

CAN QCD-ENERGY-SCALE-TURBULENCE SOURCED GRAVITATIONAL WAVES BE DETECTED THROUGH PTAs?

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15 MAY, 2024

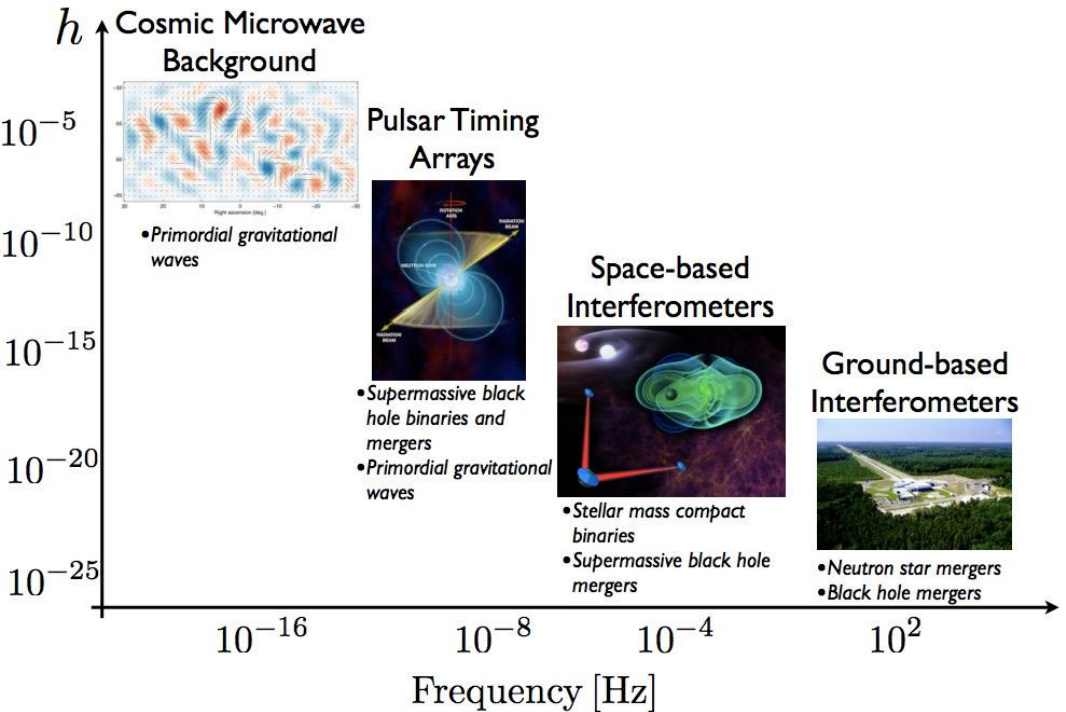
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- ANDREW LONG, PHILIP MOCZ, JENNIFER SCHOBER, MARK VOGELBERGER...

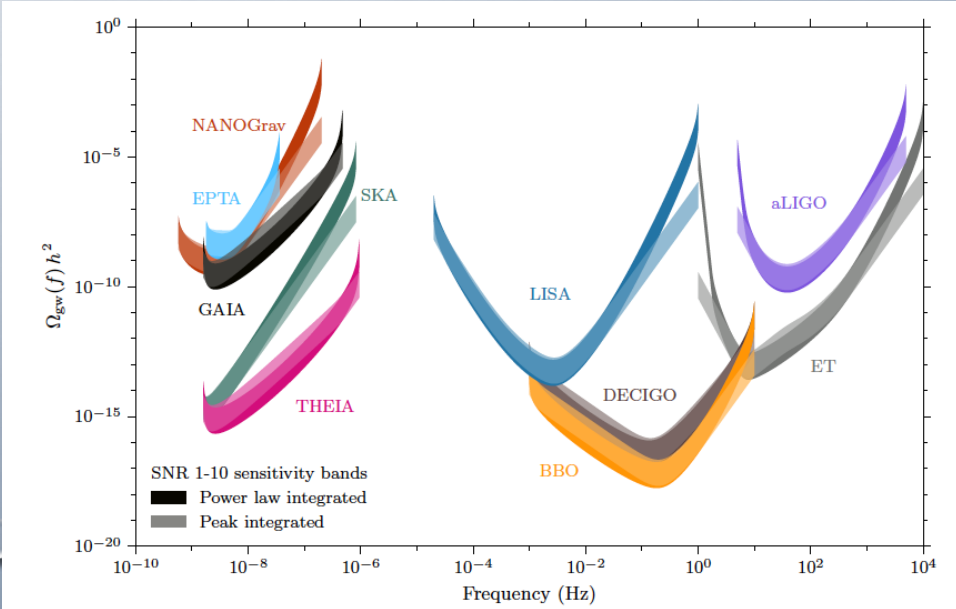
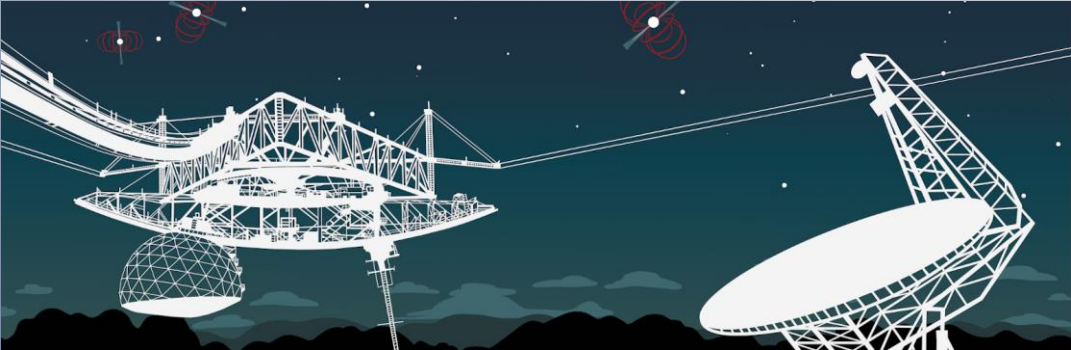
Connection with High Energy Particle Physics – the best laboratory to test the energy scales *EVEN* near the Planck scale

- The very early universe (inflation)
- Topological defects/strings
- Cosmological phase transitions
 - Bubble nucleation/collisions
 - Sound waves
 - Hydro turbulence
 - MHD turbulence

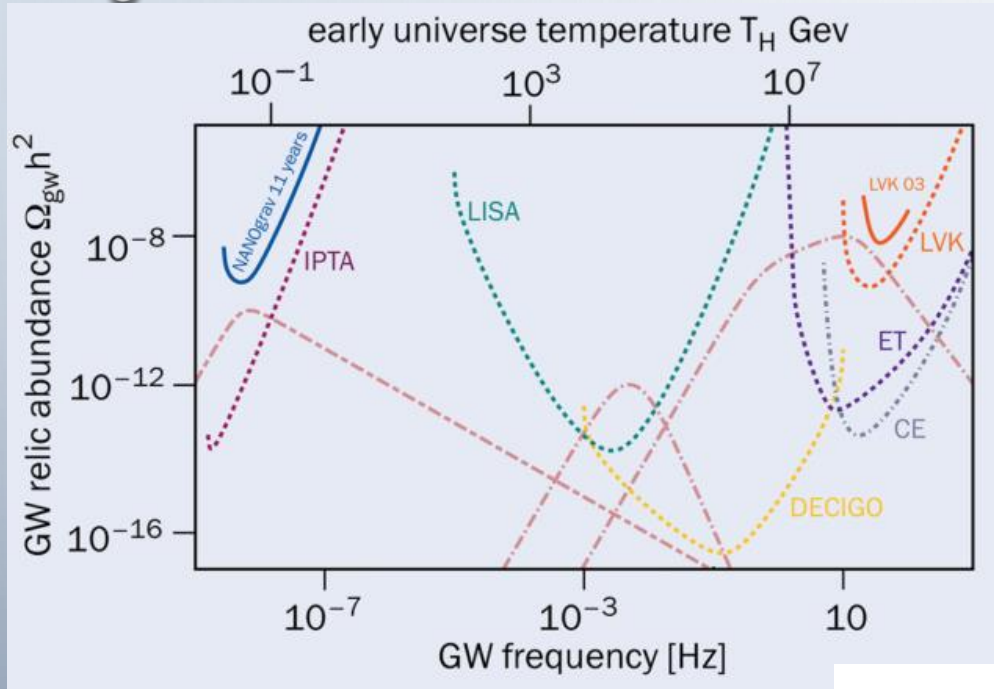
Garcia Bellido et al. 2020



Credit: NANOGrav



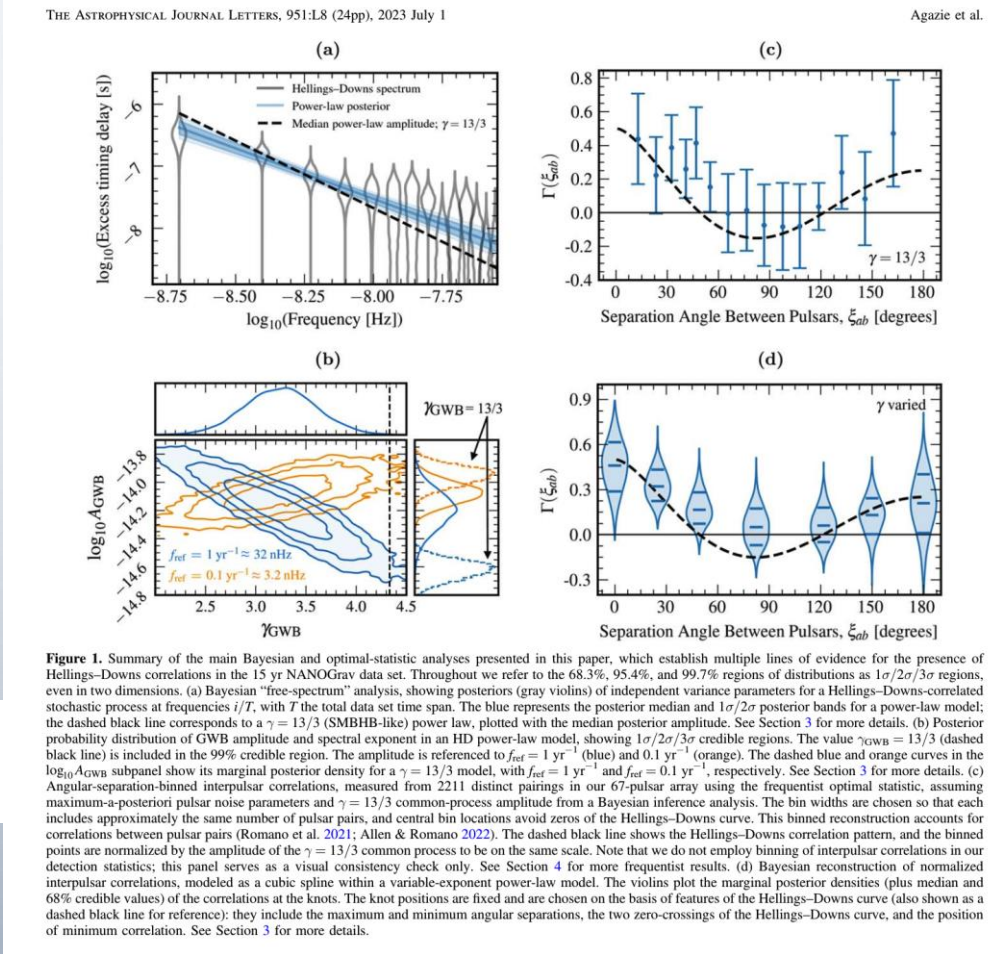
EARLY UNIVERSE – GRAVITATIONAL WAVES



Credit: CERN Courier, 2021

Sensitivity of current (solid) and future (dashed) gravitational-wave (GW) observatories to stochastic GW backgrounds (expressed in terms of the energy density fraction in the universe today). On the upper x-axis, the temperature in the early universe is given, which is obtained when the peak frequency of a GW signal is equal to the inverse of the expansion rate when GWs are emitted. Some example possible GW spectra from the early universe are also shown (pink, dashed). F. Rompineve/ arXiv:2101.12130/arXiv:2002.0461

$$\Omega_{GW}(t, f) = \frac{1}{\mathcal{E}_{crit}(t)} \frac{d\mathcal{E}_{GW}}{d \ln f}$$



Credit: NANOGrav 2023

$$\Omega_{GW}(f) = \frac{2\pi^2}{3H_0^2} f^2 h_c^2(f) = \Omega_{GW}^{yr} \left(\frac{f}{f_{yr}} \right)^{5-\gamma_{CP}}$$

NANOGrav SIGNAL – POSSIBLE SOURCES

Astrophysical:

- ✓ Super massive black hole binary (SMBHB) (Phinney 2001): $\gamma=13/3$

Cosmological:

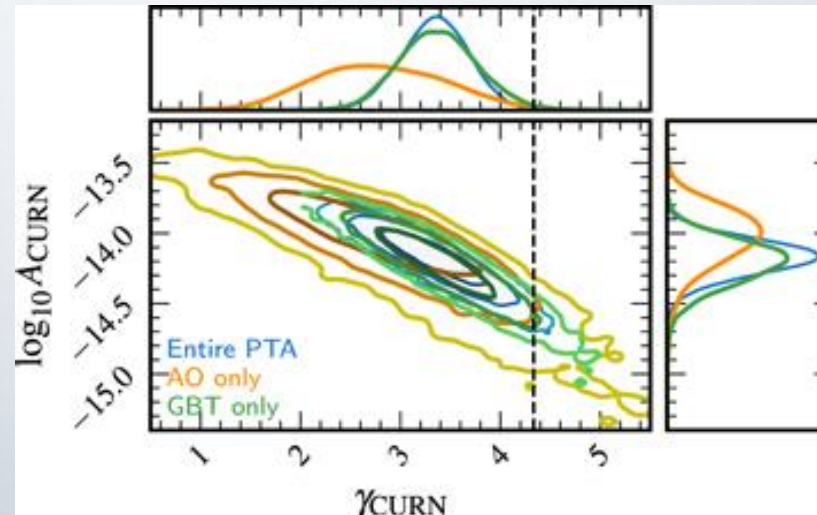
- ✓ Bubbles collisions (Kosowsky et al. 1993)
- ✓ Inflation (Vagnozzi 2020)
- ✓ Cosmic strings (Blanco-Pillado et al. 2020)
- ✓ Seed magnetic fields (Neronov et al. 2020)
- ✓ Hydrodynamic and MHD Turbulence (Brandenburg et al. 2021)

QCD energy scale

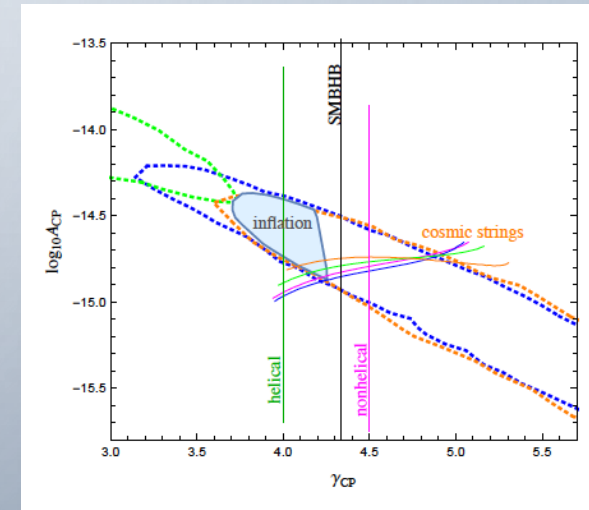
$$\frac{a_0}{a_*} = 10^{12} \left(\frac{g_{S,*}}{15}\right)^{\frac{1}{3}} \left(\frac{T_*}{150 \text{ MeV}}\right)$$

$$f_H \simeq (1.8 \times 10^{-8} \text{ Hz}) 10^{12} \left(\frac{g_*}{15}\right)^{\frac{1}{3}} \left(\frac{T_*}{150 \text{ MeV}}\right)$$

$$H_*^2 = \frac{8\pi G}{2} \epsilon_{\text{rad},*} \quad \epsilon_{\text{rad},*} = \frac{\pi^2 g_*}{30} T_*^4 \quad (c = k_B = \hbar = 1)$$



The NANOGrav 15 yr Data Set: Evidence for a Gravitational-wave Background



Clarke, et al. 2021

QCD PHASE TRANSITIONS – GRAVITATIONAL WAVES



Pulsar Timing Arrays (PTAs) are sensible to gravitational waves generated or present at QCD energy scales

Sazhin, 1978, Detweiler 1979

PHYSICAL REVIEW D

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15 MARCH 1994

Gravitational radiation from first-order phase transitions

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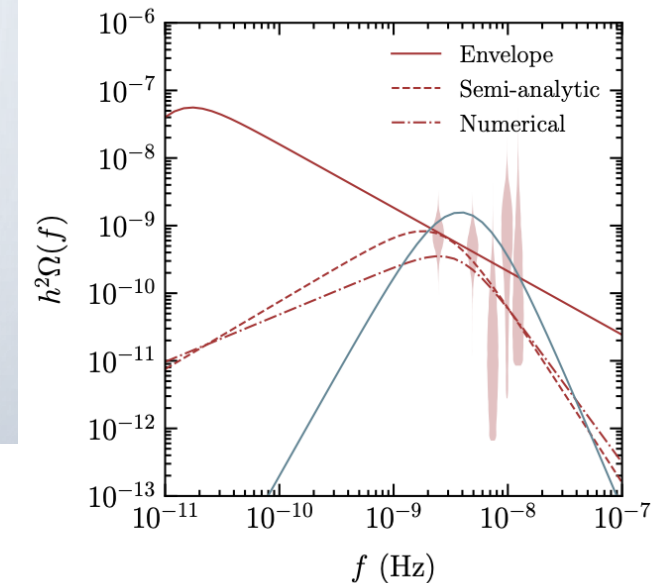
Enrico Fermi Institute, The University of Chicago, Chicago, Illinois 60637-1433

(Received 26 October 1993)

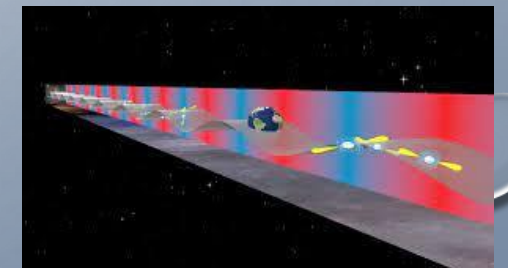
We consider the stochastic background of gravity waves produced by first-order cosmological phase transitions from two types of sources: colliding bubbles and hydrodynamic turbulence. First we discuss the fluid mechanics of relativistic spherical combustion. We then numerically collide many bubbles expanding at a velocity v and calculate the resulting spectrum of gravitational radiation in the linearized gravity approximation. Our results are expressed as simple functions of the mean bubble separation, the bubble expansion velocity, the latent heat, and the efficiency of converting latent heat to kinetic energy of the bubble walls. A first-order phase transition is also likely to excite a Kolmogoroff spectrum of turbulence. We estimate the gravity waves produced by such a spectrum of turbulence and find that the characteristic amplitude of the gravity waves produced is comparable to that from bubble collisions. Finally, we apply these results to the electroweak transition. Using the one-loop effective potential for the minimal electroweak model, the characteristic amplitude of the gravity waves produced is $h \simeq 1.5 \times 10^{-27}$ at a characteristic frequency of 4.1×10^{-3} Hz corresponding to $\Omega \sim 10^{-22}$ in gravity waves, far too small for detection. Gravity waves from more strongly first-order phase transitions, including the electroweak transition in nonminimal models, have better prospects for detection, though probably not by LIGO.

Pioneering works:

- *Winicour 1973*
- *Hogan 1982, 1986*
- *Turner & Wilczek 1990*
- *Kosowsky, Turner, Watkins. 1992*



NANOGrav 12.5yr data



NANOGrav & PHASE TRANSITIONS

PHYSICAL REVIEW LETTERS 127, 251302 (2021)

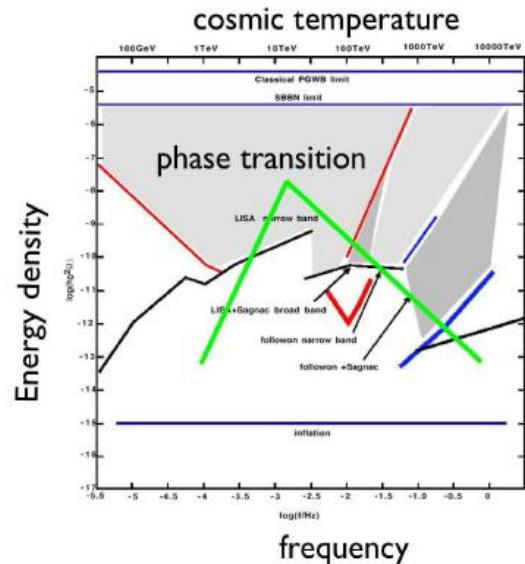
Editors' Suggestion

Featured in Physics

Searching for Gravitational Waves from Cosmological Phase Transitions with the NANOGrav 12.5-Year Dataset

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(NANOGrav Collaboration)



C. Hogan, 2006

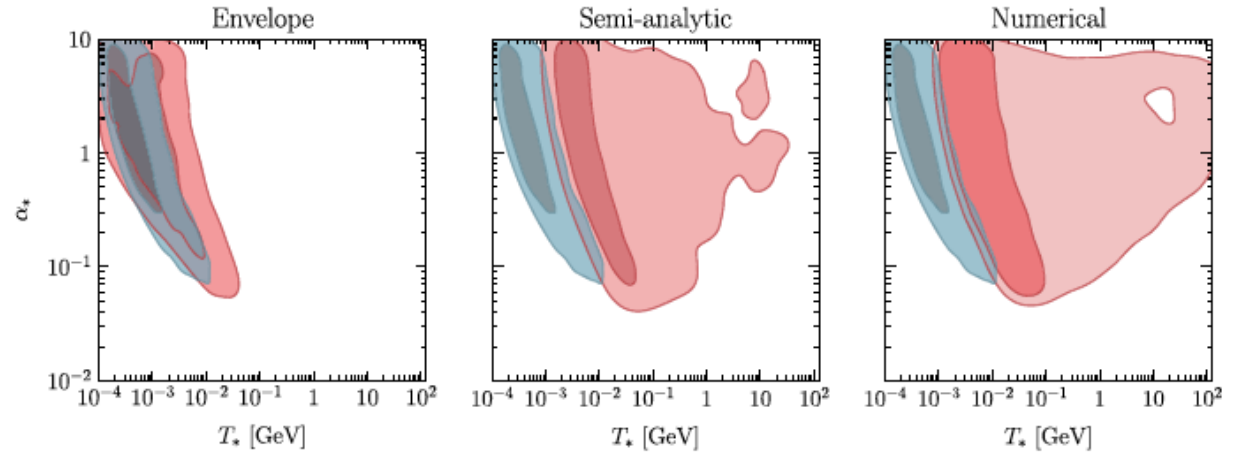


FIG. 1. In red (blue) the $1\text{-}\sigma$ (68% posterior credible level), and $2\text{-}\sigma$ (95% posterior credible level) contours for the two-dimensional posterior distributions in the (T_*, α_*) plane obtained in the BO (SWO). The BO analysis has been performed with the spectral shape computed by using the envelope approximation (left panel), semianalytic results (central panel), and numerical results (right panel). Specifically, we use $(a, b, c) = (1, 2.61, 1.5)$ for the semianalytic results, and $(a, b, c) = (0.7, 2.3, 1)$ for the numerical results.

T_* [GeV] Phase transition temperature
 α_* Phase transition strength
 H_*/β Bubble nucleation rate
 v_w Bubble wall velocity

TABLE I. Parameters for the gravitational wave spectrum of Eq. (4). The values of the parameters (a, b, c) in the spectral shape of the bubble contribution are reported in Table II.

	Bubbles [58]	Sound waves [59]
$\Delta(v_w)$	$[0.48 v_w^3 / (1 + 5.3 v_w^2 + 5 v_w^4)]$	$0.513 v_w$
κ	κ_ϕ	κ_{SW}
p	2	2
q	2	1
$S(x)$	$\{(a + b)^c / [bx^{-a/c} + ax^{b/c}]^c\}$	$x^3 [7 / (4 + 3x^2)]^{7/2}$
f_*/β	$[0.35 / (1 + 0.07 v_w + 0.69 v_w^4)]$	$(0.536 / v_w)$

GRAVITATIONAL WAVES – ANISOTROPIC STRESS

Mon. Not. R. astr. Soc. (1987) **229**, 357–370

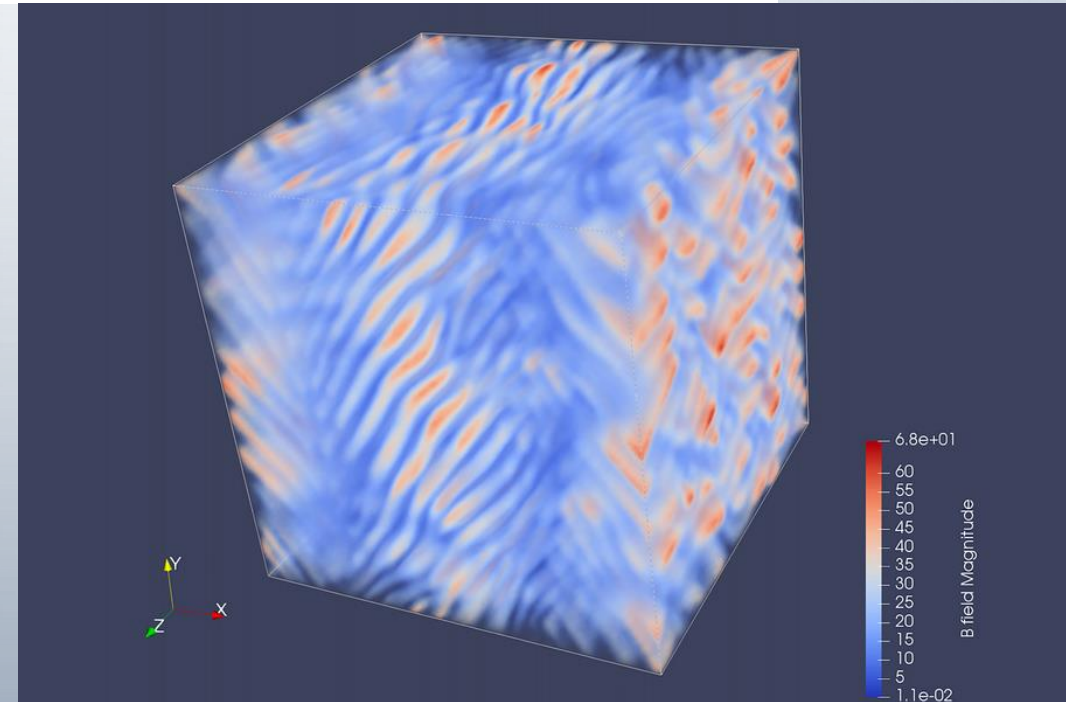
$$\nabla^2 h_{ij}(\mathbf{x}, t) - \frac{\partial^2}{\partial t^2} h_{ij}(\mathbf{x}, t) = -16\pi G S_{ij}(\mathbf{x}, t)$$

Generation of gravitational waves by the anisotropic phases in the early Universe

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M. V. Sazhin *P. K. Sternberg Astronomical Institute, Universitetskii pr. 13,
Moscow 119899, USSR*

**Magnetic fields;
Turbulence (hydro & MHD)**



Greenwald, 2022

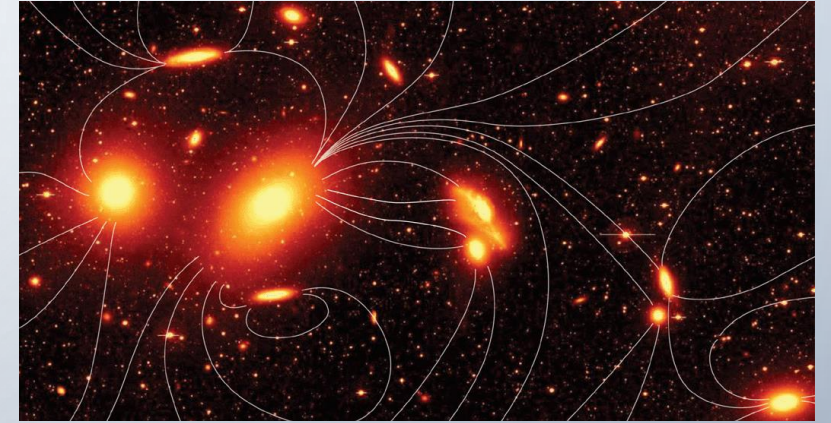
WHY PRIMORDIAL MAGNETIC FIELDS?

- cosmic seed magnetic fields
 - astrophysical seeds
 - cosmological seeds
- observations
 - Fermi data – blazars spectra

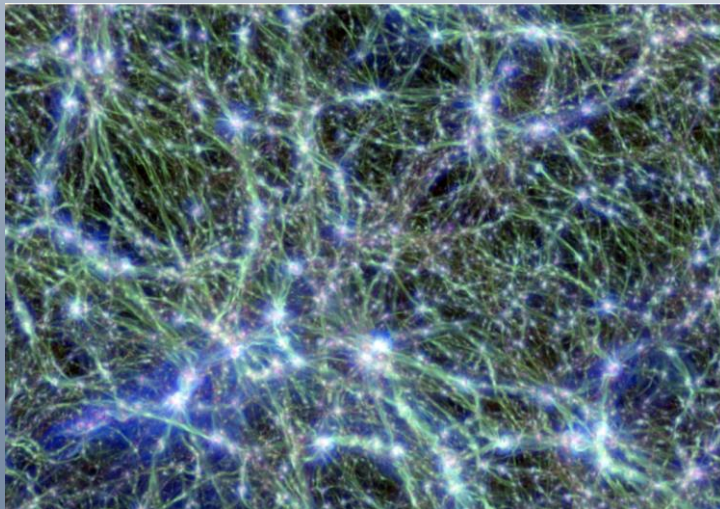


E. Fermi “*On the origin of the cosmic radiation*”, PRD, 75, 1169 (1949)

F. Hoyle in Proc. “*La structure et l’evolution de l’Universe*” (1958)



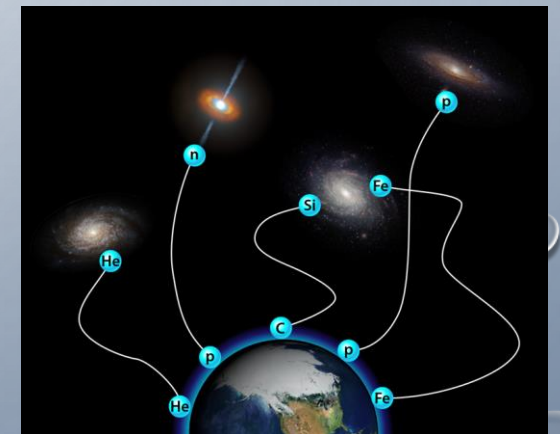
Durrer 2008



Vazza et al. 2018



Borlaff et al. 2021

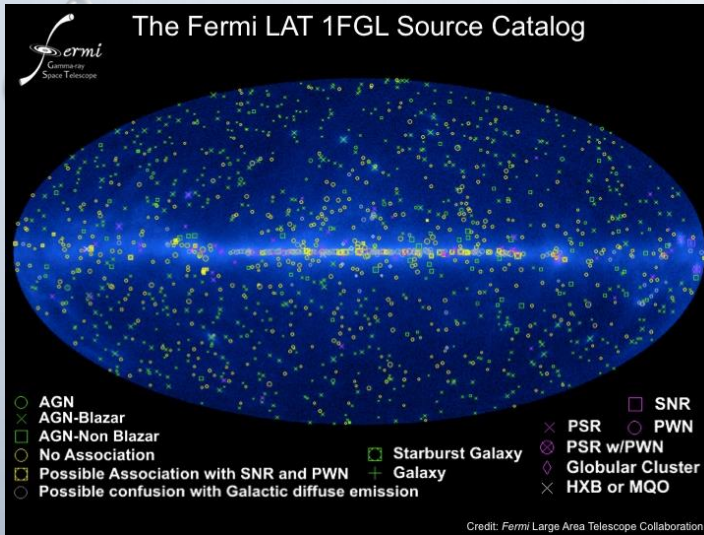


Credit: APSr

OBSERVATIONS: FERMI DATA

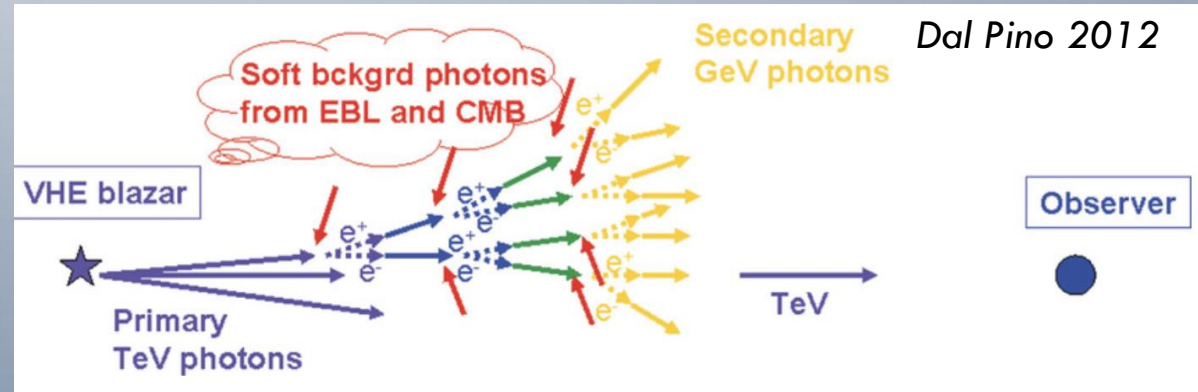
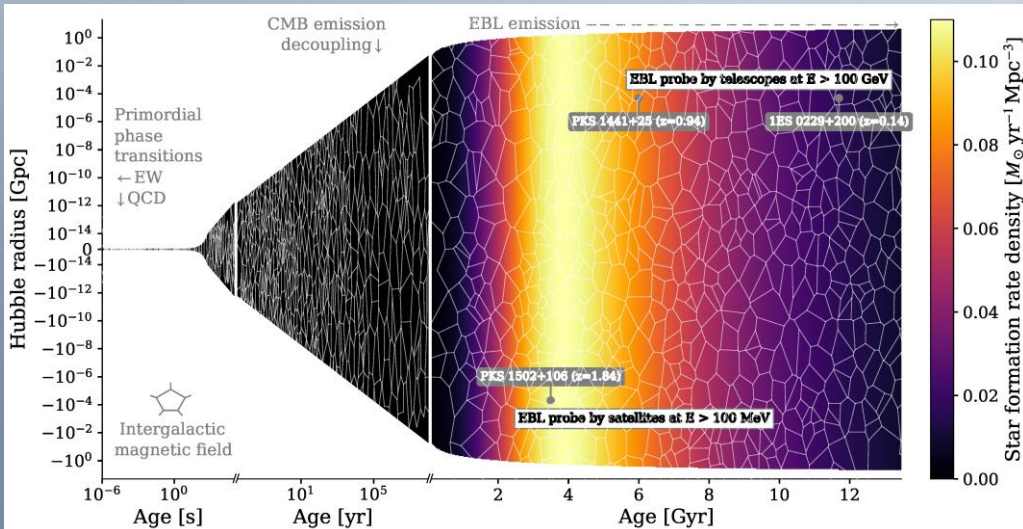
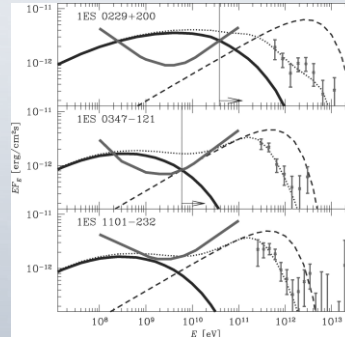
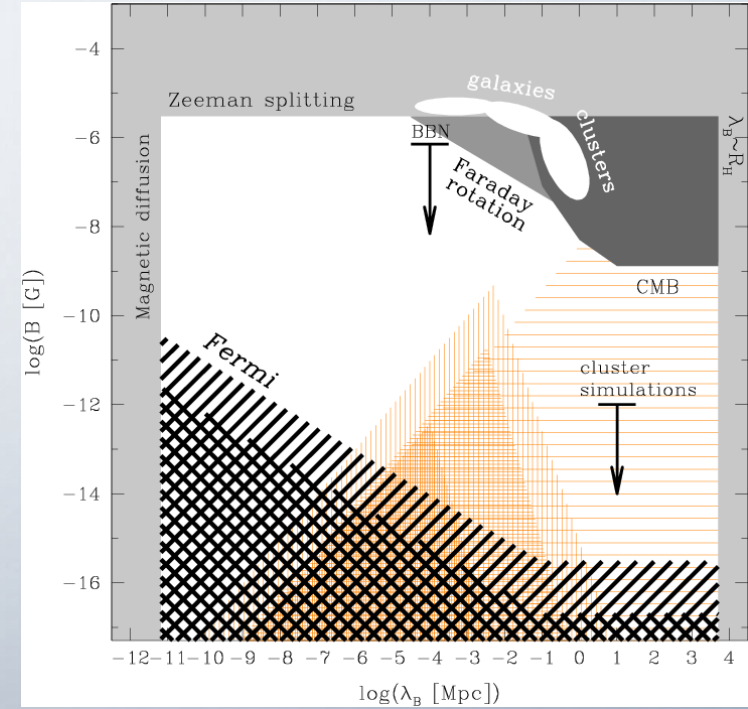
$$\gamma + \gamma \rightarrow e^+ + e^-$$

A. Neronov & E. Vovk, "Evidence for Strong Extragalactic Magnetic Fields from Fermi Observations of TeV Blazars", *Science* 328, 5974 (2010)

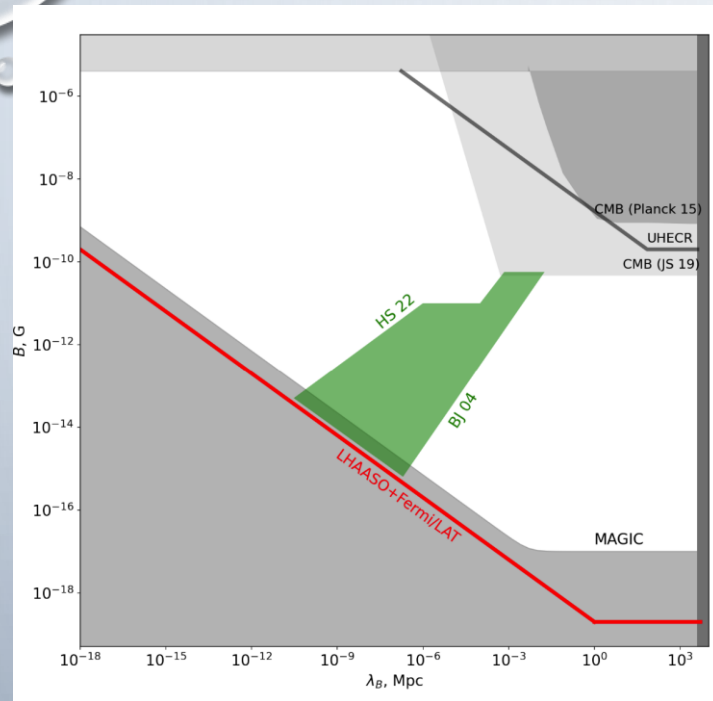


Credit FermiLAT

Bitteau, 2022



RECENT BLAZARS SPECTRA OBSERVATIONS:



E. Vovk, et al. “Constraint on intergalactic magnetic field from Fermi/LAT observations of the “pair echo” of GRB 221009A”
Astron.Astrophys. 683 A25 (2024)

Fig. 2. from Vovk et al. 2024: Lower bound on IGMF derived from the GRB 221009A (red line), compared to existing bounds from γ -ray, radio, CMB and UHECR observations and predictions of the cosmological evolution models. The CMB upper bounds are from Planck (Planck Collaboration et al. 2016) and from the analysis of Jedamzik & Saveliev (2019). UHECR upper bound is from Neronov et al. (2021). MAGIC lower bound is from Acciari et al. (2023). Green-shaded area shows the range of predictions for the endpoints of cosmological evolution of primordial magnetic fields. BJ04 is from Banerjee & Jedamzik (2004), HS22 is from Hosking & Schekochihin (2022).

Aharonian, F., et al. (Fermi-LAT, H. E. S. S), “Constraints on the intergalactic magnetic field using Fermi-LAT and H.E.S.S. blazar observations”. *Astrophys. J. Lett.* 950, L16 (2023)

V. A. Acciari et al. [MAGIC Collaboration] “A Lower Bound on Intergalactic Magnetic Fields from Time Variability of 1ES 0229+200 from MAGIC and Fermi/LAT Observations” *Astron.Astrophys.* 670 A145 (2023)

S. Archambault et al. [VERITAS Collaboration], “Search for Magnetically Broadened Cascade Emission From Blazars with VERITAS,” *Astrophys. J.* 835, 288 (2017)

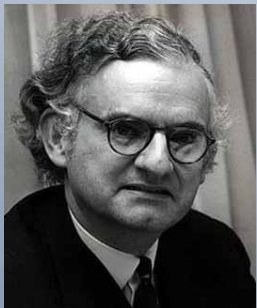
M. Ackermann, et al. [Fermi-LAT Collaboration], “The Search for Spatial Extension in High-Latitude Sources Detected by the Fermi Large Area Telescope,” *Astrophys. J. Suppl.* 237, 32 (2018)

GRAVITATIONAL WAVES FROM PRIMORDIAL TURBULENCE (AND/OR) MAGNETIC FIELDS

Gogoberidze, et al 2007

$$\nabla^2 \delta\rho(\mathbf{x}, t) - \frac{1}{c_s^2} \frac{\partial^2}{\partial t^2} \delta\rho(\mathbf{x}, t) = -\frac{\partial^2}{\partial x^i \partial x^j} T^{ij}(\mathbf{x}, t), \quad c_s^2 = \frac{\partial p}{\partial \rho}$$

$$\nabla^2 h_{ij}(\mathbf{x}, t) - \frac{\partial^2}{\partial t^2} h_{ij}(\mathbf{x}, t) = -16\pi G S_{ij}(\mathbf{x}, t), \quad c = 1$$



Aero-acoustic approximation:

- ✓ sound waves generation by turbulence
- ✓ gravitational waves generation

Lighthill, 1952;

Proudman 1952

Kosowsky, et al, 2002,

Dolgov, et al. 2002

Parameters:

τ_T turbulence lasting time

k_0 stirring scale

$M = v_0/c$ - Mach number

$R^{3/4} = k_d/k_0$ - Reynolds number

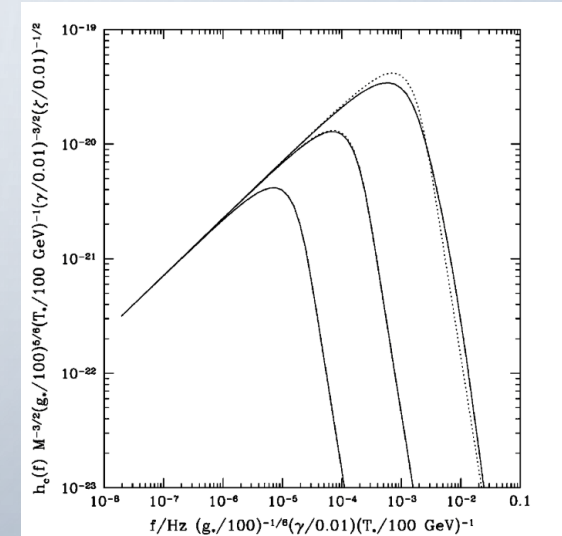
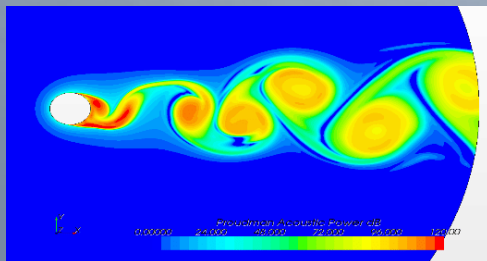


FIG. 1. The spectrum of gravitational radiation from turbulence. The three solid lines are for different Mach numbers, with $M = 0.01$, $M = 0.1$, and $M = 1$ from lowest to highest amplitude. Note that these three cases have also been scaled by a factor of $M^{-3/2}$ for display, since this is how the low-frequency tail scales with M . The dotted lines, which are virtually indistinguishable from the solid lines except for the $M = 1$ case, show the $k = 0$ approximation to the gravitational wave source.



$$f = 1.65 \times 10^{-3} \text{ Hz} \left(\frac{\omega_*}{k_0} \right) \left(\frac{g_*}{100} \right)^{1/6} \left(\frac{\gamma}{0.01} \right)^{-1} \left(\frac{T_*}{100 \text{ GeV}} \right),$$

$$\gamma H_*^{-1} = 2\pi/k_0,$$

$$\zeta H_*^{-1} = \tau_T;$$

$$h_c(f) = 1.28 \times 10^{-19} \left(\frac{100 \text{ GeV}}{T_*} \right) \left(\frac{100}{g_*} \right)^{1/3} \left(\frac{\gamma}{0.01} \right)^{3/2} \left(\frac{\zeta}{0.01} \right)^{1/2} [k_0^3 \omega_*(f) H_{ijij}(\omega_*(f), \omega_*(f))]^{1/2}$$

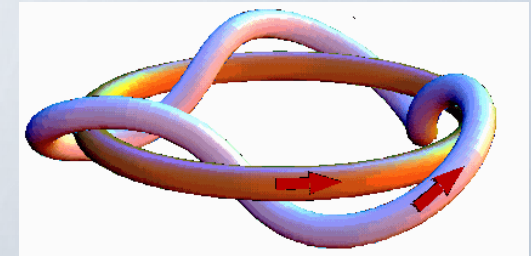
PROBING MAGNETOGENESIS SCENARIOS

- GRAVITATIONAL WAVES PROPAGATE ALMOST FREELY AND RETAIN THE INFORMATION ABOUT THE SOURCE AND PHYSICAL PROCESSES
 - FREQUENCY DETERMINES THE SOURCE CHARACTERISTIC LENGTH (TIME) SCALE
 - AMPLITUDES – THE SOURCE EFFICIENCY AND ENERGETICS.

$$f_{GW} = 2/l_s$$

$$N_b = \frac{H^{-1}}{l_s}$$

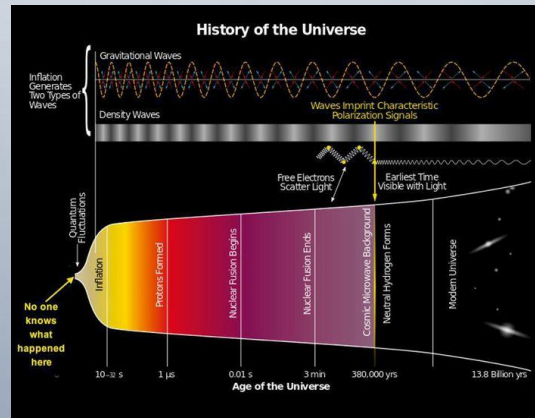
- ✓ If parity was violated
 - > helical turbulence
 - Hydrodynamics (kinetic) turbulence
 - MHD (magnetic dominant)



Magnetic helicity

$$H_B(t) = \int d^3x \mathbf{A} \cdot \nabla \times \mathbf{A},$$

- ✓ If the parity in the early universe is violated – relic gravitational waves are polarized.



WHY NUMERICAL MODELING NEEDED?

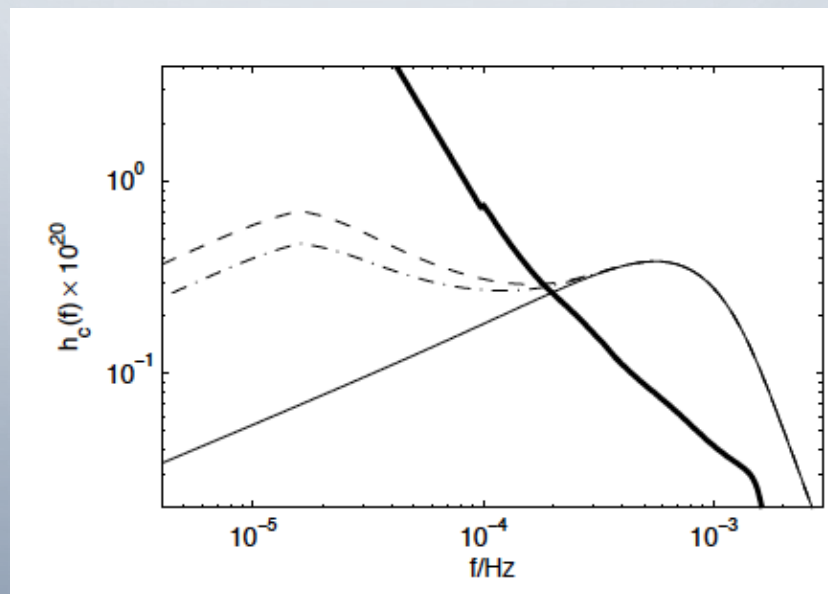
- ✓ TO ACCOUNT PROPERLY NON-LINEAR PROCESSES (MHD)
- ✓ NOT BE LIMITED BY THE SHORT DURATION OF THE PHASE TRANSITIONS
- ✓ TWO STAGES TURBULENCE DECAY
 - FORCED TURBULENCE
 - FREE DECAY
- ✓ THE SOURCE IS PRESENT TILL RECOMBINATION (AFTER THE FIELD IS FROZEN IN)
- ✓ RESULTS – STRONGLY INITIAL CONDITIONS DEPENDENT

$$\left(\frac{\partial^2}{\partial t^2} - c^2 \nabla^2 \right) h_{ij}^{\text{TT}} = \frac{16\pi G}{a^3 c^2} T_{ij}^{\text{TT}},$$

Grishchuk 1974

$$dt_{\text{phys}} = a dt$$

$$h_{ij}^{\text{TT}} = a h_{ij}^{\text{TT,phys}}$$



Gogoberidze et al. 2007

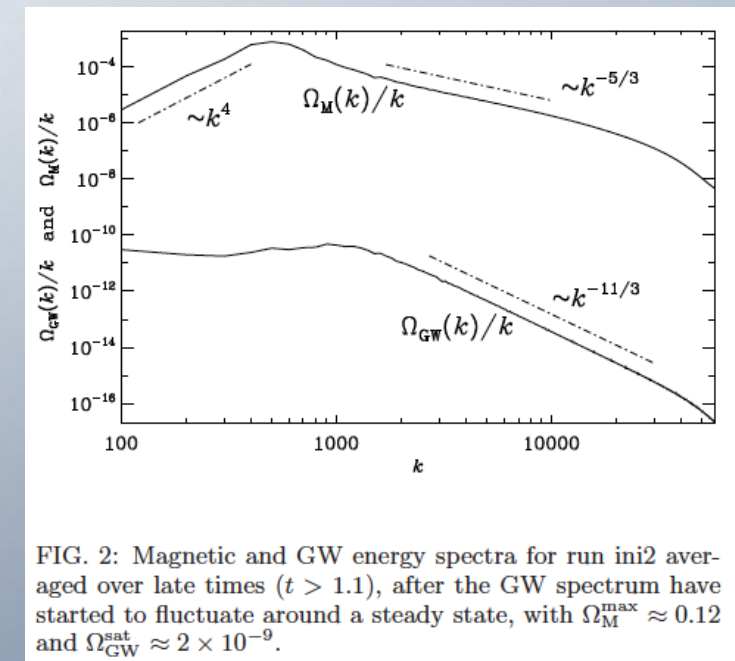


FIG. 2: Magnetic and GW energy spectra for run ini2 averaged over late times ($t > 1.1$), after the GW spectrum have started to fluctuate around a steady state, with $\Omega_{\text{M}}^{\text{max}} \approx 0.12$ and $\Omega_{\text{GW}}^{\text{sat}} \approx 2 \times 10^{-9}$.

Roper Pol et al. 2019,

WHY NUMERICAL MODELING NEEDED?

- ✓ IT IS ASSUMED THE STATIONARY TURBULENCE WHILE IN REALITY TURBULENCE DECAYS
- ✓ THREE STAGES OF GENERATION

$$\mathcal{E}_M(t) = \mathcal{E}_M^{\max} (1 + \Delta t/\tau)^{-p}$$

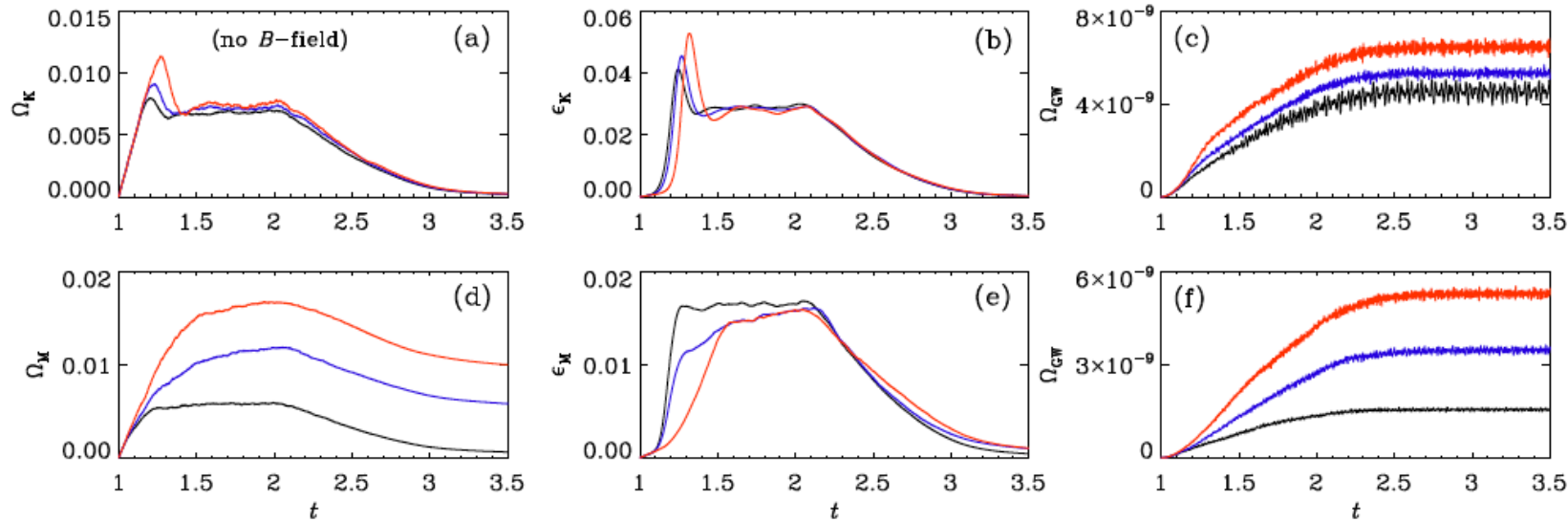


FIG. 2: Evolution of (a) Ω_K , (b) ϵ_K , and (c) Ω_{GW} for kinetically driven cases with $\sigma = 0$ (black), 0.5 (blue), and 1 (red), and of (d) Ω_M , (e) ϵ_M , and (f) Ω_{GW} for magnetically driven cases with $\sigma = 0$ (black), 0.3 (blue), and 1 (red).

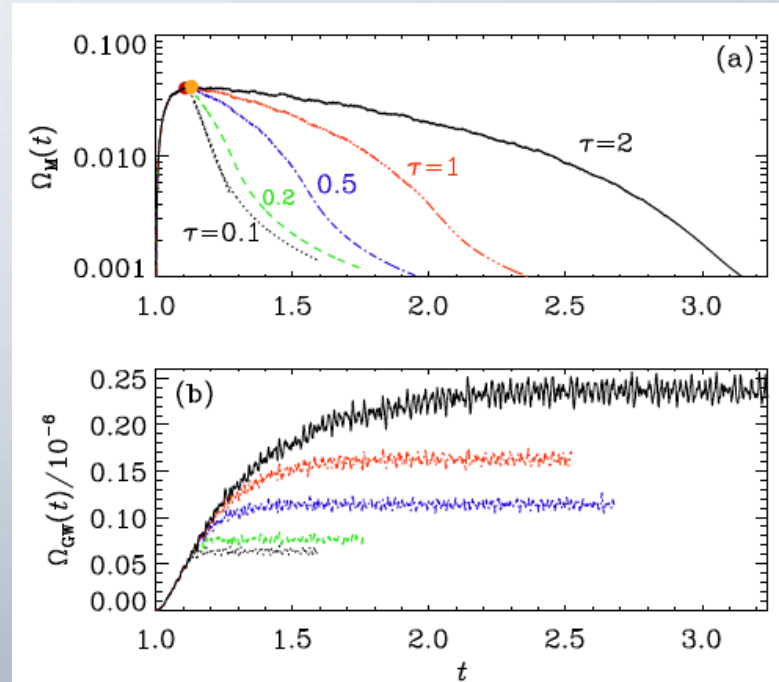


FIG. 1: Evolution of magnetic energy (top) and growth of GW energy density (bottom) for simulations where the driving is turned off at $t = 1.1$ (black dotted line), or the strength of the driving is reduced linearly in time over the duration $\tau = 0.2$ (green), 0.5 (blue), 1 (red), or 2 (black). Time is in units of the Hubble time at the moment of source activation.

PRIMORDIAL MAGNETIC FIELDS LIMITS FROM BBN

- EXTRA RADIATION LIKE ENERGY DENSITY LESS THAN $\sim 3\%$ OF THE RADIATION ENERGY DENSITY AT BBN

$$\frac{\rho_{\text{add}}}{\rho_{\text{rad}}} = 0.277 \left(\frac{\Delta N_{\text{eff}}}{0.122} \right); \quad \Delta N_{\text{eff}} = N_{\text{eff}} - N_{\text{eff}}^{\nu}$$

- THE UPPER BOUND ON THE MAGNETIC (EFFECTIVE) AMPLITUDE ORDER OF MICROGAUSS AT BBN
- ACCOUNTING FOR THE MAGNETIC FIELD DECAY:

- ❖ THE MAGNETIC ENERGY DENSITY DOES NOT EXCEED THE RADIATION ENERGY DENSITY AT THE MOMENT OF GENERATION
- ❖ BBN BOUNDS ARE SATISFIED

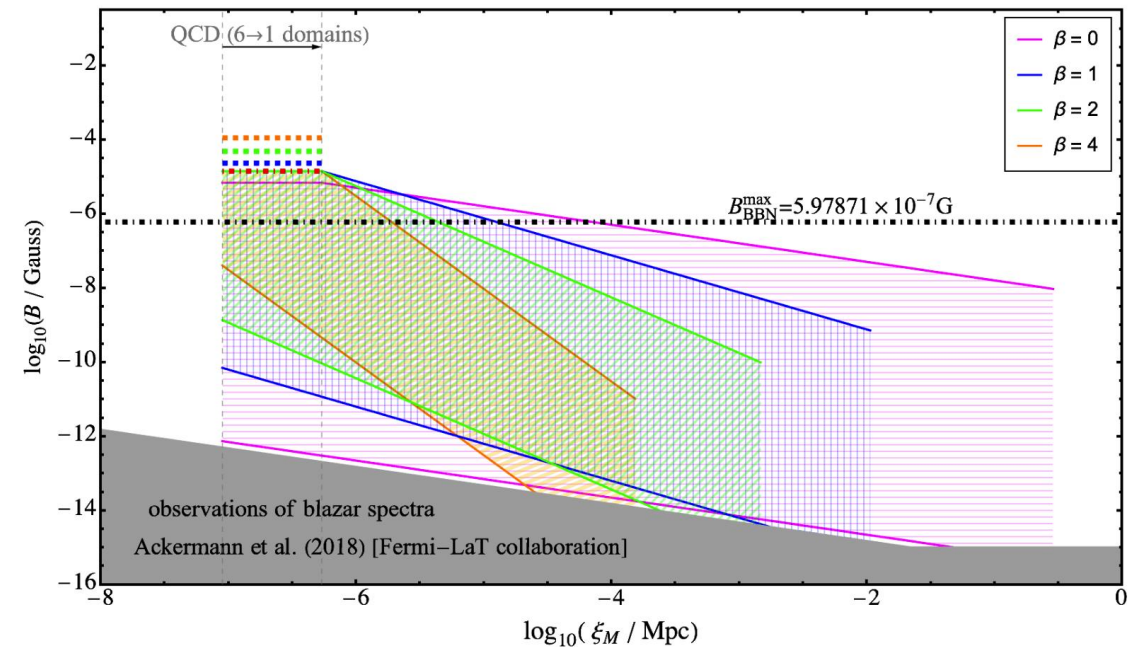


Figure 9: BBN bounds on the QCD phase transition generated initial field strength marked by horizontal dashed lines. For the non-helical cases, the initial energy density bound is more constraining than the BBN bound. Courtesy: Emma Clarke

Credit: Emma Clarke

GRAVITATIONAL WAVES FROM PRIMORDIAL TURBULENCE

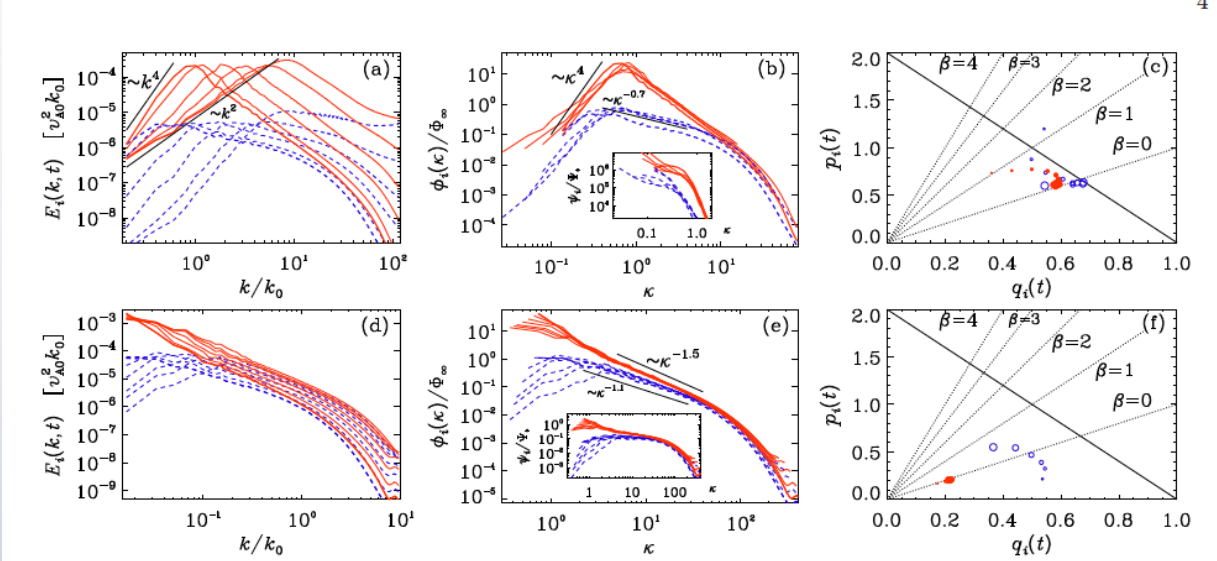
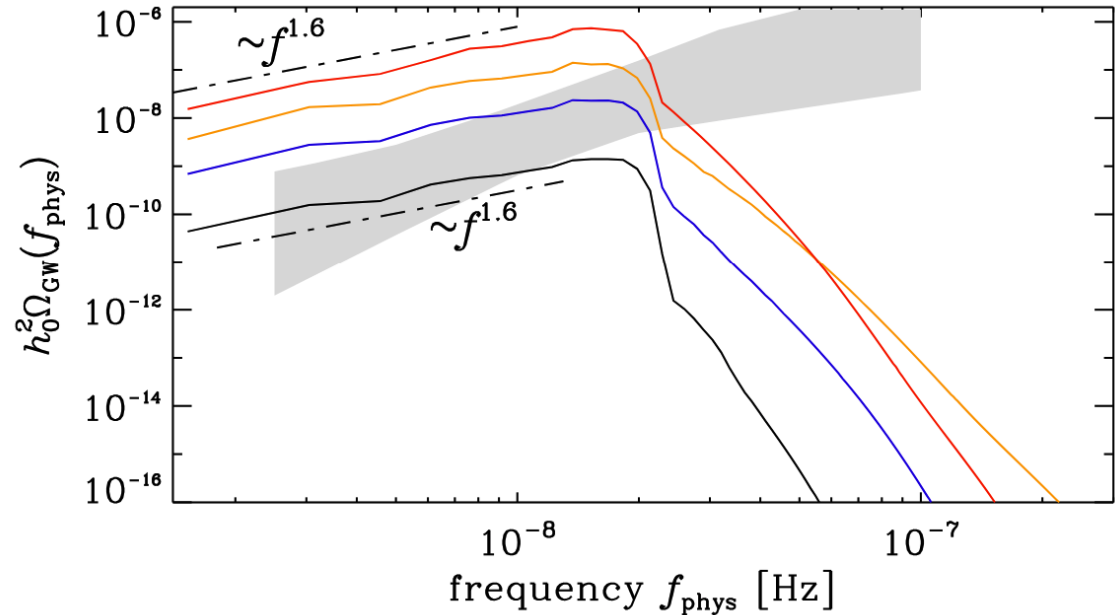
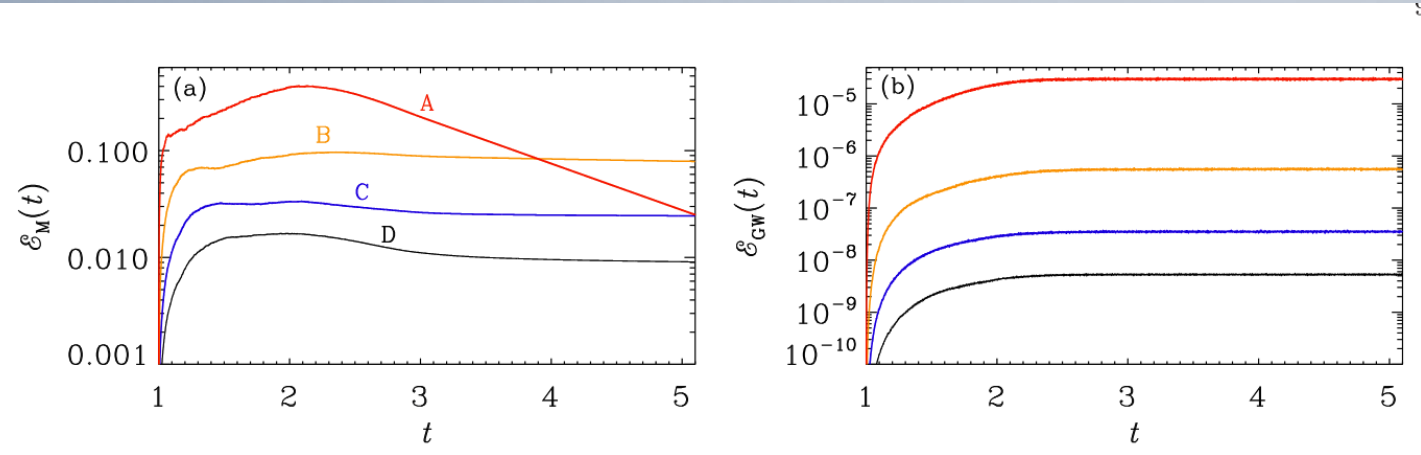


FIG. 3: E_M (solid) and E_K (dashed) in MHD with fractional helicity and $\alpha = 2$ (a), as well as full helicity and $\alpha = -1$ (d), together with compensated spectra (b,e) and the pq diagrams (c,f).



Brandenburg & Kahniashvili 2017

Kahniashvili et al. 2022

CONCLUSIONS AND TAKE HOME COMMENTS

- PTA OFFERS AN UNIQUE POSSIBILITY TO RECONSTRUCT THE INITIAL CONDITIONS AROUND QCD SCALE
- DETERMINE GRAVITATIONAL SIGNAL PROPERTIES THAT WILL ALLOW SEPARATION OF ASTROPHYSICAL AND COSMOLOGICAL BACKGROUNDS (ANISOTROPY, POLARIZATION, SPECTRAL SHAPE, NON-GAUSSIANITY...)
- IMPROVE THE MAGNETIC FIELDS OBSERVATIONS IN VOIDS AND FILAMENTS (TESTING MAGNOGENESIS MODELS)
- ADVANCE NUMERICAL SIMULATIONS TECHNIQUE TO MODEL PRIMORDIAL MAGNETIC FIELDS AND TURBULENCE; DETERMINE THE MECHANISMS INSURING THE PRESENCE OF VIABLE MAGNETIC FIELD/TURBULENT SOURCES IN THE EARLY UNIVERSE AND CORRESPONDINGLY CORRECT INITIAL CONDITIONS
- ADVANCE OUR UNDERSTANDING
 - ❖ PRIMORDIAL MAGNETOGENESIS
 - ❖ BUBBLE COLLISIONS/NUCLEATION – MORE REALISTIC MODELS
 - ❖ SOUND WAVES AS A SOURCE FOR TURBULENCE
 - ❖ AXIONS DRIVEN TURBULENCE AND AXION LIKE PARTICLES DRIVEN INFLATIONARY NEW PHYSICS

The background is a light blue gradient with several realistic water droplets of various sizes scattered around the edges. In the center, there is a faint, large circular graphic that appears to be a stylized globe or a similar abstract design.

THANK YOU!

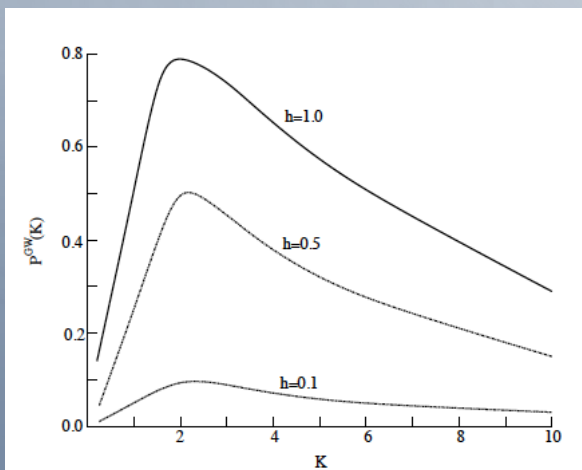
QUESTIONS? COMMENTS?

GRAVITATIONAL WAVES POLARIZATION

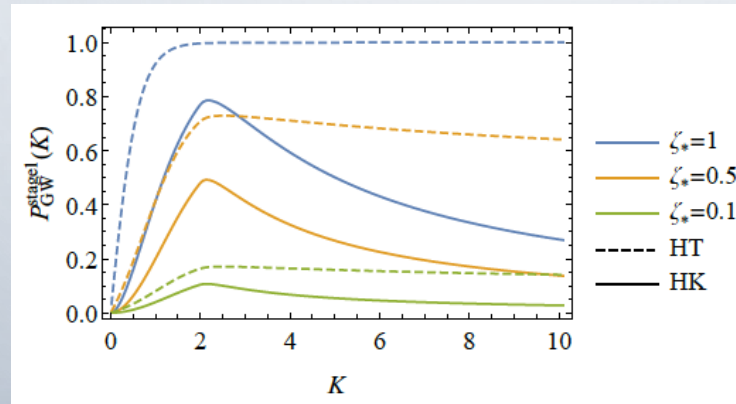
$$\mathcal{P}(k) = \frac{\langle h_+^*(\mathbf{k})h_+(\mathbf{k}') - h_-^*(\mathbf{k})h_-(\mathbf{k}') \rangle}{\langle h_+^*(\mathbf{k})h_+(\mathbf{k}') + h_-^*(\mathbf{k})h_-(\mathbf{k}') \rangle} = \frac{\mathcal{H}(k)}{H(k)}$$

- ✓ POLARIZATION SPECTRUM RETAINS INFORMATION ON PARITY VIOLATION AT LARGE WAVELENGTHS

○ INVERSE CASCADING?



Kisslinger and Kahnashvili 2015



Ellis et al. 2020

- ✓ ASSUMING STATIONARY KOLMOGOROFF LIKE TURBULENCE (HK) OR STATIONARY HELICAL KOLMOGOROFF TURBULENCE (HT)

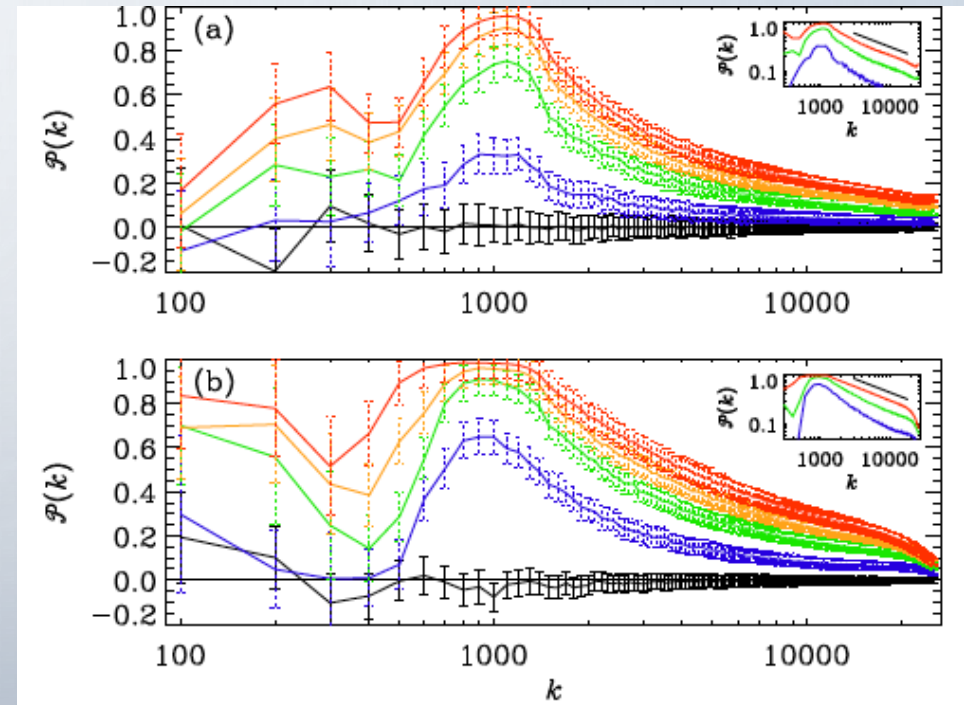


FIG. 3: Degree of circular polarization for (a) kinetically and (b) magnetically forced cases with $\sigma = 0$ (black) 0.1 (blue), 0.3 (green), 0.5 (orange), and 1 (red). Approximate error bars based on the temporal fluctuations and statistical spread for different random seeds of the forcing are shown as solid black lines for $\sigma = 0$ and as dotted lines otherwise.

Kahnashvili et al. 2020

GRAVITATIONAL WAVES FROM PRIMORDIAL TURBULENCE

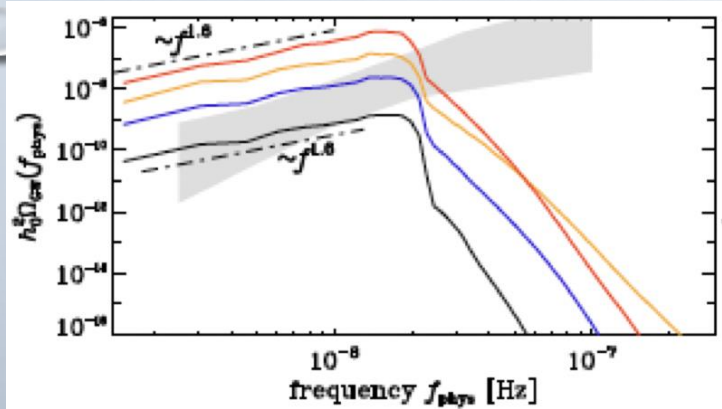


FIG. 2: Frequency spectra, $h_0^2 \Omega_{\text{GW}}(f)$, for both the QCDPT (orange, blue, and black, respectively).

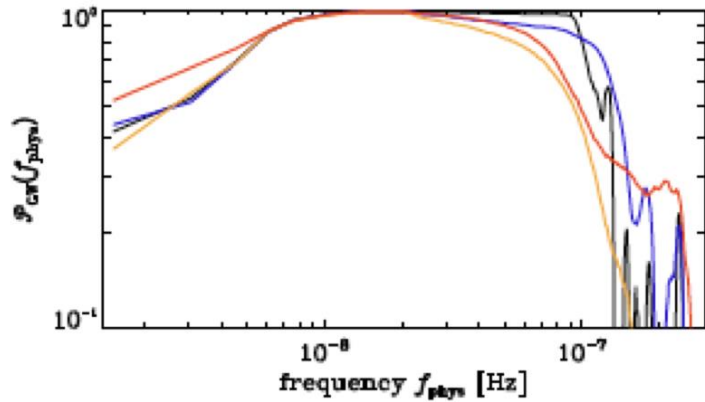


FIG. 3: Polarization spectra, $\mathcal{P}_{\text{GW}}(f)$, for the QCDPT Runs (orange, blue, and black, respectively).

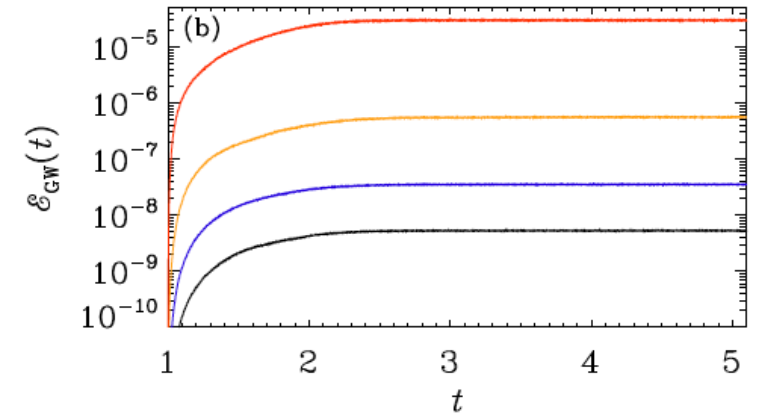
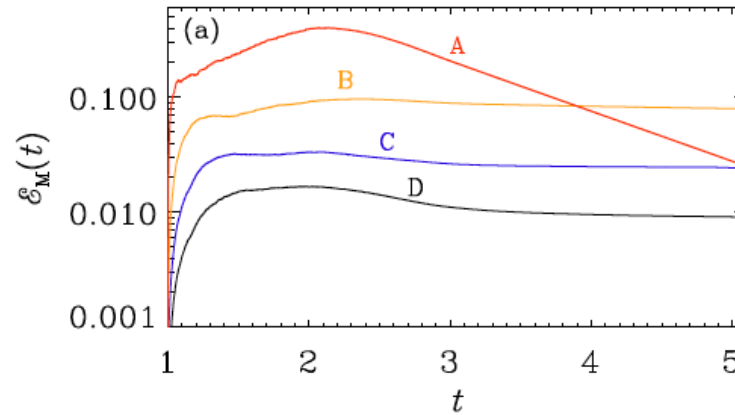


FIG. 6: Evolution of (a) $\mathcal{E}_M(t)$ and (b) $\mathcal{E}_{\text{GW}}(t)$ for Runs A–D of Table I. Note the rapid decay for Run A with the largest viscosity.

$$\mathcal{E}_{\text{GW}} = \left(q \frac{\mathcal{E}_M}{k_f} \right)^n$$