Phenomenologically Viable Froggatt-Nielsen Solutions to the Lepton Flavor Puzzle

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Outline

- Introduce the flavour puzzle and the Froggatt-Nielsen solution
- Discuss the general class of FN solutions
- Extensions to the lepton sector and neutrino masses
- Low energy decay observables
- Collider physics outlook

Background

The Flavour Puzzle

- The masses of the fermions span 6 orders of magnitude without neutrinos and 12 with neutrinos
- Theses masses are technically natural, but the dramatic hierarchies invite a BSM explanation
- Possible explanation: horizontal symmetries

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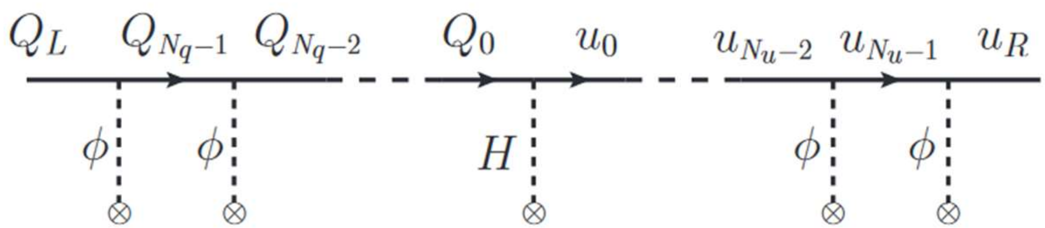
Froggatt-Nielsen



- All fermions are charged under a new $U(1)_{FN}$
- Yukawas generated by effective operators coupled to heavy flavon
- If $\frac{\langle\phi\rangle}{\Lambda} \equiv \epsilon \sim 0.1$, this can result in large hierarchies in low-energy Yukawas

$$\mathcal{L} \supset -c_{ij}^u \bar{Q}_i \tilde{H} u_j \left(\frac{\phi}{\Lambda}\right)^{|X_{Q_i}-X_{u_j}|} - c_{ij}^d \bar{Q}_i H d_j \left(\frac{\phi}{\Lambda}\right)^{|X_{Q_i}-X_{d_j}|} + h.c.$$

Determining Charges (Textures)

- Usual approach: work backwards from masses and mixings

Question: Does this miss phenomenologically viable textures?

Secondary question: Are there experimental signatures of different FN models which could probe these different textures?

The Quark Sector

$$\mathcal{L} \supset -c^u_{ij}\bar{Q}_i\tilde{H}u_j\left(\frac{\phi}{\Lambda}\right)^{|X_{Q_i}-X_{u_j}|} - c^d_{ij}\bar{Q}_iHd_j\left(\frac{\phi}{\Lambda}\right)^{|X_{Q_i}-X_{d_j}|} + h.c.$$

1. Generate all possible textures
2. Populate c_{ij} with random O(1) coefficients
3. Numerically calculate resulting masses and mixings
4. Compare to SM values, allowing ϵ to vary to minimize deviation from SM
5. Repeat steps 2-4 for many choice of c_{ij}
6. Textures which are close to the SM most frequently are considered natural

arXiv: 2306.08026v2

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Num.	X_{Q_1}	X_{Q_2}	X_{u_1}	X_{u_2}	X_{d_1}	X_{d_2}	X_{d_3}	$\mathcal{F}_2$ (%)	$\mathcal{F}_5$ (%)	ϵ
1	3	2	-4	-2	-3	-3	-3	2.7	67	0.17
2	3	2	-4	-2	-4	-3	-3	2.5	66	0.18
3	3	2	-3	-1	-3	-2	-2	1.9	56	0.12
4	3	2	-4	-1	-3	-3	-3	1.5	65	0.16
5	4	3	-4	-2	-4	-3	-3	1.2	52	0.23
6	3	2	-4	-1	-3	-3	-2	1.1	63	0.15
7	4	2	-4	-2	-4	-3	-3	1.1	47	0.21
8	3	2	-3	-1	-2	-2	-2	0.9	41	0.11
9	3	2	-3	-1	-3	-3	-2	0.9	55	0.14
10	3	2	-4	-2	-3	-3	-2	0.9	59	0.16
11	2	1	-3	-1	-2	-2	-2	0.8	52	0.06
12	4	3	-4	-1	-4	-3	-3	0.8	52	0.22
13	4	3	-4	-2	-4	-4	-3	0.8	50	0.24
14	3	2	-4	-2	-4	-3	-2	0.7	56	0.17
15	4	3	-4	-2	-3	-3	-3	0.7	43	0.22
16	4	3	-4	-1	-3	-3	-3	0.6	45	0.21
17	4	2	-4	-2	-4	-4	-3	0.6	48	0.22
18	4	2	-4	-2	-3	-3	-3	0.6	38	0.2
19	3	2	-4	-1	-3	-2	-2	0.6	56	0.14
20	3	2	-3	-2	-3	-3	-2	0.6	45	0.15

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The Lepton Sector

Lepton Extension

$$\mathcal{L} \supset -c^{\ell}_{ij}\bar{L}_i H e_j \left(\frac{\phi}{\Lambda}\right)^{|X_{L_i}-X_{e_j}|} + \quad ?$$

- In lepton sector, need to contend with how neutrinos get mass

Lepton Extension

$$\mathcal{L} \supset -c^{\ell}_{ij}\bar{L}_i H e_j \left(\frac{\phi}{\Lambda}\right)^{|X_{L_i}-X_{e_j}|} - c^{\nu}_{ij}\bar{L}_i \tilde{H} N_R \left(\frac{\phi}{\Lambda}\right)^{|X_{L_i}-X_{N_{Rj}}|} + h.c.,$$

1. Pure Dirac: all hierarchies arise due to FN charge differences. Adds RH neutrinos to the SM

Lepton Extension

$$\mathcal{L} \supset -c^{\ell}_{ij}\bar{L}_i H e_j \left(\frac{\phi}{\Lambda}\right)^{|X_{L_i}-X_{e_j}|} - \frac{c^W_{ij}}{\Lambda}\bar{L}_i{}^c \tilde{H}^* \tilde{H}^\dagger L_j \left(\frac{\phi}{\Lambda}\right)^{|X_{L_i}+X_{L_j}|} + h.c.$$

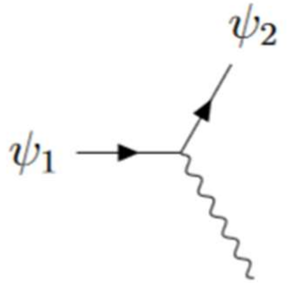
1. Pure Dirac
2. Generic high energy physics: Weinberg operator
   - $m_\nu \sim \frac{v^2}{\Lambda} \Longrightarrow \Lambda \sim 10^{14}$ GeV

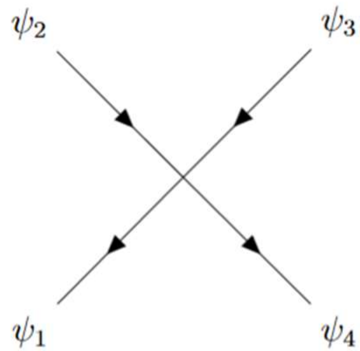
Observables

- Flavour-violating decays
- Charged lepton flavour violating collider observables

$$\frac{c_{ijkl}}{\Lambda_{eff}^2}\left(\bar{\psi}_i\psi_j\right)\left(\bar{\psi}_k\psi_l\right)$$

$$\frac{c_{ij}v}{\sqrt{2}\Lambda_{eff}^2}\bar{\psi}_{Li}\sigma^{\mu\nu}\psi_{Rj}F_{\mu\nu}+h.c.$$





$$\frac{1}{\Lambda_{eff}^2}=\frac{\epsilon^{|X_i-X_j+X_k-X_l|}}{\Lambda_F^2}$$

Results

Dirac Textures

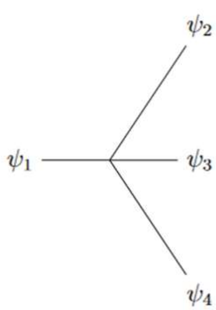
Preliminary

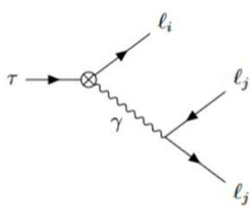
- Prefers normal ordering

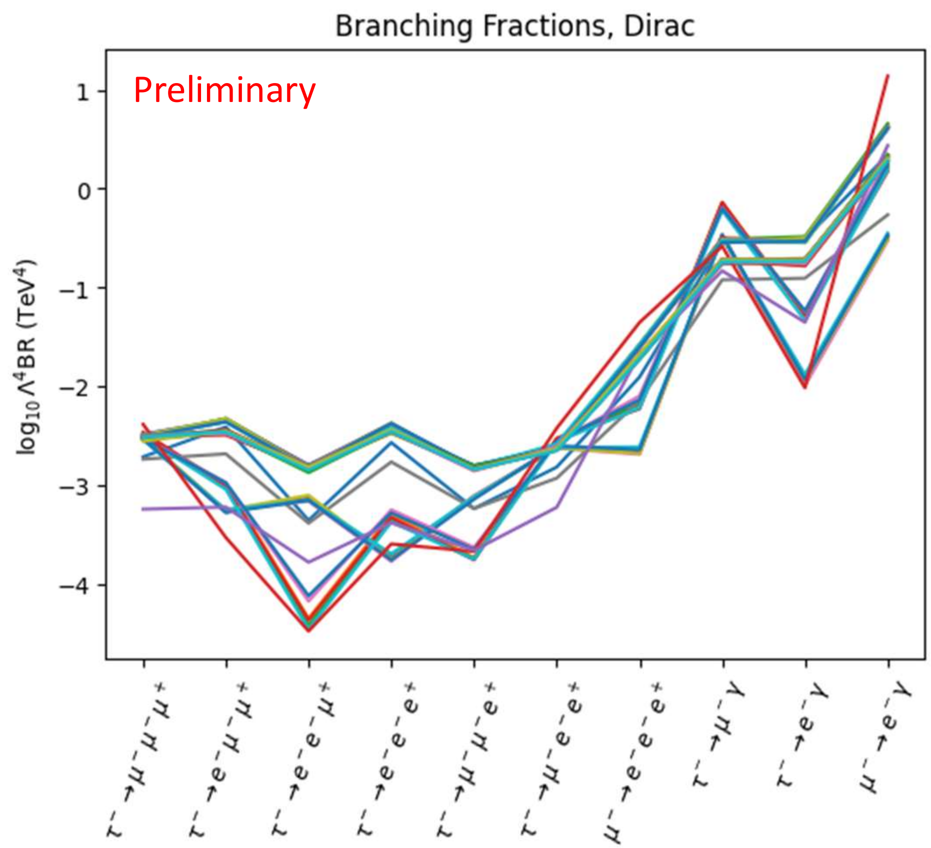
L_1	L_2	L_3	e_1	e_2	e_3	ν_1	ν_2	ν_3	F_2 (%)	F_5 (%)	ϵ_2	NO (%)	index
7.0	6.0	6.0	4.0	3.0	1.0	-9.0	-7.0	-7.0	3.947	61.736	0.102038	91.671	136.0
6.0	5.0	5.0	3.0	2.0	0.0	-9.0	-8.0	-8.0	3.855	61.179	0.101925	91.731	133.0
7.0	6.0	6.0	4.0	3.0	1.0	-8.0	-7.0	-7.0	3.855	61.179	0.101925	91.731	133.0
4.0	4.0	4.0	7.0	2.0	-1.0	-9.0	-9.0	-8.0	2.096	72.521	0.087580	100.000	101.0
4.0	4.0	4.0	7.0	6.0	-1.0	-9.0	-9.0	-8.0	2.096	72.521	0.087580	100.000	101.0
7.0	3.0	3.0	5.0	0.0	-2.0	-9.0	-9.0	-9.0	1.975	67.457	0.082372	85.802	63.0
7.0	7.0	6.0	4.0	3.0	1.0	-9.0	-8.0	-8.0	1.756	62.909	0.125789	100.000	143.0
7.0	7.0	7.0	4.0	3.0	1.0	-9.0	-9.0	-8.0	1.683	72.569	0.141574	100.000	149.0
6.0	5.0	5.0	3.0	2.0	0.0	-9.0	-9.0	-8.0	1.683	61.334	0.106958	100.000	134.0
7.0	6.0	6.0	4.0	3.0	1.0	-8.0	-8.0	-7.0	1.683	61.334	0.106958	100.000	134.0

Branching Fractions

- Assume O(1) coefficients for all SMEFT operators
- 3 body phase suppression is evident
- Decays bridging 2 generations are more suppressed



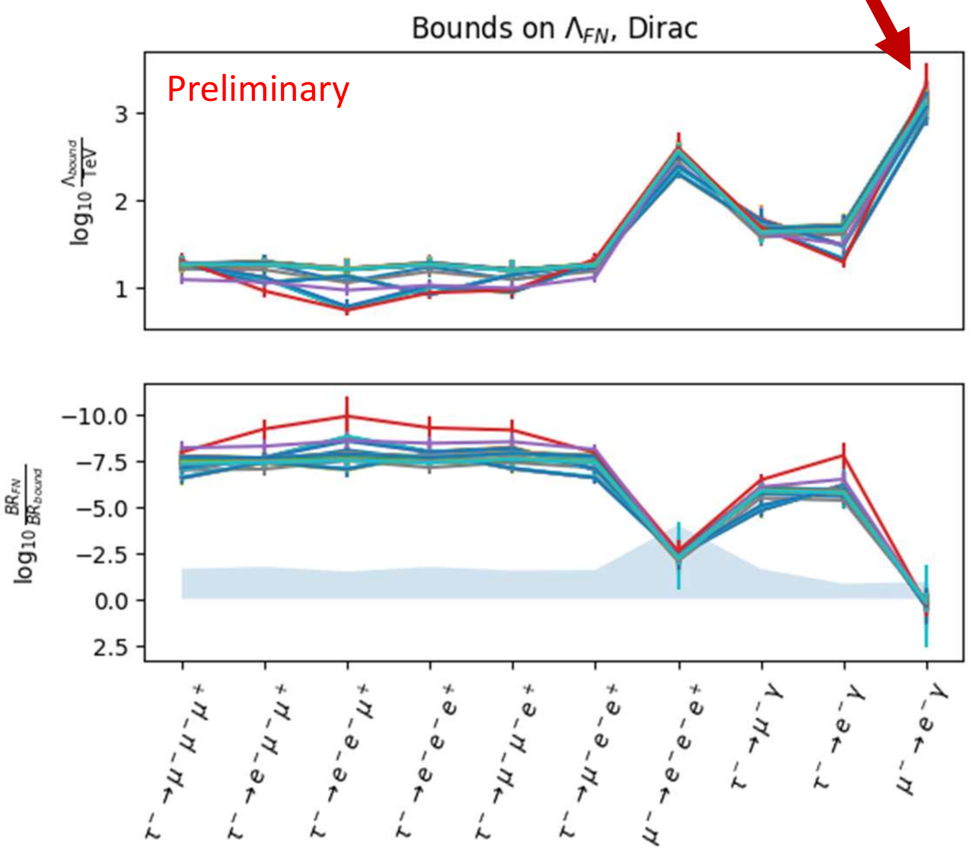


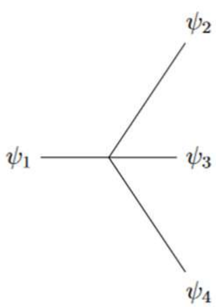


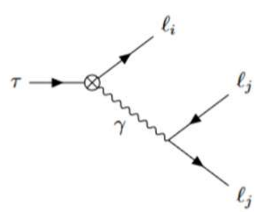
Most constraining

Flavor-Violating Decays

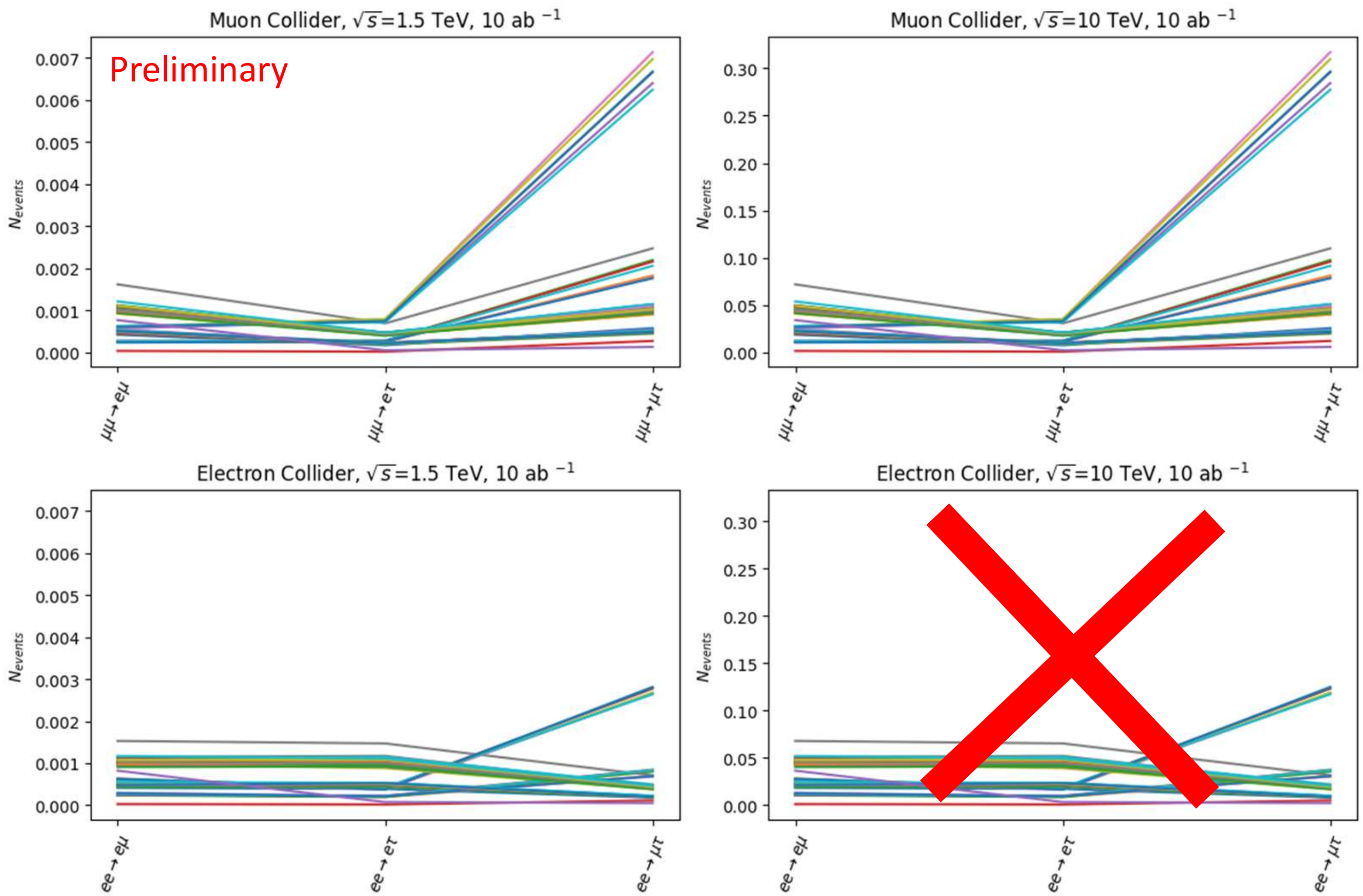
- Set Λ_{FN} based on most constraining observable
- Estimate value of all observables for this scale







Collider Observables - Dirac



Weinberg

Preliminary

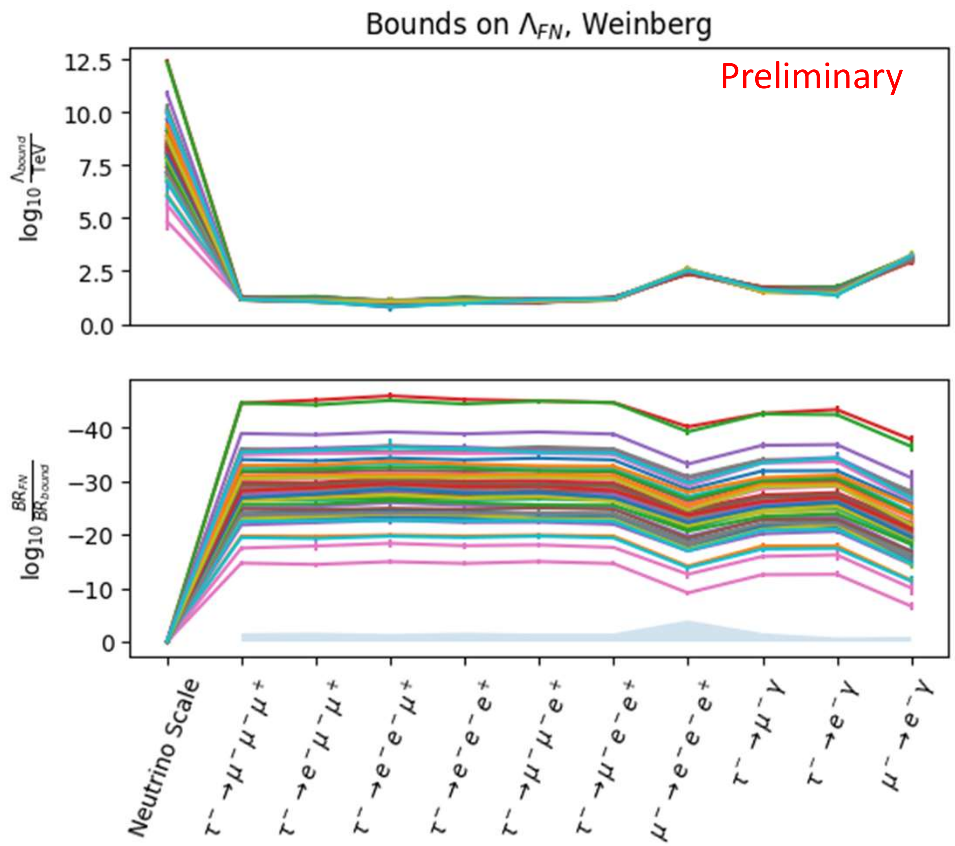
- Also prefers normal ordering

L_1	L_2	L_3	e_1	e_2	e_3	F_2 (%)	F_5 (%)	ϵ_2	$\log_{10}\Lambda$	NO (%)	index
7.0	6.0	5.0	-7.0	-2.0	0.0	14.92	83.67	0.385782	11.025863	100.0	2274.0
6.0	5.0	4.0	-7.0	-3.0	-1.0	14.68	81.36	0.373136	11.722524	100.0	2102.0
7.0	6.0	5.0	-6.0	-2.0	0.0	14.10	82.55	0.373877	10.875496	100.0	2103.0
7.0	6.0	5.0	-7.0	-3.0	0.0	13.35	83.05	0.400469	11.202661	100.0	2299.0
7.0	6.0	5.0	1.0	-1.0	-5.0	13.11	78.34	0.332569	10.321272	100.0	982.0
6.0	5.0	4.0	-6.0	-2.0	0.0	13.11	78.25	0.331636	11.265313	100.0	1854.0
5.0	4.0	3.0	-7.0	-3.0	-1.0	12.81	78.64	0.331884	12.227701	100.0	1853.0
5.0	5.0	4.0	-6.0	-2.0	0.0	12.49	82.48	0.310666	11.113967	100.0	1617.0
4.0	4.0	3.0	-7.0	-3.0	-1.0	12.22	83.42	0.310346	12.128698	100.0	1616.0
6.0	6.0	5.0	1.0	-1.0	-5.0	11.97	82.25	0.311058	10.103275	100.0	766.0

Weinberg

$$\frac{c^W_{ij}}{\Lambda}\bar{L}_i{}^c\tilde{H}^*\tilde{H}^\dagger L_j\left(\frac{\phi}{\Lambda}\right)^{|X_{L_i}+X_{L_j}|}$$

- In this case, we find Λ from the Weinberg scale
- FN suppression is not sufficient to bring Λ to observable levels in lepton decays or collider observables



Conclusions and Outlook

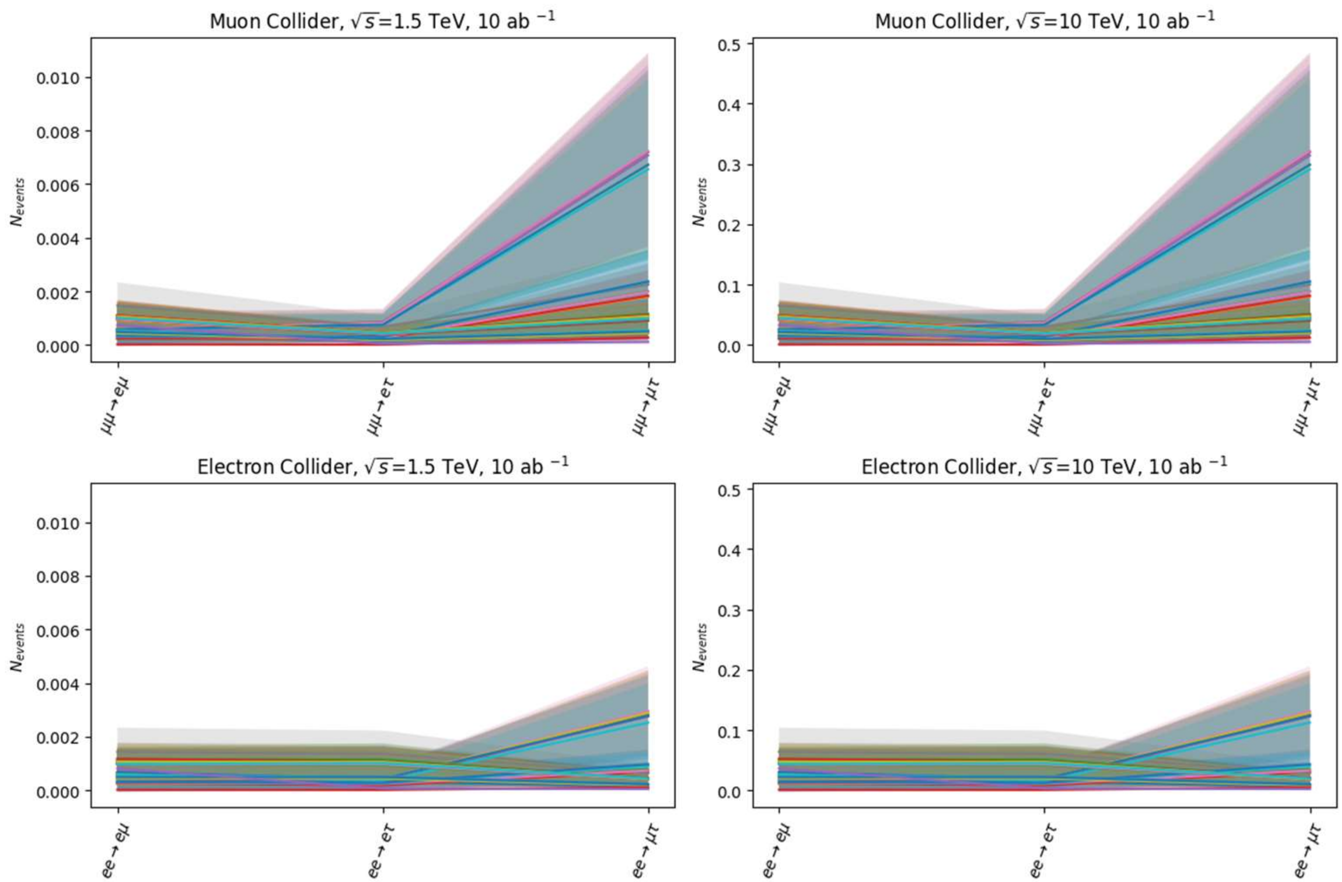
Conclusions

- We have found the most general category of leptonic FN models which can reproduce SM hierarchies
- We calculated some of the resulting observables and have found that low energy signatures tend to follow characteristic FN patterns
- FN models will be inaccessible except at extremely high energy future muon colliders
- Future work: Type 1 seesaw, lepton-quark interactions, polarization asymmetries

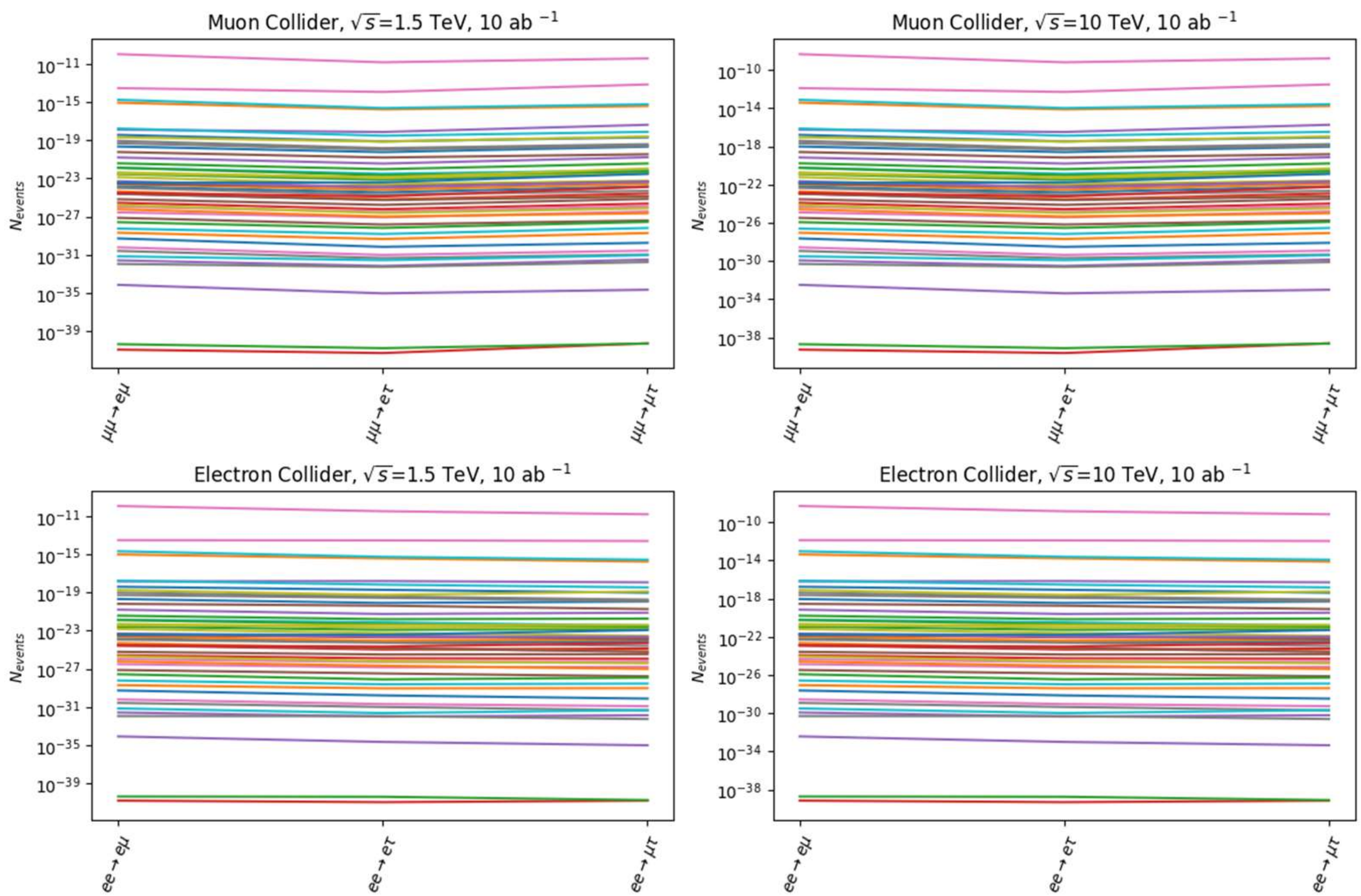
Backup Slides

Collider Observables

- Below 10 TeV, even the most optimistic scenario is unobservable
- A muon collider has greater reach than an electron collider particularly for τe final states



Weinberg Collider Observables

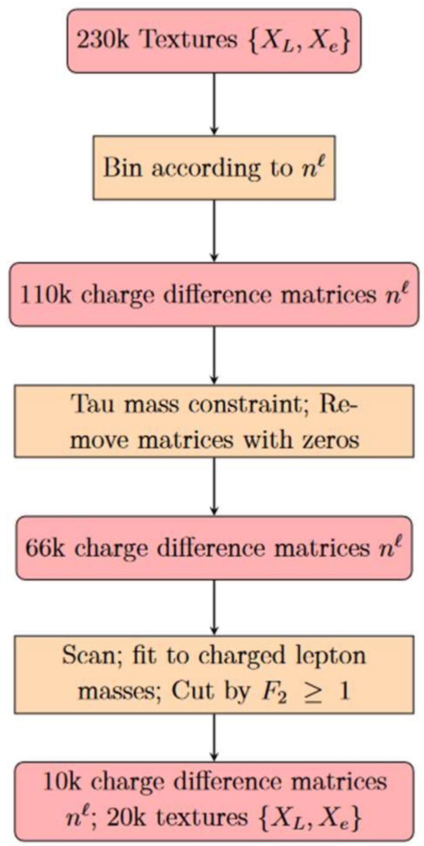


Fractional Deviation

$$\Xi_{SM} = \max_i \exp\left|\ln\left(\frac{\mu_i^{calc}}{\mu_i^{SM}}\right)\right|$$

$$\begin{aligned}\Xi_{\mu} = \theta\left(\mu_{LB}^{SM} - \mu^{calc}, 0\right) \exp\left|\ln\left(\frac{\mu^{calc}}{\mu_{LB}^{SM}}\right)\right| \\ + \theta\left(\mu^{calc} - \mu_{LB}^{SM}, 1\right) \theta\left(\mu_{UB}^{SM} - \mu^{calc}, 1\right) \\ + \theta\left(\mu^{calc} - \mu_{UB}^{SM}, 0\right) \exp\left|\ln\left(\frac{\mu^{calc}}{\mu_{LB}^{SM}}\right)\right|\end{aligned}$$

Methods



230k Textures $\{X_L, X_e\}$
Bin according to n^ℓ
110k charge difference matrices n^ℓ
Tau mass constraint; Remove matrices with zeros
66k charge difference matrices n^ℓ
Scan; fit to charged lepton masses; Cut by $F_2 \geq 1$
10k charge difference matrices n^ℓ; 20k textures $\{X_L, X_e\}$