Perturbative Unitarity
4-Point Vertices
4-Point Amplitudes
SPINAS
Constructive Standard Model (CSM)

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arXiv:2403.07977, arXiv:2403.07978, (EPJC…)arXiv:2403.07981
PRD98 (2018) 016014, PRD101 (2020) 065019, NPB993 (2023) 116278

Extremely Brief History

- PT: PRL 56 (1986) 2459: One term for maximally helicity violating amplitude.
- BCFW: PRL 94 (2005) 181602: Recursion method for any gluon amplitude.
- AHH: JHEP 11(2021) 070: Constructive Method for any mass and any spin.
- CF: PRD 98 (2018) 016014: 3-Point Constructive Vertices of SM.
- CDFHM: NPB 993 (2023) 116278: A challenge with internal photons in constructive amplitudes.
- LLT: arXiv:2312.11621: Showed that internal photons were no problem. We had missed an application of the on-shell identity. See Hsing-Yi Lai's talk.
- EGKLL: arXiv:2403.15538: Showed a momentum shift such that the amplitude vanished in the large z limit.

CSM 4-Point Vertices

Particles	Coupling	Vertex
$hhhh$	$-i\frac{3e^2 m_h^2}{4M_W^2 s_W^2}$	1
$hhZZ$	$i\frac{e^2}{2M_W^2 s_W^2}$	$[\mathbf{34}]\langle\mathbf{34}\rangle$
$hhW\bar{W}$	$i\frac{e^2}{2M_W^2 s_W^2}$	$[\mathbf{34}]\langle\mathbf{34}\rangle$

CSM 4-Point Vertices

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$hhW\bar{W}$	$i\frac{e^2}{2M_W^2 s_W^2}$	$[\mathbf{34}]\langle\mathbf{34}\rangle$

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This list is complete for the CSM:

- No gggg vertex (already known)
- No ZZWW vertex
- No WWWW vertex
- No AZWW vertex
- No AAWW vertex
- No Other “contact” terms.

The CSM is perturbatively unitary with only these 4-point vertices.

- We have checked this for all 4-point amplitudes.

CSM 4-Point Vertices

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$hhhh$	$-i\frac{3e^2 m_h^2}{4M_W^2 s_W^2}$	1
$hhZZ$	$i\frac{e^2}{2M_W^2 s_W^2}$	$[\mathbf{34}]\langle\mathbf{34}\rangle$
$hhW\bar{W}$	$i\frac{e^2}{2M_W^2 s_W^2}$	$[\mathbf{34}]\langle\mathbf{34}\rangle$

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$\mathbf{W,W \to W,W}$

$$\mathcal{M}_h^{(0,0,0,0)} = -\frac{e^2}{M_W^2 s_W^2}\mathcal{E}^2$$

$$\mathcal{M}_\gamma^{(0,0,0,0)} = \frac{4e^2}{M_W^2}\mathcal{E}^2$$

$$\mathcal{M}_Z^{(0,0,0,0)} = \frac{(-3+4c_W^2)e^2}{M_W^2 s_W^2}\mathcal{E}^2$$

CSM 4-Point Vertices

Particles	Coupling	Vertex
$hhhh$	$-i\frac{3e^2 m_h^2}{4M_W^2 s_W^2}$	1
$hhZZ$	$i\frac{e^2}{2M_W^2 s_W^2}$	$[\mathbf{34}]\langle\mathbf{34}\rangle$
$hhW\bar{W}$	$i\frac{e^2}{2M_W^2 s_W^2}$	$[\mathbf{34}]\langle\mathbf{34}\rangle$

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$$\gamma^+, \gamma^+ \to \mathbf{W}, \bar{\mathbf{W}}$$

$$\mathcal{M}_W = \frac{-2e^2[12]^2\langle\mathbf{34}\rangle^2}{(t - M_W^2)(u - M_W^2)}$$

And many more…

CSM 4-Point Vertices

Particles	Coupling	Vertex
$hhhh$	$-i\frac{3e^2m_h^2}{4M_W^2s_W^2}$	1
$hhZZ$	$i\frac{e^2}{2M_W^2s_W^2}$	$[\mathbf{34}]\langle\mathbf{34}\rangle$
$hhW\bar{W}$	$i\frac{e^2}{2M_W^2s_W^2}$	$[\mathbf{34}]\langle\mathbf{34}\rangle$

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Constructive diagrams are **not** all Feynman diagrams rewritten in this formalism:

- The absence of a 4-point vertex tells us that the contributions from that diagram have been rearranged into other constructive diagrams.
- Any diagram that includes an external photon or gluon carries the contributions from multiple Feynman diagrams.

A Complete Set of 4-Point Amplitudes in the Constructive Standard Model

I. 4-Fermion Amplitudes
   A. $\mathbf{f_1}, \bar{\mathbf{f}}_\mathbf{1}, \bar{\mathbf{f}}_\mathbf{2}, \mathbf{f_2}$
   B. $\mathbf{f_1}\bar{\mathbf{f}}_\mathbf{2}\bar{\mathbf{f}}_\mathbf{3}\mathbf{f_4}$

II. Amplitudes with an External Photon or Gluon
   A. $\mathbf{f}, \bar{\mathbf{f}}, \gamma, \mathbf{h}$ and $\mathbf{f}, \bar{\mathbf{f}}, \mathbf{g}, \mathbf{h}$
   B. $\mathbf{f}, \bar{\mathbf{f}}, \gamma, \mathbf{Z}$ and $\mathbf{f}, \bar{\mathbf{f}}, \mathbf{g}, \mathbf{Z}$
   C. $\mathbf{q_1}, \bar{\mathbf{q}}_\mathbf{2}, \mathbf{g}, \mathbf{W}$ and $\mathbf{f_1}, \bar{\mathbf{f}}_\mathbf{2}, \gamma, \mathbf{W}$
   D. $\mathbf{f}, \bar{\mathbf{f}}, \gamma, \gamma,\ \mathbf{f}, \bar{\mathbf{f}}, \mathbf{g}, \gamma$ and $\mathbf{f}, \bar{\mathbf{f}}, \mathbf{g}, \mathbf{g}$
   E. $\gamma, \mathbf{h}, \mathbf{W}, \bar{\mathbf{W}}$
   F. $\gamma, \mathbf{Z}, \bar{\mathbf{W}}, \mathbf{W}$
   G. $\gamma, \gamma, \bar{\mathbf{W}}, \mathbf{W}$
   H. $\mathbf{g}, \mathbf{g}, \mathbf{g}, \mathbf{g}$

III. Other Amplitudes without a 4-Point Vertex
   A. $\mathbf{f}, \bar{\mathbf{f}}, \mathbf{h}, \mathbf{h}$
   B. $\mathbf{f}, \bar{\mathbf{f}}, \mathbf{Z}, \mathbf{h}$
   C. $\mathbf{f_1}, \bar{\mathbf{f}}_\mathbf{2}, \bar{\mathbf{W}}, \mathbf{h}$ and $\bar{\mathbf{f}}_\mathbf{1}, \mathbf{f_2}, \mathbf{W}, \mathbf{h}$
   D. $\mathbf{f}, \bar{\mathbf{f}}, \bar{\mathbf{W}}, \mathbf{W}$
   E. $\mathbf{f}, \bar{\mathbf{f}}, \mathbf{Z}, \mathbf{Z}$
   F. $\bar{\mathbf{f}}_\mathbf{1}, \mathbf{f_2}, \mathbf{Z}, \mathbf{W}$ and $\bar{\mathbf{f}}_\mathbf{1}, \mathbf{f_2}, \mathbf{Z}, \bar{\mathbf{W}}$
   G. $\mathbf{Z}, \mathbf{h}, \bar{\mathbf{W}}, \mathbf{W}$
   H. $\mathbf{Z}, \mathbf{Z}, \mathbf{Z}, \mathbf{Z}$
   I. $\mathbf{Z}, \mathbf{Z}, \bar{\mathbf{W}}, \mathbf{W}$
   J. $\mathbf{W}, \mathbf{W}, \bar{\mathbf{W}}, \bar{\mathbf{W}}$

IV. Amplitudes with a 4-Point Vertex
   A. $\mathbf{h}, \mathbf{h}, \mathbf{h}, \mathbf{h}$
   B. $\mathbf{h}, \mathbf{h}, \mathbf{Z}, \mathbf{Z}$ and $\mathbf{h}, \mathbf{h}, \bar{\mathbf{W}}, \mathbf{W}$

arXiv:2403.07978

SPINAS: Spinor Amplitude Subroutines for Constructive Diagram Evaluations

SPINAS is a C++ package created for the implementation and numerical computation of phase-space points of constructive amplitudes in particle physics.

(EPJC…)arXiv:2403.07981

SPINAS: Spinor Amplitude Subroutines for Constructive Diagram Evaluations

This package contains a suite of classes and methods for handling particles, propagators, spinor products, and processes. SPINAS is structured to offer straightforward usability while ensuring maximum efficiency.

(EPJC…)arXiv:2403.07981

SPINAS: Spinor Amplitude Subroutines for Constructive Diagram Evaluations

validation of this package, including both a validation of individual components of the package and a comparison of a complete set of Standard Model processes with Feynman diagrams.

(EPJC…)arXiv:2403.07981

SPINAS: Spinor Amplitude Subroutines for Constructive Diagram Evaluations

$q,\bar{q} \to q,\bar{q}$ Neutral	
$u,\bar{u} \to c,\bar{c}$	$u,c \to u,c$
$u,\bar{u} \to u,\bar{u}$	$u,u \to u,u$
$u,\bar{u} \to s,\bar{s}$	$u,s \to u,s$
$u,\bar{u} \to d,\bar{d}$	$u,d \to u,d$
$d,\bar{d} \to s,\bar{s}$	$d,s \to d,s$
$d,\bar{d} \to d,\bar{d}$	$d,d \to d,d$
$q,\bar{q} \to l,\bar{l}$ Neutral	
$u,\bar{u} \to e,\bar{e}$	$u,e \to u,e$
$u,\bar{u} \to \nu_e,\bar{\nu}_e$	$u,\nu_\mu \to \nu_\mu,u$
$d,\bar{d} \to e,\bar{e}$	$d,e \to e,d$
$d,\bar{d} \to \nu_e,\bar{\nu}_e$	$d,\nu_\mu \to \nu_\mu,d$
$l,\bar{l} \to l,\bar{l}$ Neutral	
$e,\bar{e} \to \mu,\bar{\mu}$	$e,\mu \to e,\mu$
$e,\bar{e} \to e,\bar{e}$	$e,e \to e,e$
$e,\bar{e} \to \nu_\mu,\bar{\nu}_\mu$	$e,\nu_\mu \to e,\nu_\mu$
$e,\bar{e} \to \nu_e,\bar{\nu}_e$	$e,\nu_e \to \nu_e,e$
$\nu_e,\bar{\nu}_e \to \nu_\mu,\bar{\nu}_\mu$	$\nu_e,\nu_\mu \to \nu_e,\nu_\mu$
$\nu_e,\bar{\nu}_e \to \nu_e,\bar{\nu}_e$	$\nu_e,\nu_e \to \nu_e,\nu_e$

$q,\bar{q} \to b,\bar{b}$ Neutral without g Continued		
$u,\bar{u} \to Z,Z$	$u,Z \to Z,u$	
$u,\bar{u} \to W,\bar{W}$	$u,W \to W,u$	
$d,\bar{d} \to h,h$	$d,h \to d,h$	
$d,\bar{d} \to \gamma,h$	$d,\gamma \to d,h$	
$d,\bar{d} \to Z,h$	$d,Z \to d,h$	
$d,\bar{d} \to \gamma,\gamma$	$\gamma,\gamma \to d,\bar{d}$	$d,\gamma \to \gamma,d$
$\gamma,Z \to d,\bar{d}$	$\gamma,d \to Z,d$	
$d,\bar{d} \to Z,Z$	$d,Z \to Z,d$	
$\bar{d},d \to W,\bar{W}$	$W,d \to W,d$	
$q,\bar{q} \to b,\bar{b}$ Neutral with g		
$u,g \to u,h$	$h,g \to u,\bar{u}$	
$g,Z \to u,\bar{u}$	$g,u \to Z,u$	
$g,\gamma \to u,\bar{u}$	$g,u \to \gamma,u$	
$g,g \to u,\bar{u}$	$g,u \to g,u$	
$d,g \to d,h$	$h,g \to d,\bar{d}$	
$g,Z \to d,\bar{d}$	$g,d \to Z,d$	
$g,\gamma \to d,\bar{d}$	$d,\gamma \to g,d$	
$g,g \to d,\bar{d}$	$\bar{d},g \to g,\bar{d}$	

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SPINAS: Spinor Amplitude Subroutines for Constructive Diagram Evaluations

$f, \bar{f} \to f, \bar{f}$ Charged		
$u, \bar{d} \to t, \bar{b}$	$u, b \to d, t$	
$u, \bar{d} \to \nu_\tau, \bar{\tau}$	$u, \tau \to \nu_\tau, d$	
$\mu, \bar{e} \to \bar{\nu}_e, \nu_\mu$	$\mu, \nu_e \to e, \nu_\mu$	
$l, \bar{l} \to b, \bar{b}$ Neutral		
$e, \bar{e} \to h, h$	$e, h \to e, h$	
$e, \bar{e} \to \gamma, h$	$e, \gamma \to e, h$	
$e, \bar{e} \to Z, h$	$e, Z \to e, h$	
$e, \bar{e} \to \gamma, \gamma$	$e, \gamma \to \gamma, e$	$\gamma, \gamma \to e, \bar{e}$
$\gamma, Z \to \bar{e}, e$	$\gamma, e \to Z, e$	
$e, \bar{e} \to Z, Z$	$e, Z \to Z, e$	
$e, \bar{e} \to W, \bar{W}$	$e, W \to W, e$	
$\nu_e, \bar{\nu}_e \to Z, h$	$\nu_e, Z \to \nu_e, h$	
$\nu_e, \bar{\nu}_e \to Z, Z$	$\nu_e, Z \to Z, \nu_e$	
$\nu_e, \bar{\nu}_e \to W, \bar{W}$	$\nu_e, W \to W, \nu_e$	
$q, \bar{q} \to b, \bar{b}$ Neutral without g		
$u, \bar{u} \to h, h$	$u, h \to u, h$	
$u, \bar{u} \to \gamma, h$	$u, \gamma \to u, h$	
$u, \bar{u} \to Z, h$	$u, Z \to u, h$	
$u, \bar{u} \to \gamma, \gamma$	$\gamma, \gamma \to u, \bar{u}$	$u, \gamma \to \gamma, u$
$\gamma, Z \to u, \bar{u}$	$\gamma, u \to Z, u$	

$f, \bar{f} \to b, \bar{b}$ Charged		
$\bar{e}, \nu_e \to W, h$	$h, \nu_e \to W, e$	
$\bar{e}, \nu_e \to \gamma, W$	$\gamma, \nu_e \to W, e$	
$\bar{e}, \nu_e \to Z, W$	$Z, \nu_e \to W, e$	
$u, \bar{d} \to W, h$	$u, h \to W, d$	
$\bar{u}, d \to \gamma, \bar{W}$	$\gamma, W \to u, \bar{d}$	
$d, W \to g, u$	$g, W \to u, \bar{d}$	
$\bar{u}, d \to Z, \bar{W}$	$W, d \to Z, u$	
$b, \bar{b} \to b, \bar{b}$		
$h, h \to h, h$		
$h, h \to Z, Z$	$h, Z \to Z, h$	
$h, h \to W, \bar{W}$	$h, W \to W, h$	
$\gamma, h \to W, \bar{W}$	$\gamma, W \to W, h$	
$Z, h \to W, \bar{W}$	$Z, W \to W, h$	
$\gamma, \gamma \to W, \bar{W}$	$\gamma, W \to \gamma, W$	
$\gamma, Z \to W, \bar{W}$	$\gamma, W \to Z, W$	
$Z, Z \to Z, Z$		
$Z, Z \to W, \bar{W}$	$Z, W \to Z, W$	
$W, W \to W, W$	$W, \bar{W} \to W, \bar{W}$	
$g, g \to g, g$		

SPINAS: Spinor Amplitude Subroutines for Constructive Diagram Evaluations

$f, \bar{f} \to f, \bar{f}$ Charged		
$u, \bar{d} \to t, \bar{b}$	$u, b \to d, t$	
$u, \bar{d} \to \nu_\tau, \bar{\tau}$	$u, \tau \to \nu_\tau, d$	
$\mu, \bar{e} \to \bar{\nu}_e, \nu_\mu$	$\mu, \nu_e \to e, \nu_\mu$	
$l, \bar{l} \to b, \bar{b}$ Neutral		
$e, \bar{e} \to h, h$	$e, h \to e, h$	
$e, \bar{e} \to \gamma, h$	$e, \gamma \to e, h$	
$e, \bar{e} \to Z, h$	$e, Z \to e, h$	
$e, \bar{e} \to \gamma, \gamma$	$e, \gamma \to \gamma, e$	$\gamma, \gamma \to e, \bar{e}$
$\gamma, Z \to \bar{e}, e$	$\gamma, e \to Z, e$	
$e, \bar{e} \to Z, Z$	$e, Z \to Z, e$	
$e, \bar{e} \to W, \bar{W}$	$e, W \to W, e$	
$\nu_e, \bar{\nu}_e \to Z, h$	$\nu_e, Z \to \nu_e, h$	
$\nu_e, \bar{\nu}_e \to Z, Z$	$\nu_e, Z \to Z, \nu_e$	
$\nu_e, \bar{\nu}_e \to W, \bar{W}$	$\nu_e, W \to W, \nu_e$	
$q, \bar{q} \to b, \bar{b}$ Neutral without g		
$u, \bar{u} \to h, h$	$u, h \to u, h$	
$u, \bar{u} \to \gamma, h$	$u, \gamma \to u, h$	
$u, \bar{u} \to Z, h$	$u, Z \to u, h$	
$u, \bar{u} \to \gamma, \gamma$	$\gamma, \gamma \to u, \bar{u}$	$u, \gamma \to \gamma, u$
$\gamma, Z \to u, \bar{u}$	$\gamma, u \to Z, u$	

$f, \bar{f} \to b, \bar{b}$ Charged		
$\bar{e}, \nu_e \to W, h$	$h, \nu_e \to W, e$	
$\bar{e}, \nu_e \to \gamma, W$	$\gamma, \nu_e \to W, e$	
$\bar{e}, \nu_e \to Z, W$	$Z, \nu_e \to W, e$	
$u, \bar{d} \to W, h$	$u, h \to W, d$	
$\bar{u}, d \to \gamma, \bar{W}$	$\gamma, W \to u, \bar{d}$	
$d, W \to g, u$	$g, W \to u, \bar{d}$	
$\bar{u}, d \to Z, \bar{W}$	$W, d \to Z, u$	
$b, \bar{b} \to b, \bar{b}$		
$h, h \to h, h$		
$h, h \to Z, Z$	$h, Z \to Z, h$	
$h, h \to W, \bar{W}$	$h, W \to W, h$	
$\gamma, h \to W, \bar{W}$	$\gamma, W \to W, h$	
$g, g \to g, g$		

- Agreement with Feynman diagrams both on-shell and off-shell.

$$-2\,\frac{e^2}{s}\left(\langle\mathbf{13}\rangle[\mathbf{24}]+[\mathbf{13}]\langle\mathbf{24}\rangle+[\mathbf{14}]\langle\mathbf{23}\rangle+\langle\mathbf{14}\rangle[\mathbf{23}]\right)$$

```
-two*e*e*(
   a13a.v(ds1,ds3)*s24s.v(ds2,ds4)
   + a14a.v(ds1,ds4)*s23s.v(ds2,ds3)
   + s13s.v(ds1,ds3)*a24a.v(ds2,ds4)
   + s14s.v(ds1,ds4)*a23a.v(ds2,ds3)
   )/pDenS;
```

Summary

CSM 4-Point Vertices

Particles	Coupling	Vertex
$hhhh$	$-i\frac{3e^2 m_h^2}{4M_W^2 s_W^2}$	1
$hhZZ$	$i\frac{e^2}{2M_W^2 s_W^2}$	$[\mathbf{34}]\langle\mathbf{34}\rangle$
$hhW\bar{W}$	$i\frac{e^2}{2M_W^2 s_W^2}$	$[\mathbf{34}]\langle\mathbf{34}\rangle$

- Complete set of 4-point vertices.
- Perturbatively unitary
- Complete set of 4-point amplitudes.
- SPINAS: C++ package to calculate phase-space points for constructive amplitudes.
- Very large set of validations.