# Higher order QCD corrections to top-quark pair production in the SMEFT

based on arXiv:2309.16758 [hep-ph], N. Kidonakis, AT

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# Motivation

- At the LHC, the top quark plays an important role in the search for new physics (in BSM frameworks like SMEFT)
- It is very important to have precise determination of the top-quark production rates by going at higher order in QCD
- Soft-gluon corrections are an important subset of QCD corrections, dominate at LHC energies and provide excellent approximations at NLO and NNLO to the full calculation (for $t\bar{t}$ production in the SM, see e.g. [N. Kidonakis, 1806.03336])
- SMEFT contributions have been computed and automated at NLO, here we take a step further by going at approximate NNLO (aNNLO)
- We consider the chromomagnetic SMEFT operator and we calculate total cross sections and $p_T$ distributions at aNNLO in QCD
- Other applications of resummation, see N. Kidonakis talks on May 14 ($pp \to H^+H^-$) and May 16 ($pp \to t\bar{t}W$)

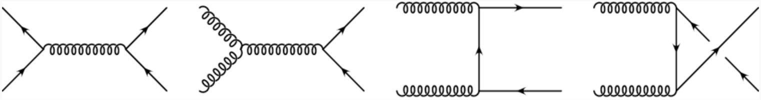
# Soft-gluon corrections and resummation

# $t\bar{t}$ production in the SM

- Partonic processes contributing to $t\bar{t}$ at LHC

$$f_1(p_1) + f_2(p_2) \to t(p_t) + \bar{t}(p_{\bar{t}}) + X$$

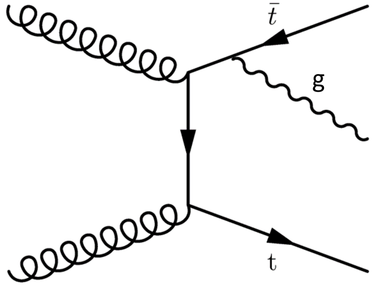
- Mandelstam variables: $s = (p_1 + p_2)^2$, $t = (p_1 - p_t)^2$, and $u = (p_2 - p_t)^2$
- LO partonic channels are $q\bar{q} \to t\bar{t}$ and $gg \to t\bar{t}$



- At NLO we have one-loop virtual diagrams

# NLO real emission term

- At NLO we also have additional gluon emission with momentum $p_g$




- Define the 1PI kinematic variable

$$s_4 = s + t + u - 2m_t^2 = (p_{\bar{t}} + p_g)^2 - m_t^2$$

- When $p_g \to 0$ (soft gluon limit) we approach the so-called partonic threshold and we have $s_4 \to 0$

# Soft-gluon corrections

- In fixed-order (FO) calculations of the hadronic cross section, the soft divergent terms of real and virtual contributions cancel
- BUT this cancellation is incomplete in the sense that numerically large log reminders are left behind and appear systematically to all orders in perturbation theory

$$\alpha_s^n[(\log^k(s_4/m_t^2))/s_4]_+$$

- These soft-gluon contributions can be resummed to all orders in the eikonal approximation and by going to Laplace space (phase space factorization)

$$\alpha_s^n \log^{k+1} N$$

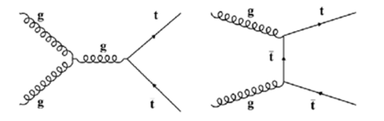
with $0 \leq k \leq 2n-1$

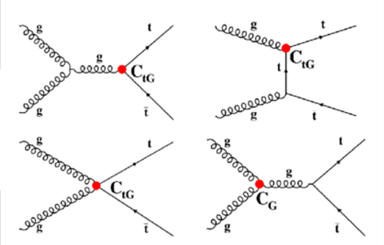
# $t\bar{t}$ production in the SMEFT

- We consider SM + the chromomagnetic dipole operator

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \frac{c_{tG}}{\Lambda^2} g_S \bar{q}_{3L} \sigma_{\mu\nu} T^A t_R \tilde{\varphi} G^{\mu\nu}_A + \text{h.c.}$$

- LO Feynmann diagrams




Soft-gluon contributions can be resummed as in the SM (universal)

# Approximate higher order results

- Re-factorization + RG evolution leads to resummation [arXiv:2008.09914]

$$
\begin{aligned}
d\tilde{\hat{\sigma}}^{\rm resum}_{ab\to t\bar{t}}(N,\mu_F) &= \exp\left[\sum_{i=a,b} E_i(N_i)\right] \exp\left[\sum_{i=a,b} 2\int_{\mu_F}^{\sqrt{s}} \frac{d\mu}{\mu}\gamma_{i/i}(N_i)\right] \\
&\times {\rm tr}\left\{ H_{ab\to t\bar{t}}\left(\alpha_s(\sqrt{s})\right) \bar{P}\exp\left[\int_{\sqrt{s}}^{\sqrt{s}/N} \frac{d\mu}{\mu}\,\Gamma^{\dagger}_{S\,ab\to t\bar{t}}\left(\alpha_s(\mu)\right)\right]\right. \\
&\left.\times \tilde{S}_{ab\to t\bar{t}}\left(\alpha_s\left(\frac{\sqrt{s}}{N}\right)\right) P\exp\left[\int_{\sqrt{s}}^{\sqrt{s}/N} \frac{d\mu}{\mu}\,\Gamma_{S\,ab\to t\bar{t}}\left(\alpha_s\right)\right]\right\}
\end{aligned}
$$

- By expanding the resummed cross section in the Laplace space at some specific order we generate our *approximate FO results*
- We transform the corrections back to momentum space (no prescription needed)
- We match to NLO results

# Results

# Total cross section

- Cross section is a polynomial of second degree in the Wilson coefficient $c_{tG}$

$$\sigma(c_{tG}) = \beta_0 + \frac{c_{tG}}{(\Lambda/1\mathrm{TeV})^2}\beta_1 + \frac{c_{tG}^2}{(\Lambda/1\mathrm{TeV})^4}\beta_2$$

- $\beta_0$ is the SM cross section, $\beta_1$ is the SM-SMEFT interference and $\beta_2$ is the pure SMEFT term
- Complete LO and NLO QCD results at 13 and 13.6 TeV for $\beta_i$ are calculated using MadGraph5_aMC@NLO
- We use MSHT20 pdf and set $\mu_F = \mu_R = \mu$
- Central results are obtained by setting $\mu = m_t = 172.5$ GeV
- Scale uncertainties are obtained by varying $\mu$ in the range $m_t/2 \leq \mu \leq 2m_t$
- Pdf uncertainties are also computed

# $K$-factors of $\beta_i$ terms

- Considering MSHT20 NNLO pdf at 13 TeV, the NLO over LO $K$-factors are :

$$\frac{\beta_0^{\rm NLO}}{\beta_0^{\rm LO}} = 1.50\,, \qquad \frac{\beta_1^{\rm NLO}}{\beta_1^{\rm LO}} = 1.50\,, \qquad \frac{\beta_2^{\rm NLO}}{\beta_2^{\rm LO}} = 1.49$$

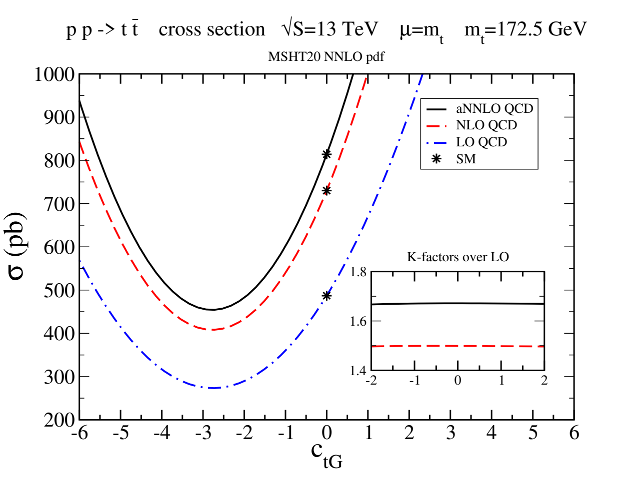
while the aNNLO over LO $K$-factors are:

$$\frac{\beta_0^{\rm NNLO}}{\beta_0^{\rm LO}} = 1.67\,, \qquad \frac{\beta_1^{\rm aNNLO}}{\beta_1^{\rm LO}} = 1.67\,, \qquad \frac{\beta_2^{\rm aNNLO}}{\beta_2^{\rm LO}} = 1.66$$

- $K$-factor similarity between SM and SMEFT contributions of chromomagnetic operator (first presented at NLO in 1503.08841) holds also at aNNLO [1]

[1] NB: This is a scale-dependent and operator-dependent statement.

# Cross section at 13 TeV




Flat NLO and NNLO $K$-factors, (*) is the SM result

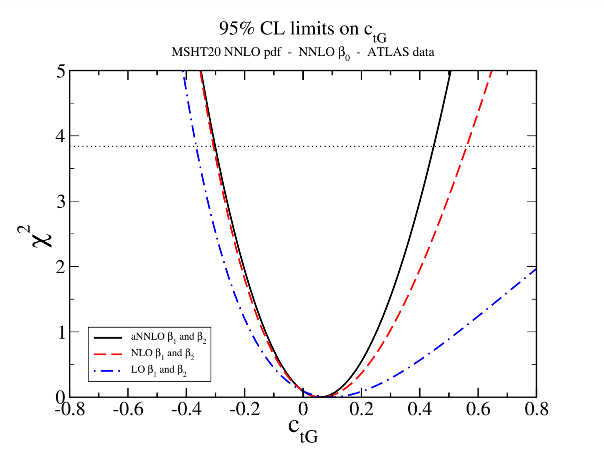
# 95% CL exclusion limits on $c_{tG}$

- We construct the following chi-squared function:

$$\chi^2(c_{tG}) = \frac{[\sigma_{\mathrm{exp}} - \sigma(c_{tG})]^2}{\delta\sigma_{\mathrm{exp}}^2 + \delta\sigma(c_{tG})^2}$$

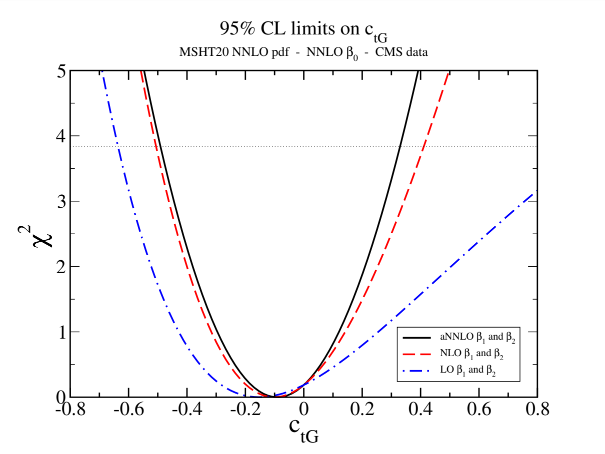
- We consider as $\sigma_{\mathrm{exp}}$ the most recent 13 TeV ATLAS [arXiv:2303.15340] and CMS [arXiv:2108.02803] results of $829 \pm 15$ pb and $791 \pm 25$ pb
- We compare with $\sigma(c_{tG}) = \beta_0 + c_{tG}\beta_1 + c_{tG}^2\beta_2$ (we set $\Lambda = 1$ TeV )
- We use for the SM contribution $\beta_0$ both the NNLO QCD result and the aN$^3$LO QCD result [arXiv:2306.06166]

# ATLAS data




From NLO to aNNLO the negative limit values reduce by about 2% and the positive limit values reduce by about 25%

# CMS data




From NLO to aNNLO the negative limit values reduce by about 3% and the positive limit values reduce by about 35%

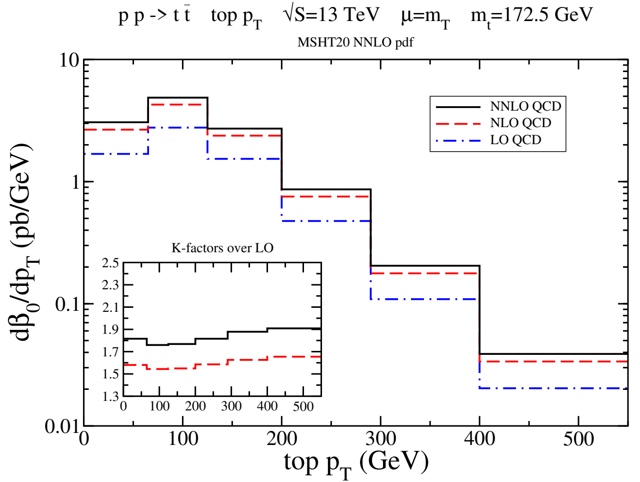
# $p_T$-distributions

- Differential cross section is a polynomial of second degree in $c_{tG}$

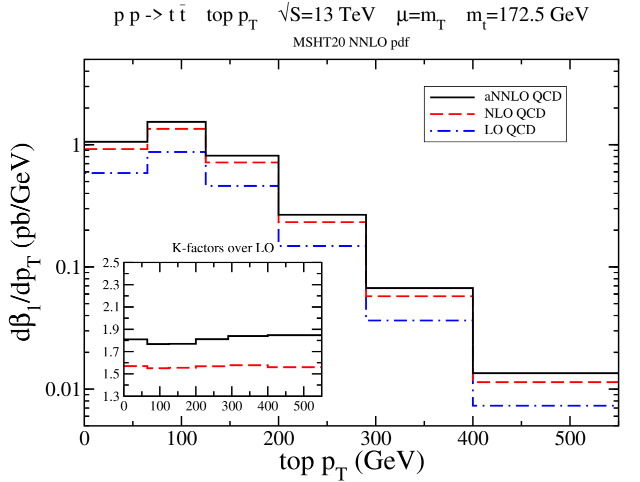
$$\frac{d\sigma(c_{tG})}{dp_T} = \frac{d\beta_0}{dp_T} + \frac{c_{tG}}{(\Lambda/1\mathrm{TeV})^2}\frac{d\beta_1}{dp_T} + \frac{c_{tG}^2}{(\Lambda/1\mathrm{TeV})^4}\frac{d\beta_2}{dp_T}$$

- Complete LO and NLO QCD results at 13 and 13.6 TeV for $\beta_i$ are calculated using MadGraph5_aMC@NLO
- We use MSHT20 NNLO pdf and set $\mu_F = \mu_R = \mu$
- Central results are obtained by setting $\mu = m_T = (p_T^2 + m_t^2)^{1/2}$
- Scale uncertainties are obtained by varying $\mu$ in the range $m_T/2 \leq \mu \leq 2m_T$
- Pdf uncertainties are also computed

# $d\beta_0/dp_T$ at 13 TeV

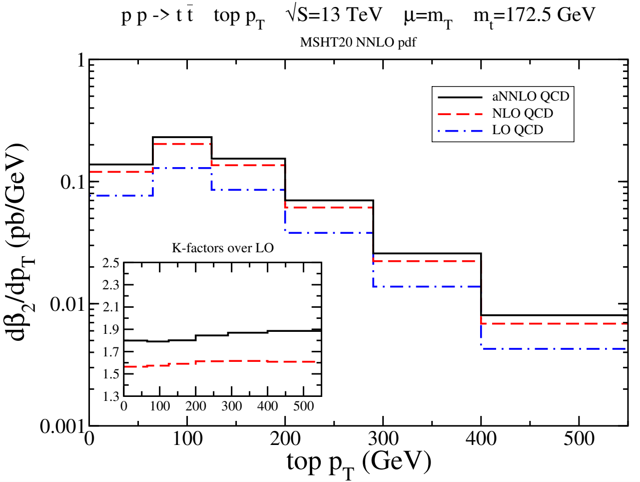


# $d\beta_1/dp_T$ at 13 TeV




SM-SMEFT $K$-factor similarity not true, especially for high $p_T$ bins

# $d\beta_2/dp_T$ at 13 TeV




SM-SMEFT $K$-factor similarity not true, especially for high $p_T$ bins

# Summary

- We have added for the first time soft-gluon corrections at aNNLO to the complete QCD NLO result for $t\bar{t}$ cross section, in the presence of the chromomagnetic dipole operator.
- The additional aNNLO QCD corrections are similar and significant for both SM and SMEFT contributions, accounting for another 17% enhancement of the cross section, and reduce theoretical uncertainties from scale variation.
- In setting constraints, aNNLO corrections improve the lower bound on the $c_{tG}$ coefficient by 2 to 5%, while the upper bound reduces by 23 to 35%, depending on the considered experimental input and SM prediction.

# Summary

- We have also computed SM and SMEFT contributions to the top-quark $p_T$ distribution up to aNNLO in QCD.
- These contributions further significant enhancements at aNNLO, similar to the total cross section case.
- Appreciable differences in SM and SMEFT $K$-factors for the $p_T$ distribution.
- Including these aNNLO contributions is crucial to improve further the sensitivity on SMEFT operators in global-fit analyses that use total cross section and differential distributions.

# Thank you

# BACK UP

# Eikonal approximation

Let us first of all introduce the eikonal approximation [89, 90]. We consider a Feynman diagram with the emission of a soft photon/gluon from an external particle with $p^2 = m^2$ as shown in Fig. 2.3. In QED the structure of this matrix element is given by

$$\mathcal{M} = \tilde{\mathcal{M}} i \frac{(\not{p} + \not{k} + m)}{(p+k)^2 - m^2} (-ie\gamma_\mu)\, u(p). \tag{2.28}$$

When the photon or gluon is sufficiently soft, $k^2$ compared to $p \cdot k$ is small and may be neglected in the propagator denominator. Analogously $\not{k}$ is omitted in the numerator. Making use of the Dirac equation gives

$$\mathcal{M} = \tilde{\mathcal{M}} i \frac{\not{p} + m}{2p \cdot k} (-ie\gamma_\mu)\, u(p) = \tilde{\mathcal{M}} \frac{i}{p \cdot k} (-iep_\mu)\, u(p). \tag{2.29}$$

Thus the eikonal propagator and photon-fermion vertex are given by

$$\frac{i}{p \cdot k + i\epsilon}, \qquad -iep_\mu. \tag{2.30}$$

# Soft-gluon resummation

- Hadronic cross section is a convolution of the partonic cross section with the pdf

$$d\sigma_{pp\to t\bar{t}}(c_{tG}) = \sum_{a,b} \int dx_a\, dx_b\, \phi_{a/p}(x_a,\mu_F)\, \phi_{b/p}(x_b,\mu_F)\, d\hat{\sigma}_{ab\to t\bar{t}}(s_4,\mu_F,c_{tG})$$

- Laplace transform

$$d\hat{\sigma}_{ab\to t\bar{t}}(s_4,\mu_F,c_{tG}) \to \tilde{d\hat{\sigma}}_{ab\to t\bar{t}}(N,\mu_F,c_{tG})$$

- Re-factorization + RG evolution leads to resummation [arXiv:2008.09914]

$$\begin{aligned} d\tilde{\hat{\sigma}}^{\rm resum}_{ab\to t\bar{t}}(N,\mu_F) \quad = \quad & \exp\left[\sum_{i=a,b} E_i(N_i)\right] \exp\left[\sum_{i=a,b} 2\int_{\mu_F}^{\sqrt{s}} \frac{d\mu}{\mu}\gamma_{i/i}(N_i)\right] \\ & \times {\rm tr}\left\{ H_{ab\to t\bar{t}}\left(\alpha_s(\sqrt{s})\right) \bar{P}\, \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/N} \frac{d\mu}{\mu}\, \Gamma^{\dagger}_{S\, ab\to t\bar{t}}\left(\alpha_s(\mu)\right)\right]\right. \\ & \left. \times \tilde{S}_{ab\to t\bar{t}}\left(\alpha_s\left(\frac{\sqrt{s}}{N}\right)\right)\; P\, \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/N} \frac{d\mu}{\mu}\, \Gamma_{S\, ab\to t\bar{t}}\left(\alpha_s\right)\right]\right\} \end{aligned}$$

# Soft-gluon resummation

- Re-factorization + RG evolution leads to resummation

$$
\begin{aligned}
d\tilde{\hat{\sigma}}^{\rm resum}_{ab\to t\bar{t}}(N,\mu_F) \quad = \quad & \exp\left[\sum_{i=a,b} E_i(N_i)\right] \exp\left[\sum_{i=a,b} 2\int_{\mu_F}^{\sqrt{s}} \frac{d\mu}{\mu}\gamma_{i/i}(N_i)\right] \\
& \times {\rm tr}\left\{H_{ab\to t\bar{t}}\left(\alpha_s(\sqrt{s})\right) \bar{P}\exp\left[\int_{\sqrt{s}}^{\sqrt{s}/N} \frac{d\mu}{\mu}\,\Gamma^{\dagger}_{S\,ab\to t\bar{t}}\left(\alpha_s(\mu)\right)\right]\right. \\
& \left.\times \tilde{S}_{ab\to t\bar{t}}\left(\alpha_s\left(\frac{\sqrt{s}}{N}\right)\right)\; P\exp\left[\int_{\sqrt{s}}^{\sqrt{s}/N} \frac{d\mu}{\mu}\,\Gamma_{S\,ab\to t\bar{t}}\left(\alpha_s\right)\right]\right\}
\end{aligned}
$$

- The first exponential resums collinear and soft contributions from incoming partons (universal contributions)
- The second exponential expresses the factorization-scale dependence in terms of the anomalous dimension $\gamma_{i/i}$ of pdf
- Resummation of noncollinear soft-gluon emission is performed via the soft anomalous dimensions $\Gamma_{S\,q\bar{q}\to t\bar{t}}$ and $\Gamma_{S\,gg\to t\bar{t}}$

# Approximate NNLO results 13 TeV

| SM and SMEFT contributions to $t\bar{t}$ cross sections at LHC 13 TeV | | | |
|---|---|---|---|
| | $\beta_0$ (pb) | $\beta_1$ (pb) | $\beta_2$ (pb) |
| LO (LO pdf) | $575^{+186+7}_{-132-7}$ | $184^{+59+3}_{-42-2}$ | $33.4^{+11.3+0.7}_{-7.9\ \ -0.5}$ |
| LO (NLO pdf) | $488^{+143+8}_{-104-8}$ | $156^{+45+2}_{-33-3}$ | $28.1^{+8.7+0.6}_{-6.2-0.4}$ |
| LO (NNLO pdf) | $487^{+142+10}_{-103-6}$ | $155^{+46+4}_{-32-2}$ | $28.1^{+8.6+0.7}_{-6.1-0.4}$ |
| NLO (NLO pdf) | $730^{+86+13}_{-86-11}$ | $233^{+27+4}_{-27-4}$ | $41.9^{+4.8+0.8}_{-5.0-0.7}$ |
| NLO (NNLO pdf) | $730^{+85+14}_{-86-10}$ | $232^{+27+5}_{-27-3}$ | $41.8^{+4.8+1.0}_{-5.0-0.6}$ |
| aNNLO (NNLO pdf) | $814^{+28+16}_{-46-11}$ | $259^{+9\ \ +6}_{-15-3}$ | $46.6^{+1.6+1.1}_{-2.6-0.7}$ |

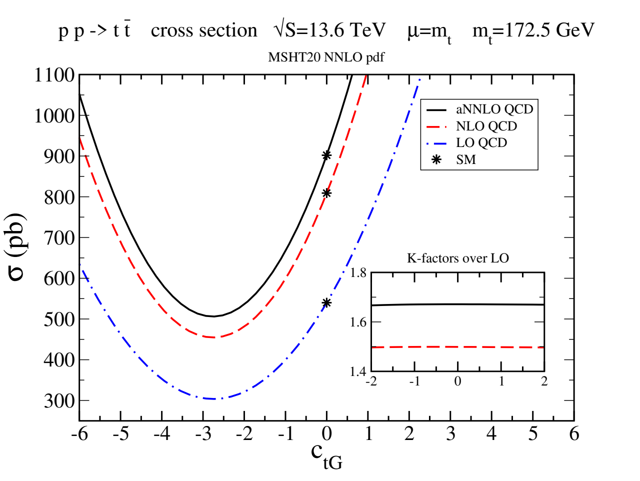
Scale uncertainties are similar for SM and SMEFT: they are roughly $+12\%$ $-12\%$ at NLO and around $+3.4\%$ $-5.5\%$ at aNNLO for both LHC energies. Pdf uncertainties are much smaller than the scale uncertainties.

# Approximate NNLO results 13.6 TeV

| SM and SMEFT contributions to $t\bar{t}$ cross sections at LHC 13.6 TeV | | | |
|---|---|---|---|
| | $\beta_0$ (pb) | $\beta_1$ (pb) | $\beta_2$ (pb) |
| LO (LO pdf) | $638^{+203+11}_{-145-8}$ | $204^{+65+4}_{-46-3}$ | $37.3^{+12.4+0.7}_{-8.8\ -0.6}$ |
| LO (NLO pdf) | $540^{+156+9}_{-114-8}$ | $172^{+50+3}_{-36-2}$ | $31.4^{+9.5+0.6}_{-6.9-0.6}$ |
| LO (NNLO pdf) | $540^{+155+10}_{-113-7}$ | $172^{+49+3}_{-36-2}$ | $31.3^{+9.4+0.7}_{-6.8-0.5}$ |
| NLO (NLO pdf) | $810^{+95+14}_{-95-12}$ | $258^{+30+4}_{-30-4}$ | $46.7^{+5.4+0.9}_{-5.6-0.8}$ |
| NLO (NNLO pdf) | $809^{+94+16}_{-94-11}$ | $257^{+29+5}_{-30-3}$ | $46.6^{+5.3+1.0}_{-5.5-0.7}$ |
| aNNLO (NNLO pdf) | $902^{+31+18}_{-50-12}$ | $287^{+10+6}_{-16-3}$ | $52.0^{+1.8+1.1}_{-2.9-0.8}$ |

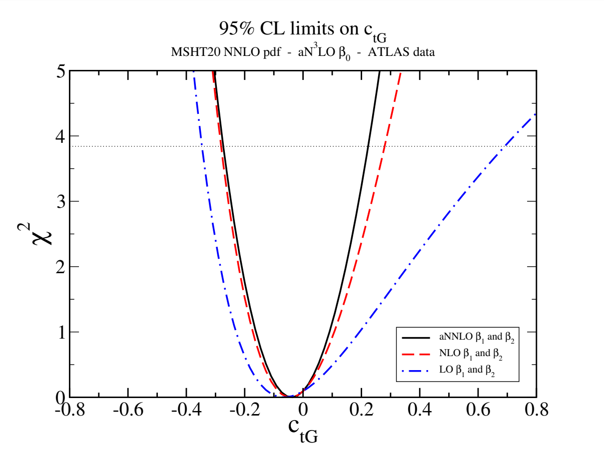
Scale uncertainties are similar for SM and SMEFT: they are roughly $+12\%$ $-12\%$ at NLO and around $+3.4\%$ $-5.5\%$ at aNNLO for both LHC energies. Pdf uncertainties are much smaller than the scale uncertainties.

# Cross section at 13.6 TeV



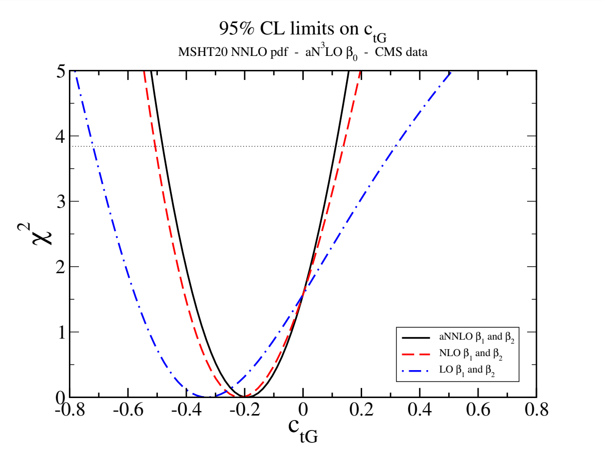

Flat NLO and NNLO $K$-factors, (*) is the SM result

# ATLAS data




At aNNLO the negative limit values reduce by about 3% and the positive limit values reduce by about 25%

# CMS data




At aNNLO the negative limit values reduce by about 5% and the positive limit values reduce by about 23%