Scattering Amplitudes of Massive Spin-2 Kaluza-Klein States with Matter

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With

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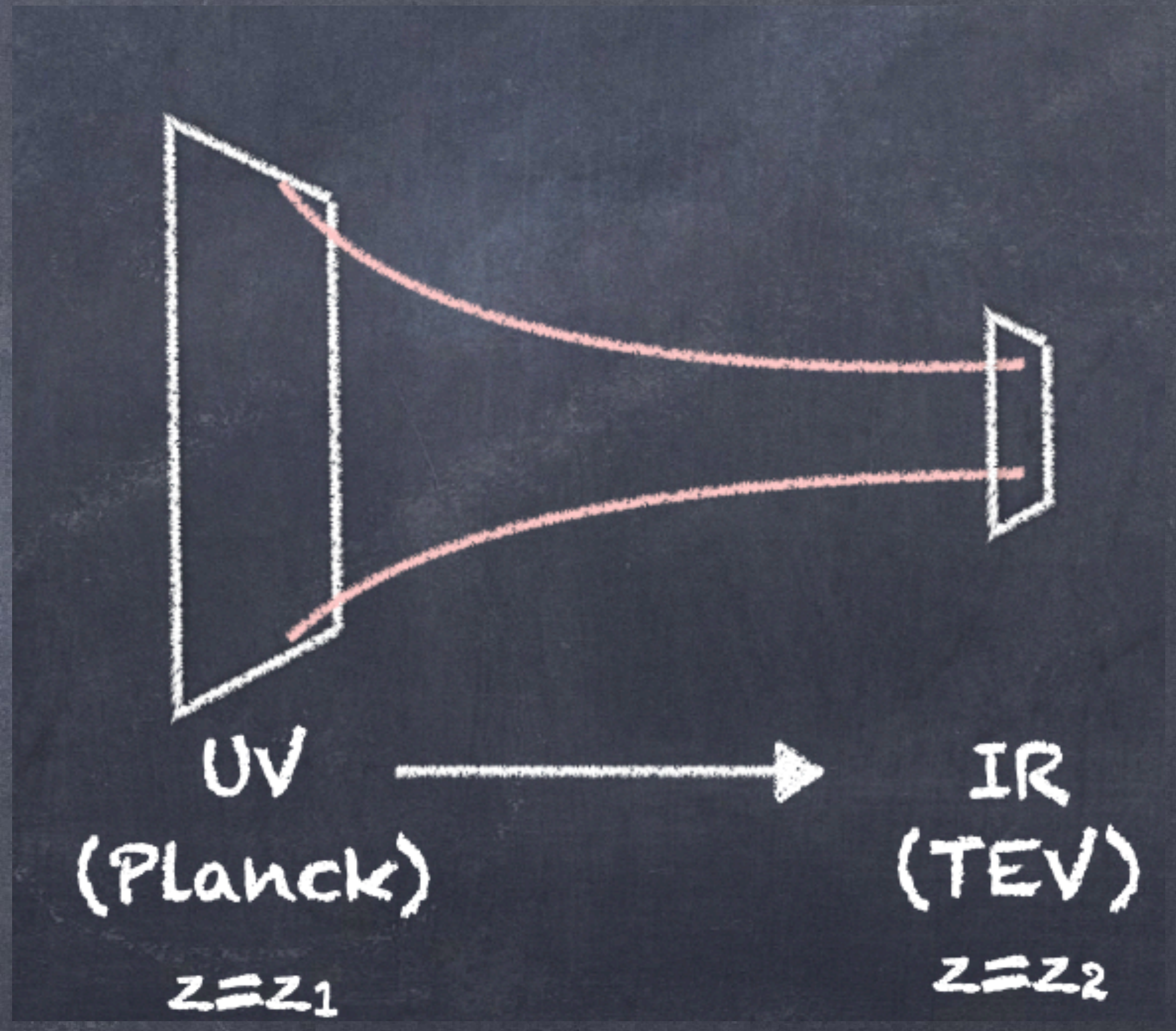
arXiv:1903.05650, 1910.06159, 2002.12458, 2104.08169, 2206.10628, 2207.02887, 2311.00770, 2312.08576

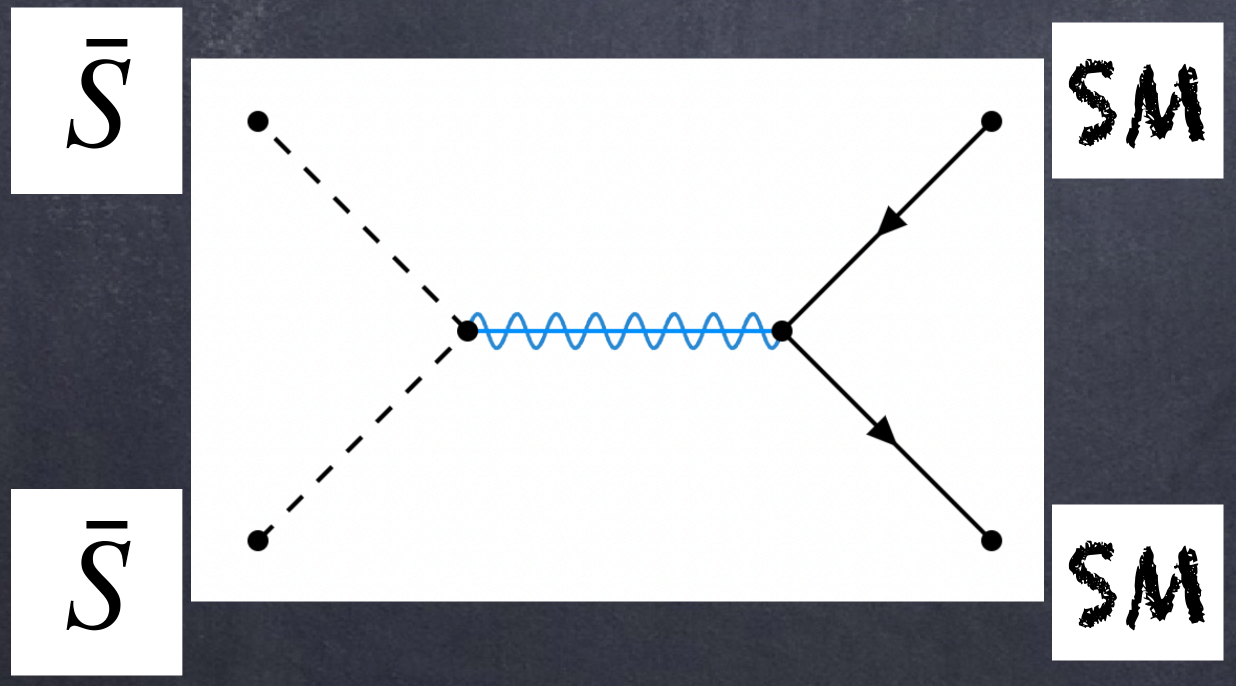
MICHIGAN STATE UNIVERSITY

Motivation

- Can Dark Matter only have gravitational interactions?
- Yes, but interactions are Planck scale suppressed.
- Can Dark Matter only have gravitational interactions and and still behave like a WIMP?
- Yes!



Simple Example with a brane scalar:

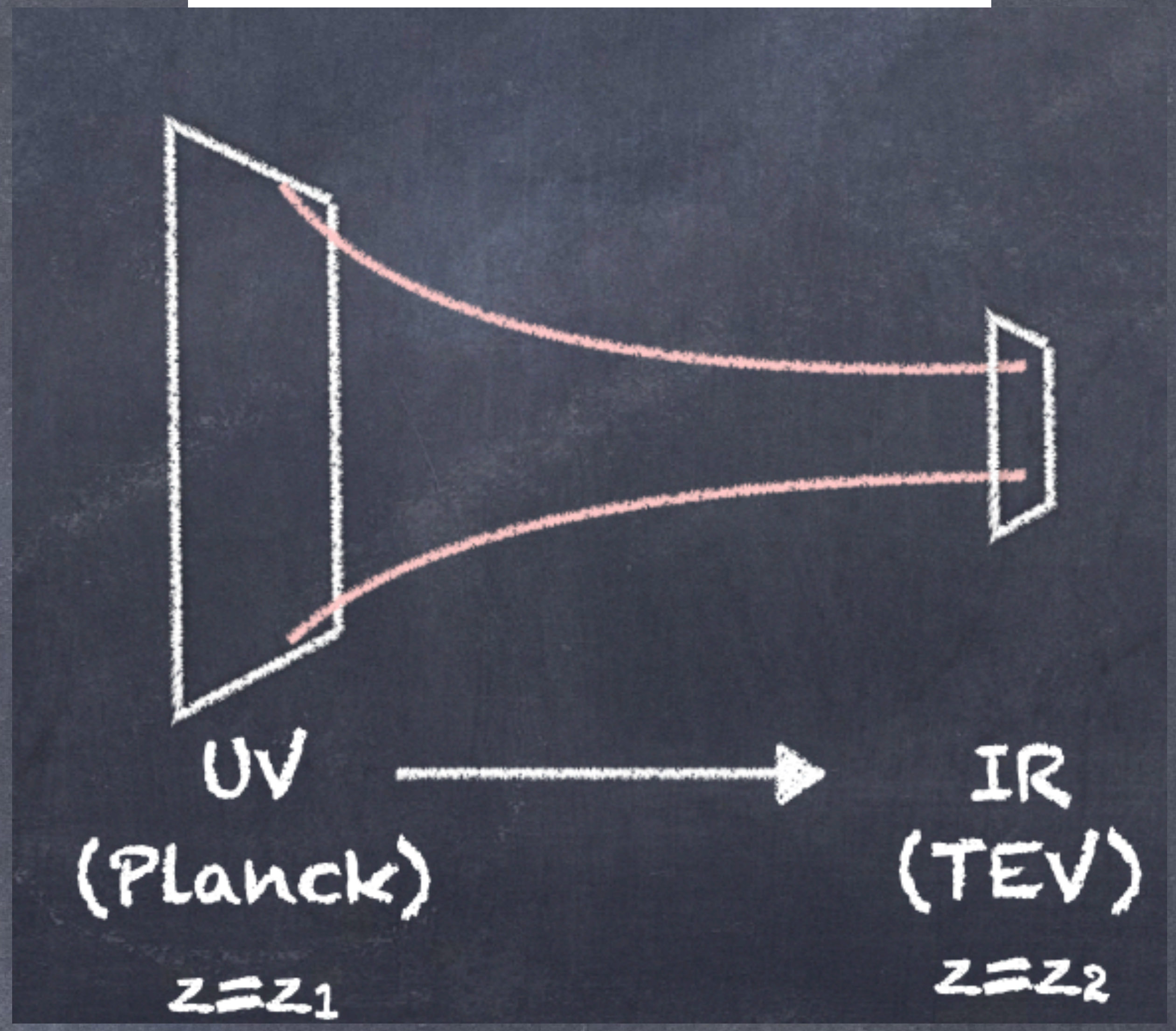


$$h^{\mu\nu}(T^{\rm SM}_{\mu\nu} + T^{\rm DM}_{\mu\nu})$$

$$\mathcal{L}_{\bar{S},\rm brane} = \int_{z_1}^{z_2} dz\, \sqrt{\bar{G}} \left(\frac{1}{2} \bar{G}^{MN} \partial_M \bar{S} \partial_N \bar{S} - \frac{1}{2} M_{\bar{S}}^2 \bar{S}^2 \right) e^{-2A(z)} \delta(z - \bar{z}),$$

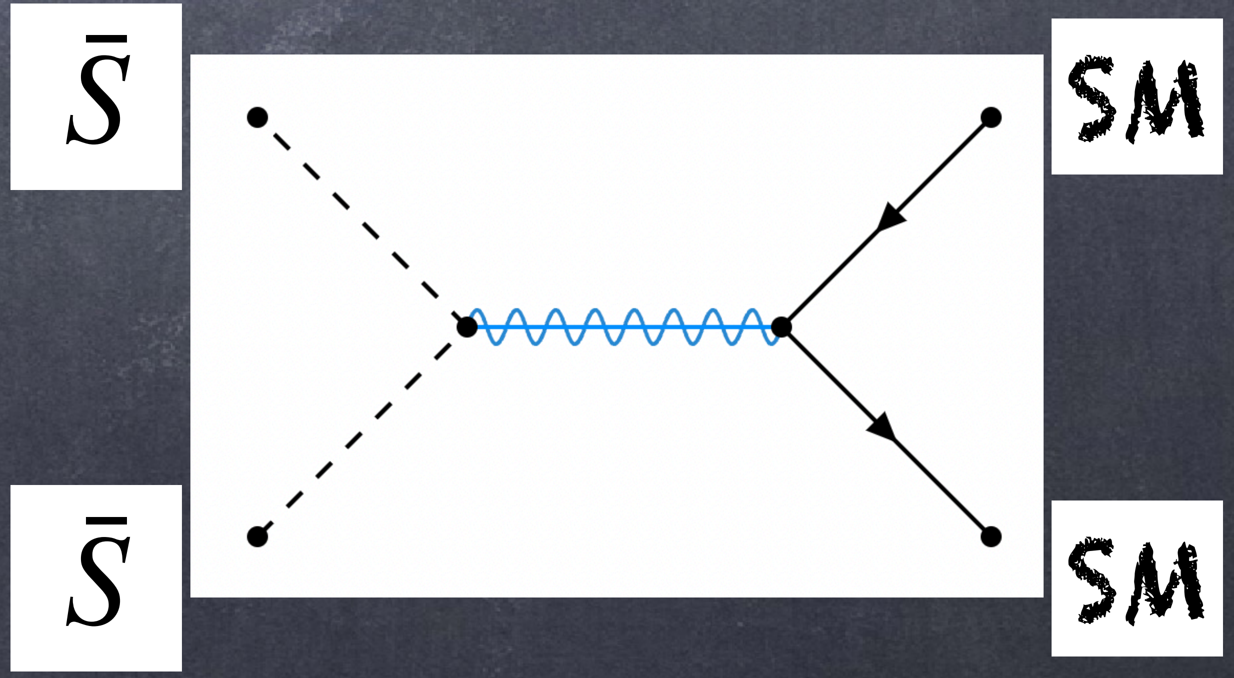
Particle Content

$$A(z) = -\ln(kz)$$



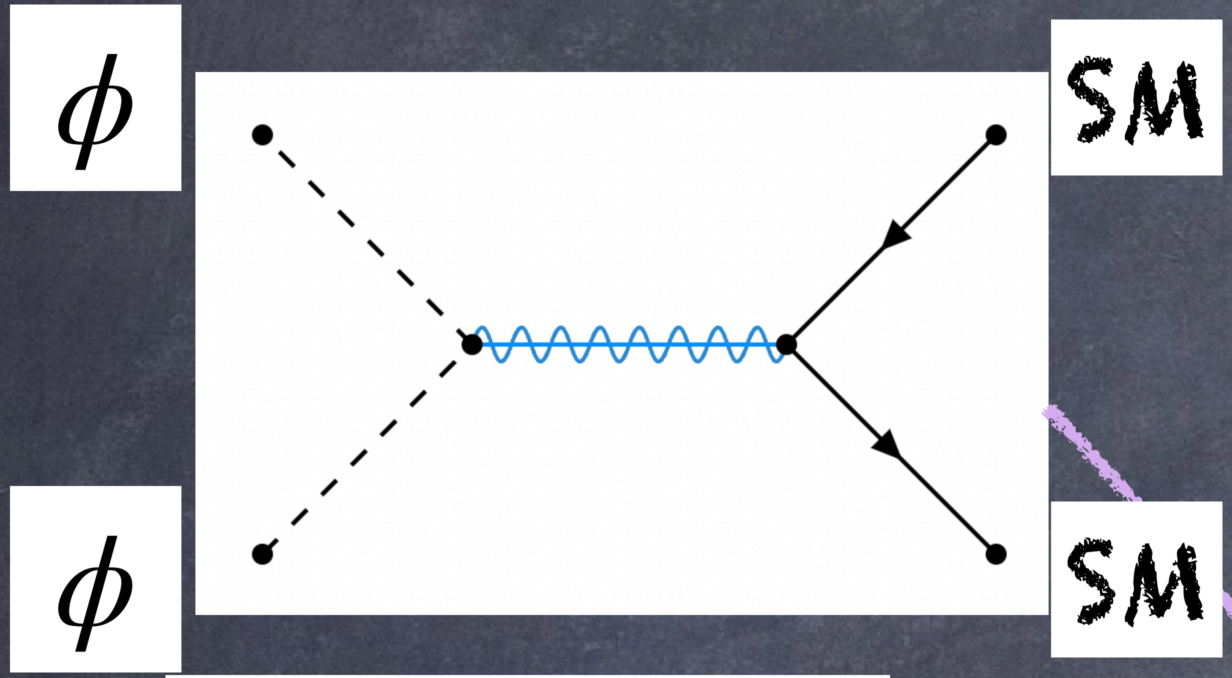
- Tower of massive spin-2 states $h_{\mu\nu}$
- Radion (r)
  - Goldberger-Wise Scalar to stabilize extradimension
- SM +DM
  - DM in Bulk or localized on TeV Brane
  - DM : Scalar, Fermion or Vector
- SM +DM couple to $\frac{1}{\Lambda} h^{\mu\nu}(T^{(SM)}_{\mu\nu} + T^{(DM)}_{\mu\nu})$
- $\Lambda \simeq M_{pl}\, e^{-kr_c}$ (Usually a few TeV or more)

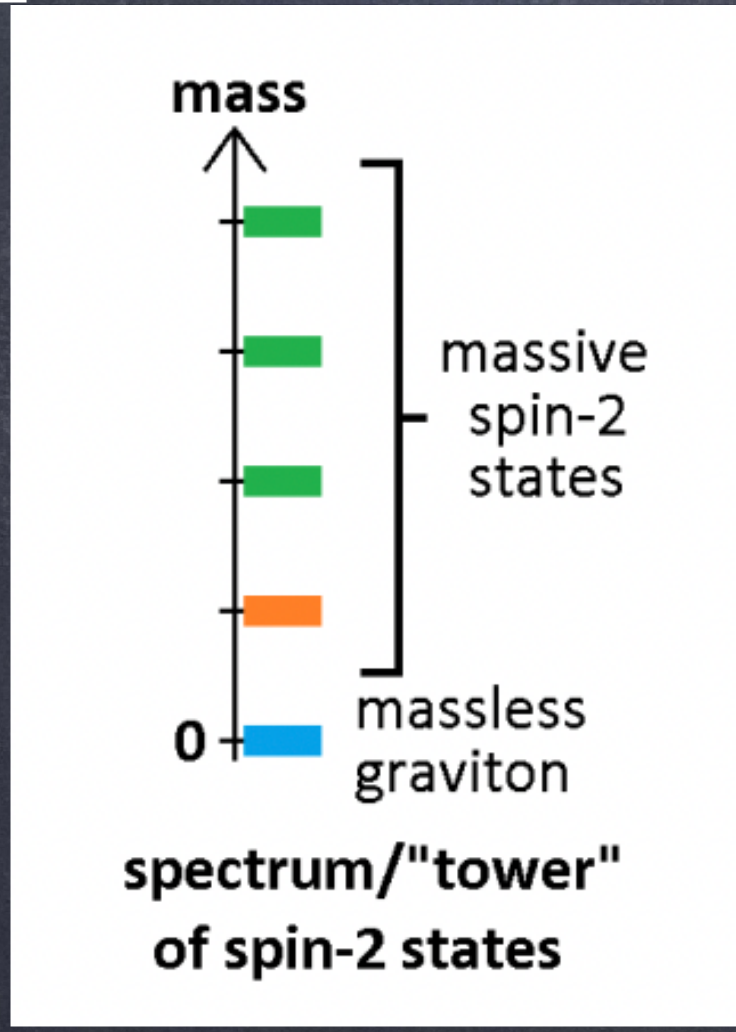
$$G_{MN} = e^{2A(z)} \begin{pmatrix} e^{-\kappa\hat{\varphi}/\sqrt{6}}(\eta_{\mu\nu} + \kappa\hat{h}_{\mu\nu}) & \frac{\kappa}{\sqrt{2}}\hat{A}_\mu \\ \frac{\kappa}{\sqrt{2}}\hat{A}_\mu & -\left(1 + \frac{\kappa}{\sqrt{6}}\hat{\varphi}\right)^2 \end{pmatrix}$$

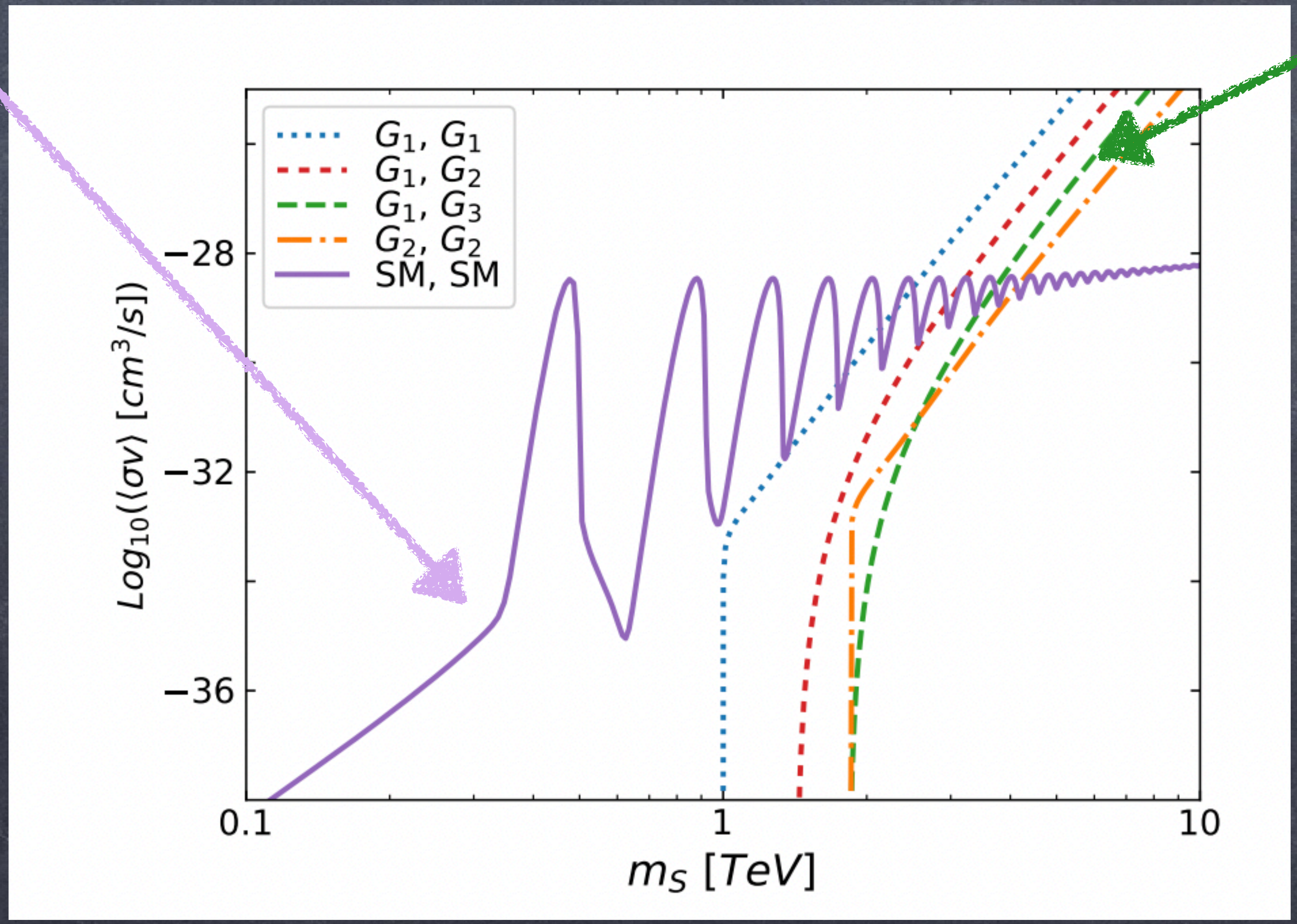


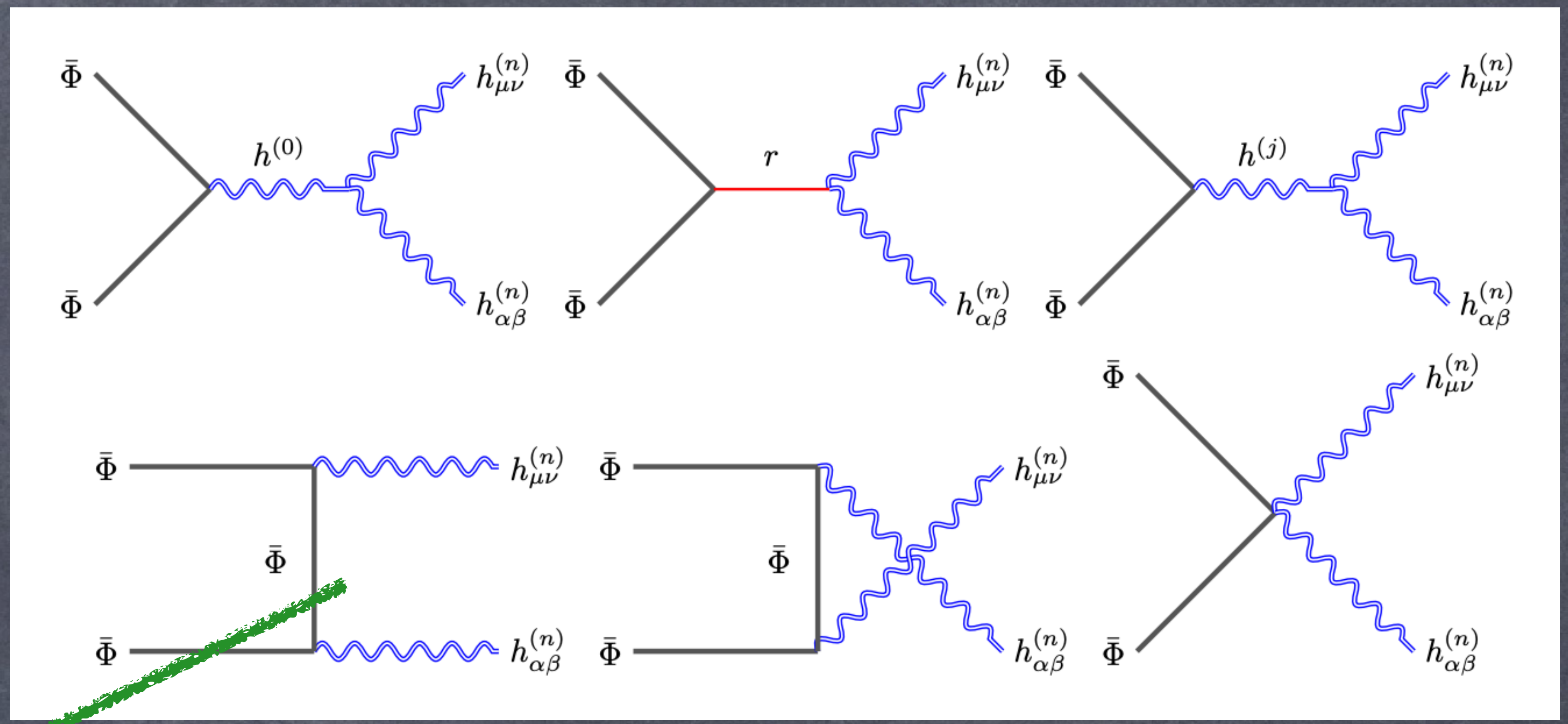
$$h^{\mu\nu}(T^{\rm SM}_{\mu\nu} + T^{\rm DM}_{\mu\nu})$$

Let's Calculate the Relic









Bad High Energy Growth

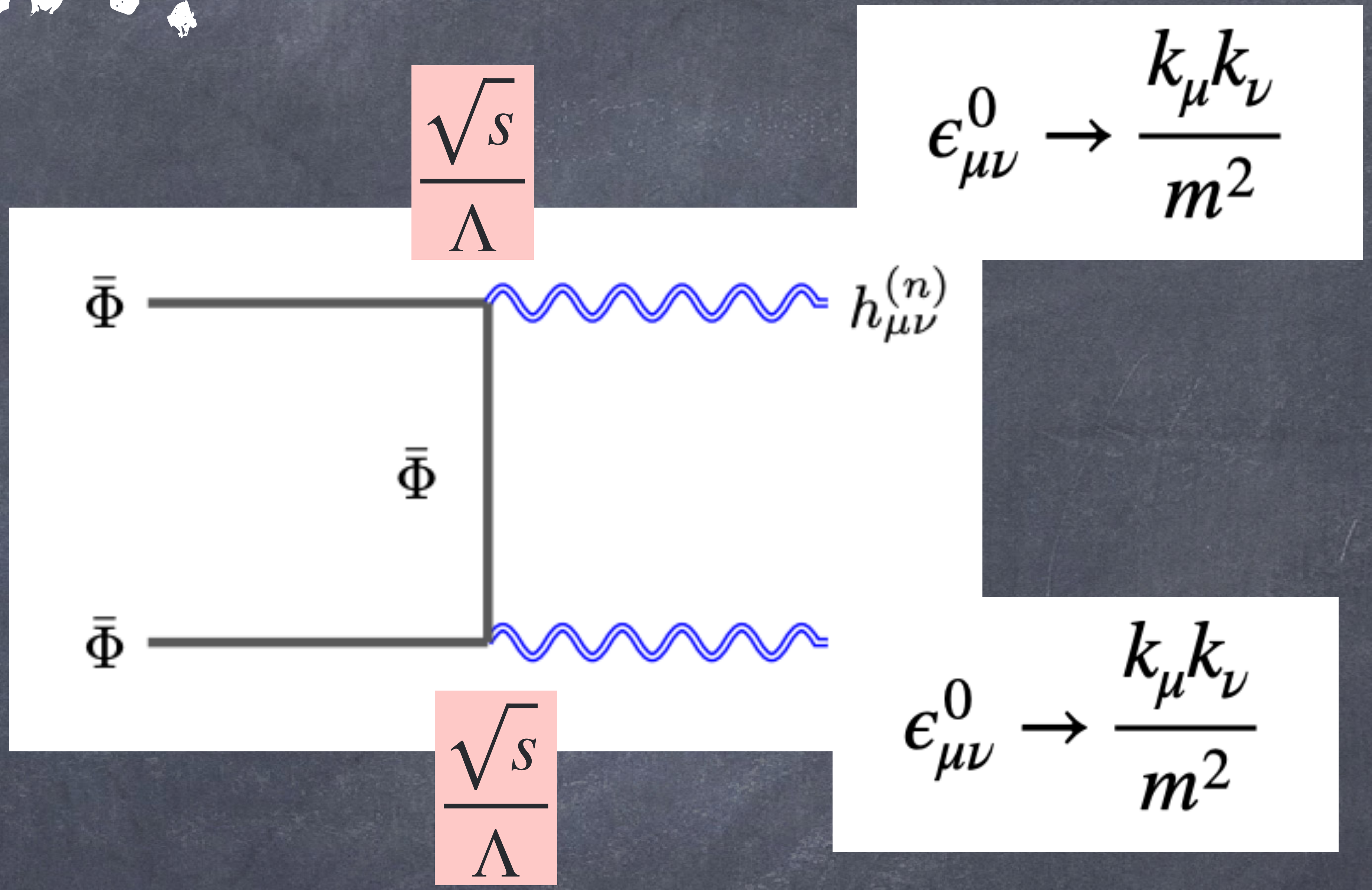
$< \sigma v > \propto s^6$ or E^{12} ?

arXiv: 1511.03278, 1709.09688, 1907.04340, 2004.14403

Why the bad high energy growth?

- Counting energy from polarization and vertices(*), we see that the energy of the amplitude should grow like s^3 (E^6)

- $\Longrightarrow$Unitarity is violated at a scale much smaller than Λ



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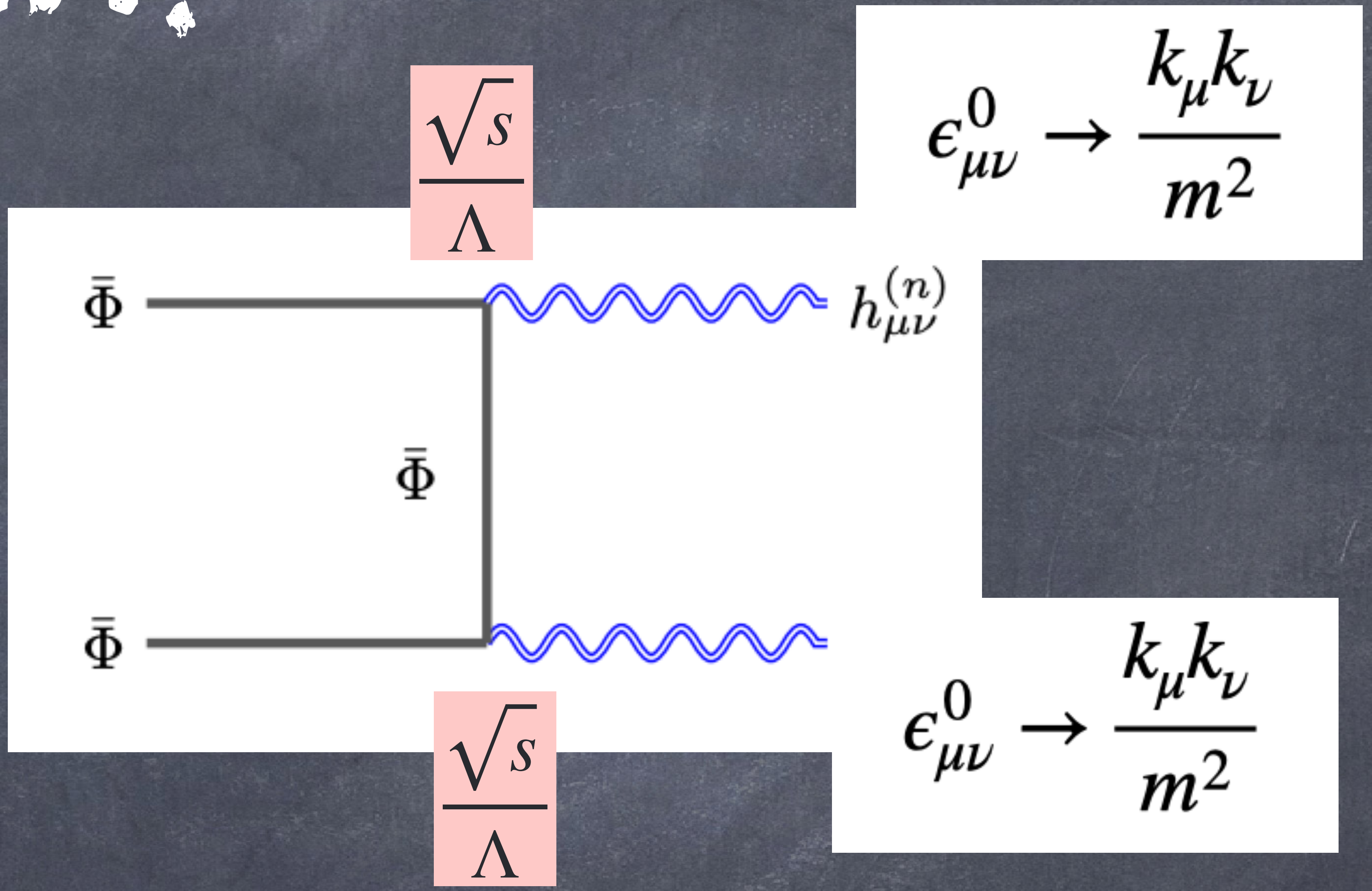

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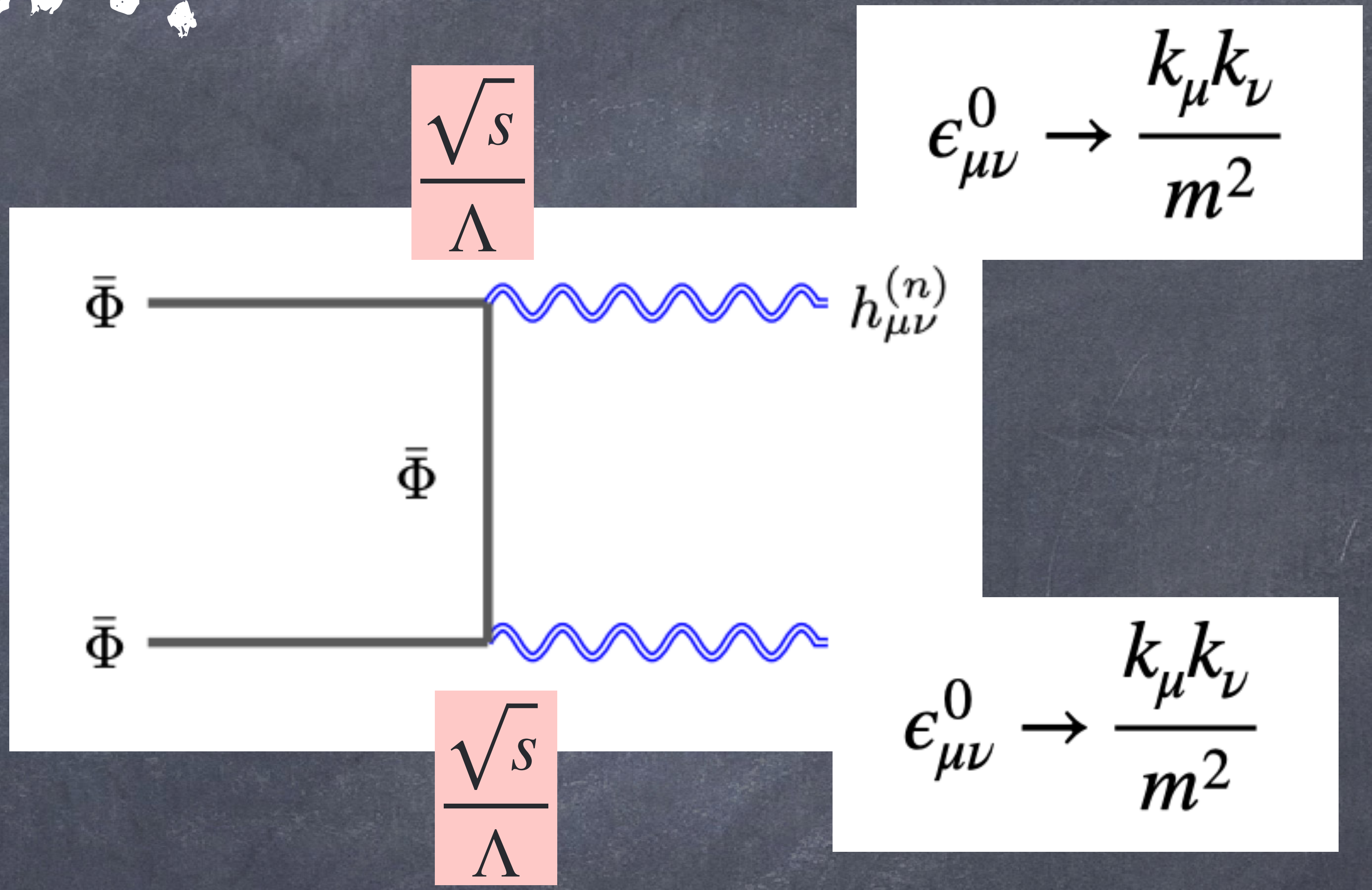

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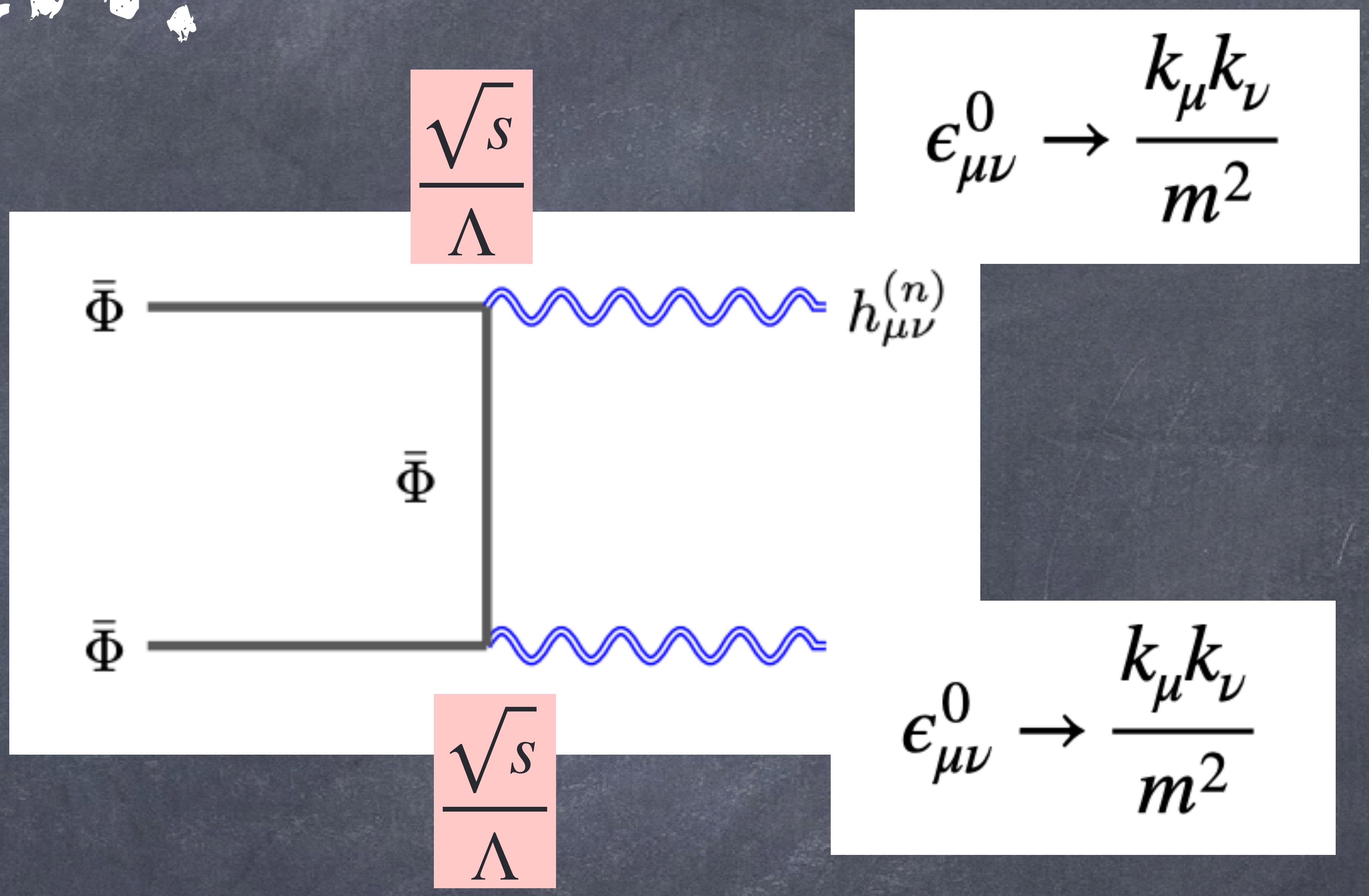


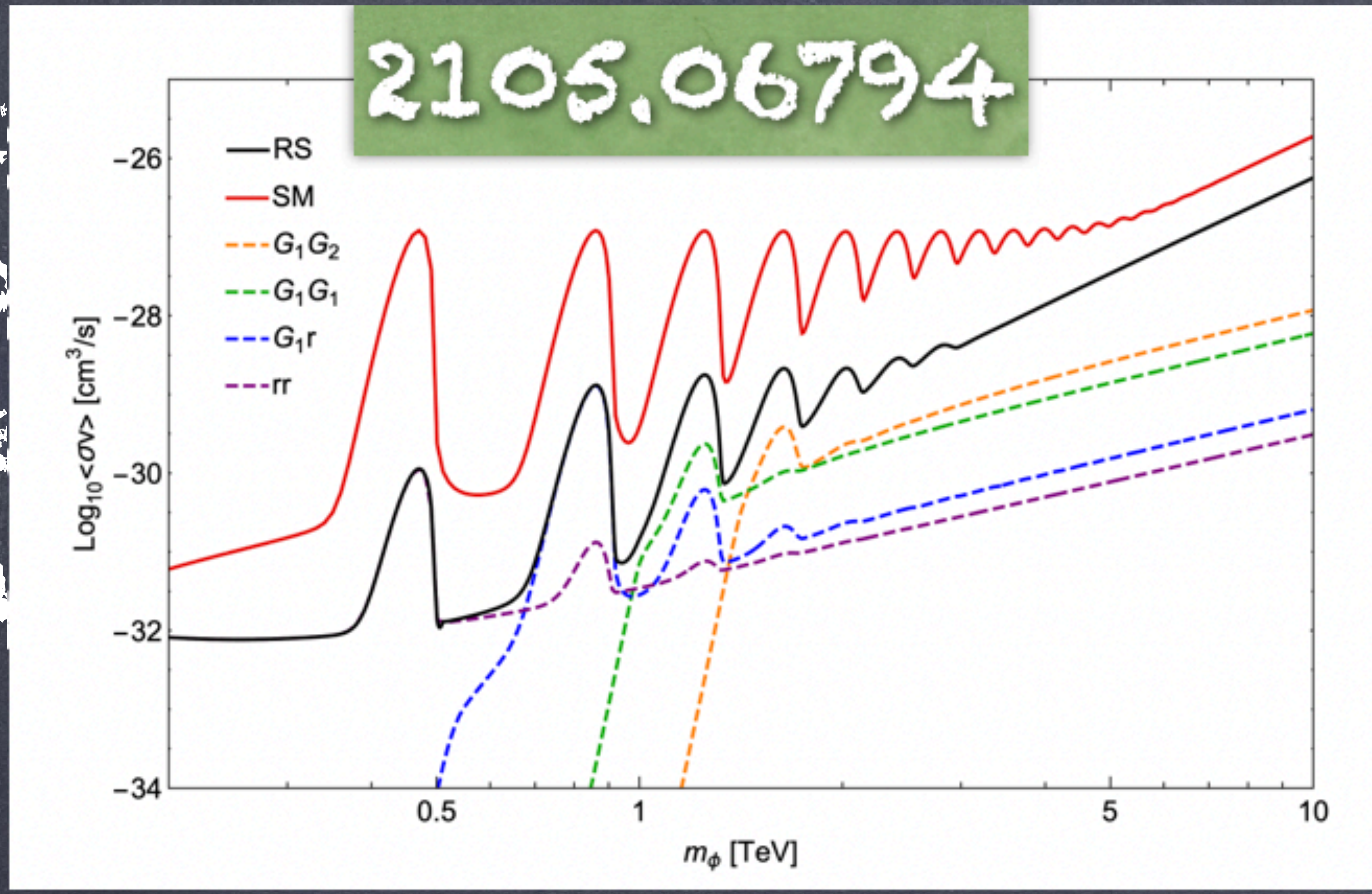
- $\Longrightarrow$Unitar[…]s violated at a scale mu[…]aller than Λ

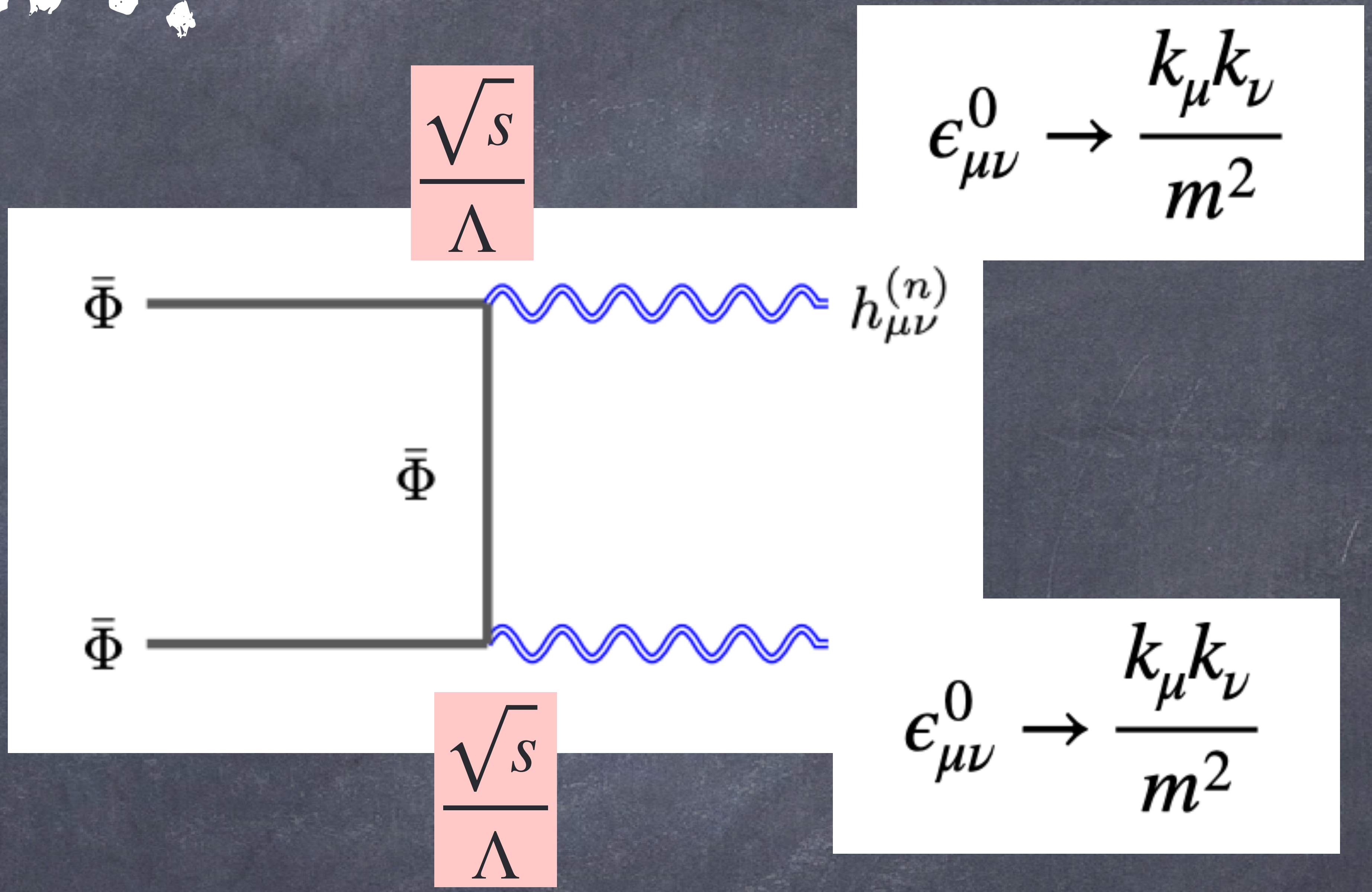
Diffeomorphism invariance protects amplitude from bad high energy growth! How?

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_{(\mu}\xi_{\nu)}$$

Why the bad high energy growth?

- Co... po...ces(*), we... of the an... w like s^3





- $\Longrightarrow$Unitarity is violated at a scale much smaller than Λ

Diffeomorphism invariance protects amplitude from bad high energy growth! How?

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_{(\mu}\xi_{\nu)}$$

Unitarity in Extra Dimensional Models

Tower of States and Gauges

- Unitary Gauge: This talk
  - One radion field (r)
  - Tower of Gravitons (h)
    - One massless Graviton + Tower of Massive Gravitons
- 't-Hooft-Feynman Gauge: next Talk by Xing Wang
  - Vector Goldstone Modes ($A^{(n)}$)
  - Scalar Goldstone Modes ($\pi^{(n)}$)

KK decomposition of 5D fields

$$\hat{h}_{\mu\nu}(x^\alpha, z) = \sum_{n=0}^{\infty} \hat{h}^{(n)}_{\mu\nu}(x^\alpha) f^{(n)}(z),$$

$$\hat{A}_\mu(x^\alpha, z) = \sum_{n=1}^{\infty} \hat{A}^{(n)}_\mu(x^\alpha) g^{(n)}(z),$$

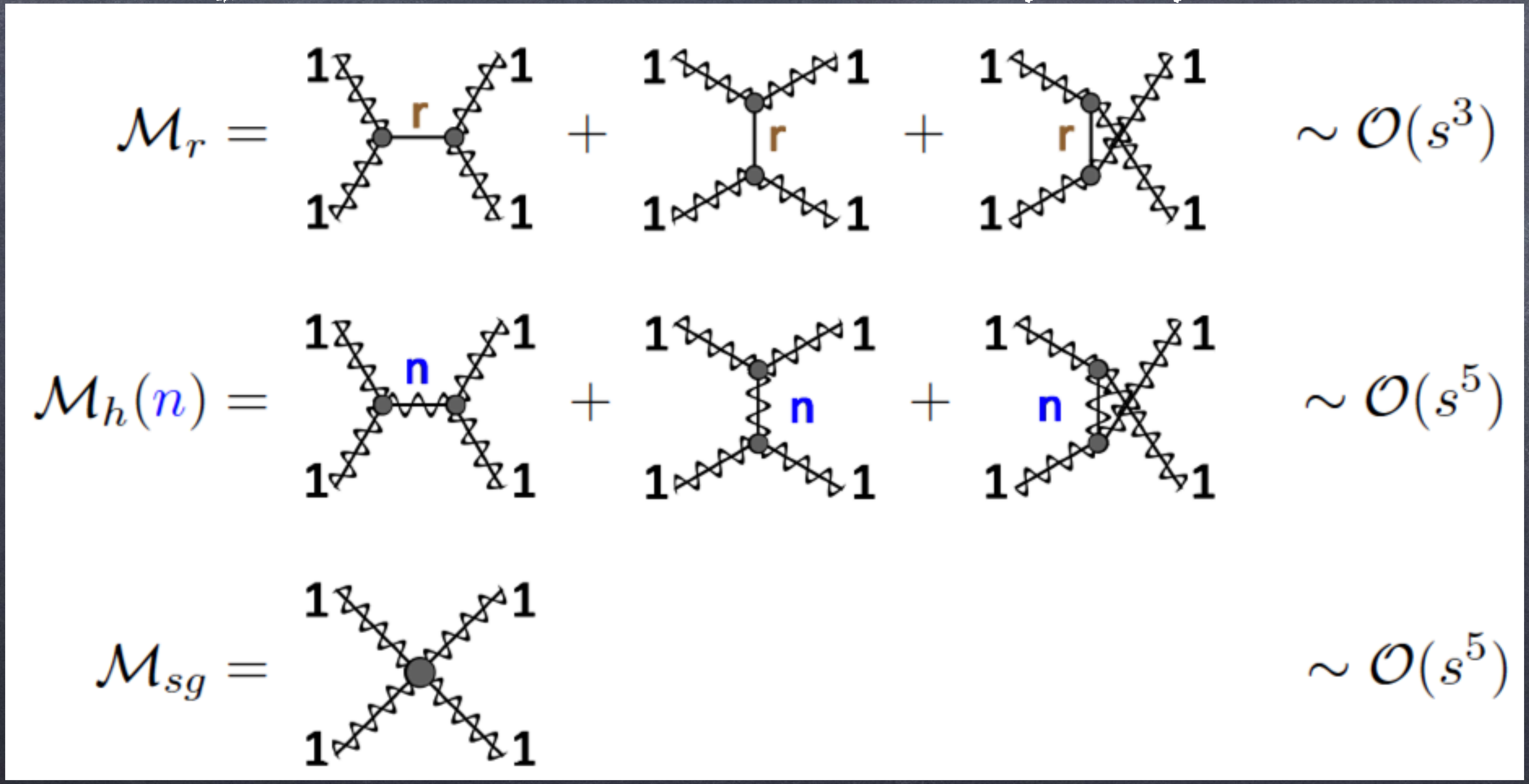
$$\hat{\varphi}(x^\alpha, z) = \hat{r}(x^\alpha) k^{(0)}(z) + \sum_{n=1}^{\infty} \hat{\pi}^{(n)}(x) k^{(n)}(z)$$

Solutions of of 5D WF

$$\begin{aligned} f^{(n)}(z) &= C^{(n)}_h z^2 \left[Y_1(m_n z_2) J_2(m_n z) - J_1(m_n z_2) Y_2(m_n z)\right] \\ g^{(n)}(z) &= C^{(n)}_A z^2 \left[Y_1(m_n z_2) J_1(m_n z) - J_1(m_n z_2) Y_1(m_n z)\right] \\ k^{(n)}(z) &= C^{(n)}_\varphi z^2 \left[Y_1(m_n z_2) J_0(m_n z) - J_1(m_n z_2) Y_0(m_n z)\right] \end{aligned}$$

Unitarity in Extra Dimensions

- Let's look at $2 \to 2$ scattering of gravitons - worst high energy behavior using simple power counting



Preliminaries

- Decompose the 5D graviton field into a tower of 4D fields

$$\hat{h}_{\mu\nu}(x^\alpha, z) = \sum_{n=0}^{\infty} \hat{h}^{(n)}_{\mu\nu}(x^\alpha) f^{(n)}(z),$$

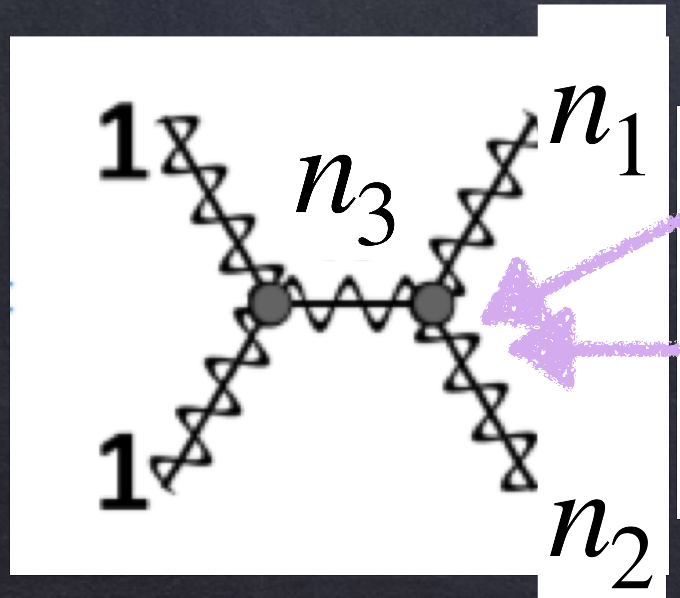
- Determine the fifth dimensional wave function for each mode from the Sturm-Liouville problem

$$\begin{cases} \partial_z f^{(n)} = m_n g^{(n)} \\ (-\partial_z - 3A')g^{(n)} = m_n f^{(n)} \end{cases} \qquad \begin{cases} (\partial_z + A')g^{(n)} = m_n k^{(n)} \\ (-\partial_z - 2A')k^{(n)} = m_n g^{(n)} \end{cases}$$

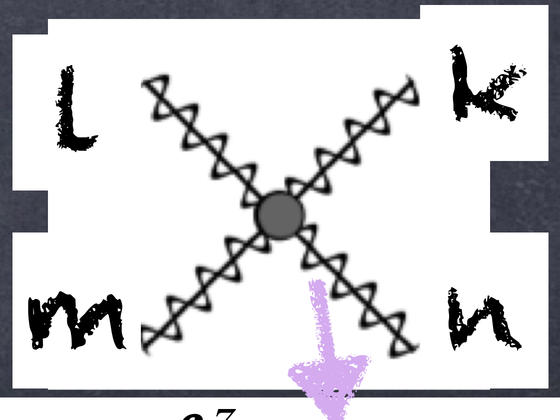
- Wave functions are orthogonal and complete

$$\left\langle f^{(n)} f^{(m)} \right\rangle = \int_{z_1}^{z^2} dz e^{3A(z)} f^{(n)}(z) f^{(m)}(z) = \delta_{mn}$$

- Substitute wave functions into overlap integrals that set the couplings of the vertices



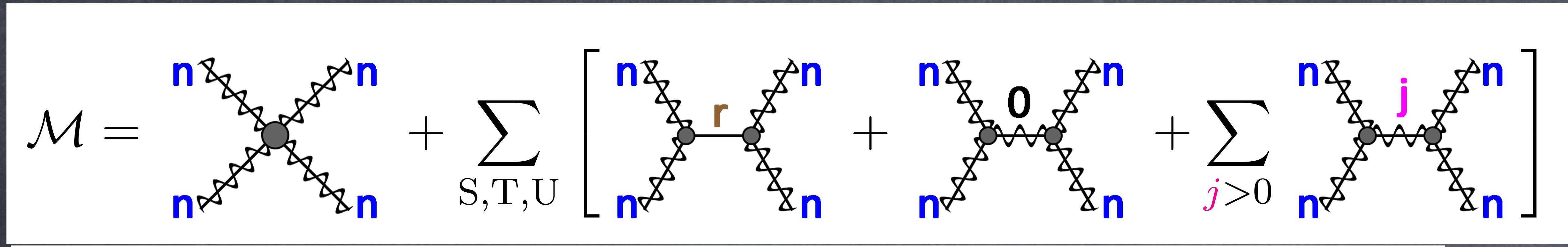
$$\begin{aligned} a_{n_1 n_2 n_3} &= \langle f^{(n_1)} f^{(n_2)} f^{(n_3)} \rangle , \\ b_{\bar{n}_1 \bar{n}_2 n_3} &= \langle (\partial_z f^{(n_1)})(\partial_z f^{(n_1)}) f^{(n_3)} \rangle , \\ b_{\bar{n}_1 \bar{n}_2 r} &= \langle (\partial_z f^{(n_1)})(\partial_z f^{(n_1)}) k^{(0)} \rangle , \end{aligned}$$



$$a_{klmn} = \left\langle f^{(k)} f^{(l)} f^{(m)} f^{(n)} \right\rangle = \int_{z_1}^{z_2} dz e^{3A(z)} f^{(k)}(z) f^{(l)}(z) f^{(m)}(z) f^{(n)}(z)$$

SUM RULES

More on this in 't-Hooft-Feynman Gauge in next talk from Xing Wang



$$\mathcal{M}(s,\theta) = \sum_{\sigma \in \frac{1}{2}\mathbb{Z}} \overline{\mathcal{M}}^{(\sigma)}(\theta) \cdot s^{\sigma}$$

$$\mathcal{M}^{(5)}(\cos\theta) = -\frac{\kappa^2}{\pi r_c}\frac{(7+\cos 2\theta)\sin^2\theta}{2304 m_n^8} \cdot \left(a_{nnnn} - \sum_j a_{nnj}^2\right)$$

$$\mathcal{M}^{(4)}(\cos\theta) = \frac{\kappa^2}{\pi r_c}\frac{(7+\cos 2\theta)^2}{27648 m_n^8} \cdot \left(4m_n^2 a_{nnnn} - 3\sum_j m_j^2 a_{nnj}^2\right)$$

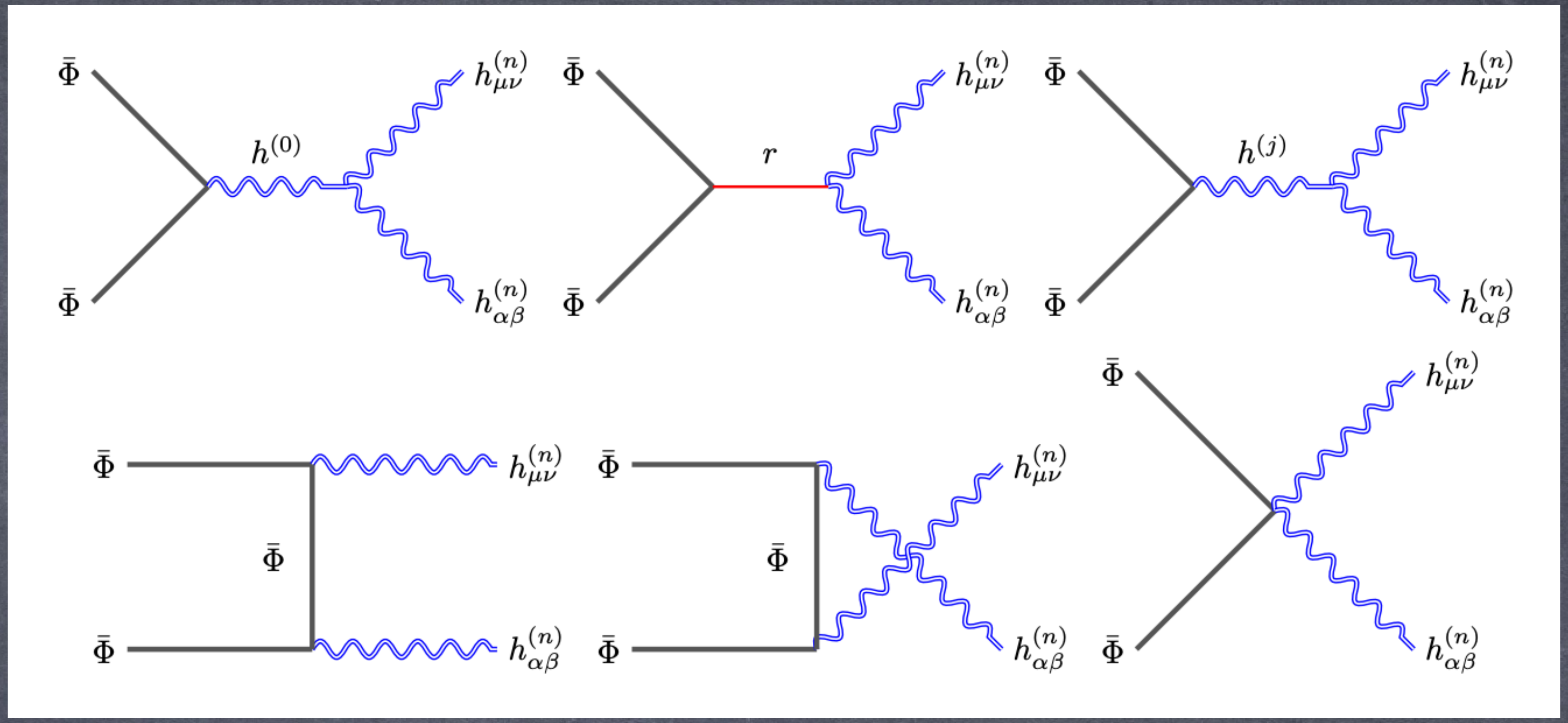
$$\overline{\mathcal{M}}^{(3)} = \frac{5\,\kappa^2\,\sin^2\theta}{1152\,\pi r_c\, m_n^4}\left\{\sum_j \frac{m_j^4}{m_n^4} a_{nnj}^2 - \frac{16}{15} a_{nnnn} - \frac{4}{5}\left[\frac{9\,b_{nnr}^2}{(m_n r_c)^4} - a_{nn0}^2\right]\right\}.$$

$$\overline{\mathcal{M}}^{(2)} = \frac{\kappa^2\,[7+\cos(2\theta)]}{864\,\pi r_c\, m_n^2}\left\{\sum_j \left[\frac{m_j^2}{m_n^2} - \frac{5}{2}\right]\frac{m_j^4}{m_n^4} a_{nnj}^2 + \frac{8}{3} a_{nnnn} - 2\left[\frac{9\,b_{nnr}^2}{(m_n r_c)^4} - a_{nn0}^2\right]\right\}$$

- At each order of S, relations between overlap integrals ensure that the bad high energy growth vanishes
- Example: $\sum_j a_{nnj}^2 = a_{nnnn}$
- Possible to prove analytically using completeness and orthogonality
- Finally, as expected, we find that the amplitude grows as $\mathcal{O}(s)$ $\Longrightarrow$ Unitarity violated at $\sim \Lambda \simeq M_{pl}\, e^{-kr_c}$

arXiv: 2002.12458, 2206.10628

Brane Scalar



- Amplitudes appear to grow as E^6
- Cancellations due to sum rules ensure the amplitudes grow only as E^2,
  - i.e. $\tilde{M}^{(6)} = \tilde{M}^{(4)} = 0$ and $\tilde{M}^{(2)} \neq 0$

$$\widetilde{\mathcal{M}}^{(6)} = \frac{\kappa^2(1-\cos 2\theta)}{192 m_n^4}\left[\left(f^{(n)}(\bar{z})\right)^2 - \sum_{j=0}^{\infty} a_{nnj} f^{(j)}(\bar{z})\right]$$

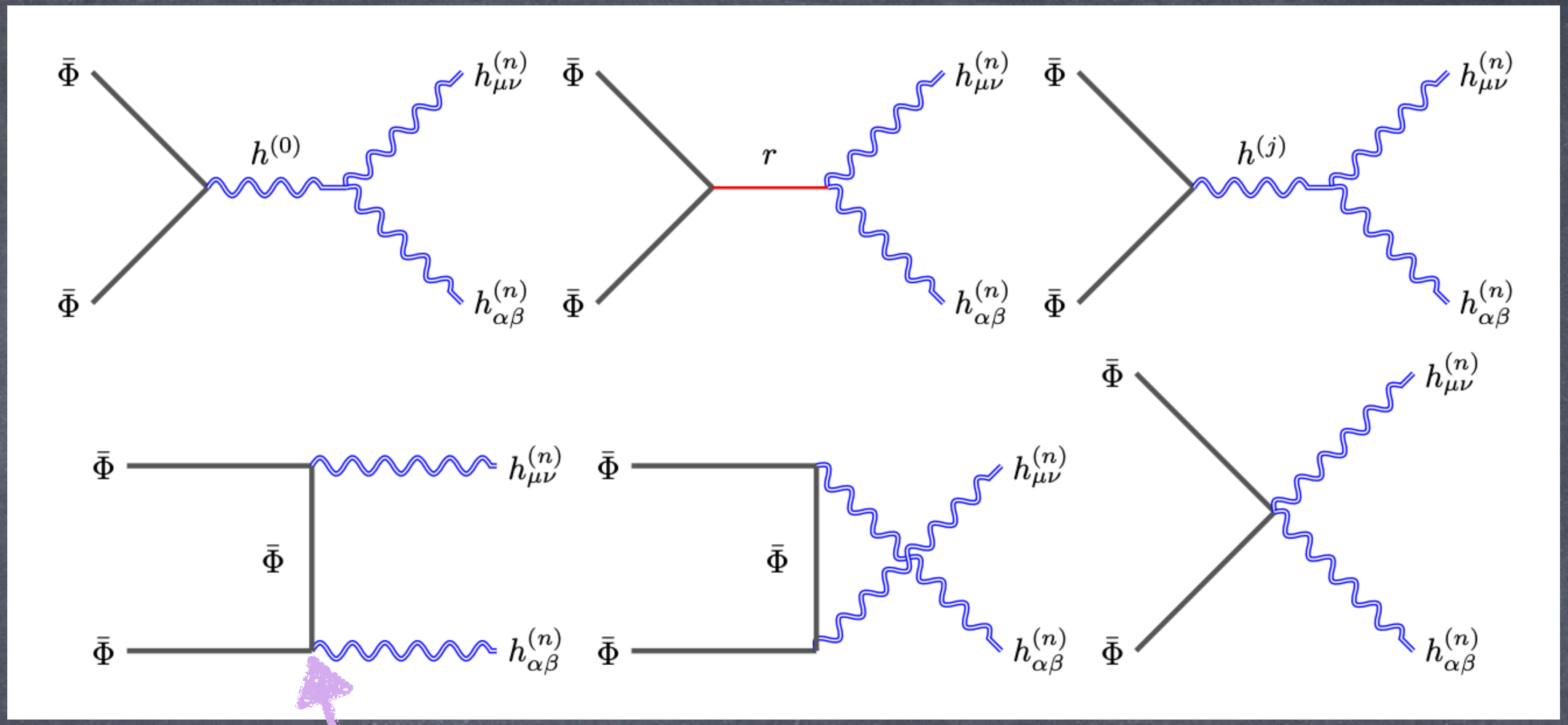
$$\widetilde{\mathcal{M}}^{(4)} = -\frac{\kappa^2}{72 m_n^4}\left\{3 b_{\bar{n}\bar{n}r} k^{(0)}(\bar{z}) - m_n^2\left[f^{(n)}(\bar{z})\right]^2 - m_n^2 a_{nn0} f^{(0)}(\bar{z})\right\}$$

$$\widetilde{\mathcal{M}}^{(2)} = -\frac{\kappa^2(3\cos 2\theta + 1)}{96}\left[f^{(n)}(\bar{z})\right]^2 \quad \longleftrightarrow \quad \widetilde{\mathcal{M}}^{(2)} = -\frac{\kappa^2(3\cos 2\theta + 1)}{96}\left[k^{(n)}(\bar{z})\right]^2$$

Connection to Goldstone Boson Equivalence

Bulk Scalar

- Now also consider the 5D Wavefunctions of Bulk Fields.
- Amplitudes appear to grow as E^6
- Cancellations due to sum rules ensure the amplitudes grow only as E^2,
  - i.e. $\tilde{M}^{(6)} = \tilde{M}^{(4)} = 0$ and $\tilde{M}^{(2)} \neq 0$



$$a^{\Phi_1\Phi_2}_{n_1n_2n_3} = \langle f^{(n_1)} f^{(n_2)}_{\Phi_1} f^{(n_3)}_{\Phi_2} \rangle_{\Phi_1} ,$$
$$a^{\Phi_1\Phi_2}_{n_1n_2n_3n_4} = \langle f^{(n_1)} f^{(n_2)} f^{(n_3)}_{\Phi_1} f^{(n_4)}_{\Phi_2} \rangle_{\Phi_1} ,$$
$$a^{\Phi_1\Phi_2}_{n_1n_2r} = \langle f^{(n_1)}_{\Phi_1} f^{(n_2)}_{\Phi_2} k^{(0)} \rangle_{\Phi_1} ,$$

$$\widetilde{\mathcal{M}}^{(6)} = \frac{\kappa^2}{192 m_n^4} \left[(3\cos 2\theta + 5) \sum_{j=0}^{\infty} \left(a^S_{nmj}\right)^2 + (\cos 2\theta - 1) \sum_{j=0}^{\infty} a_{nnj} a^S_{jmm} - 4(\cos 2\theta + 1) a^S_{nnmm} \right]$$

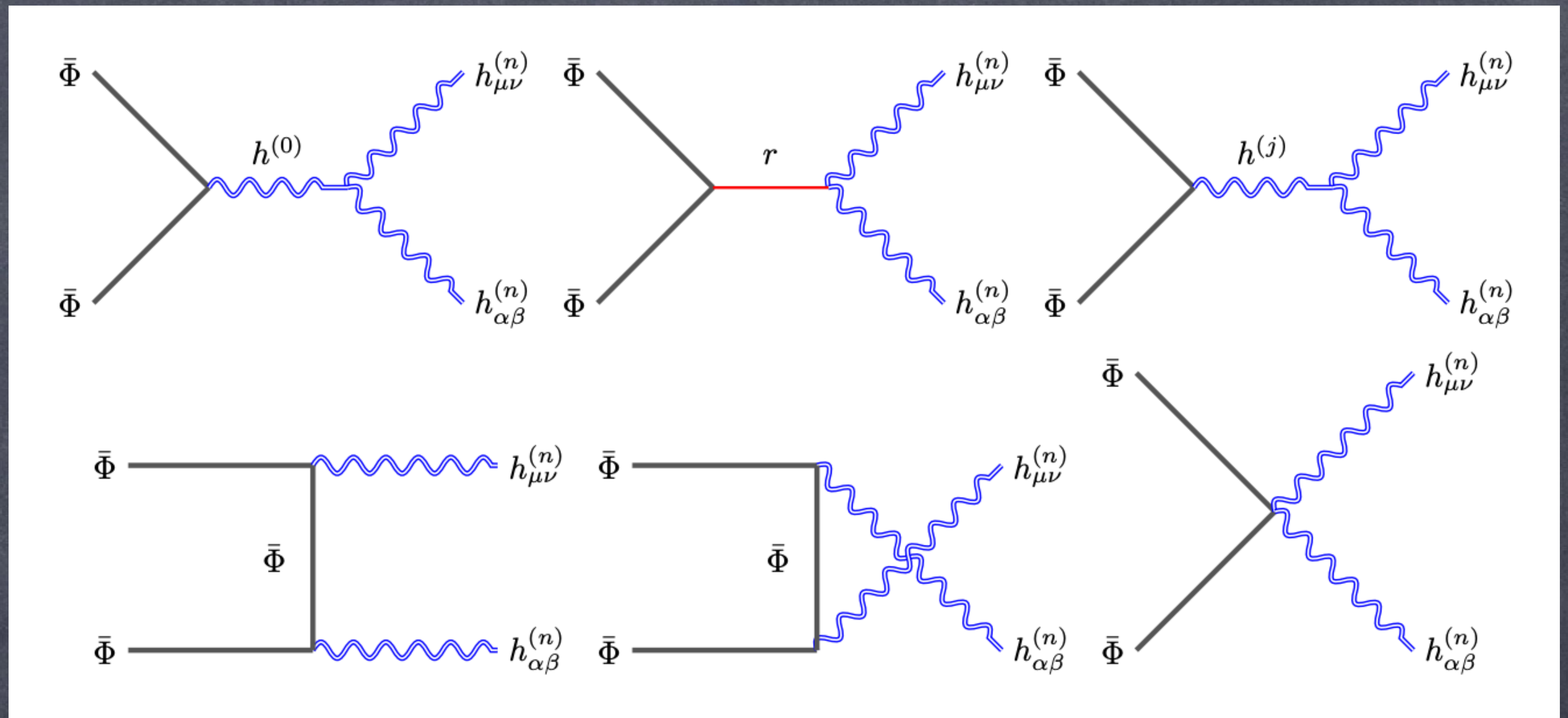
$$\widetilde{\mathcal{M}}^{(4)} = \frac{\kappa^2}{192 m_n^4} \Bigg\{ (5 - \cos 2\theta) \sum_{j=0}^{\infty} m_j^2 a_{nnj} a^S_{jmm} - 2m_n^2 (5 - \cos 2\theta) a^S_{nnmm}$$
$$-2(\cos 2\theta + 3) \sum_{j=0}^{\infty} m^2_{S,j} \left(a^S_{nmj}\right)^2 + 2m^2_{S,m} (\cos 2\theta + 3) a^S_{nnmm} + 16 b^S_{\bar{n}\bar{n}mm} \Bigg\} .$$

$$\widetilde{\mathcal{M}}^{(2)} = \frac{\kappa^2 (1 - \cos 2\theta)}{32} \langle k^{(n)} k^{(n)} f_S^{(m)} f_S^{(m)} \rangle_S ,$$

Connection to Goldstone Boson Equivalence

Proven Sum Rules and Cancellations Down to $E^{(2)}$

	Brane	Bulk
Scalar	✓	✓
Vector	✓	✓
Fermion	✓	✓



Also done for a stabilized RS model with Goldberger-Wise Scalaralr

arXiv: 2311.00770

Summary

- Naive power counting implies bad high energy growth for amplitudes involving massive gravitons
- We have shown using sum rules how cancellations proceed so that eventually all amplitudes grow as O(s)
- What are the symmetries that are behind these sum rules and cancellations? - See next talk by Xing Wang
- What next?
  - Now that we have all the correct amplitudes, work in progress on evaluation of the relic density in these models.

Amplitudes	Brane	Bulk
Scalar	O(s)	O(s)
Vector	O(s)	O(s)
Fermion	O(s)	O(s)