Goldstone boson equivalence theorems for KK gravitons in warped five-dimensional theories

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Energy growth in unitary gauge

• Massive spin-2 propagator

$$
\mathcal{P}_h^{\mu\nu\rho\sigma} = \frac{i}{p^2 - m^2} \frac{1}{2} \left[\left(\eta^{\mu\rho} - \frac{p^\mu p^\rho}{m^2} \right) \left(\eta^{\nu\sigma} - \frac{p^\nu p^\sigma}{m^2} \right) + \cdots \right] \sim \mathcal{O}(E^2/m^2)
$$

• Longitudinal polarization tensor $\epsilon_0^{\mu\nu}$ = 1 $\overline{\sqrt{6}}$ $\overline{(\ }$ $\epsilon_+^{\mu} \epsilon_-^{\nu} + \epsilon_-^{\mu} \epsilon_+^{\nu} + 2 \epsilon_0^{\mu} \epsilon_0^{\nu}$ 0 $\overline{)}$ $\sim \mathcal{O}(E^2/m^2)$

• Ricci scalar contains two derivatives: vertices $\sim \mathcal{O}(E^2)$

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't Hooft-Feynman Gauge

- As in the gauge theories, power counting is much more transparent in the 't Hooft-Feynman gauge. $\overline{131}$ *ne gauge theories, power counting is much more* $\frac{1}{2}$ in the $\frac{1}{2}$ Heath Foundation
- Goldstone modes have the same masses as the gauge bosons. \ddotsc conditions are given by Eq.(III.69), and Eq.(III.69) or in the three combinations, \overline{a} \overline{d} *z*1 $\frac{1}{2}$ and the same masses as the gauge where

$$
\text{maximize}_{k} \sum_{k} \text{minimize}_{k} \text{
$$

- In warped extra dimension model, KK masses are determined by transcendental equations $\frac{1}{2}$ note that for the $\frac{1}{2}$ section there are f $\frac{1}{2}$ in and $\frac{1}{2}$ $Y_1(m_nz_2)J_1(m_nz_1)-J_1(m_nz_2)Y_1(m_nz_1)=0.$
- How come Goldstones have the same non-trivial masses?

Quantum Mechanical SUSY

• For two Hamiltonians

 $H_1 = A^{\dagger} A$ $H_2 = A A^{\dagger}$

• SUSY doublet $\Psi = (\psi_1, \psi_2)^T$, with supercharges

$$
Q = \begin{pmatrix} 0 & 0 \\ A & 0 \end{pmatrix}, \qquad Q^{\dagger} = \begin{pmatrix} 0 & A^{\dagger} \\ 0 & 0 \end{pmatrix}
$$

$$
Q = \begin{pmatrix} 0 & 0 \\ A & 0 \end{pmatrix}, \qquad Q^{\dagger} = \begin{pmatrix} 0 & A^{\dagger} \\ 0 & 0 \end{pmatrix}
$$

$$
[Q, H] = [Q^{\dagger}, H] = 0,
$$

$$
[(-1)^{F}, H] = 0,
$$

$$
\{Q, (-1)^{F}\} = \{Q^{\dagger}, (-1)^{F}\} = 0.
$$

• They have degenerate eigenvalues, except for the ground state

Metric Perturbation in Feynman Gauge \sim ation principle for the action for the complete the coordinate Matric Darturbation in Equipmen Course incuit i dituluation in reyunian Gauge pendix A outlines our notation, while appendix B gives the 't-Hooft-Feynman gauge Feynman rules needed for the com-**L** \int_{0}^{L} \int_{0}^{L} \int_{0}^{L} \int_{0}^{L} where *LEH* and *L*CC are the usual Einstein-Hilbert and cosmological constant terms respectively. The *L* term is a

• 5D Metric in conformal coordinates Random extra-dimensional theory, extending theory, extending the results of Du α total derivative term required for a well defined variational principle for the action. *L*GF is gauge fixing term and *L*^m

$$
G_{MN} = e^{2A(z)} \begin{pmatrix} e^{-\kappa \hat{\varphi}/\sqrt{6}} (\eta_{\mu\nu} + \kappa \hat{h}_{\mu\nu}) & \frac{\kappa}{\sqrt{2}} \hat{A}_{\mu} \\ \frac{\kappa}{\sqrt{2}} \hat{A}_{\mu} & -\left(1 + \frac{\kappa}{\sqrt{6}} \hat{\varphi}\right)^2 \end{pmatrix}
$$

• KK decomposition $A(z) = -\ln(kz)$

 $\frac{1}{\infty}$ • KK decomposition

$$
h_{\mu\nu}(x^{\alpha}, z) = \sum_{n=0}^{\infty} h_{\mu\nu}^{(n)}(x^{\alpha}) f^{(n)}(z)
$$

$$
A_{\mu}(x^{\alpha}, z) = \sum_{n=1}^{\infty} A_{\mu}^{(n)}(x^{\alpha}) g^{(n)}(z)
$$

$$
\varphi(x^{\alpha}, z) = \sum_{n=0}^{\infty} \varphi^{(n)}(x) k^{(n)}(z)
$$

Xing Wang, UCSD xiw006@physics.ucsd.edu satisfies the background geometry bulk Einstein equation For Fields, the radion field, and the radion field, and the mass visit α *ziw006@physics.ucsd.edu* is the internal compact coordinate, *z*1*,*² are the locations of the orbifold fixed points, and the mode wavefunctions

KK spectra in Feynman gauge

• $h_{\mu\nu}^{(n)}$, $A_{\mu}^{(n)}$, and $\varphi^{(n)}$ have degenerate spectra, except for the massless state

 $h^{(n)}_{\mu\nu}$ and $A^{(n)}_{\mu}$ form a SUSY pair $A^{(n)}_{\mu}$ and $\varphi^{(n)}$ form another SUSY pair

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Goldberger-Wise Model

• Similar "eaten" pattern for GW model

(See arXiv: [2207.02887](https://arxiv.org/abs/2207.02887) for details)

Residual Symmetry of 5D Diffeomorphism The 5D Lagrangian is invariant under an infinitesimal coordinate transformation, $Symmetry$ of $5D$ Diffeomorphism \mathcal{A}

- $x^M \mapsto \overline{x}^M = x^M + \xi^M$ • For a infinitesimal coordinate transformation
- $E = \frac{1}{2}$ • Expand the parameter using wave functions

$$
\xi_{\mu}(x^{\alpha}, z) = \sum_{n=0}^{\infty} \xi_{\mu}^{(n)}(x^{\alpha}) f^{(n)}(z),
$$

$$
\theta(x^{\alpha}, z) \equiv \xi^{5}(x^{\alpha}, z) = \sum_{n=0}^{\infty} \theta^{(n)}(x^{\alpha}) g^{(n)}(z) ,
$$

ˆ ˆ $\frac{1}{2}$ • KK modes transform as

$$
h_{\mu\nu}^{(n)} \mapsto h_{\mu\nu}^{(n)} - \partial_{\mu} \tilde{\xi}_{\nu}^{(n)} - \partial_{\nu} \tilde{\xi}_{\mu}^{(n)} + m_n \eta_{\mu\nu} \tilde{\xi}^{(n)},
$$

\n
$$
A_{\mu}^{(n)} \mapsto A_{\mu}^{(n)} - \sqrt{2} m_n \tilde{\xi}_{\mu}^{(n)} + \partial_{\mu} \tilde{\xi}^{(n)},
$$

\n
$$
\varphi^{(n)} \mapsto \varphi^{(n)} - \sqrt{6} m_n \tilde{\xi}^{(n)}
$$

Massive Graviton Propagators such that the above identities relating amplitudes can be written as, *v P*ugueoi. *.* (44) FIG. 2. Schwa Graviton, Pr

• Unitary gauge **F** be a 5D graviton propagator propagator that has the tensor structure of a 5D mass σ • Unitary gauge *and y* and you are the manufactured with $\frac{1}{2}$ and $\frac{1}{2}$

FIG. 2. Schematic Feynman diagrams involving one internal KK graviton, one KK vector Goldstone boson, or one KK scalar

$$
\mathcal{P}_h^{\mu\nu\rho\sigma} = \frac{i}{p^2 - m^2} \frac{1}{2} \left[\left(\eta^{\mu\rho} - \frac{p^{\mu}p^{\rho}}{m^2} \right) \left(\eta^{\nu\sigma} - \frac{p^{\nu}p^{\sigma}}{m^2} \right) + \cdots \right]
$$

where the primality gauge • Feynman gauge *^Y* ⌫ + *T* ' *^Y* ⌘ *^T*^ˆ Correspondingly, we can parametrize the scattering matrix *TMN* as in Eq. (43) so that the internal propagators can **be a 5D vritten as 5D graviton propagator structure of a 5D massless construction propagator structure of a 5D massless construction propagator structure of a 5D mass constructure of a 5D massless graviton. For a scatteri**

$$
\mathcal{P}_{h}^{\mu\nu\rho\sigma} = \frac{i}{p^{2} - m_{n}^{2}} \frac{1}{2} \left(\eta^{\mu\rho} \eta^{\nu\sigma} + \eta^{\mu\sigma} \eta^{\nu\rho} - \eta^{\mu\nu} \eta^{\rho\sigma} \right)
$$

$$
\mathcal{P}_{A}^{\mu\nu} = \frac{-i\eta^{\mu\nu}}{p^{2} - m_{n}^{2}},
$$

$$
\mathcal{P}_{\varphi} = \frac{i}{p^{2} - m_{n}^{2}}
$$

Xing Wang, UCSD

Xing Wang, UCSD xiw006@physics.ucsd.edu $\mathcal S$ model (see Sec. IV for a brief discussion of the GW model). To leading order, the Goldstone boson Equivalence boson Equivalence boson Equivalence boson Equivalence boson Equivalence boson Equivalence boson Equivale x_{I} and y_{I} and y_{II} and y_{II} are x_{II} and y_{II} and y_{II} and y_{II} and y_{II} are y_{II} and y_{II} and y_{II} are y_{II} and y_{II} and y_{II} are y_{II} a *.* (47)

Polarization Tensors and Ward Identities **Polarization Tensors and W** This extends the Goldstone boson Equivalence the Goldstone boson Equivalence theorem given by Hang and He for Didi ization terisors and vvard identities. *zation lensors and vvard identities* The polarization vectors have the energy dependency, when *E m*,

• For a scattering involving a longitudinal KK graviton • For a scallering involving a longitudinal KK graviton udinal KK graviton $\frac{1}{1}$ $\frac{1}{2}$ K gravito • For a scattering involving a longitudinal KK graviton \mathcal{L} ering frouving a forigitudinal NN graviton \mathcal{L} T thus, the longitudinal polarization tensor depends on the energy quadratically at high-energies, $\frac{1}{2}$ ✏ *µ*⌫

T ^A A ^µ

 J O Q \ket{n} aphysics.ucsd.edu The Goldstone boson Equivalence the scattering $\frac{1}{2}$ states is: the scattering amplitude of the KKK $\frac{1}{2}$ $T_{\rm eff}$ rewrite the longitudinal polarization tensor as $T_{\rm eff}$ as a rewrite tensor as $T_{\rm eff}$

multiplying by (⇤ *^m*²

Equivalence Theorem for KK Gravitons **1** α ¹ equals that of the vector KK α ¹ α ² α ² α ² α ² Equivalence Theorem for KK Cravitons gravitons with helicities *±*1 equals that of the vector KK Goldstone boson in the high energy limit up to a overall and the longitudinal scattering amplitude can be expressed as α be expressed as α

$$
T_{\mu\nu}^{h} \epsilon_0^{\mu\nu} = T^{\varphi} - i\sqrt{3} T_{\mu}^A \tilde{\epsilon}_0^{\mu} + T_{\mu\nu}^h \tilde{\epsilon}_0^{\mu\nu}
$$

$$
\mathcal{O}(m/E)
$$

$$
\mathcal{O}(1)
$$

$$
\mathcal{O}(s)
$$

$$
\mathcal{O}\left(\frac{1}{s}\right)
$$

$$
\mathcal{O}(s)
$$

$$
\mathcal{M}\left[h_L^{(n_1)}h_L^{(n_2)}\cdots\right] = \mathcal{M}\left[\varphi^{(n_1)}\varphi^{(n_2)}\cdots\right] + \mathcal{O}(s^0)
$$

$$
\mathcal{M}\left[h_{\pm 1}^{(n_1)}h_{\pm 1}^{(n_2)}\cdots\right] = (-i)^{N_{\text{in}}}(i)^{N_{\text{out}}}\mathcal{M}\left[A_{\pm}^{(n_1)}A_{\pm}^{(n_2)}\cdots\right] + \mathcal{O}(s^0)
$$

M -
or torus, Randall-Sundrum, and Goldberger-Wise models. • True for torus, Randall-Sundrum, and Goldberger-Wise mo W also note that one can organize the above results into a more compact way by introducing 5 Similarly, using the definitions of the helicity *±*1 polarization tensors \overline{a} *,* (35) • True for torus, Randall-Sundrum, and Goldberger-Wise models.

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Equivalence Theorem for KK Gravitons A . Scattering of two K α scalars into two K α scalars into two α K gravitons into two helicity- 0 **neorer** \mathbf{r} m for KK Gravitons Finally, similar to the behavior of brane scalars and fermions, the leading non-vanishing contribution to the ampli- $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ LYU ivalei h *k*(*n*) (¯*z*) i2 *,* (112) *[±]*⌥ ⁼ 2(cos 2✓ 1)

For the first example, we consider the scattering of two KK bulk scalar into two longitudinal KK gravitons, • Scattering of a pair of bulk scalar into KK gravitons ha ann an chaidh an
Tagairtí *S,nf*(*n*) in a manner consistent with an equivalence theorem.

$$
S^{(n_1)}S^{(n_2)} \to h_L^{(n_3)}h_L^{(n_4)}
$$

 $\kappa^2 s$
 $\kappa^2 (1 - \cos 2\theta) / L^{(n)} L^{(n)} f^{(n)} + \mathcal{O}(\epsilon^0)$ $\{a_{b_{\bar{n}}\bar{n}r}\left(b_{\bar{m}\bar{n}r}^S + \frac{1}{3}M_B^2a_{\bar{m}m}^{M_S}\right)\}$
 $\{22\}$ $(4\ 265\ 26)$ $(6\ 26\ 26)$ $(7\ 26\ 26)$ $(8\ 26\ 26)$ $(8\ 26\ 26)$ $(9\ 26)$ $(1\ 26\ 26)$ = κ^2 *s* $\frac{\partial^2 S}{\partial^2} (1 - \cos 2\theta) \langle k^{(n)} k^{(n)} f_S^{(n)} f_S^{(n)} \rangle + \mathcal{O}(s^0),$

(Unitary gauge result <u>arXiv: 2311.00770</u>) Xing Wang, UCSD

 $\left(b_{\bar{m}\bar{m}r}^S + \frac{1}{3}\right]$

 $-144b_{\bar{n}\bar{n}r}$

 $-8\left[(3\cos 2\theta + 1)m_n^2 + 4m_{S,m}^2\right]b_{\bar{n}\bar{n}mm}^S + 16m_n^2m_{S,m}^2a_{nn0}a_{0mm}^S$

 $\left\{\frac{1}{3}M_S^2a^{M_S}_{mmr}\right\}\right\}.$

S(*n*1)

Xing Wang, UCSD xiw006@physics.ucsd.edu *^S*(*n*2) ! *^h*(*n*3) *^L ^h*(*n*4) *L* = *M ^S*(*n*2) ! '(*n*3) We take the matter Lagrangian for a real bulk scalar *S* with a mass *M^S* to be (the metric *GMN* is defined in GSD metric. Following the determinant of the state of the state of the state of the notation in Ref. Eq. and the notation in Ref. GSD metric. For the state of the GSD metric. For the state of the GSD metric. The state σ simplicity, we consider a model with no bulk potential or brane-localized scalar interactions.

A Robust Method Including Subleading Terms Similarly, using the definitions of the helicity *±*1 polarization tensors *µ*⌫ \mathbf{I} $\overline{}$ *µ ±*✏ ⌫ ⁰ + ✏ *µ* 0 ✏ ⌫ *,* (35) \overline{a} Including Subleading lerms states as an appropriate combination of Goldstone boson amplitudes. A Robust Method Including Subleading Terms propagators in the 't Hooft-Feynman gauge behave like 1*/p*², eliminating the problematic high-energy behavior coming ² *^µ ^h*⌫ *ⁿ*() I st 111 \rightarrow \downarrow *ⁿ ^A^µ* and A Robust Method Including Sublead

• Ward identities also tells us what the subleadin
 $T^h_{\mu\nu} \epsilon_0^{\mu\nu} = T^\varphi - i \sqrt{3} T^A_\mu \tilde{\epsilon}_0^\mu + T^h_{\mu\nu} \tilde{\epsilon}_0^{\mu\nu}$ $rac{1}{2}$ b are the polarization vectors of the polarization vectors of the external spin-
2 and 2 *^h ^T^µ*⌫ ⁼ Because of the mass degeneracy of *^hn* $A Rob$ ARO
ARO
W AF
 \cdot A Dabugt Mathad Including Cublanding Tarms (36) show that we can replace the matrix elements involving external helicity-0 and helicity-1 massive spin-2 states – A Boburt Mothod Including Subloading Torms gauge and applies the Ward identities described above to rewrite any matrix elements involving problematic external from unitary gauge projection operators in the property in the property in the United States in Taxana is in Equation of Taxana in the United States in Equation of Taxana is in Equation of Taxana in the United States in Eq A NODUST METHOG INCIUDING SUDICACING IELINS – $S₁$ scattering and of using $S₁$ integrating Terms gauge and applies the Ward identities described above to rewrite any matrix elements involving problematic external from unitary gauge projection operators in the propagators in the other hand, the \sim d Including Subleading Terms the states whose polarization tensors have potentially large high-energy behavior – by a combination of amplitudes sport-2 scattering amplitudes. Instead of using the amplituding state of using the amplitudes to us the amplit gauge and applies the Ward identities described above to rewrite any matrix elements involving problematic external

• Ward identities also tells us what p2 *µ* • Ward identities also tells us what the subleading terms are space and gives an explicit expression for the residual terms not captured by the leading-order expression. from unitary gauge projection operators in the propagators. On the other hand, the Ward identities in Eqs. (33) and • Ward identities also tells us what the subleading terms are $\frac{1}{2}$ tells $\mathbf{1}$ n n *m* th ng ter
 µ✏ ⌫ and identities also tells us what the subleading tern • Ward identitie
 T^h_μ involving the corresponding the corresponding to the c whose behavior and the sensus where the subsequently terms are • Ward identities also tells us what the subleading terms are propagators in the term of the three three bediedering terminative involved identities also talle us what the subleading terms are whose behavior and the substitutes is mild. The substantial term is and • Ward identities also tells us what the subleading terms are propagators in the 't Hooft-Feynman gauge behave like 1*/p*², eliminating the problematic high-energy behavior coming involving the corresponding Goldstone bosons and a residual "spin-2" polarization vector (˜✏*^µ*⌫ in those equations) tells us what the subleading terms are so tells us what the subleading terms are propagators in the 't Hooft-Feynman gauge behave like 1*/p*², eliminating the problematic high-energy behavior coming

$$
T_{\mu\nu}^h \epsilon_0^{\mu\nu} = -i T_{\mu}^A \epsilon_1^{\mu} + T_{\mu\nu}^h \tilde{\epsilon}_{\pm 1}^{\mu\nu}
$$
\nWhat identities also tells us what the subleading terms are

\n
$$
T_{\mu\nu}^h \epsilon_0^{\mu\nu} = T^{\varphi} - i \sqrt{3} T_{\mu}^A \tilde{\epsilon}_0^{\mu} + T_{\mu\nu}^h \tilde{\epsilon}_{\pm 1}^{\mu\nu}
$$

Truncation Error

• If only a finite number of intermediate KK states are included

Xing Wang, UCSD xiw006@physics.ucsd.edu FIG. 5. The truncation error as a function of the number of included intermediate-state KK modes *N*cut-o↵ for the scattering

Truncation Error

• If only a finite number of intermediate KK states are included

Xing Wang, UCSD xiw006@physics.ucsd.edu $F = \frac{1}{2}$ minimum number of intermediate K modes one must include the accuracy of trunc $\frac{1}{2}$ (solid trunc $\frac{1}{2}$ (solid

Summary

- QM SUSY ensures that KK gravitons and their corresponding Goldstone modes have degenerate spectra.
- Transparent power-counting in Feynman gauge
- The scattering amplitudes of the longitudinal KK gravitons equal those of the Goldstone bosons at O(s), for torus, RS, and GW models.
- Ward identities allow us to exactly compute the amplitudes without large cancellation

GW Model and QM SUSY − 1 √ 6 0 φ
0 φ! (δ/γ)
0 φ! (δ/γ) $C(M/M_2)$ and $C(M)$ $C(M)$ The terms involving the computation of the computation Q GW Model and OM SUSY where, the fields Q and Q

$$
\hat{\Psi}(x^{\alpha}, z) = \begin{pmatrix} \hat{\varphi}(x^{\alpha}, z) \\ \hat{\phi}(x^{\alpha}, z) \end{pmatrix}
$$
 55-component of metric
GW scalar

$$
{\cal L}_{\phi/\varphi\hbox{-} \phi/\varphi}=\qquad \hat{\Psi} \,\Bigg[\,\, -\frac{1}{2}\Box + \frac{1}{2} \overline{\cal D} \Lambda \overline{\cal D}^\dagger \Bigg]\,\hat{\Psi}
$$

$$
\overline{\mathcal{D}} = \begin{pmatrix} \partial_z + A' & -\frac{1}{\sqrt{6}} \phi_0' \\ \frac{1}{\sqrt{6}} \phi_0' & -\frac{1}{\phi_0'} (\partial_z + 2A') \phi_0' \end{pmatrix}, \quad \overline{\mathcal{D}}^{\dagger} = \begin{pmatrix} -(\partial_z + 2A') & \frac{1}{\sqrt{6}} \phi_0' \\ -\frac{1}{\sqrt{6}} \phi_0' & \phi_0' (\partial_z + A') \frac{1}{\phi_0'} \end{pmatrix} \qquad \Lambda = \begin{pmatrix} 2 \\ -1 \end{pmatrix}
$$

$$
\overline{\mathcal{D}}\Lambda\overline{\mathcal{D}}^{\dagger}
$$
 is a complicated 2x2 matrix, but
\n
$$
\overline{\mathcal{D}}^{\dagger}\overline{\mathcal{D}} = \begin{pmatrix}\n-(\partial_z + 2A')(\partial_z + A') + \frac{1}{6}\phi_0'^2 \\
\frac{-(\partial_z + 2A')(\partial_z + A') + \frac{1}{6}\phi_0'^2}{\partial_z'^2} \\
\frac{-\phi_0'(\partial_z + A') + \frac{1}{6}\phi_0'^2}{\phi_0'^2} \\
\frac{-\phi_0'(\partial_z + A') + \frac{1}{6}\phi_0'^2}{\phi_0'^2}\n\end{pmatrix}
$$
\nSpectrum that is degenerate to KK gravitons [7]

xiw006@physics.ucsd.edu
Xiw006@physics.ucsd.edu Specifient in the first equation are to use since it is independent of the bulk potential v ~ 17 King Wang, UCSD \cap t $\frac{1}{2}$ ∕
degener

acting on a two-component doublet

 $h^{(n)}_{\mu\nu}$ **WWW** $T^h_{\mu\nu}$ $\mu\nu$ www. $\left(\frac{T_{\mu\nu}^n}{2}\right)$: *· · p* $A_{\mu}^{(n)}$ and T_{μ}^{A} μ
 μ
 σ 31100 fiving term *· · p* $\varphi^{(n)}$ ------ $\left(T^{\varphi}\right)$: *· ·* Polarization Tensors and Ward Identities $F_{\mu}^{(n)} \; = \; - \left(\, \partial^{\nu} h_{\mu \nu}^{(n)} - \frac{1}{2} \partial_{\mu} h^{(n)} + \frac{1}{\sqrt{2}} m_{n} A_{\mu}^{(n)} \, \right),$ $\sqrt{2}$ and $\sqrt{2}$ after the Lehmann-Symmermann-Symmermann (LSZ) and $\sqrt{2}$ *D^µ*⌫⇢ *^h* = ² (⌘*^µ*⇢⌘⌫ ⁺ ⌘*^µ*⌘⌫⇢ ⌘*^µ*⌫⌘⇢) (⇤ *^m*² *ⁿ*)*,* (5) $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ D $\left\{ \frac{d}{dx} \right\}$ $\left\{ \frac{d}{dx} \right\}$ $h_{\mu\nu}^{(n)}$ www. $T_{\mu\nu}^{(n)}$: $A_{\mu}^{(n)}$ www. $T_{\mu}^{(1)}$: $\varphi^{(n)}$ ------(T^{φ}) : with a KK decomposed gauge fixing terms in Feynman gauge are $\mathcal{L}_{\text{GF}} = \sum$ *n* $F^{(n)\mu}F^{(n)}_{\mu}-F^{(n)}_{5}\nabla^{(n)}_{5}$ $\frac{5}{5}$, $\frac{5}{5}$, $\frac{5}{5}$ $\sqrt{2}$ $\partial^{\nu}h^{(n)}_{\mu\nu}$ $\binom{n}{\mu\nu} - \frac{1}{2}$ 2 $\partial_\mu h^{(n)}$ + 1 $\overline{\sqrt{2}}$ $m_n A_\mu^{(n)}$ ◆ *,* (9) $F_5^{(n)} = \sqrt{ }$ 1 2 $m_n h^{(n)} - \frac{1}{\sqrt{n}}$ $\overline{\sqrt{2}}$ $\partial^{\mu}A^{(n)}_{\mu} +$ $\sqrt{3}$ 2 $m_n\varphi^{(n)}$! *.* (10)

Plugging in the gauge fixing condition in Eqs. (9) and (10), we have the following identities for the time-ordered • Ward identities can be derived by

$$
\langle \mathbf{T} \left(\partial^{\nu} (h_{\mu\nu}^{(n)} - \frac{1}{2} \eta_{\mu\nu} h^{(n)}) + \frac{1}{\sqrt{2}} m_n A_{\mu}^{(n)} \right) \Phi \rangle = 0,
$$

$$
\langle \mathbf{T} \left(\frac{1}{2} m_n h^{(n)} - \frac{1}{\sqrt{2}} \partial^{\mu} A_{\mu}^{(n)} + \sqrt{\frac{3}{2}} m_n \varphi^{(n)} \right) \Phi \rangle = 0.
$$

Because of the mass degeneracy of *hⁿ* multiplying by (⇤ *^m*² Xing Wang, UCSD

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A Robust Method Including Subleading Terms Similarly, using the definitions of the helicity *±*1 polarization tensors *µ*⌫ \mathbf{I} $\overline{}$ *µ ±*✏ ⌫ ⁰ + ✏ *µ* 0 ✏ ⌫ *,* (35) from unitary gauge projection operators in the projection operators. On the \sim A Kobust Method including Subleading lerms the states whose polarization tensors have potentially large high-energy behavior – by a combination of amplitudes Λ Dobust Mothod Including Cublooding Tomps involving the corresponding Goldstone bosons and a residual "spin-2" polarization vector (˜✏*^µ*⌫ in those equations)

• Ward identities also tells us what p2 *µ* involving the corresponding Goldstone bosons and a residual "spin-2" polarization vector (˜✏*^µ*⌫ in those equations) • Ward identities also tells us what the subleading terms are $\mathcal{N}(\mathcal{N})$ in unitary gauge; the scattering amplitudes will converge as fast as fast as the overlap integrals will converge as fast as fast as the overlap integrals of the overlap integrals with a structure as fast as • Ward identities also tells us what the subleading terms are

$$
T^h_{\mu\nu}\epsilon^{\mu\nu}_0=T^\varphi-i\sqrt{3}\,T^A_\mu\tilde{\epsilon}^\mu_0+T^h_{\mu\nu}\tilde{\epsilon}^{\mu\nu}_0
$$

$$
T^h_{\mu\nu}\epsilon^{\mu\nu}_{\pm 1}=-iT^A_\mu\epsilon^\mu_\pm+T^h_{\mu\nu}\tilde{\epsilon}^{\mu\nu}_{\pm 1}
$$

$$
\mathcal{M}\left[S^{(n_1)}S^{(n_2)} \to h_{\lambda_3}^{(n_3)}h_{\lambda_4}^{(n_4)}\right] = \sum_{\mathcal{F}_i = \phi, A, h} \left(\prod_{i=3}^4 \zeta_{\lambda_i}^*(\mathcal{F}_i)\right) \mathcal{M}\left[S^{(n_1)}S^{(n_2)} \to \mathcal{F}_{3,\lambda_3}^{(n_3)}\mathcal{F}_{4,\lambda_4}^{(n_4)}\right]
$$

$$
\zeta_{\lambda}(\phi) = \begin{cases} 1 & \lambda = 0 \\ 0 & \text{else} \end{cases}, \qquad \zeta_{\lambda}(A) = \begin{cases} -i\sqrt{3} & \lambda = 0 \\ -i & \lambda = \pm 1 \\ 0 & \text{else} \end{cases}, \qquad \zeta_{\lambda}(h) = 1.
$$

$$
\tilde{\epsilon}_{\lambda}^{\mu} = \begin{cases} \epsilon_{\lambda}^{\mu} & \lambda = \pm \\ \tilde{\epsilon}_{0}^{\mu} & \lambda = 0 \end{cases}, \qquad \tilde{\epsilon}_{\lambda}^{\mu\nu} = \begin{cases} \epsilon_{\lambda}^{\mu\nu} & \lambda = \pm 2 \\ \tilde{\epsilon}_{\lambda}^{\mu\nu} & \text{else} \end{cases}
$$

Xing Wang, UCSD xiw006@physics.ucsd.edu *±* N) $\sqrt{5}$ ics.ucsa.eau $\overline{}$ While we have the above identities for one external Kill world above it to the case of the case of the case of α Sing Wang, UCSD and two Kiw006@physics.ucsd.edu

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Infinite Parameter Lie Algebra $\frac{1}{2}$ α and α α α α

$$
[P_n^{\mu}, P_m^{\nu}] = 0,
$$

\n
$$
[M_n^{\mu\nu}, P_m^{\sigma}] = \sum_l \langle f^{(n)} f^{(m)} f^{(l)} \rangle \left(\eta^{\mu\sigma} P_l^{\nu} - \eta^{\nu\sigma} P_l^{\mu} \right),
$$

\n
$$
[M_n^{\mu\nu}, M_m^{\rho\sigma}] = \sum_l \langle f^{(n)} f^{(m)} f^{(l)} \rangle \left(\eta^{\mu\rho} M_l^{\nu\sigma} + \eta^{\nu\sigma} M_l^{\mu\rho} - \eta^{\mu\sigma} M_l^{\nu\rho} - \eta^{\nu\rho} M_l^{\mu\sigma} \right),
$$

\n
$$
[Q_n, Q_m] = \sum_l \left[\frac{m_m - m_n}{2} \left(\langle f^{(m)} g^{(n)} g^{(l)} \rangle + \langle g^{(m)} f^{(n)} g^{(l)} \rangle \right) + \frac{m_m + m_n}{2} \left(\langle f^{(m)} g^{(n)} g^{(l)} \rangle - \langle g^{(m)} f^{(n)} g^{(l)} \rangle \right) \right] Q_l,
$$

\n
$$
[Q_n, P_m^{\mu}] = \sum_l m_m \langle g^{(m)} g^{(n)} f^{(l)} \rangle P_l^{\mu},
$$

\n
$$
[Q_n, M_m^{\mu\nu}] = \sum_l m_m \langle g^{(m)} g^{(n)} f^{(l)} \rangle M_l^{\mu\nu}.
$$

Xing Wang, UCSD xiw006@physics.ucsd.edu mode eigenequations [15–17]. Note that the masses of the eigenmodes *mⁿ* and the overlap integrals defined by Eq. (55)

20 In the last expressions, which require evaluating derivatives of the mode functions, we use the SUSY structure of the