Simple calculation of the Coulomb-nuclear corrections in *pp* and $\bar{p}p$ scattering

Loyal Durand (UW-Madison) & Phuoc Ha (Towson U)

Outline

I. Introduction

II. Theoretical Background

III. Examples of the Simple Calculation of the Mixed-Coulomb-Nuclear Corrections

- Exponential-type models for the nuclear scattering amplitude f_N
- Accuracy of the Coulomb-Nuclear Corrections for exponential models

IV. Applications: Fit to High-Energy Data

V. Conclusions

I. Introduction

- Coulomb and nuclear interactions in the scattering of charged particles has been studied by many authors over more than fifty years.
- Particular emphasis is placed on the use of Coulomb-nuclear interference (CNI) effects to determine the ratio $\rho = \mathrm{Re}(f_N)/\,\mathrm{Im}(f_N)$ in *pp* and $\bar{p}p$ scattering.
- Direct information on $\mathrm{Re}(f_N)$, the real part of the nuclear scattering amplitude, is obtained at much larger angles very near the observed dips in the differential cross sections.
- Cahn's work [Z. Phys. C 15, 253 (1982)] and Kundrát and Lokajiček's work [Z. Phys. C 63, 619 (1994)] seem to have become standard in the analysis of CNI effects at high energies.
- Their method is based on the use of the Fourier-Bessel convolution theorem to calculate the corrections that involve Coulomb and nuclear interactions simultaneously, and to include the effects of the nucleon charge form factors.
- The results, which involve delicate manipulations in their derivation to avoid singularities, are not transparent.

I. Introduction

- We show here that the full *pp* and $\bar{p}p$ scattering amplitudes can be written as the sum of a Coulomb and form-factor related term, the pure strong interaction or "nuclear" amplitude, and a mixed Coulomb-nuclear correction term.
- The mixed Coulomb-nuclear correction term:

  - ❑ It involves the inverse Fourier-Bessel transform of the nuclear scattering amplitude.
  - ❑ It can be evaluated analytically for the exponential-type models used to fit the observed cross sections at very high energies and small momentum transfers.

- The correction term is small and easily calculated.
- This approach provides a substantial improvement in simplicity and clarity over the methods most commonly used at present.

II. Theoretical Background

- In the absence of significant spin effects, the spin-averaged differential cross section for pp and $\bar{p}p$ scattering can be written in terms of a **single spin-independent amplitude**

$$f(s,q^2) = i\int_0^\infty db\, b\,\left(1 - e^{2i\left(\delta_c^{tot}(b,s)+\delta_N(b,s)\right)}\right)J_0(qb) \quad (1)$$

full Coulomb phase shift including the effects of the finite charge structure of the proton $\delta_c^{tot}(b,s) = \delta_c(b,s) + \delta_c^{FF}(b,s)$

The nuclear phase shift

- where $q^2 = -t$: the square of the invariant momentum transfer, b : the impact parameter,

$\eta \sim \alpha = \frac{1}{137}; \mu^2 = 0.71\text{GeV}^2$

$$\delta_c(b,s) = \eta(\ln pb + \gamma)\ ,$$

$$\delta_c^{FF}(b,s) = \sum_{m=0}^{3} \frac{\eta}{2^m\Gamma(m+1)}(\mu b)^m K_m\,(\mu b)\ ,$$

For $\bar{p}p$ scattering, η → - η

for the standard proton charge form factor $F_Q(q^2) = \frac{\mu^4}{(q^2+\mu^2)^2}$

- We can divide out a common Coulomb phase factor $(4p^2/q^2)^{i\eta}$ from all terms in Eq. (1).
- $\delta_c \rightarrow \delta'_c(b,s) = \eta(\ln(qb/2) + \gamma)$

L.Durand and P. Ha, PRD 102, 036025 (2020)

Theoretical Background

- Eq (1) can be written as

$$f(s,q^2) = f_c(s,q^2) + f_c^{FF}(s,q^2) + f_N(s,q^2) + f_{N,c}^{Corr}(s,q^2) \qquad (2)$$

Coulomb amplitude without form factors (FFs)

Term accounts for the effects of the FFs on the Coulomb scattering

The pure nuclear amplitude

The mixed Coulomb-Nuclear term

$$f_c(s,q^2) = i\int_0^\infty db\, b\,(1 - e^{2i\delta'_c(s,b)})J_0(qb)$$

$$f_c^{FF}(s,q^2) = i\int_0^\infty db\, b e^{2i\delta'_c(s,b)}(1 - e^{2i\delta_c^{FF}(s,b)})J_0(qb)$$

For $\bar{p}p$ scattering, η → - η

- For *pp* scattering, we find (for η $/F_Q^2(q^2) << 1$)

$$F_Q(q^2) = \frac{\mu^4}{(q^2+\mu^2)^2}$$

$$f_c(s,q^2) + f_c^{FF}(s,q^2) = -\frac{2\eta}{q^2}F_Q^2(q^2) + O(\eta^2, i\eta^2) \qquad (3)$$

Theoretical Background

- The pure nuclear amplitude

$$f_N(s,q^2) = i\int_0^\infty db\, b\left(1 - e^{2i\delta_N(s,b)}\right)J_0(qb) \qquad (4)$$

- The mix Coulomb-nuclear term

$$f_{N,c}^{Corr}(s,q^2) = i\int_0^\infty db\, bJ_0(qb)\left(e^{2i\delta'_c(s,b)+2i\delta_c^{FF}(s,b)} - 1\right)\hat{f}_N(b,s) \qquad (5)$$

Where $\hat{f}_N(b,s)$ is the inverse Fourier-Bessel transformation of $f_N(s,q^2)$

$$\hat{f}_N(b,s) = i\left(1 - e^{2i\delta_N(s,b)}\right) = \int_0^\infty dq'q'f_N(s,q'^2)J_0(q'b) \qquad (6)$$

- Both integrals in Eq (5) and Eq (6) are expected to converge very rapidly for realistic models of the nuclear amplitude.
- In Eq. (5), the order of integration is crucial.
- *This simple result has been missed in previous work leading to unnecessary complications.*

Theoretical Background

- With our normalization, the differential cross section is

$$\frac{d\sigma}{dq^2}(s,q^2) = \pi|f(s,q^2)|^2$$

$$= \pi\left(|f_1|^2 + \frac{2|f_1||f_N|}{(1+\rho^2)^{1/2}}(sin\Phi_1 + \rho cos\Phi_1) + |f_N|^2\right) \qquad (7)$$

$\rho(s,q^2)$ = Re $f_N(s,q^2)$/ Im $f_N(s,q^2)$

Φ_1 is the phase of $f_1(s,q^2)$

$$f(s,q^2) = f_1(s,q^2) + f_N(s,q^2)$$

$$f_1(s,q^2) = f_c(s,q^2) + f_c^{FF}(s,q^2) + f_{N,c}^{Corr}(s,q^2)$$

III. Examples: Exponential-type models for f_N

- Consider the simple exponential model:

$$f_N^{exp}(s,q^2) = (\mathrm{i}+\rho)\sqrt{A/\pi}\; e^{-\frac{1}{2}Bq^2} \qquad (8)$$

with A, B, and ρ functions of s but independent of b.

- This model has been used over very wide range of energies to fit experimental data on pp and $\bar{p}p$ differential cross sections at small q^2 to determine the forward slope parameters B, the total cross sections σ_{tot} , and the ratios ρ .
- For this model, the inverse Fourier-Bessel transform in Eq (4) is

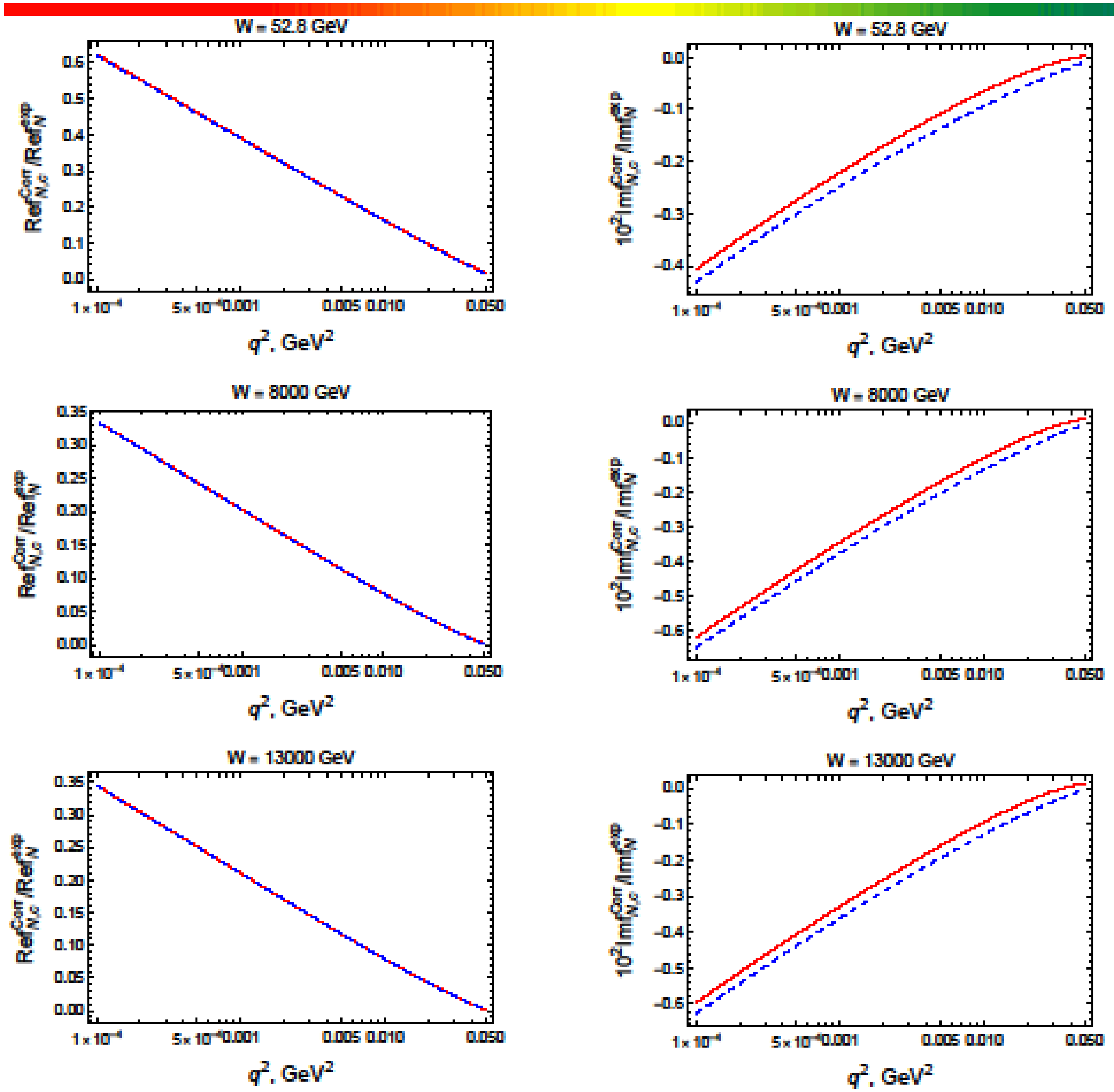
$$\hat{f}_N^{exp}(b,s) = (\mathrm{i}+\rho)\int_0^\infty dq q\sqrt{A/\pi}\; e^{-\frac{1}{2}Bq^2} J_0(qb) = (\mathrm{i}+\rho)\sqrt{A/\pi}\,\frac{1}{B}\; e^{-b^2/2B} \qquad (9)$$

- The simple exponential model can be extended to:

$$f_N^{exp'}(s,q^2) = (\mathrm{i}+\rho)\sqrt{A/\pi}\; e^{-\frac{1}{2}Bq^2}(1 + \tfrac{1}{2}\mathrm{C}q^4 - \tfrac{1}{2}\mathrm{D}q^6 + \ldots) \qquad (10)$$

to include the curvature corrections.

Accuracy of the Coulomb-Nuclear Corrections for Exponential-type models



Left column: Ratios of the real parts of the mixed Coulomb-nuclear corrections calculated using the eikonal model (solid red line) and the exponential model (dashed blue line) to the real part of the exponential model.

Right column: Ratios of the imaginary parts of the mixed Coulomb-nuclear corrections calculated using the eikonal model (solid red line) and the exponential model (dashed blue line) to the imaginary part of the exponential model.

The agreement of the results for the real parts is excellent.

Applications: Fits to High Energy Data

- For ρ(s, q^2) constant, we perform the least squares fits to the data for the differential cross sections (using data up to a maximum value q^2_{max}).
- The total cross section was derived via the optical theorem $\sigma^2_{tot} = \frac{16\pi A}{1+\rho^2}$.
- Below are the results of our fits to the ISR data at W=23.5 GeV, 53 GeV, and 62.3 GeV and to the TOTEM data at 8 TeV and 13 TeV.

$\sqrt{s}$ (GeV)	df	A (mb GeV^{-2})	B (GeV^{-2})	ρ	σ_{tot} (mb)	χ^2/df
13000	96	640.2	20.62	0.07655	111.6	1.424
8000	21	545.	19.94	0.08342	102.9	0.8044
62.3	22	96.5	12.77	0.05612	43.39	1.436
52.8	34	92.52	12.9	0.07891	42.42	2.56
23.5	31	80.12	11.23	0.01987	39.59	0.9132

Applications: Fits to High Energy Data

- The results of our fits are compared with experimental data

$\sqrt{s}$ (GeV)	B (GeV^{-2})	B^{Exp} (GeV^{-2})	ρ	ρ^{Exp}	σ_{tot} (mb)	σ^{Exp}_{tot} (mb)
13000	20.62	20.36 ± 0.19	0.07655	0.09 ± 0.01	111.6	110.5 ± 2.4
8000	19.94	20.47 ± 0.14	0.08342	0.12 ± 0.03	102.9	102.9 ± 2.3
62.3	12.77	13.02 ± 0.27	0.05612	0.095 ± 0.011	43.39	43.55 ± 0.31
52.8	12.9	12.87 ± 0.14	0.07891	0.077 ± 0.009	42.42	42.38 ± 0.15
30.6	12.24	12.2 ± 0.3	0.03586	0.034 ± 0.008	40.17	40.11 ± 0.17
23.5	11.23	11.8 ± 0.3	0.01987	0.022 ± 0.014	39.59	39.65 ± 0.22

IV. Conclusions

- The very simple calculation of the Coulomb-nuclear corrections to the *pp* and $\bar{p}p$ scattering amplitudes using a reasonable model for f_N is quite accurate at high energies.
- The results agrees with those obtained in the comprehensive eikonal model [PRD 92, 014030 (20150); PRD 99, 014009 (2019)].
- This approach provides a substantial improvement in simplicity and clarity over the methods most commonly used at present.

Extra slides. Background

- Consider p-p and pbar-p scattering at high energies. Neglecting the small effects of the nucleon spins, we describe the scattering amplitude and cross sections in an impact parameter.
- The spin-independent eikonal scattering amplitude and differential elastic scattering amplitude are

$$f(s,t) = i\int_0^\infty dbb\,(1-e^{i\chi(b,s)})J_0(b\sqrt{-t}), \qquad (1)$$

$$\frac{d\sigma}{dt}(s,t) = \pi\left|f(s,t)\right|^2. \qquad (2)$$

$s = W^2 = 4(p^2 + m^2)$ - square of total energy in the C.M. system,
p – the C.M. momentum of either incident particle,
b = j/p , j is the partial wave angular momentum,
$t = -2p^2 (1 - \cos\theta)$ – the invariant 4 - momentum transfer.

Extra slides. Eikonal Model

$\chi(b,s) = \chi_R + i\,\chi_I$ - the eikonal function

$$\sigma_{elas}(s) = 2\pi \int_0^\infty dbb \left|1 - e^{i\chi}\right|^2 \quad , \qquad (3)$$

$$\sigma_{tot}(s) = 4\pi \,\mathrm{Im}\, f(s,0) = 4\pi \int_0^\infty dbb\,(1 - \cos\chi_R e^{-\chi_I}), \qquad (4)$$

$$\sigma_{inelas}(s) = \sigma_{tot} - \sigma_{elas} = 2\pi \int_0^\infty dbb\,(1 - e^{-2\chi_I}), \qquad (5)$$

$$\rho = \mathrm{Re}\, f(s,0) / \mathrm{Im}\, f(s,0)$$

$$= -\int_0^\infty dbb\, e^{-\chi_I} \sin\chi_R \Big/ \int_0^\infty dbb\,(1 - \cos\chi_R e^{-\chi_I}), \qquad (6)$$

$$\mathrm{B} = \frac{d}{dt}\left[\ln \frac{d\sigma}{dt}(s,t)\right]$$

$$\approx \frac{1}{2} \int_0^\infty dbb^3 (1 - e^{-\chi_I}) \Big/ \int_0^\infty dbb(1 - e^{-\chi_I}). \qquad (7)$$

Extra slides. Eikonal Fit: An update

$$\chi_{p\bar{p}}(b,W) = (\chi_E(b,W) + \chi_O(b,W))/2\ , \quad (8)$$

$$\chi_{pp}(b,W) = (\chi_E(b,W) - \chi_O(b,W))/2\ . \quad (9)$$

where

$$\chi_E(b,W) = i\left[\sigma_{qq}(\tilde{W})A(b,\mu_{qq}) + \sigma_{qg}(\tilde{W})A(b,\mu_{qg}) + \sigma_{gg}(\tilde{W})A(b,\mu_{gg})\right],\ (10)$$

$$\chi_O(b,W) = -C_5\Sigma_{gg}\left(\frac{m_0}{\tilde{W}}\right)^{2-2\alpha_1} A(b,\mu_{odd})\ . \quad (11)$$

$$\tilde{W} = We^{-i\pi/4}$$

A(b, μ) – overlap functions, σ_{ij} – describing interactions between components i and j.

- We fit data on total cross sections for W ≥ 5.3 GeV and the elastic cross sections, ρ and B for energies W ≥ 10 GeV.
- Fix σ_{tot} at W = 4 GeV to match the results obtained from low-energy data.

Extra slides. Eikonal Fit: An update

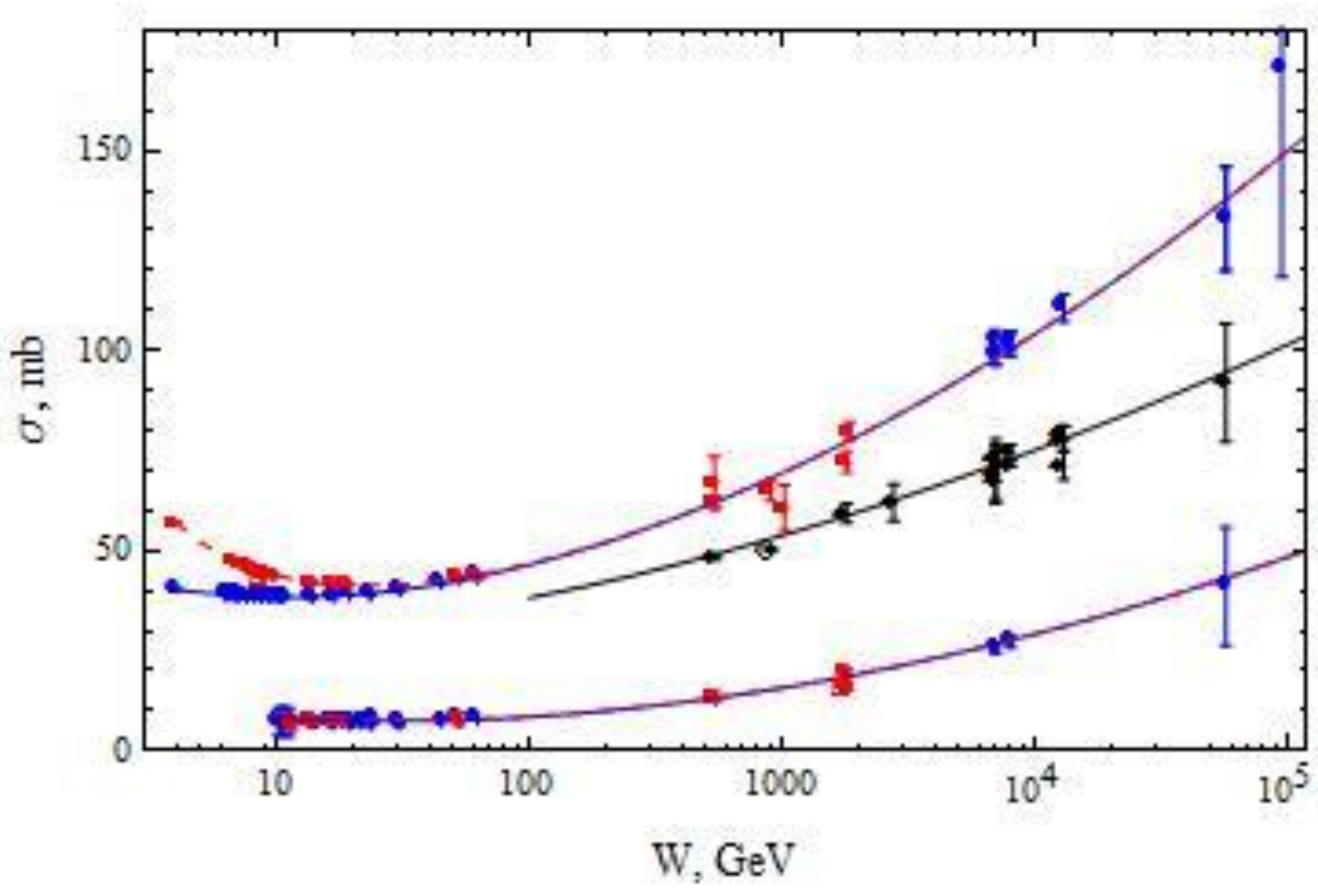
- 9 parameter fit
- 189 datum points
- Seive algorithm eliminates 14 outliers
- dof = 166; χ^2 /dof = 0.985
- $\mathcal{R}$ χ^2 /dof =1.08
- Fixed parameters

$$m_0 = 0.6 \text{ GeV}, W_0 = 4 \text{ GeV},$$
$$\mu_{\text{gg}} = 0.705 \text{ GeV}, \mu_{\text{qq}} = 0.89 \text{ GeV},$$
$$\mu_{\text{odd}} = 0.6 \text{ GeV}, \alpha_{\text{S}} = 0.5,$$
$$\Sigma_{gg} = 19.635 \text{ GeV}^{-2}.$$

- The fitted parameters

$$C_0 = 6.790 \pm 0.07$$
$$C_1 = 26.80 \pm 0.02$$
$$C_2 = -0.187 \pm 0.0004$$
$$C_3 = -2.480 \pm 0.004$$
$$C_4 = 13.75 \pm 0.013$$
$$C_5 = -26.13 \pm 0.02$$
$$\alpha_1 = 0.3188 \pm 0.0003$$
$$\alpha_2 = 0.4866 \pm 0.0001$$
$$\beta_1 = 0.1474 \pm 0.0002$$

Extra slides. Comprehensive Fit: An update



Fits, top to bottom, to the total, inelastic, and elastic scattering cross sections