Chiral Nelson–Barr Models: Quality and Cosmology

Phys. Rev. D 107 (2023), 11 [arXiv:2212.03882]

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DPF-Pheno, PITT/CMU, May 15, 2024

The Strong CP Problem

We know CP is violated in the quark sector of the Standard Model (e.g. in the Kaon system) — parameterized by $\delta_{\mathrm{CKM}} \sim 65^\circ$.

There is another CP violating parameter in the SM — the θ-term:

$$\frac{\theta}{16\pi^2} \int \mathrm{d}^4x \, \mathrm{tr} \, G_{\mu\nu} \widetilde{G}^{\mu\nu}$$

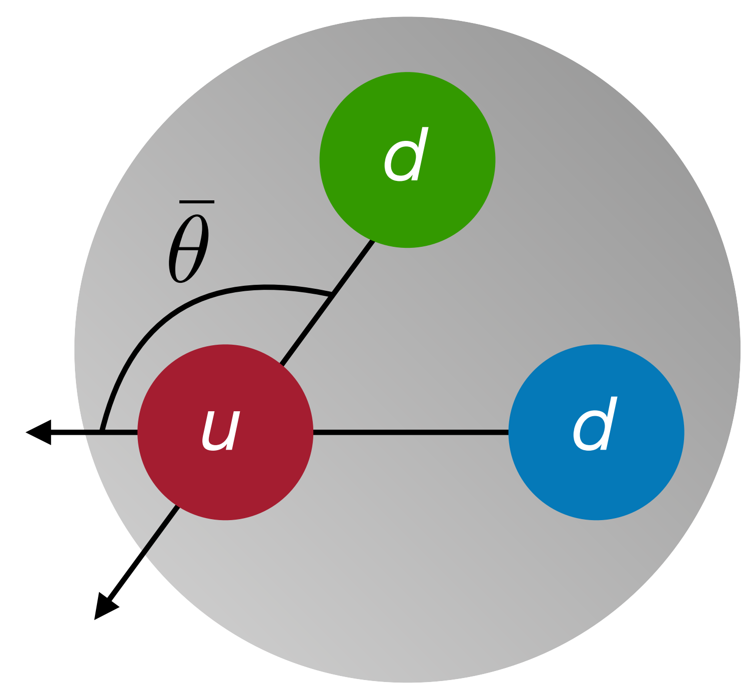
Due to the chiral anomaly, the *invariant* angle is:

$$\bar{\theta} \equiv \theta + \arg\det\left(m_u \, m_d\right)$$

This leads to an electric dipole moment for the neutron. Experimentally:

$$\bar{\theta} \lesssim 10^{-10}$$

Begging for a dynamical or symmetry-based explanation!



Is CP a Spontaneously Broken Symmetry?

Some hints in the SM:

- CP Violation only observed in flavor-changing processes.
- Running of $\bar{\theta}$ arises only at seven loops in the SM.
  finite contributions appear at four loops, (Khriplovich, 1986)

Moreover, there are arguments that (3+1)-dimensional CP can arise from the spacetime symmetries of superstring theory (Strominger-Witten, 1985)
(c.f. Dine, Leigh, MacIntire, 1992 and Choi, Kaplan, Nelson, 1993)

Formally, we say a theory has an *exact* parity symmetry if it can be defined on *non-orientable* manifolds.
(CP is simply parity combined with an internal symmetry)

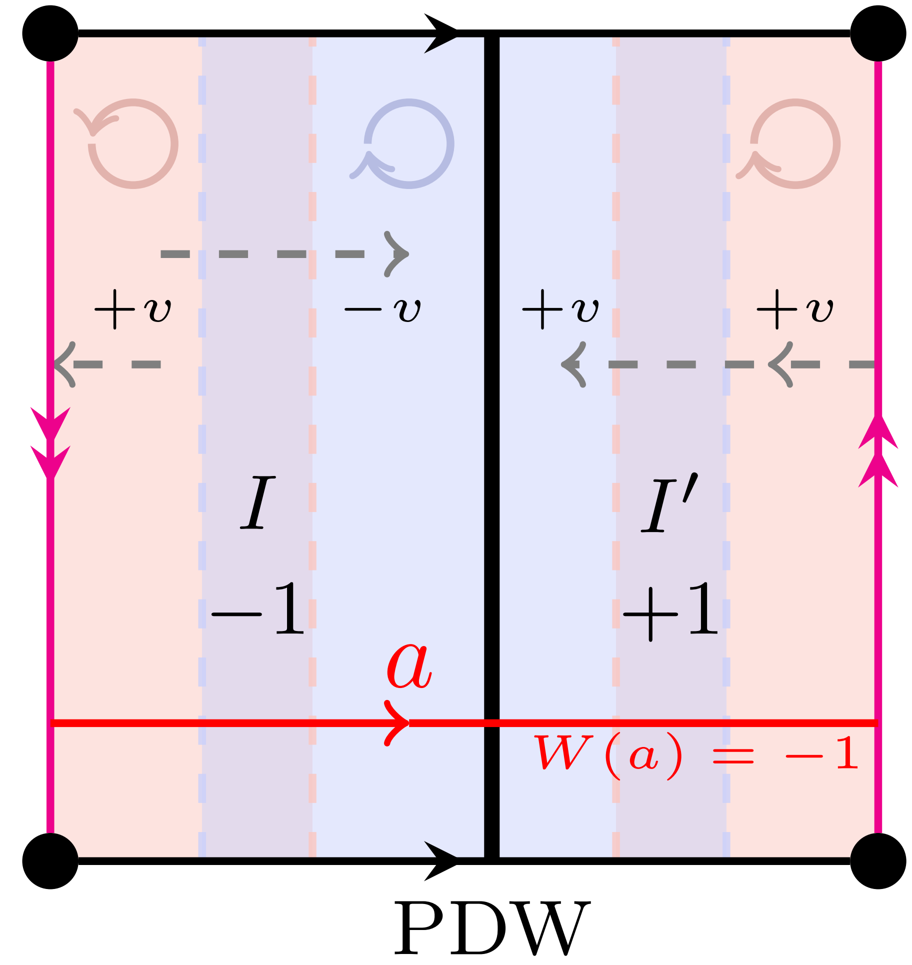
This careful definition allows us to derive important phenomenological consequences…

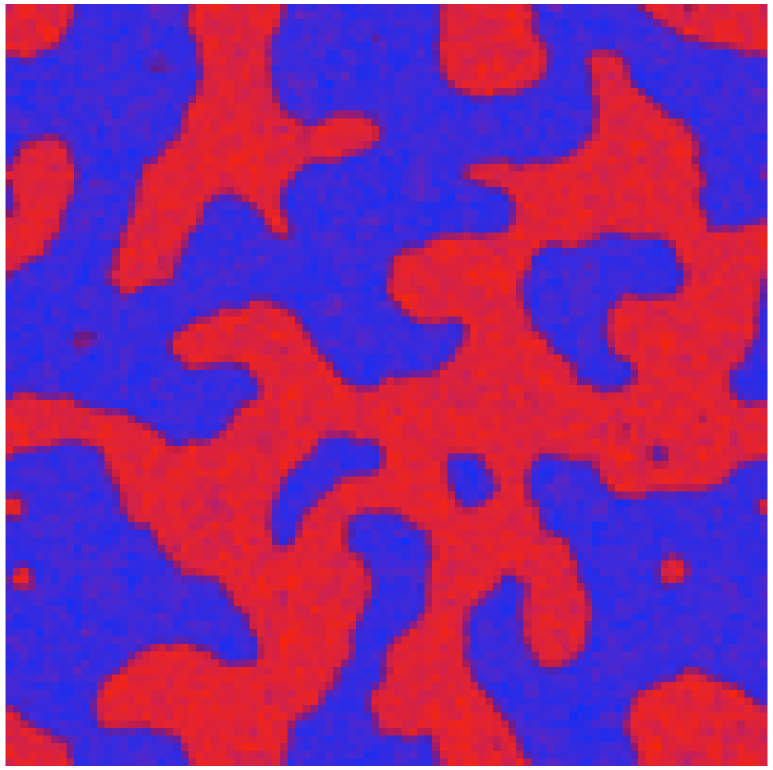
CP Domain Walls

J. McNamara and M. Reece, [2212.00039]

As with any Z_2 symmetry, we can think about *Parity (or CP) Domain Walls*, which interpolate between the distinct vacua.

These form in Cosmology via the Kibble–Zurek mechanism at scale of CP breaking.





[1010.2328]

In contrast to ordinary discrete symmetries these domain walls *cannot* be dynamically destroyed.

(Key fact: vortices with "parity winding" do not exist, as all closed 1-manifolds are orientable)

$\Longrightarrow$ To avoid DWs dominating the energy density, they must be *inflated away*.

The Nelson–Barr Mechanism

A. Nelson (*PLB* 136, 1984), S. Barr (*PRL* 53, 1984)

Challenge: spontaneously break CP, generating an $\mathcal{O}(1)$ CKM phase (not suppressed by powers of v/Λ_{CP}) without large corrections to $\bar{\theta}$.

Best illustrated via the minimal model by Bento, Branco and Parada: Introduce a set of vector-like quarks D, $\bar{D}$ transforming like $\bar{d}$, along with N (pseudo)scalars, η_a.

Impose CP and a Z_N symmetry which acts as:

$$\eta_a \to \mathrm{e}^{2\pi i k/N}\eta_a, \quad D \to \mathrm{e}^{-2\pi i k/N} D, \quad \bar{D} \to \mathrm{e}^{2\pi i k/N}\bar{D}$$

The allowed down-type mass terms are:

$$\mathcal{L} \supset \mu_D D\bar{D} + (\lambda_d)^i{}_j Q_i H^c \bar{d}^j - f_i^a \eta_a D \bar{d}^i + \mathrm{h.\,c.}$$

$\mu_D,\ f,\ \lambda_d \in \mathbb{R}$

Note the $Q_i H^c \bar{D}$ term is forbidden by the Z_N.

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The resulting mass matrix is:

$$\mathcal{L} \supset \begin{pmatrix} Q & D \end{pmatrix} \overbrace{\begin{pmatrix} \lambda_d v/\sqrt{2} & 0 \\ \underbrace{\sum_a f_i^a \langle \eta_a \rangle}_{\equiv F_i} & \mu_D \end{pmatrix}}^{\mathcal{M}_0} \begin{pmatrix} \bar{d} \\ \bar{D} \end{pmatrix}$$

Vanishes by CP symmetry

$$\Longrightarrow \bar{\theta} = \cancel{\theta_0} + \arg\det \mathcal{M}_0 = 0$$

The effective mixing matrix for the down-type quarks, however, is:

$$(m_0^2)^i_j = (m_d)^i_k \underbrace{\left(\delta^k_l + \frac{F^{\dagger\,k} F_l}{F_p F^{\dagger\,p} + \mu_D^2}\right)}_{\text{An } \mathcal{O}(1) \text{ complex phase!}} (m_d^T)^l_j$$

The Nelson–Barr "Quality Problem"

The Nelson–Barr mechanism ensures that $\Delta\bar{\theta} \equiv 0$ at the *renormalizable* level, but irrelevant operators can spoil this solution.
(Similar in spirit to the Axion Quality Problem)

In our minimal model, we have already at dimension-5:

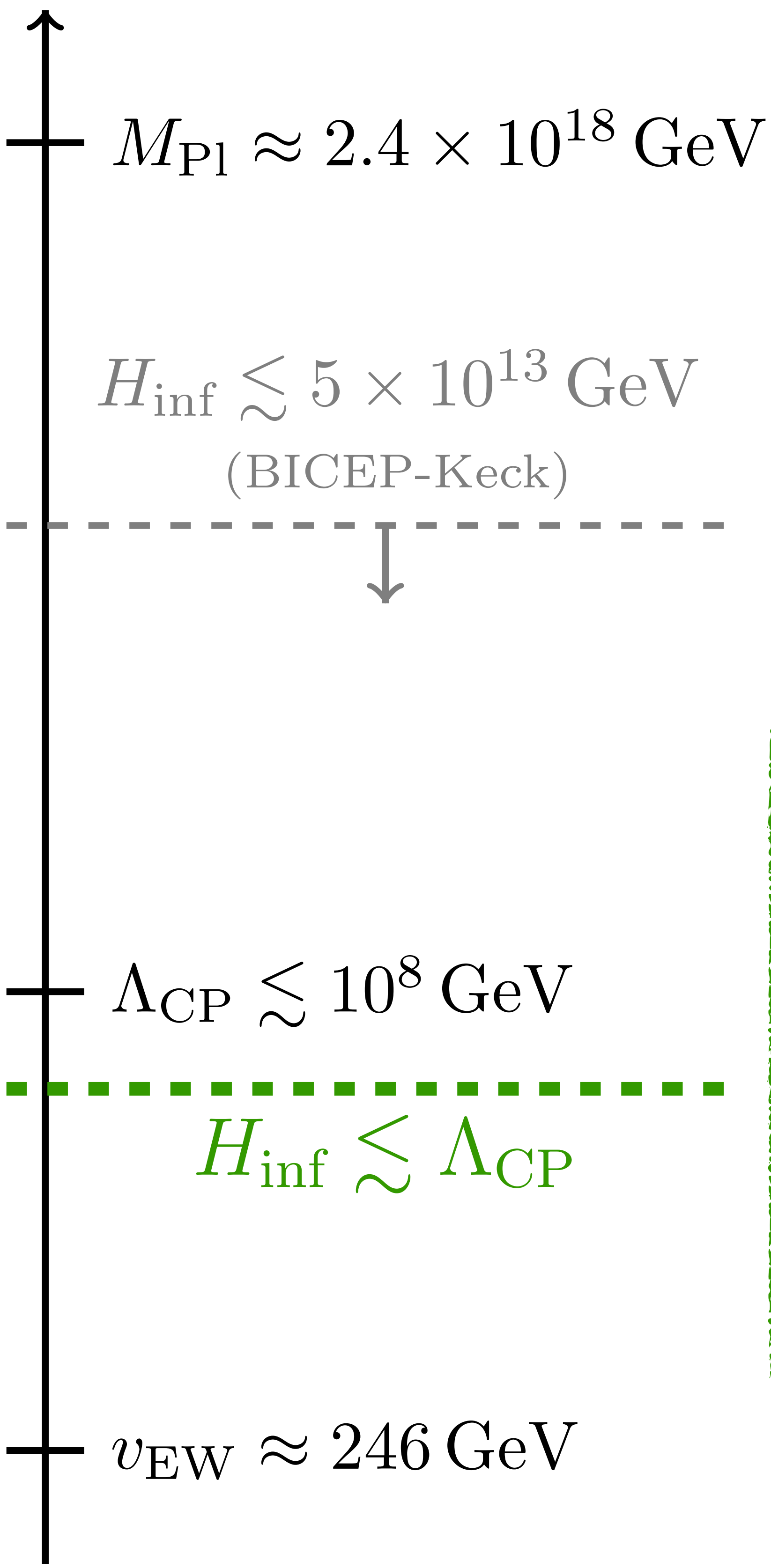
$$\frac{1}{\Lambda_{\mathrm{EFT}}}\eta_a^\dagger \eta_b D\bar{D}, \qquad \frac{1}{\Lambda_{\mathrm{EFT}}}\eta_a^\dagger Q_i H^c \bar{D}$$

$$\begin{pmatrix} Q & D \end{pmatrix} \begin{pmatrix} \lambda_d v/\sqrt{2} & 0 \\ \sum_a f_i^a \langle \eta_a \rangle & \mu_D \end{pmatrix} \begin{pmatrix} \bar{d} \\ \bar{D} \end{pmatrix}$$

These contribute $\Delta\bar{\theta} \sim \Lambda_{\mathrm{CP}}/\Lambda_{\mathrm{EFT}}$ — this sets an ***upper bound*** on the scale of spontaneous CP breaking!

$$\Lambda_{\mathrm{EFT}} = M_{\mathrm{Pl}} \implies \Lambda_{\mathrm{CP}} \lesssim 10^8\,\mathrm{GeV}$$

Late-Time Spontaneous CP Breaking?



To avoid Domain Wall problem, Inflation must occur ***after*** spontaneous CP breaking:

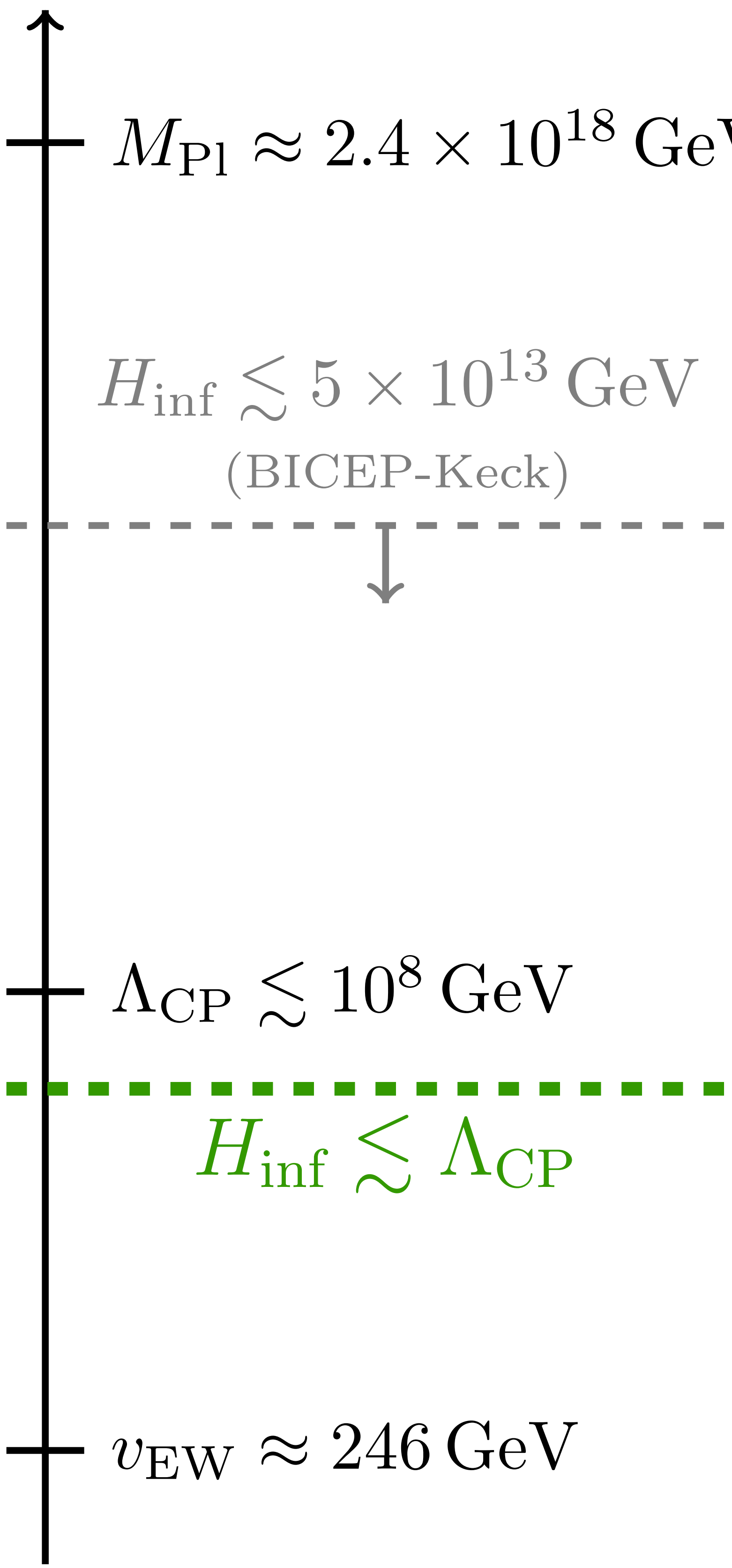
$$H_{\rm inf},\ T_{\rm reh}\ <\ \Lambda_{\rm CP} \lesssim 10^8\ {\rm GeV}$$

This constrains a lot of potential dynamics and signatures in the early Universe!

Inflation (assuming single-field):

- The tensor-to scalar ratio is bounded by $H_{\rm inf}$ to be $r \lesssim 2 \times 10^{-13}$ (current bound is 0.036)
- The slow-roll parameter ϵ is likewise constrained to be $r/16 \lesssim 10^{-14}$ — the potential must be *extremely* flat (fine-tuning problem)

Late-Time Spontaneous CP Breaking?



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Baryogenesis:

- Many models involve asymmetric decays of a heavy state that thermalized with the SM bath — mass must be *lower* than $T_{\rm reh}$
- Concretely, in thermal leptogenesis, this is the "Davidson–Ibarra bound":

$$\varepsilon \lesssim \frac{3}{8\pi}\frac{M_N m_\nu}{v^2} \implies T_{\rm reh} \gtrsim 10^{8-10}\,{\rm GeV}$$

Chiral Nelson–Barr Models

One way of ameliorating this tension is to introduce a new symmetry that forbids the dimension-5 operators. This works if D, $\bar{D}$ transform *chirally* under a new $\mathrm{U}(1)_X$:

$$D \to \mathrm{e}^{-i\alpha_X} D, \qquad \bar{D} \to \mathrm{e}^{-5i\alpha_X} \bar{D}$$

We must also introduce a new $\mathrm{U}(1)_X$-charged scalar ρ, whose vev plays the role of the vector-like mass, μ_D:

$$\mathcal{L} \supset -y_D \rho\, D\bar{D} \qquad \mu_D = y_D \langle\rho\rangle$$

The rest of the Nelson–Barr mechanism is entirely unchanged, and the $\mathrm{U}(1)_X$ can replace the Z_N.

Chiral Nelson–Barr Models

All the mixed anomalies involving $\mathrm{U}(1)_X$ can be eliminated by introducing a single extra set of chiral fermions, B, $\bar{B}$:

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)_X$
D	$\mathbf{3}$	–	$-1/3$	-1
$\bar{D}$	$\bar{\mathbf{3}}$	–	$+1/3$	-5
B	$\mathbf{3}$	–	$+1/3$	$+1$
$\bar{B}$	$\bar{\mathbf{3}}$	–	$-1/3$	$+5$
ρ	–	–	0	$+6$
η_a	–	–	0	$+2$

Vector-like under the SM, and “pairwise” vector-like under $\mathrm{U}(1)_X$

This works because we take X to be a linear combination of hypercharge and SM Baryon number: $X = -4Y + (B - L)$. This naturally extends the charge assignments to the rest of the SM.

Chiral Nelson–Barr Models

With these assignments, there are no additional renormalizable interactions other than:

$$\mathcal{L} \supset -y_D \rho D\bar{D} - y_B \rho^\dagger B\bar{B} + \mathrm{h.\,c.}$$

All possible dimension-5 operators are forbidden, but the quality problem arises again at dimension-6:

$$\eta_a^\dagger \eta_b \rho\, D\bar{D}, \qquad \eta_a^\dagger \rho\, Q_i H^c \bar{D}, \qquad \eta_a \eta_b \eta_c^\dagger D\bar{d}_j$$

A rough estimate is that these operators contribute:

$$\Delta\bar{\theta} \simeq \frac{1}{y_D} \frac{\Lambda_{\mathrm{CP}}^2}{\Lambda_{\mathrm{EFT}}^2} \implies \Lambda_{\mathrm{CP}} \lesssim 10^{13}\ \mathrm{GeV}$$

High enough to recover most of the Cosmology we're interested in!

Summary and Conclusions

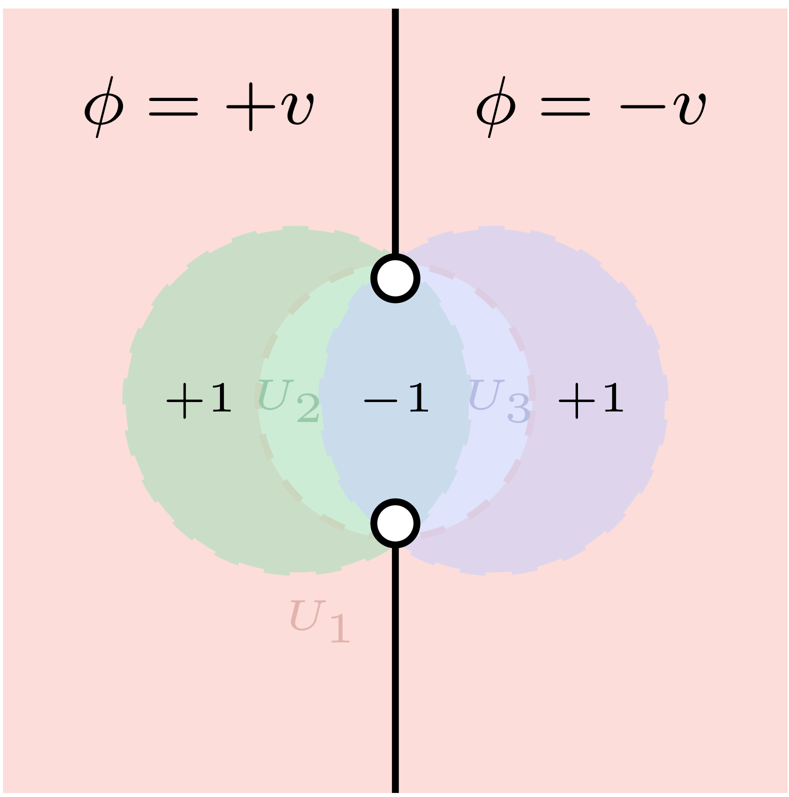
- An exact, spontaneously broken CP symmetry is an interesting alternative to axion solutions of the strong CP problem.
- Spontaneous CP breaking leads to exactly stable domain walls, which puts important constraints on our Cosmological history, especially when combined with the “quality” problem in Nelson–Barr models.
- Additional chiral symmetries can relax the quality problem and allow for high-scale inflation.
- Lots of open model-building questions!
    - Are models of baryogenesis compatible with Spontaneous CP breaking?
    - What about complete models of flavor?
    - Are there other ways of avoiding constraints from the CP domain walls? (Sequestered breaking?)

Backup

Details on Topological Stability of CP Domain Walls

J. McNamara and M. Reece, [arXiv:2212.00039]

Treating parity as a gauge symmetry, the choice of background field is *fixed* by the choice of manifold. If parity is spontaneously broken by a pseudoscalar field, Domain Walls arise as a *topological requirement:*



Question of stability boils down to whether there exist *vortex* solutions, which insert a twist (gauge transformation), and allow the domain walls to end.

This amounts to a codimension-2 boundary condition in the (semiclassical) gravitational path integral, but requires a “parity Wilson line” with $W_P(C) = -1$.

All closed 1-manifolds are orientable, so no such boundary condition exists.

Loop Corrections in the BBP Model

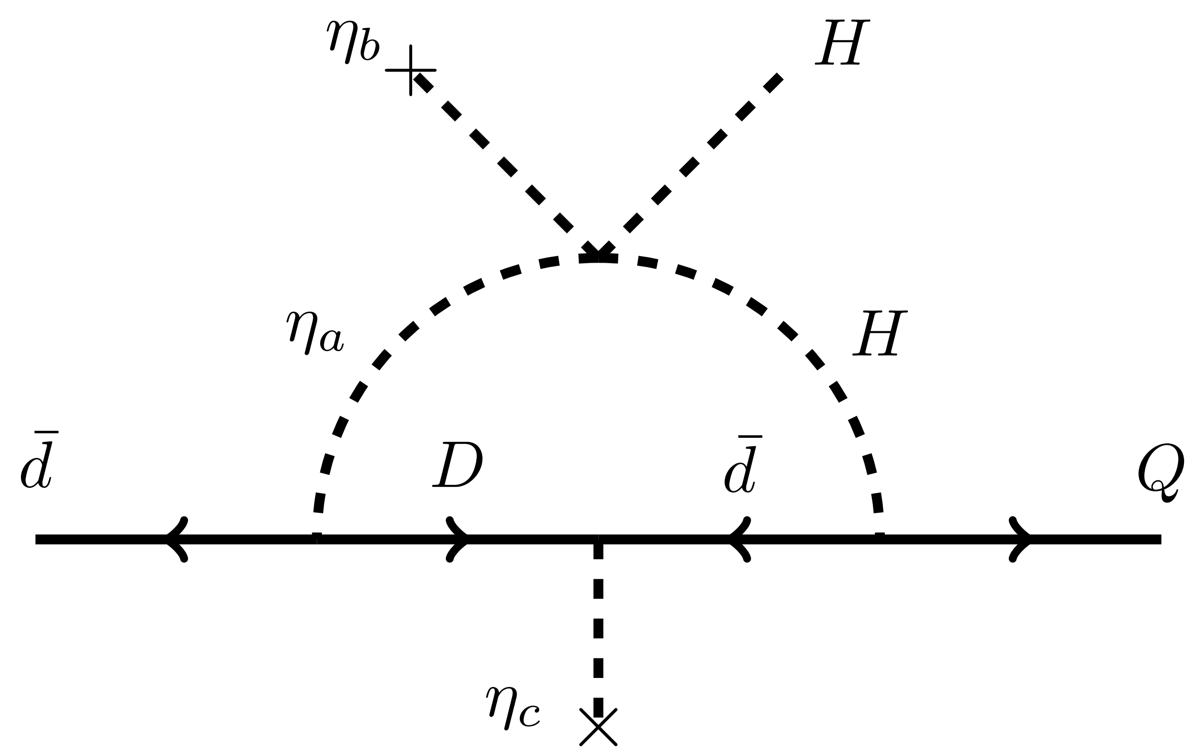
Warning: this minimal example requires some fine-tuning:

$$\mathcal{L} \supset \underbrace{\lambda_{ab}\eta_a\eta_b^\dagger H^\dagger H}_{} + \gamma_{abcd}\eta_a\eta_b^\dagger\eta_c\eta_d^\dagger$$

Correction to the Higgs mass $\propto \Lambda_{\rm CP}^2$

Also corrections to $\bar{\theta}$ at one-loop:

$$\Delta\bar{\theta} \sim \frac{1}{8\pi^2} f_k f_k \lambda \frac{\Lambda_{\rm CP}^2}{m_\eta^2} \log\left(\frac{v^2}{\Lambda_{\rm CP}^2}\right)$$



These could be ameliorated e.g., with supersymmetry, (see Dine, Draper [1506.05433])

We assume $\lambda \lesssim 10^{-8}$ to ignore these loop corrections. This is radiatively safe for $f \sim 0.1$.

Flavor Ansätze and the Quality Problem

A more careful analysis of the dimension-5 contributions to $\Delta\bar{\theta}$ reveals a dependence on the flavor-structure of the new couplings to D, $\bar{D}$. E.g.,

$$\mathcal{L} \supset \frac{h_a^i}{\Lambda_{\mathrm{EFT}}} \eta_a^\dagger Q_i H^c \bar{D} \qquad \Longrightarrow \qquad \Delta\bar{\theta} \sim \frac{\langle \eta_a \rangle \langle \eta_b \rangle}{\mu_D \Lambda_{\mathrm{EFT}}} \left(f_i^b (\lambda_d^{-1})^i{}_j h_a^j \right)$$

Treating the couplings f_i, h^j as spurions under the SM flavor group, $\mathrm{SU}(3)_Q \times \mathrm{SU}(3)_u \times \mathrm{SU}(3)_d$, we see that $f_i^b h_a^j \sim (\bar{\mathbf{3}}_Q, \mathbf{3}_d) \sim (\lambda_d)_i{}^j$

This lets us consider simple flavor Ansätze:

- "Minimal Flavor Violation": $f_i^b (\lambda_d^{-1})^i{}_j h_a^j \sim 1$, recover $\Lambda_{\mathrm{CP}} \lesssim 10^8$ GeV bound
- New flavor-aligned spurion, $f_i^b (\lambda_d^{-1})^i{}_j h_a^j \lesssim 1/y_d \approx 10^5$, requires $\Lambda_{\mathrm{CP}} \sim 1 - 10$ TeV?

Sequestered CP Breaking?

Another possibility: break CP first in a sequestered hidden sector ($\Lambda_{\mathrm{CP}} \lesssim 10^8$ GeV constraint doesn't apply), then inflate away the stable domain walls

Subsequently, small interactions with the visible sector make *effective* explicit CPV

$\Longrightarrow$ visible sector CP breaking gives *unstable* domain walls

Challenges:

- Keep explicit $\bar{\theta}$ term small enough?
- Short enough lifetime for DWs?
- Gravitational Wave signatures?