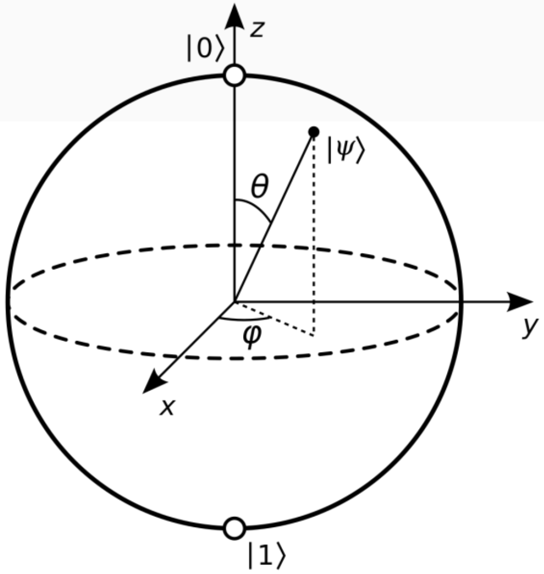
Sachdev-Ye-Kitaev (SYK) model on a noisy quantum computer

Bharath Sambasivam, Syracuse University

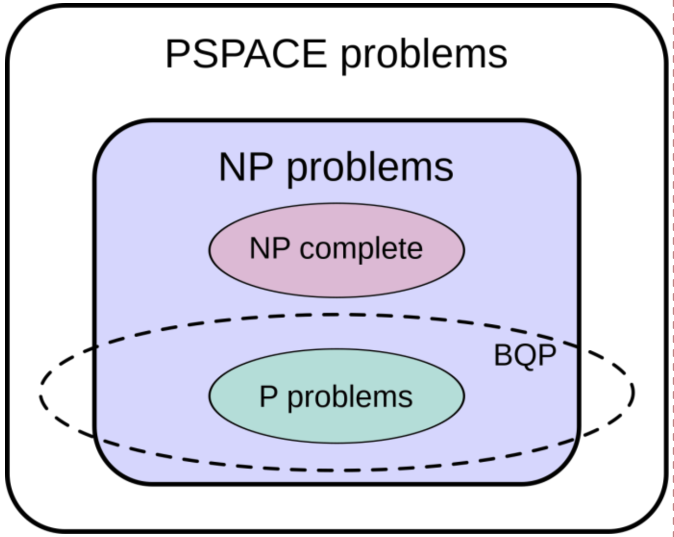
DPF-PHENO 2024

Based on 2311.17991 (PRD, 1 May 2024) with Dr. Muhammad Asaduzzaman (U Iowa), Dr. Raghav Jha (Jefferson Lab)

Quantum Computing- What? and Why?



- Quantum computers use qubits to represent states
- An N qubit state lives in a 2^N dimensional Hilbert space; measurement "collapses" it to one basis state
- "Quantum Advantage" due to superposition + entanglement
- Need to track 2^N complex numbers, classically
- Becomes hard to do for $\sim\mathcal{O}(100)$ qubits



P↔NP

BQP ↔QMA

Image: https://en.wikipedia.org/wiki/Quantum_complexity_theory, https://en.wikipedia.org/wiki/Bloch_sphere

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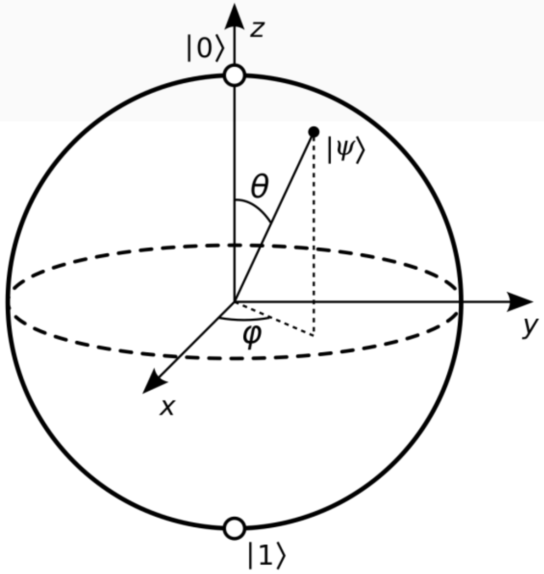

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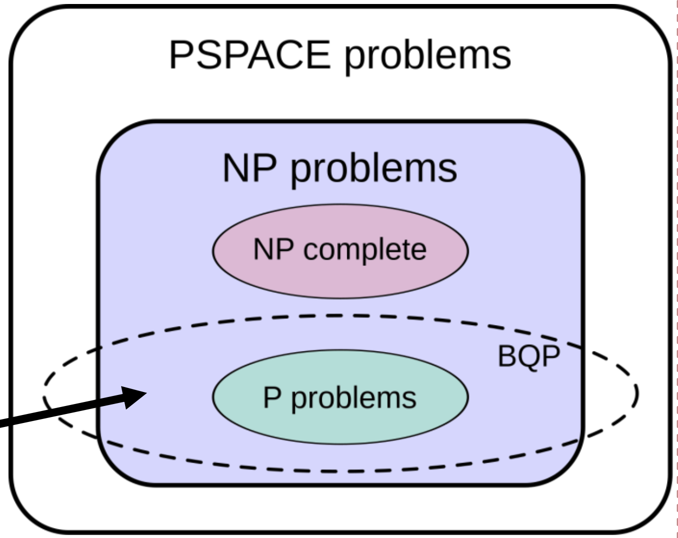
Are there any interesting HEP problems here?



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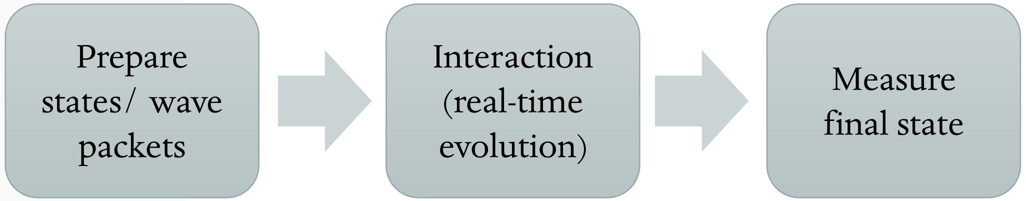
Scattering and Dynamics

- Lattice methods are valuable for computing static observables in field theories even in strong coupling
- Dynamical observables like scattering amplitudes are harder to measure
- Requires 3 ingredients

$$e^{-iHt}$$



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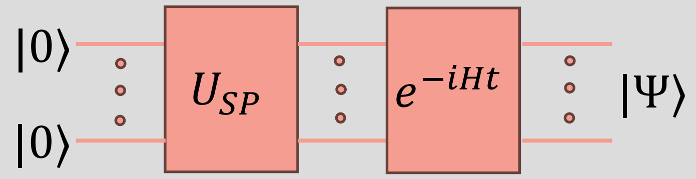
$$e^{-iHt}$$

Prepare states/ wave packets → Interaction (real-time evolution) → Measure final state

Operator	Gate(s)	Matrix
Pauli-X (X)	X	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Pauli-Y (Y)	Y	$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$
Pauli-Z (Z)	Z	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Hadamard (H)	H	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
Phase (S, P)	S	$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$
$\pi/8$ (T)	T	$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$
Controlled Not (CNOT, CX)		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$
Controlled Z (CZ)	Z	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$
SWAP		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
Toffoli (CCNOT, CCX, TOFF)		$\begin{bmatrix} 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&1&0 \end{bmatrix}$

On a Quantum Computer



- Dynamics of non-relativistic many-body systems is in **BQP**
- **Jordan, Lee, Preskill** showed advantage for scalar ϕ^4 scattering

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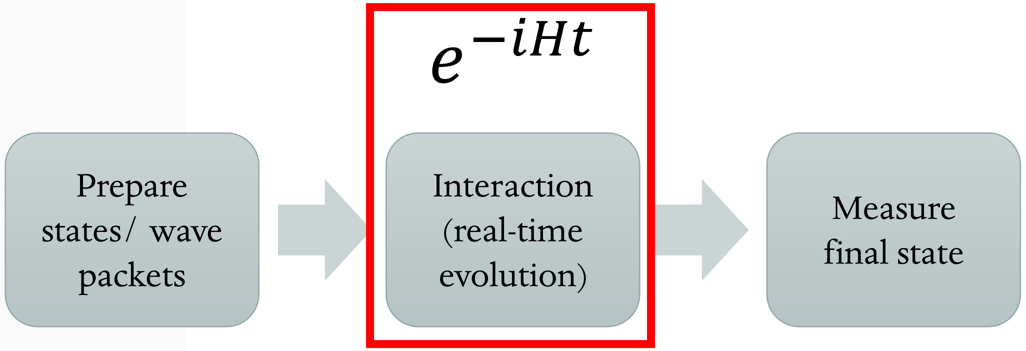
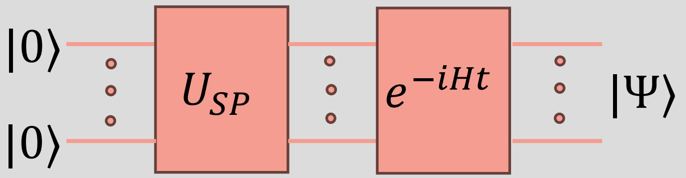


Focus of this talk for SYK model

Ingredient 0: Writing down digitized Hamiltonian

On a Quantum Computer



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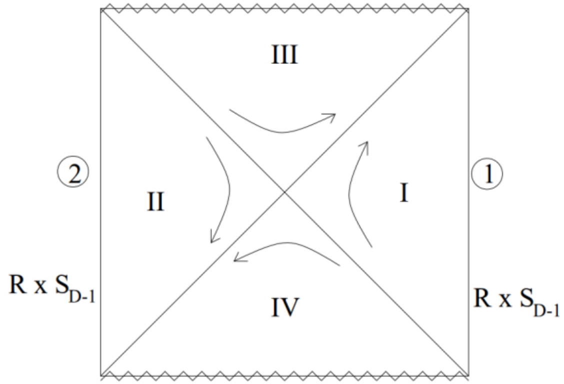
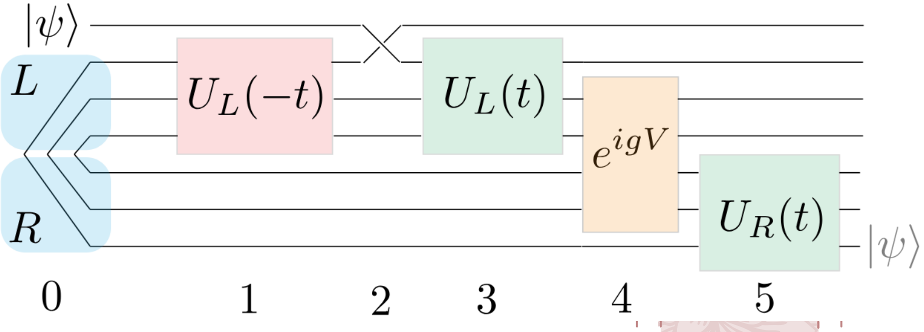
Sachdev-Ye-Kitaev (SYK) model



- SYK is a (0+1)D quantum mechanical model of N_f Majorana fermions $\{\chi_i, \chi_j\} = \delta_{i,j}$

$$H = \frac{1}{4!}\sum_{i,j,k,l} J_{ijkl}\, \chi_i \chi_j \chi_k \chi_l \qquad \sigma = \sqrt{\frac{3!J^2}{N_f^3}}$$

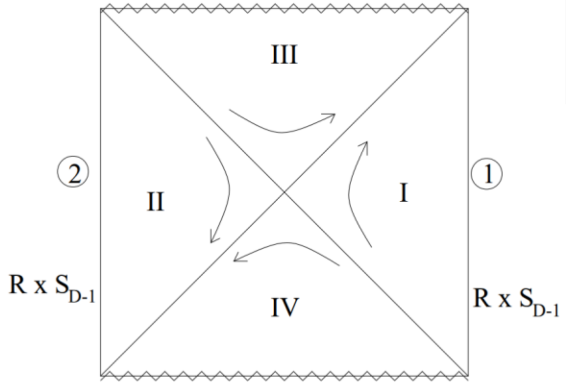
- Time reversal symmetry $\mathcal{T}\chi_a\mathcal{T}^{-1} = \chi_a$; Fermion number (mod 2) operator symmetry
- $N_f \to \infty, \beta J \gg 1$, the model develops and approximate conformal symmetry (time reparametrization)
- It is a chaotic system for $N_f > 4$
- Has been used as a mean field model for non-Fermi liquids
- This is related to near AdS_2 gravity and eternal blackholes via holography!



Goal: Do e^{-iHt} for SYK model

Papers: 1503.01409, 1908.11351, 2205.14081
Image: https://arxiv.org/abs/hep-th/0106112

Sachdev-Ye-Kitaev (SYK) model



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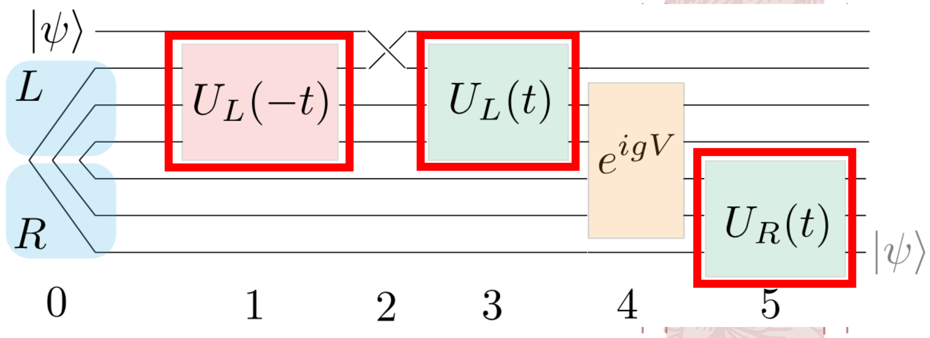

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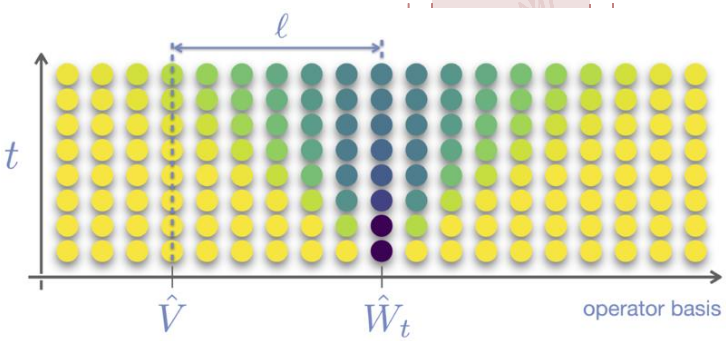
Scrambling and OTOCs

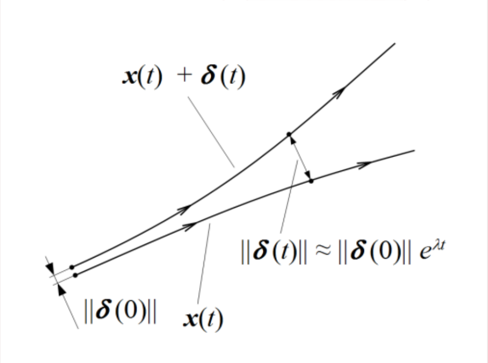


Scrambling: The quantum information stored between a small number of d.o.f's gets spread to an exponentially large number of d.o.f's

- Chaos in quantum mechanical systems can be quantified by the Out-of-Time-Ordered Correlator (OTOC)

$$O(t) = -\langle [W(t), V(0)]^\dagger [W(t), V(0)] \rangle = 2\left(1 - \langle W^\dagger(t) V^\dagger(0) W(t) V(0) \rangle\right)$$

- The OTOC grows as $e^{\lambda_L t}$ when $t_d < t < t_*$ for chaotic systems, and saturates beyond
- Chaos bound conjecture: $\lambda_L \leq \frac{2\pi k_B T}{\hbar}$ for quantum mechanical systems at finite temperature T
- This is saturated by blackholes, which have a temperature $T_H = \frac{\hbar c^3}{8\pi G M k_B}$
- The low-temperature, large N_f SYK model also saturates this bound, making it interesting to study



Trotter evolution

This work

2-qubit gate Complexity	Work
$\mathcal{O}(N^{10}t^2/\epsilon)$	1607.08560 (2016)
$\mathcal{O}(N^{8}t^2/\epsilon)$	2008.02303 (2020)
$\mathcal{O}(N^{5}t^2/\epsilon)$	2311.17991 (2023)

$$H = \sum_j \alpha_j \prod_i \sigma_i^j$$

$$e^{-iHt} = \left(\prod_j e^{-i\prod_i \frac{\sigma_i^j t}{r}}\right)^r + \mathcal{O}\left(\sum_{j<k} \left\|\left[\prod_i \sigma_i^j, \prod_i \sigma_i^k\right]\right\| t^2/r^2\right)$$

- Gate costs can be reduced by grouping Pauli terms into commuting clusters

$$H = \sum_i H_i \qquad [H_i, H_j] \neq 0 \quad \text{for} \quad i \neq j$$

$$H_i = \sum_j \alpha_j \prod_k \sigma_k^j \qquad \left[\prod_k \sigma_k^j, \prod_l \sigma_l^m\right] = 0 \;\forall\, m, j$$

- Then exponentiate each piece and diagonalize them simultaneously

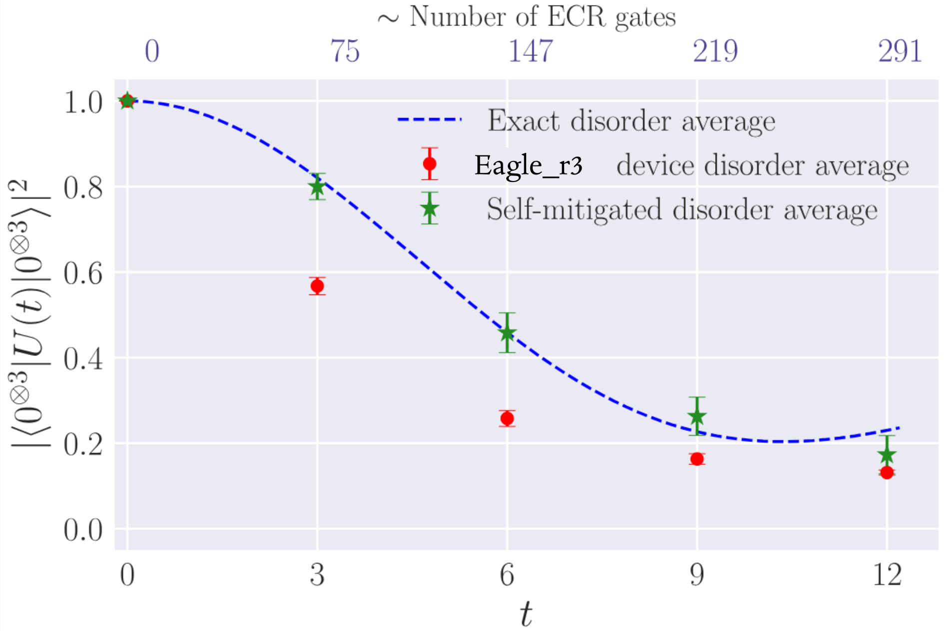
$$e^{-iHt} = e^{-iHr\delta t} \approx \left(\prod_i e^{-iH_i\delta t}\right)^r$$

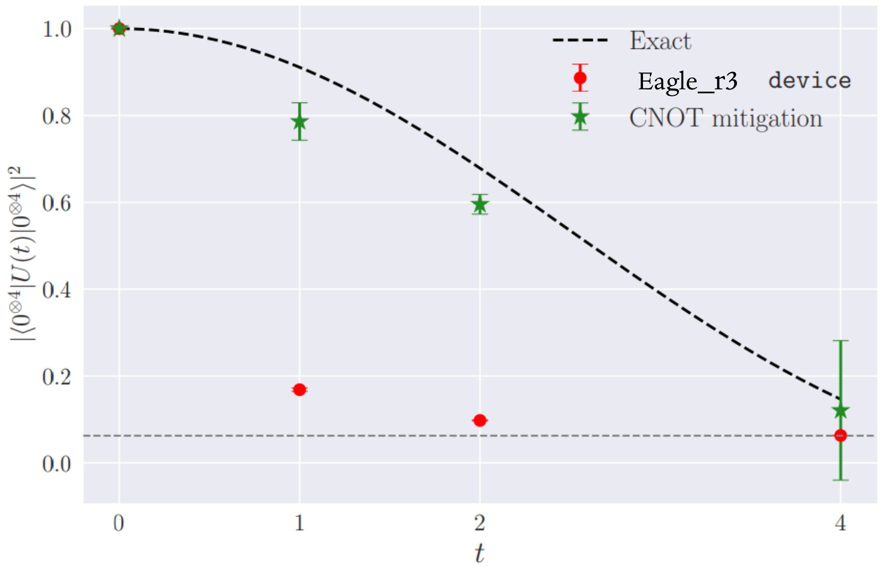
N_f	Pauli Strings	Clusters	2-qubit gates
4	1	1	2
6	15	5	30
8	70	6	110
10	210	23	498
12	495	57	1504
14	1001	92	3560
16	1820	116	6812
18	3060	175	11962
20	4845	246	19984

Effective trotter parameter for SYK is $\sigma\ \delta t \sim J\delta t/N_f^{3/2}$. So, we take $\delta t \sim \mathcal{O}(1)$

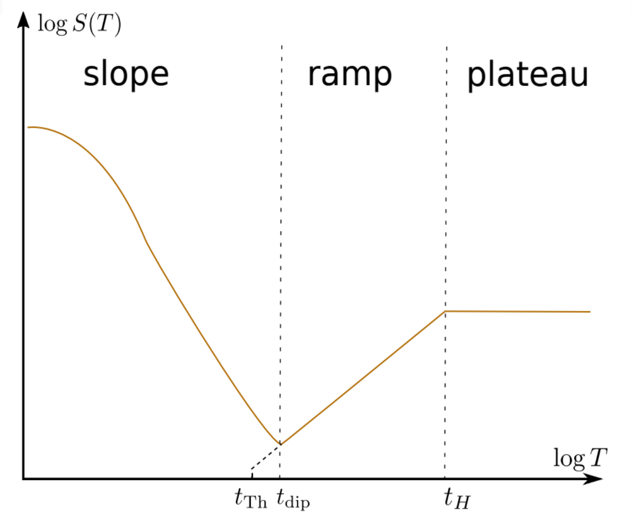
Paper: https://dl.acm.org/doi/10.1145/359094.359101
Paper: 2305.11847

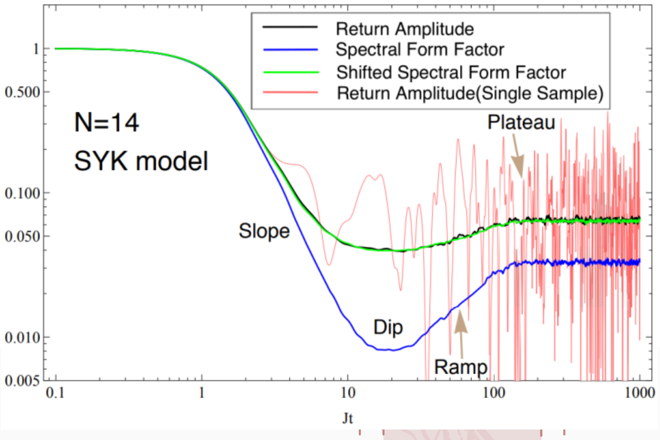
Return Probability



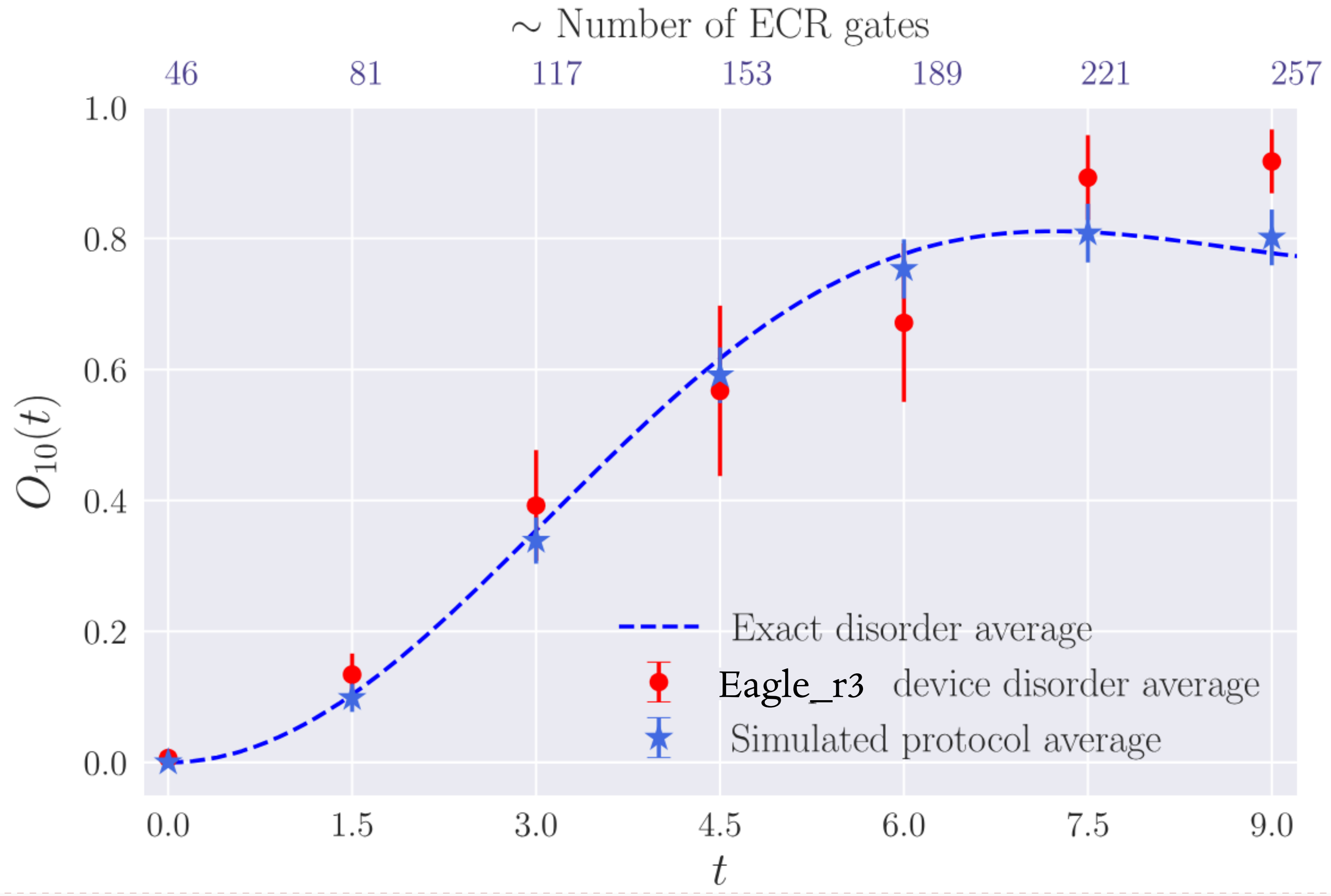


$N_f = 6$ (3 qubits) $N_f = 8$ (4 qubits)





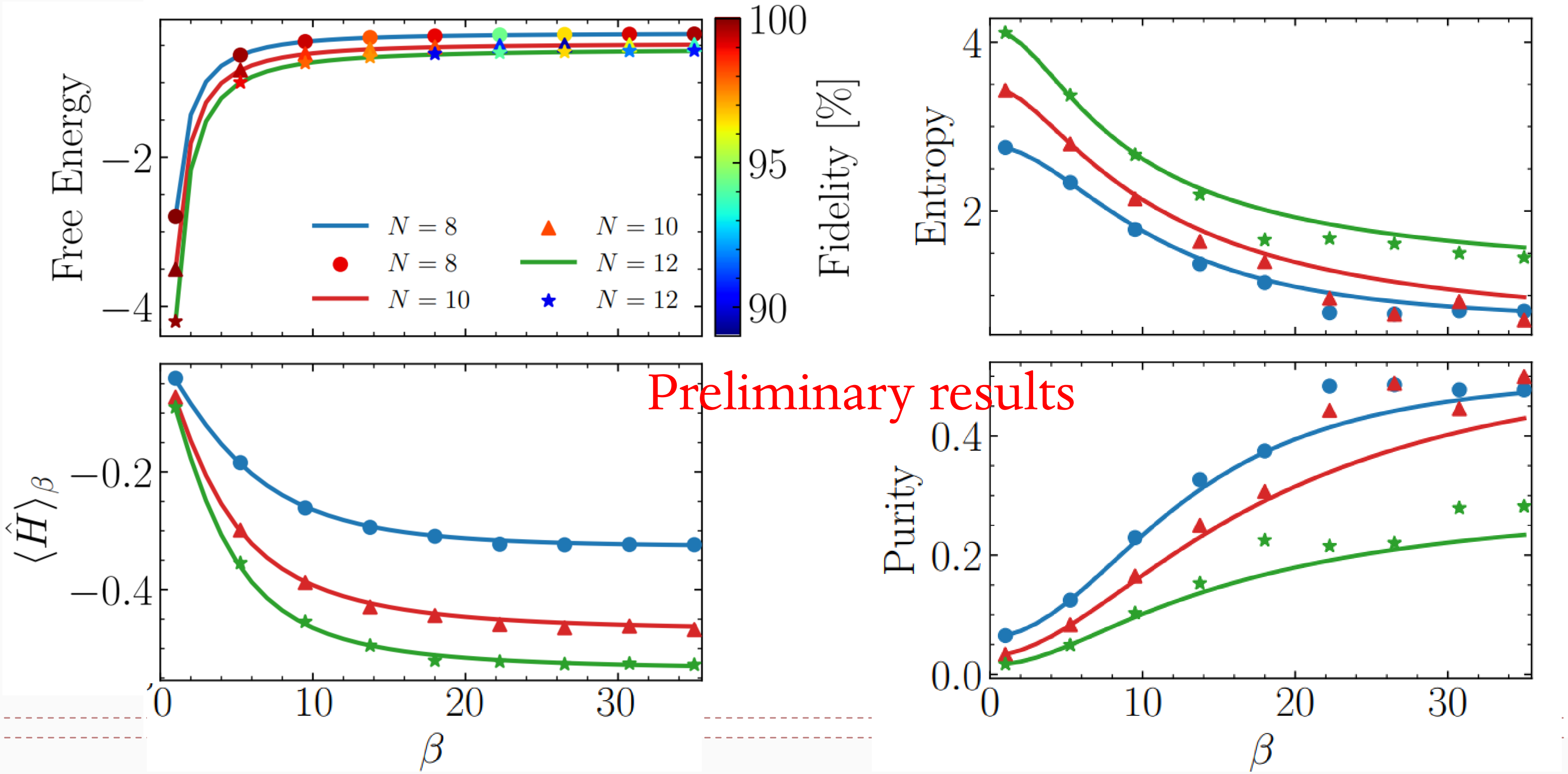
OTOC ($N_f = 6$)


~ Number of ECR gates
46
81
117
153
189
221
257
1.0
0.8
0.6
0.4
0.2
0.0
$O_{10}(t)$
0.0
1.5
3.0
4.5
6.0
7.5
9.0
t
Exact disorder average
Eagle_r3 device disorder average
Simulated protocol average

Conclusions and Outlook

- The SYK model is a simple fermionic model that is interesting for
  - Holography
  - Chaos
- We studied the SYK model on quantum computers and propose an improved scaling of $\mathcal{O}(N^5 t^2/\epsilon)$ using graph-coloring techniques
- We did hardware simulation of small systems of size $N_f = 6, 8$ and used error mitigation techniques
- We see great agreement with exact evolution for both the return probability and the OTOC
- Non-trotter-based approaches such as variational methods and Cartan tricks can also be explored
- Ongoing work: Studying finite-temperature Gibbs state preparation for the SYK model

Thermal Gibbs States- simulator results



Preliminary results

2405.XXXX with Jack Araz, Raghav Jha, Felix Ringer

Thank You

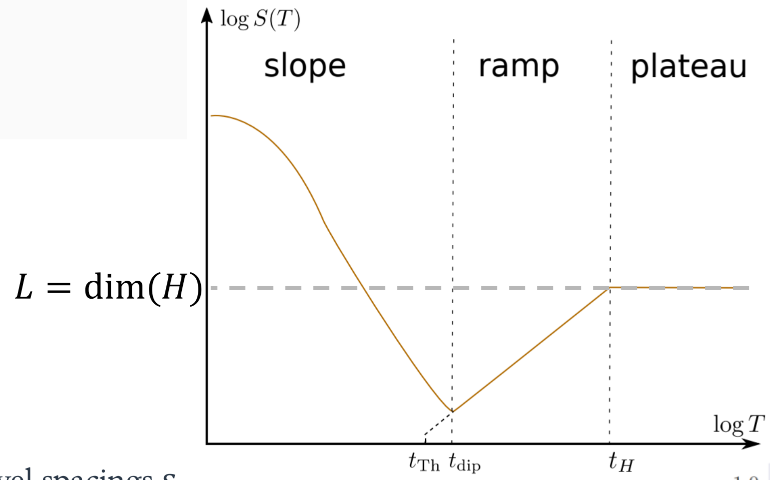
bsambasi@syr.edu

2311.17991 (PRD, 1 May 2024), 2405.XXXX

https://zenodo.org/records/10202045

Backup slides

Features of SYK



- Diagnose chaos by looking at nearest neighbor energy level spacings s
  - Integrable: Follows e^{-s}
  - Chaotic: Follow GUE, GOE, or GSE depending on symmetries. All 3 realizable in SYK!
- The spectral form factor has interesting features at characteristic time scales

$$S(t) = \frac{\langle Z(it)Z(-it)\rangle}{\langle Z(0)\rangle^2}$$

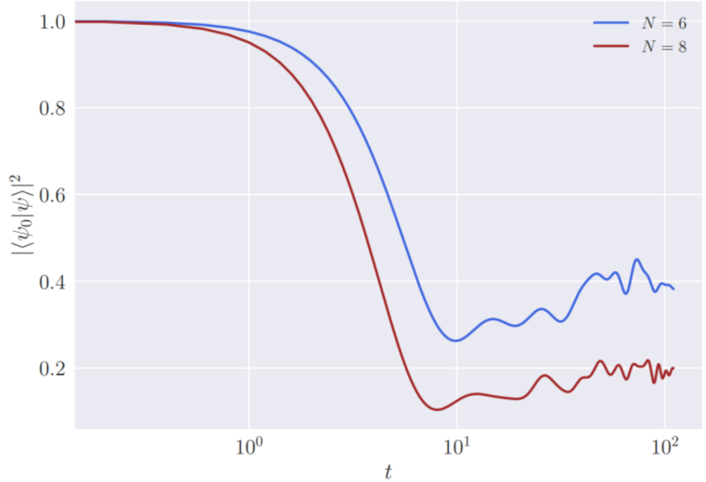
- In this work, we compute the return probability

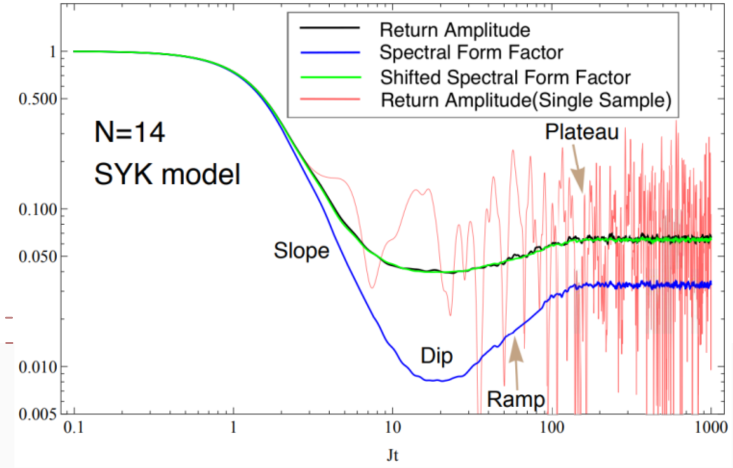
$$p_0 = \left|\langle\psi_0|e^{-iHt}|\psi_0\rangle\right|^2$$

- In the semi-classical regime, they have the same features

$$\left\langle\left|\langle\psi_0|e^{-iHt}|\psi_0\rangle\right|^2\right\rangle_{GUE} = \int dH\, e^{-\frac{L}{2}Tr\, H^2}\left|\langle\psi_0|e^{-iHt}|\psi_0\rangle\right|^2 = \frac{1}{L(L+1)}\left(\langle S(t)\rangle_{GUE} + L\right)$$

$N_f mod\ 8$	Ensemble
2 or 6	GUE
0	GOE
4	GSE





Paper: 1902.02025

Digitization

Jordan-Wigner mapping

$$\chi_{2k-1} = \frac{1}{\sqrt{2}}\left(\prod_{j=1}^{k-1} Z_j\right) X_k \mathbb{1}^{\otimes(N-2k)/2} \qquad \chi_{2k} = \frac{1}{\sqrt{2}}\left(\prod_{j=1}^{k-1} Z_j\right) Y_k \mathbb{1}^{\otimes(N-2k)/2}$$

- N_f Majorana fermions requires $\frac{N_f}{2}$ qubits using the Jordan-Wigner mapping to Pauli Strings
- The number of Pauli strings is then $\binom{N_f}{4}$ and grows like $\sim N^q$

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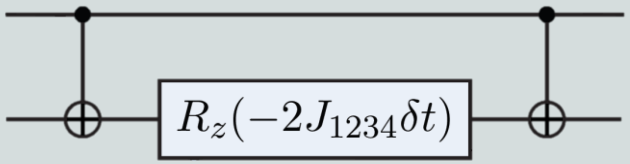
$N_f = 4$

$$H_4 = J_{1234}\, \chi_1 \chi_2 \chi_3 \chi_4$$
$$\chi_1 = X\mathbb{1}, \chi_2 = Y\mathbb{1}, \chi_3 = ZX, \chi_4 = ZY$$
$$H_4 = J_{1234}(X\mathbb{1}).(Y\mathbb{1}).(ZX).(ZY) = -J_{1234}ZZ$$

$$e^{-iH_4\delta t} \equiv$$

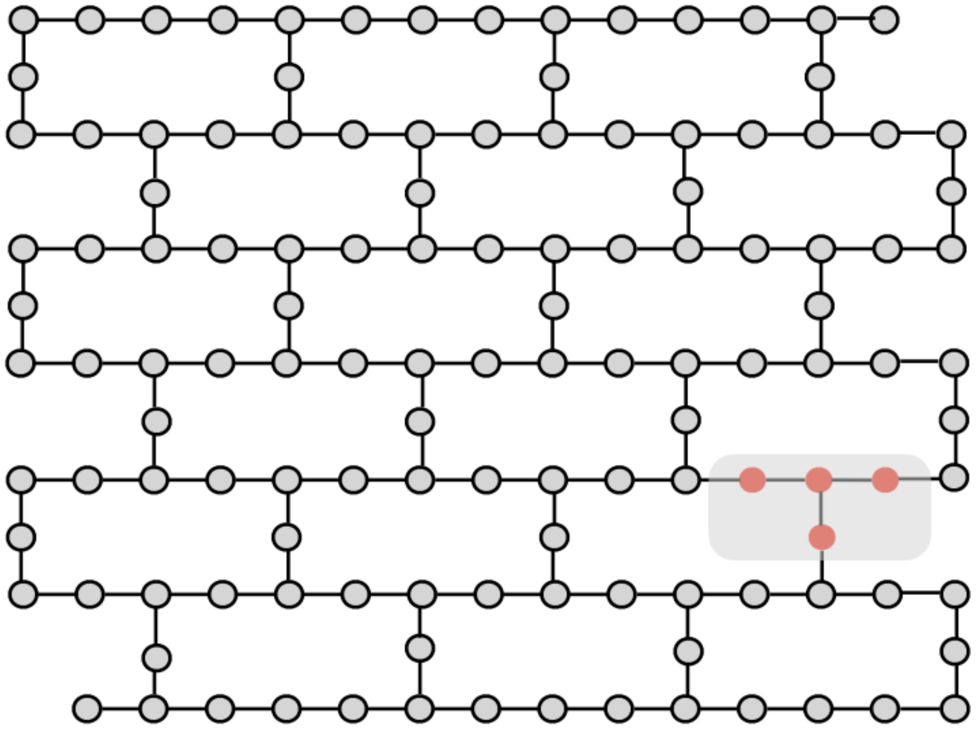


IBM processors

- We use eagle_r3 processor from IBM
- Basis gates: ECR, ID,RZ, SX, X

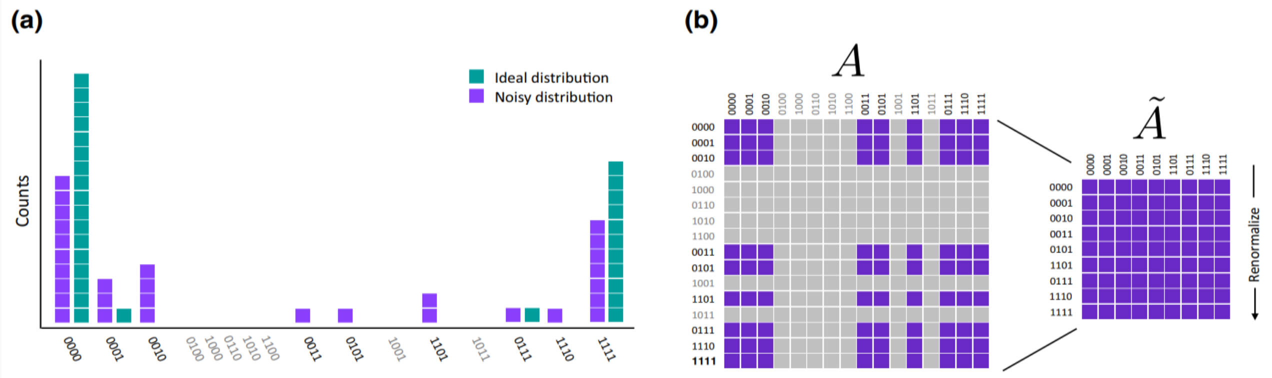
$$ECR \equiv \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 1 & -i \\ 0 & 0 & -i & 1 \\ 1 & i & 0 & 0 \\ i & 1 & 0 & 0 \end{pmatrix}$$

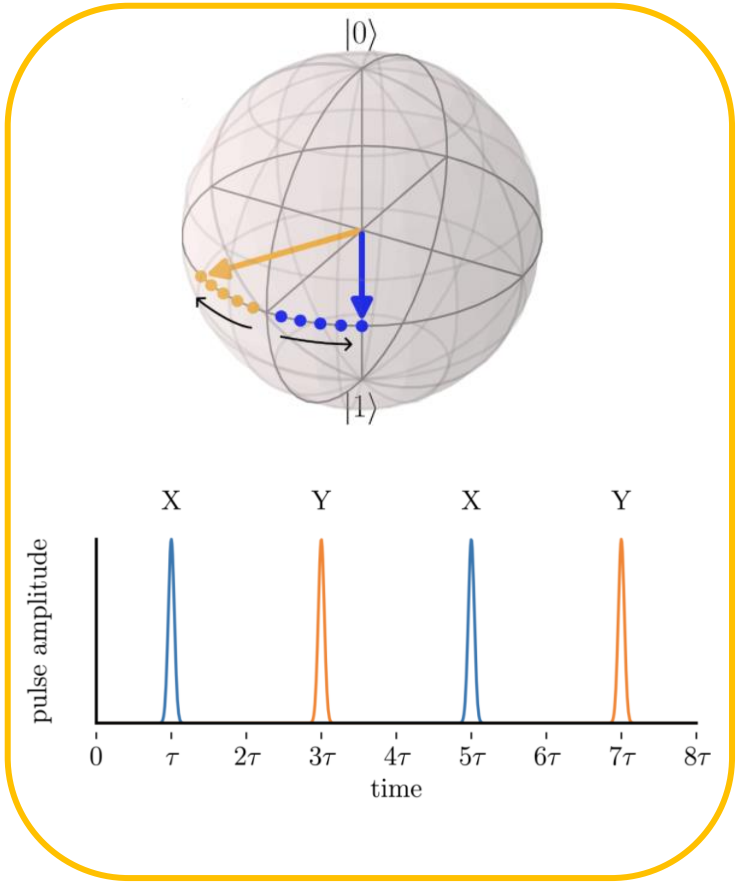
- Fixed chip topology requires extra SWAP gates
- Gate application times are short



Error Mitigation Recipe (Return Probability)

- M3 protocol- For readout errors
- Dynamical decoupling- For decoherence errors
- Pauli Twirling- For converting coherent errors to incoherent/ stochastic errors, described by the depolarizing channel
- Mitigation Circuits/ Self-Mitigation- For depolarizing noise





Papers: 2108.12518, 2207.03670

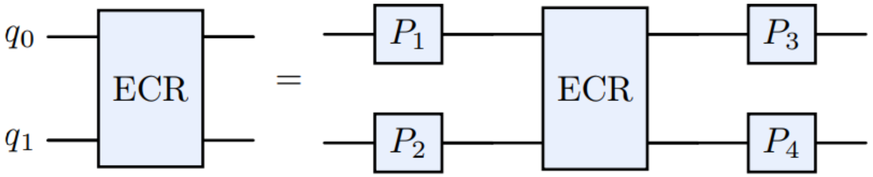
Image: https://aws.amazon.com/blogs/quantum-computing/suppressing-errors-with-dynamical-decoupling-using-pulse-control-on-amazon-braket/

Pauli Twirling with ECR gates

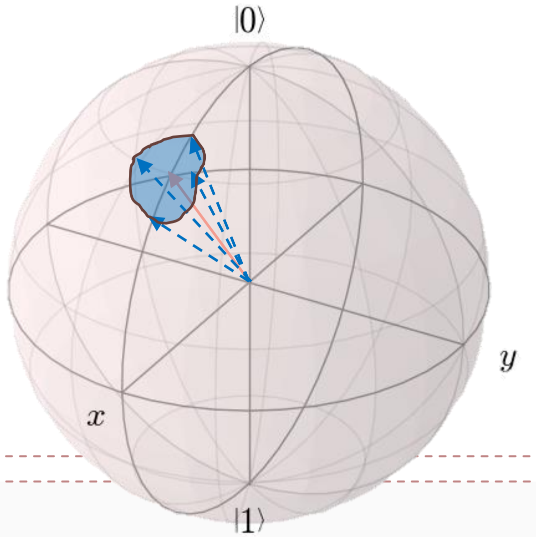
- 16 single-qubit conjugations leave the ECR gate invariant
- `Twirl' each ECR gate using a randomly chosen conjugation
- Usual approach- 1 physics circuit with N_s shots
- Instead, do n randomly twirled physics circuits with $\frac{N_s}{n}$ shots
- Each circuit has a different trajectory to the same final state in the absence of noise

Coherent Noise⟶Stochastic Noise

$$\mathcal{E}(\rho) = (1-p)\rho + \frac{p}{2}\mathbb{I}$$



P_1	$\mathbb{1}$	X	X	$\mathbb{1}$	Y	Z	Z	Y	Z	Y	$\mathbb{1}$	$\mathbb{1}$	Y	Z	X	X
P_2	$\mathbb{1}$	$\mathbb{1}$	Z	Z	X	X	Y	Y	$\mathbb{1}$	$\mathbb{1}$	X	Y	Z	Z	Y	X
P_3	$\mathbb{1}$	X	X	$\mathbb{1}$	Y	Z	Z	Y	Y	Z	X	X	Z	Y	$\mathbb{1}$	$\mathbb{1}$
P_4	$\mathbb{1}$	$\mathbb{1}$	Z	Z	X	X	Y	Y	Z	Z	Y	X	$\mathbb{1}$	$\mathbb{1}$	X	Y



Papers: 1512.01098

Depolarizing Error

- Good noise model for stochastic noise is the depolarizing quantum channel

$$\mathcal{E}(\rho) = (1-p)\rho + \frac{p}{2}\mathbb{I}$$

- Given error probability p we can reconstruct noiseless expectation value of

$$\mathcal{O} = \sum_j c_j \prod_i \sigma_i^j = c_0\mathbb{I} + \mathcal{O}'$$

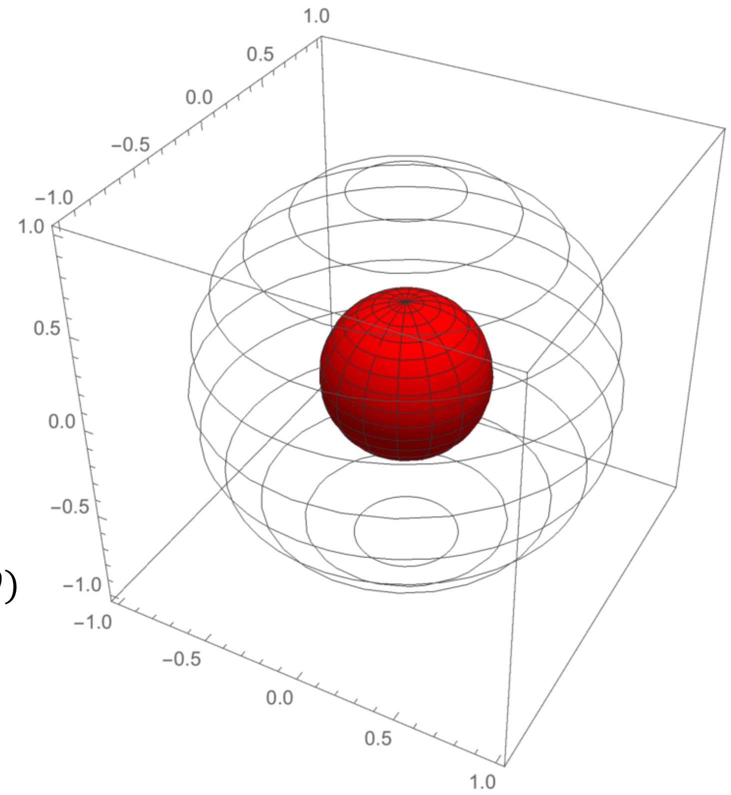
- Noisy expectation value

$$\overline{\langle\mathcal{O}\rangle} = Tr(\mathcal{E}(\rho)\mathcal{O}) = (1-p)\langle\mathcal{O}\rangle + \frac{p}{2^N}Tr(\mathcal{O})$$

- We can extract the noiseless expectation value

$$\langle\mathcal{O}\rangle = \frac{\overline{\langle\mathcal{O}\rangle} - c_0 p}{(1-p)}$$

How do you estimate p?



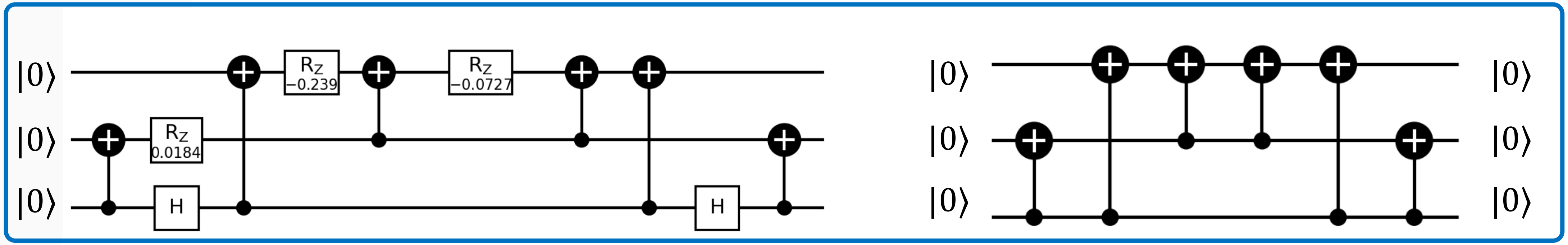
Papers: 2103.08591

Image: https://www.researchgate.net/publication/326638661_An_Invitation_to_Quantum_Channels

Mitigation Circuits

Broad idea: Construct mitigation circuits with a known outcome that have a similar structure to the physics circuit

Physics circuit | Mitigation circuit



CNOT-only mitigation

$$p = 1 - p_{|0\rangle}$$

Self-mitigation

Papers: 2205.09247, 2103.08591

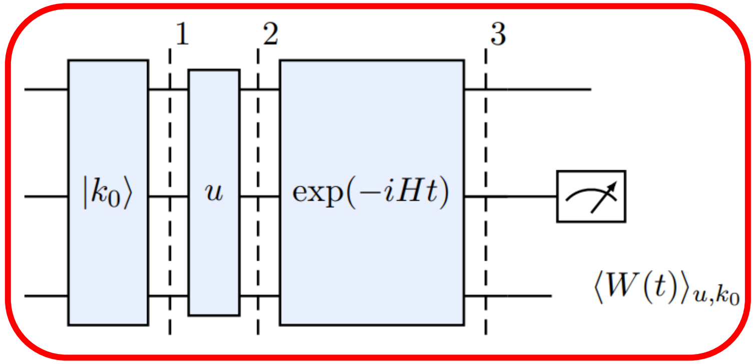
OTOC local protocol

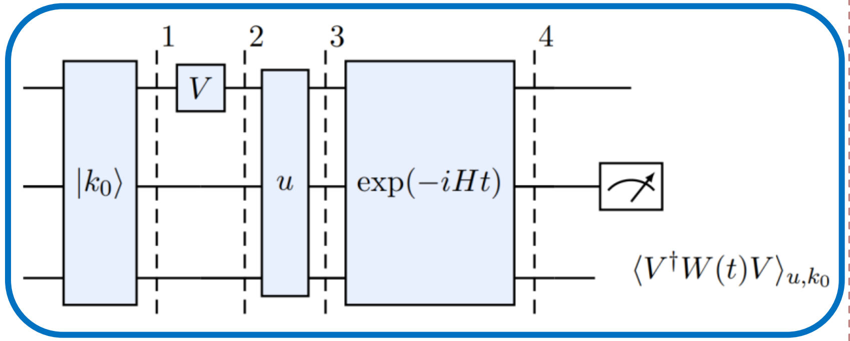
- We use a randomized local protocol to compute OTOC
- Compute OTOC through statistical correlations of observables measured on random states sampled from a distribution

$$O(t) \sim Tr\left(W(t)V^{\dagger}(0)W(t)V(0)\right)$$

$$O(t) \sim \frac{\overline{\langle W_i(t)\rangle_{u,k_0}\left\langle V_j^{\dagger}W_i(t)V_j\right\rangle_{u,k_0}}}{\overline{\langle W_i(t)\rangle_{u,k_0}^2}}$$

- Replace the trace with an average over random states
- Resilient to depolarizing noise! Why? Average over random states, ratio of observables





quantum Variational Quantum Thermalizer (qVQT)

$$\rho_{VQC1} = \left(\sum_i a_i(\vec{\phi})|b_i\rangle\right)\left(\sum_j a_j^*(\vec{\phi})\langle b_j|\right)$$

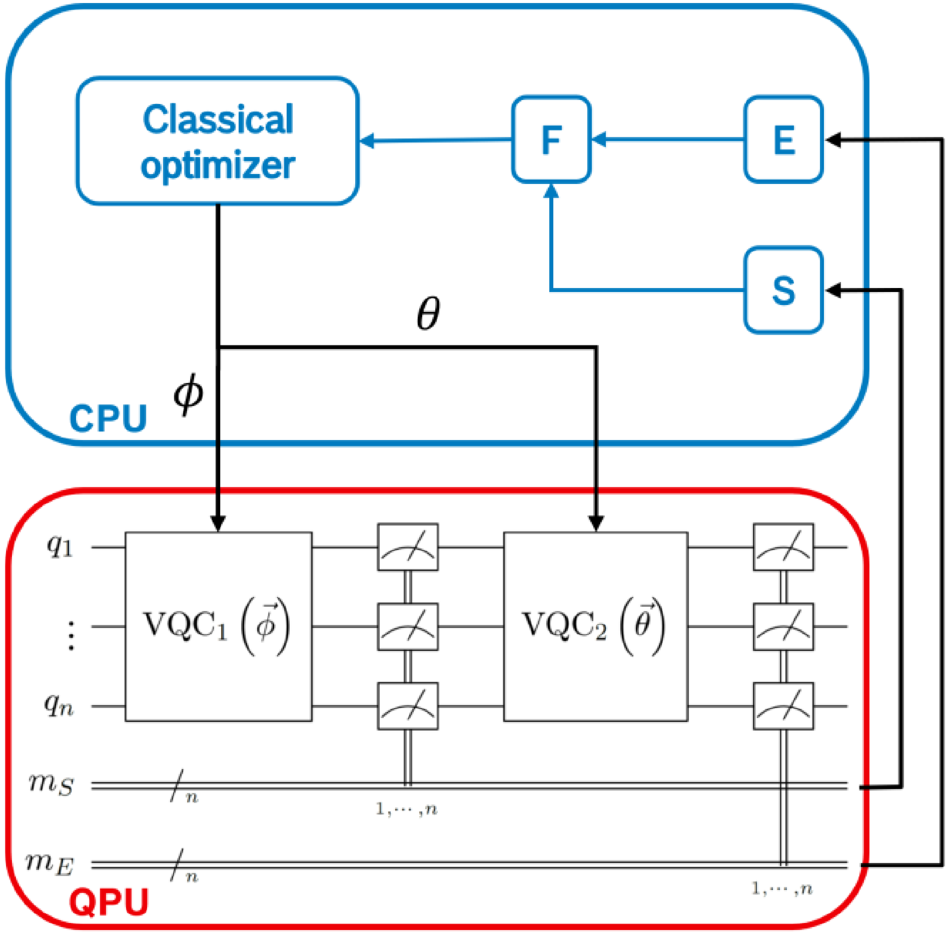
$$\rho_{mm} = \sum_i |a_i(\vec{\phi})|^2 |b_i\rangle\langle b_i| \qquad S = \sum_i |a_i(\vec{\phi})|^2 \log|a_i(\vec{\phi})|^2$$

$$\rho_{VQC2} = \sum_i |a_i(\vec{\phi})|^2 |\psi_i(\vec{\theta})\rangle\langle\psi_i(\vec{\theta})|$$

$$E = \langle H\rangle = Tr(\rho_{VQC2}H) = \sum_i |a_i(\vec{\phi})|^2 \langle\psi_i(\vec{\theta})|H|\psi_i(\vec{\theta})\rangle$$

$$F = E - TS$$

Minimize the Free Energy over $(\vec{\phi}, \vec{\theta})$



paper: https://arxiv.org/pdf/2208.07621