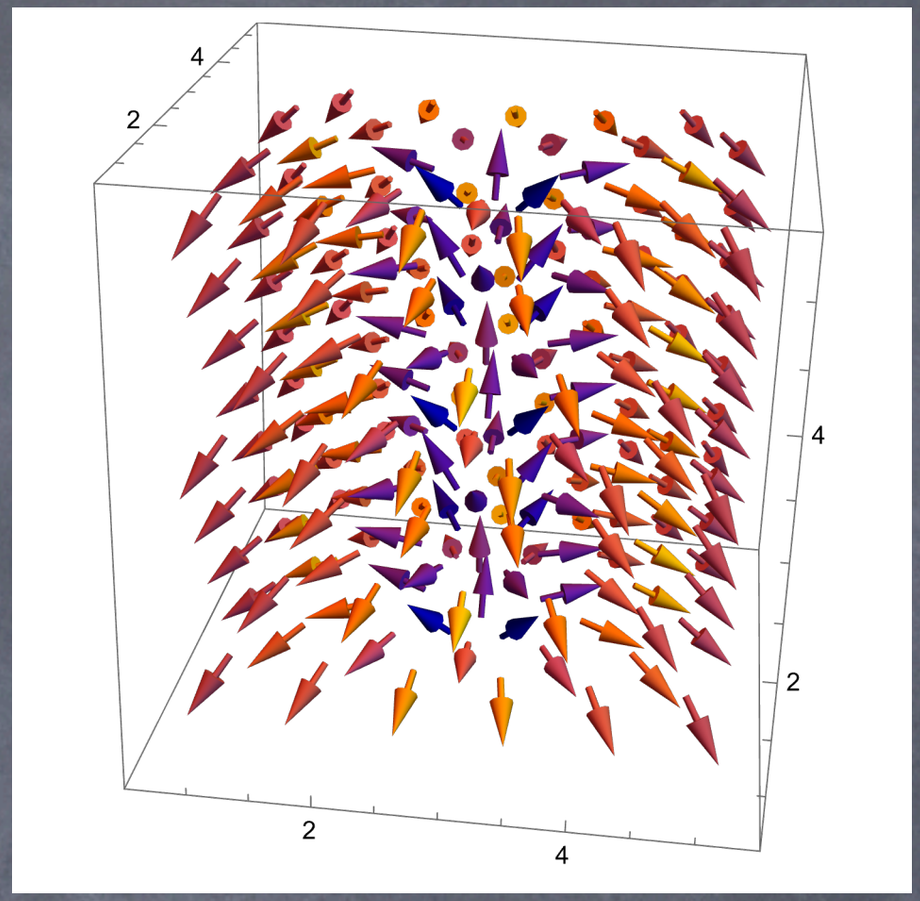
$$\phi_1 = v\cos\theta g(r)$$

$$\phi_2 = v\sin\theta g(r)$$

$$\phi_3 = vh(r)$$

Color-Flavor Flux Tube





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- interested in strongly-coupled dark sector