Predictive Dirac neutrino spectrum with parity solution to the strong CP problem in $SU(5)_L \times SU(5)_R$

Anil Thapa

University of Virginia

In collaboration with K S Babu and R N Mohapatara

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Shortcomings of the Standard Model

Neutrino masses are predicted to be zero in SM, but neutrino oscillates! $\boxed{\Longrightarrow M_\nu \neq 0!}$

Open questions

> Octant of θ_{23} ?

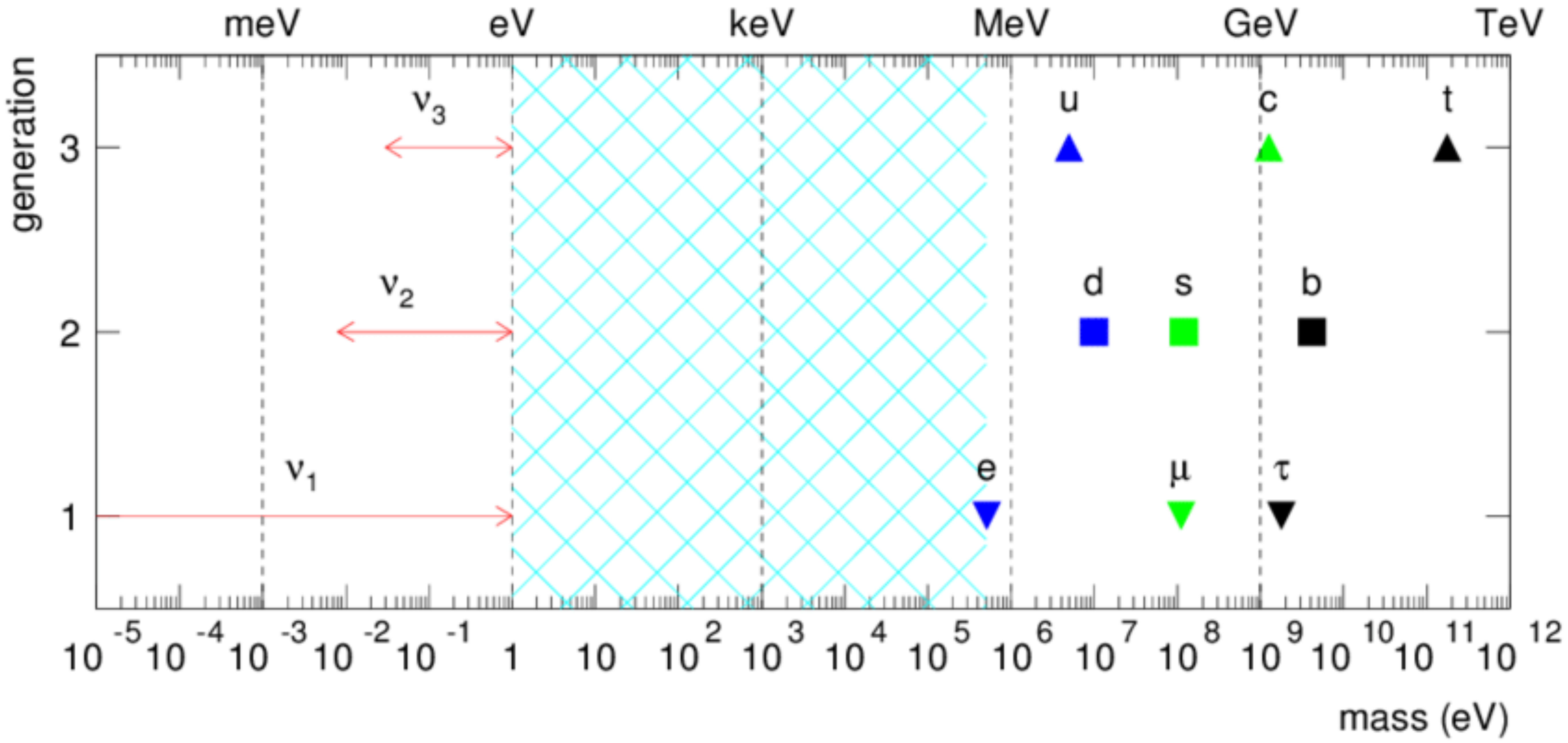
> Absolute mass scale and mass hierarchy?

> Are neutrinos their own antiparticle? Dirac vs Majorana

> Is there CP Violation in lepton sector, $P(\nu_\mu \to \nu_e) \neq P(\bar{\nu}_\mu \to \bar{\nu}_e)$?

> Why is neutrino mass so tiny?

Nonzero neutrino masses
$\Longrightarrow$ existence of new fundamental fields



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Anil Thapa (UVA)

Shortcomings of the Standard Model

- QCD lagrangian allows term that violates Parity P and Time Reversal T symmetries, thus CP symmetry:

$$\mathscr{L}_{\text{QCD}} = -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} + \theta\frac{g_s^2}{32\pi^2}G^a_{\mu\nu}\tilde{G}^{a\mu\nu} + \bar{q}\left(i\gamma^\mu D_\mu - m_q e^{i\theta_q\gamma_5}\right)q$$

g_s = strong coupling constant

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc}G^b_\mu G^c_\nu$$

- Any chiral rotation of the quark field, $q \to e^{i\alpha\gamma_5}q$ would lead to redefinition of the the new parameter $\theta \to \theta + \alpha$ due to anomalous nature of this rotation,

$\Longrightarrow$ only invariant physical quantity is $\bar{\theta} = \theta + \theta_q$

or $\bar{\theta} = \boxed{\theta + \text{ArgDet}[M_Q]}$ with multiple flavors of the quark

No reason for them to cancel

- $\bar{\theta}$ induces neutron electric dipole moment (neutron EDM) $d_n \sim 3 \times 10^{-16}\ \bar{\theta}$ e cm

- Current bound on neutron is $d_n < 3 \times 10^{-26}$ e cm

The mass parameters can in principle have arbitrary phases, and one would expect $\bar{\theta} \sim \mathcal{O}(1)$

Why is $\bar{\theta}$ so small?

Strong CP Problem

Solutions to the Strong CP Problem

Massless up quark

$\bar{\theta} = \theta + \mathrm{ArgDet}[M_Q]$

- chiral rotations,
  $u \to e^{i\alpha\gamma_5} u \Longrightarrow \theta \to \theta + \alpha$
  can remove it.
- $m_u = 0$ is inconsistent with experimental data as well as lattice calculations.

H. Georgi and I. Mc Arthur'81
K. Choi, C.W. Kim and W.K. Sze'88

The Axion

Make $\bar{\theta}$ a dynamical filed.

A global chiral U(1) symmetry is introduced that is spontaneously broken. Effective interaction of axion:

$$\mathscr{L} \supset \left(\frac{a}{f_a} + \theta\right) \frac{1}{32\pi^2} G\tilde{G}$$

Axion effective potential is such that vacuum solution relaxes to $\bar{\theta} = 0$

R.D. Peccei and H.R. Quinn'77
F. Wilczek'78, S. Weinberg'78

P or CP

Make P or CP exact symmetry broken spontaneously such a way that the determinant of the quark mass matrix is real.

$$\bar{\theta} = 0$$

A. Nelson'84 and S.M. Barr'84
Babu and Mohapatra, '90

Dirac Neutrinos from Left-Right Symmetry

- Fermion representation:

$$Q_L\ (3,2,1,1/3) = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \qquad Q_R\ (3,1,2,1/3) = \begin{pmatrix} u_R \\ d_R \end{pmatrix}$$

$$L_L\ (1,2,1,-1) = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \qquad L_R\ (1,1,2,-1) = \begin{pmatrix} \nu_R \\ e_R \end{pmatrix}$$

$$\boxed{\begin{array}{c} SU(2)_L \times SU(2)_R \times U(1)_X \\ \downarrow \langle H_R^0 \rangle \\ SU(2)_L \times U(1)_Y \\ \downarrow \langle H_L^0 \rangle \\ U(1)_{\text{em}} \end{array}}$$

- Higgs sector for symmetry breaking is very simple:

$$H_L\ (1,2,1,1) = \begin{pmatrix} H_L^+ \\ H_L^0 \end{pmatrix}_L \qquad H_R\ (1,1,2,1) = \begin{pmatrix} H_R^+ \\ H_R^0 \end{pmatrix}_R$$

- Vector-like fermion introduced to realize "universal seesaw" for charged fermion masses

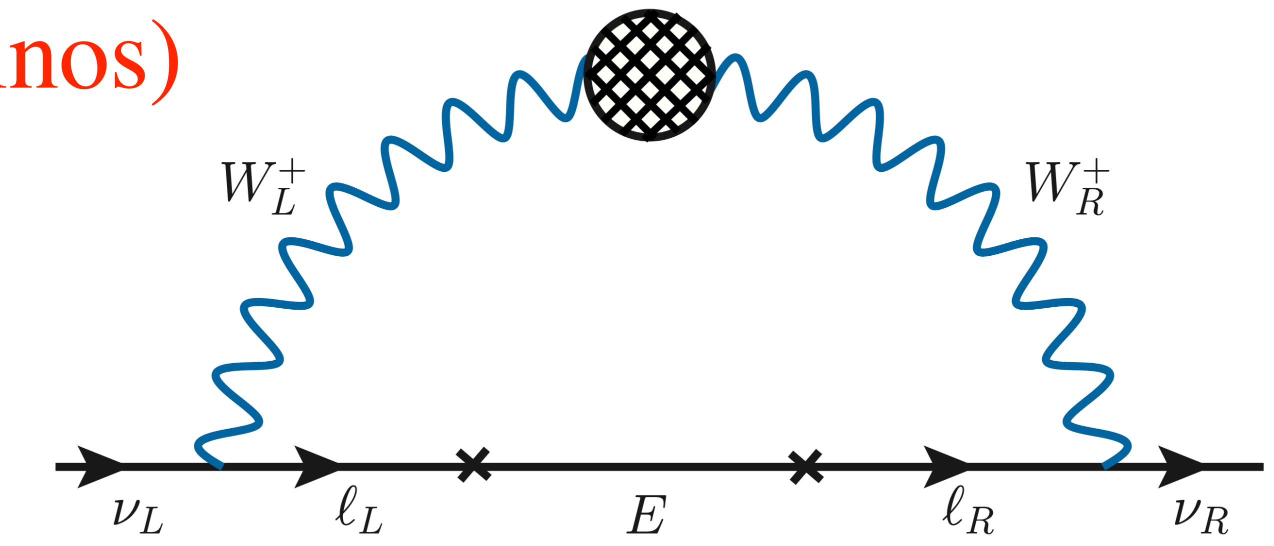
$$U\ (3,1,1,4/3), \quad D\ (3,1,1,-2/3), \quad E\ (1,1,1,-2)$$

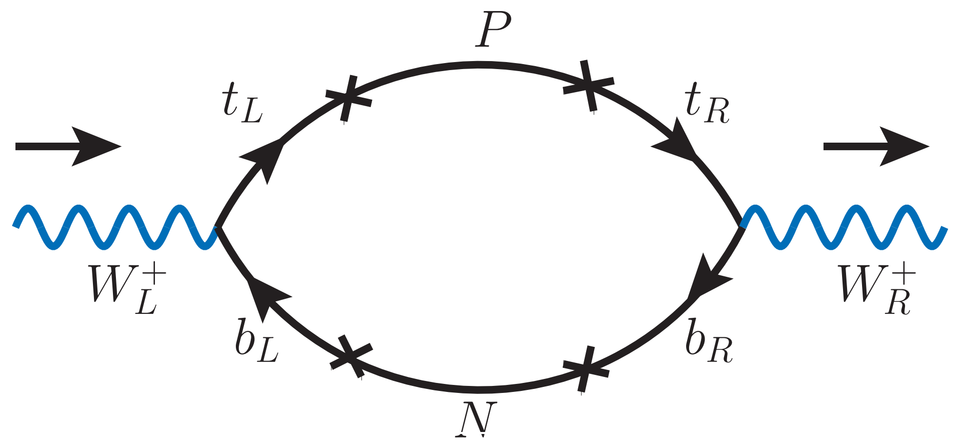
[Davidson, Wali '87]

$$\boxed{M_F = \begin{pmatrix} 0 & y\,\kappa_L \\ y^\dagger\,\kappa_R & M \end{pmatrix} \implies m_{u_i} \approx \frac{y^2 \kappa_L \kappa_R}{M}}$$

Seesaw for charged fermion masses (no seesaw for neutrinos)

- Dirac neutrinos arise naturally at two loop





[Babu, He, Su, **Thapa** '22]

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Embedding in $SU(5)_L \times SU(5)_R$

- The fermion spectrum of the model has a natural embedding in $SU(5)_L \times SU(5)_R$ unification

- All left-handed (right-handed) fermions of the SM fit into $\mathbf{10} + \bar{\mathbf{5}}$ of $SU(5)_L$ $(SU(5)_R)$

- The remaining vector-like quarks and leptons fill rest of the multiples

$D_{L,R}$ $\quad$ $L_{L,R}$

$$\psi_{L,R} = \begin{pmatrix} D_1^c \\ D_2^c \\ D_3^c \\ e \\ -\nu \end{pmatrix}_{L,R}$$

$U_{L,R}$ $\quad$ $Q_{L,R}$ $\quad$ $E_{L,R}$

$$\chi_{L,R} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & U_3^c & -U_2^c & u_1 & d_1 \\ -U_3^c & 0 & U_1^c & u_2 & d_2 \\ U_2^c & -U_1^c & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & E^c \\ -d_1 & -d_2 & -d_3 & -E^c & 0 \end{pmatrix}_{L,R}$$

- Parity can be imposed under which $\psi_L \leftrightarrow \psi_R$ and $\chi_L \leftrightarrow \chi_R$

GUT Symmetry Breaking and Gauge Coupling Unification

- With the SM particles, we obtain following beta function coefficients with properly normalized gauge couplings: $b_1 = \frac{41}{26}, \quad b_2 = -\frac{19}{6}, \quad b_3 = -\frac{7}{2}$

- If $SU(5) \times SU(5)$ directly break to the SM group, where g_i meet at a single value. $\alpha_{\rm GUT} = 2\,\alpha_3 = \alpha_2 = \frac{13}{3}\alpha_1$

  $\Longrightarrow \sin^2\theta_W = 3/16$

  $\Longrightarrow$ Cannot reconcile value measured at EW scale

  $\Longrightarrow$ An intermediate symmetry is needed

$$\sin^2\theta_W(m_t) = \frac{3}{16}\left[1 + \frac{\alpha}{6\pi}\left\{-\frac{185}{3}\log\frac{M_G}{m_t}\right\}\right]$$

To break $SU(5)_L \times SU(5)_R$ spontaneously to $SU(3)_c \times U(1)_{\rm em}$ we choose the following Higgs multiplets

$\{\Sigma_L(\mathbf{75},1) + \Sigma_R(1,\mathbf{75})\},$

Why not $(\mathbf{24},1)$+$(1,\mathbf{24})$?

> allows $(\mathbf{24},1)H_R^\dagger \Phi H_L$ and $(\mathbf{24},1)\eta^\dagger\Phi\Phi$

that spoils strong CP solution

$\{H_L(\mathbf{5},1) + H_R(1,\mathbf{5})\}, \quad \Phi(\bar{\mathbf{5}},\mathbf{5}),$

Required for fermion mass generation

Required for symmetry breaking

$\eta(\overline{\mathbf{15}},\mathbf{15})$

Required for gauge coupling unification

Why not $(\overline{\mathbf{10}},\mathbf{10})$?

> allows rapid proton decay

> spoils strong CP

> makes g_{5R} nonperturbative

[Babu, Mohapatra, **Thapa**, '24]

GUT Symmetry Breaking and Gauge Coupling Unification

$$
\begin{aligned}
&SU(5)_L \times SU(5)_R \\
&\quad \downarrow M_G \sim \langle \Sigma_L \rangle \\
&SU(3)_{CL} \times SU(2)_L \times U(1)_L \times SU(5)_R \\
&\quad \downarrow M_I \sim \langle \Phi \rangle, \langle H_R \rangle \\
&SU(3)_C \times SU(2)_L \times U(1)_Y \\
&\quad \downarrow M_W \sim \langle H_L \rangle \\
&SU(3)_C \times U(1)_{\text{em}}
\end{aligned}
$$

- The evolution of the gauge couplings are governed by the following RGEs

$$
\boxed{16\pi^2 \frac{dg_i}{dt} = g_i^3 b_i + \frac{g_i^3}{16\pi^2}\left[\sum_j b_{ij} g_j^2 - \sum_k C_{ik} \mathrm{Tr}\left(Y_k^\dagger Y_k\right)\right]}
$$

- $\sin^2\theta_W$ at one-loop accuracy (ignoring threshold effect from VLF)

$$
\sin^2\theta_W(m_t) = \frac{3}{16}\left[1 + \frac{\alpha}{6\pi}\left\{-\frac{185}{3}\log\frac{M_I}{m_t} + (46 + \boxed{39})\log\frac{M_G}{M_I}\right\}\right]
$$

$(\bar{\mathbf{3}}, \mathbf{2}, -1/6, \mathbf{15}) \supset (\overline{\mathbf{15}}, \mathbf{15})$

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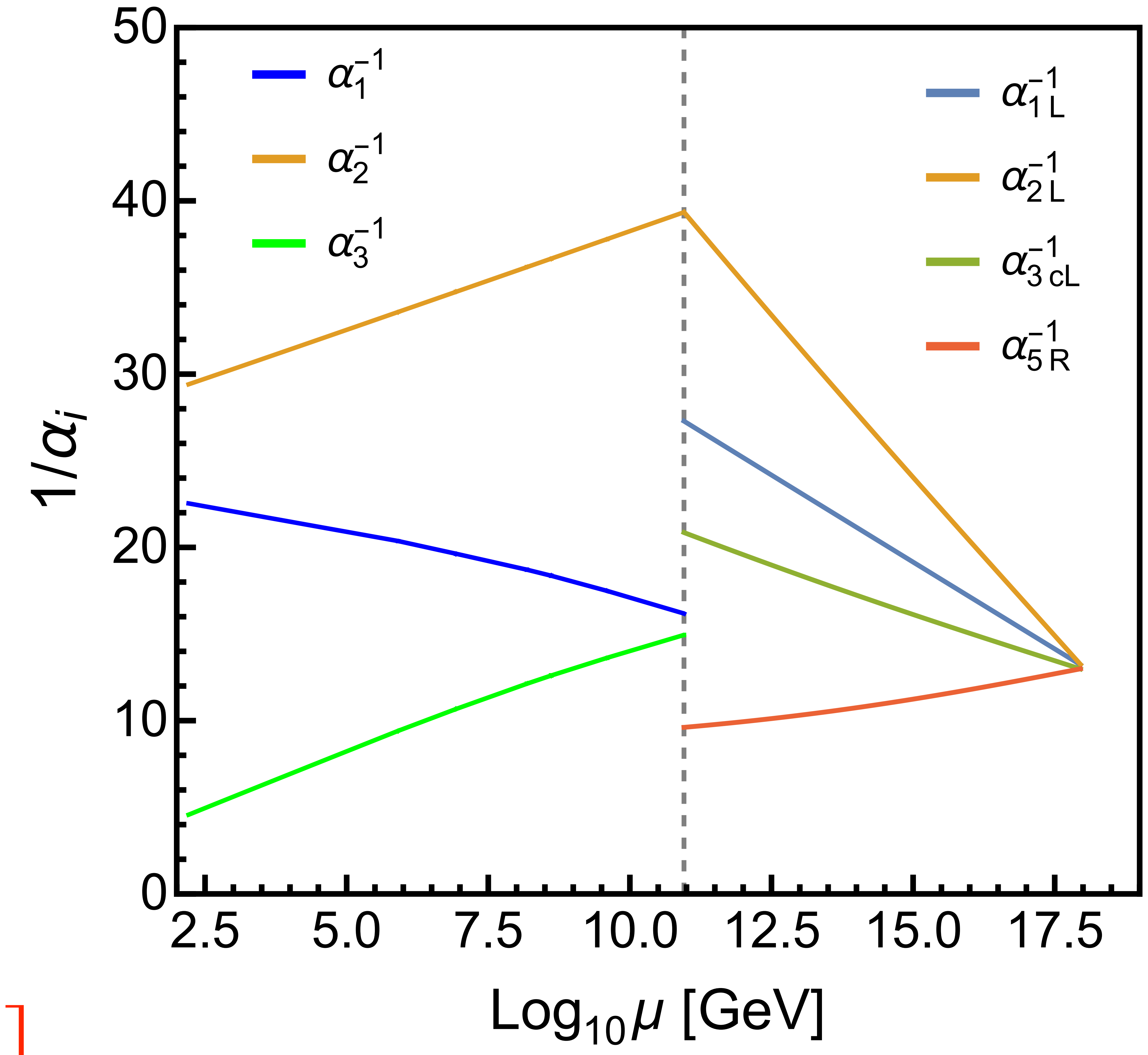

$$\boxed{\begin{array}{c} M_I = 9.02 \times 10^{10}\ \text{GeV} \\ M_G = 8.0 \times 10^{17}\ \text{GeV} \\ \alpha_G^{-1} = 13.18 \end{array}}$$

Fermion Mass Generation

$$-\mathscr{L}_{\rm Yuk} = \frac{(Y_u^\star)_{ij}}{4}\epsilon_{\alpha\beta\gamma\delta\rho}\left\{\chi_{Li}^{\alpha\beta}\chi_{Lj}^{\gamma\delta}H_L^\rho + \chi_{Ri}^{\alpha\beta}\chi_{Rj}^{\gamma\delta}H_R^\rho\right\} + \sqrt{2}\,(Y_\ell^\star)_{ij}\left\{\psi_{Li\alpha}\chi_{Lj}^{\alpha\beta}H_{L\beta}^\star + \psi_{Ri\alpha}\chi_{Rj}^{\alpha\beta}H_{R\beta}^\star\right\} + (Y_D^\star)_{ij}\,\overline{\Psi}_{Li}^\alpha\,\Phi_\alpha^\beta\,\psi_{Rj\beta}$$

- After spontaneous symmetry breaking, the masses of fermions read as

$$M_u = \begin{pmatrix} 0 & Y_u\,\kappa_L \\ Y_u^\dagger\,\kappa_R & 0 \end{pmatrix}, \qquad M_\ell = \begin{pmatrix} 0 & Y_\ell\,\kappa_L \\ Y_\ell^\dagger\,\kappa_R & 0 \end{pmatrix}, \qquad M_d = \begin{pmatrix} 0 & Y_\ell^T\,\kappa_L \\ Y_\ell^\star\,\kappa_R & Y_D v_\phi \end{pmatrix}$$

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Crucial for the model to be compatible with proton decay with $SU(5)_R$ intermediate symmetry.

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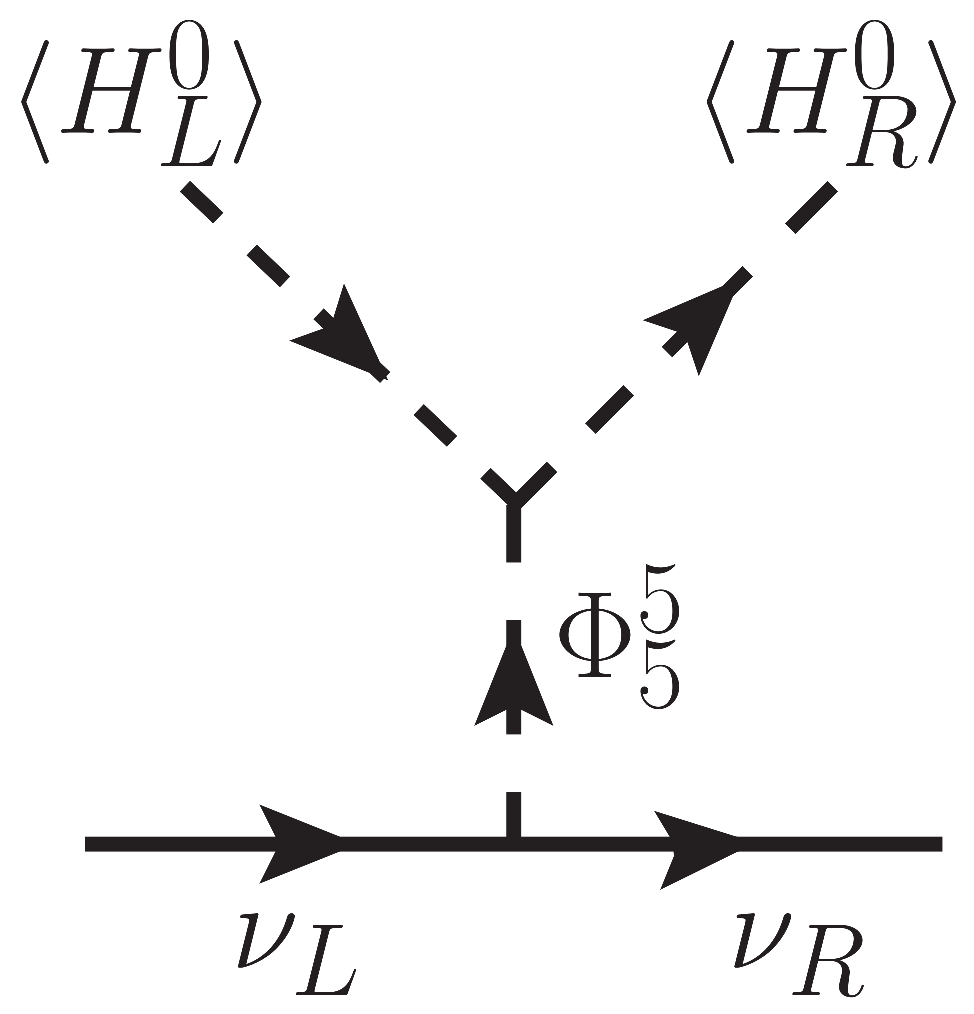
Crucial for the model to be compatible with proton decay with $SU(5)_R$ intermediate symmetry.

- Small Dirac neutrinos masses are induced naturally at the tree level via type-II Dirac seesaw



$$\boxed{\mathscr{L}_{\nu-\text{mass}}^{\text{Dirac}} = \frac{\overline{\nu}_L\nu_R\langle H_L^0\rangle\langle H_R^0\rangle}{M_G} \implies Y_\nu^{\text{Dirac}} \sim \frac{M_I}{M_G} \approx 10^{-7}}$$

Majorana mass for ν_R is forbidden by unbroken $B-L$ symmetry

Preditions for Neutrino Oscillations

- In the basis where Y_u and Y_ℓ are diagonal, down-type quark mass matrix M_d read as

$$M_u = \begin{pmatrix} 0 & \hat{M}_u \kappa_L \\ \hat{M}_u \frac{\kappa_R}{\kappa_L} & 0 \end{pmatrix}, \qquad M_\ell = \begin{pmatrix} 0 & \hat{M}_\ell \kappa_L \\ \hat{M}_\ell \frac{\kappa_R}{\kappa_L} & 0 \end{pmatrix},$$

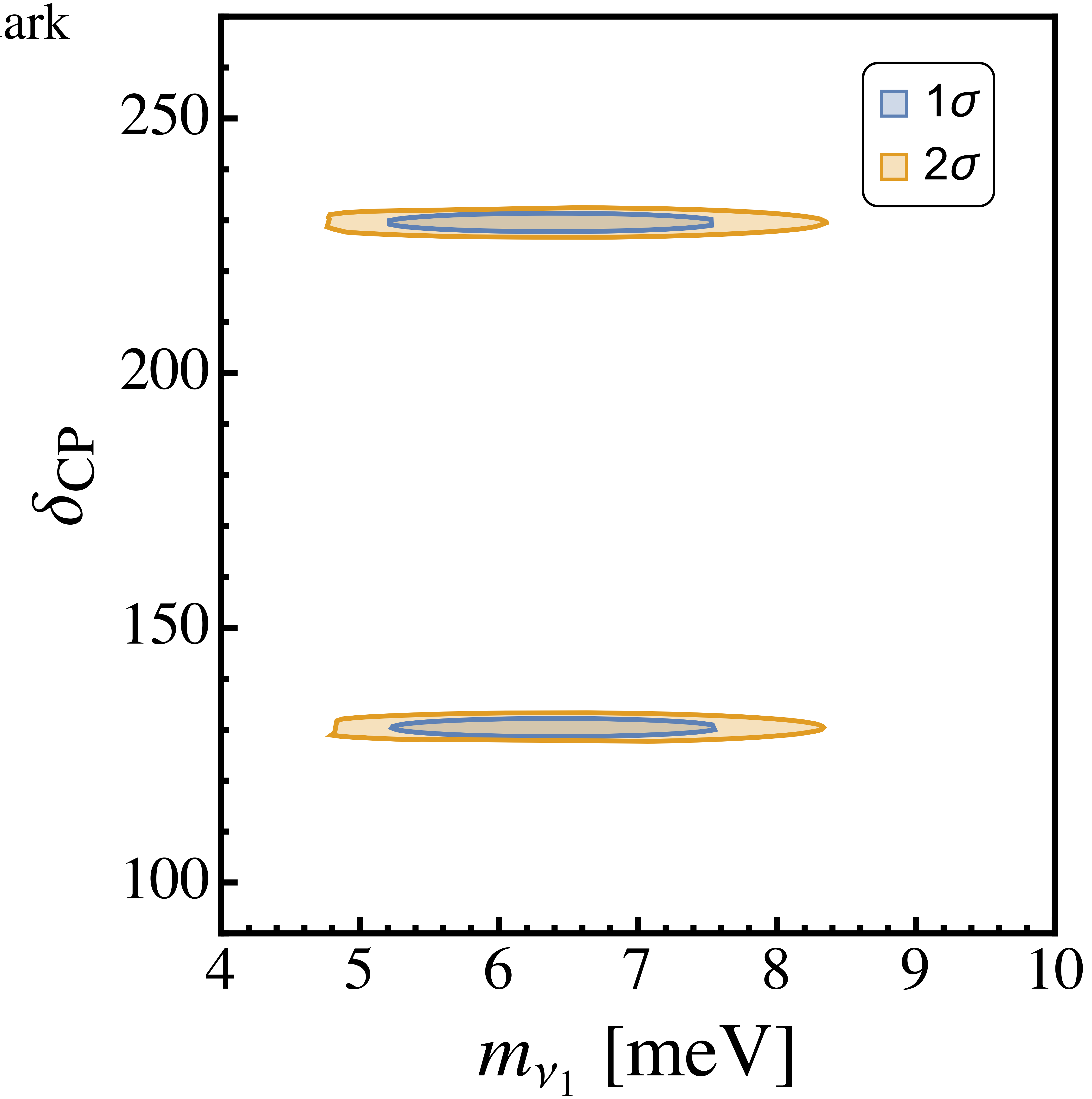
$$M_d = \begin{pmatrix} 0 & \hat{M}_\ell \\ \hat{M}_\ell \frac{\kappa_R}{\kappa_L} & \frac{v_\phi}{v_\nu} U^*_{\rm PMNS} \hat{M}_\nu U^T_{\rm PMNS} \end{pmatrix}$$

- Only one parameter in M_d to fit three light down-quark masses

$\Longrightarrow$ Predicts $\delta_{\rm CP}$ and lightest neutrino mass m_{ν_1}

$$\boxed{\begin{array}{c} \delta_{CP} = (130.4 \pm 1.2)^\circ \text{ or } (229.6 \pm 1.2)^\circ \\ m_{\nu_1} = (4.8 - 8.4)\ \text{meV} \end{array}}$$

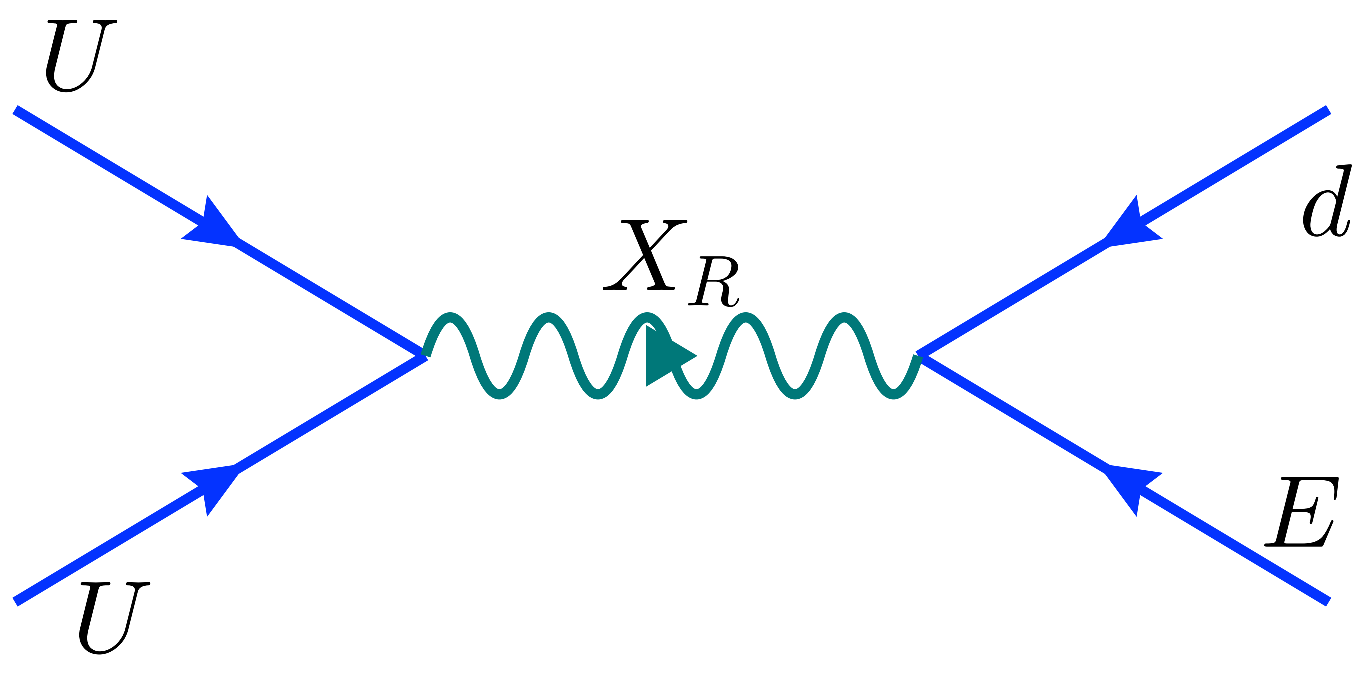
$\Longrightarrow$ Only normal hierarchy

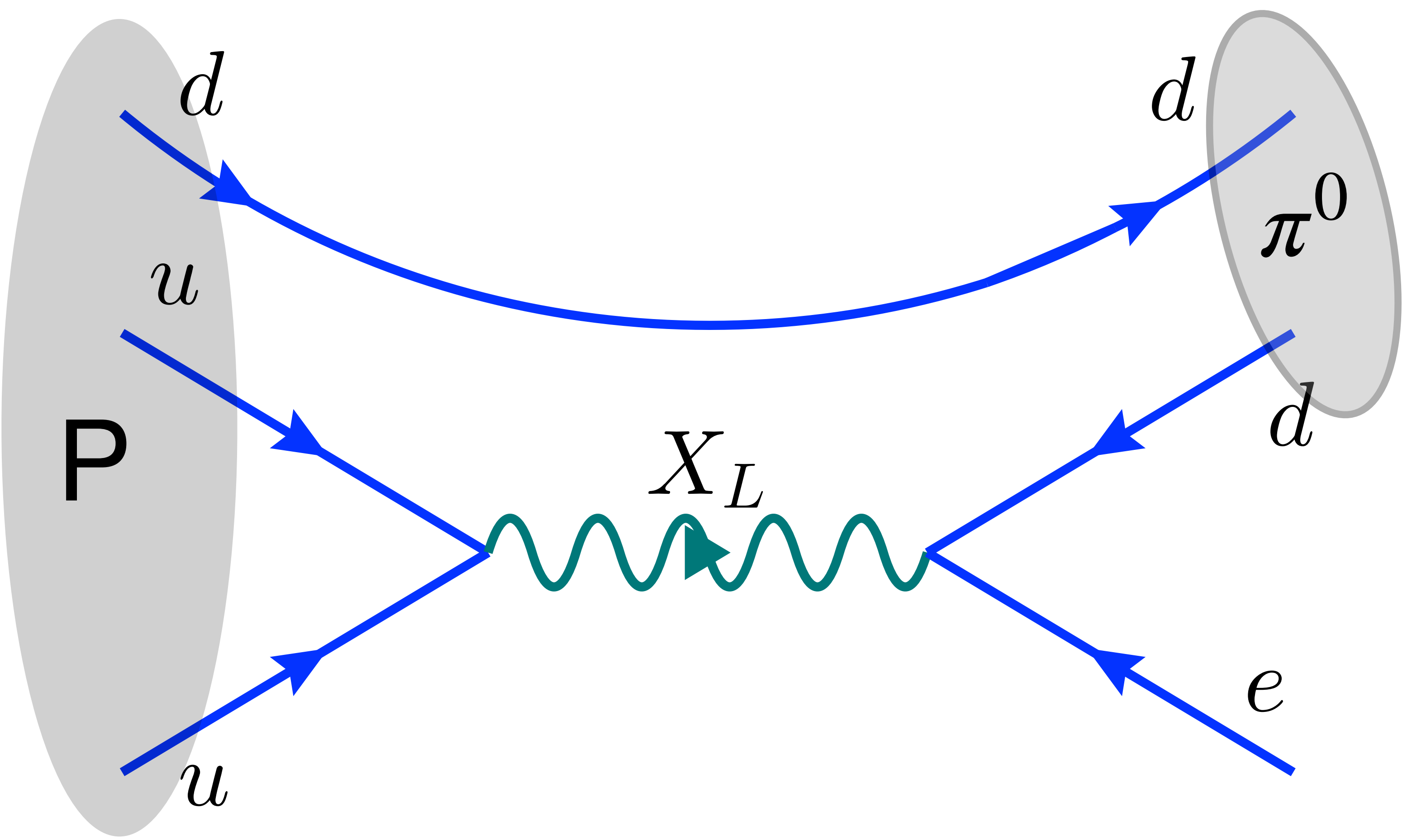


[Babu, Mohapatra, **Thapa**, '24]

Proton Decay

- Gauge bosons of $SU(5)_R$ with masses $M_{X_R,Y_R} \simeq M_I \sim 10^{11}$ GeV do not lead to proton decay owing to the structure of the zeros in (2,2) blocks of M_u and M_ℓ
- These couplings involve at least one heavy field
- Same is true with $H_R(1,\mathbf{5})$ Higgs field which has mass of order M_I





- B-violating interactions of X_L and Y_L gauge bosons of $SU(5)_L$ with masses of order $M_G = (7 \times 10^{16} - 8 \times 10^{17})$ GeV mediate proton decay.
- The leading decay mode of proton is $p \to e^+\pi^0$ with lifetime $\tau_p \approx (10^{38} - 10^{42})$ years. (Well beyond the reach of forthcoming experiments like JUNO, Hyperkamiokande, and DUNE)

Parity Solves the Strong CP Problem

quark mass matrix

$$\bar{\theta} = \boxed{\theta} + \text{Arg Det}\ \boxed{[M_Q]}$$

$M_Q \propto$ parity breaking VEVs, need to make sure the determinant is real.

$G^a_{\mu\nu}\tilde{G}^{a\mu\nu} \propto \overrightarrow{E}_{\text{color}} \cdot \overrightarrow{B}_{\text{color}}$

θ is odd under parity, therefore in parity symmetric theory it would vanish.

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$\Longrightarrow$ Det $[M_Q]$ = Det $[M_u M_d] \equiv$ Real

$\Longrightarrow$ $\bar{\theta} = 0$ at tree level

All the Higgs potential parameters with the fields [$\{\Sigma_L(\mathbf{75},1) + \Sigma_R(1,\mathbf{75})\}$, $\{H_L(\mathbf{5},1) + H_R(1,\mathbf{5})\}$, $\Phi(\bar{\mathbf{5}},\mathbf{5})$, $\eta(\overline{\mathbf{15}},\mathbf{15})$] are real with parity. Thus CP conserving vacuum is admitted, where all the VEVs are real.

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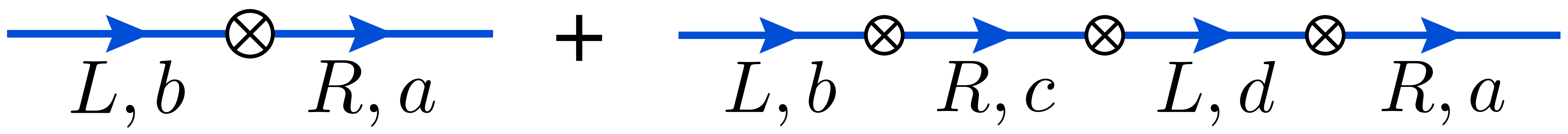
- Quantum corrections would in general induce $\bar{\theta} \neq 0$, but this may be within experimentally allowed range $\bar{\theta} \leq 1.19 \times 10^{-10}$ arising from neutron EDM limits.

Vanishing of one loop $\bar{\theta}$ contributions

- Convenient to work in the flavor basis, where the mass matrices M_u and M_d are treated as part of the interaction Lagrangian.

$\Longrightarrow$ need to sum all possible chirality flip in the propagator



$$+ \;\ldots\ldots = \bar{f}_R \left(M_d^\dagger \frac{k^2}{k^2 - M_d M_d^\dagger} \right) f_L \qquad f_{L,R} = \begin{pmatrix} d \\ D \end{pmatrix}_{L,R}$$

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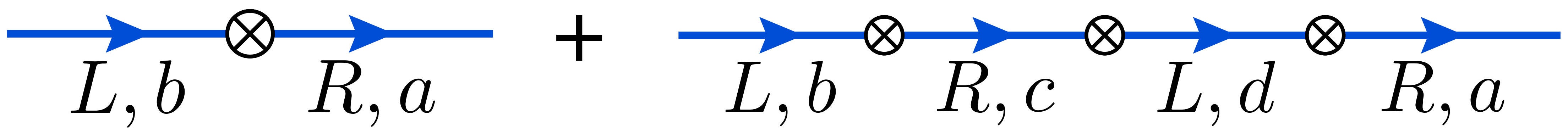

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- Loop-corrected quark mass matrix

tree level quark mass for $q = u, d$
where Arg Det $[M_q^{(0)}] = 0$

$C = C_1 + C_2 + \ldots.$ contribution
from 1-loop, 2-loop, ..

$$M_q = M_q^{(0)} + \delta M_q = M_q^{(0)}(1 + C)$$

L : light sector
H : heavy sector

$$\delta M_q = \begin{pmatrix} \delta M_{LL}^q & \delta M_{LH}^q \\ \delta M_{HL}^q & \delta M_{HH}^q \end{pmatrix}$$

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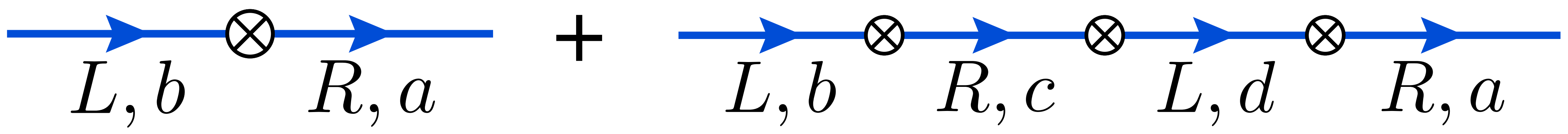

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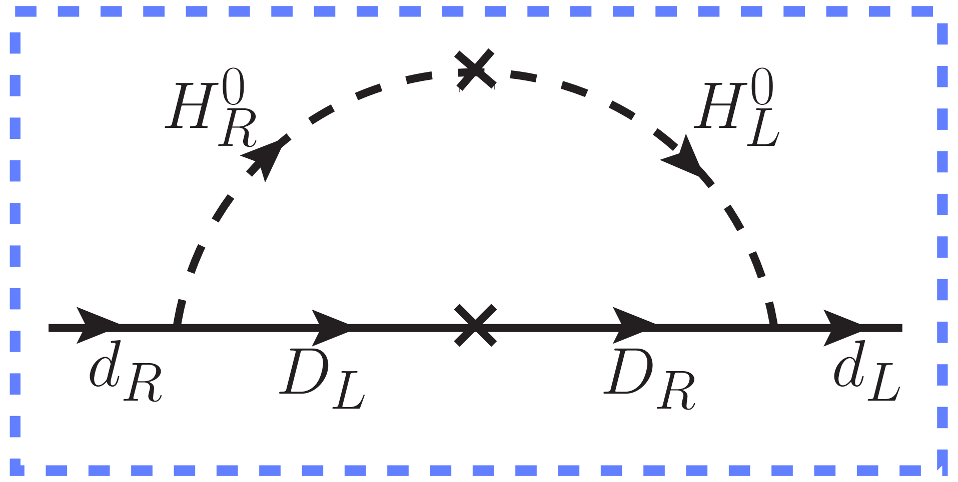
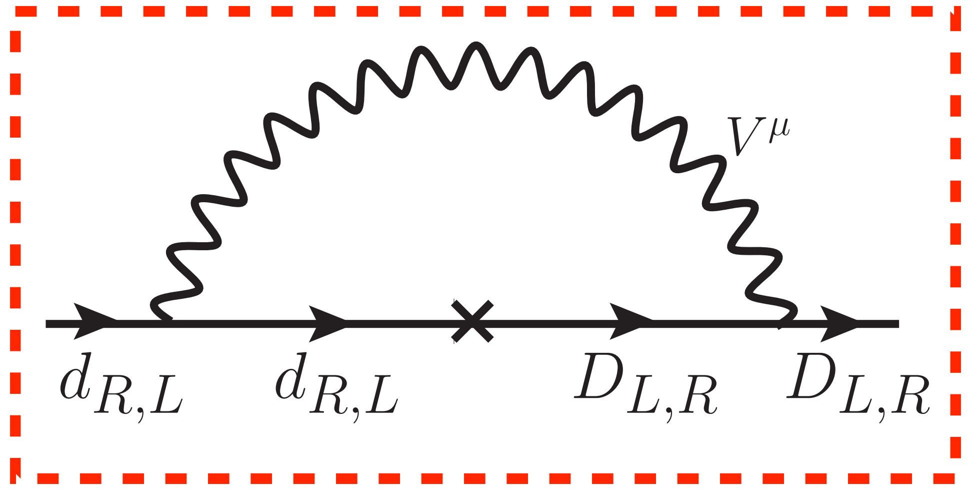




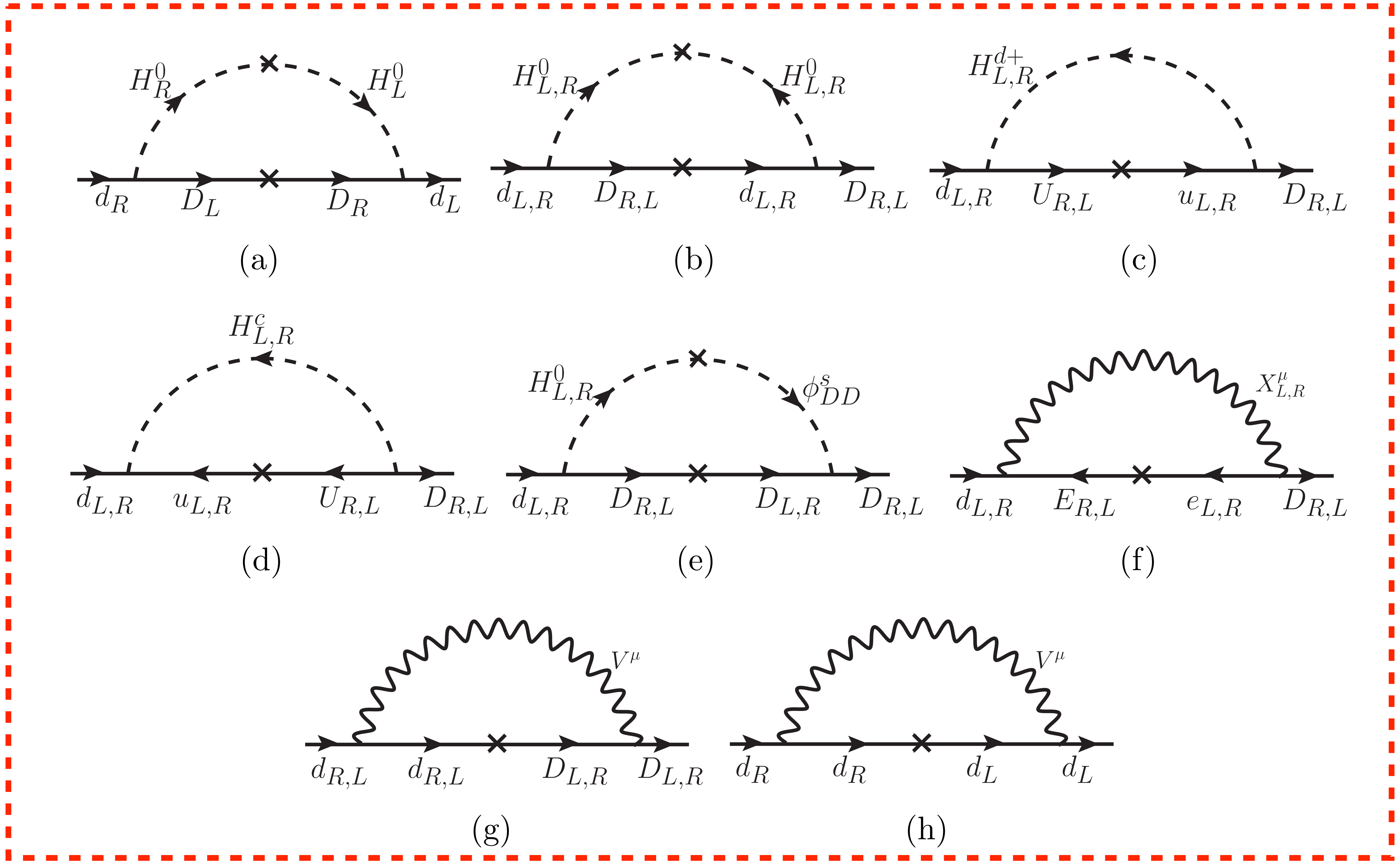
- $\bar{\theta}$ is given by

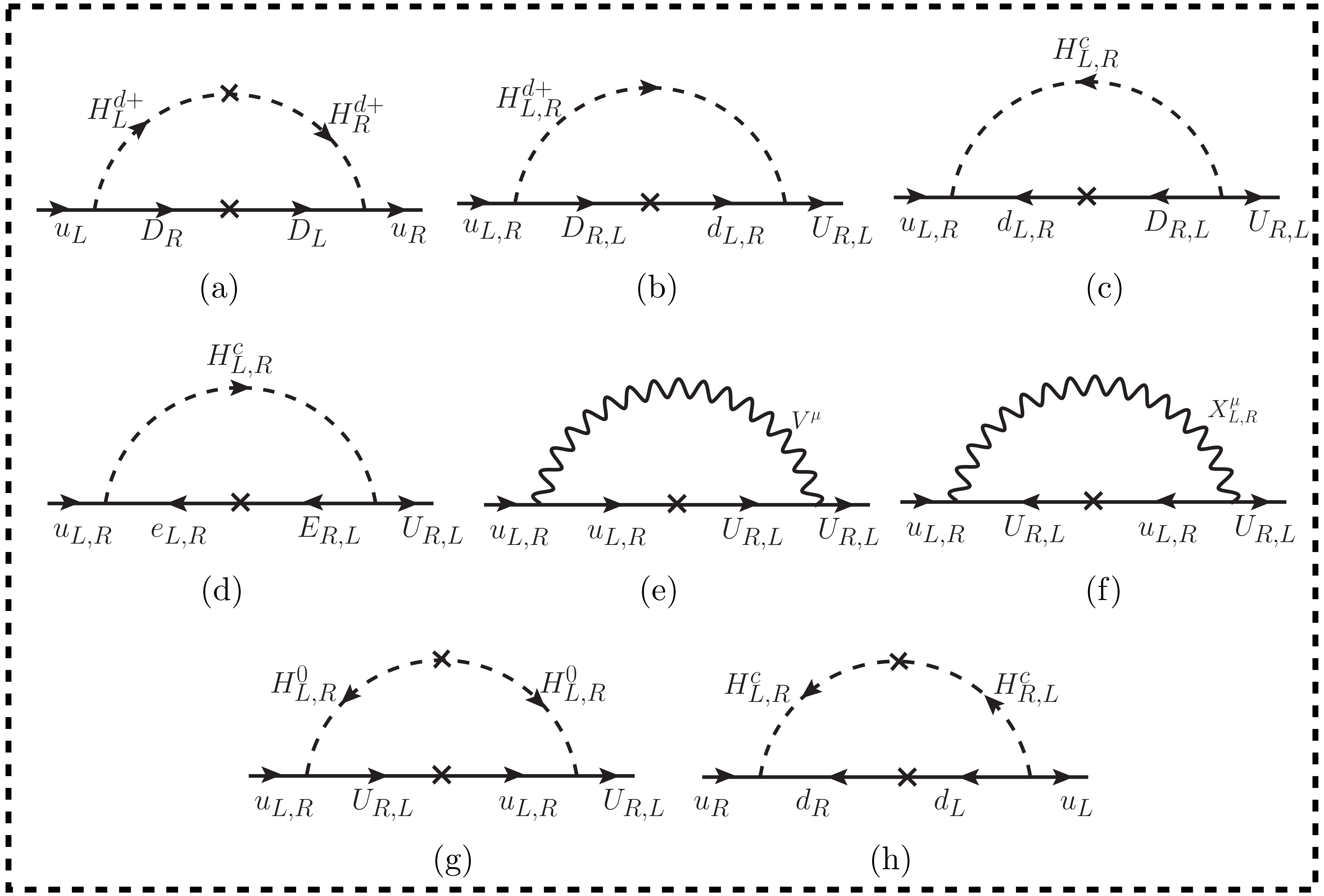
$$\bar{\theta} = \text{Im Tr}C_1 + \text{Im Tr}(C_2 - \frac{1}{2}C_1^2) + \ldots$$

$$\bar{\theta} = \text{Im Tr}\left[-\frac{v_\phi}{\kappa_L\kappa_R}\delta M^d_{LL}(Y_d^\dagger)^{-1}Y_D Y_d^{-1} + \frac{1}{\kappa_L}\delta M^d_{LH} Y_d^{-1} + \frac{1}{\kappa_R}\delta M^d_{HL}(Y_d^\dagger)^{-1}\right]$$

Vanishing of one loop $\bar{\theta}$

[Babu, Mohapatra, **Thapa**, ’24]





Each diagram individually gives
$\bar{\theta} = 0$

Summary

- Universal LRSM has natural embedding in $SU(5)_L \times SU(5)_R$
- Open questions in neutrino oscillations
  - > Absolute mass scale and mass hierarchy? ✓
    $m_{\nu_1} = (4.8 - 8.4)$ meV and Normal hierarchy
  - > Are neutrinos their own antiparticle? ✓
    Dirac neutrino via type-II seesaw
  - > Is there CP Violation in lepton sector? ✓
    Predicts $\delta_{CP} = (130.4 \pm 1.2)^\circ$ or $(229.6 \pm 1.2)^\circ$
  - > Why is neutrino mass so tiny? ✓
    Dirac mass suppressed by $\mathcal{O}\left(M_I/M_G\right) \approx 10^{-7}$
- The model solves strong CP problem without the need for an axion
  $\bar{\theta} = 0$ at tree level and one-loop level.
- No $0\nu\beta\beta$ and suppressed proton decay

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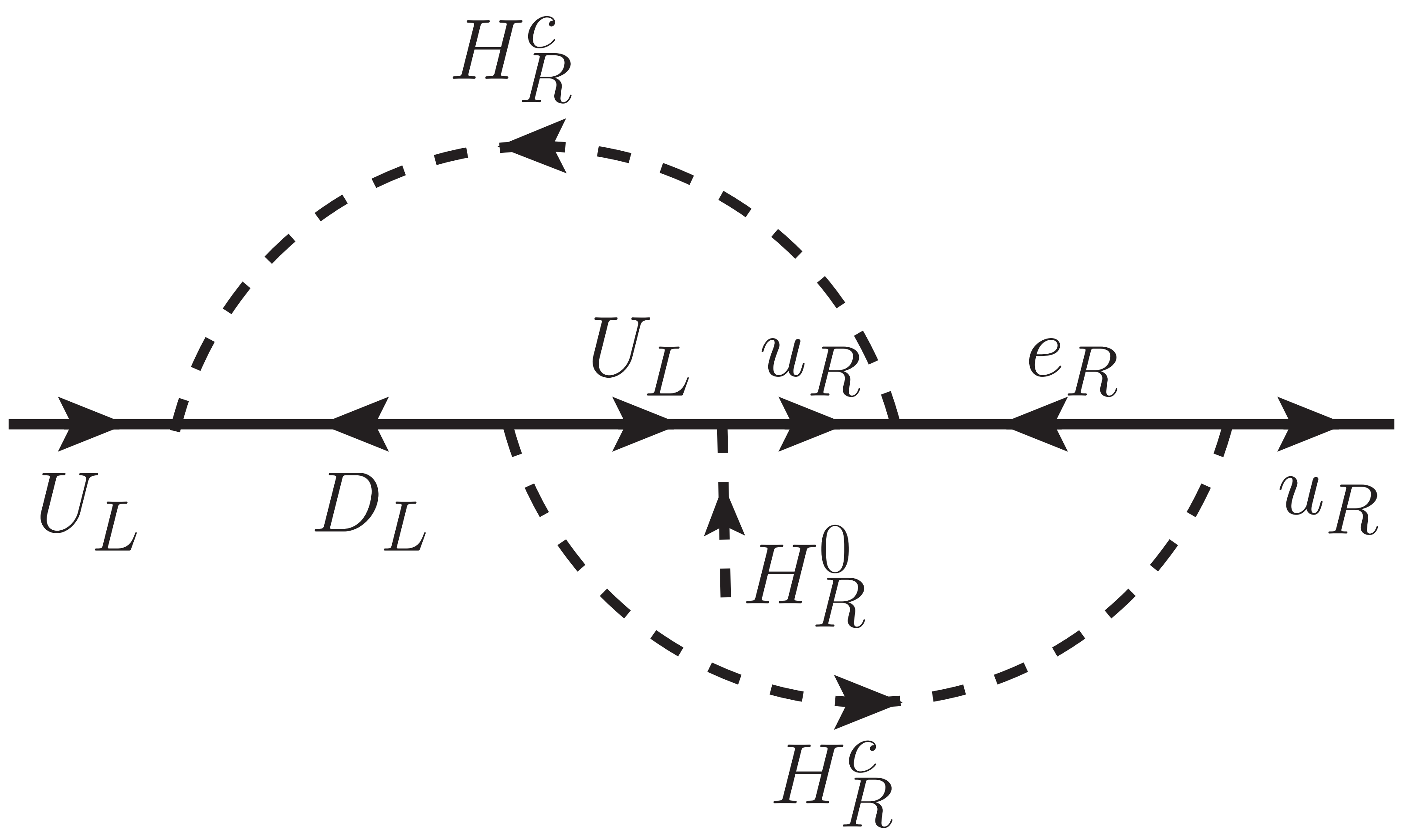
Thank you for your time

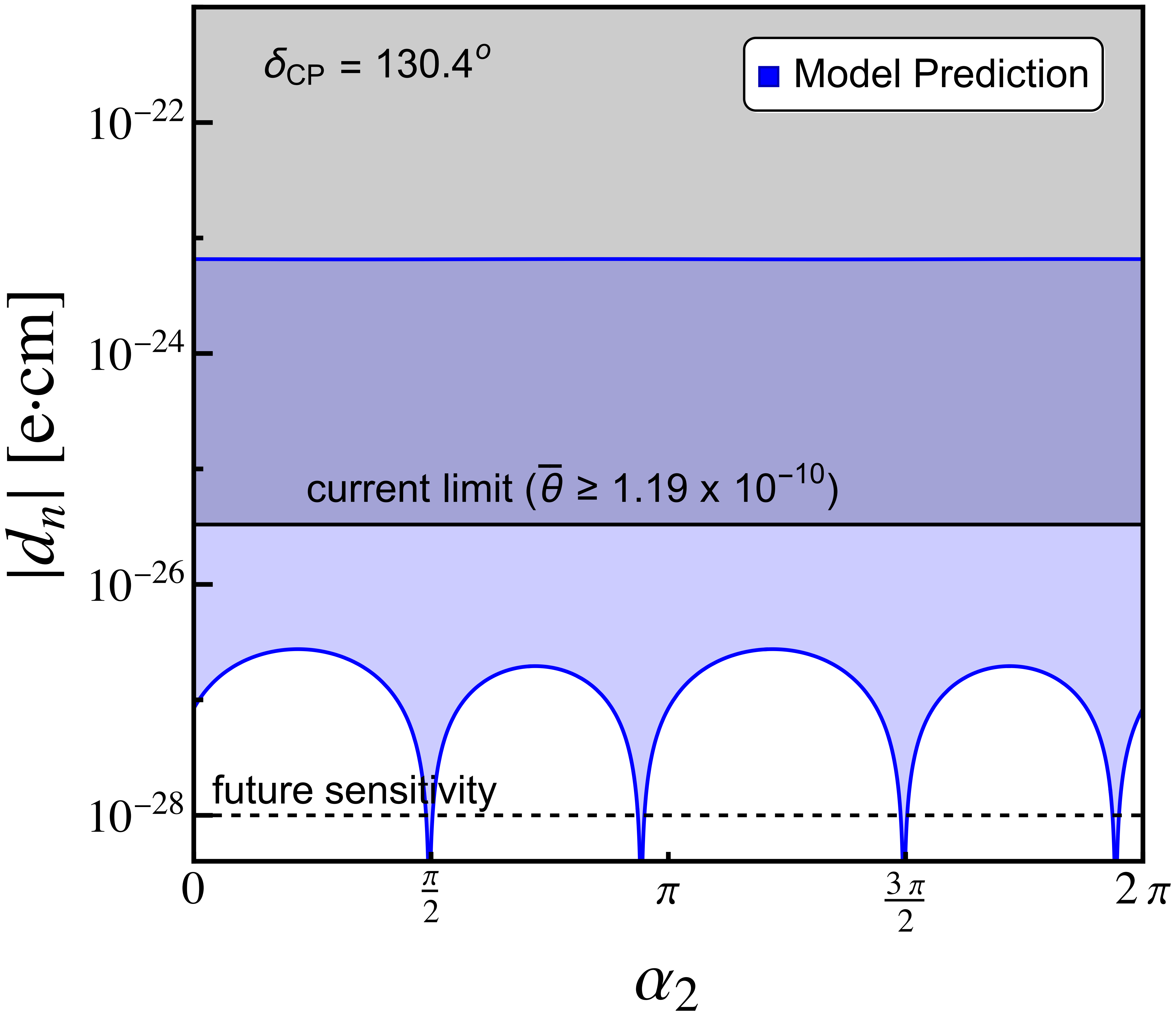
Backup Slides

Two loop contribution to $\bar{\theta}$



$$\bar{\theta} \approx \frac{8}{(4\pi)^4} \mathrm{Im\,Tr} \left[Q^* U_{\mathrm{PMNS}}^\dagger \hat{Y}_\ell^2 U_{\mathrm{PMNS}} Q^2 \hat{Y}_u U_{\mathrm{PMNS}}^T \hat{Y}_\ell^2 U_{\mathrm{PMNS}}^* (\hat{Y}_u)^{-1} \right] \ln \left(\frac{\mathrm{M}_{\mathrm{H_L^c}}}{\mathrm{M}_{\mathrm{H_R^c}}} \right)$$

All parameters are known expect for $Q = \mathrm{diag}\,.(e^{i\alpha_1}, e^{i\alpha_2}, 1)$ with $\alpha_1 = 0.128 + n\pi/2 \ (n = 0,1,2,..)$



Renormalization group evolution of $\bar{\theta}$

- There is the possibility that extrapolation of the Yukawa couplings by the RGE from the GUT scale to the weak scale could generate a nonzero $\bar{\theta}$

- The induced $\bar{\theta}$ via RGE from the up-quark sector read as

$$\delta(\bar{\theta}) = \mathrm{Im}\,\mathrm{Tr}\left[\frac{d}{dt}\left(Y_{uL}Y^{\dagger}_{uR}\right)\left(Y_{uL}Y^{\dagger}_{uL}\right)^{-1}\right]$$

$$\boxed{\begin{aligned}\beta^{(1)}\left(Y_{uL}\right) &= +\frac{3}{2}Y_{uL}Y^{\dagger}_{uL}Y_{uL} - \frac{3}{2}Y_{dL}Y^{\dagger}_{dL}Y_{uL} + 3\,\mathrm{Tr}\left(Y^{\dagger}_{uL}Y_{uL}\right)Y_{uL} + 3\,\mathrm{Tr}\left(Y^{\dagger}_{dL}Y_{dL}\right)Y_{uL} + \mathrm{Tr}\left(Y^{\dagger}_{lL}Y_{lL}\right)Y_{uL} - \frac{17}{20}g^2_{1L}Y_{uL} - \frac{9}{4}g^2_{2L}Y_{uL} - 8g^2_{3L}Y_{uL}\\ &\vdots\end{aligned}}$$

- $\frac{d}{dt}\left(Y_{uL}Y^{\dagger}_{uR}\right)$ is a hermitian matrix $\Longrightarrow$ does not generate $\bar{\theta}$ if the initial $\bar{\theta}$ is zero

Fermion mass fitting

- Redefine the down-type quarks (d, D) and the charged leptons (e, E) to go from the original basis to new basis such that $\hat{M}_\ell$ and $\hat{M}_u$ are diagonal

$d_L = V_R P^* d'_L, \quad d_R = V_R P^* d'_R, \quad D_L = Q U^T_{\rm PMNS} D'_L, \quad D_R = Q U^T_{\rm PMNS} D'_R e_L = Q^* U^\dagger_{\rm PMNS} e'_L, \quad e_R = Q^* U^\dagger_{\rm PMNS}$
$e'_R, \quad \nu_L = Q^* \nu'_L, \quad \nu_R = Q^* \nu'_R E_L = V^*_R P E'_L, \quad E_R = V^*_R P E'_R\,.$

$$M_u = \begin{pmatrix} 0 & \hat{M}_u \kappa_L \\ \hat{M}_u \frac{\kappa_R}{\kappa_L} & 0 \end{pmatrix}, \qquad M_\ell = \begin{pmatrix} 0 & \hat{M}_\ell \kappa_L \\ \hat{M}_\ell \frac{\kappa_R}{\kappa_L} & 0 \end{pmatrix}, \qquad M_d = \begin{pmatrix} 0 & \hat{M}_\ell \\ \hat{M}_\ell \frac{\kappa_R}{\kappa_L} & \frac{v_\phi}{v_\nu} U^*_{\rm PMNS} \hat{M}_\nu U^T_{\rm PMNS} \end{pmatrix}$$

$$\xi^\dagger_L M_d \xi_R = \text{diag}\,.\left(m_d, m_s, m_b, m_{D_1}, m_{D_2}, m_{D_3}\right) \text{ where } \xi_{L,R} = \begin{pmatrix} \xi^{11} & \xi^{12} \\ \xi^{21} & \xi^{22} \end{pmatrix}_{L,R}$$

- CKM matrix is given by $V_{\rm CKM} = P'^* V_R P^* \xi^{11}_L Q'^*$

unspecified unitary matrix V_R, thus $V_{\rm CKM}$ is unconstrained

$$\boxed{m_{D_1}(M_I) = 1.05 \times 10^7 \text{ GeV} \qquad m_{D_2}(M_I) = 1.62 \times 10^8 \text{ GeV} \qquad m_{D_1}(M_I) = 4.38 \times 10^9 \text{ GeV}}$$