Is the Charm heavy?

Margarita Gavrilova, Cornell University

Based on MG, Y. Grossman and S. Schacht

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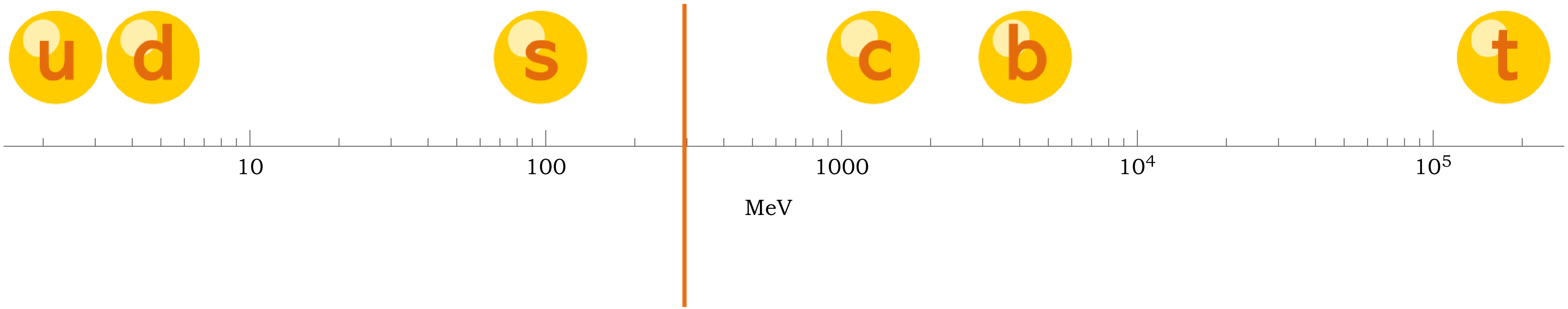
$c\bar{u}$ *

*quark content of the charm meson, $D^0 = (c\bar{u})$

Is the Charm Heavy?

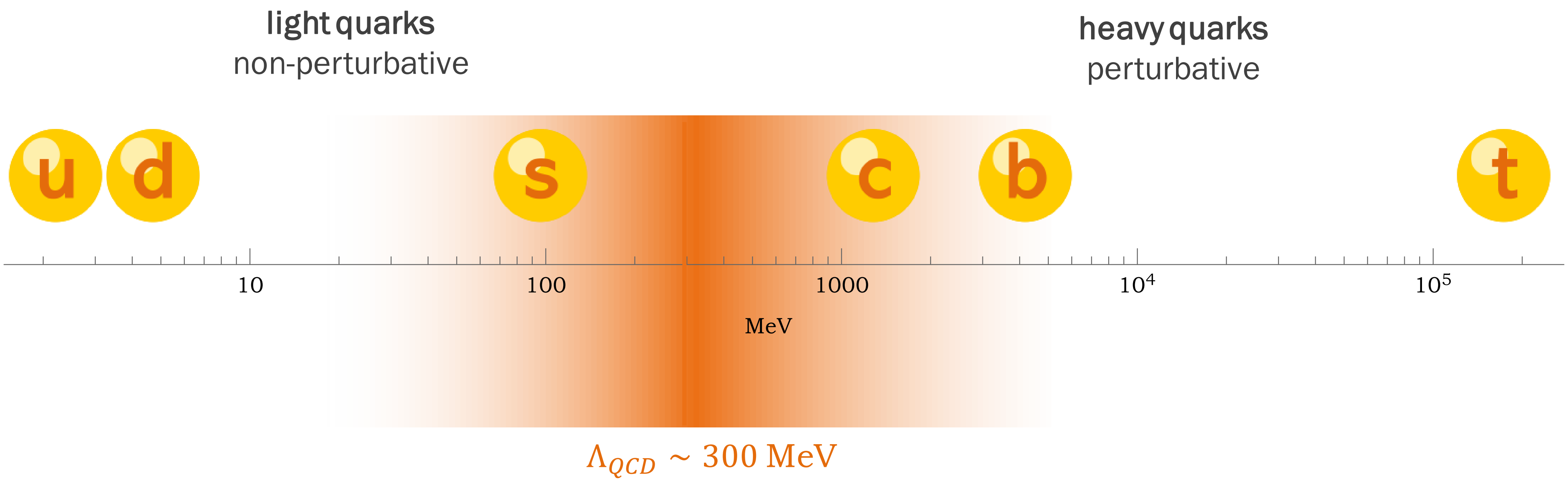
light quarks
non-perturbative

heavy quarks
perturbative



$\Lambda_{QCD} \sim 300\ \mathrm{MeV}$

Is the Charm Heavy?



Fundamental question:

How to deal with QCD in charm physics?

(or in other words “Is the Charm heavy?”)

How to deal with QCD in charm physics?

1. Lattice QCD

2. Analytical methods (solving QCD)

} not there yet

3. **Learning from experiment (flavor physics)**

- find theoretical parameters that are sensitive to non-perturbative effects in charm
- measure the values of these parameters

The fundamental problem

number of observables $<$ number of theory parameters

The fundamental problem

number of observables < number of theory parameters 😭

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approximate $SU(3)_F$ of QCD,
the symmetry between u, d and s quarks

The fundamental problem

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↓ approximate $SU(3)_F$ of QCD,
the symmetry between u, d and s quarks

number of observables $\geq$ number of theory parameters 😊

the values of the theory parameters can be extracted from experiment!

The plan

1. Find theory parameters sensitive to non-perturbative QCD in charm
2. Use flavor symmetries to reduce the number of independent theory parameters
3. Extract the values of the parameters of interest from experimental data

$D \to \pi\pi$

- We consider the following system of 6 **charm decays**:

$$
\begin{array}{lll}
D^0 \to \pi^+\pi^- & D^0 \to \pi^0\pi^0 & D^+ \to \pi^+\pi^0 \\
\overline{D}^0 \to \pi^+\pi^- & \overline{D}^0 \to \pi^0\pi^0 & D^- \to \pi^-\pi^0
\end{array}
$$

3 decays and their CP-conjugates

- These decays are related by **isospin**, $SU(2)$ subgroup of $SU(3)_F$ that relates $\boldsymbol{u}$ **and** $\boldsymbol{d}$
- The particles in the initial and final state form **isospin multiples**:

$$
\begin{bmatrix} D^+ \\ D^0 \end{bmatrix} = \begin{bmatrix} c\bar{d} \\ c\bar{u} \end{bmatrix}, \quad \begin{bmatrix} \overline{D}^0 \\ D^- \end{bmatrix} = \begin{bmatrix} \bar{c}u \\ \bar{c}d \end{bmatrix}, \quad \begin{bmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{bmatrix} = \begin{bmatrix} u\bar{d} \\ \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) \\ d\bar{u} \end{bmatrix}
$$

isospin doublets

isospin triplet

Observables

- We consider the following system of 6 **charm decays**:

$$\begin{array}{lll} D^0 \to \pi^+\pi^- & D^0 \to \pi^0\pi^0 & D^+ \to \pi^+\pi^0 \\ \overline{D}^0 \to \pi^+\pi^- & \overline{D}^0 \to \pi^0\pi^0 & D^- \to \pi^-\pi^0 \end{array}$$

3 decays and their CP-conjugates

1. Averaged branching ratios

phase space — decay amplitude — CP-conjugate decay amplitude

$$\mathcal{B}^{+-},\, \mathcal{B}^{00},\, \mathcal{B}^{+0}: \quad \mathcal{B}^f = \frac{1}{2}\mathcal{P}^f\left(|A^f|^2 + |\overline{A}^f|^2\right), \quad f = +-,\, 00,\, +0$$

2. CP-asymmetries

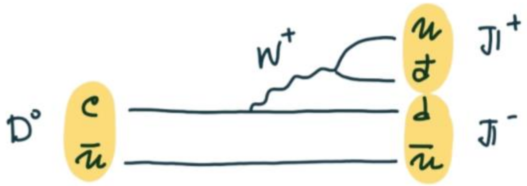
$$a_{CP}^{+-},\, a_{CP}^{00},\, a_{CP}^{+0}: \quad a_{CP}^f = \frac{|A^f|^2 - |\overline{A}^f|^2}{|A^f|^2 + |\overline{A}^f|^2}, \quad f = +-\,, 00,\, +0$$

Why CP-violation?

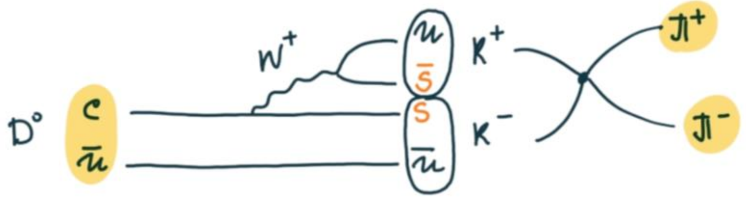
different CKM-factors → different weak phase → CPV

CP-violation is an interference effect:

$$D^0 \xrightarrow{V_{cd}^* V_{ud}} \pi^+\pi^-$$



$$D^0 \xrightarrow{V_{cs}^* V_{us}} K^+K^-, \ldots \xrightarrow{\text{QCD}} \pi^+\pi^-$$



rescattering!

Theoretical parametrization of the amplitudes:

$$A^f = (-V_{cd}^* V_{ud}) \times T^f - \left(\frac{V_{cb}^* V_{ub}}{2}\right) \times P^f, \quad f = +-,\ 00,\ +0$$

$$a_{CP}^f = \text{CKM} \times \left|\frac{P^f}{T^f}\right| \times \sin\delta^f$$

strong phase = phase of P/T

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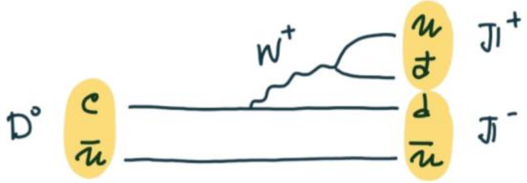
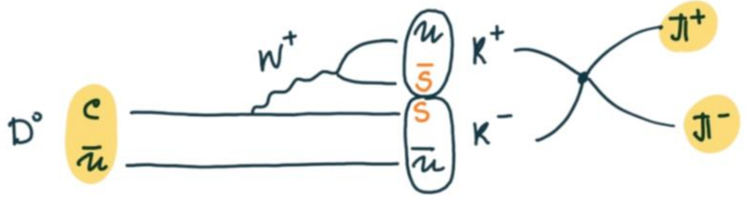
 ~ 



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strong phase = phase of P/T

Penguin vs Tree

- The P/T ratio is a measure of rescattering in $D \to \pi\pi$

$$\left|\frac{P^f}{T^f}\right|_{\text{no rescatt.}} \ll 1 \qquad\qquad \left|\frac{P^f}{T^f}\right|_{\text{rescatt.}} \gtrsim 1$$

perturbative — non-perturbative

- Measuring P/T

$$a^f_{CP} = \text{CKM} \times \left|\frac{P^f}{T^f}\right| \times \sin\delta^f$$

without isospin, we can only measure the product of P/T and the strong phase

$$a^f_{CP} = \text{CKM} \times \left|\frac{P^f}{T^f}\right| \times \sin\delta^f$$

with isospin and the BR measurements, we can separate the two contributions!

Summary of the result

- Assuming isospin, we can extract the magnitude of P/T and its phase from the measurements of branching fractions and CP-asymmetries (don't need any assumptions about the strong phase!)

$$\left|\frac{P^f}{T^f}\right| = F(\mathcal{B}^{+-},\, \mathcal{B}^{00},\, \mathcal{B}^{+0},\, a_{CP}^{+-},\, a_{CP}^{00})$$

$$\sin\delta^f = f(\mathcal{B}^{+-},\, \mathcal{B}^{00},\, \mathcal{B}^{+0},\, a_{CP}^{+-},\, a_{CP}^{00})$$

- Isospin is expected to hold at order 1%, thus the relations have theoretical **precision of order few percent**
- Although, at the time, we have essentially no experimental information about $\sin\delta^{00}$, we still **can extract non-trivial information about r^{00}** due to its correlation to r^{+-}

$$\frac{r^{00}}{r^{+-}} = \sqrt{\frac{1}{2}\frac{\mathcal{B}^{+-}}{\mathcal{P}^{+-}}\frac{\mathcal{P}^{00}}{\mathcal{B}^{00}}}, \qquad r^f \equiv \left|\frac{P^f}{T^f}\right|$$

What does the data say?

Parameter	**Current data** (LHCb, Belle, CLEO)	**Future data** (LHCb Upgrade II, Belle-II)
r^{00}	$5.2^{+13.3}_{-2.4}$	$5.2^{+1.6}_{-1.2}$
r^{+-}	$5.5^{+14.2}_{-2.7}$	$5.5^{+1.8}_{-1.3}$

- |P/T| is large, future data will significantly reduce the errors!
- $r^f = 1$ hypothesis is at $\sim 2.6\sigma$
- $r^f = 0$ hypothesis is at $\sim 3.8\sigma$

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Soo… Is the Charm Heavy? – the data hints that its not!

Backup

Flavor symmetry

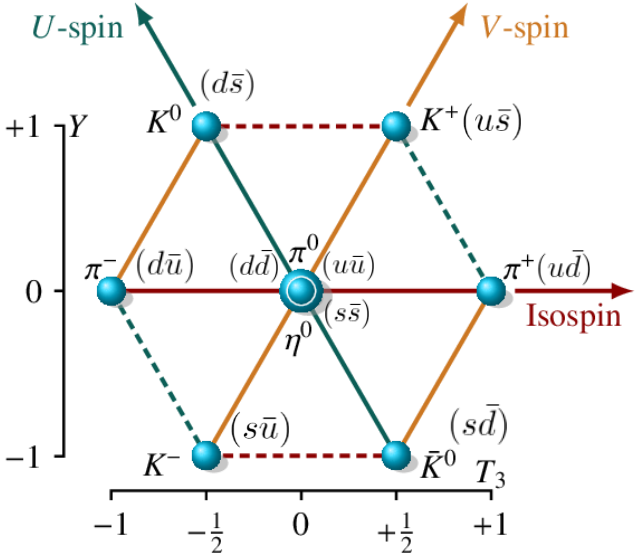
- QCD has an approximate $SU(3)$ flavor symmetry of light quarks u, d, s
- $SU(3)$ flavor contains an $SU(2)$ subgroup

  Isospin (u, d): λ_1 λ_2 λ_3

- $SU(3)$ flavor is broken by quark masses $m_u \neq m_d \neq m_s$
- The breaking of the isospin can be parametrized by $\varepsilon \sim \Delta m/\Lambda_{QCD} \sim 1\%$
- This means that predictions based on isospin have theoretical uncertainty of the order few %

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$



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3 decays and their CP-conjugates

- These decays are related by **isospin**, $SU(2)$ subgroup of $SU(3)_F$ that relates $\boldsymbol{u}$ **and** $\boldsymbol{d}$

* isospin is an **approximate symmetry** of QCD
* isospin is **broken by quark masses** $m_u \neq m_d$
* the breaking parameter $\varepsilon \sim 1\%$

→ our predictions based on isospin will have theoretical uncertainty only of order few %

Effective Hamiltonian

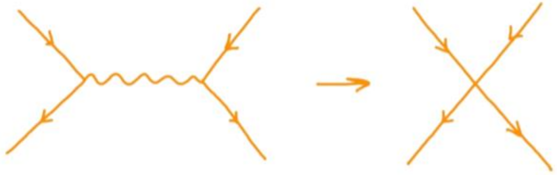
$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}}\left(\sum_{q=d,s}\lambda_q\left(C_1 Q_1^q + C_2 Q_2^q\right)\right) \equiv \sum_{q=d,s}\lambda_q \mathcal{O}^q\,,$$

$$Q_1^q \equiv (\bar{u}\gamma_\mu(1-\gamma_5)q)\,(\bar{q}\gamma_\mu(1-\gamma_5)c)\,,$$

$$Q_2^q \equiv (\bar{q}\gamma_\mu(1-\gamma_5)q)\,(\bar{u}\gamma_\mu(1-\gamma_5)c)\,,$$

$$\langle\mathcal{O}^q\rangle^f \equiv \langle f|\mathcal{O}^q\left|D^0\right\rangle\,,$$

CKM: $V_{cq}^* V_{uq}$



- integrated out EW bosons
- integrated out b
- neglected E&M interactions

then the decay amplitudes can be written as

$$A(D^0\to f) = \lambda_d\langle\mathcal{O}^d\rangle^f + \lambda_s\langle\mathcal{O}^s\rangle^f = -\lambda_d\left(\langle\mathcal{O}^s\rangle^f - \langle\mathcal{O}^d\rangle^f\right) - \frac{\lambda_b}{2}\left(2\langle\mathcal{O}^s\rangle^f\right)$$

unitarity of the CKM: $\lambda_d + \lambda_s + \lambda_b = 0$

hierarchy of the CKM is such that $\left|\frac{\lambda_b}{\lambda_d}\right| \sim 10^{-3}$ and thus the observables can be written as series in λ_b/λ_d

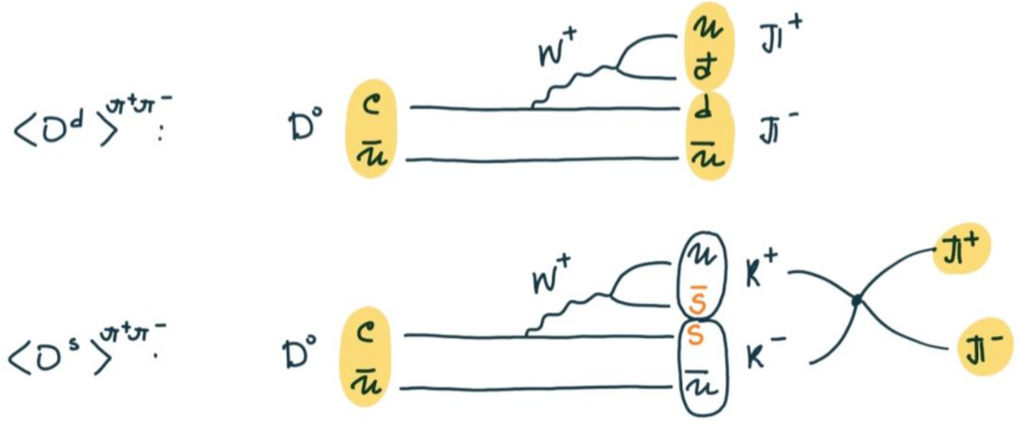
Penguin over tree ratio

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left(\sum_{q=d,s} \lambda_q \left(C_1 Q_1^q + C_2 Q_2^q \right) \right) \equiv \sum_{q=d,s} \lambda_q \mathcal{O}^q \,,$$
$$Q_1^q \equiv (\bar{u}\gamma_\mu(1-\gamma_5)q)\,(\bar{q}\gamma_\mu(1-\gamma_5)c)\,,$$
$$Q_2^q \equiv (\bar{q}\gamma_\mu(1-\gamma_5)q)\,(\bar{u}\gamma_\mu(1-\gamma_5)c)\,,$$
$$\langle \mathcal{O}^q \rangle^f \equiv \langle f | \mathcal{O}^q \,| D^0 \rangle \,,$$

What is the physical significance of r^f?

$$r^f \equiv \left| \frac{2\langle \mathcal{O}^s \rangle^f}{\langle \mathcal{O}^s \rangle^f - \langle \mathcal{O}^d \rangle^f} \right|$$

For concreteness, let us consider $D^0 \to \pi^+\pi^-$



rescattering

=

Penguin over tree ratio

What is the physical significance of r^f?

$$r^f \equiv \left| \frac{2\langle \mathcal{O}^s \rangle^f}{\langle \mathcal{O}^s \rangle^f - \langle \mathcal{O}^d \rangle^f} \right|$$

$r^f_{no\ rescatt.} =$  $\ll 1$ (subscript: $no\ rescatt.$)

perturbative

$r^f_{with\ rescatt.} =$  \gtrsim 1 (subscript: $with\ rescatt.$)

non-perturbative

Closed-form expressions

$$\sin\arg(P/T)^{00} = \frac{-\mathrm{sign}\,(a_{CP}^{00})}{\sqrt{1+\frac{1}{\sin^2\delta_d}\left(\frac{a_{CP}^{+-}}{a_{CP}^{00}}\sqrt{\frac{1}{2}\frac{\mathcal{B}^{+-}}{\mathcal{P}^{+-}}\frac{\mathcal{P}^{00}}{\mathcal{B}^{00}}}+\cos\delta_d\right)^2}}\,,$$

$$\sin\arg(P/T)^{+-} = \frac{-\mathrm{sign}\,(a_{CP}^{+-})}{\sqrt{1+\frac{1}{\sin^2\delta_d}\left(\frac{a_{CP}^{00}}{a_{CP}^{+-}}\sqrt{2\frac{\mathcal{P}^{+-}}{\mathcal{B}^{+-}}\frac{\mathcal{B}^{00}}{\mathcal{P}^{00}}}+\cos\delta_d\right)^2}}\,,$$

$$|P/T|^{00} = \frac{1}{|\mathrm{Im}\,(-\lambda_b/\lambda_d)\,|}\sqrt{(a_{CP}^{00})^2+\frac{(a_{CP}^{+-}\sqrt{\mathcal{B}^{+-}\mathcal{P}^{00}}+a_{CP}^{00}\sqrt{2\mathcal{B}^{00}\mathcal{P}^{+-}}\cos\delta_d)^2}{2\mathcal{B}^{00}\mathcal{P}^{+-}\sin^2\delta_d}}\,,$$

$$|P/T|^{+-} = \frac{1}{|\mathrm{Im}\,(-\lambda_d/\lambda_d)|}\sqrt{\left(a_{CP}^{+-}\right)^2+\frac{\left(a_{CP}^{00}\sqrt{2\mathcal{B}^{00}\mathcal{P}^{+-}}+a_{CP}^{+-}\sqrt{\mathcal{B}^{+-}\mathcal{P}^{00}}\cos\delta_d\right)^2}{\mathcal{B}^{+-}\mathcal{P}^{00}\sin^2\delta_d}}\,.$$

Numerical results

Direct CP Asymmetries		
a_{CP}^{+0}	$+0.004 \pm 0.008$	[79–82]
a_{CP}^{00}	-0.0002 ± 0.0064	[a][79, 83, 84]
a_{CP}^{+-}	0.00232 ± 0.00061	[2]
Branching Ratios		
$\mathcal{B}(D^0 \to \pi^+\pi^0)$	$(1.247 \pm 0.033) \cdot 10^{-3}$	[85]
$\mathcal{B}(D^0 \to \pi^+\pi^-)$	$(1.454 \pm 0.024) \cdot 10^{-3}$	[85]
$\mathcal{B}(D^0 \to \pi^0\pi^0)$	$(8.26 \pm 0.25) \cdot 10^{-4}$	[85]
Further Numerical Inputs		
$\mathrm{Im}\,(\lambda_b/(-\lambda_d))$	$(-6.1 \pm 0.3) \cdot 10^{-4}$	[85]

TABLE I. Experimental input data. We use the decay times and masses from Ref. [85]. [a]Our extraction from $A_{CP}(D^0 \to \pi^0\pi^0) = -0.0003 \pm 0.0064$ [79] and $\Delta Y = (-1.0 \pm 1.1 \pm 0.3) \cdot 10^{-4}$ [52].

a_{CP}^{+-}	$(2.32 \pm 0.07) \cdot 10^{-3}$
a_{CP}^{00}	$(-2 \pm 9) \cdot 10^{-4}$

TABLE II. Future data scenario employing the current central values and using prospects for the errors from Table 6.5 of Ref. [86] (300 fb^{-1}) and Table 122 of Ref. [87] (50 ab^{-1}) for $D^0 \to \pi^+\pi^-$ and $D^0 \to \pi^0\pi^0$, respectively. All other input data is left as specified in Table I

Parameter	Current data	Future data scenario
r_t	3.43 ± 0.06	3.43 ± 0.06
$\cos\delta_t$	0.06 ± 0.02	0.06 ± 0.02
$\cos\delta_d$	-0.68 ± 0.01	-0.68 ± 0.01
$\lvert\sin\delta^{00}\rvert$	0^{+1}_{-0}	$0.06^{+0.20}_{-0.06}$
$\lvert\sin\delta^{+-}\rvert$	$0.7^{+0.3}_{-0.5}$	$0.69^{+0.21}_{-0.16}$
r^{00}	$5.2^{+13.3}_{-2.4}$	$5.2^{+1.6}_{-1.2}$
r^{+-}	$5.5^{+14.2}_{-2.7}$	$5.5^{+1.8}_{-1.3}$

TABLE III. Numerical results for current and hypothetical future data. In the future data scenario, the results for r_t, $\cos\delta_t$ and $\cos\delta_d$ are identical to the ones with current data, as these depend only on the branching ratio data which is not modified in the future data scenario compared to current data. Furthermore, in the future data scenario $\sin\delta^{+-} < 0$. The overall additional relative systematic uncertainty of $\mathcal{O}(10\%)$ due to the universality assumption of ΔY for the extraction of the direct CP asymmetries comes on top of the errors shown here, see text for details.