Production of fully-heavy tetraquark

at the LHC and electron-ion colliders

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arxiv: 2304.11142, 2311.08292, 2404.13889

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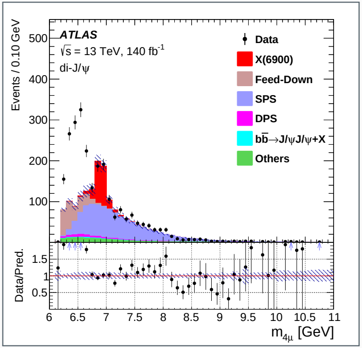
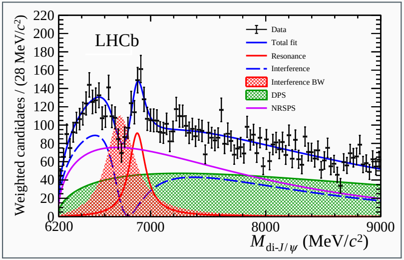
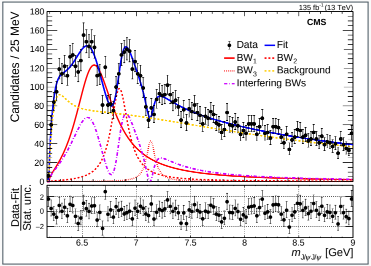
Discovery of $X(6900)$ at the LHC

- Resonance near 6.9 GeV discovered in the di-J/ψ invariant mass spectrum: **$\mathbf{X(6900)}$**
- First discovered by LHCb in 2020, later confirmed by ATLAS and CMS in 2023
- First strong candidate for compact fully-charmed tetraquark T_{4c}

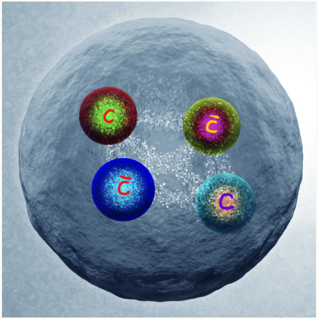






FULLY-HEAVY TETRAQUARK: T_{4c}

- All heavy quarks: $Q\bar{Q}Q\bar{Q}$
  $\Rightarrow$light quarks and gluons suppressed in Fock states
- Easier to deal with theoretically
- Mass spectra and decay properties thoroughly studied



Credit: ATLAS

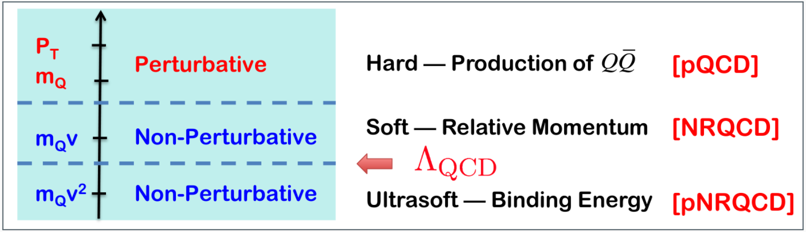
- Production mechanism less explored:
  - Duality relations 1511.05209, 2009.02100
  - Color evaporation models 1101.5881, 1611.00348

  $\Rightarrow$ **Rough estimations!**

  - **nonreletivistic QCD (NRQCD)** (post-2020): based on rigorous NRQCD factorization
    - LHC 2009.08376, 2009.08450, 2010.09082, **2304.11142**, **2404.13889**
    - B factory 2011.03039, 2104.03887
    - Electron-ion colliders **2311.08292**, **2404.13889**

NRQCD FACTORIZATION for fully-heavy tetraquarks

- Heavy masses of four quarks means they are produced at shorter distance
- Relative velocity v between quarks are small



- Factorize the cross section

$$\mathrm{d}\sigma(H) = \sum_n F_n(\Lambda) \langle 0|\mathcal{O}_n^H(\Lambda)|0\rangle$$

  - $F_n(\Lambda)$: Short-distance coefficients (SDCs)
  - $\langle 0|\mathcal{O}_n^H(\Lambda)|0\rangle$: Long-distance matrix elements (LDMEs)

LDMEs & NRQCD operators for S-wave T_{4c} 2009.08450, 2104.03887

The nonperturbative vacuum-to-vacuum LDMEs $\langle 0|\mathscr{O}_n^H(\Lambda)|0\rangle$ are given

NRQCD production operators for $J^{PC}=0^{++},1^{+-},2^{++}$ tetraquarks

$$\mathscr{O}_{3,3}^{(J)}=\mathcal{O}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{(J)}\sum_X|T_{4c}^J+X\rangle\langle T_{4c}^J+X|\mathcal{O}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{(J)\dagger}$$

$$\mathscr{O}_{6,6}^{(J)}=\mathcal{O}_{\mathbf{6}\otimes\bar{\mathbf{6}}}^{(J)}\sum_X|T_{4c}^J+X\rangle\langle T_{4c}^J+X|\mathcal{O}_{\mathbf{6}\otimes\bar{\mathbf{6}}}^{(J)\dagger}$$

$$\mathscr{O}_{3,6}^{(J)}=\mathcal{O}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{(J)}\sum_X|T_{4c}^J+X\rangle\langle T_{4c}^J+X|\mathcal{O}_{\mathbf{6}\otimes\bar{\mathbf{6}}}^{(J)\dagger}$$

leading order NRQCD local operators for $J^{PC}=0^{++},1^{+-},2^{++}$ states

$$\mathcal{O}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{(0)}=-\frac{1}{\sqrt{3}}[\psi_a^T(i\sigma^2)\sigma^i\psi_b][\chi_c^\dagger\sigma^i(i\sigma^2)\chi_d^*]\ \boxed{\mathcal{C}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{ab;cd}}\longleftarrow \text{color projection}$$

$$\mathcal{O}_{\mathbf{6}\otimes\bar{\mathbf{6}}}^{(0)}=[\psi_a^T(i\sigma^2)\psi_b][\chi_c^\dagger(i\sigma^2)\chi_d^*]\ \mathcal{C}_{\mathbf{6}\otimes\bar{\mathbf{6}}}^{ab;cd},\qquad \boxed{\mathcal{O}_{\mathbf{6}\otimes\bar{\mathbf{6}}}^{(J>0)}=0}$$

$$\mathcal{O}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{i}=\frac{\mathrm{i}}{\sqrt{2}}\epsilon^{ijk}\left(\psi_a^\dagger\sigma^j\mathrm{i}\sigma^2\psi_b^{\dagger T}\right)\left(\chi_c^t\mathrm{i}\sigma^2\sigma^k\chi_d\right)\mathcal{C}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{ab;cd}$$

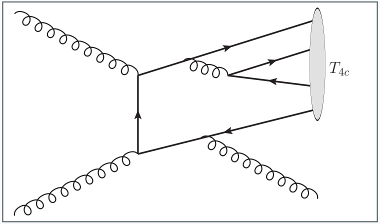
$$\mathcal{O}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{\alpha\beta;(2)}=[\psi_a^T(i\sigma^2)\sigma^m\psi_b][\chi_c^\dagger\sigma^n(i\sigma^2)\chi_d^*]\ \Gamma^{\alpha\beta;mn}\ \mathcal{C}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{ab;cd}$$

SDCs $F_n(\Lambda)$: insensitive to IR behaviors, perturbatively calculable

- perturbatively calculate both sides of the factorization formula with free four-quark states i.e. replacing the $|T_{4c}\rangle$ hadron state with four free quarks $|cc\bar{c}\bar{c}\rangle$
- then we match both sides to obtain the SDCs

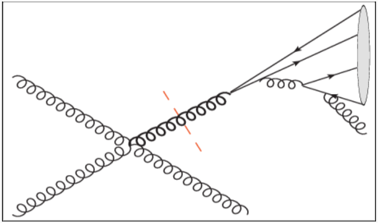
Production of T_{4c} at the LHC

Fixed order:



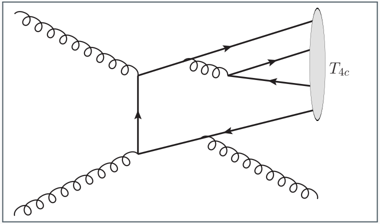
$$\mathrm{d}\sigma(pp \to T_{4c} + X)$$
$$= \sum_{i,j=q,g} \int_0^1 dx_1\, dx_2 f_{i/p}(x_1, \mu_F)\, f_{j/p}(x_2, \mu_F)$$
$$\times\, \mathrm{d}\hat{\sigma}_{ij \to T_{4c}+X}(x_1 x_2 s, \mu_F)$$

Fragmentation: Feng, **YH**, Jia, Sang, et al., 2009.08450



$$\mathrm{d}\sigma\,(pp \to T_{4c}\,(p_\mathrm{T}) + X)$$
$$= \sum_i \int_0^1 \mathrm{d}x_a \int_0^1 \mathrm{d}x_b \int_0^1 \mathrm{d}z\; f_{a/p}(x_a, \mu)\, f_{b/p}(x_b, \mu)$$
$$\times\, \mathrm{d}\hat{\sigma}\,(ab \to i(p_T/z) + X, \mu)\, D_{i \to T_{4c}}\,(z, \mu) + \mathcal{O}(1/p_T)$$

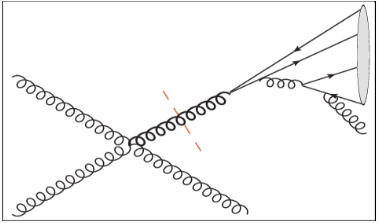
Fixed order:



$$\mathrm{d}\sigma(pp \to T_{4c} + X) = \sum_{i,j=q,g} \int_0^1 dx_1\, dx_2 f_{i/p}(x_1, \mu_F)\, f_{j/p}(x_2, \mu_F) \times \mathrm{d}\hat{\sigma}_{ij\to T_{4c}+X}(x_1 x_2 s, \mu_F)$$

$$\frac{\mathrm{d}\hat{\sigma}(T_{4c}^{(J)} + X)}{\mathrm{d}\hat{t}} = \frac{2M_{T_{4c}}}{m_c^{14}} \left[F_{3,3}^{(J)} \left\langle \mathcal{O}_{3,3}^{(J)} \right\rangle + 2F_{3,6}^{(J)} \left\langle \mathcal{O}_{3,6}^{(J)} \right\rangle + F_{6,6}^{(J)} \left\langle \mathcal{O}_{6,6}^{(J)} \right\rangle \right]$$

Fragmentation: Feng, **YH**, Jia, Sang, et al., 2009.08450

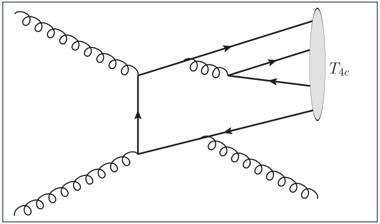


$$\mathrm{d}\sigma\,(pp \to T_{4c}\,(p_{\mathrm{T}}) + X) = \sum_i \int_0^1 \mathrm{d}x_a \int_0^1 \mathrm{d}x_b \int_0^1 \mathrm{d}z\, f_{a/p}(x_a, \mu)\, f_{b/p}(x_b, \mu) \times \mathrm{d}\hat{\sigma}\,(ab \to i(p_T/z) + X, \mu)\, D_{i\to T_{4c}}\,(z, \mu) + \mathcal{O}(1/p_T)$$

- Dominates at high p_T
- Only valid when $p_T \gg M_{T_{4c}}$

Two production mechanisms at the LHC

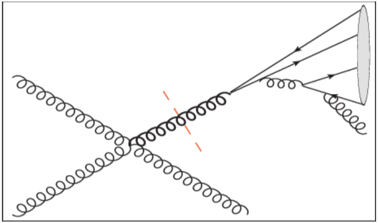
Fixed order:



$$\mathrm{d}\sigma(pp \to T_{4c} + X) = \sum_{i,j=q,g} \int_0^1 dx_1\, dx_2 f_{i/p}(x_1, \mu_F)\, f_{j/p}(x_2, \mu_F) \times \mathrm{d}\hat{\sigma}_{ij\to T_{4c}+X}(x_1 x_2 s, \mu_F)$$

$$\frac{\mathrm{d}\hat{\sigma}(T_{4c}^{(J)} + X)}{\mathrm{d}\hat{t}} = \frac{2M_{T_{4c}}}{m_c^{14}} \left[F_{3,3}^{(J)} \left\langle \mathcal{O}_{3,3}^{(J)} \right\rangle + 2F_{3,6}^{(J)} \left\langle \mathcal{O}_{3,6}^{(J)} \right\rangle + F_{6,6}^{(J)} \left\langle \mathcal{O}_{6,6}^{(J)} \right\rangle \right]$$

Fragmentation: Feng, **YH**, Jia, Sang, et al., 2009.08450



$$\mathrm{d}\sigma\,(pp \to T_{4c}\,(p_\mathrm{T}) + X) = \sum_i \int_0^1 \mathrm{d}x_a \int_0^1 \mathrm{d}x_b \int_0^1 \mathrm{d}z\, f_{a/p}(x_a, \mu)\, f_{b/p}(x_b, \mu) \times \mathrm{d}\hat{\sigma}\,(ab \to i(p_T/z) + X, \mu)\, D_{i\to T_{4c}}\,(z, \mu) + \mathcal{O}(1/p_T)$$

$$D_{g\to T_{4c}}\,(z, \mu_\Lambda) = \frac{1}{m^9} \Big\{ d_{3,3} \left[g \to T_{4c}^{(J)} \right] \left\langle \mathcal{O}_{3,3}^{(J)} \right\rangle + d_{6,6} \left[g \to T_{4c}^{(J)} \right] \left\langle \mathcal{O}_{6,6}^{(J)} \right\rangle + d_{3,6} \left[g \to T_{4c}^{(J)} \right] 2\mathrm{Re} \left[\left\langle \mathcal{O}_{3,6}^{(J)} \right\rangle \right] \Big\} + \cdots$$

Feng, **YH**, Jia, Sang, et al., 2304.11142

- Results of SDCs

$$F_{3,3}^{0^{++}} = \frac{2209\pi^4 m_c^6 \alpha_s^5 \left(\hat{s}\hat{t} + \hat{s}^2 + \hat{t}^2\right)^4}{15552\hat{s}^5(-\hat{t})^3 \left(\hat{s} + \hat{t}\right)^3} + \mathcal{O}\left(\frac{m_c^7}{p_T^7}\right),$$

$$F_{3,6}^{0^{++}} = \sqrt{6} F_{3,3}^{0^{++}} + \mathcal{O}\left(\frac{m_c^7}{p_T^7}\right),$$

$$F_{6,6}^{0^{++}} = \frac{2}{3} F_{3,3}^{0^{++}} + \mathcal{O}\left(\frac{m_c^7}{p_T^7}\right),$$

$$F_{3,3}^{1^{+-}} = \frac{60025\pi^4 m_c^8 \alpha_s^5 \left(\hat{s}\hat{t} + \hat{s}^2 + \hat{t}^2\right)^2}{34992\hat{s}^4\hat{t}^2 \left(\hat{s} + \hat{t}\right)^2} + \mathcal{O}\left(\frac{m_c^9}{p_T^9}\right),$$

$$F_{3,3}^{2^{++}} = \frac{17617\pi^4 m_c^6 \alpha_s^5 \left(\hat{s}\hat{t} + \hat{s}^2 + \hat{t}^2\right)^4}{38880\hat{s}^5(-\hat{t})^3 \left(\hat{s} + \hat{t}\right)^3} + \mathcal{O}\left(\frac{m_c^7}{p_T^7}\right),$$

- Scaling:
  - C-even states 0^{++} & 2^{++}: $\sim p_T^{-6}$
  - C-odd state 1^{+-}: $\sim p_T^{-8}$
  - Comparing with fragmentation which scales as p_T^{-4}, fixed-order for C-even states are suppressed by p_T^2

LDMEs

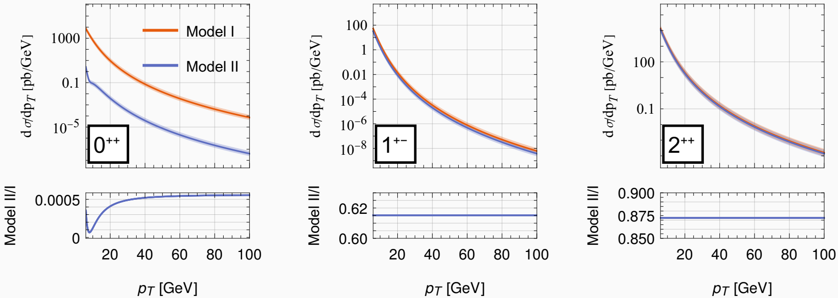
- NRQCD LDMEs are nonperturbative quantities and need lattice input
- in this talk we use potential models to estimate the LDMEs
  - LDMEs in terms of wave functions at the origin:

$$\left\langle O^{(0)}_{C_1,C_2} \right\rangle \approx 16\,\psi^{(0)}_{C_1}(\mathbf{0})\psi^{(0)*}_{C_2}(\mathbf{0}),$$
$$\left\langle O^{(1)}_{C_1,C_2} \right\rangle \approx 48\,\psi^{(1)}_{C_1}(\mathbf{0})\psi^{(1)*}_{C_2}(\mathbf{0}),$$
$$\left\langle O^{(2)}_{C_1,C_2} \right\rangle \approx 80\,\psi^{(2)}_{C_1}(\mathbf{0})\psi^{(2)*}_{C_2}(\mathbf{0}).$$

  - $\psi(\mathbf{0})$ denotes the four-body Schrödinger wave function at the origin;
  - the color structure labels C_1 and C_2 can be either 3 or 6, representing the $\mathbf{3} \otimes \mathbf{3}$ and the $\mathbf{6} \otimes \mathbf{6}$ diquark-antidiquark configuration.

LDME $[\mathrm{GeV}^9]$	0^{++}			1^{+-}	2^{++}
	$\left\langle O^{(0)}_{3,3} \right\rangle$	$\left\langle O^{(0)}_{3,6} \right\rangle$	$\left\langle O^{(0)}_{6,6} \right\rangle$	$\left\langle O^{(1)}_{3,3} \right\rangle$	$\left\langle O^{(2)}_{3,3} \right\rangle$
Model I Lü et al. 2020	0.0347	0.0211	0.0128	0.0780	0.072
Model II Liu et al. 2020	0.0187	-0.0161	0.0139	0.0480	0.0628

- $pp \to T_{4c} + X$ at $\sqrt{s} = 13\ \mathrm{TeV}$, $|y(T_{4c})| < 5$, `CT14lo` PDF
- Theoretical uncertainties: scale variations $M_T/2 \leq \mu \leq 2M_T$



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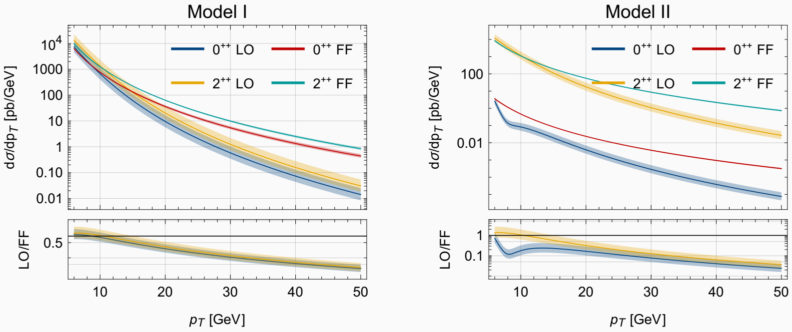
- Possible to identify a better model with future measurements



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- Comparing w/ fragmentation contributions Feng, **YH**, Jia, Sang, et al., 2009.08450
- Fragmentation mechanism is enhanced by p_T^2 comparing to LO fixed-order and overtakes fixed-order at large p_T

Integrated x-sections and event yields

- Luminosity: $3000\ \text{fb}^{-1}$

	J^{PC}	Model I		Model II	
		σ [nb]	$N_{\text{events}}/10^9$	σ [nb]	$N_{\text{events}}/10^9$
fixed order $p_T \geq 6$ GeV	0^{++}	12 ± 8	37 ± 25	0.002 ± 0.001	0.006 ± 0.004
	1^{+-}	0.07 ± 0.04	0.21 ± 0.12	0.044 ± 0.025	0.13 ± 0.07
	2^{++}	23 ± 16	70 ± 49	20 ± 14	61 ± 43
gluon frag. $p_T \geq 20$ GeV	0^{++}	0.24	0.71	0.43	1.3
	2^{++}	1.6×10^{-4}	4.7×10^{-4}	0.37	1.1
		σ [fb]	$N_{\text{events}}/10^4$	σ [fb]	$N_{\text{events}}/10^4$
charm frag. $p_T \geq 20$ GeV	0^{++}	93 ± 56	28 ± 17	0.077 ± 0.047	0.023 ± 0.014
	1^{+-}	60 ± 36	18 ± 11	37 ± 22	11 ± 7
	2^{++}	76 ± 47	23 ± 14	66 ± 41	20 ± 12

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Feng, **YH**, Jia, Sang, et al., 2009.08450

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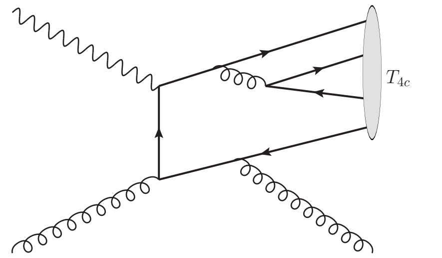
Photoproduction of T_{4c}

at electron-ion colliders

- Low-virtuality photon
  ⇒effective photon approximation (EPA): photon is treated as on-shell, $2 \to 3$ reduced to $2 \to 2$



Inclusive cross section of T_{4c} production with EPA

$$\frac{d\sigma}{dz dp_T} = \sum_i \int_{x_\gamma^{\min}}^1 \mathrm{d}x_\gamma \frac{2x_i p_T}{z(1-z)} f_{\gamma/e}(x_\gamma) f_{i/p}(x_i,\mu) \frac{\mathrm{d}\hat{\sigma}(\gamma + i \to T_{4c} + j;\mu)}{\mathrm{d}\hat{t}}$$

Photon flux Kniehl, Kramer, Spira, hep-ph/9610267

$$f_{\gamma/e}(x_\gamma) = \frac{\alpha}{2\pi}\left[\frac{1+(1-x_\gamma)^2}{x_\gamma}\ln\frac{Q^2_{\max}}{Q^2_{\min}(x_\gamma)} + 2m_e^2 x_\gamma\left(\frac{1}{Q^2_{\max}} - \frac{1}{Q^2_{\min}(x_\gamma)}\right)\right]$$

where $Q^2_{\min}(x_\gamma) = m_e^2 x_\gamma^2/(1-x_\gamma)$.

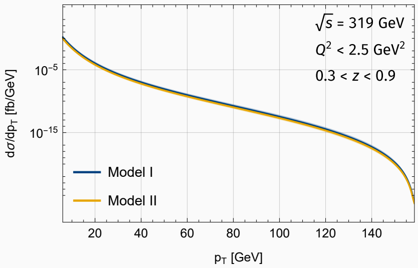
NRQCD factorization for T_{4c} photoproduction

$$\frac{\mathrm{d}\hat{\sigma}(\gamma g \to T_{4c}^{(1)} + X)}{\mathrm{d}\hat{t}} = \frac{2M_{T_{4c}}}{m_c^{14}} F_{3,3}^{(1)}(\hat{s},\hat{t}) \left\langle O_{3,3}^{(1)} \right\rangle$$

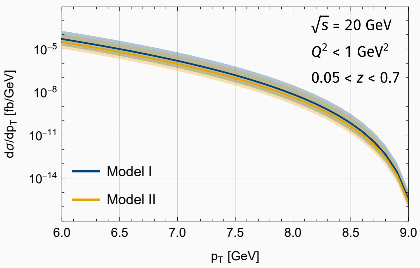
- Dominant partonic channel $\gamma + g \to T_{4c} + g$ can only produce 1^{+-} due to C-parity conservation

p_T DISTRIBUTION at HERA and EicC

- Adopting the same LDMEs as before
- $|y| < 5$, CT14lo PDF
- cuts on elasticity z to remove diffractive and resolved photon contributions



(a) HERA: $\sqrt{s} = 319\ \mathrm{GeV}$



(b) EicC: $\sqrt{s} = 20\ \mathrm{GeV}$

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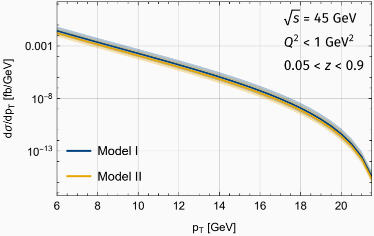
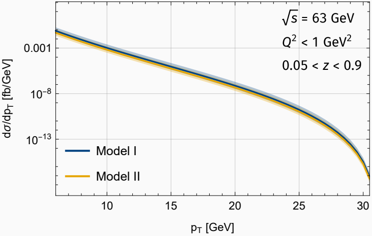
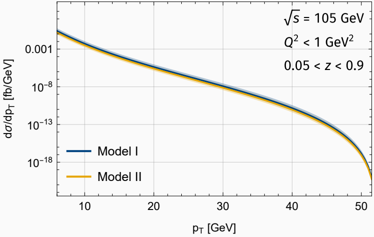


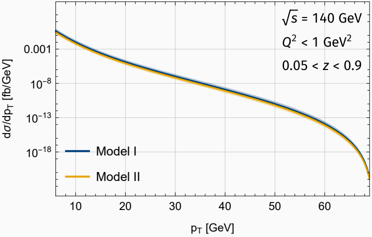
(c) $\sqrt{s} = 45$ GeV



(d) $\sqrt{s} = 63$ GeV



(e) $\sqrt{s} = 105$ GeV



(f) $\sqrt{s} = 140$ GeV

X-SECTIONS AND EVENT YIELDS

- Luminosity: 468 pb^{-1} for HERA
  assume 100 fb^{-1} for EIC and 50.5 fb^{-1} for EicC

	$\sqrt{s}$ [GeV]	p_T range [GeV]	Model I		Model II	
			σ [fb]	N	σ [fb]	N
EIC	44.7	6-20	0.022	2.2	0.014	1.4
	63.2	6-20	0.069	6.9	0.043	4.3
	104.9	6-20	0.25	25	0.15	15
	140.7	6-20	0.45	45	0.28	28
HERA	319	6-20	0.7	31	0.94	0.4
EicC	20	6-9	0.00002	0.0008	0.000009	0.0005

- Possible at EIC
- Small event yields at HERA (low lumi) and EicC (low energy)

Effect of charm fragmentation

	$\sqrt{s}$ [GeV]	p_T range[GeV]	J^{PC}	Model I	Model II
				σ [fb]	σ [fb]
HERA	319	≥ 20	0^{++}	1.1×10^{-3}	8.3×10^{-7}
			1^{+-}	6.6×10^{-4}	4.1×10^{-4}
			2^{++}	6.4×10^{-4}	5.6×10^{-4}
EIC	140.7	≥ 20	0^{++}	8.2×10^{-5}	6.4×10^{-8}
			1^{+-}	5.0×10^{-5}	3.1×10^{-5}
			2^{++}	4.5×10^{-5}	3.9×10^{-5}

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- Effects from charm fragmentation are extremely small

Summary

- Introduced the model-independent NRQCD factorization framework for T_{4c} production
- Predicted the p_T distributions of various S-wave T_{4c} states within the NRQCD factorization framework at the LHC and electron-ion colliders such as HERA, EIC and EicC
- Compared the p_T scaling of fixed order and fragmentation mechanism at the LHC
- The prospect of T_{4c} production at the LHC is promising
- Estimated the event yields at HERA, EIC and EicC: only EIC has shown potential
- Charm fragmentation is in general very small

Thanks for your attention!

Backup

NRQCD operators & LDMEs

$$O^{(J)}_{3,3} = \mathcal{O}^{(J)}_{\mathbf{3}\otimes\mathbf{3}} \sum_X |T^J_{4c} + X\rangle\langle T^J_{4c} + X|\mathcal{O}^{(J)\dagger}_{\mathbf{3}\otimes\mathbf{3}},$$

$$O^{(0)}_{6,6} = \mathcal{O}^{(0)}_{\mathbf{6}\otimes\mathbf{6}} \sum_X |T^0_{4c} + X\rangle\langle T^0_{4c} + X|\mathcal{O}^{(0)\dagger}_{\mathbf{6}\otimes\mathbf{6}},$$

$$O^{(0)}_{3,6} = \mathcal{O}^{(0)}_{\mathbf{3}\otimes\mathbf{3}} \sum_X |T^0_{4c} + X\rangle\langle T^0_{4c} + X|\mathcal{O}^{(0)\dagger}_{\mathbf{6}\otimes\mathbf{6}},$$

$$\mathcal{O}^{(0)}_{\mathbf{3}\otimes\mathbf{3}} = -\frac{1}{\sqrt{3}}[\psi_a^T(i\sigma^2)\sigma^i\psi_b][\chi_c^\dagger\sigma^i(i\sigma^2)\chi_d^*]\,\mathcal{C}^{ab;cd}_{\mathbf{3}\otimes\mathbf{3}},$$

$$\mathcal{O}^{i;(1)}_{\mathbf{3}\otimes\mathbf{3}} = -\frac{i}{\sqrt{2}}\left[\psi_a^T(i\sigma^2)\sigma^j\psi_b\right]\left[\chi_c^\dagger\sigma^k(i\sigma^2)\chi_d^*\right]\,\epsilon^{ijk}\,\mathcal{C}^{ab;cd}_{\mathbf{3}\otimes\mathbf{3}},$$

$$\mathcal{O}^{ij;(2)}_{\mathbf{3}\otimes\mathbf{3}} = [\psi_a^T(i\sigma^2)\sigma^m\psi_b][\chi_c^\dagger\sigma^n(i\sigma^2)\chi_d^*]\,\Gamma^{ij;mn}\,\mathcal{C}^{ab;cd}_{\mathbf{3}\otimes\mathbf{3}},$$

$$\mathcal{O}^{(0)}_{\mathbf{6}\otimes\mathbf{6}} = [\psi_a^T(i\sigma^2)\psi_b][\chi_c^\dagger(i\sigma^2)\chi_d^*]\,\mathcal{C}^{ab;cd}_{\mathbf{6}\otimes\mathbf{6}}.$$

where

$$\Gamma^{kl;mn} \equiv \frac{1}{2}(\delta^{km}\delta^{ln} + \delta^{kn}\delta^{lm} - \frac{2}{3}\delta^{kl}\delta^{mn}),$$

$$\mathcal{C}^{ab;cd}_{\mathbf{3}\otimes\mathbf{3}} \equiv \frac{1}{2\sqrt{3}}(\delta^{ac}\delta^{bd} - \delta^{ad}\delta^{bc}), \mathcal{C}^{ab;cd}_{\mathbf{6}\otimes\mathbf{6}} \equiv \frac{1}{2\sqrt{6}}(\delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc}).$$

Matching the SDCs

- **Vacuum-saturation approximation (VSA):** drop summation over X in the state projection $\mathcal{P} = \sum_X |T_{4c}(P) + X\rangle\langle T_{4c}(P) + X|$

$$\langle 0|\mathscr{O}|0\rangle \to \sum_X \left\langle 0\middle|\mathcal{O}^{(J)}_{\text{color}}\middle|T^{(J)}_{4c} + X\right\rangle \left\langle T^{(J)}_{4c} + X\middle|\mathcal{O}^{(J)\dagger}_{\text{color}}\middle|0\right\rangle \to \left|\left\langle 0\middle|\mathcal{O}^{(J)}_{\text{color}}\middle|T^{(J)}_{4c}\right\rangle\right|^2$$

1. Replace physical (bounded) tetraquark state $|T^J_{4c}\rangle$ with a free 4-quark state $|cc\bar{c}\bar{c}\rangle$:

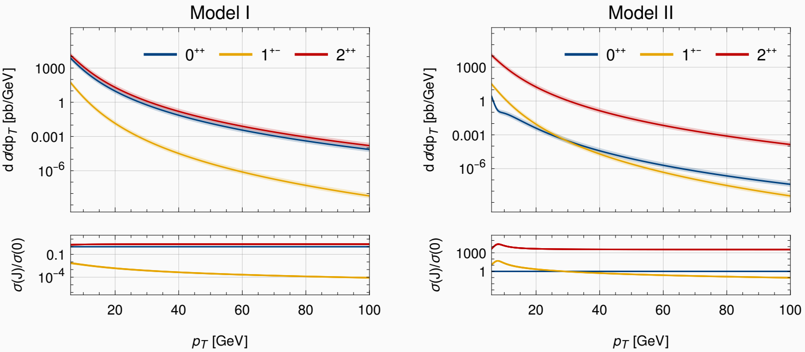
(VSA applied) $$D_{g\to \cancel{T_{4c}}}(z,\mu_\Lambda) = \frac{d_{3,3}\left[g \to cc\bar{c}\bar{c}^{(J)}\right]}{m^9}\left|\left\langle \cancel{T^J_{4c}}\middle|\mathcal{O}^{(J)}_{\bar{\mathbf{3}}\otimes\mathbf{3}}\middle|0\right\rangle\right|^2 + \cdots$$

$$\Rightarrow D_{g\to cc\bar{c}\bar{c}}(z,\mu_\Lambda) = \frac{d_{3,3}\left[g \to cc\bar{c}\bar{c}^{(J)}\right]}{m^9}\left|\langle cc\bar{c}\bar{c}|\mathcal{O}^{(J)}_{\bar{\mathbf{3}}\otimes\mathbf{3}}|0\rangle\right|^2 + \cdots$$

   - For simplicity, we choose the angular momentum eigenstates of the free 4-quark states.

2. Perturbatively calculate both sides of the factorization formula (with QCD and NRQCD)
3. Solve the factorization formula to determine the **SDCs**

Comparing T_{4c} with different Js



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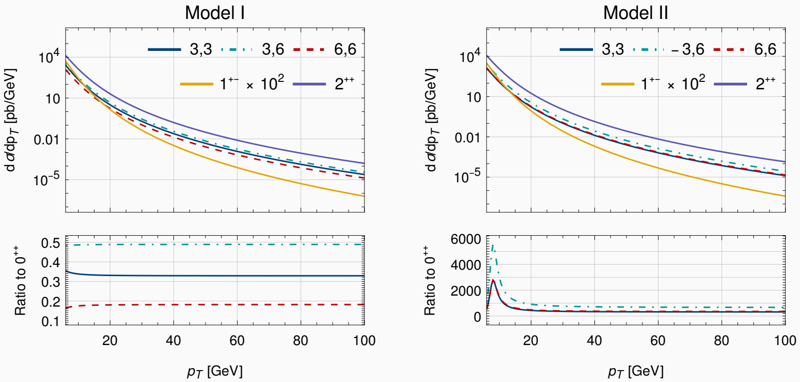
Dissecting the 0^{++} contribution



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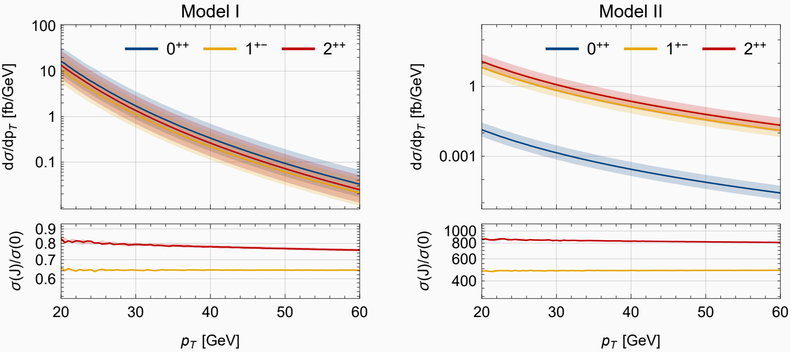
- large cancellation from 0^{++} state in model II

Effect of charm fragmentation



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