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Chiral Property of Nucleon Interpolating Currents and θ-dependent Observables

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Outline

- Introduction
  - Nucleon interpolating currents interpolates the QCD description and the phenomenological description of the nucleon.
  - Physical observables coming from the QCD θ parameter can only appear in the form of $m_* \bar{\theta} = m_*(\theta_m + \theta_G)$.
- Chiral property of interpolating currents
  - The arbitrary combination $j^\beta = j_1 + \beta j_2$, $\beta \in \mathbb{R}$ leads to unphysical θ-dependence in the chiral limit.
  - In particular, the choice $\beta = 0$ used in lattice QCD calculations does not guarantee physical results.
  - The interpolating currents $j^\pm = j_1 \pm j_2$ has well-defined chiral property.
  - The procedure to extract physical θ-dependence for $\beta = \pm 1$ is discussed.
- EDM sum rules
  - The θ-induced neutron EDM is calculated within the QCD sum rule framework.

Interpolating currents

- A nucleon can simultaneously be described as quarks propagating in the QCD vacuum and as a phenomenological field with appropriate quantum numbers.
- Interpolating currents attempt to connect these two descriptions by constructing currents out of quarks that has the same quantum numbers as the nucleon.
- For the neutron two linearly independent interpolating currents can be found:

$$j_1(x) = 2\epsilon_{ijk}(d_i^T C\gamma^5 u_j)d_k, \quad j_2(x) = 2\epsilon_{ijk}(d_i^T C u_j)\gamma^5 d_k.$$

  In general one can consider linear combinations of them:

$$j^\beta(x) = j_1(x) + \beta j_2(x)$$

- The interpolating current is expected to have an overlap with the neutron state:

$$\langle\Omega|j(x)|n\rangle = \lambda_n u_n$$

- Allows one to use the knowledge about quarks to infer information about the nucleons.

$\longrightarrow$ Application in QCD sum rule and Lattice QCD.

QCD sum rule

- The product of the quarks can be written as an Operator Product Expansion, in the existence of quark condensate $\langle \bar{q}q \rangle$,

$$S^q_{ij}(x) = \langle \Omega | T \{ q^i(x), \bar{q}^j(0) \} | \Omega \rangle = \frac{i \not{x} \delta_{ij}}{2\pi^2 x^4} - \frac{m_q \delta_{ij}}{4\pi^2 x^2} - \frac{\delta_{ij}}{12} \langle \bar{q}q \rangle + \cdots$$

$\longrightarrow$ Allows calculating the correlator of interpolating currents

$$\Pi_{\text{OPE}}(p) = i \int d^4x e^{ipx} \left\langle \Omega \left| T \left\{ j^\beta(x), \bar{j}^\beta(0) \right\} \right| \Omega \right\rangle .$$

- On the other hand we have the phenomenological description of the neutron

$$\Pi_{\text{Pheno}}(p) = i \int d^4x e^{ipx} \langle \Omega | T \{ n(x), n(0) \} | \Omega \rangle .$$

- $\Pi_{\text{OPE}}(p)$ and $\Pi_{\text{Pheno}}(p)$ are compared to extract nucleon properties.
- Borel transform is performed before comparison to suppress excited states.

The θ angle

- The θ angle parameterizes CP violation in the strong sector:

$$\mathcal{L}_\theta = \theta \frac{\alpha_s}{8\pi} G^a_{\mu\nu} \tilde{G}^a_{\mu\nu}$$

- Under the $\mathrm{U}(1)_A$ rotation $q \to e^{i\theta_A \gamma^5} q$, θ can be distributed among θ_m and θ_G:

$$\mathcal{L}_\theta = -\theta_m m_* \bar{q} i \gamma^5 q + \theta_G \frac{\alpha_s}{8\pi} G^a_{\mu\nu} \tilde{G}^a_{\mu\nu}, \quad m_* = \frac{m_u m_d}{m_u + m_d}$$

  - Only the combination $\bar{\theta} = \theta_m + \theta_G$ is physical.
  - Physical dependence on θ vanishes at the chiral limit $m_{u,d} \to 0$

Does observables calculated with interpolating currents respect these properties?

Unphysical θ dependence in the chiral limit: neutron mass and neutron EDM

Taking $m_u = m_d \to 0$, in the θ_G + EM background,

$$S_q = e^{i\gamma\theta_G/4}\left(\frac{i\not{x}}{2\pi^2 x^4} - \frac{1}{12}\langle\bar{q}q\rangle - \frac{\tilde{\chi}_q}{24}F\cdot\sigma\right)e^{i\gamma\theta_G/4}$$

- Neutron mass
  - The OPE side gives

$$\Pi_n^\beta\Big|_{\mathbb{1},\gamma_5} = \frac{\langle\bar{q}q\rangle}{16\pi^2}p^2\log\left(-\frac{p^2}{\mu^2}\right)(1-\beta)\left[6(1+\beta)e^{i\theta_G\gamma_5/2} + (1-\beta)e^{-i\theta_G\gamma_5/2}\right],$$

  - Terms linear in θ_G can be absorbed as an overall phase, unphysical θ^2-dependence cannot be canceled
- Neutron EDM
  - The OPE side again contains the unphysical θ_G:

$$\Pi_n^\beta\Big|_{\{\not{p},\{\not{p},F\cdot\sigma/2\}\}} = -\frac{(1-\beta)^2\tilde{\chi}_u}{96\pi^2}\log\left(-\frac{p^2}{\mu^2}\right)$$

$$\Pi_n^\beta\Big|_{\{\not{p},\{\not{p},iF\cdot\sigma\gamma_5/2\}\}} = -\frac{\theta_G}{2}\frac{(1-\beta)^2\tilde{\chi}_u}{96\pi^2}\log\left(-\frac{p^2}{\mu^2}\right)$$

  - Lattice procedure (M. Abramczyk et al. 2017, T. Bhattacharya et al. 2021) suggests subtracting the phase of the two point function from the phase of the three point function to remove the overall phase, which gives

$$\Pi_n^\beta\Big|_{\{\not{p},\{\not{p},iF\cdot\sigma\gamma_5/2\}\}} + \alpha_n^\beta \times \Pi_n^\beta\Big|_{\{\not{p},\{\not{p},F\cdot\sigma/2\}\}} \propto \frac{(1-\beta)^2(1+\beta)}{7+5\beta}\theta_G$$

  - The unphysical θ-dependence does not go away with this procedure unless $\beta = \pm 1$

Chiral property of interpolating currents

What's the origin of the unphysical θ-dependence?

- The starting point for calculating the current correlators is the QCD Lagrangian + a source term η_a:

$$\mathcal{L}_\eta = \mathcal{L}_{\text{QCD}} + \bar{\eta} j^\beta + (\text{h.c.})$$

  - If j^β transforms covariantly under $\text{U}(1)_A$ rotation, we can reabsorb the chiral phase in the source η_a to keep the Lagrangian invariant.
  - If j^β does not transform covariantly under $\text{U}(1)_A$ rotation, $\text{U}(1)_A$ symmetry of the original theory will not be preserved.

- Under $\text{U}(1)_A$ rotation, j^β transforms as two separate pieces:

$$j^\beta \to \frac{1+\beta}{2} e^{3i\theta_A \gamma_5} j^+ + \frac{1-\beta}{2} e^{-i\theta_A \gamma_5} j^-$$

$\longrightarrow$ Only the choice $\beta = \pm 1$ guarantees the physical result.

The procedure for getting physical results for $\beta = \pm 1$

For $\beta = \pm 1$ two procedures based on chirality channels allows the extraction of physical results:

- Use the chirality conserving structure (with an odd number of γ_μ) in the correlator.
- Use the chirality flipping structure (with an even number of γ_μ) in the correlator, and subtract the chiral phase computed from the two-point function.

EDM sum rules

- For $\beta = +1$ we focus on the chirality conserving structure $\{\not{p}, iF\cdot\sigma\gamma^5\}$, we get

$$\mathcal{B}\left[\Pi_n^+\right]_{\{\not{p},iF\cdot\sigma\gamma_5\}} = \frac{4e_d - e_u}{16\pi^2} m_*\bar{\theta}\chi\langle\bar{q}q\rangle$$

- For $\beta = -1$ we focus on the chirality flipping structure $\{\not{p}, \{\not{p}, iF\cdot\sigma\gamma_5/2\}\}$, direct calculation gives

$$\mathcal{B}\left[\Pi_n^-\right]_{\mu} = \frac{\tilde{\chi}_u}{24\pi^2} + \frac{e_u m_u}{32\pi^4}\left[\log\left(\frac{M^2}{\mu_{\rm IR}^2}\right) - \frac{e_d}{e_u}\right]$$

$$\mathcal{B}\left[\Pi_n^-\right]_{\tilde{d}} = \frac{\tilde{\chi}_u}{24\pi^2}\frac{m_*}{m_u}\theta_G - \frac{e_u m_* \theta_m}{32\pi^4}\left[\log\left(\frac{M^2}{\mu_{\rm IR}^2}\right) - \frac{e_d}{e_u}\right]$$

Subtracting the phase of the two-point function $\alpha_n^- = -\frac{m_*}{m_u}\theta_G - \frac{m_* M^2}{4\pi^2\langle\bar{q}q\rangle}\bar{\theta}$ gives

$$\mathcal{B}\left[\Pi_n^-\right]_d \equiv \mathcal{B}\left[\Pi_n^-\right]_{\tilde{d}} + \alpha_n^- \times \mathcal{B}\left[\Pi_n^-\right]_{\mu} = -\left[\frac{\chi_u M^2}{96\pi^4} + \frac{e_u}{32\pi^4}\left(\log\left(\frac{M^2}{\mu_{\rm IR}^2}\right) - \frac{e_d}{e_u}\right)\right] m_*\bar{\theta}$$

- Both procedure leads to the physical combination $m_*\bar{\theta}$.
- Numerically, after normalizing on MDM,

$$d_n|_{\beta=+1} \simeq 2\times10^{-16}\, e\,{\rm cm}\times\bar{\theta}\times\left(\frac{|\chi|}{6\,{\rm GeV}^{-2}}\right), \quad d_n|_{\beta=-1} \sim (0.5-1.5)\times10^{-16}\, e\,{\rm cm}\times\bar{\theta}$$

Summary

- Chiral property of the interpolating currents requires using $\beta = \pm 1$ currents.
- The choice $\beta = 0$ is problematic in calculating θ-dependent observables.
- Procedures to get physical results are defined and checked with the θ-induced neutron EDM calculation.
- Numerically $d_n(\theta)$ consistently fall within the size of $\sim 10^{-16}\, e\,\mathrm{cm} \times \bar{\theta}$.

Thank you!