Phase Transition of **Co**nformal **F**reeze-**I**n Dark Matter

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A conformal dark sector

Consider a dark sector described by a CFT between energy scales, $\Lambda_{\rm UV}$ and $M_{\rm gap}$

containing a scalar operator $\mathcal{O}_{\rm CFT}$

- ➢ Relevant, with scaling dimension d < 4 (non-integers allowed)
- ➢ Not charged under SM gauge
- ➢ Charged under global $\mathbb{Z}_2$

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Dark sector populated through freeze-in via coupling to SM, given by

$$\mathcal{L}_{\mathrm{int}} = \frac{\lambda_{\mathrm{CFT}}}{\Lambda_{\mathrm{CFT}}^{D-4}} \mathcal{O}_{\mathrm{SM}} \mathcal{O}_{\mathrm{CFT}} \qquad D = d_{\mathrm{SM}} + d \qquad \lambda_{\mathrm{CFT}} \ll 1$$

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Explicitly breaks conformal symmetry and global $\mathbb{Z}_2$

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Confinement

Interaction term with relevant operator $\mathcal{O}_{\mathrm{CFT}} \rightarrow$ conformal symmetry broken in the IR

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In all cases of $\mathcal{O}_{\rm SM}$, this is the leading deformation.

Phenomenological Constraints:

Gap scale from Naive Dimensional Analysis (NDA):

$$M_{\rm gap} \sim c(\mu_{\rm UV})^{1/(4-d)} = (\frac{\lambda_{\rm CFT}}{\Lambda_{\rm CFT}^{d-2}} v^2)^{1/(4-d)}$$

in keV-MeV range, below which the dark sector spontaneously confines.

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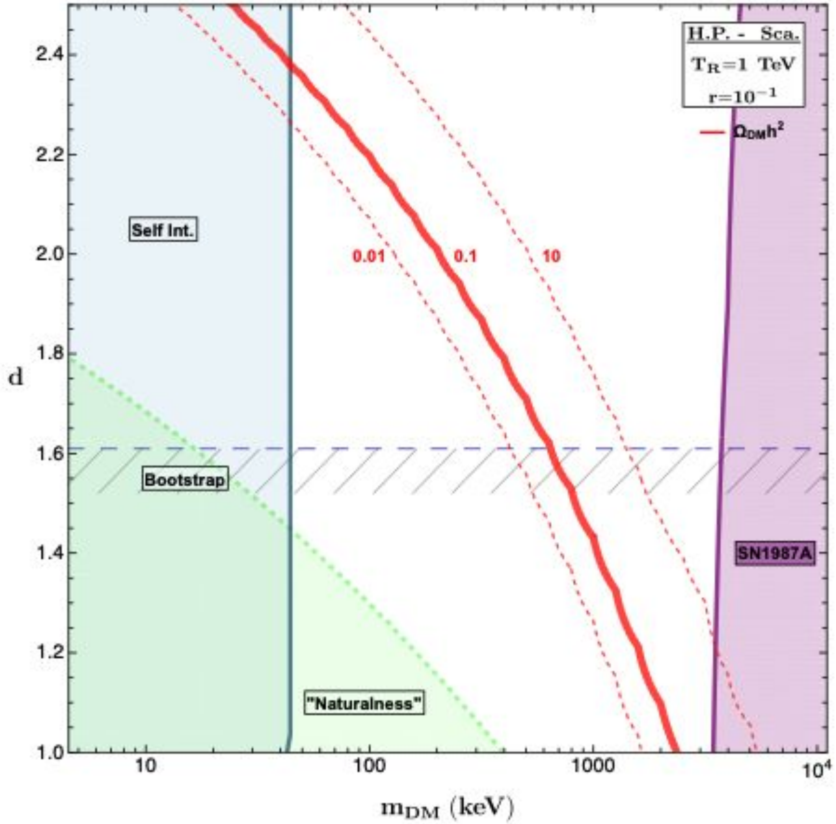

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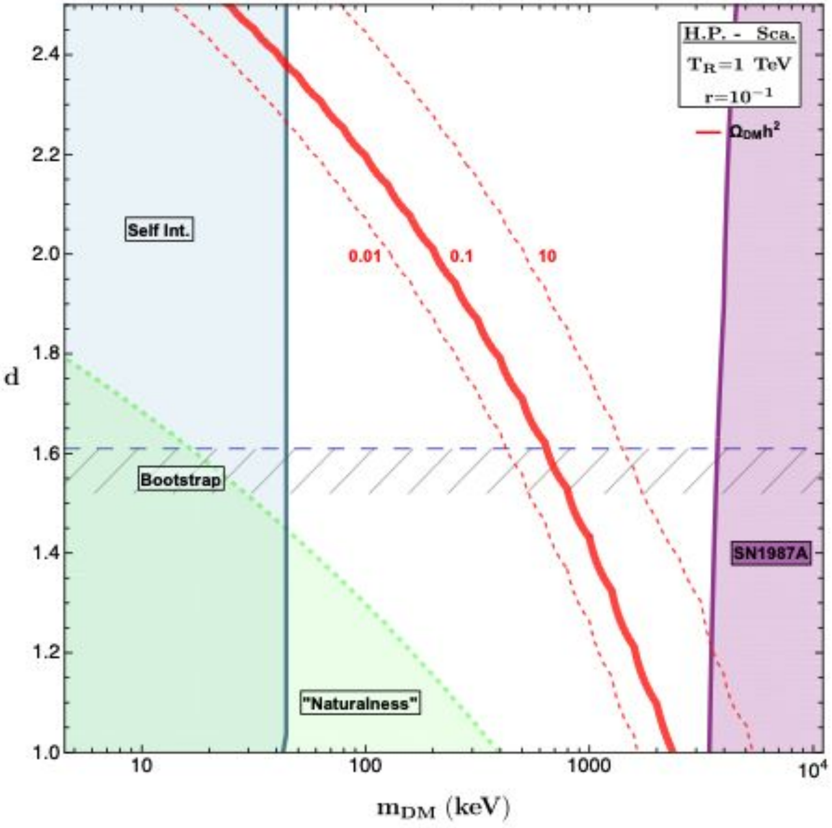

Holographic Dual in 5D

5D gravitational action in a slice of AdS space

$$S = -\int \mathrm{d}^4x\, \mathrm{d}y \sqrt{|g|}\left(2M_5^3 R + \Lambda\right) + \sqrt{|g_{\mathrm{ind}}|}\Lambda_{UV}\delta(y) + \sqrt{|g_{\mathrm{ind}}|}\Lambda_{IR}\delta(y - y_r)$$

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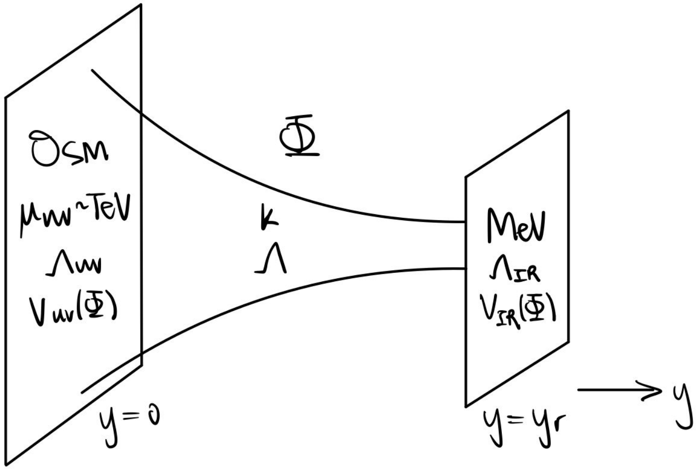
Assume small backreaction, we find the RS metric

$$\mathrm{d}s^2 = e^{-2ky}\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu - \mathrm{d}y^2$$

$$M_{\mathrm{Pl}}^2 \sim M_5^3/k$$

$$\Lambda = -24M_5^3k^2$$

$$\Lambda_{\mathrm{UV}} = -\Lambda_{\mathrm{IR}} = 24M_5^3k$$



$\mathcal{O}_{\text{CFT}} \leftrightarrow \Phi$

Bulk scalar dual to $\mathcal{O}_{\text{CFT}}$

$$S_\Phi = \int \mathrm{d}^4x\, \mathrm{d}y \sqrt{|g|}\left(\tfrac{1}{2} g^{MN} \partial_M \Phi \partial_N \Phi - \tfrac{1}{2} m^2 \Phi^2\right) - \sqrt{|g_{\text{ind}}|} V_{\text{UV}}(\Phi)\delta(y) - \sqrt{|g_{\text{ind}}|} V_{\text{IR}}(\Phi)\delta(y - y_c)$$

where brane localized potentials

$$V_{\text{UV}}(\Phi) = \tfrac{1}{2} m_{\text{UV}} \Phi^2 + \gamma k^{1/2} \mu_{\text{UV}}^2 \Phi \quad \text{and} \quad V_{\text{IR}}(\Phi) = \tfrac{1}{2} m_{\text{IR}} \Phi^2$$

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$\gamma \propto \mu_{\mathrm{UV}}^{d-4}c(\mu_{\mathrm{UV}})$ Small explicit breaking of $\mathbb{Z}_2$

C. Csáki, M. Geller, Z. Heller-Algazi, A. Ismail, 2301.10247

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Bulk mass is related to scaling dimension by $d = 2 + \sqrt{4 + m^2/k^2} \equiv 2 + \nu$

4D Effective Potential

Zero mode solution to the scalar EoM ~ radion

$$(\Box + m^2)\Phi = 0$$
$$2\Phi'(0) = m_{\rm UV}\Phi(0) + \gamma k^{1/2}\mu_{\rm UV}^2$$
$$-2\Phi'(y_r) = m_{\rm IR}\Phi(y_r)$$

Integrate out the 5th dimension and identify 4D dilaton field $\chi \equiv \mu_{\rm UV} e^{-ky_r}$, we find the effective potential and vev, written in a convenient form

$$V(\chi) = \frac{24M_5^3\lambda}{k^3}\langle\chi\rangle^4\left[(\chi/\langle\chi\rangle)^4 - 1 - \frac{(\chi/\langle\chi\rangle)^{2\nu} - 1}{\nu/2}\right]$$
$$\langle\chi\rangle \sim \mu_{\rm UV}\gamma^{1/(4-d)} \sim c(\mu_{\rm UV})^{1/(4-d)}$$

C. Csáki, M. Geller, Z. Heller-Algazi, A. Ismail, 2301.10247
P. Creminelli, A. Nicolis, R. Rattazzi, hep-th/0107141
K. Agashe, P. Du, M. Ekhterachian, S. Kumar, R. Sundrum, 2010.04083

Phase Transition

High temperature phase: black brane solution ≈ AdS-Schwartzchild, with free energy

$$F_{\text{deconf}} = V_0 - 2\pi^4 (M_5/k)^3 T^4$$

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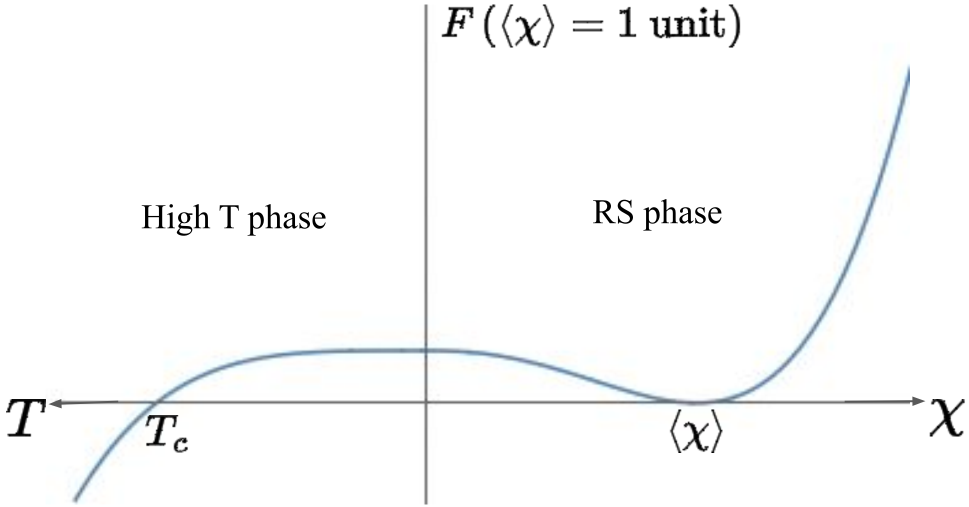

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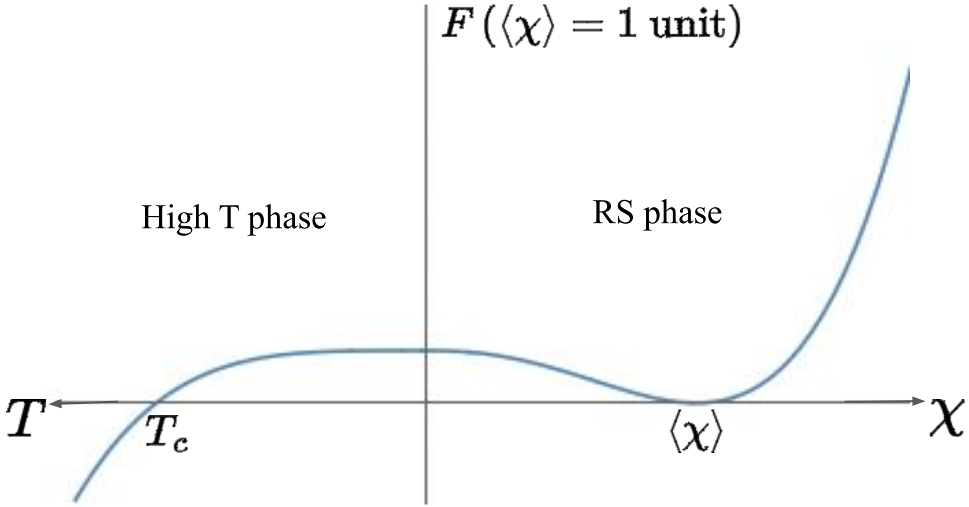
Critical temperature

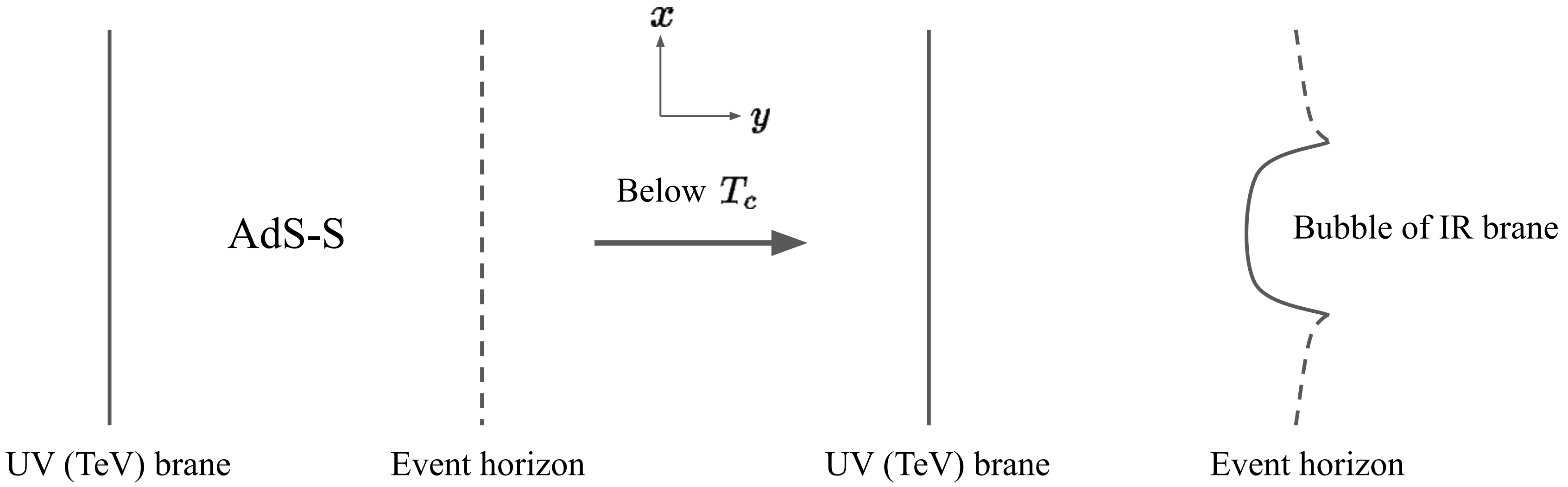
$$F_{\text{deconf}}(T_c) = V_\chi(\langle\chi\rangle)$$

$$\implies T_c = \frac{\langle\chi\rangle}{\pi}\left(12\lambda \frac{2-\nu}{\nu}\right)^{1/4}$$



Phase Transition

The confinement-deconfinement phase transition can be interpreted in 5D as bubbles of IR brane nucleating from the event horizon.



Phase Transition

Phase transition completes when

$$T^4 e^{S_b} \sim \Gamma > H^4 \sim \left(\frac{2\pi^4 M_5^3 T_c^2}{3k^3 M_{\rm Pl}}\right)^2$$

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For $T_c \sim M_{\rm gap} \sim MeV$,

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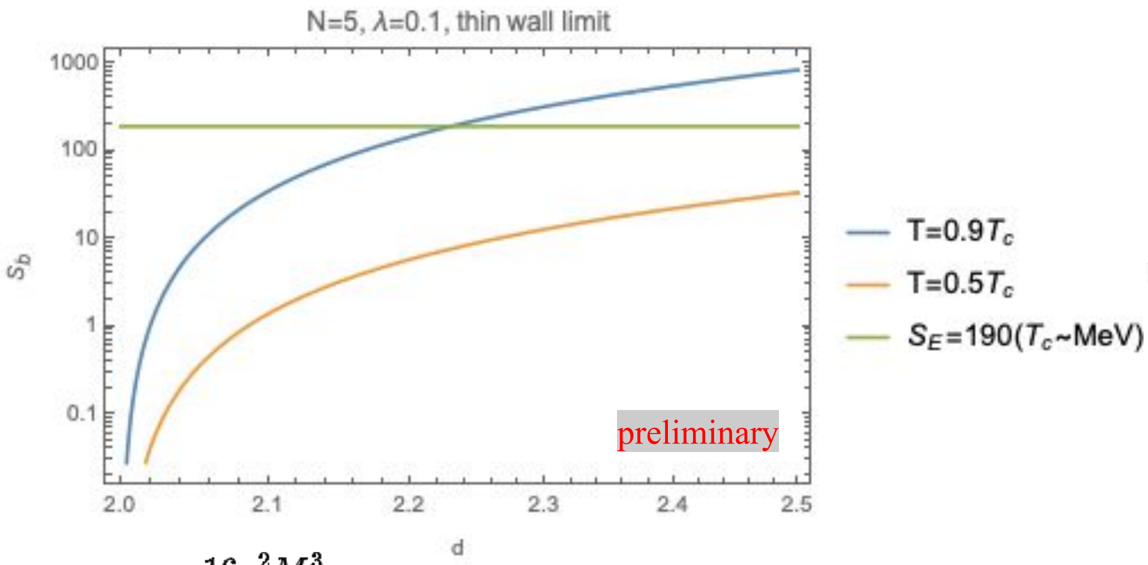
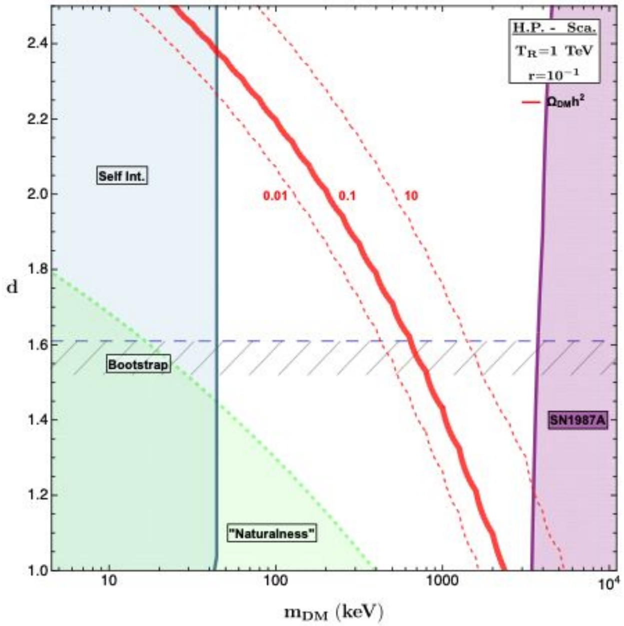
We consider a regime without significant supercooling, i.e. $T_n \lesssim T_c$. Estimate S_b in the thin-wall limit using the $O(3)-$ symmetric bounce.

Other notable constraints: validity of dilaton EFT, small backreaction, zero mode stability.

Phase Transition



$$N^2 \equiv \frac{16\pi^2 M_5^3}{k^3}$$



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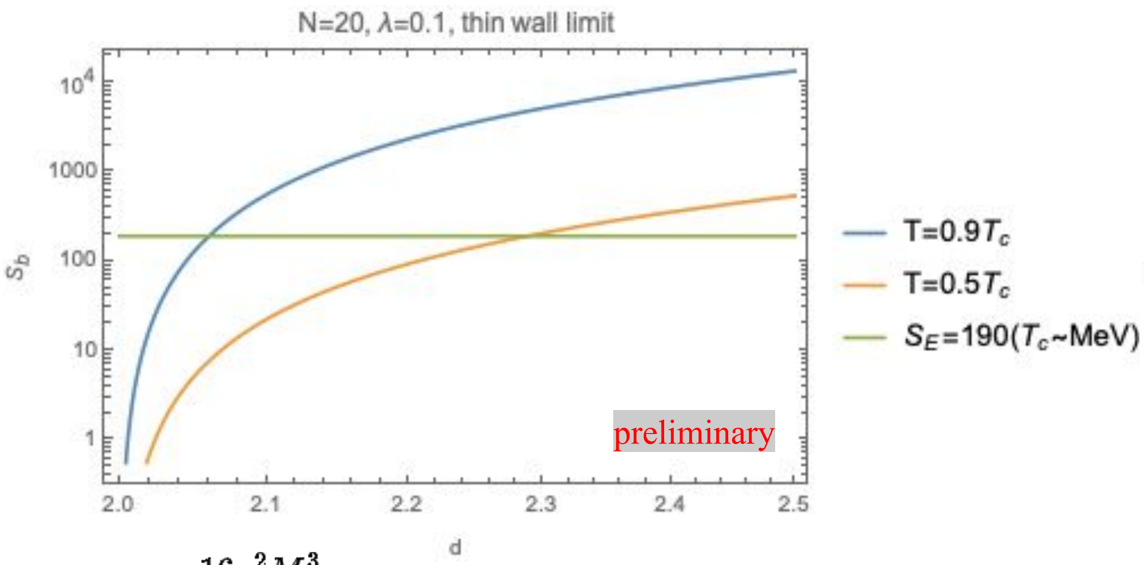

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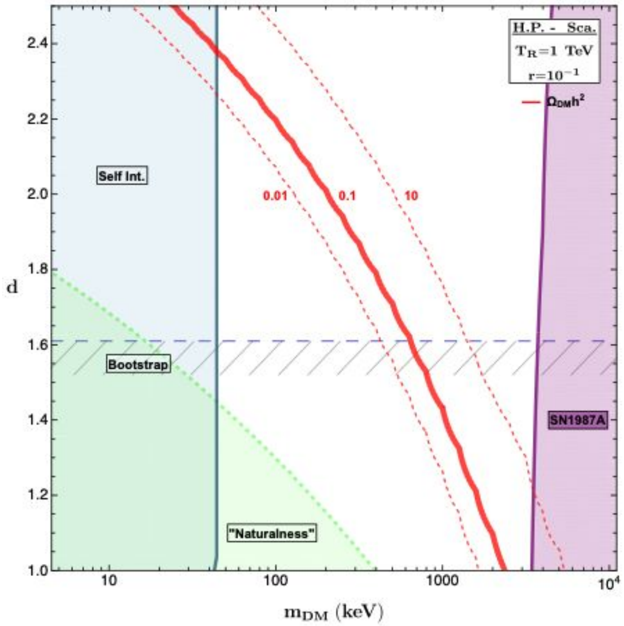

Summary

- A conformal dark sector deformed by a relevant operator $\mathcal{O}_{\mathrm{CFT}}$, resulting in broken conformal symmetry in the IR at $M_{\mathrm{gap}} \sim MeV$
- (d > 2) Dual to a 5D bulk scalar field which stabilizes RS geometry - *Relevant Dilaton Stabilization*;
- Phase transition completion does NOT significantly limit the model parameter space;

Ongoing work

- A dual for d < 2 can be obtained by fine tuning the UV mass;
- Gravitational wave with significant supercooling?
- A UV theory featuring a 3-brane model.

Backup

C. Csáki, M. Geller, Z. Heller-Algazi, A. Ismail, 2301.10247

Confinement as SSB by Dilaton Potential

We can find the dilaton effective potential by restoring scaling invariance and $\mathbb{Z}_2$ symmetry and perform spurion analysis

$$V_{\text{eff}}(\chi) = \lambda\chi^4 - \lambda_2 \mu_{UV}^{2(4-d)} \chi^{2d-4}\,, \quad \lambda_2 \propto (\mu_{\text{UV}}^{d-4} \underline{c(\mu_{\text{UV}})})^2$$

$$[c(\mu_{\text{UV}})] = 4-d$$

This potential has a minimum at

$$\langle\chi\rangle = \mu_{\text{UV}}\left(\frac{(d-2)\lambda_2}{2\lambda}\right)^{1/2(4-d)} = M_{\text{gap}} \sim c(\mu_{\text{UV}})^{1/(4-d)} = \left(\frac{\lambda_{\text{CFT}}}{\Lambda_{\text{CFT}}^{d-2}} v^2\right)^{1/(4-d)}$$

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Dilaton action

$$S_\chi = \int \mathrm{d}^4x \boxed{\frac{12M_5^3}{k^3}}(\partial\chi)^2 - V(\chi)$$

$$\propto N^2$$

Operator scaling dimension: d < 2 case

The relation between the scalar bulk mass **m** and CFT operator scaling dimension **d** is

$$d(d-4) = m^2 \quad (k=1)$$

Quadratic equation! $\rightarrow d_\pm = 2 \pm \nu\,, \quad \nu = \sqrt{4+m^2}$

Closer look at how to find the scaling dimension of an operator:

$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle \sim \frac{1}{|x-y|^{2d}} \implies [\mathcal{O}] = d$$

T. Hartman, L. Rastelli, hep-th/0602106
P. Minces, V. O. Rivelles, hep-th/9907079
D. B. Kaplan, J. Lee, and D. T. Son, 0905.4752

Operator scaling dimension: d < 2 case

In terms of 4 momentum, 2-pt function of our model in IR brane $\to \infty$ limit:

$$\langle \mathcal{O}(p)\mathcal{O}(p')\rangle \sim \frac{\delta(p+p')}{(m_{\mathrm{UV}}/2 - d_-) - c\,p^{2\nu}}$$

d_+ $\qquad\qquad$ d_-

Stable fixed point $\qquad\qquad$ Fine tune UV mass, going against RG flow

$$m_{\mathrm{UV}} - 2d_- = 0$$

2 boundary theories, one for each scaling dimension. To access d_- scaling, UV mass term needs to be fine tuned.