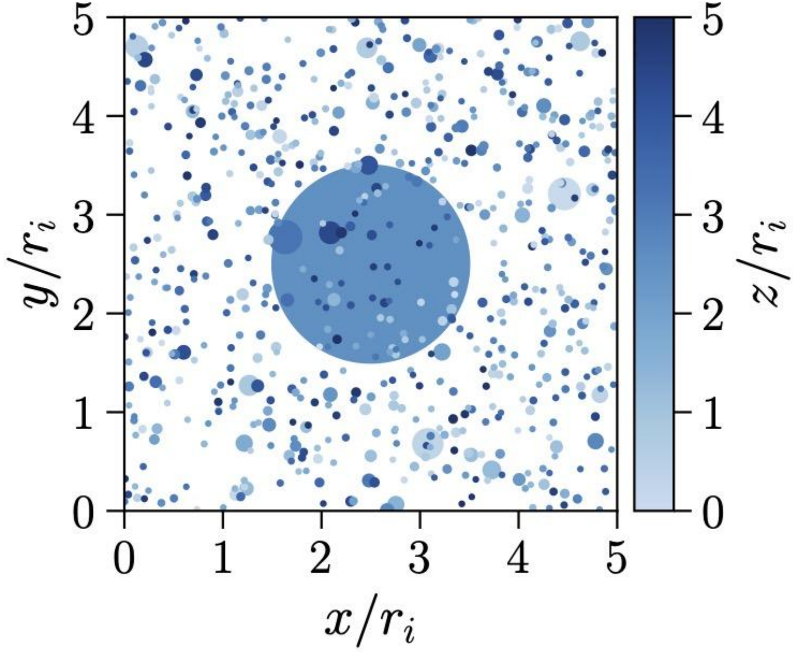
Dark Radiation Isocurvature from Cosmological Phase Transitions

Mitchell Weikert



DPF-Pheno
May 14th, 2024
Pittsburgh

Based on 2402.13309:
Matthew R. Buckley, Peizhi Du, Nicolas Fernandez, &
MJW

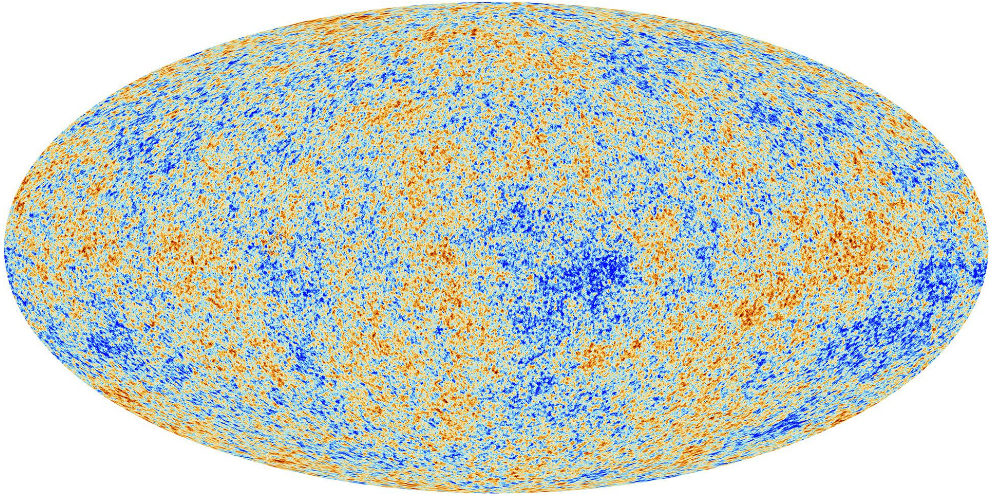
Why are Cosmological Phase Transitions Interesting?

- Many models of Physics beyond the SM predict early universe first order phase transitions
  - Electroweak baryogenesis, dark SU(N) gauge theories, etc…

Why are Cosmological Phase Transitions Interesting?

- Many models of Physics beyond the SM predict early universe first order phase transitions
    - Electroweak baryogenesis, dark SU(N) gauge theories, etc…
- Such phase transitions are often accompanied by observable signatures which can be used to learn about the associated physics
    - **Generic signature: production of gravitational waves from bubble collisions**
    - Also can produce light particles that do not interact with the SM
        - Both behave as dark radiation

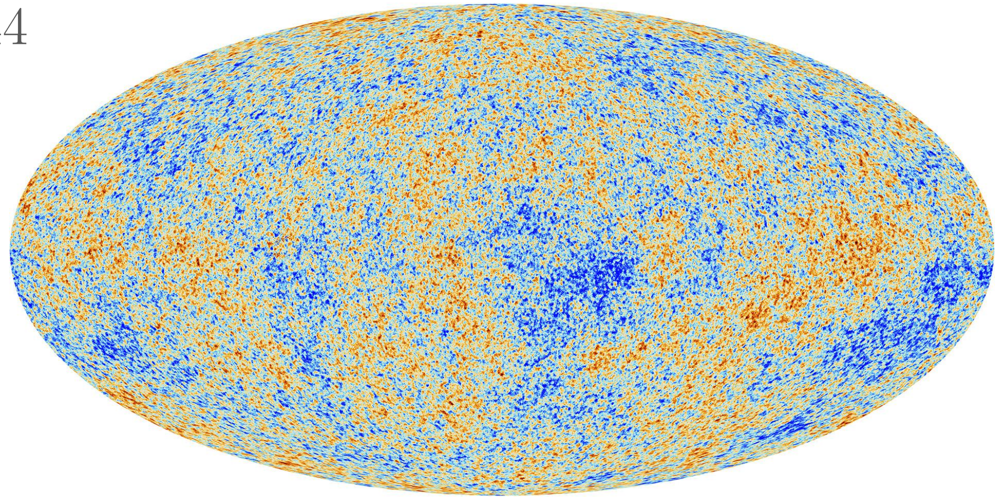
Cosmic Microwave Background



Cosmic Microwave Background

- Dark radiation contributes to energy density, which influences CMB photons through the metric
  - Constraints on dark radiation (assuming adiabatic): $\Delta N_{\text{eff}} < 0.3$ [1807.06209]

$$\Delta N_{\text{eff}} = \frac{\rho_{\text{dr}}}{\rho_\nu} N_{\text{eff}} \qquad N_{\text{eff}} = 3.044$$



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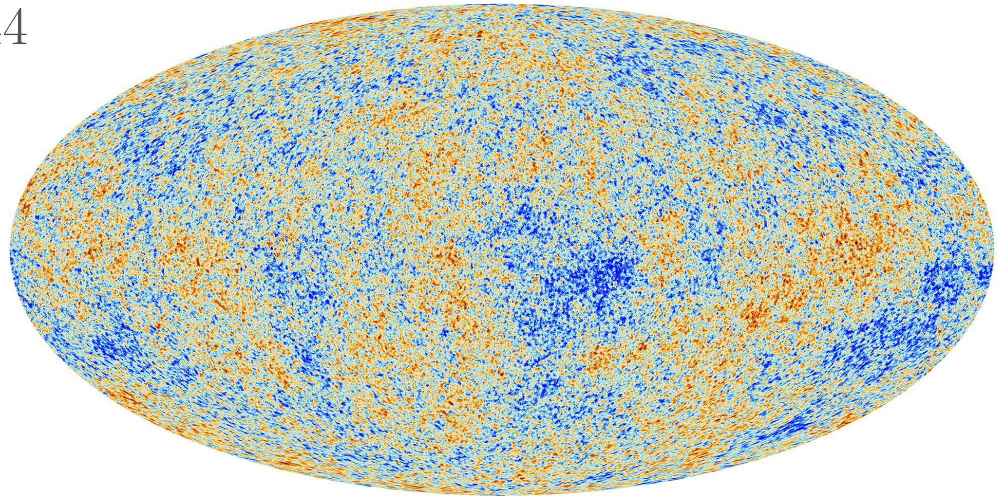
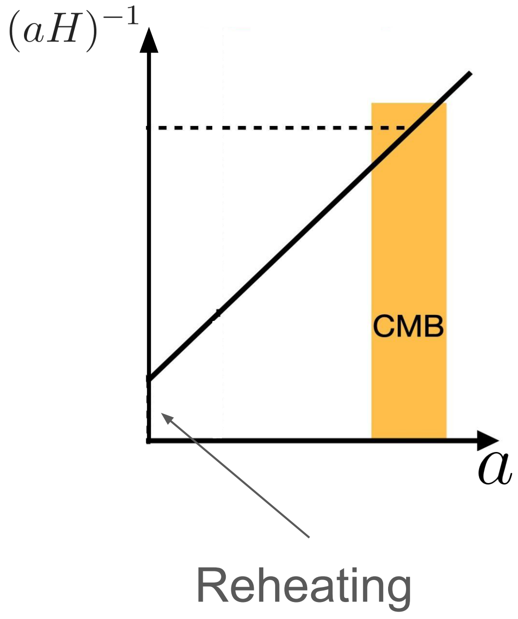
FOPTs can produce DR isocurvature modes which may be constrained more stringently!



Inflationary Phase Transition

- Consider comoving horizon



Inflationary Phase Transition

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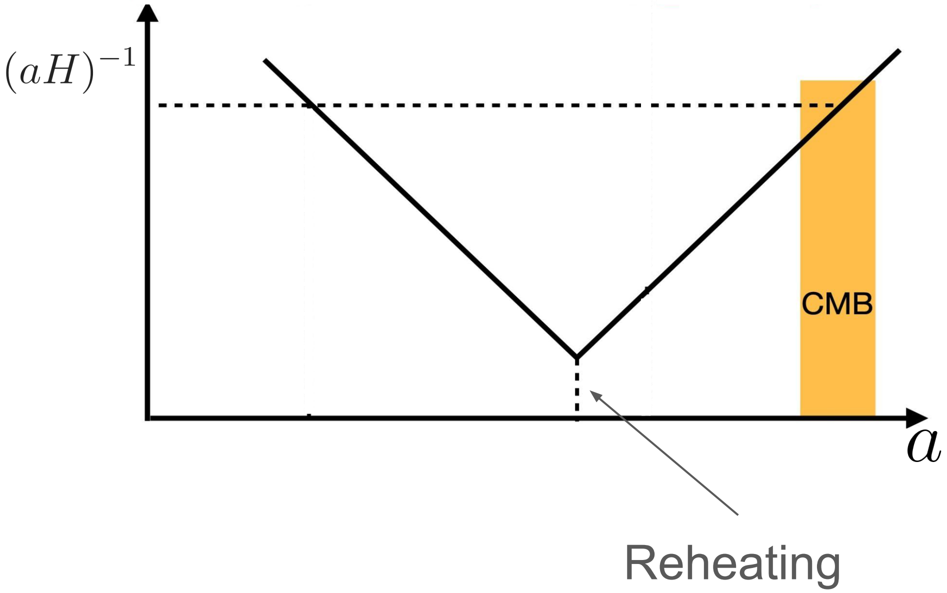
- Single field inflation driven by ϕ
  - $H \approx H_{\text{inf}} \to \text{const}$



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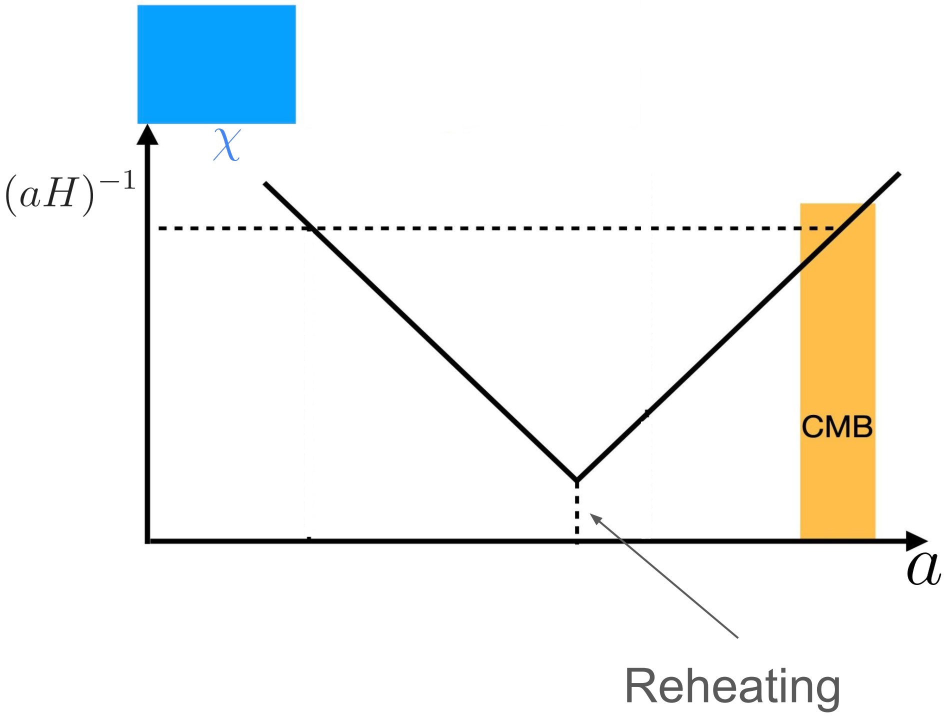

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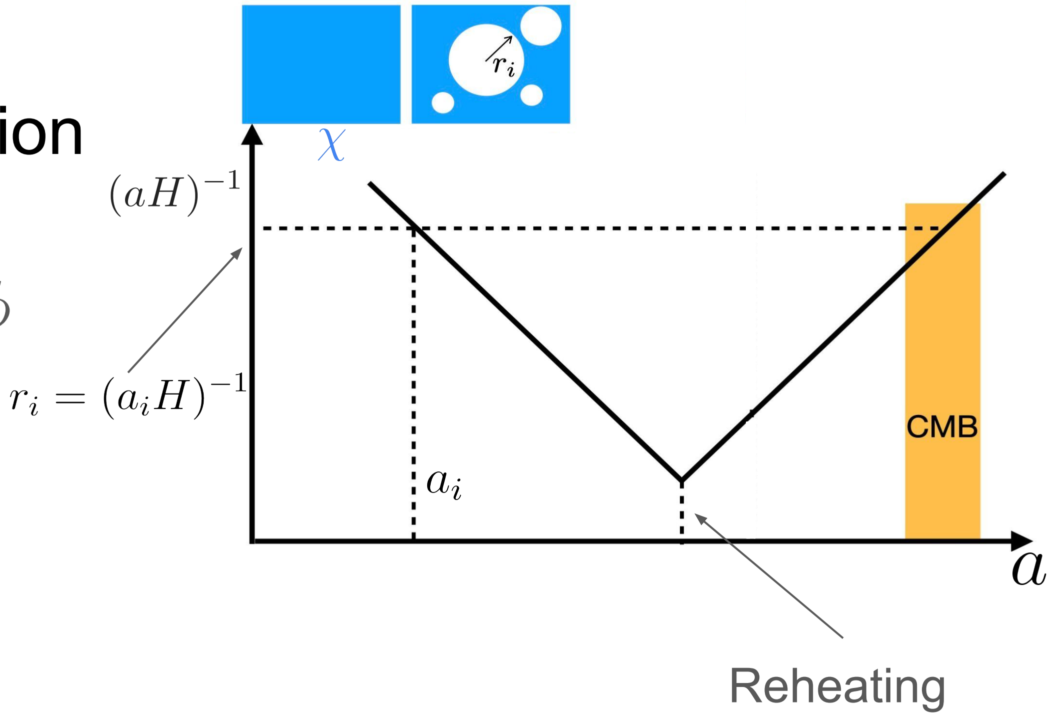

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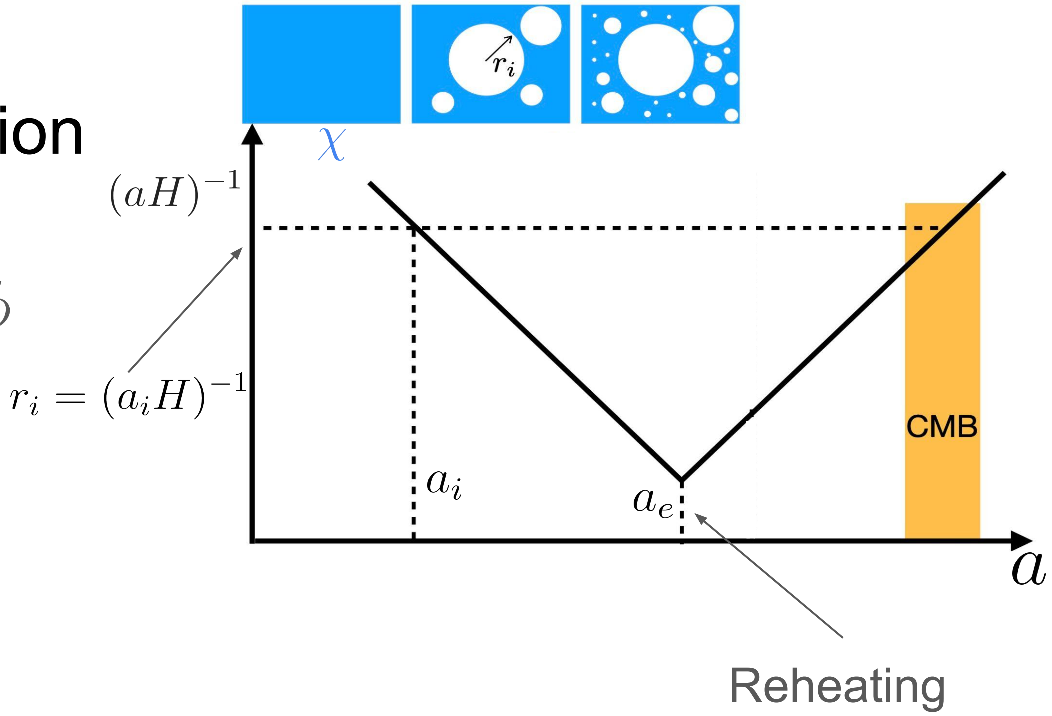
$$\gamma_{\rm PT} \equiv \frac{\Gamma_{\rm PT}}{H_{\rm inf}^4} \ll 1 \Longrightarrow$$ PT remains incomplete



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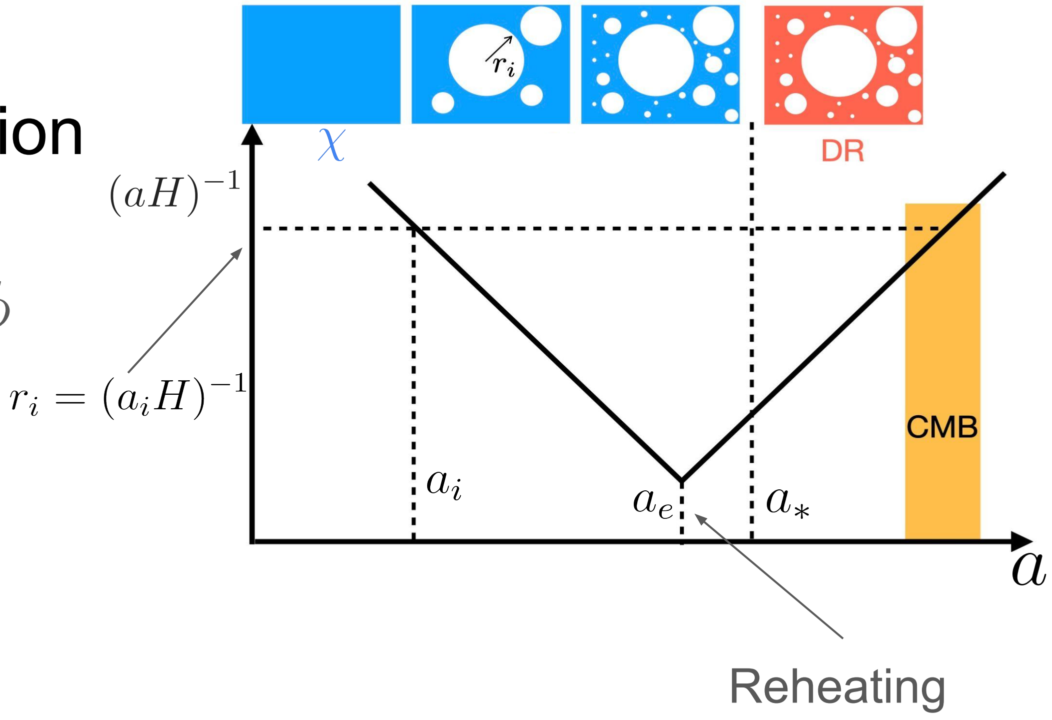
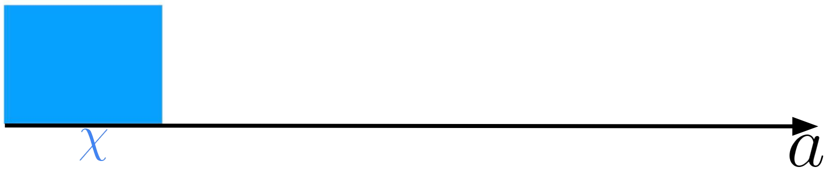
$$\gamma_{\rm PT} \equiv \frac{\Gamma_{\rm PT}}{H_{\rm inf}^4} \ll 1 \Rightarrow$$ PT remains incomplete

- PT completes: $\chi \longrightarrow$ DR
    - Large-scale features in CMB



PT with Non-Thermal Trigger



$$V(\chi, \phi) = -\frac{1}{2}m^2\chi^2 + \frac{\mu(\phi)}{3}\chi^3 + \frac{\lambda}{4}\chi^4$$

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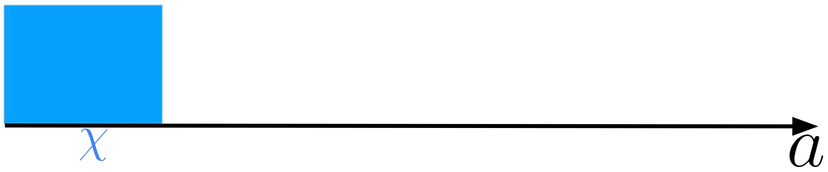
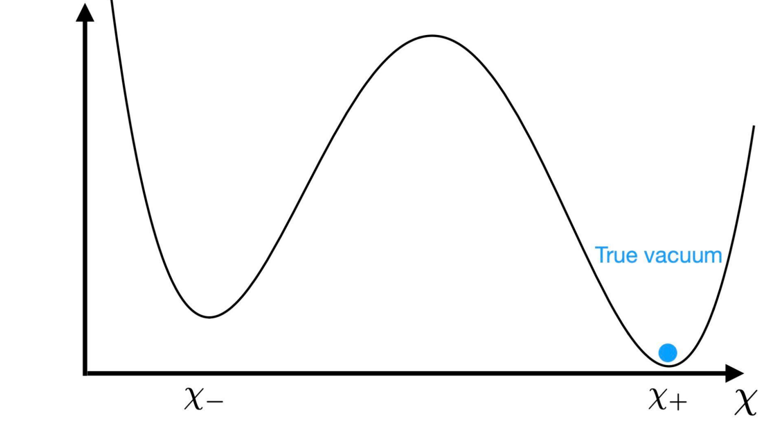


$$V(\chi,\phi) = -\frac{1}{2}m^2\chi^2 + \frac{\mu(\phi)}{3}\chi^3 + \frac{\lambda}{4}\chi^4$$

$V(\chi,\phi)$ $\mu(\phi) < 0$ $\phi < \phi_i$



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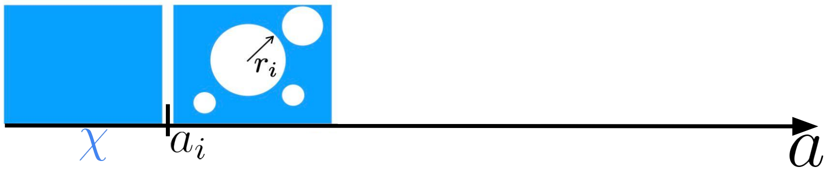

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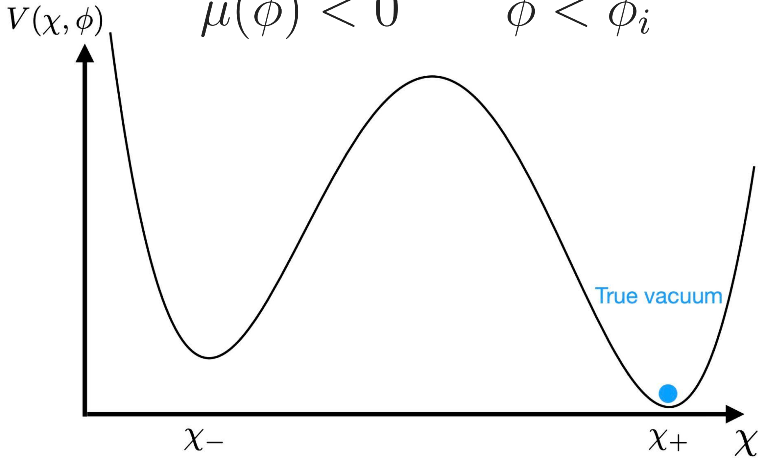

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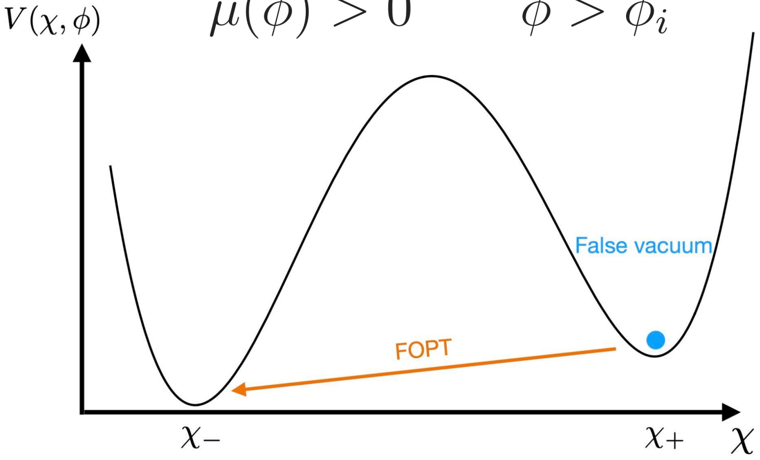

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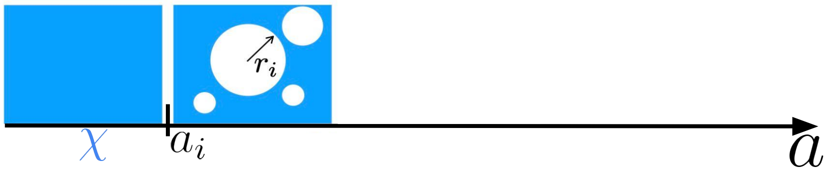

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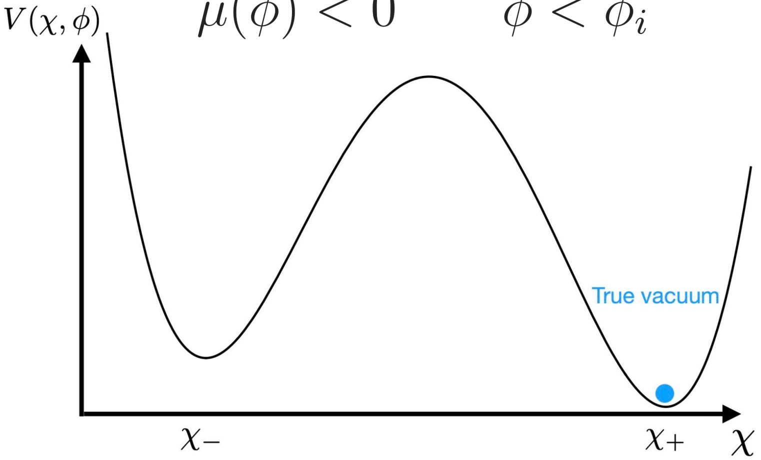

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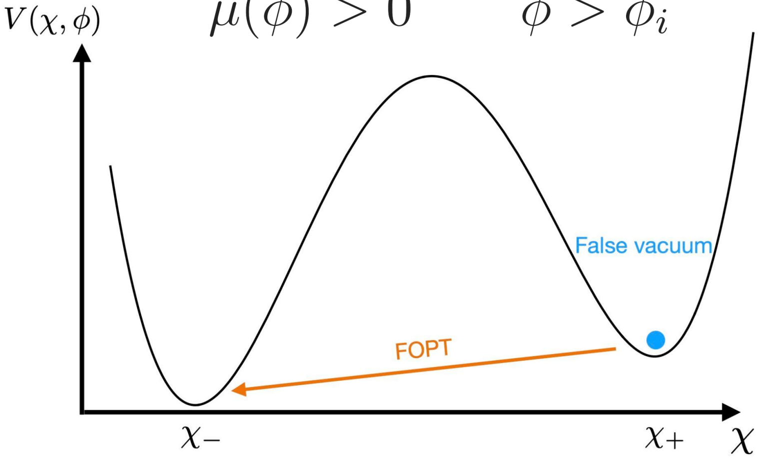
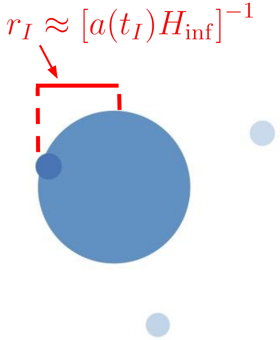


$$\mu(\phi) = \mu\ \tanh\left(\frac{\phi - \phi_i}{\Delta\phi}\right)$$

$\Rightarrow$

ΔV and $\Gamma_{\rm PT}$ are approximately constant

Energy Density Distribution for χ



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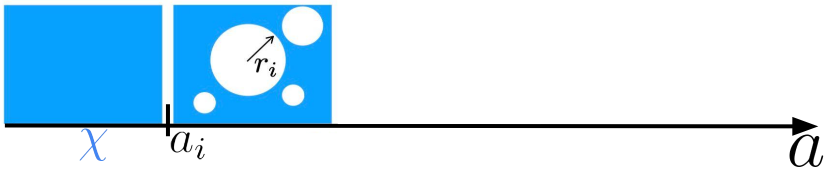
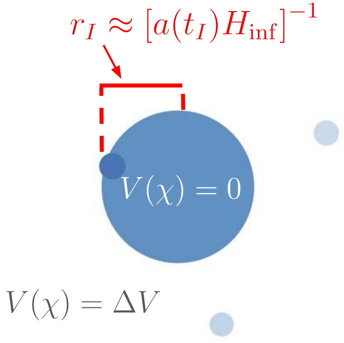


$$\rho_\chi(\mathbf{x},t) = \Delta V \left[1 - \sum_{I:t_I<t} \Theta(r_I - |\mathbf{x} - \mathbf{x_I}|)\right]$$

t_I and $\mathbf{x_I}$ randomly sampled



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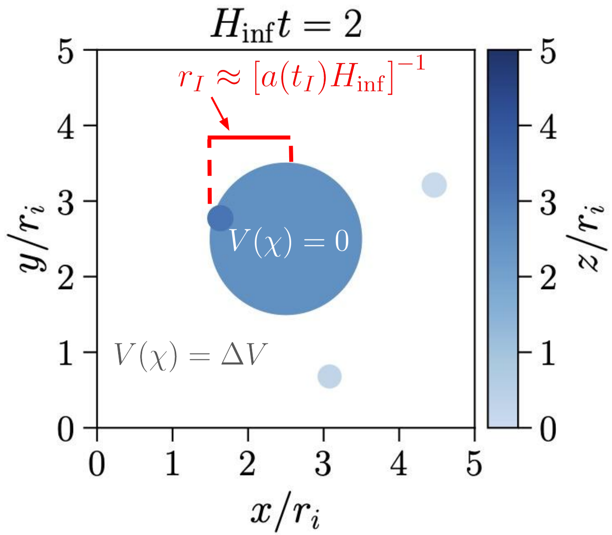

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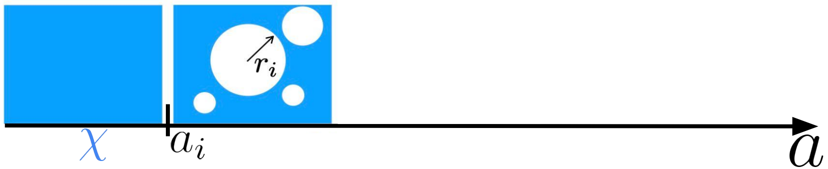

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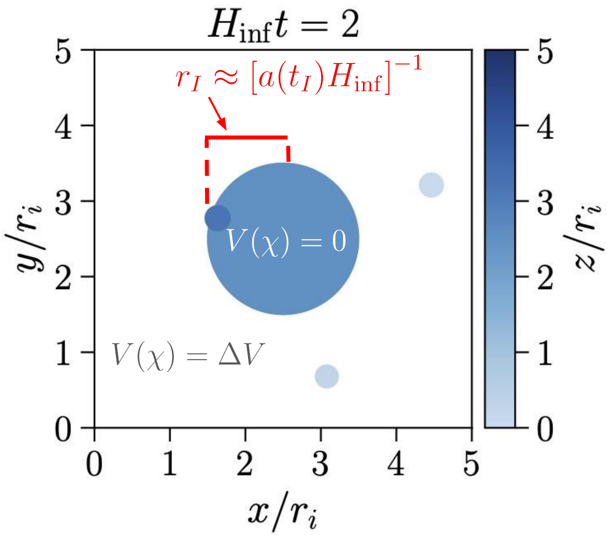

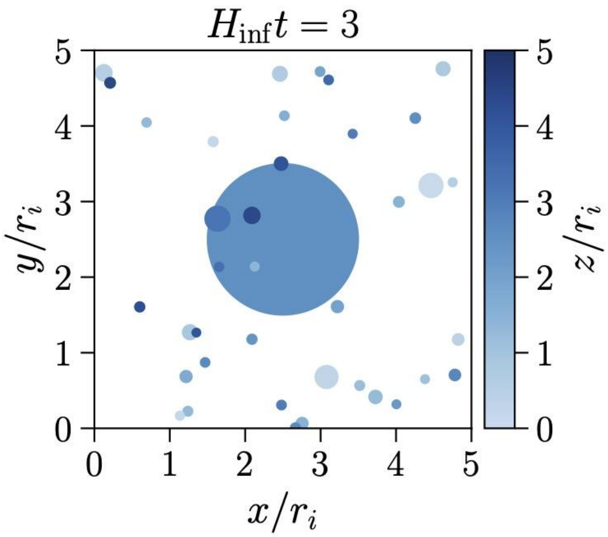

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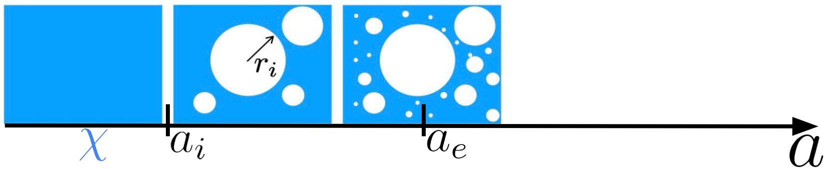

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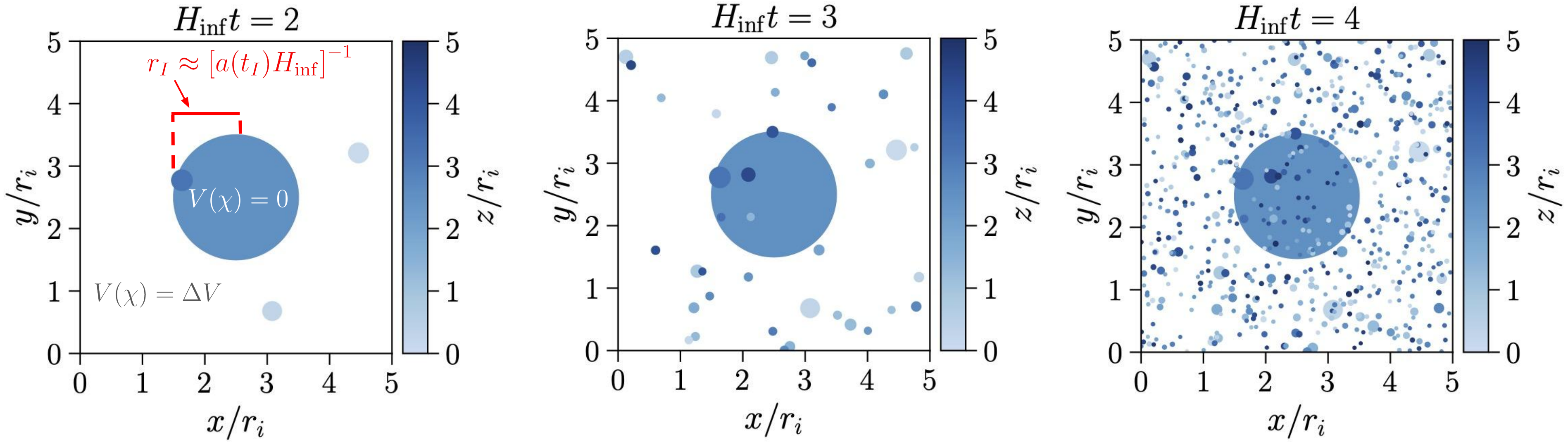
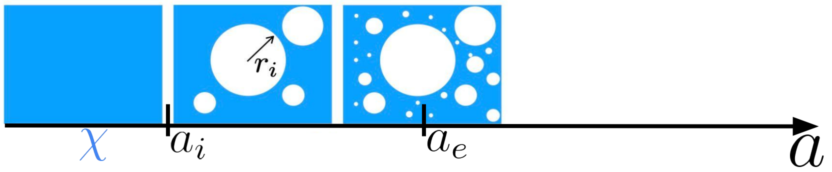


Isocurvature Power Spectrum



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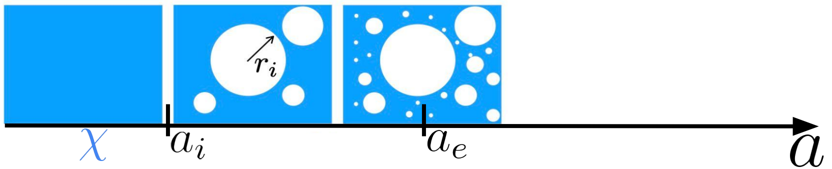
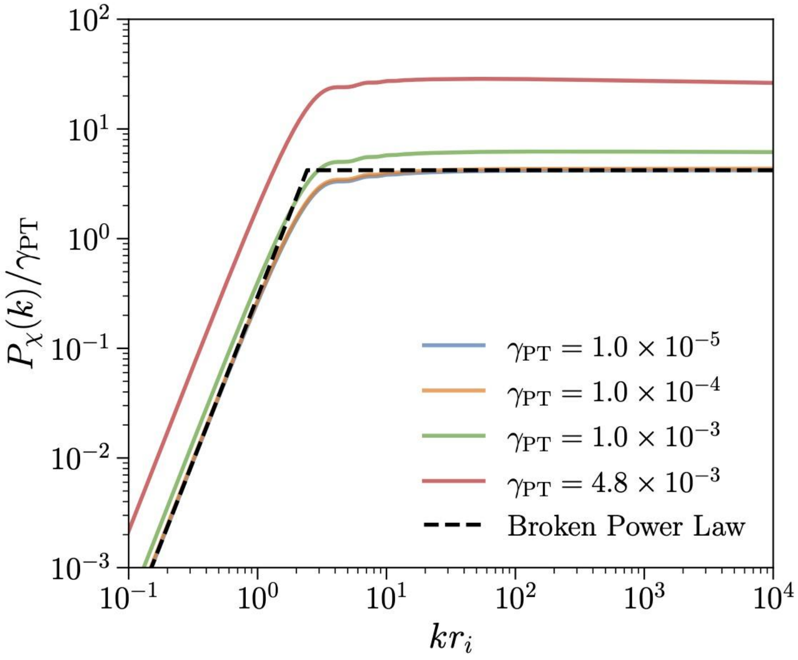


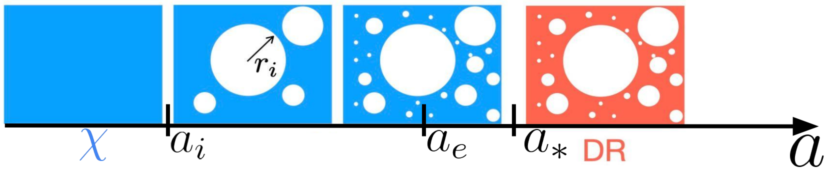
Isocurvature Power Spectrum

$$\rho_\chi(\mathbf{x},t) = \Delta V \left[1 - \sum_{I:t_I<t} \Theta(r_I - |\mathbf{x}-\mathbf{x_I}|)\right] \Rightarrow \langle \delta_\chi(\mathbf{k})\delta_\chi(\mathbf{k}')\rangle \equiv 2\pi^2 \frac{(2\pi)^3\delta^3(\mathbf{k}+\mathbf{k}')}{k^3} P_\chi(k)$$



$$P_\chi(k) \approx \gamma_{\rm PT} \begin{cases} \frac{8}{27}(kr_i)^3 & kr_i \ll 1 \\ 4.2 & kr_i \gg 1 \end{cases}$$

Isocurvature Power Spectrum



$$\rho_\chi(\mathbf{x},t) = \Delta V\left[1 - \sum_{I:t_I<t}\Theta(r_I - |\mathbf{x}-\mathbf{x_I}|)\right]$$

⇒

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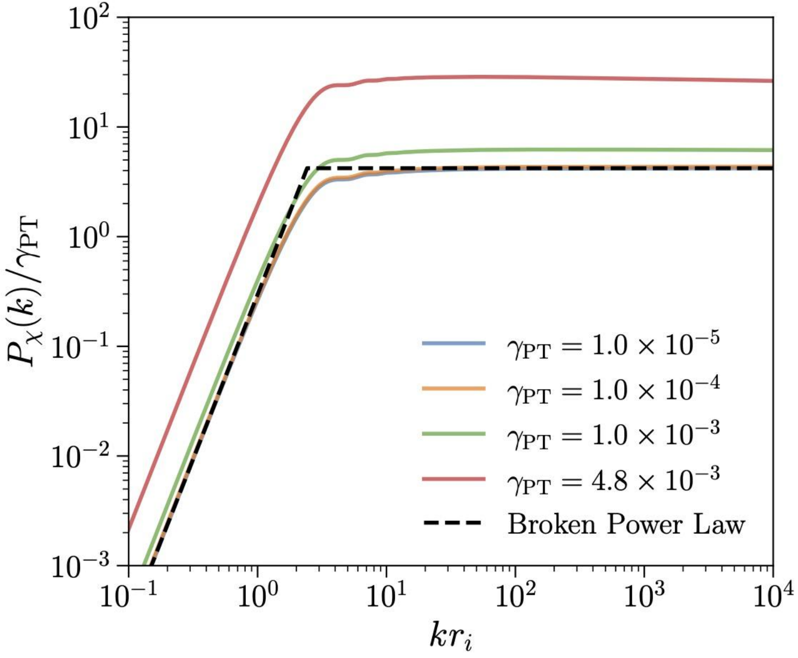
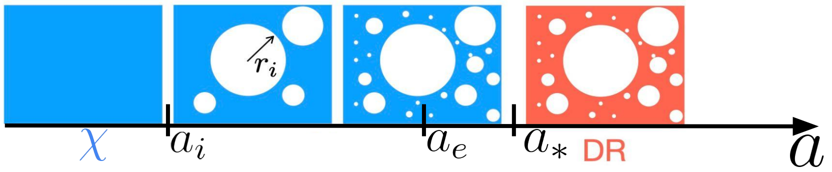


$$P_\chi(k) \approx \gamma_{\rm PT}\begin{cases}\frac{8}{27}(kr_i)^3 & kr_i \ll 1\\ 4.2 & kr_i \gg 1\end{cases}$$

The PT completes when $\dfrac{\Gamma_{\rm PT}}{H^4} = \gamma_{\rm PT}\left(\dfrac{T_{\rm rh}}{T}\right)^8 \approx 1$

⇒ **Uncorrelated Isocurvature**

Isocurvature Power Spectrum



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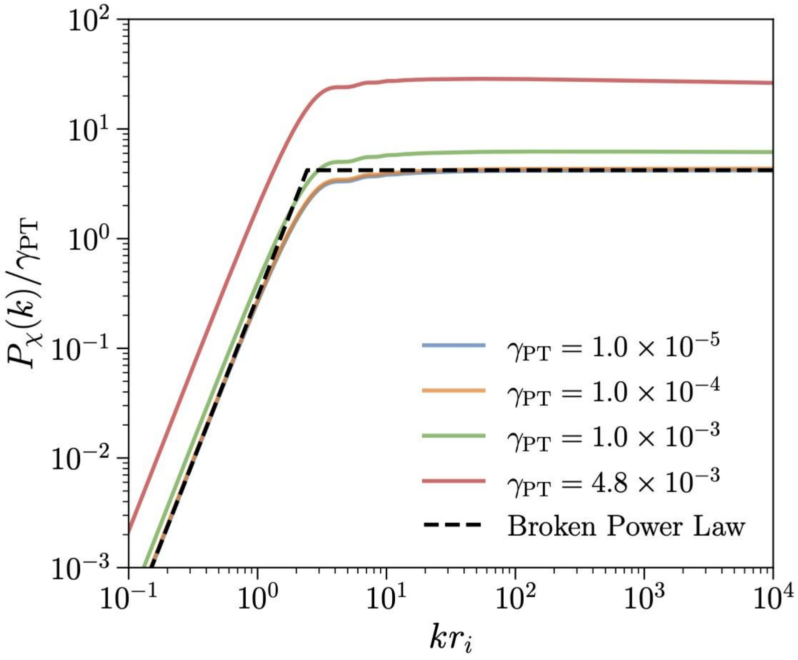

$$P_{\rm iso}(k) \approx P_\chi(k) \approx \gamma_{\rm PT} \begin{cases} \frac{8}{27}(kr_i)^3 & kr_i \ll 1 \\ 4.2 & kr_i \gg 1 \end{cases}$$

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⟹ **Uncorrelated Isocurvature**

$$P_{\rm iso}(k) = f_{\rm iso}^2 A_s \begin{cases} (k/k_i)^3 & k \le k_i \\ 1 & k > k_i \end{cases}$$

Temperature Anisotropy Angular Power Spectrum

$$\Delta(\mathbf{x}, \hat{\mathbf{n}}, \tau) \equiv \frac{\Delta T}{\bar{T}}$$

Fourier and Legendre Transform

$$\Delta_\ell(\mathbf{k}, \tau) = c^{\mathrm{ad}}(\mathbf{k})\Delta_\ell^{\mathrm{ad}}(k, \tau) + c^{\mathrm{iso}}(\mathbf{k})\Delta_\ell^{\mathrm{iso}}(k, \tau)$$

Only curvature initially

Only isocurvature initially

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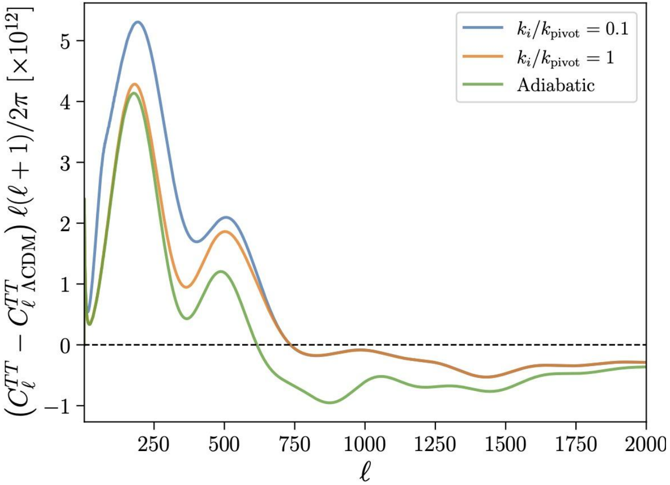
$$\propto \Delta N_{\rm eff}^2$$

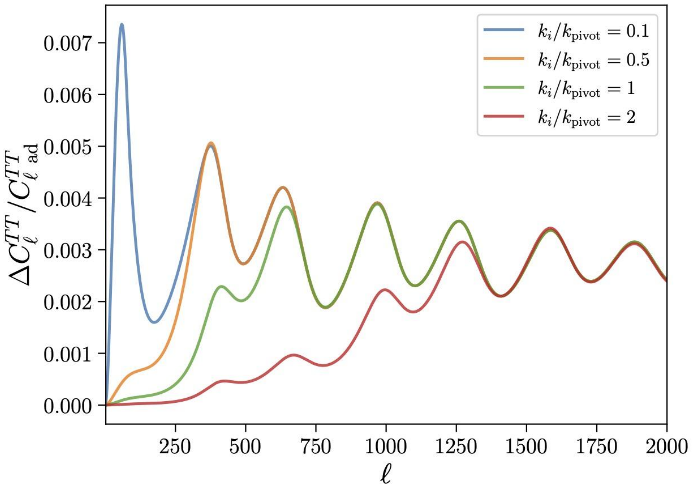
$$k_i \sim r_i^{-1}$$

Calculation of Angular Power Spectrum with CLASS

$$f_{\rm iso} = 10$$
$$\Delta N_{\rm eff} = 0.1$$

$$k_{\rm pivot} \equiv 0.05\,{\rm Mpc}^{-1}$$
$$\ell \sim 500 \times (k/k_{\rm pivot})$$





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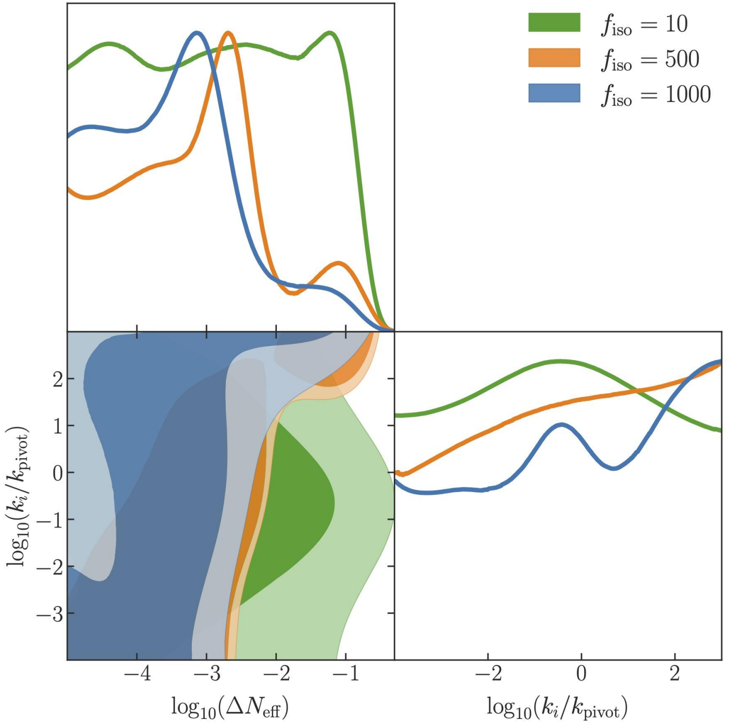
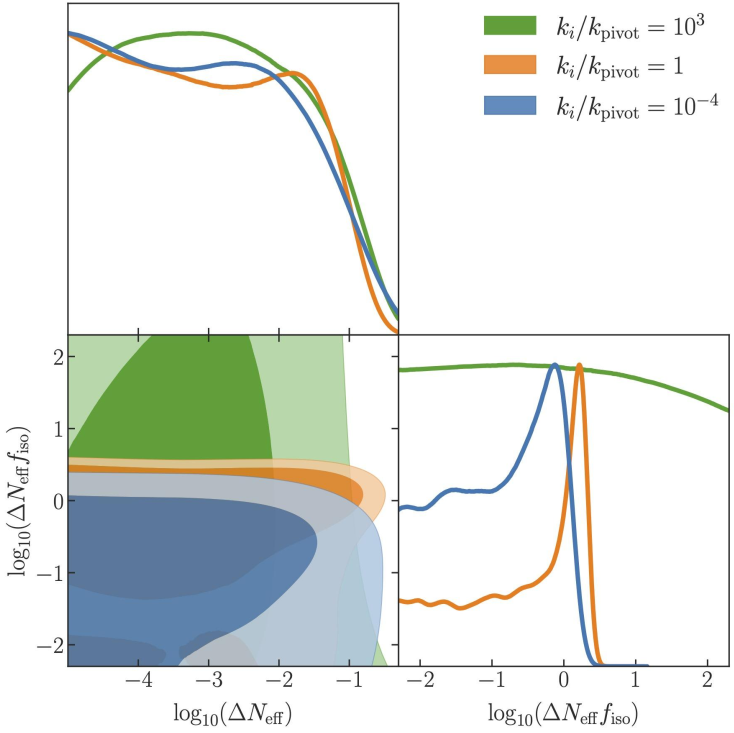
MCMC Results with Planck+BAO





$f_{\rm iso}$ Fixed (Left)

-Small $f_{\rm iso}$: preference for intermediate values of k_i

-large $f_{\rm iso}$: limits weaken at large k_i

k_i Fixed (Right)

-Limits depend on $\Delta N_{\rm eff} f_{\rm iso}$ for small $\Delta N_{\rm eff}$ and $k_i \lesssim k_{\rm pivot}$

Conclusion

- We set constraints on an FOPT that produces DR isocurvature
- PT must start during inflation and remain incomplete until after reheating
- When the PT completes, bubble collisions produce dark radiation with the imprint of bubbles nucleated during inflation
    - Leads to DR isocurvature modes observable in CMB
- Associated signal: gravitational waves from when the PT completes
    - If a gravitational wave signal is ever observed, our constraints can be interpreted as constraints on the scale of inflation

Thank You!

χ Power Spectrum (Details)

$$\bar{\rho}_\chi(t) = \Delta V p_{\text{false}}(t) = \Delta V e^{-t/\tau_{\text{PT}}}$$

$$\tau_{\text{PT}}^{-1} \equiv \frac{4\pi}{3}\gamma_{\text{PT}} H_{\text{inf}}$$

$$\langle \delta_\chi(\mathbf{k})\delta_\chi(\mathbf{k}')\rangle = e^{2t_e/\tau_{PT}} \frac{(4\pi)^2}{k^3 k'^3} N \int d^4x \, p_1(x) e^{-i(\mathbf{k}+\mathbf{k}')\cdot \boldsymbol{x}} \mathcal{A}(kr(t))\mathcal{A}(k'r(t))$$

$$p_1(x) = \frac{1}{\mathcal{V}}\left(\frac{1}{N}\frac{dN}{dt}\right)$$

$$\mathcal{A}(y) \equiv \sin y - y\cos y$$

$$\frac{dN}{dt} = \mathcal{V}_{\text{false}}(t) a(t)^3 \Gamma_{\text{PT}}$$

$$\mathcal{V}_{\text{false}}(t) = \mathcal{V} p_{\text{false}}(t) = \mathcal{V} e^{-t/\tau_{\text{PT}}}$$

DR Isocurvature Power Spectrum

$$dt = a d\tau$$

The PT completes after reheating when $\frac{\Gamma_{\rm PT}}{H^4} = \gamma_{\rm PT}\left(\frac{T_{\rm rh}}{T}\right)^8 \approx 1$ at conformal time τ_*

All remaining energy density in χ is quickly converted to DR

Working the gauge $\hat{\delta}_\gamma = 0$

$$\hat{\delta}_{\rm dr}(\mathbf{k}, \tau_*) \approx \delta_\chi(\mathbf{k}, t_*) \approx \delta_\chi(\mathbf{k}, t_e)$$

$$\mathcal{S}_{{\rm dr},\gamma} = \frac{3}{4}\left(\delta_{\rm dr} - \delta_\gamma\right) = \frac{3}{4}\hat{\delta}_{\rm dr}$$

$$P_{\rm iso}(k) = \frac{16}{9} P_{\mathcal{S}}(k) \approx P_\chi(k)$$

χ density is approximately gauge invariant as

$$\delta\hat{\rho}_a = \delta\rho_a + \rho_a' \delta\tau$$

DR Isocurvature and Adiabatic Modes Formalism

The perturbation variables $X \in [h, \eta, \delta_a, \theta_a, \sigma_a, F_{a,\ell} \ldots]$ can be written as

$$X(\mathbf{k},\tau) = c^{\mathrm{ad}}(\mathbf{k}) X^{\mathrm{ad}}(k,\tau) + c^{\mathrm{iso}}(\mathbf{k}) X^{\mathrm{iso}}(k,\tau)$$

Where the initial conditions for the modes satisfy

$$\delta^{\mathrm{ad}}_{\mathrm{dr}} = \delta^{\mathrm{ad}}_{\nu} = \delta^{\mathrm{ad}}_{\gamma} \qquad \delta^{\mathrm{iso}}_{\mathrm{dr}} = 1, \quad \delta^{\mathrm{iso}}_{\gamma} = \delta^{\mathrm{iso}}_{\nu} = -\frac{R_{\mathrm{dr}}}{1-R_{\mathrm{dr}}}$$

$$\zeta = \eta - \mathcal{H}\frac{\delta\rho}{\rho'} = c^{\mathrm{ad}}(\mathbf{k}) \qquad \mathcal{S}_{\mathrm{dr},\gamma} = -3\mathcal{H}\left(\frac{\delta\rho_{\mathrm{dr}}}{\rho'_{\mathrm{dr}}} - \frac{\delta\rho_{\gamma}}{\rho'_{\gamma}}\right) \approx \frac{3}{4}c^{\mathrm{iso}}(\mathbf{k})$$

And the power spectra are defined as $\langle c^A(\mathbf{k}) c^A(\mathbf{k}')\rangle = \frac{2\pi^2}{k^3}(2\pi)^3\delta(\mathbf{k}+\mathbf{k}')P_A(k)$

$$P_{\mathrm{ad}} = P_{\zeta} \qquad P_{\mathrm{iso}} = \frac{16}{9}P_{\mathcal{S}}$$

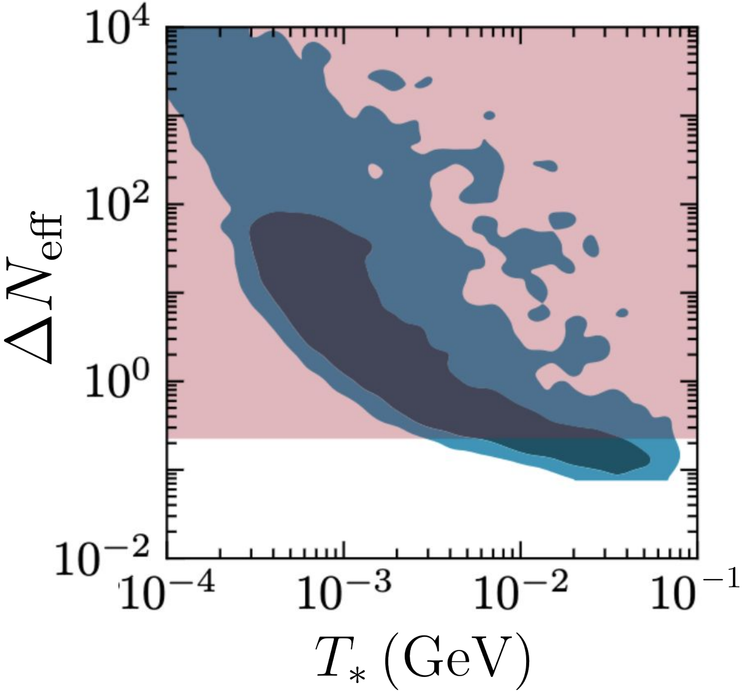
NANOGrav Signal

Asymptotic limits for small $\Delta N_{\rm eff}$

$$\Delta N_{\rm eff} f_{\rm iso} < \beta(k_i) \sim \mathcal{O}(1)$$

$$\Delta N_{\rm eff} < 2.8 \times 10^{-5} \left(\frac{T_{\rm rh}}{T_*}\right)^4 \left(\frac{\beta(k_i)}{1.25}\right)$$

[2306.09411]



Estimated Non-Gaussianity Constraints

$$\langle c_{\rm iso}(\mathbf{k_1}) c_{\rm iso}(\mathbf{k_2}) c_{\rm iso}(\mathbf{k_3})\rangle \equiv (2\pi)^3 \delta^3(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3}) B_{\rm iso}(k_1+k_2+k_3)$$

$$B_{\rm iso}(k,k,k) \approx -0.97 \times f_{\rm iso}^2 A_s \frac{1}{k^6}$$

Naive mapping from neutrino density isocurvature suggests:

$$f_{\rm iso}^2 \Delta N_{\rm eff}^3 \lesssim 2 \times 10^{-4}$$

A dedicated search for non Gaussianity in the equilateral configuration is needed