Gauge Invariant Constraints on Gravitational Waves from A First-Order EWPT

AMHERST CENTER FOR FUNDAMENTAL INTERACTIONS
Physics at the interface: Energy, Intensity, and Cosmic frontiers
University of Massachusetts Amherst

Manuel Díaz
In collaboration with: Michael Ramsey-Musolf and Leon Friedrich

DPF-PHENO 2024

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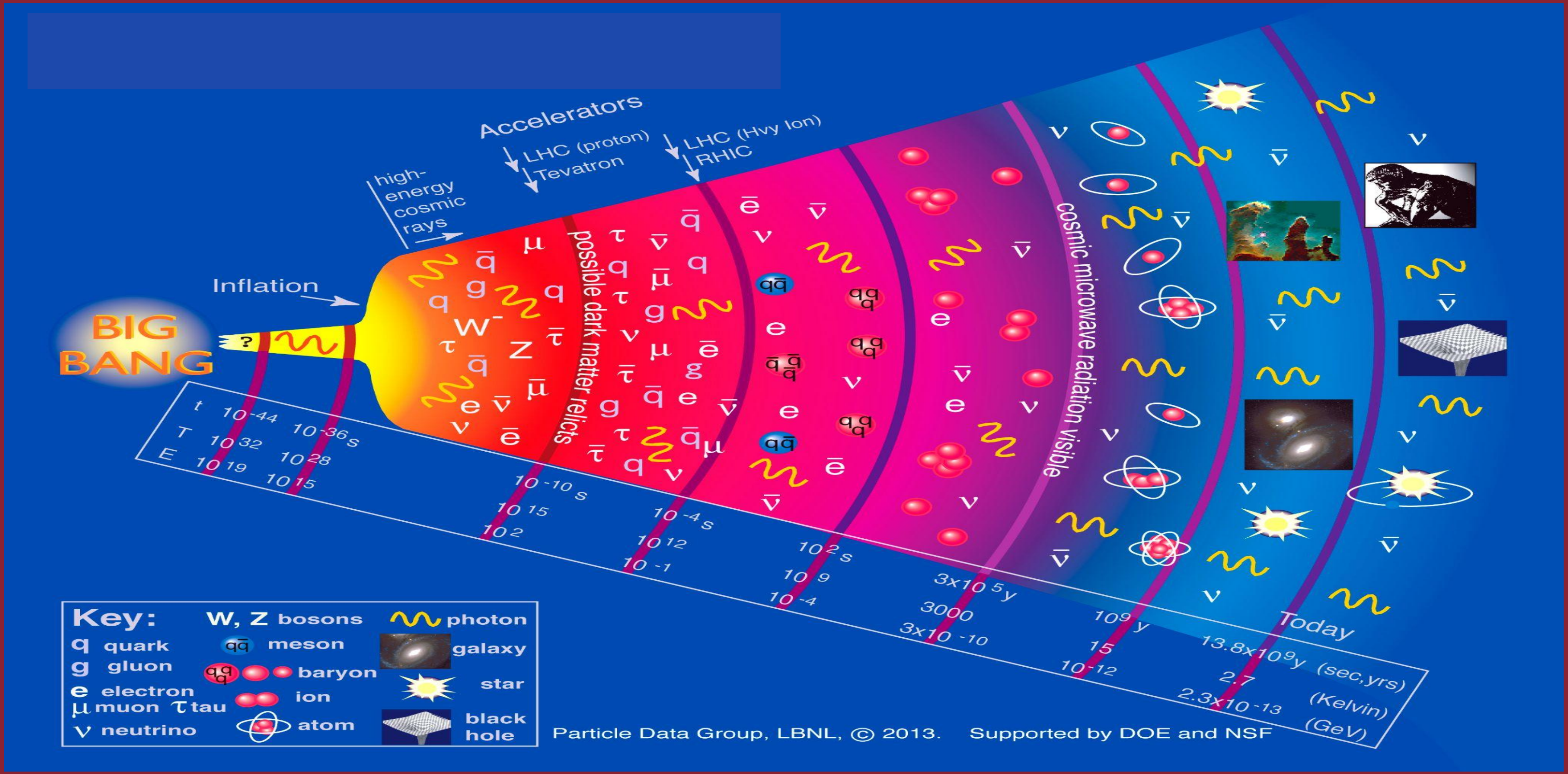
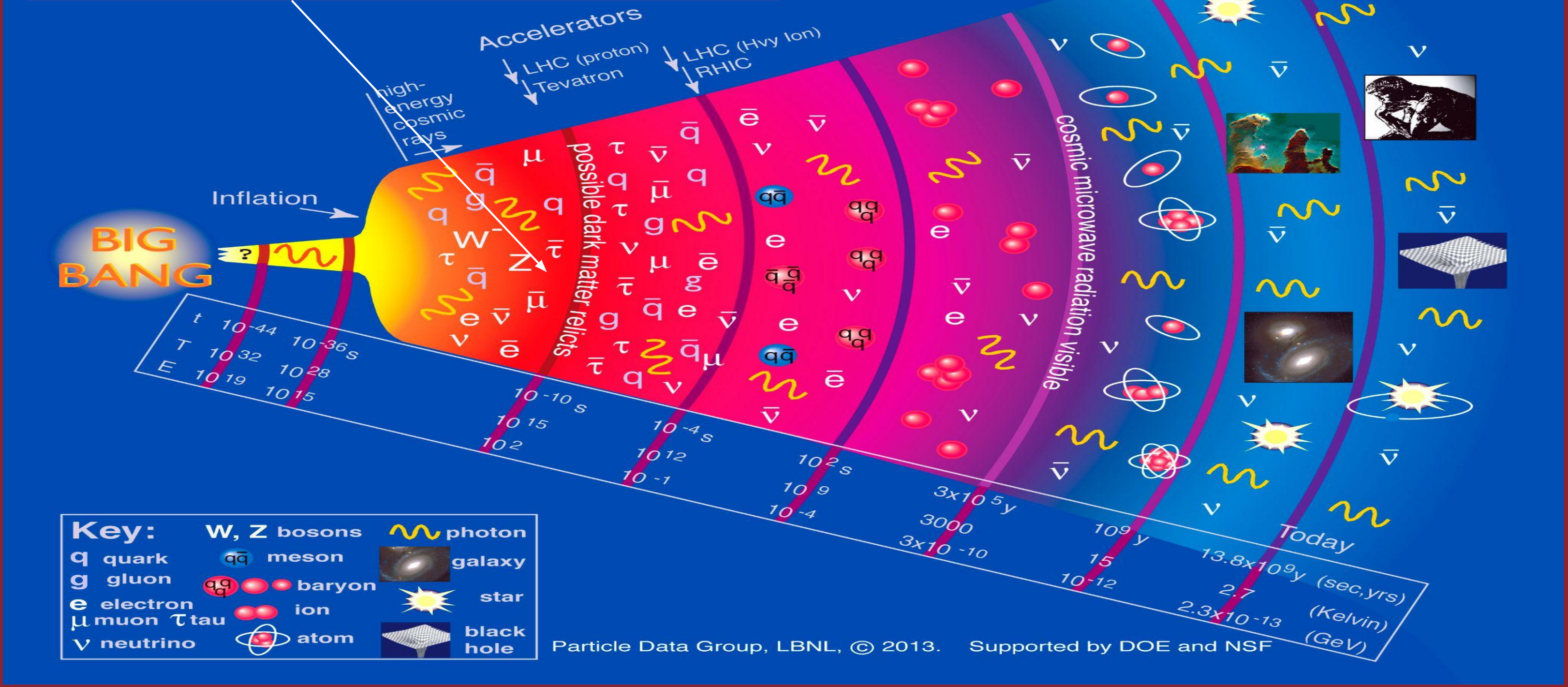
IV. **Implications:** Our results therefore show that such a gauge-invariant framework should be reliably applicable to BSM physics models.

Cosmic History



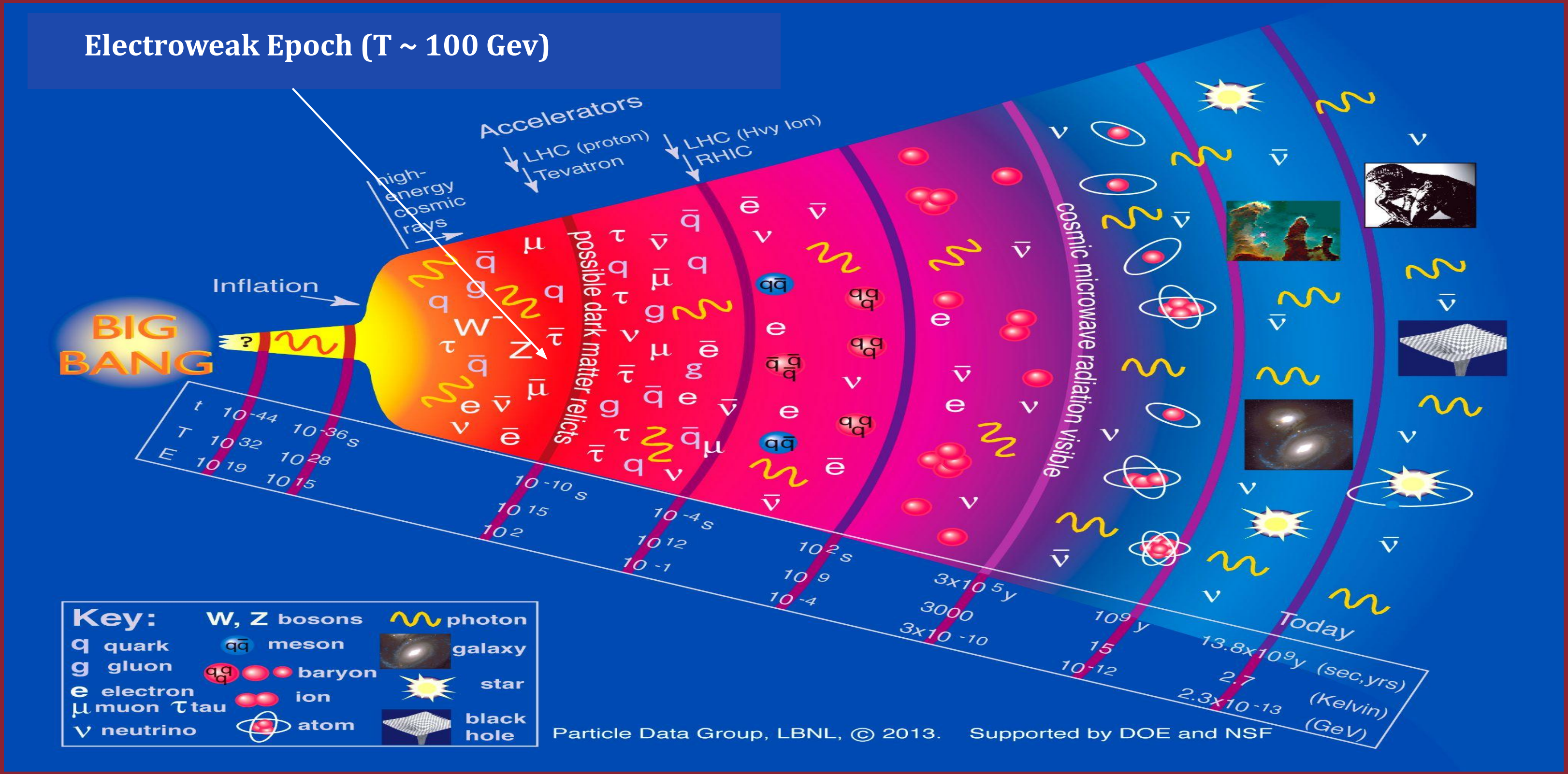
Cosmic History

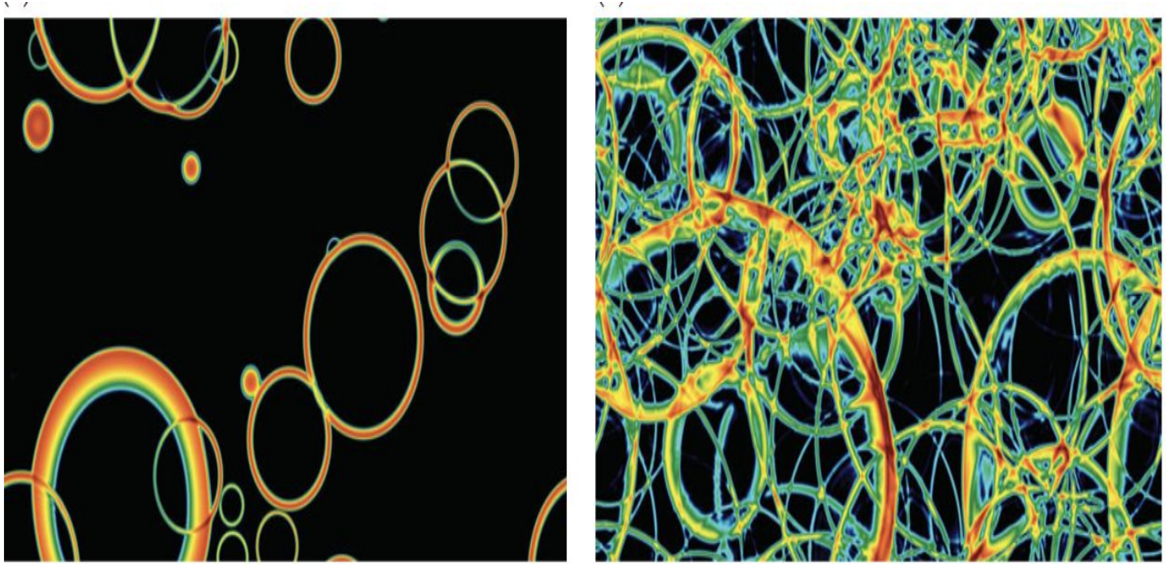
Electroweak Epoch (T ~ 100 Gev)



Cosmic History

Was there a FOPT during the electroweak epoch?

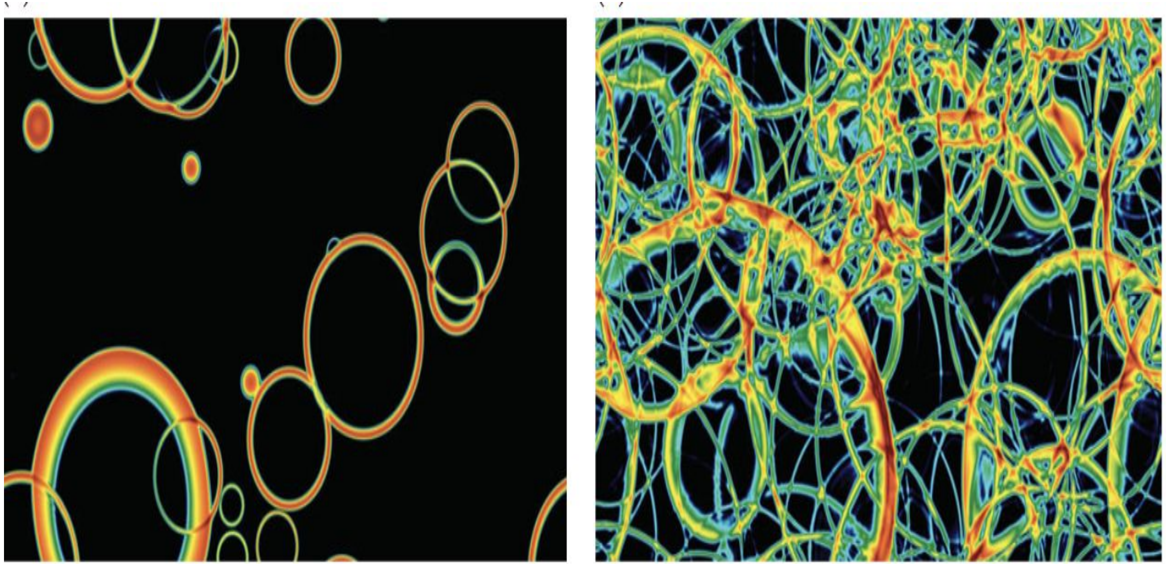




Credit: David Weir arXiv:1705.01783 [hep-ph]

Gravitational Waves from a First Order EWPT

1. Bubble Collisions



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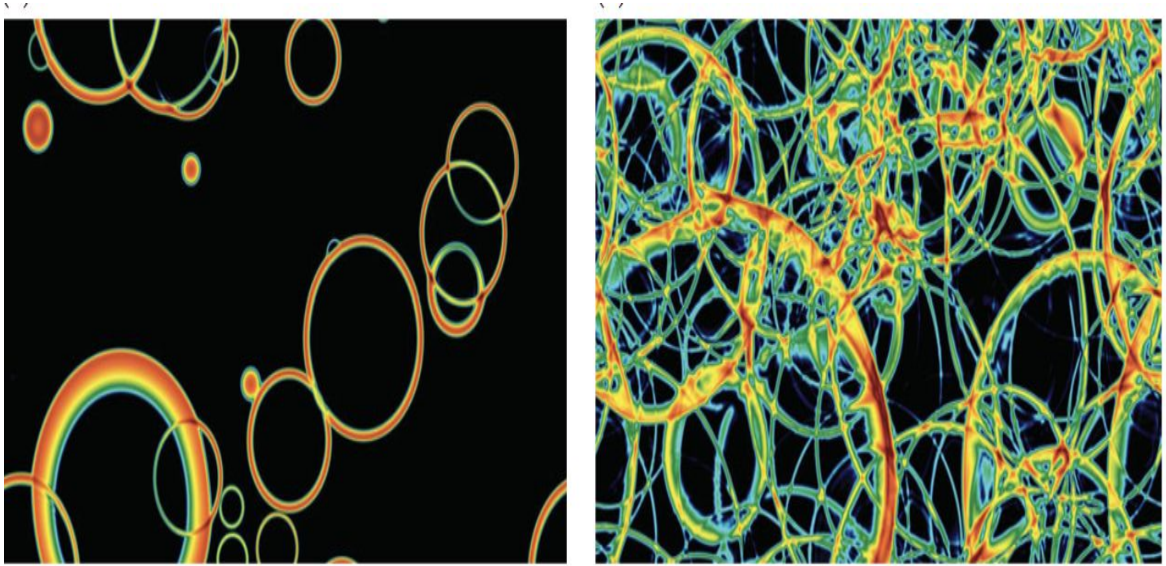
Gravitational Waves from a First Order EWPT

1. Bubble Collisions

2. Acoustic Waves



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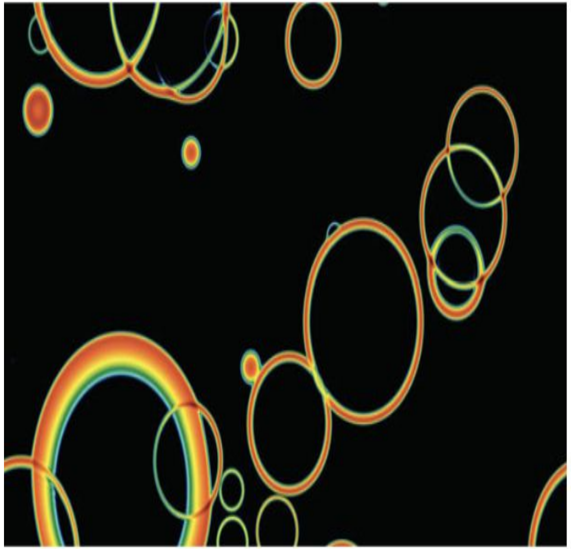
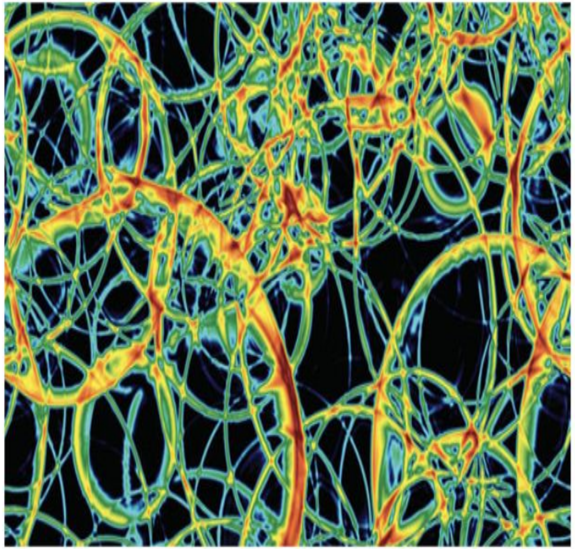
Gravitational Waves from a First Order EWPT

1. **Bubble Collisions**

2. **Acoustic Waves**

3. **Turbulence**





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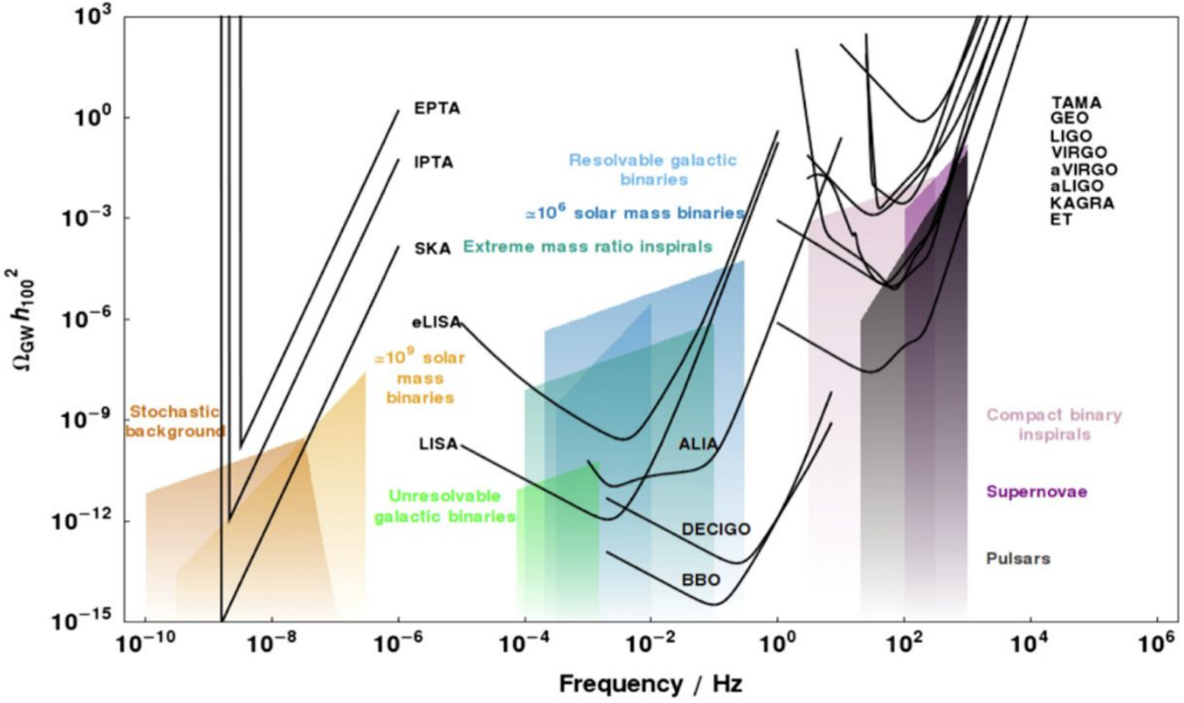
Gravitational Waves from a First Order EWPT

The energy density of GWs vs. frequency for a variety of detectors and sources.



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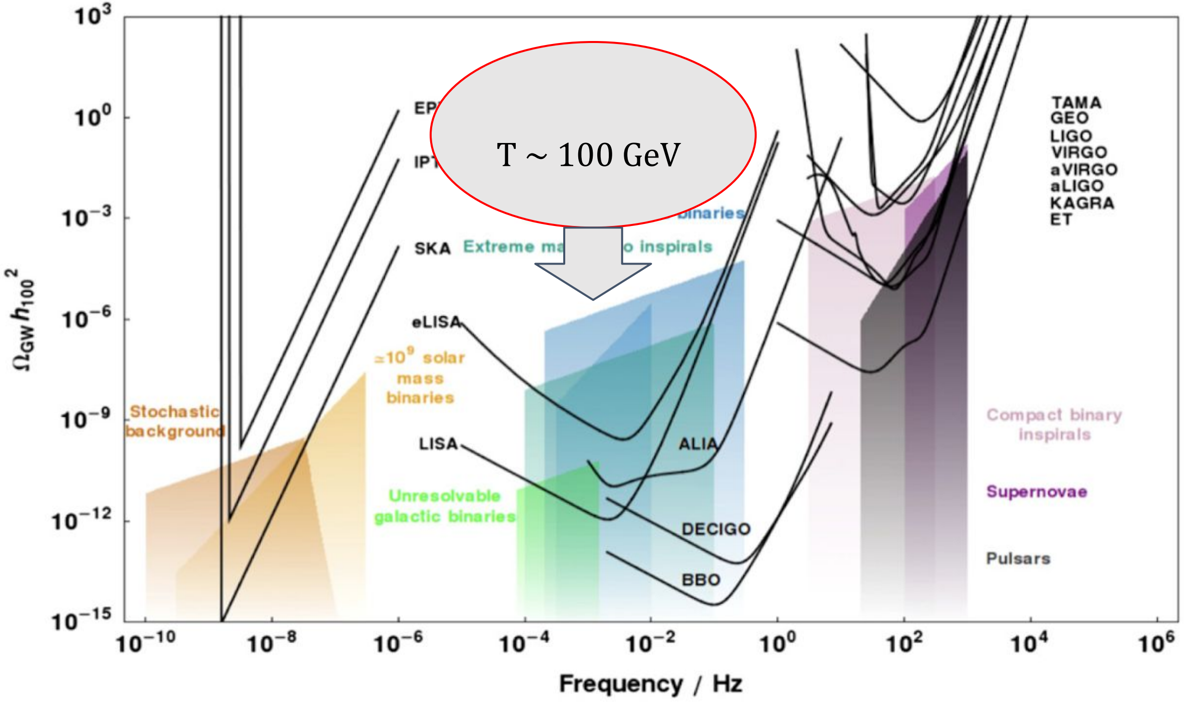

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Gravitational Wave Detectors

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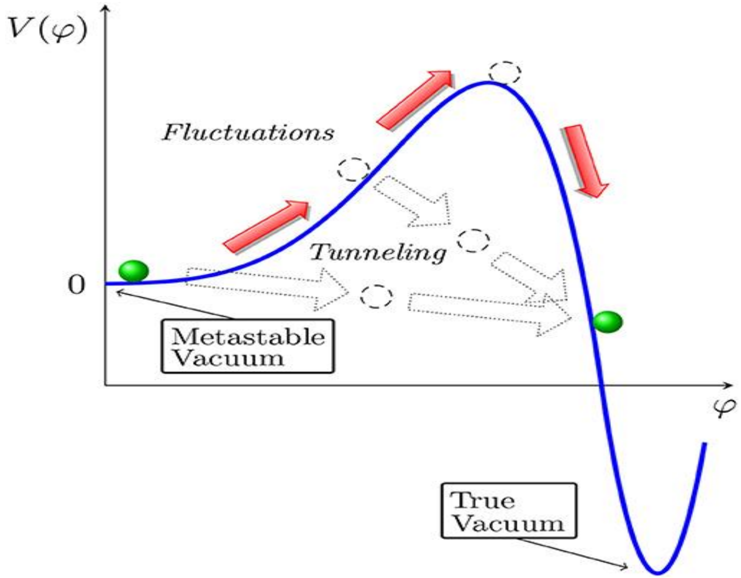
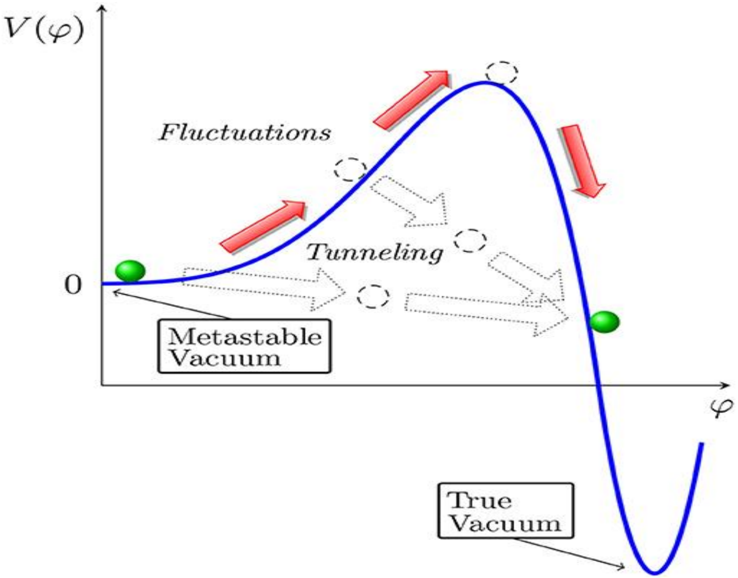


Gravitational Wave Constraints

Credit: Tommi Markkanen,arXiv:1809.06923 [astro-ph.CO]

Tunneling Rate

$$\Gamma = Ae^{-S_3/T}$$



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The inverse duration of the phase transition is given by:

$$\frac{\beta}{H_*} = T_* \frac{d}{dT}\left(\frac{S_3(T)}{T}\right)_{T=T_*}, \text{ where}$$

T_* is the temperature at which the phase transition occurs and $H_* = H(T_*)$ is the Hubble parameter at T_*.

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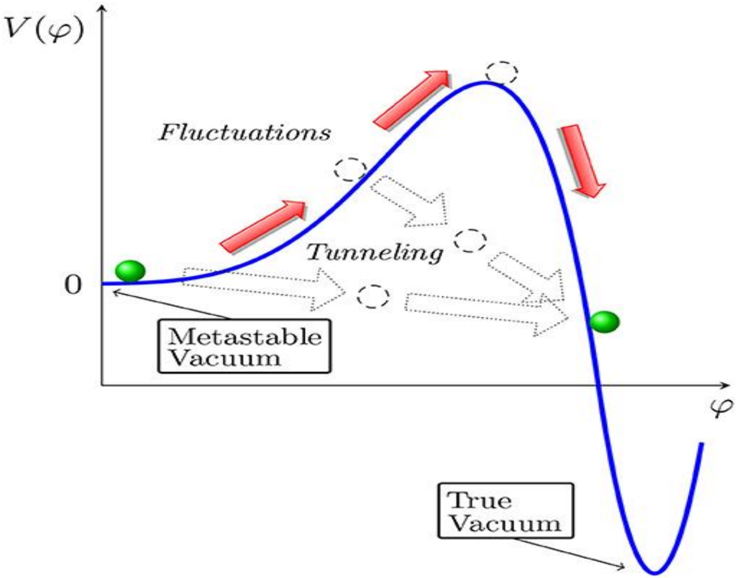

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The strength of the phase transition is given by:

$$\alpha = \frac{30L(T_*)}{\pi^2 g_*(T_*)T_*^4}, \text{ where}$$

$$L(T_*) = \left[T\frac{d}{dT}V[\eta(T),T]\right]_{T=T_*}$$

$\eta(T)$ is the VEV of the true vacuum at temperature T.

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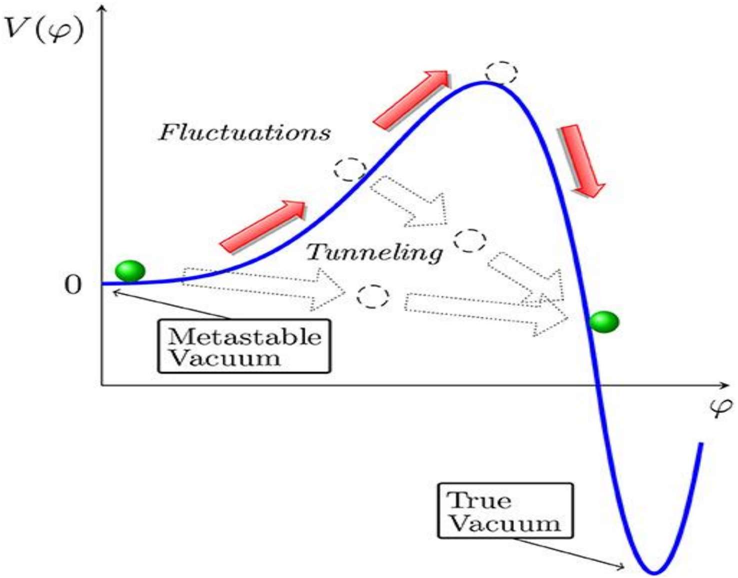

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Gravitational Wave Constraints

Using the imaginary time formalism in QFT we have that:

$$Z = Tr\left[e^{-\beta H}\right] = \int_{PBC} \mathcal{D}\phi e^{-\int_0^\beta d\tau \int d^3x \, \mathcal{L}_{\mathrm{E}}[\phi(x)]}$$

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$$\int \frac{d^4k}{(2\pi)^4} \rightarrow \frac{1}{\beta} \sum_n \int \frac{d^3k}{(2\pi)^3}$$

Finite Temperature QFT

High-T expansion using the bosonic one-loop function:

$$J_b(x) \equiv \frac{1}{2}\sum\!\!\!\!\!\!\int_P \ln(P^2 + x) = -\frac{\pi^2 T^4}{90} + \frac{T^2 x}{24} - \frac{T x^{3/2}}{12\pi} + \mathcal{O}(x^2) \; .$$

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=> Valid for: $\mu \sim gT$

Dimensional Reduction:

Scale	Validity	Dimension	Lagrangian	Fields	Parameters
Hard	πT	$d+1$	$\mathcal{L}_{\text{4d}}$	$B_\mu, \Phi,$	μ^2, λ, g
		$\downarrow$ Step 1: *Integrate out* $n \neq 0$ *Matsubara modes*			
Intermediate	gT	d	$\mathcal{L}_{\text{3d}}$	$B_{3,i}, B_0, \Phi_3$	$\mu_3^2, \lambda_3, g_3, m_\text{D}, h_3, \kappa_3$

3d Effective Field Theory

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Thermal loops require daisy resummation of zero-mode masses:

In practice amounts to replacing field-dependent masses with thermal masses.

3d Effective Field Theory

Naive Calculation vs. Our Approach

PROBLEM! => Finite temperature QFT calculations yield gauge dependent results

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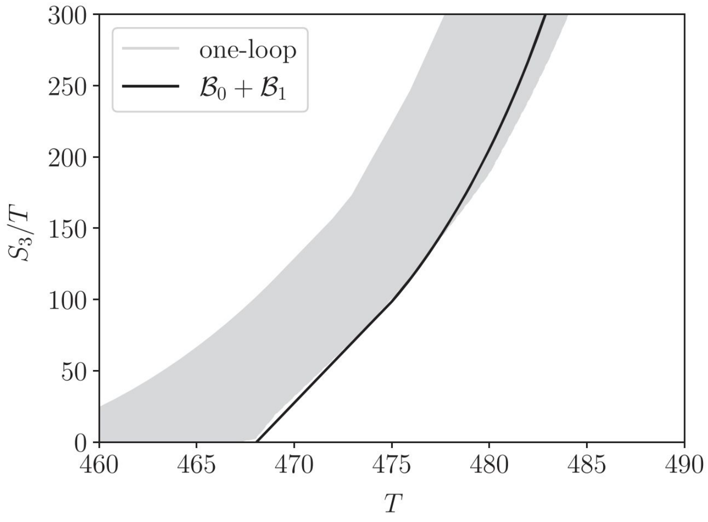
U(1) Abelian Higgs model Example: Naive Calculation vs. Our Approach



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Nucleation at Finite Temperature: A Gauge-Invariant Perturbative Framework

Johan Löfgren,[1,*] Michael J. Ramsey-Musolf,[2,3,4,5,†] Philipp Schicho,[6,‡] and Tuomas V. I. Tenkanen[4,5,7,§]

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[2]*Amherst Center for Fundamental Interactions, Department of Physics, University of Massachusetts, Amherst, Massachusetts 01003, USA*
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[4]*Tsung-Dao Lee Institute and School of Physics and Astronomy, Shanghai Jiao Tong University, 800 Dongchuan Road, Shanghai 200240, China*
[5]*Shanghai Key Laboratory for Particle Physics and Cosmology, Key Laboratory for Particle Astrophysics & Cosmology (MOE), Shanghai Jiao Tong University, Shanghai 200240, China*
[6]*Department of Physics and Helsinki Institute of Physics, P.O. Box 64, FI-00014 University of Helsinki, Finland*
[7]*Nordita, KTH Royal Institute of Technology and Stockholm University, Roslagstullsbacken 23, SE-106 91 Stockholm, Sweden*

Figure 1 in

Naive Calculation vs. Our Approach

The key to going beyond the naive perturbative calculations :
Keep terms satisfying a power counting (Arnold & Espinosa 1994) in g wherein $V_{\rm LO}$ has a radiatively generated barrier.

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$$\lambda \sim g^3 \,, \qquad \mu_{\rm eff}^2 \sim g^3 T^2 \,, \qquad \phi \sim T \sim \frac{\mu}{g} \,.$$

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$$V^{\rm eff} = V^{\rm eff}_{g^3} + V^{\rm eff}_{g^4} + V^{\rm eff}_{g^{9/2}} + \dots \, ,$$
$$Z = 1 + Z_g + Z_{g^{3/2}} + \dots \, .$$

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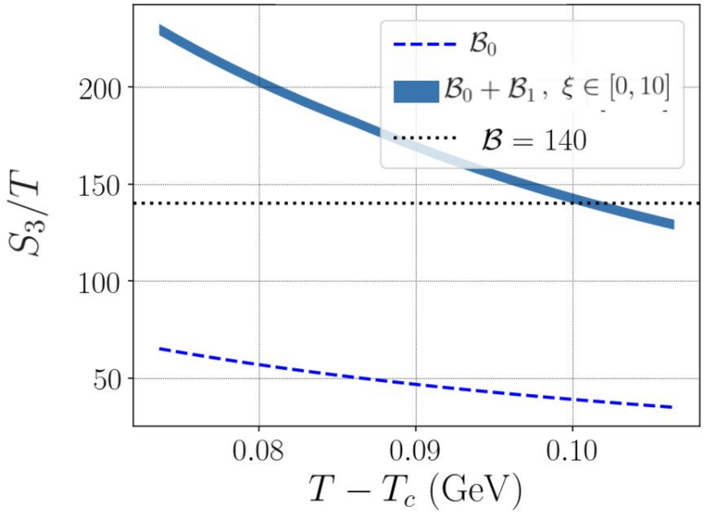
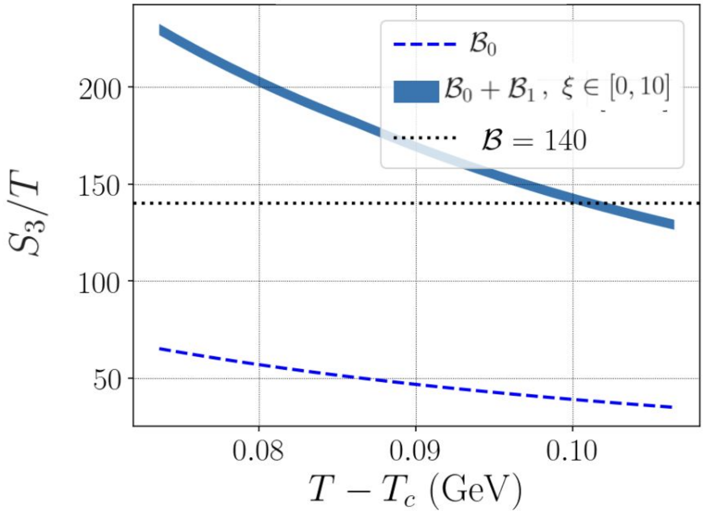
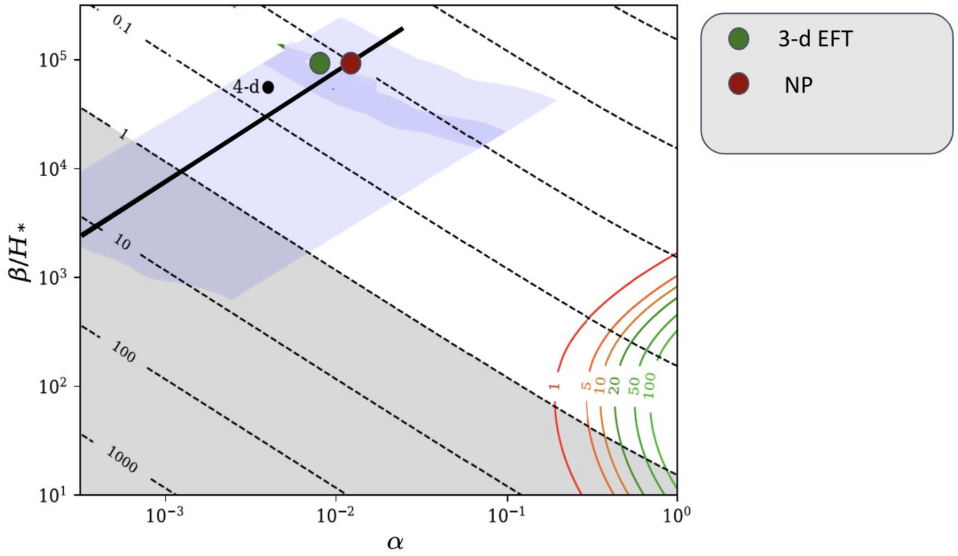
$$S_3 = \mathcal{B}_0 + \mathcal{B}_1$$
$$\mathcal{B}_0 = \int d^3x \left(\frac{1}{2} (\partial_i \phi_b)^2 + V_{\mathrm{LO}}(\phi_b) \right)$$
$$\mathcal{B}_1 = \int d^3x \left(\frac{1}{2} Z_{\mathrm{NLO}} (\partial_i \phi_b)^2 + V_{\mathrm{NLO}}(\phi_b) \right)$$

Power Counting



Gauge-Invariance





Gauge-Invariant Gravitational Wave Constraints

The inverse duration of the phase transition is given by: $\frac{\beta}{H_*}$

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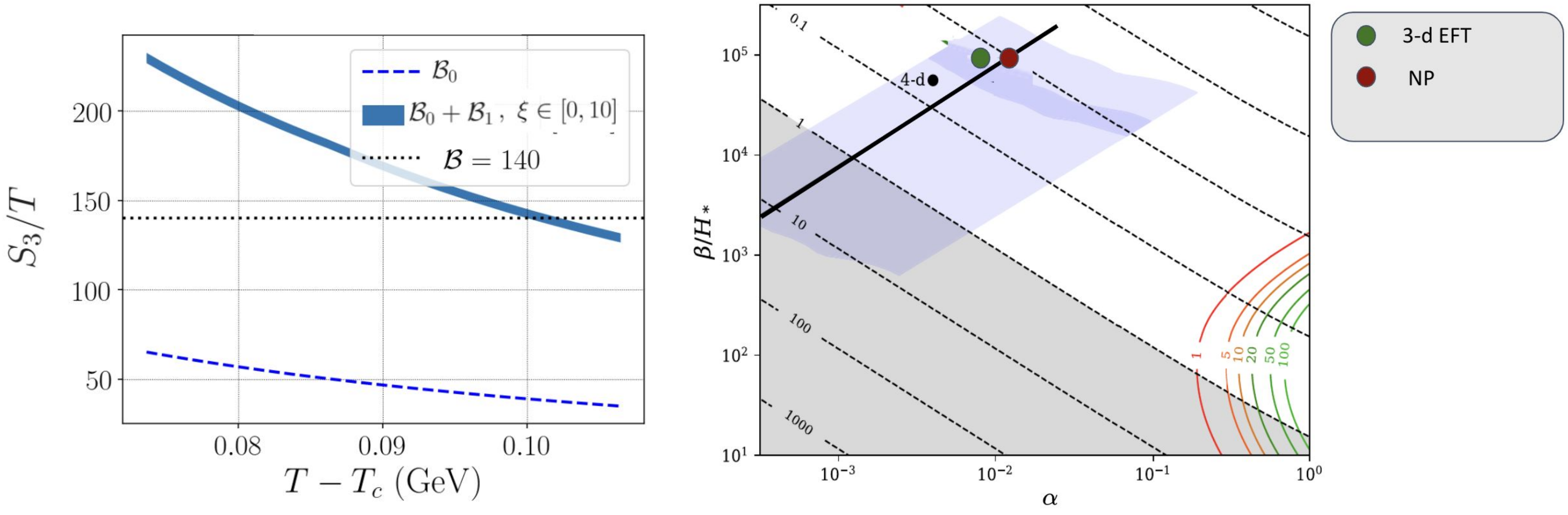

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The strength of the phase transition is given by: α

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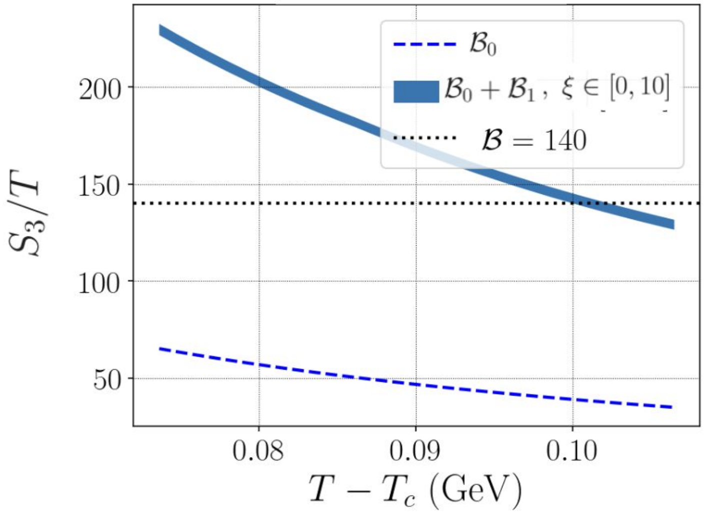

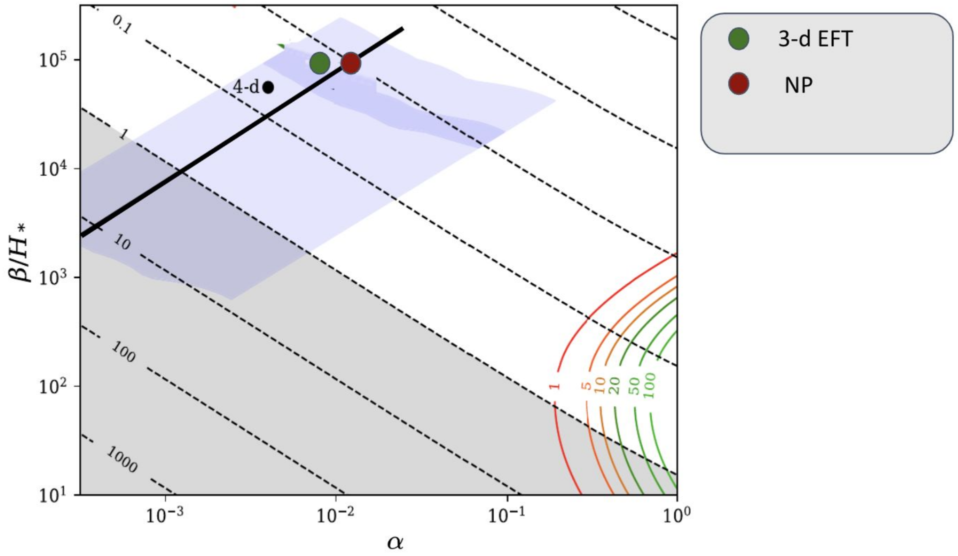

Gauge-Invariant Gravitational Wave Constraints

Thank You For Your Attention!

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References

1. arXiv:2112.08912 [hep-ph] Michael Ramsey-Musolf, Tuomas V. I. Tenkanen, etc

2. arXiv:1903.11604 [hep-ph] Michael Ramsey-Musolf, Tuomas V. I. Tenkanen, etc

3. arXiv:1705.01783 [hep-ph] David J. Weir

4. arXiv:1809.06923 [astro-ph.CO] Tommi Markkanen, etc

5. arXiv: 1408.0740 [gr-qc] Christopher Moore, etc.

6. arXiv:0709.2773 [hep-ph] M. Vepsalainen

7. Finite Temperature Field Theory by A. Das