Goldstone boson equivalence theorems for KK gravitons in warped five-dimensional theories

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Based on arXiv: 2312.08576 and 2207.02887, arXiv: 2311.00770)

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Energy growth in unitary gauge

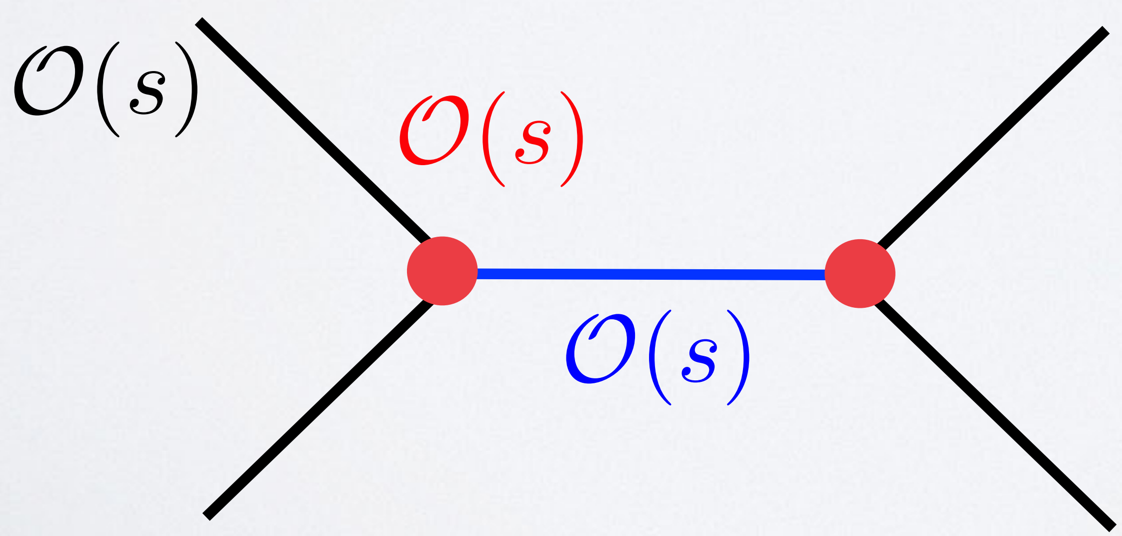
- Massive spin-2 propagator

$$\mathcal{P}_h^{\mu\nu\rho\sigma} = \frac{i}{p^2 - m^2}\frac{1}{2}\left[\left(\eta^{\mu\rho} - \frac{p^\mu p^\rho}{m^2}\right)\left(\eta^{\nu\sigma} - \frac{p^\nu p^\sigma}{m^2}\right) + \cdots\right] \sim \mathcal{O}(E^2/m^2)$$

- Longitudinal polarization tensor

$$\epsilon_0^{\mu\nu} = \frac{1}{\sqrt{6}}\left(\epsilon_+^\mu \epsilon_-^\nu + \epsilon_-^\mu \epsilon_+^\nu + 2\epsilon_0^\mu \epsilon_0^\nu\right) \sim \mathcal{O}(E^2/m^2)$$

- Ricci scalar contains two derivatives: vertices $\sim \mathcal{O}(E^2)$



$$\xrightarrow{?} \mathcal{O}(s)$$

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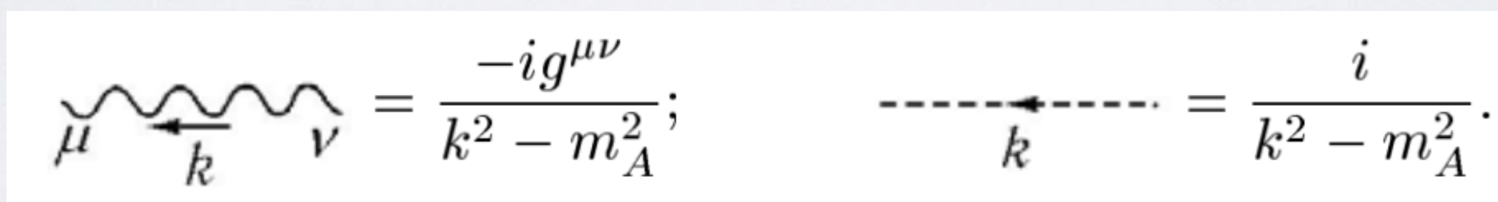
't Hooft-Feynman Gauge

- As in the gauge theories, power counting is much more transparent in the 't Hooft-Feynman gauge.
- Goldstone modes have the same masses as the gauge bosons.



- In warped extra dimension model, KK masses are determined by transcendental equations

$$\mathrm{Y}_1(m_n z_2)\mathrm{J}_1(m_n z_1) - \mathrm{J}_1(m_n z_2)\mathrm{Y}_1(m_n z_1) = 0$$

- How come Goldstones have the same non-trivial masses?

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Quantum Mechanical SUSY

- For two Hamiltonians

$$H_1 = A^\dagger A \qquad\qquad H_2 = AA^\dagger$$

- SUSY doublet $\Psi = (\psi_1, \psi_2)^T$, with supercharges

$$Q = \begin{pmatrix} 0 & 0 \\ A & 0 \end{pmatrix}, \qquad Q^\dagger = \begin{pmatrix} 0 & A^\dagger \\ 0 & 0 \end{pmatrix}$$

$$\begin{aligned} H &= \{Q, Q^\dagger\}, \\ \{Q, Q\} &= \{Q^\dagger, Q^\dagger\}, \\ [Q, H] &= [Q^\dagger, H] = 0, \\ [(-1)^F, H] &= 0, \\ \{Q, (-1)^F\} &= \{Q^\dagger, (-1)^F\} = 0. \end{aligned}$$

- They have degenerate eigenvalues, except for the ground state

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Metric Perturbation in Feynman Gauge

- 5D Metric in conformal coordinates

$$G_{MN} = e^{2A(z)} \begin{pmatrix} e^{-\kappa\hat{\varphi}/\sqrt{6}}(\eta_{\mu\nu} + \kappa\hat{h}_{\mu\nu}) & \frac{\kappa}{\sqrt{2}}\hat{A}_\mu \\ \frac{\kappa}{\sqrt{2}}\hat{A}_\mu & -\left(1 + \frac{\kappa}{\sqrt{6}}\hat{\varphi}\right)^2 \end{pmatrix}$$

$$A(z) = -\ln(kz)$$

- KK decomposition

$$h_{\mu\nu}(x^\alpha, z) = \sum_{n=0}^{\infty} h_{\mu\nu}^{(n)}(x^\alpha) f^{(n)}(z)$$

$$A_\mu(x^\alpha, z) = \sum_{n=1}^{\infty} A_\mu^{(n)}(x^\alpha) g^{(n)}(z)$$

$$\varphi(x^\alpha, z) = \sum_{n=0}^{\infty} \varphi^{(n)}(x) k^{(n)}(z)$$

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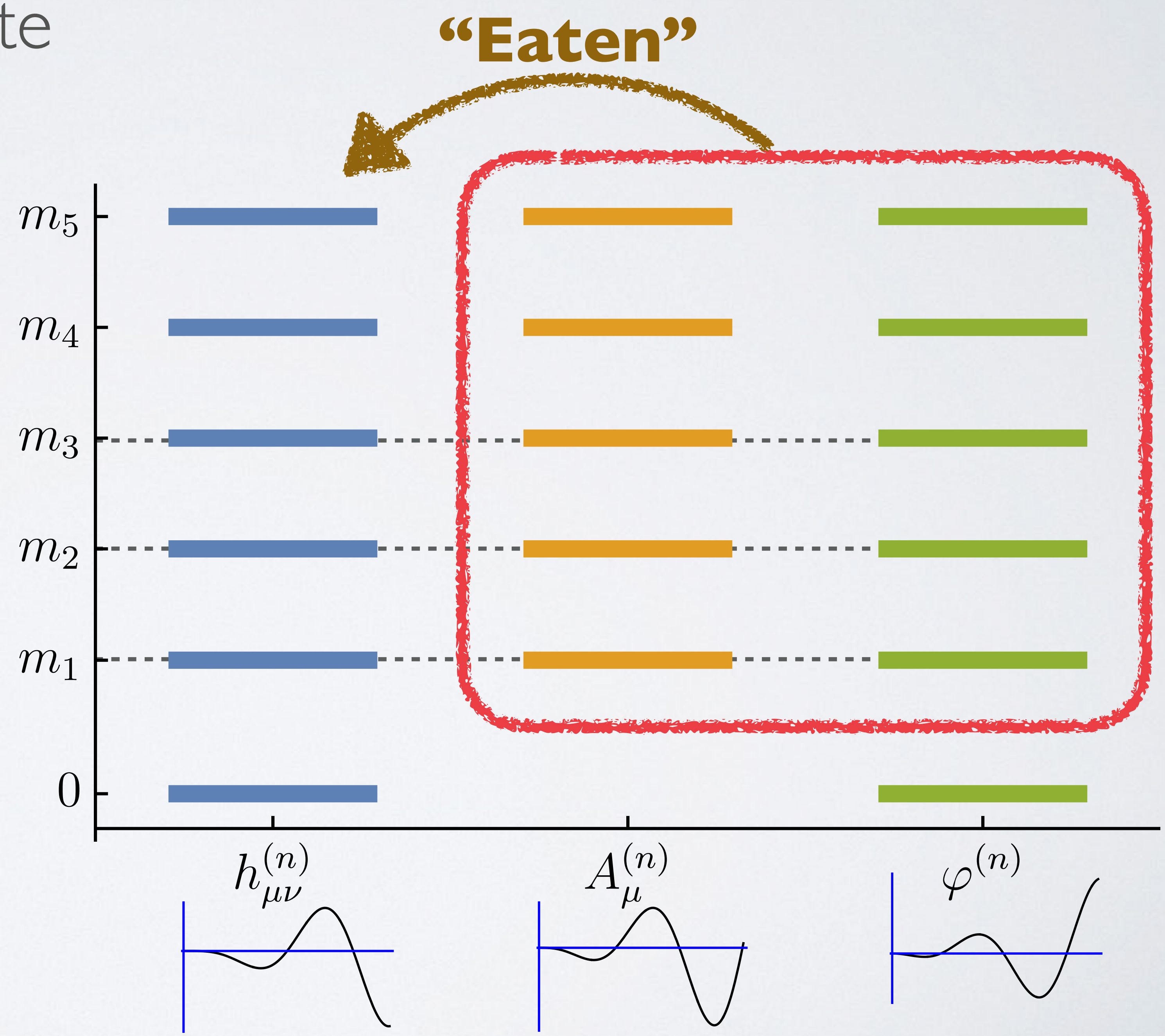
KK spectra in Feynman gauge

- $h_{\mu\nu}^{(n)}$, $A_\mu^{(n)}$, and $\varphi^{(n)}$ have degenerate spectra, except for the massless state

$h_{\mu\nu}^{(n)}$ and $A_\mu^{(n)}$ form a SUSY pair

$A_\mu^{(n)}$ and $\varphi^{(n)}$ form another SUSY pair



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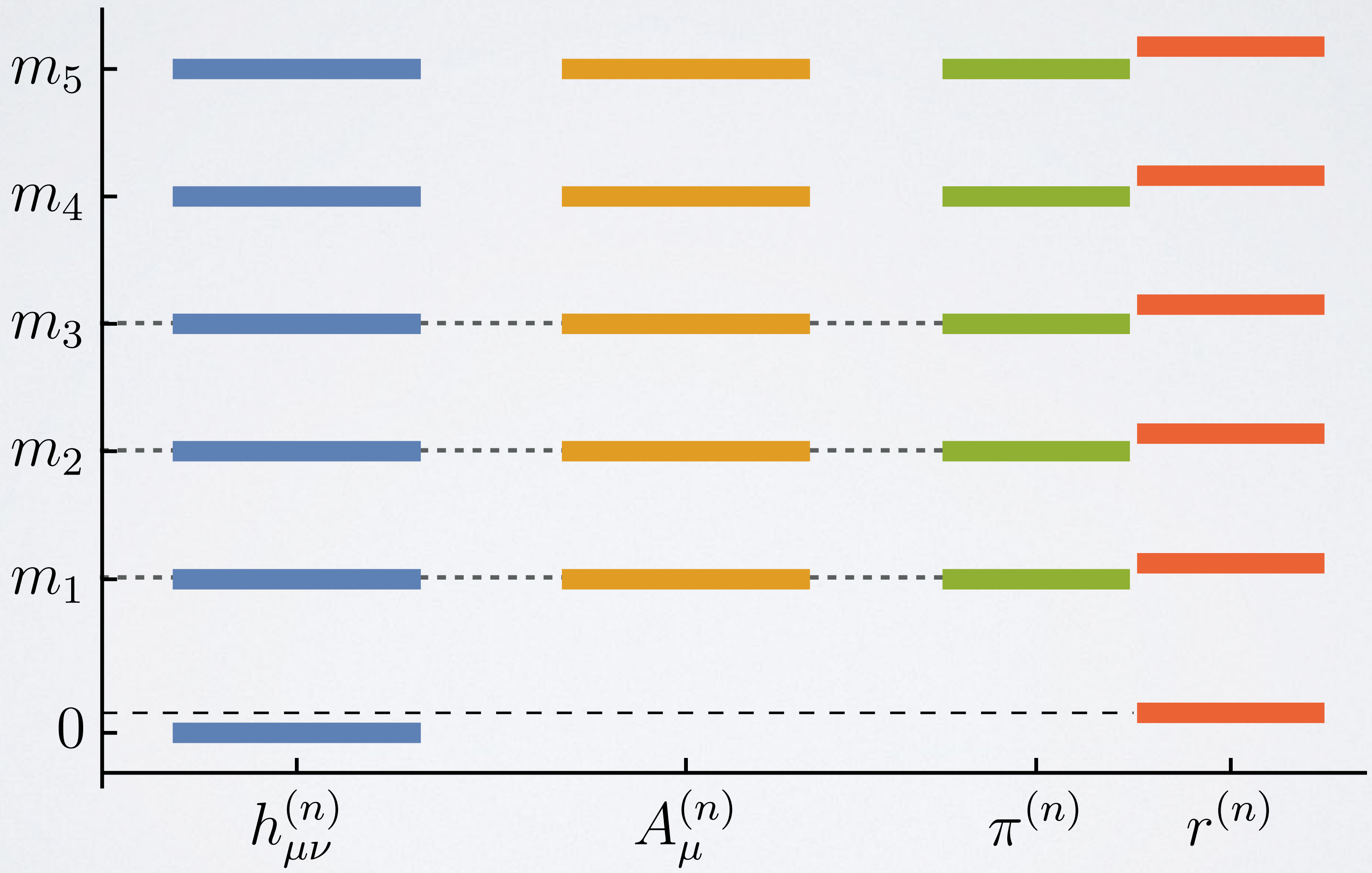
Goldberger-Wise Model

- Similar "eaten" pattern for GW model

(See arXiv: 2207.02887 for details)



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Residual Symmetry of 5D Diffeomorphism

- For a infinitesimal coordinate transformation

$$x^M \mapsto \overline{x}^M = x^M + \xi^M$$

- Expand the parameter using wave functions

$$\xi_\mu(x^\alpha, z) = \sum_{n=0}^{\infty} \xi_\mu^{(n)}(x^\alpha) f^{(n)}(z),$$

$$\theta(x^\alpha, z) \equiv \xi^5(x^\alpha, z) = \sum_{n=0}^{\infty} \theta^{(n)}(x^\alpha) g^{(n)}(z) \ ,$$

- KK modes transform as

$$\begin{aligned} h_{\mu\nu}^{(n)} &\mapsto h_{\mu\nu}^{(n)} - \partial_\mu \tilde{\xi}_\nu^{(n)} - \partial_\nu \tilde{\xi}_\mu^{(n)} + m_n \eta_{\mu\nu} \tilde{\xi}^{5(n)}, \\ A_\mu^{(n)} &\mapsto A_\mu^{(n)} - \sqrt{2} m_n \tilde{\xi}_\mu^{(n)} + \partial_\mu \tilde{\xi}^{5(n)}, \\ \varphi^{(n)} &\mapsto \varphi^{(n)} - \sqrt{6} m_n \tilde{\xi}^{5(n)} \end{aligned}$$

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Massive Graviton Propagators

- Unitary gauge

$$\mathcal{P}_h^{\mu\nu\rho\sigma} = \frac{i}{p^2 - m^2}\frac{1}{2}\left[\left(\eta^{\mu\rho} - \frac{p^\mu p^\rho}{m^2}\right)\left(\eta^{\nu\sigma} - \frac{p^\nu p^\sigma}{m^2}\right) + \cdots\right]$$

- Feynman gauge

$$\mathcal{P}_h^{\mu\nu\rho\sigma} = \frac{i}{p^2 - m_n^2}\frac{1}{2}\left(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \eta^{\mu\nu}\eta^{\rho\sigma}\right)$$

$$\mathcal{P}_A^{\mu\nu} = \frac{-i\eta^{\mu\nu}}{p^2 - m_n^2},$$

$$\mathcal{P}_\varphi = \frac{i}{p^2 - m_n^2}$$

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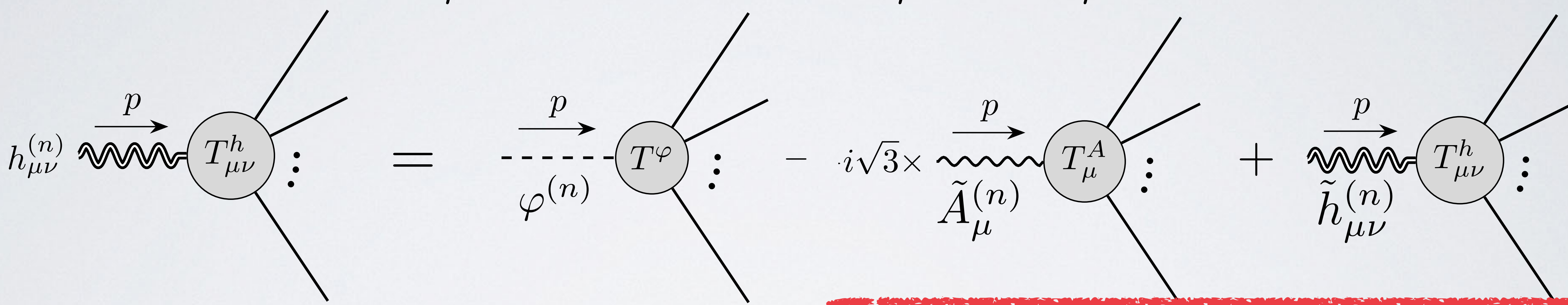
Polarization Tensors and Ward Identities

- For a scattering involving a longitudinal KK graviton

$$T^h_{\mu\nu}\epsilon_0^{\mu\nu} = T^\varphi - i\sqrt{3}\,T^A_\mu\tilde{\epsilon}_0^\mu + T^h_{\mu\nu}\tilde{\epsilon}_0^{\mu\nu}$$



where $\tilde{\epsilon}_0^\mu \equiv -\dfrac{m}{E+|\mathbf{p}|}(1, -\mathbf{p}/|\mathbf{p}|) \sim \mathcal{O}(m/E)$

$$\tilde{\epsilon}_0^{\mu\nu} \equiv \sqrt{\frac{3}{2}}\tilde{\epsilon}_0^\mu\tilde{\epsilon}_0^\nu \sim \mathcal{O}\left(\frac{m^2}{E^2}\right)$$

Bad energy growth subtracted.

- Similarly, for KK graviton with helicity ± 1

$$T^h_{\mu\nu}\epsilon^{\mu\nu}_{\pm 1} = -iT^A_\mu\epsilon^\mu_\pm + T^h_{\mu\nu}\tilde{\epsilon}^{\mu\nu}_{\pm 1}$$

where $\tilde{\epsilon}^{\mu\nu}_{\pm 1} \equiv \dfrac{1}{\sqrt{2}}\left(\epsilon^\mu_\pm\tilde{\epsilon}^\nu_0 + \tilde{\epsilon}^\mu_0\epsilon^\nu_\pm\right) \sim \mathcal{O}\left(\dfrac{m}{E}\right)$

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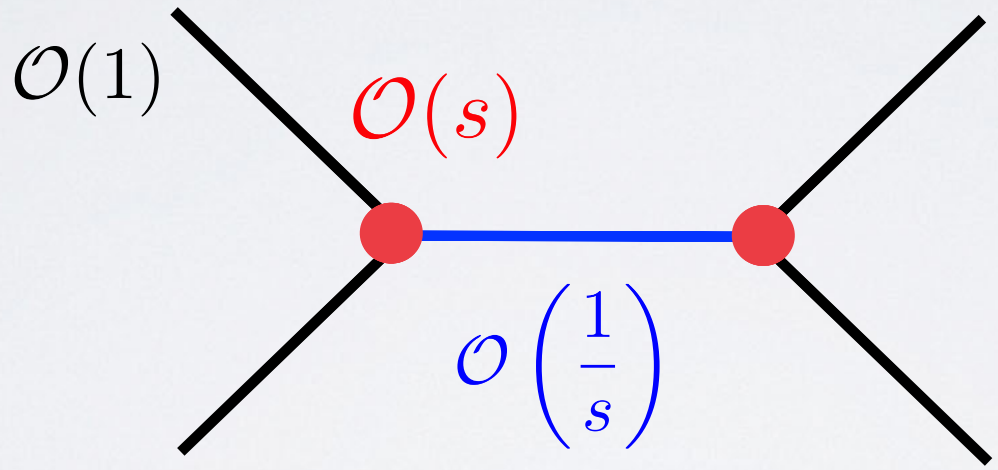
Equivalence Theorem for KK Gravitons

$$T^h_{\mu\nu}\epsilon_0^{\mu\nu} = T^\varphi - i\sqrt{3}\,T^A_\mu \tilde{\epsilon}_0^\mu + T^h_{\mu\nu}\tilde{\epsilon}_0^{\mu\nu}$$

$\tilde{\epsilon}_0^{\mu\nu} \to \mathcal{O}(m^2/E^2)$

$\tilde{\epsilon}_0^\mu \to \mathcal{O}(m/E)$



$\sim \mathcal{O}(s)$

$$\mathcal{M}\left[h_L^{(n_1)} h_L^{(n_2)} \cdots\right] = \mathcal{M}\left[\varphi^{(n_1)} \varphi^{(n_2)} \cdots\right] + \mathcal{O}(s^0)$$

$$\mathcal{M}\left[h_{\pm 1}^{(n_1)} h_{\pm 1}^{(n_2)} \cdots\right] = (-i)^{N_{\rm in}} (i)^{N_{\rm out}} \mathcal{M}\left[A_\pm^{(n_1)} A_\pm^{(n_2)} \cdots\right] + \mathcal{O}(s^0)$$

- True for torus, Randall-Sundrum, and Goldberger-Wise models.

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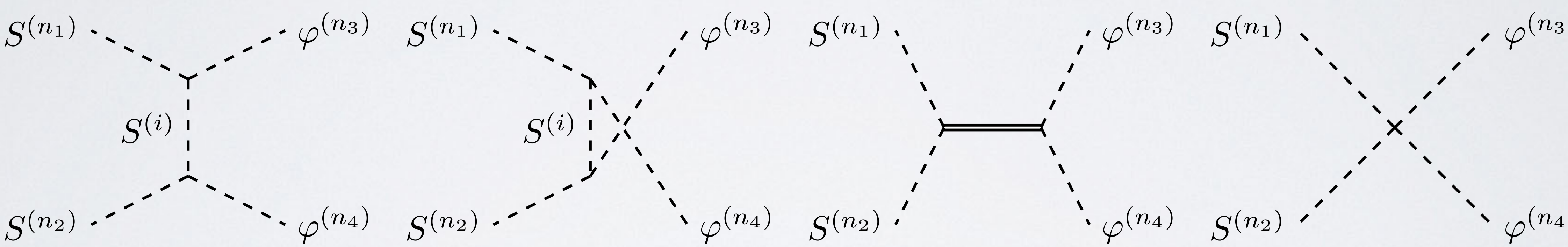
Equivalence Theorem for KK Gravitons

- Scattering of a pair of bulk scalar into KK gravitons

$$S^{(n_1)}S^{(n_2)} \to h_L^{(n_3)}h_L^{(n_4)}$$



$$\mathcal{M}\left[S^{(n_1)}S^{(n_2)} \to \varphi^{(n_3)}\varphi^{(n_4)}\right] = -\frac{\kappa^2 s}{96}(3\cos 2\theta + 5)\sum_{i=0}^{\infty}\langle k^{(n)}k^{(n)}f^{(i)}\rangle\langle f^{(i)}f_S^{(n)}f_S^{(n)}\rangle + \frac{\kappa^2 s}{12}\langle k^{(n)}k^{(n)}f_S^{(n)}f_S^{(n)}\rangle + \mathcal{O}(s^0)$$

$$= \frac{\kappa^2 s}{32}(1-\cos 2\theta)\langle k^{(n)}k^{(n)}f_S^{(n)}f_S^{(n)}\rangle + \mathcal{O}(s^0),$$

$$\widetilde{\mathcal{M}}^{(2)} = \frac{\kappa^2}{576 m_n^4}\Big\{24\sum_{j=0}^{\infty} m_{S,j}^4\left(a_{nmj}^S\right)^2 - (3\cos 2\theta + 1)\sum_{j=0}^{\infty} m_j^2 a_{nnj}a_{jmm}^S + \left[2(3\cos 2\theta+1)m_n^4 + 16 m_n^2 m_{S,m}^2 - 24 m_{S,m}^4\right]a_{nnmm}^S - 8\left[(3\cos 2\theta+1)m_n^2 + 4m_{S,m}^2\right]b_{\bar{n}\bar{n}mm}^S + 16 m_n^2 m_{S,m}^2 a_{nn0}a_{0mm}^S - 144 b_{\bar{n}\bar{n}r}\left(b_{\bar{m}\bar{m}r}^S + \frac{1}{3}M_S^2 a_{mmr}^{M_S}\right)\Big\}.$$

(Unitary gauge result arXiv: 2311.00770)

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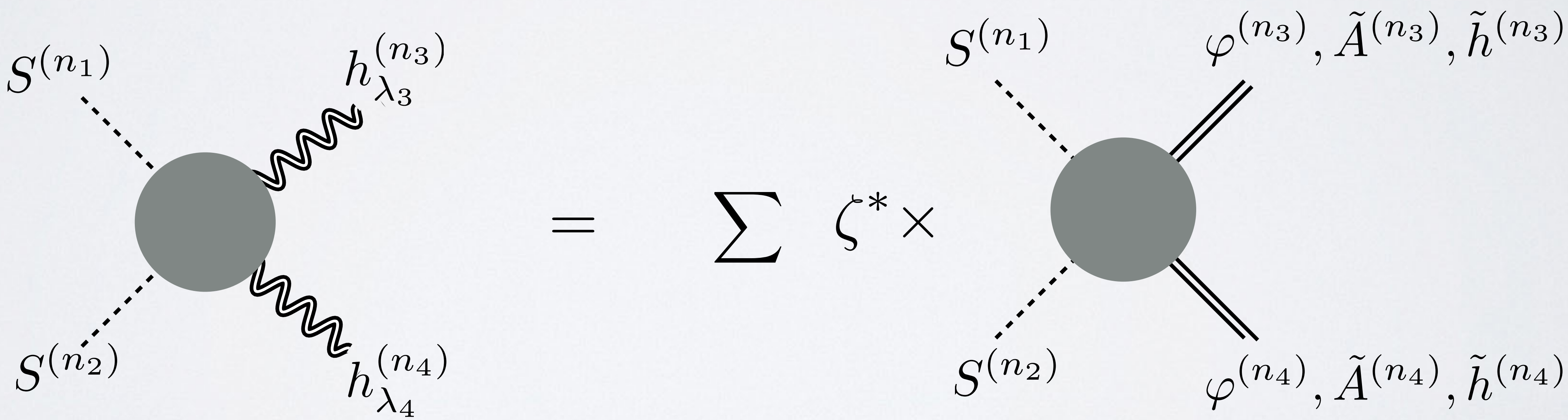
A Robust Method Including Subleading Terms

- Ward identities also tells us what the subleading terms are

$$T^h_{\mu\nu}\epsilon^{\mu\nu}_0 = T^\varphi - i\sqrt{3}\,T^A_\mu \tilde{\epsilon}^\mu_0 + T^h_{\mu\nu}\tilde{\epsilon}^{\mu\nu}_0$$

$$T^h_{\mu\nu}\epsilon^{\mu\nu}_{\pm 1} = -iT^A_\mu \epsilon^\mu_\pm + T^h_{\mu\nu}\tilde{\epsilon}^{\mu\nu}_{\pm 1}$$



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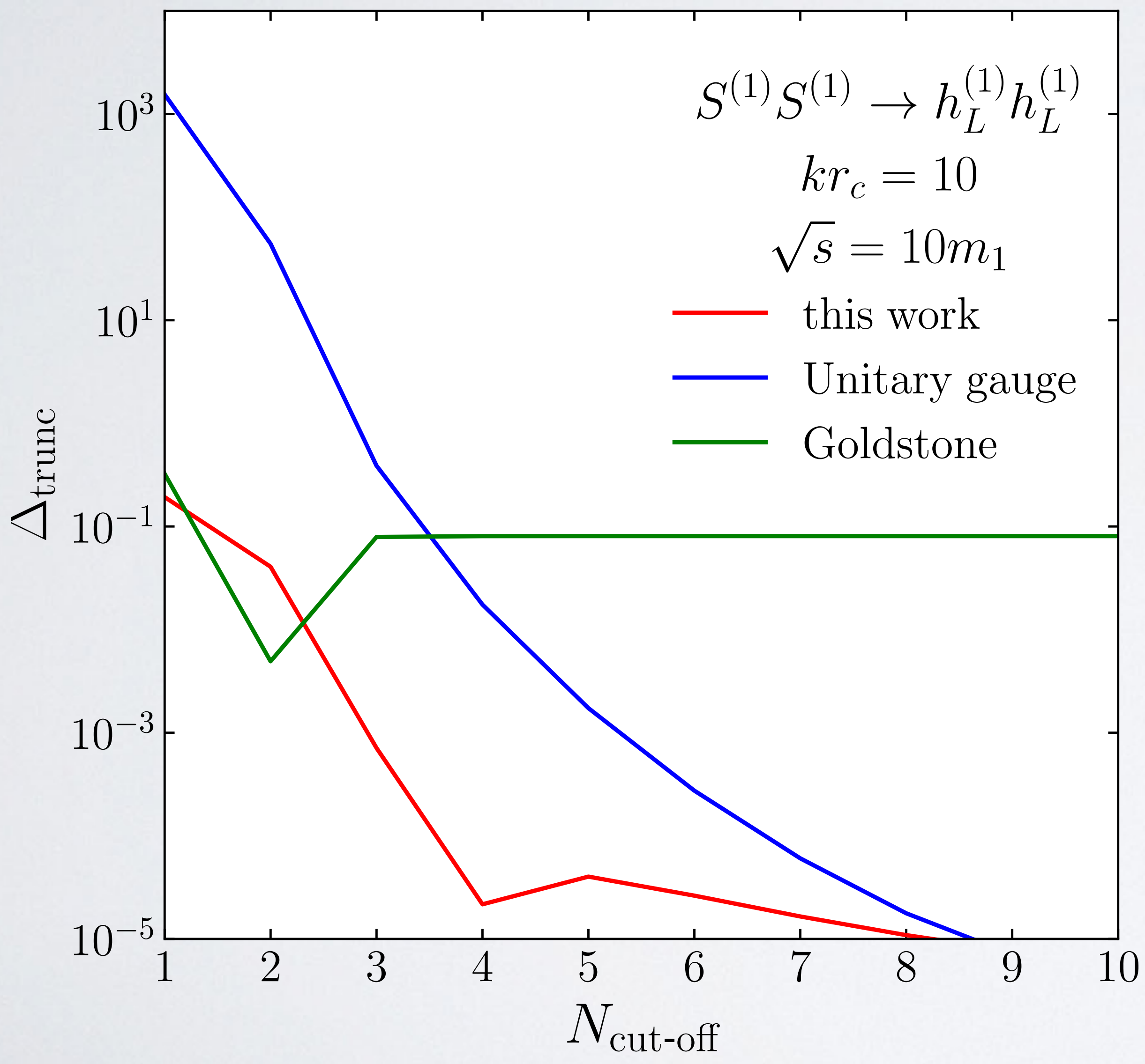
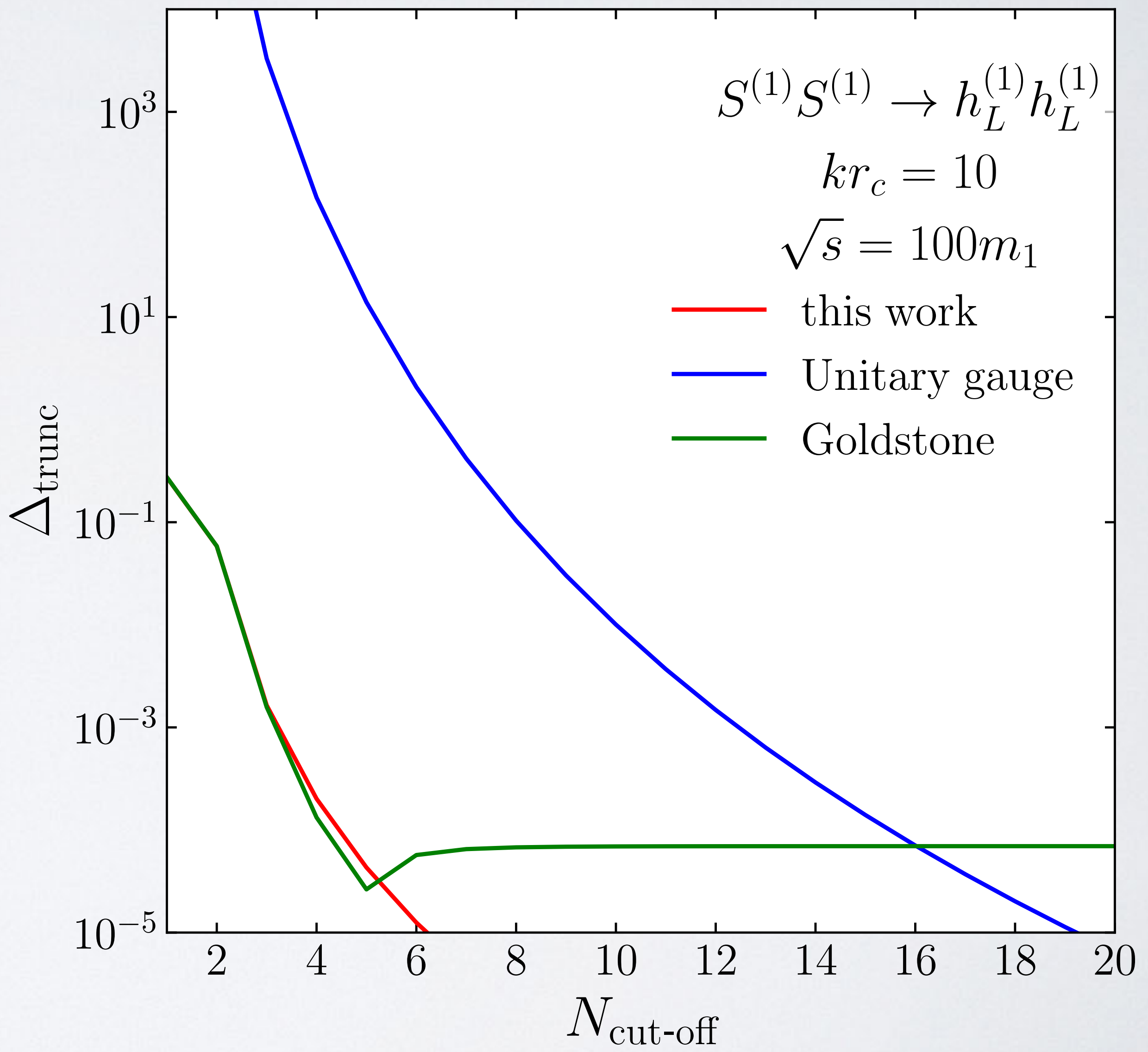
Truncation Error

- If only a finite number of intermediate KK states are included





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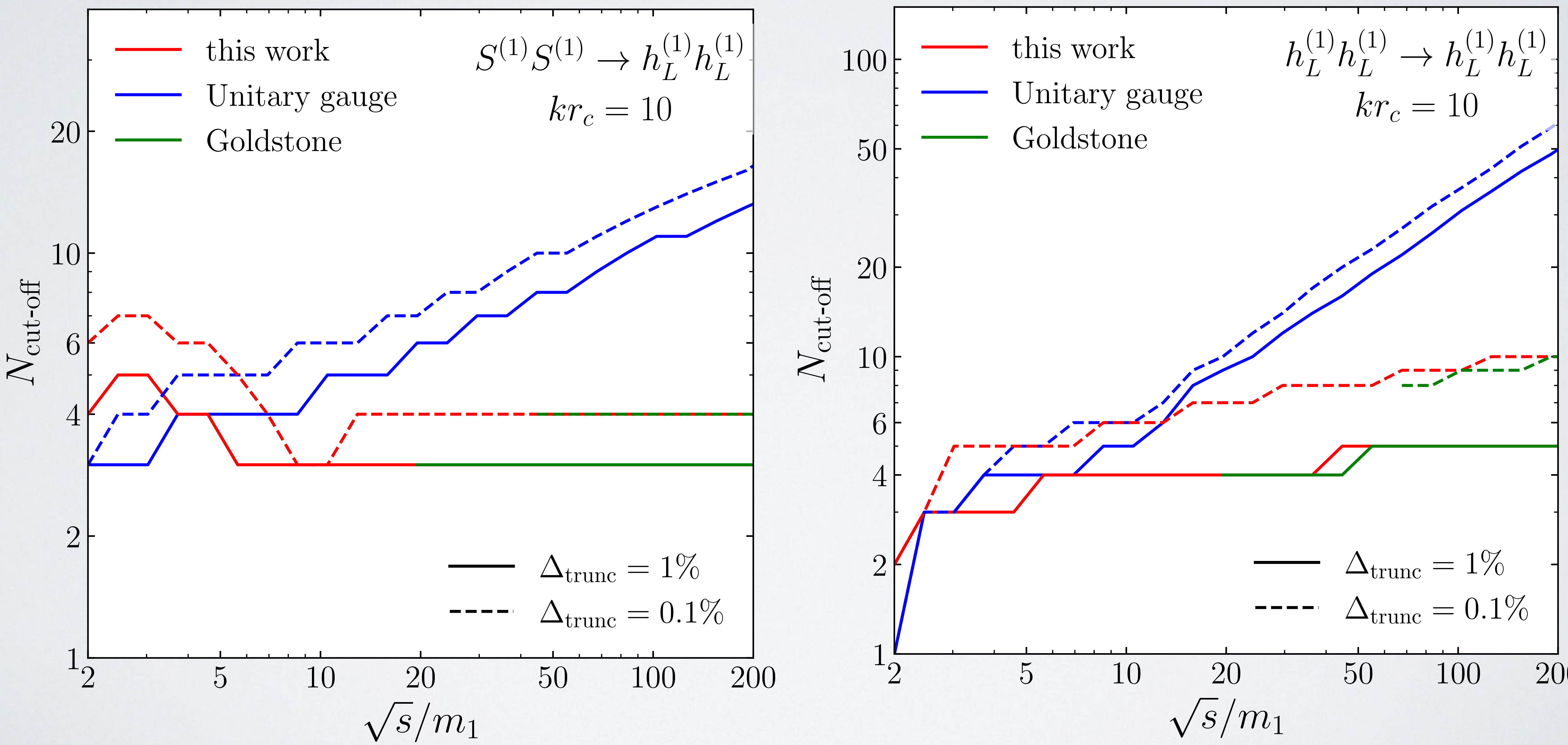

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Summary

- QM SUSY ensures that KK gravitons and their corresponding Goldstone modes have degenerate spectra.
- Transparent power-counting in Feynman gauge
- The scattering amplitudes of the longitudinal KK gravitons equal those of the Goldstone bosons at O(s), for torus, RS, and GW models.
- Ward identities allow us to exactly compute the amplitudes without large cancellation

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GW Model and QM SUSY

$$\hat{\Psi}(x^\alpha, z) = \begin{pmatrix} \hat{\varphi}(x^\alpha, z) \\ \hat{\phi}(x^\alpha, z) \end{pmatrix}$$

55-component of metric

GW scalar

$$\mathcal{L}_{\phi/\varphi\text{-}\phi/\varphi} = \hat{\Psi}\left[-\frac{1}{2}\Box + \frac{1}{2}\overline{\mathcal{D}}\Lambda\overline{\mathcal{D}}^\dagger\right]\hat{\Psi}$$

$$\overline{\mathcal{D}} = \begin{pmatrix} \partial_z + A' & -\frac{1}{\sqrt{6}}\phi_0' \\ \frac{1}{\sqrt{6}}\phi_0' & -\frac{1}{\phi_0'}(\partial_z + 2A')\phi_0' \end{pmatrix}, \quad \overline{\mathcal{D}}^\dagger = \begin{pmatrix} -(\partial_z + 2A') & \frac{1}{\sqrt{6}}\phi_0' \\ -\frac{1}{\sqrt{6}}\phi_0' & \phi_0'(\partial_z + A')\frac{1}{\phi_0'} \end{pmatrix} \qquad \Lambda = \begin{pmatrix} 2 & \\ & -1 \end{pmatrix}$$

$\overline{\mathcal{D}}\Lambda\overline{\mathcal{D}}^\dagger$ is a complicated 2x2 matrix, but

Spectrum of physical GW scalars

$$\overline{\mathcal{D}}^\dagger\overline{\mathcal{D}} = \begin{pmatrix} -(\partial_z + 2A')(\partial_z + A') + \frac{1}{6}\phi_0'^2 & \\ & -\phi_0'(\partial_z + A')\frac{1}{\phi_0'^2}(\partial_z + 2A')\phi_0' + \frac{1}{6}\phi_0'^2 \end{pmatrix}$$

Spectrum that is degenerate to KK gravitons

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Polarization Tensors and Ward Identities

- KK decomposed gauge fixing terms in Feynman gauge are

$$\begin{aligned}
\mathcal{L}_{\rm GF} &= \sum_n F^{(n)\mu} F^{(n)}_\mu - F^{(n)}_5 F^{(n)}_5, \\
F^{(n)}_\mu &= -\left(\partial^\nu h^{(n)}_{\mu\nu} - \frac{1}{2}\partial_\mu h^{(n)} + \frac{1}{\sqrt{2}} m_n A^{(n)}_\mu\right), \\
F^{(n)}_5 &= -\left(\frac{1}{2} m_n h^{(n)} - \frac{1}{\sqrt{2}}\partial^\mu A^{(n)}_\mu + \sqrt{\frac{3}{2}} m_n \varphi^{(n)}\right).
\end{aligned}$$

- Ward identities can be derived by

$$\begin{aligned}
&\langle \mathbf{T}\left(\partial^\nu (h^{(n)}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h^{(n)}) + \frac{1}{\sqrt{2}} m_n A^{(n)}_\mu\right)\Phi\rangle = 0, \\
&\langle \mathbf{T}\left(\frac{1}{2} m_n h^{(n)} - \frac{1}{\sqrt{2}}\partial^\mu A^{(n)}_\mu + \sqrt{\frac{3}{2}} m_n \varphi^{(n)}\right)\Phi\rangle = 0.
\end{aligned}$$

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$$T^h_{\mu\nu}\epsilon^{\mu\nu}_0 = T^\varphi - i\sqrt{3}\,T^A_\mu\tilde{\epsilon}^\mu_0 + T^h_{\mu\nu}\tilde{\epsilon}^{\mu\nu}_0$$

$$T^h_{\mu\nu}\epsilon^{\mu\nu}_{\pm1} = -iT^A_\mu\epsilon^\mu_\pm + T^h_{\mu\nu}\tilde{\epsilon}^{\mu\nu}_{\pm1}$$

$$\mathcal{M}\left[S^{(n_1)}S^{(n_2)} \to h^{(n_3)}_{\lambda_3}h^{(n_4)}_{\lambda_4}\right] = \sum_{\mathcal{F}_i=\phi,A,h}\left(\prod_{i=3}^{4}\zeta^*_{\lambda_i}(\mathcal{F}_i)\right)\mathcal{M}\left[S^{(n_1)}S^{(n_2)} \to \mathcal{F}^{(n_3)}_{3,\lambda_3}\mathcal{F}^{(n_4)}_{4,\lambda_4}\right]$$

$$\zeta_\lambda(\phi) = \begin{cases} 1 & \lambda = 0 \\ 0 & \text{else} \end{cases}, \qquad \zeta_\lambda(A) = \begin{cases} -i\sqrt{3} & \lambda = 0 \\ -i & \lambda = \pm1 \\ 0 & \text{else} \end{cases}, \qquad \zeta_\lambda(h) = 1.$$

$$\tilde{\epsilon}^\mu_\lambda = \begin{cases} \epsilon^\mu_\lambda & \lambda = \pm \\ \tilde{\epsilon}^\mu_0 & \lambda = 0 \end{cases}, \qquad \tilde{\epsilon}^{\mu\nu}_\lambda = \begin{cases} \epsilon^{\mu\nu}_\lambda & \lambda = \pm2 \\ \tilde{\epsilon}^{\mu\nu}_\lambda & \text{else} \end{cases}$$

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Infinite Parameter Lie Algebra

$$
\begin{aligned}
[P_n^\mu, P_m^\nu] &= 0, \\
[M_n^{\mu\nu}, P_m^\sigma] &= \sum_l \langle f^{(n)} f^{(m)} f^{(l)} \rangle \left(\eta^{\mu\sigma} P_l^\nu - \eta^{\nu\sigma} P_l^\mu\right), \\
[M_n^{\mu\nu}, M_m^{\rho\sigma}] &= \sum_l \langle f^{(n)} f^{(m)} f^{(l)} \rangle \left(\eta^{\mu\rho} M_l^{\nu\sigma} + \eta^{\nu\sigma} M_l^{\mu\rho} - \eta^{\mu\sigma} M_l^{\nu\rho} - \eta^{\nu\rho} M_l^{\mu\sigma}\right), \\
[Q_n, Q_m] &= \sum_l \Bigg[\frac{m_m - m_n}{2} \left(\langle f^{(m)} g^{(n)} g^{(l)} \rangle + \langle g^{(m)} f^{(n)} g^{(l)} \rangle\right) \\
&\quad + \frac{m_m + m_n}{2} \left(\langle f^{(m)} g^{(n)} g^{(l)} \rangle - \langle g^{(m)} f^{(n)} g^{(l)} \rangle\right)\Bigg] Q_l, \\
[Q_n, P_m^\mu] &= \sum_l m_m \langle g^{(m)} g^{(n)} f^{(l)} \rangle P_l^\mu, \\
[Q_n, M_m^{\mu\nu}] &= \sum_l m_m \langle g^{(m)} g^{(n)} f^{(l)} \rangle M_l^{\mu\nu}.
\end{aligned}
$$

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