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Neutrinos Are Darkly Different

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Neutrinos Are Darkly Different —— from What?

The Standard Model is a theory of interactions (Strong, Weak, Electromagnetic) and of fermion charges but it is a model of fermion masses

$$m_f = Y_f < H^0 >$$

And all **charged** fermion `flavors' are conventionally defined by the mass of each fermion

But not the neutrinos

Also, the charged fermion `flavors' are not aligned with the weak interaction currents defined by the transitions from up-quarks to down-quarks or vice versa

First identified by Cabibbo and enshrined in the CKM matrix, this misalignment is considered to be small. But is it?

Universal Current Coupling

The charged current weak interactions identify (***somehow***) a particular up-type quark with a particular down-type quark but only the difference between the misalignment between the `current' eigenstates and mass eigenstates for up-quarks and for down-quarks must be `small' for consistency with experiment

(one Higgs)

(Quark) Mass in the SM

(leptons later)

$$\Sigma_i\{Y_{Ui}\overline{\frac{(1+\gamma_5)}{2}\Psi_{Ui}} < \phi^0, \phi^+ > \frac{(1-\gamma_5)}{2}\begin{bmatrix} \Psi_{Ui} \\ \Psi_{Di} \end{bmatrix} + \text{h.c.}\}$$

+

$$\Sigma_i\{Y_{Di}\overline{\frac{(1+\gamma_5)}{2}\Psi_{Di}} < \phi^-, -(\phi^0)^* > \frac{(1-\gamma_5)}{2}\begin{bmatrix} \Psi_{Ui} \\ \Psi_{Di} \end{bmatrix} + \text{h.c.}\}$$

For Weyl spinors in Majorana form the mass term is

$$\Psi_M = \begin{bmatrix} \xi \\ -\sigma_2\xi^* \end{bmatrix} \qquad \overline{\Psi_M}\Psi_M = -(\xi^\dagger\sigma_2\xi^* + \xi^T\sigma_2\xi)$$

Applied to Dirac this requires

Wigner-Weyl rep

$$\Psi_D = \begin{bmatrix} \xi \\ \chi \end{bmatrix}$$

where

$$\xi = \zeta_a + \zeta_b$$

and

$$\chi = \sigma_2(-\zeta_a^* + \zeta_b^*)$$

So that $$\overline{\Psi_D}\Psi_D = -(\chi^\dagger\xi + \xi^\dagger\chi)$$

But IF the Higgs DOES NOT KNOW the difference between the different right-chiral parts

A combined way to understand both the masses and the CKM matrix

Phenomenological Analysis of Quark and Charged Lepton Masses

Jarlskog suggested, Kaus & Meshkov showed, consistency of mass values with universal Higgs' coupling and perturbative BSM corrections

Kaus & Meshkov proposed specific pattern of BSM corrections

Many `textures' were considered: Frtizsch, ….

We follow Cabbibo in extracting BSM corrections from data phenomenologically rather than presuming any particular theory for the BSM components structure

Recall pairing (democratic) matrix of nuclear physics:

$$M_{dem} = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{bmatrix} \times M_0$$

Here for
3X3 case

is diagonalized by the tri-bi-maximal matrix (TBM)

Yes, this will
be important
but mores later!

$$TBM_q = \begin{bmatrix} \frac{2}{\sqrt{6}} & 0 & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \end{bmatrix}$$

(Phase of 2nd column
not conventional, arb.
mix of 1st & 2nd allowed)

to

$$TBM^\dagger \times M_{dem} \times TBM = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and we recall the
smallness of
2 of the fermion
mass values
relative to the large
value

Applying TBM to undiagonalize the observed diagonal fermion mass matrices by

$$M_{rvsd} = TBM \times M_{diag} \times TBM^T$$

produces revised mass matrices

$$M_{+\frac{2}{3}rvsd} = 162900 \times \begin{bmatrix} 0.335235 & 0.331435 & 0.333331 \\ 0.331435 & 0.335235 & 0.333331 \\ 0.333331 & 0.333331 & 0.333339 \end{bmatrix}$$

$$M_{-\frac{1}{3}rvsd} = 2890 \times \begin{bmatrix} 0.343017 & 0.323986 & 0.332997 \\ 0.323986 & 0.343017 & 0.332997 \\ 0.332997 & 0.332997 & 0.334007 \end{bmatrix}$$

$$M_{-1rvsd} = 1776.82 \times \begin{bmatrix} 0.363114 & 0.303649 & 0.333237 \\ 0.303649 & 0.363114 & 0.333237 \\ 0.333237 & 0.333237 & 0.333525 \end{bmatrix}$$

Very good (4%) ‘democratic’ approximation!

Is there a small expansion parameter in this future?

Transformation of general BSM-corrected mass matrix from current eigenstates to mass eigenstates

Factor off M_0 :

$$M_{int} \propto \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \epsilon \times$$

Fulll U(3) of BSM corrections

$$TBM^\dagger \begin{bmatrix} \sqrt{\frac{2}{3}}y_0 + y_3 + \frac{1}{\sqrt{3}}y_8 & y_1 - Iy_2 & y_4 - Iy_5 \\ y_1 + Iy_2 & \sqrt{\frac{2}{3}}y_0 - y_3 + \frac{1}{\sqrt{3}}y_8 & y_6 - Iy_7 \\ y_4 + Iy_5 & y_6 + Iy_7 & \sqrt{\frac{2}{3}}y_0 - \frac{2}{\sqrt{3}}y_8 \end{bmatrix} TBM$$

Different ϵ and y_j for u-quarks and for d-quarks

The weak interaction only defines which left-chiral part of which current u-quark(j) and which current d-quark(j) transform into each other.

Effect of Mass Diagonalization on Universal Weak Interaction

$$U^\dagger\, M\, U\, U^\dagger\, f_c = M_{diag}\, f_m \qquad U^\dagger\, f_c = f_m$$

$$f_x = \begin{bmatrix} f_{x1} \\ f_{x2} \\ f_{x3} \end{bmatrix}$$

Universal Weak Interaction

$$H_{current} = \bar{f}_a 1 f_b \qquad f_{xm} = U_x^\dagger f_x$$

$$U_{fx} = TBM_f\, U_{fx,3\to 2}\, R2(\omega_{fx})$$

Small Corrections

$$\bar{f}_{uc} U_u U_u^\dagger 1 U_d U_d^\dagger f_{dc} = \bar{f}_{um} CKM\, f_{dm}$$

parallel to PDG form

$$CKM = U_u^\dagger U_d$$

but $TBM_u = TBM_d$ so

NB: The TBM factors cancel out: $TBM_q^\dagger\, TBM_q = 1$

For each, after TBM, block diagonalize, then do 2X2:

$$X_{tot} = TBM \times X_{3\to2} \times X_{2x2}$$

$$X_{3\to2} = \begin{bmatrix} 1 & 0 & \alpha \\ 0 & 1 & \beta \\ -\alpha^* & -\beta^* & 1 \end{bmatrix} \qquad X_{2x2} = \begin{bmatrix} \cos(\omega) & \sin(\omega) & 0 \\ -\sin(\omega) & \cos(\omega) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$X_{3\to2} \times X_{2x2} =$$

[Other phases transform away due to no CPV in 2x2]

$$\begin{bmatrix} \cos(\omega) & \sin(\omega) & \epsilon y_b e^{I\zeta} \\ -\sin(\omega) & \cos(\omega) & -\epsilon y_c \\ -\epsilon(y_b e^{I\zeta}\cos(\omega) + y_c\sin(\omega)) & -\epsilon(y_b e^{I\zeta}\sin(\omega) - y_c\cos(\omega)) & 1 \end{bmatrix}$$

Ignoring higher order (ε[1]) corrections in the 2X2 block

$$UV^\dagger = \quad CKM_{BSM} = \begin{bmatrix} \cos(\Theta_C) & \sin(\Theta_C) & \mathcal{R}[(1,3)] + \imath\frac{XZ}{Q} \\ -\sin(\Theta_C) & \cos(\Theta_C) & Q \\ \mathcal{R}[(3,1)] + \imath\frac{XZ}{Q}\cos(\Theta_C) & \mathcal{R}[(3,2)] + \imath\frac{XZ}{Q}\sin(\Theta_C) & 1 \end{bmatrix}$$

where we have recognized that ω_d - ω_u = Θ_C and define†

$$A_{du} = \epsilon_d y_{bd} e^{I\zeta_d} - \epsilon_u y_{bu} e^{I\zeta_u} \qquad B_{du} = -(\epsilon_d y_{cd} - \epsilon_u y_{cu})$$

and $\quad X = B_{du}\ ,\ Y = Re(A_{du})\ ,\ Z = Im(A_{du})$

with $\quad Q = \sqrt{[\cos(\omega_u)X + \sin(\omega_u)Y]^2 + [\sin(\omega_u)Z]^2}$

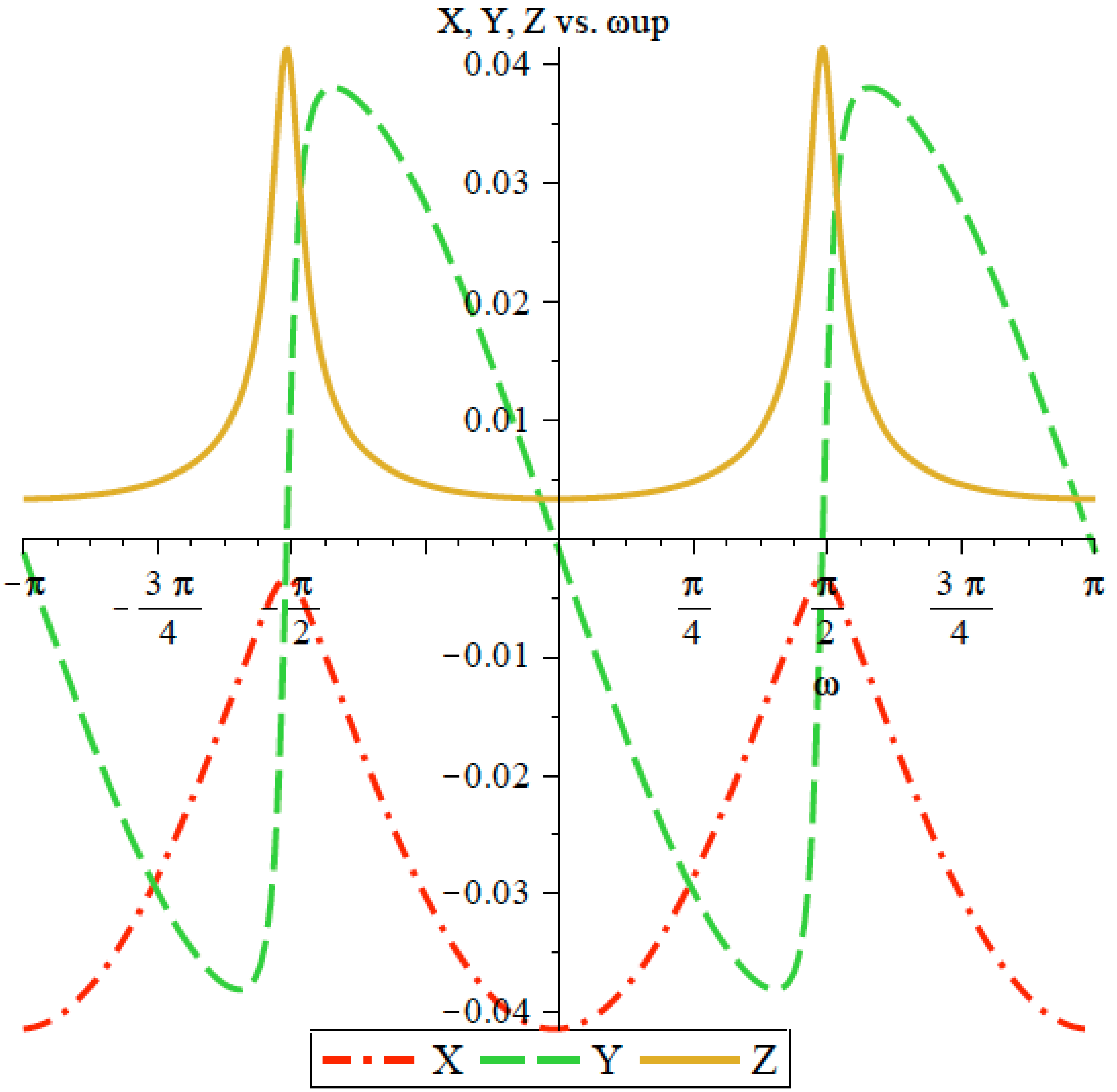
† This demonstrates that the CKM depends only on the difference between up-quark and down-quark properties

$y_{ij} = mess!$

$\mathcal{R}(i,j) =$ Quadratic functions of X, Y, Z and trig functions of ω_u, ω_d

Fit to PDG values of masses and CKM for X, Y, Z as functions of ω_u

Graphically



ω_u

$$X, Y, Z, \omega_f : (y_{fj})$$

$$\theta_C = \omega_d - \omega_u$$

Since CKM values fitted, agreement with Jarlskog CPV invariant guaranteed, for any value of ω_u.

Note: These values are all of order ε or less so the parameters are all "natural" and the corrections perturbative

So there is no inconsistency for quarks and charged leptons can be treated the same way

BUT neutrinos are DIFFERENT

The flavors of quarks and charged leptons are defined by their masses.

The flavors of neutrinos are defined by their weak coupling to leptons.

Neutrino `flavors’ are defined by their weak current interaction with charged lepton mass eigenstates

Therefore they couple to the mass lepton basis and not the current lepton basis

“…their eyes filled with a wild surmise, silent, upon speak, in Darien.” — W.B.Yeats

So if the neutrino mass matrix is not also almost democratic, this difference omits one approximately TBM factor and the cancellation as in quarks does not occur, leaving

PMNS ~ TBM

But does this occur?

$$H_{current} = \bar{\ell}_c 1 \nu_c = \bar{\ell}_c U_{\ell c} U^\dagger_{\ell c} \nu_c = \bar{\ell}_m \nu_\ell$$

But to match charged leptons $f_\ell = \begin{bmatrix} e \\ \tau \\ \mu \end{bmatrix}$

Malice aforethought?

so $TBM_\ell = \begin{bmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$

and by definition $\nu_\ell = \begin{bmatrix} \nu_e \\ \nu_\tau \\ \nu_\mu \end{bmatrix}$ hence $U^\dagger_{\ell c} \nu_c = \nu_\ell$

That is $$PMNS \sim U_{\ell c}^{\dagger} \sim TBM_{\ell}^{\dagger}$$

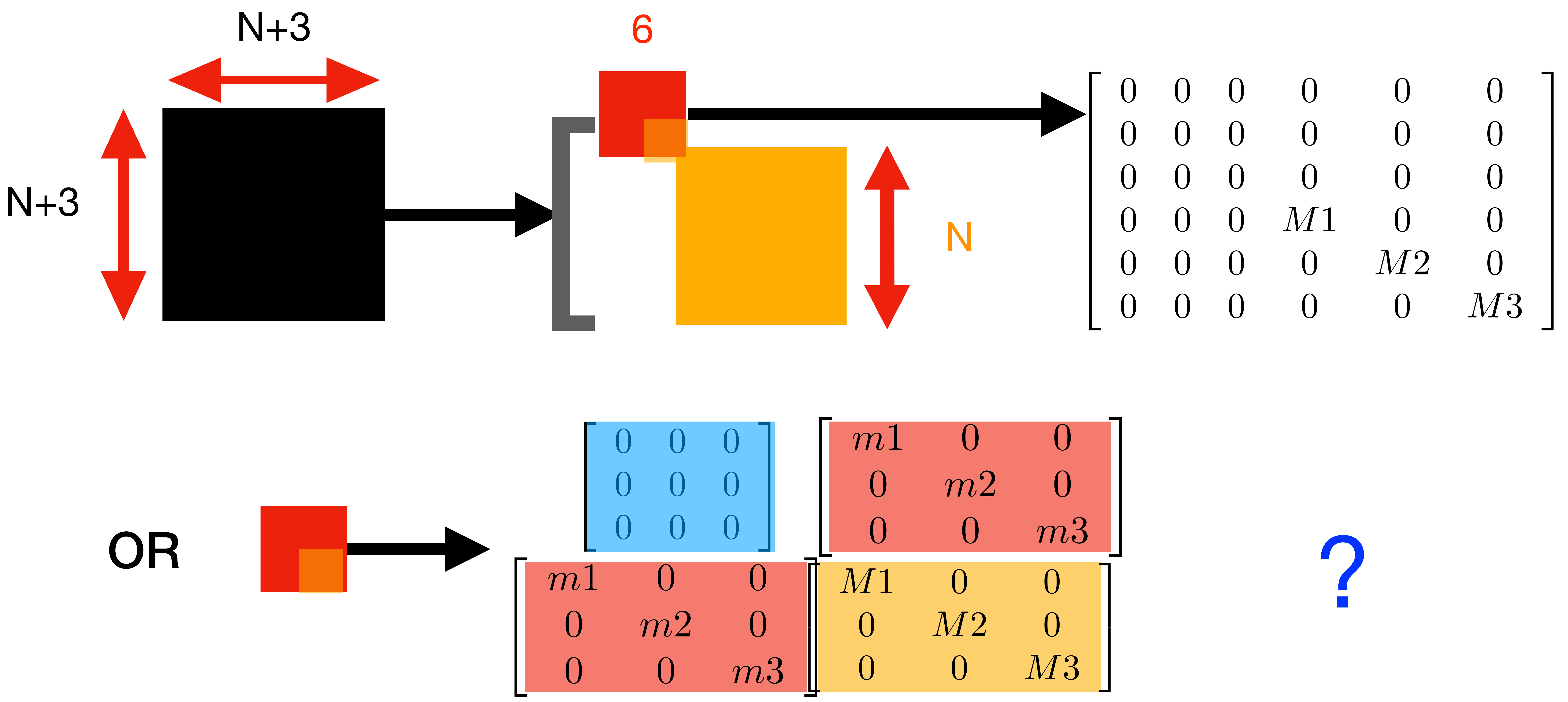
but since $$U_{\ell c} = TBM_{\ell}\, U_{\ell,blk}\, R2(\omega_{\ell})$$

and we know from experiment that $$PMNS\, \nu_{\ell} = \nu_m = \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}$$

It must be that $$\nu_c = \begin{bmatrix} 1+\epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{21} & 1+\epsilon_{22} & \epsilon_{23} \\ \epsilon_{31} & \epsilon_{32} & 1+\epsilon_{33} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}$$

and so **neutrinos are different** **Why?**

Is it because the right-chiral parts are the C-conjugates of the left-chiral parts so they must match and the masses are Majorana?
But then the weak quantum number mismatch requires a different Higgs!
Or are they different but dark matter that "connects" to the SM somehow?



Dark Matter Effect

CONCLUSION

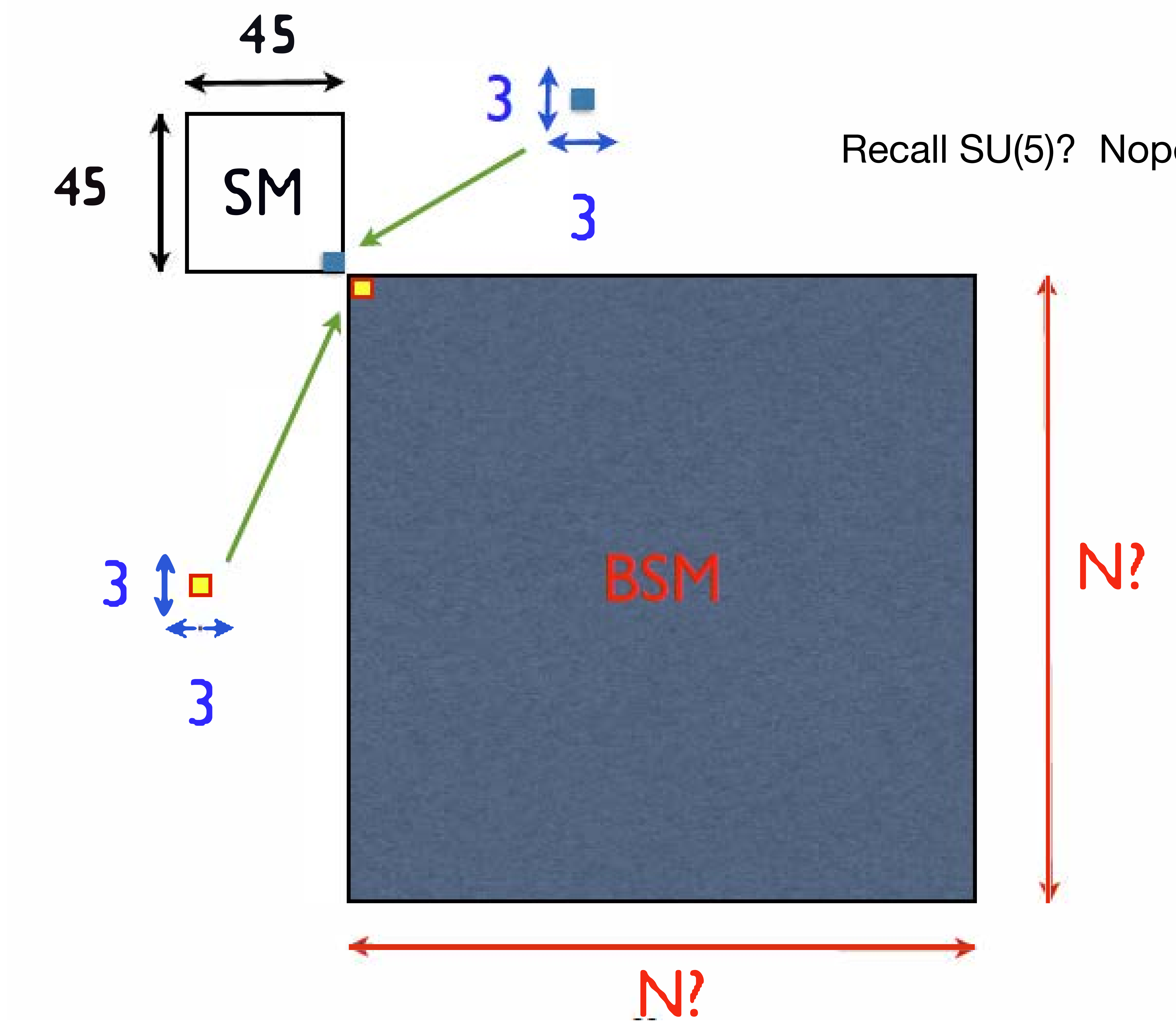
The agreement of the assumption that the Higgs coupling to the right chiral components of the charged fundamental fermions is the same for each of the current basis fields of a given charge, except for small BSM corrections, with the measured fermion mass (flavor) spectrum and the CKM matrix is remarkable.

Because of the different nature of their flavor definition, the extension to the neutrinos demonstrates that the PMNS matrix can be almost fully accounted for by the transformation of the charged leptons from a current basis to the mass basis for their field operators.

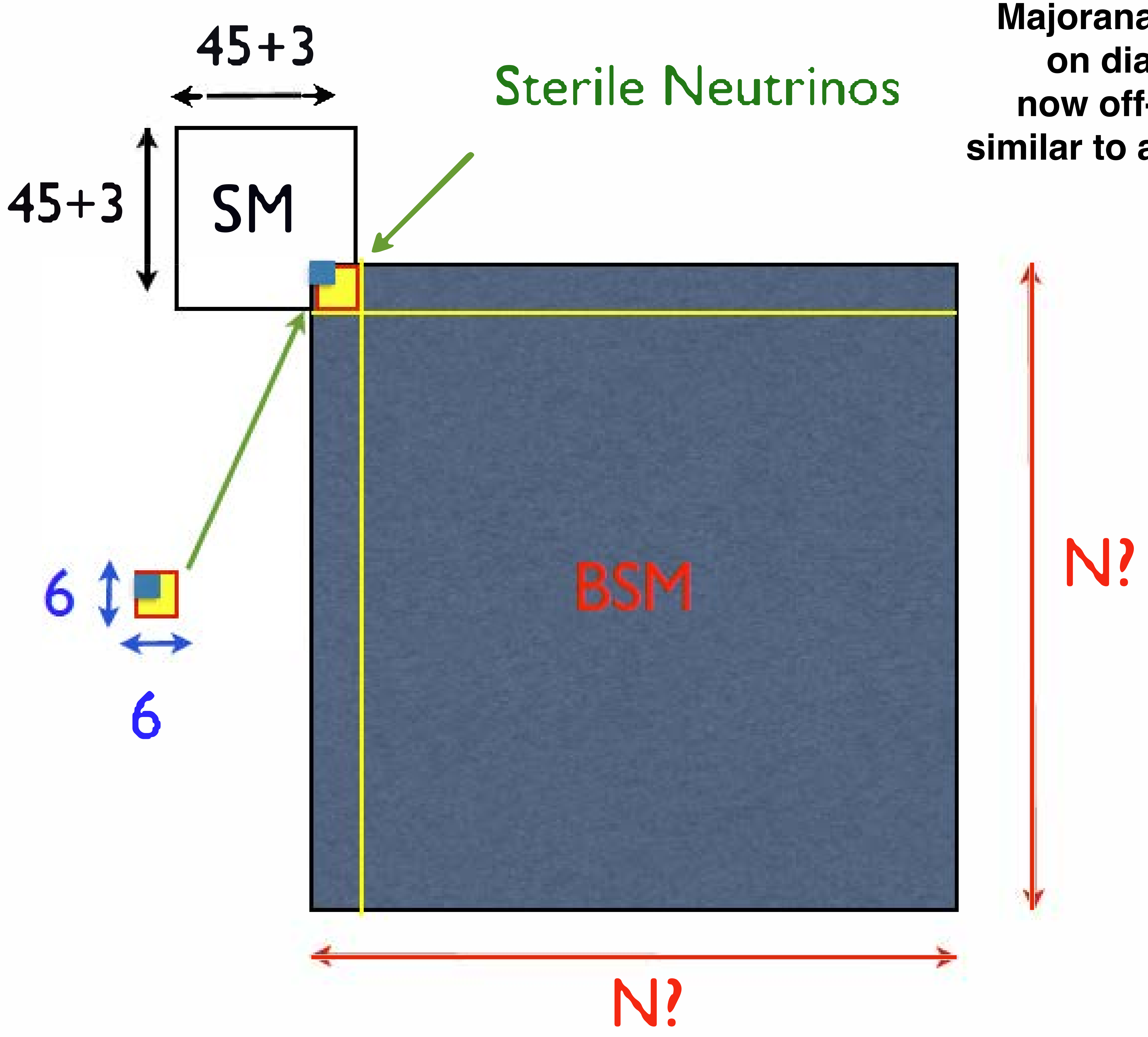
In addition to being capable of describing all of these features, this result suggests that information on the coupling of luminous matter to dark matter may be accessed from available data in the luminous sector.

Dark Matter BSM?

Recall SU(5)? Nope, no proton decay



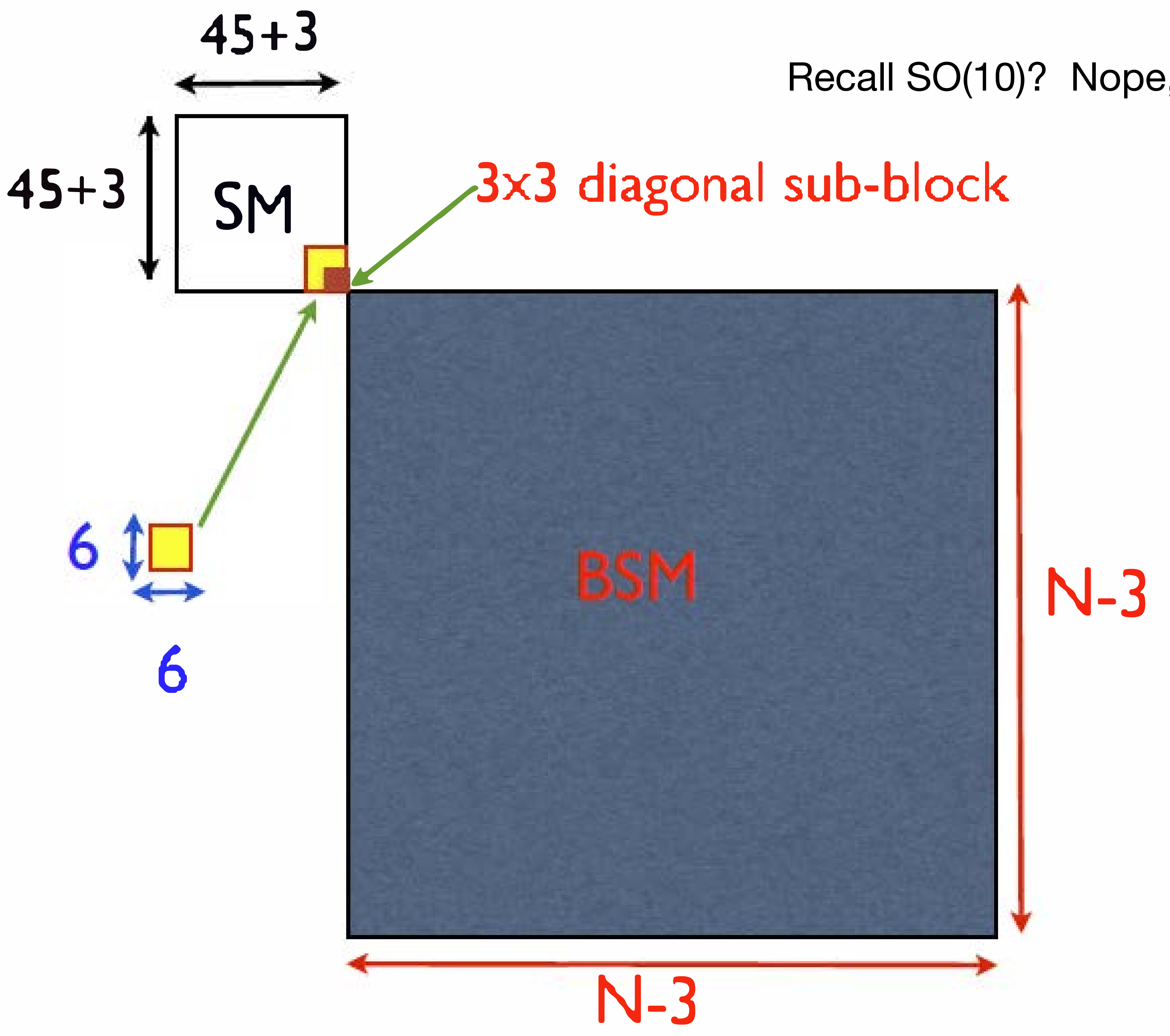
Use BSM Dark Matter?



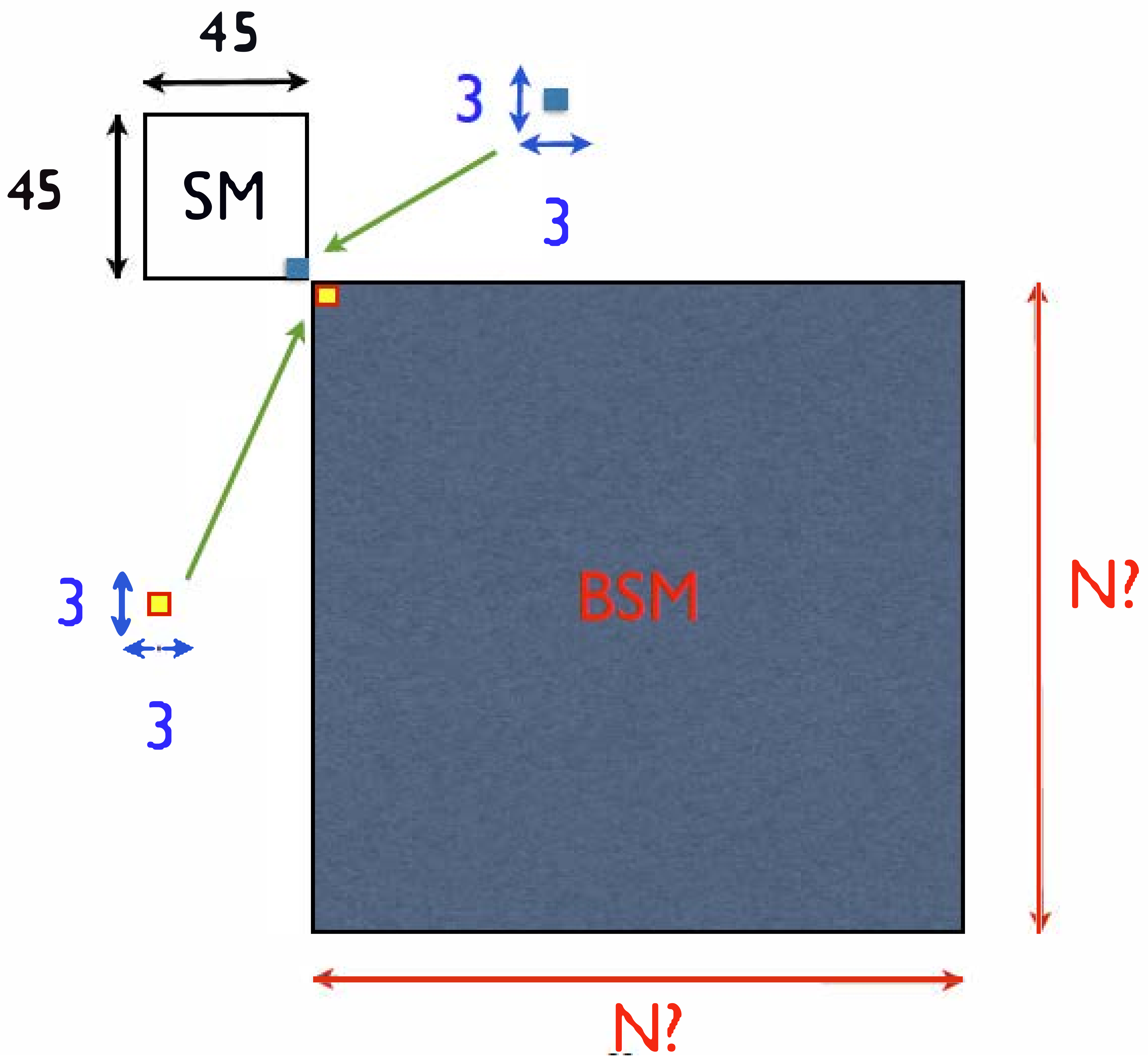
Majorana zeroes for neutrinos on diagonal remain but now off-diagonal blocks are similar to all other known fermions?
NOPE!

Assume 3 rows and columns of zeroes to proceed further

Recall SO(10)? Nope, no proton decay



Dark Matter BSM?



So have we learned something about a (small) corner of the dark matter Weyl spinors?

as well as very small corrections to matter from dark matter interactions

Masses, CKM and muon (g-2)?

- Back-ups

Cabibbo: Universality of Weak Interactions

Why are kaon decay rates so much less than expected from pion decay rates?
Universalize phenomenologically with an angle between strangeness conserving and strangeness violating decay rates.
Small value of angle implies small correction to strangeness conserving decay rates.

Higgs' Universality of fermion mass term couplings?

NOT Y_q for each mass q but same Y_f for all members of each charge f.

This is possible

Quarks and Charged Leptons
have a common weak interaction structure related to their masses:

Their flavors are defined by their masses.
However, the mass eigenstates for quarks are not
exactly parallel to their weak interaction currents
via charged weak boson transformations

This is described by the CKM matrix which implements overall universality of weak interactions

insert CKM page

Separately, their mass spectra have spectacular variation:
enormous, large and tiny
exemplified by top, charm and up quarks

insert mass page

Both features can be addressed by assuming there are weak interaction current eigenstates
that connect a specific up-quark to a specific down-quark all with the same unit strength

show inverse TBM,
describe universality
starting point and
needed corrections

Charged leptons have the same description but neutrinos are different

flavor defined by charged
lepton coupling, not
neutrino mass

show effect of missing TBM factor

Mass Terms in the Standard Model

$$Y_U \Sigma_{ij} \{ \overline{\frac{(1+\gamma_5)}{2} \Psi_{Ui}} < \phi^0, \phi^+ > \frac{(1-\gamma_5)}{2} \begin{bmatrix} \Psi_{Uj} \\ \Psi_{Dj} \end{bmatrix} + \text{h.c.} \}$$

$$Y_D \{ \Sigma_{ij} \overline{\frac{(1+\gamma_5)}{2} \Psi_{Di}} < \phi^-, -(\phi^0)^* > \frac{(1-\gamma_5)}{2} \begin{bmatrix} \Psi_{Uj} \\ \Psi_{Dj} \end{bmatrix} + \text{h.c.} \}$$

Left-chiral (Weyl) u symmetry-linked to left-chiral (Weyl) d **but** no SM quantum numbers distinguish one right-chiral (Weyl) u **or** d from another

Absent other (BSM) information, the chiral bases **can be chosen** to produce this form of the mass matrix as above

Masses of fundamental fermions at weak scale*: (all in MeV/c²)

$$m_u = 1.28 \pm 0.45 \quad m_c = 619 \pm 84 \quad m_t = 162900 \pm 2800$$

$$m_d = 2.92 \pm 1.22 \quad m_s = 55 \pm 15 \quad m_b = 2890 \pm 90$$

$$m_e = 0.510999 \quad m_\mu = 105.658 \quad m_\tau = 1776.82 \pm 0.16$$

$\epsilon\delta M_0$ $\qquad$ ϵM_0 $\qquad$ $m = 3M_0$

Largest Y_f is 1/3 of SM value

Scaling out largest masses and using central values only:

$$M_{+\frac{2}{3}} = 162900 \times \begin{bmatrix} 7.86 \times 10^{-6} & 0 & 0 \\ 0 & 3.80 \times 10^{-3} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M_{-\frac{1}{3}} = 2890 \times \begin{bmatrix} 1.01 \times 10^{-3} & 0 & 0 \\ 0 & 1.90 \times 10^{-2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M_{-1} = 1776.82 \times \begin{bmatrix} 2.8759 \times 10^{-4} & 0 & 0 \\ 0 & 5.9465 \times 10^{-2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\delta_u \sim 2 \cdot 10^{-3}$$
$$\delta_d \sim 5 \cdot 10^{-2}$$
$$\delta_\ell \sim 5 \cdot 10^{-3}$$

*Zhi-zhong Xing, He Zhang and Shun Zhou, *Phys. Rev. D77* (2008) 113016

We have diagonalized by

$$X_{2x2}^{\dagger} X_{3\to 2}^{\dagger} (TBM^{\dagger}) \mathcal{M}_{dem} (TBM) X_{3\to 2} X_{2x2}$$

This means that we inserted unity in the mass term as

$$\overline{q_{current}} X_{tot} X_{tot}^{\dagger} \mathcal{M}_{dem} X_{tot} X_{tot}^{\dagger} q_{current} = \overline{q_{mass}} \mathcal{M}_{diag} q_{mass}$$

so we can identify

$$X_{tot}^{\dagger} q_{current} = q_{mass} \quad \text{or} \quad q_{current} = X_{tot} q_{mass}$$

Applying separately for up-quarks (U) and down-quarks (V)

$$\overline{u_{current}} = \overline{u_{mass}} X^{\dagger}_{[tot,u]} = \overline{u_{mass}} U$$

$$d_{current} = X_{[tot,d]} d_{mass} = V^{\dagger} d_{mass}$$

in terms relevant to the PDG description of CKM:

$$\mathcal{L}_W = -\frac{g_W}{\sqrt{2}} \overline{u_{current}} \gamma^{\mu} W^{+}_{\mu} \frac{(1-\gamma_5)}{2} d_{current}$$

Thus $U\,V^{\dagger} = X^{\dagger}_{[tot,u]} X_{[tot,d]}$

or $U\,V^{\dagger} = X^{\dagger}_{[2x2,u]} X^{\dagger}_{[3\rightarrow 2,u]} X_{[3\rightarrow 2,d]} X_{[2x2,d]}$

which shows that the TBM and TBM† factors cancel out

$$X_{tot} = TBM \times X_{3\to 2} \times R2_\omega$$
$$= TBM \times \begin{bmatrix} 1 & 0 & \epsilon x_a e^{\imath\zeta} \\ 0 & 1 & -\epsilon x_b \\ -\epsilon x_a e^{-\imath\zeta} & \epsilon x_b & 1 \end{bmatrix} \times \begin{bmatrix} \cos(\omega) & \sin(\omega) & 0 \\ -\sin(\omega) & \cos(\omega) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$y_0 = \frac{1}{\sqrt{6}}(1+\delta)$$
$$y_3 = \frac{1}{2\sqrt{3}}[2\sqrt{2}x_b - (1-\delta)\sin(2\omega)]$$
$$y_8 = \frac{1}{2\sqrt{3}}[2\sqrt{2}x_a\cos(\zeta) + (1-\delta)\cos(2\omega)]$$
$$y_1 = \frac{1}{6}[2\sqrt{2}x_a\cos(\zeta) - 2\cos(2\omega)(1-\delta) - (1+\delta)]$$

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$$y_4 = \frac{1}{6}[-\sqrt{2}x_a\cos(\zeta) + \sqrt{6}x_b + (\cos(2\omega) + \sqrt{3}\sin(2\omega))(1-\delta) - (1+\delta)]$$
$$y_6 = \frac{1}{6}[-\sqrt{2}x_a\cos(\zeta) - \sqrt{6}x_b + (\cos(2\omega) - \sqrt{3}\sin(2\omega))(1-\delta) - (1+\delta)]$$
$$y_2 = 0\,,\; y5 = y7 = -\frac{1}{\sqrt{2}}x_a\sin(\zeta)$$

$$X = -(\epsilon_d x_{bd} - \epsilon_u x_{bu})$$

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$$Y = \epsilon_d x_{ad}\cos(\zeta_d) - \epsilon_u x_{au}\cos(\zeta_u)$$
$$Z = \epsilon_d x_{ad}\sin(\zeta_d) - \epsilon_u x_{au}\sin(\zeta_u)$$

$$CKM_1(1,3) = (\epsilon_d x_{ad}e^{\imath\zeta_d} - \epsilon_u x_{au}e^{\imath\zeta_u})\cos(\omega_u) + (\epsilon_d x_{bd} - \epsilon_u x_{bu})\sin(\omega_u)$$
$$CKM_1(2,3) = -(\epsilon_d x_{bd} - \epsilon_u x_{bu})\cos(\omega_u) + (\epsilon_d x_{ad}e^{\imath\zeta_d} - \epsilon_u x_{au}e^{\imath\zeta_u})\sin(\omega_u)$$
$$CKM_1(3,1) = -(\epsilon_d x_{ad}e^{-\imath\zeta_d} - \epsilon_u x_{au}e^{-\imath\zeta_u})\cos(\omega_d) - (\epsilon_d x_{bd} - \epsilon_u x_{bu})\sin(\omega_d)$$
$$CKM_1(3,2) = (\epsilon_d x_{bd} - \epsilon_u x_{bu})\cos(\omega_d) - (\epsilon_d x_{ad}e^{-\imath\zeta_d} - \epsilon_u x_{au}e^{-\imath\zeta_u})\sin(\omega_d)$$

$$CKM_1(1,3) = Y\cos(\omega_u) - X\sin(\omega_u) + \imath Z\cos(\omega_u)$$
$$CKM_1(2,3) = Y\sin(\omega_u) + X\cos(\omega_u) + \imath Z\sin(\omega_u)$$
$$CKM_1(3,1) = -Y\cos(\omega_d) + X\sin(\omega_d) + \imath Z\cos(\omega_d)$$
$$CKM_1(3,2) = -Y\sin(\omega_d) - X\cos(\omega_d) + \imath Z\sin(\omega_d)$$

$$U\,V^\dagger = \begin{bmatrix} \cos(\Theta_C) & \sin(\Theta_C) & CKM_{1,3} \\ -\sin(\Theta_C) & \cos(\Theta_C) & CKM_{2,3} \\ CKM_{3,1} & CKM_{3,2} & 1 \end{bmatrix}$$

$$\Theta_C \approx \omega_d - \omega_u$$

$$\mathcal{R}[(1,3)] = \frac{(2\cos(\omega_u)^2 - 1)XY - \sin(\omega_u)\cos(\omega_u)(X^2 - Y^2 - Z^2)}{Q}$$

$$\mathcal{R}[(3,1)] = \frac{X^2\cos(\omega_u)\sin(\omega_d) - (Y^2 + Z^2)\sin(\omega_u)\cos(\omega_d) - XY\cos(\omega_u + \omega_d)}{Q}$$

$$\mathcal{R}[(3,2)] = -\frac{X^2\cos(\omega_u)\cos(\omega_d) + (Y^2 + Z^2)\sin(\omega_u)\sin(\omega_d) + XY\sin(\omega_u + \omega_d)}{Q}$$

$$CKM_1(1,3) = (\epsilon_d x_{ad} e^{\imath\zeta_d} - \epsilon_u x_{au} e^{\imath\zeta_u})\cos(\omega_u) + (\epsilon_d x_{bd} - \epsilon_u x_{bu})\sin(\omega_u)$$

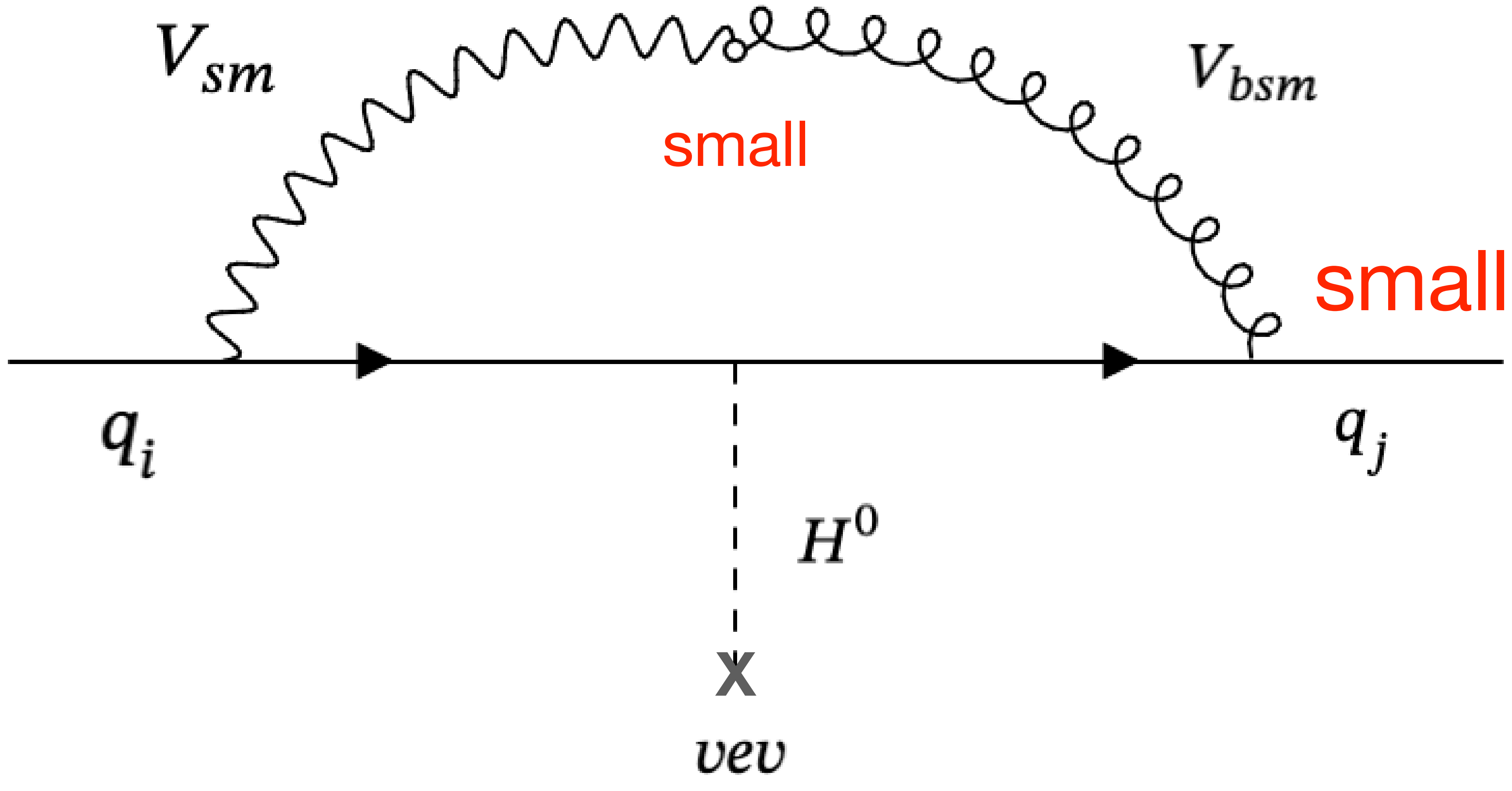
$$CKM_1(2,3) = -(\epsilon_d x_{bd} - \epsilon_u x_{bu})\cos(\omega_u) + (\epsilon_d x_{ad} e^{\imath\zeta_d} - \epsilon_u x_{au} e^{\imath\zeta_u})\sin(\omega_u)$$

$$CKM_1(3,1) = -(\epsilon_d x_{ad} e^{-\imath\zeta_d} - \epsilon_u x_{au} e^{-\imath\zeta_u})\cos(\omega_d) - (\epsilon_d x_{bd} - \epsilon_u x_{bu})\sin(\omega_d)$$

$$CKM_1(3,2) = (\epsilon_d x_{bd} - \epsilon_u x_{bu})\cos(\omega_d) - (\epsilon_d x_{ad} e^{-\imath\zeta_d} - \epsilon_u x_{au} e^{-\imath\zeta_u})\sin(\omega_d)$$

Zero correction to mass matrix before Higgs' symmetry breaking

Current mixing for quarks and charged leptons



Current mixing for neutrinos?

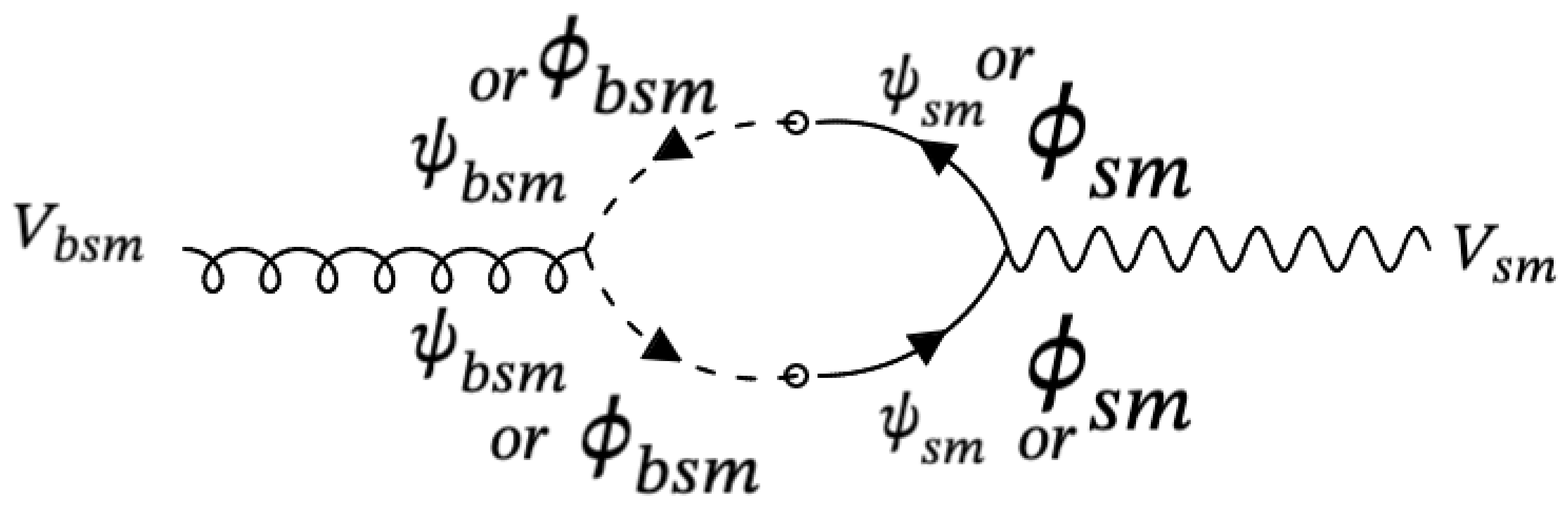
Dirac or Majorana?

See-saw?

BSM physics provides for deviations

via current mixing of
gauge boson interactions
due to mass mixing of
fermions or scalars of each type

Small mass mixing
produces
small loop effect

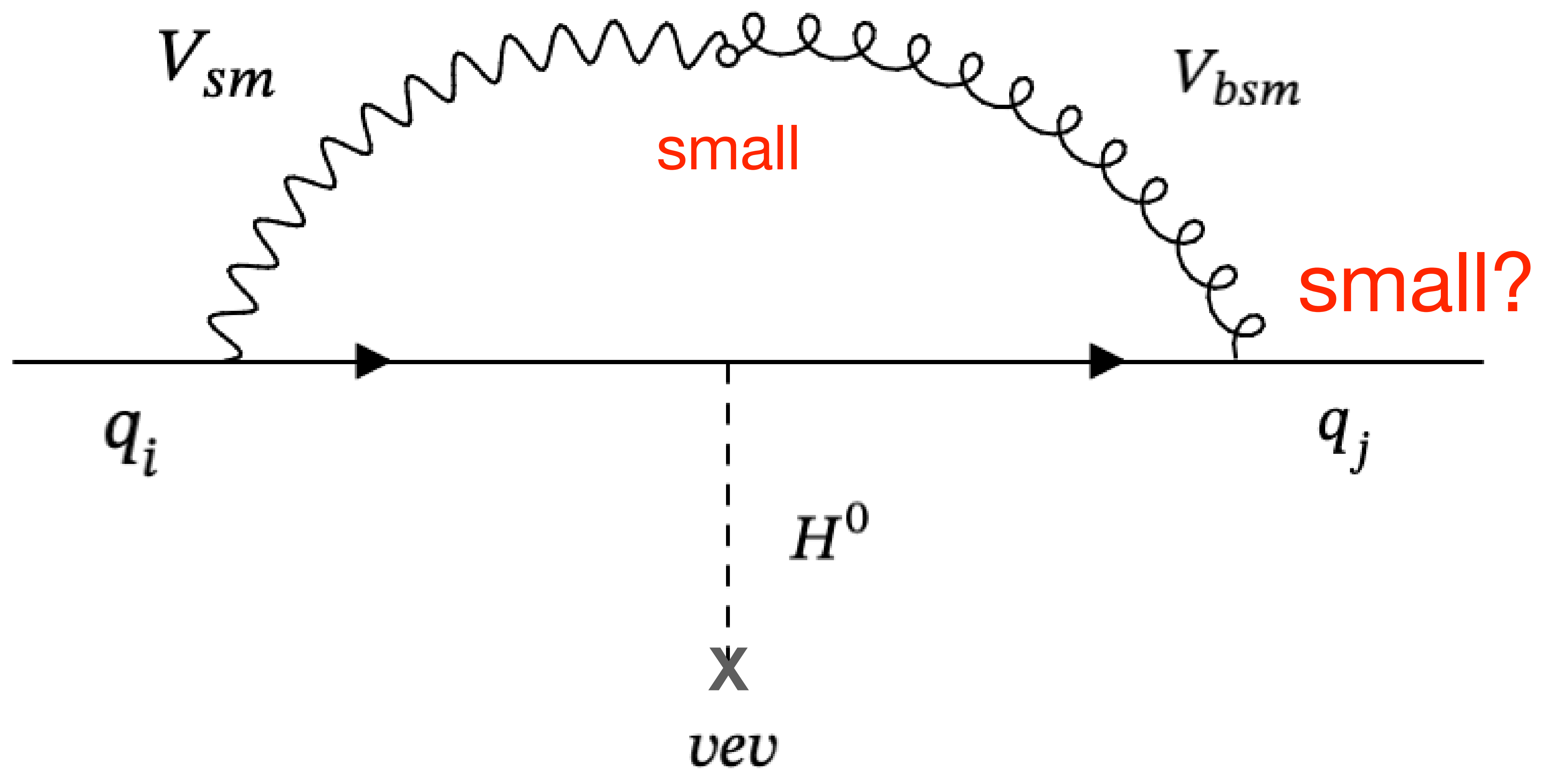


V{bsm} is drawn
suggestively to
consider that it
may describe a
strong interaction
with dark matter

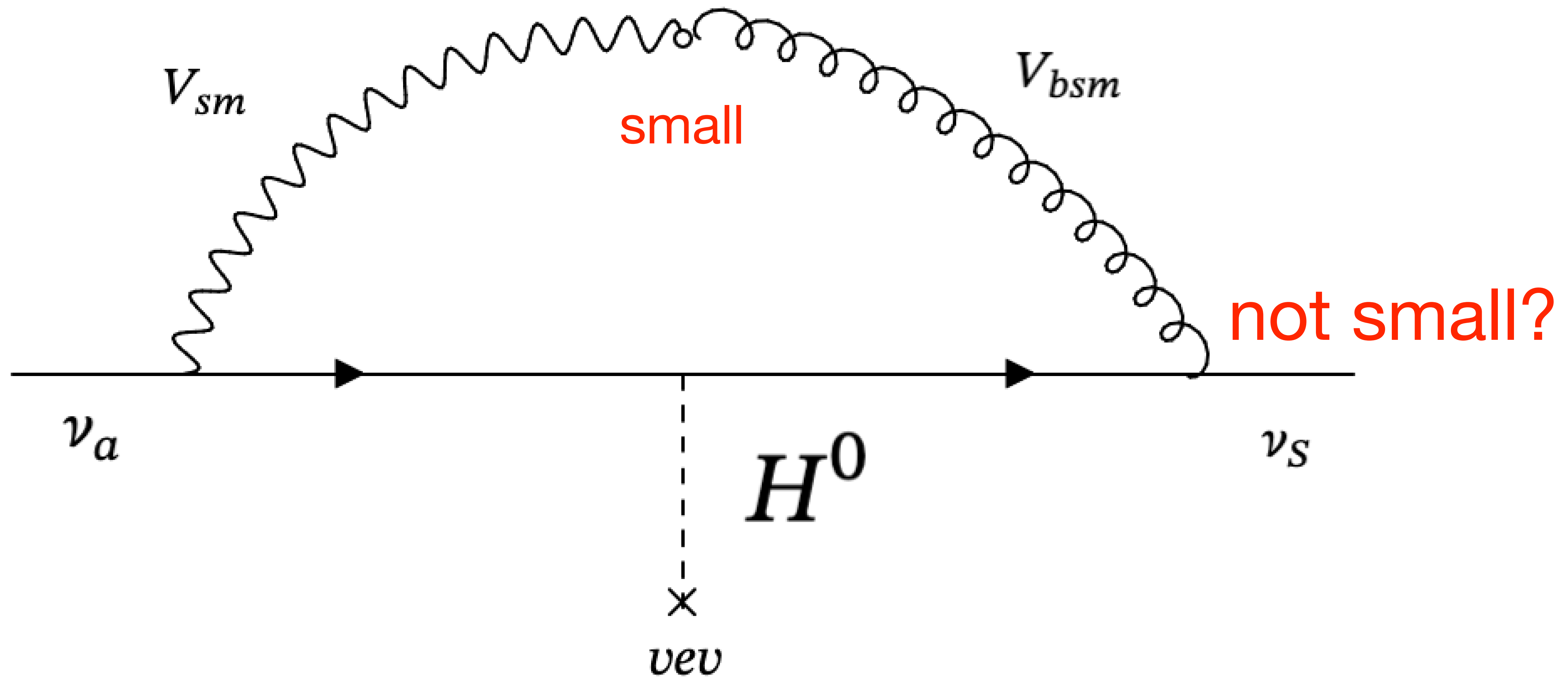
or change V to S on both sides
to mix scalar interactions

Zero correction to mass matrix before Higgs' symmetry breaking

Current mixing for quarks and charged leptons



Current mixing for neutrinos



2022 pdg

sinsqtheta12 = 0.307+ 0.013 – 0.012

deltam21sq= 7.53± 0.18 $\times 10^{-5} eV^2$

sinsqtheta23= 0.539± 0.022 Inverted

0.546± 0.021 Normal

deltam32sq= – 2.536± 0.034 Inverted $\times 10^{-3} eV^2$

2.453± 0.033 Normal

So the mass pattern works but: m2 has the all entries ~ equal amplitude eigenvector
Rearranging columns is no problem but then 1 and 3 are the closest mass eigenstates

Something else/more is going on — neutrinos are (more!) different