Proper Treatment of Flux Uncertainties in Neutrino Cross Section Measurement

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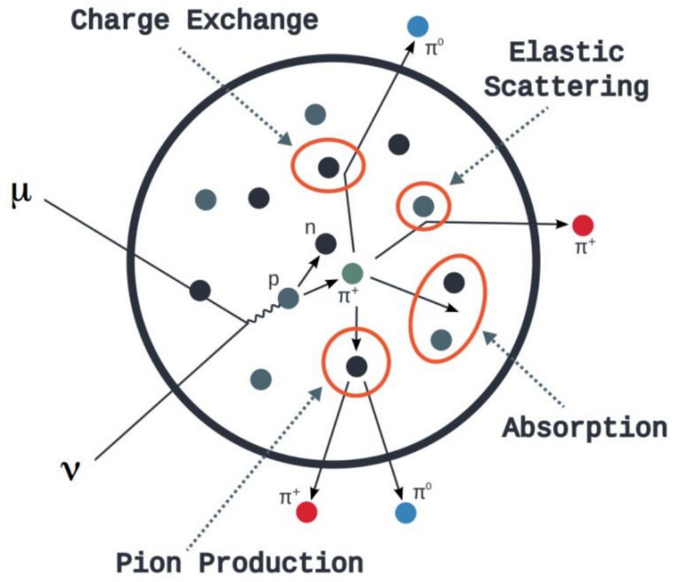
Issue first raised to the neutrino cross section community in PhysRevD.102.113012

Challenges in Neutrino Interaction Modeling

- Wide range of energies
  - Spans QE, RES, DIS
- Range of nuclear targets across experiments
  - Hydrogen, Deuterium, Carbon, Argon, Iron, Lead
- Complex QCD physics inside nucleus
  - Nuclear initial state
  - Nucleon-nucleon correlations
  - Final state interactions

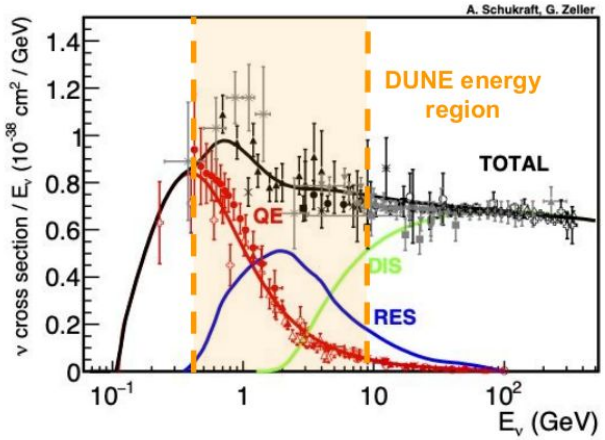


Credit: T. Golan

Flux-Averaged Cross Sections

- Accelerator neutrino experiments do not directly observe the incoming neutrino
  - Reconstructing Enu introduces model dependence that we want to avoid
- Cross-section measurements are flux-averaged over the beam flux they are exposed to
  - Wide energy-range beams means cross section varies significantly across measured phase space

RevModPhys.84.1307



$$S \overset{\text{def}}{=} \frac{\int F(E_\nu)\cdot\sigma(E_\nu)\cdot dE_\nu}{\Phi}$$

$$\Phi = \int \bar{F}(E_\nu)\cdot dE_\nu$$

Signal S
True neutrino flux distribution F
Estimated flux distribution $\bar{F}$
Total estimated flux Φ
Neutrino energy E_ν
Cross section σ

How to Measure Flux-Averaged Cross Sections

- Directly measure N events
  - Subtract background B
  - Correct for efficiency ϵ and smearing D
  - Scale by number of nuclei T
  - Scale by total flux prediction Φ

$$S \stackrel{\text{def}}{=} \frac{\int F(E_\nu)\cdot\sigma(E_\nu)\cdot dE_\nu}{\Phi}$$

$$N_i = B_i + T\cdot\Sigma_j \int_j F(E_\nu)\cdot\sigma(E_\nu)\cdot D_{ij}\cdot\epsilon_{ij}\cdot dE_\nu$$

- Flux uncertainties present in Φ, B, ϵ
  - B can vary with E_ν
  - ϵ can vary with E_ν within each bin

- Potential for low model dependence

Signal S
Measured event count N
Estimated background B
Estimated efficiency ϵ
Detector smearing D
True neutrino flux distribution F
Estimated flux distribution $\overline{F}$

Number of target nuclei T
Total estimated flux Φ
Neutrino energy E_ν
Cross section σ
Reco bin i
Truth bin j

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- Potential for low model dependence

$$\frac{\Sigma_i(N_i\text{-}B_i)\cdot(\epsilon\cdot D)^{-1}_{ij}}{T\cdot\Phi} = \frac{\int_j F(E_\nu)\cdot\sigma(E_\nu)\cdot dE_\nu}{\text{��}} = S_j$$

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Comparing Measurements to Predictions

- Prediction uses estimated flux $\overline{F}$
  - Note: $\overline{F}$ contains uncertainties as well as central value
- Now both measurement and prediction contain flux uncertainties!
  - Prediction contains full flux uncertainties
  - Measurement has norm from Φ and some shape effects in B, ϵ
- Exact correlation cannot be easily determined
  - No measurement to date provides sufficient info (as far as I know)

$$S_{j,meas} = \frac{\Sigma_i (N_i - B_i)\cdot(\epsilon\cdot D)^{-1}_{ij}}{T\cdot\Phi} = \frac{\int_j F(E_\nu)\cdot\sigma(E_\nu)\cdot dE_\nu}{\text{��}}$$

$$S_{j,pred} = \frac{\int_j \overline{F}(E_\nu)\cdot\sigma(E_\nu)\cdot dE_\nu}{\Phi}$$

$$\Phi = \int \overline{F}(E_\nu)\cdot dE_\nu$$

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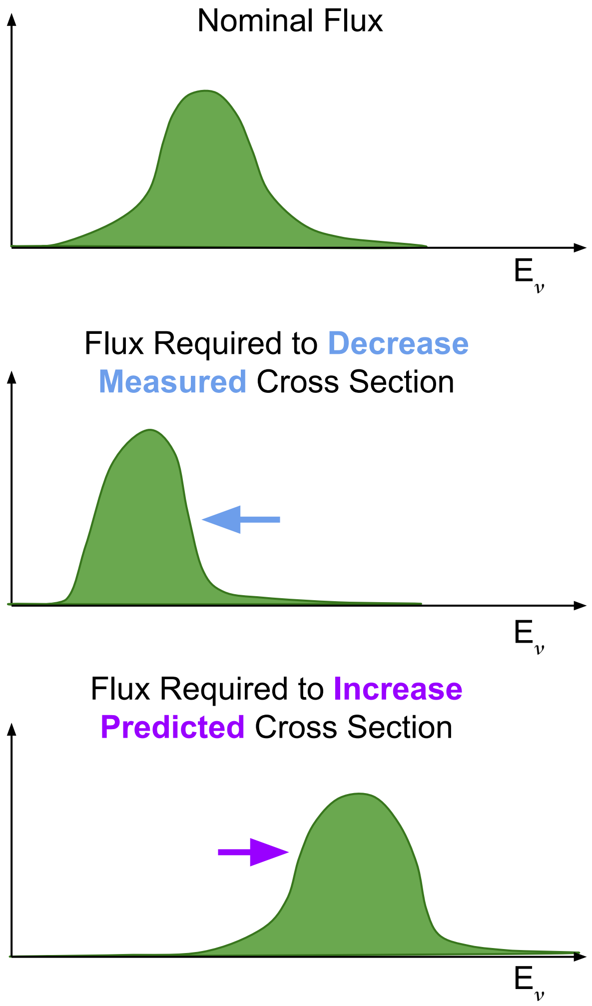
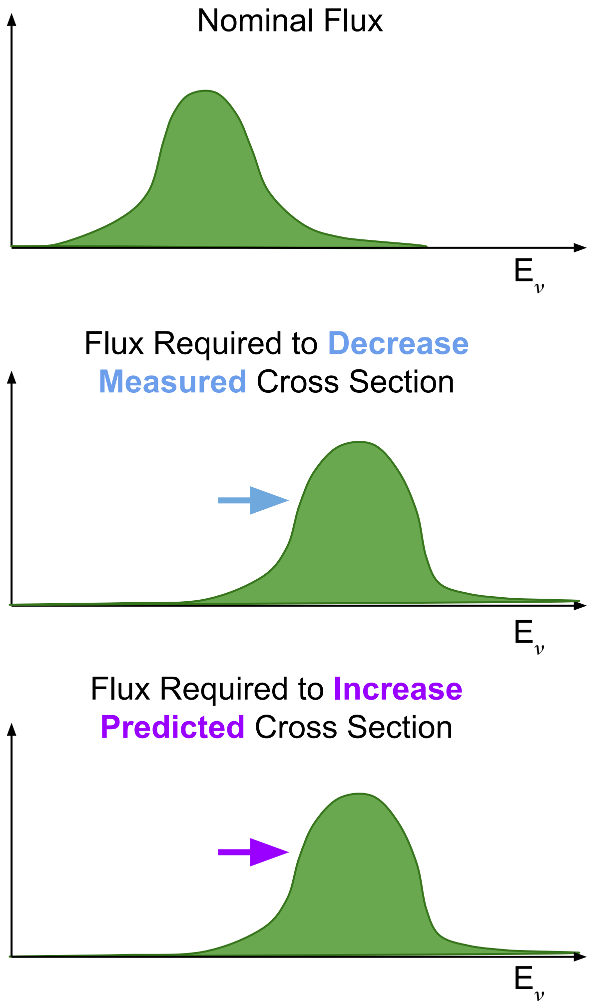
Example 1: Under-Estimate χ^2

- Consider a total cross section measurement in 1 bin
    - Suppose there is a 10% data excess
- Assume a cross section prediction $\sigma \propto E_\nu$
- Assume a background prediction B $\propto$ $1/E_\nu$
- Ignoring correlations under-estimates χ^2:
    - Meas and pred uncertainties can address tension, but require pulling flux in **opposite directions**
    - Correct treatment requires **larger** deviation from



Example 2: Over-Estimate χ^2

- Consider a total cross section measurement in 1 bin
  - Suppose there is a 10% data excess
- Assume a cross section prediction $\sigma \propto E_\nu$
- Assume a background prediction $B \propto E_\nu$
- Ignoring correlations over-estimates χ^2:
  - Uncertainties are under-counted when added in quadrature: arise from the same flux deviation
  - Correct treatment requires **smaller** deviation from nominal flux



Solutions to the Flux Treatment Problem

- Provide more info to allow correlations be determined
  - Publish full set of flux universes and extracted cross section for each
  - Theorist could compute predicted cross section for each flux universe, construct joint covariance between meas and pred cross section

- Extremely messy and difficult
  - Asks a lot of work from theorists
  - Perhaps a standardized framework could be written to allow plug-in and compute

- Alternative approach: measure **nominal**-flux-averaged cross section

Nominal-Flux-Averaged Measurements

Include estimated flux entirely in measurement

- Measurement contains all flux uncertainties
- Prediction only requires nominal flux estimate
- Much easier to make comparison to theory

$$\tilde{S} \stackrel{\text{def}}{=} \frac{\int \bar{F}(E_\nu)\cdot\sigma(E_\nu)\cdot dE_\nu}{\Phi}$$

Nominal-flux-averaged signal S
Nominal-flux-averaged signal $\tilde{S}$
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$$= \frac{(\text{number of events reconstructed in bin i that originate in bin j})}{(\text{number of events that originate in bin j})} \cdot \underbrace{T\cdot\Phi}_{\tilde{F}_j} \cdot \underbrace{\frac{\int_j \bar{F}(E_\nu)\cdot\sigma(E_\nu)\cdot dE_\nu}{\Phi}}_{\tilde{S}_j}$$

$\tilde{F}_j \cdot \tilde{S}_j$

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$$= \Sigma_j \quad M_{ij} \quad \cdot \quad \tilde{F}_j \quad \cdot \quad \tilde{S}_j$$

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$$N_i - B_i = \Sigma_j M_{ij} \cdot \tilde{F}_j \cdot \tilde{S}_j$$

Summary

- Cross section measurements are vital to improving our neutrino interaction modeling
  - We need to be able to accurately compare measurements to predictions

- Industry standard real-flux-averaged cross section contains complicated correlations between meas and pred
  - Existing measurements contain insufficient info for accurate comparison
  - In theory info release is possible - flux universes, each cross section extracted, let theorists construct covariance across joint distribution
  - However, this is messy and asks a lot of work on theorists

- Nominal-flux-averaged cross section allows for direct comparison to prediction