# Multiphonon Processes in Spin-Dependent Dark-Matter Scattering

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# Outline

1. Why phonons + DM

2. Derivation of Multiphonon scattering

3. Results!

# Outline

# The Dark Matter Landscape



Nuclear recoils
(LZ, XENON1T, …)

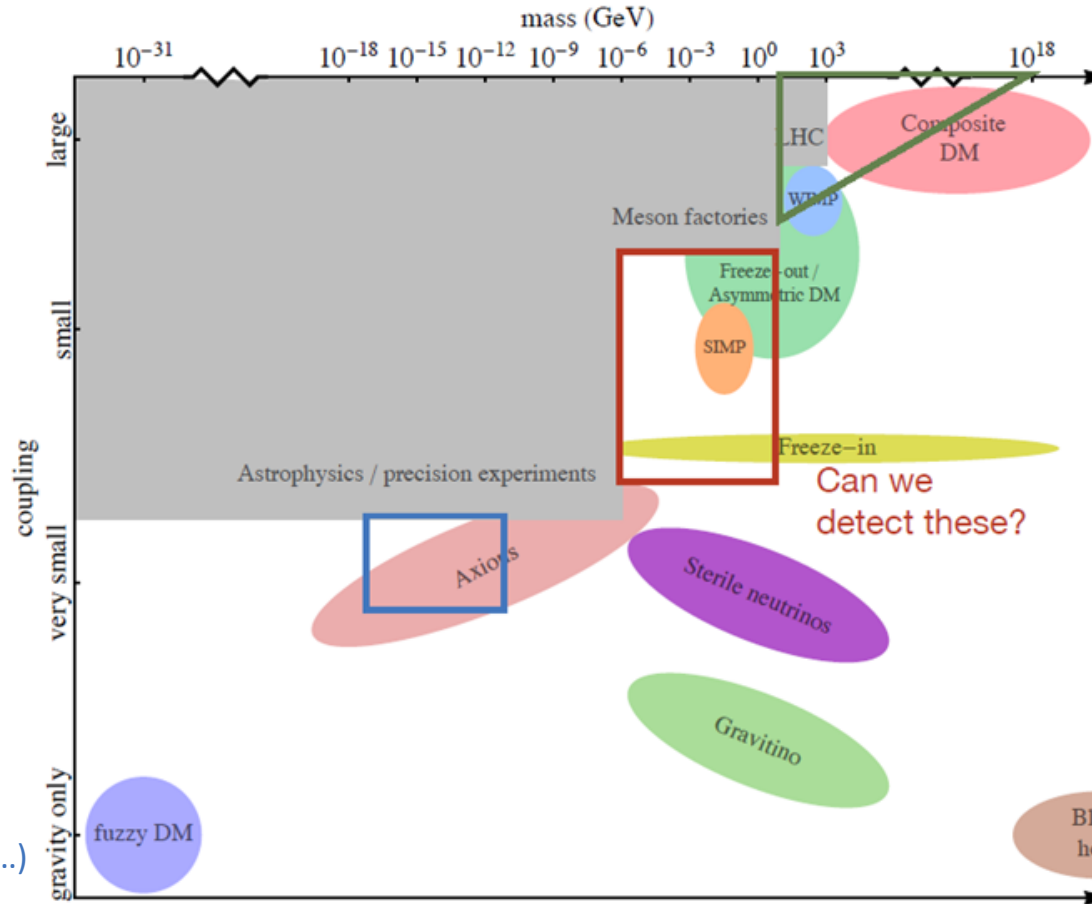Resonant cavities (ADMX, DMradio…)

Figure by S. Knapen

# The Dark Matter Landscape



Resonant cavities (ADMX, MADMAX, DMradio…)

Nuclear recoils (LZ, XENON1T, …)

Can we detect these?

Figure by S. Knapen

# Outline

1. Why phonons + DM

2. Derivation of Multiphonon scattering

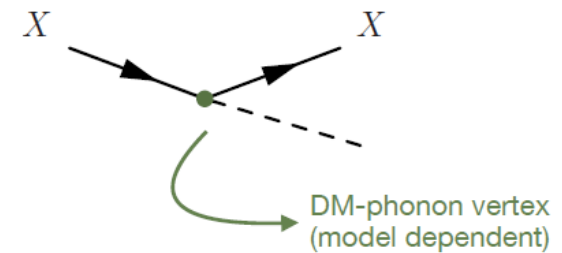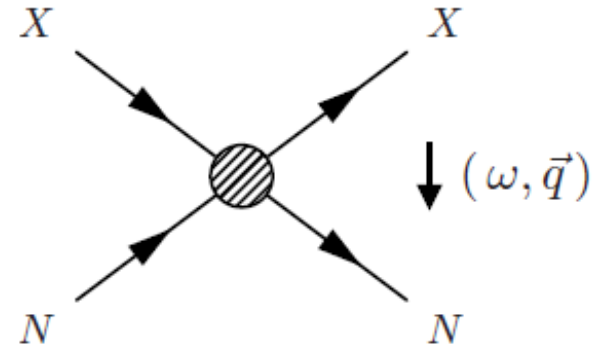3. Results!

# DM-Multiphonon Expansion

Nuclear Recoil:

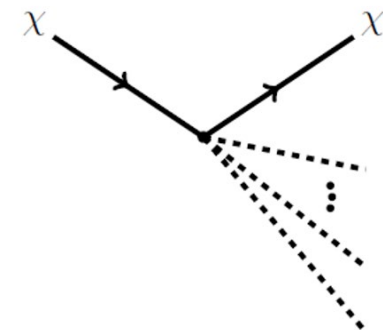$$\omega = \frac{q^2}{2m_N}$$

Phonon Regime:

$$q \ll \sqrt{2m_N\omega}$$

- Momentum is a good expansion parameter
- Single phonon vs multiphonon

S. Knapen, T. Lin, M. Pyle, K. Zurek: 1712.06598
S. Griffin, S. Knapen, T. Lin, M. Pyle, K. Zurek: 1807.10291
B. Campbell-Deem, P. Cox, S. Knapen, T. Lin, T. Melia: 1911.0348
B. Campbell-Deem, S. Knapen, T. Lin, E. Villarama: 2205.02250

$(\omega, \vec{q})$



DM-phonon vertex (model dependent)

$\mathcal{O}(q),\ \mathcal{O}(q^2)$ or $\mathcal{O}(q^4)$

(Depends on DM model & phonon branch)



$\mathcal{O}(q^{2n})$

# Multiphonon Rate

*Fermi's Golden Rule*

$\lambda_i$ & $\lambda_f$ denote initial & final spin & phonon states

$$\frac{d\sigma}{d^3q\,d\omega} \sim \sum_{i,f} \left| \sum_d^N \langle \lambda_f | \mathcal{O}(q) e^{iq\cdot r_d} | \lambda_i \rangle \right|^2 \delta(E_f - \omega)$$

$d$ labels each atom in the crystal

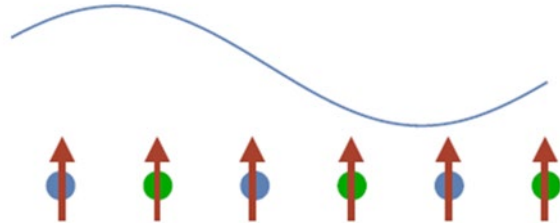$\mathcal{O}(q)$ is some spin dependent operator

$\mathcal{O}(q) = 1$ is spin independent (already been done: 2205.0225)
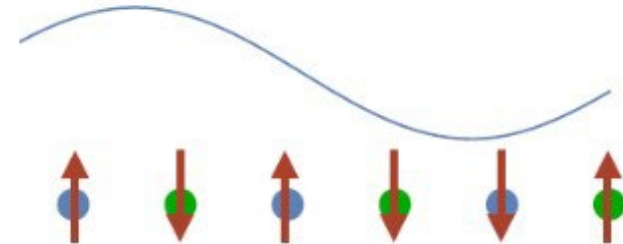
$r_d$ is the position of each atom

# Spin Dependent Interactions



Nuclear Spin Polarized

Similar to spin independent case, with a form factor correction

(see T. Trickle et. al. 2009.13534)



Nuclear Spin Unpolarized

Cross section doesn't average away:

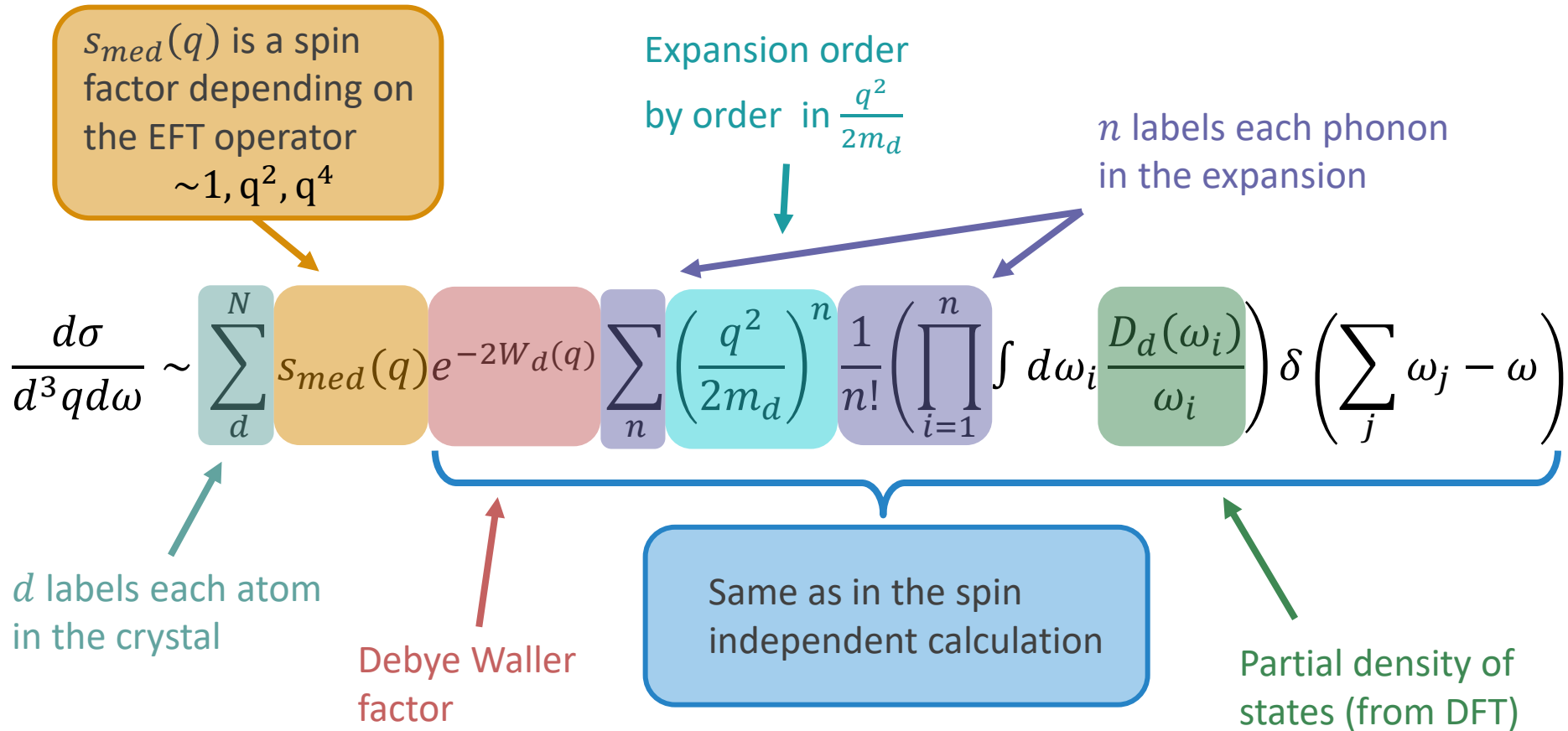$d = d'$ (incoherent) terms contribute $\sim \langle S_N^2 \rangle$

Modify spin independent calculation!

Figures by S. Knapen

# Approximations

- Harmonic Approximation:
  - Decompose into a sum of harmonic oscillators weighted by the phonon density of states
  - Good for crystals with few anharmonicities

- Isotropic crystal

- Nuclei spins are distributed spherically symmetrically

# Multiphonon Rate

$s_{med}(q)$ is a spin factor depending on the EFT operator $\sim 1, q^2, q^4$

Expansion order by order in $\frac{q^2}{2m_d}$

$n$ labels each phonon in the expansion

$$\frac{d\sigma}{d^3q\,d\omega} \sim \sum_{d}^{N} s_{med}(q) e^{-2W_d(q)} \sum_{n} \left(\frac{q^2}{2m_d}\right)^n \frac{1}{n!} \left(\prod_{i=1}^{n} \int d\omega_i \frac{D_d(\omega_i)}{\omega_i}\right) \delta\left(\sum_{j}\omega_j - \omega\right)$$

$d$ labels each atom in the crystal

Debye Waller factor

Same as in the spin independent calculation

Partial density of states (from DFT)

# Outline
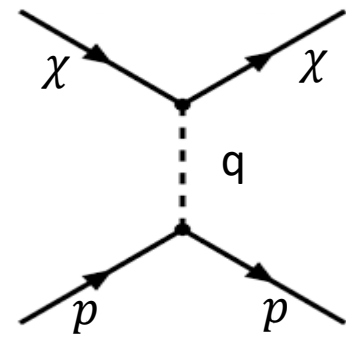
1. Why phonons + DM

2. Derivation of Multiphonon scattering

3. Results!

# Model Dependence: EFT operators



## Scalar mediator $\phi$

$$\mathcal{L} \supset g_\chi \phi \bar{\chi}\chi + g_p \phi \bar{N}\gamma^5 N$$
$$\mathcal{L}_{NR} \sim (q \cdot S_N)$$

$$s_\phi \sim q^2 \left( \overline{\frac{f_d^2 \langle S_d^2 \rangle}{m_d^2}} \right)$$

## Pseudoscalar mediator $a$

$$\mathcal{L} \supset g_\chi a \bar{\chi}\gamma^5\chi + g_p a \bar{N}\gamma^5 N$$
$$\mathcal{L}_{NR} \sim (q \cdot S_N)(q \cdot S_\chi)$$

$$s_a \sim q^4 \frac{\langle S_\chi^2 \rangle}{m_\chi^2} \left( \overline{\frac{f_d^2 \langle S_d^2 \rangle}{m_d^2}} \right)$$
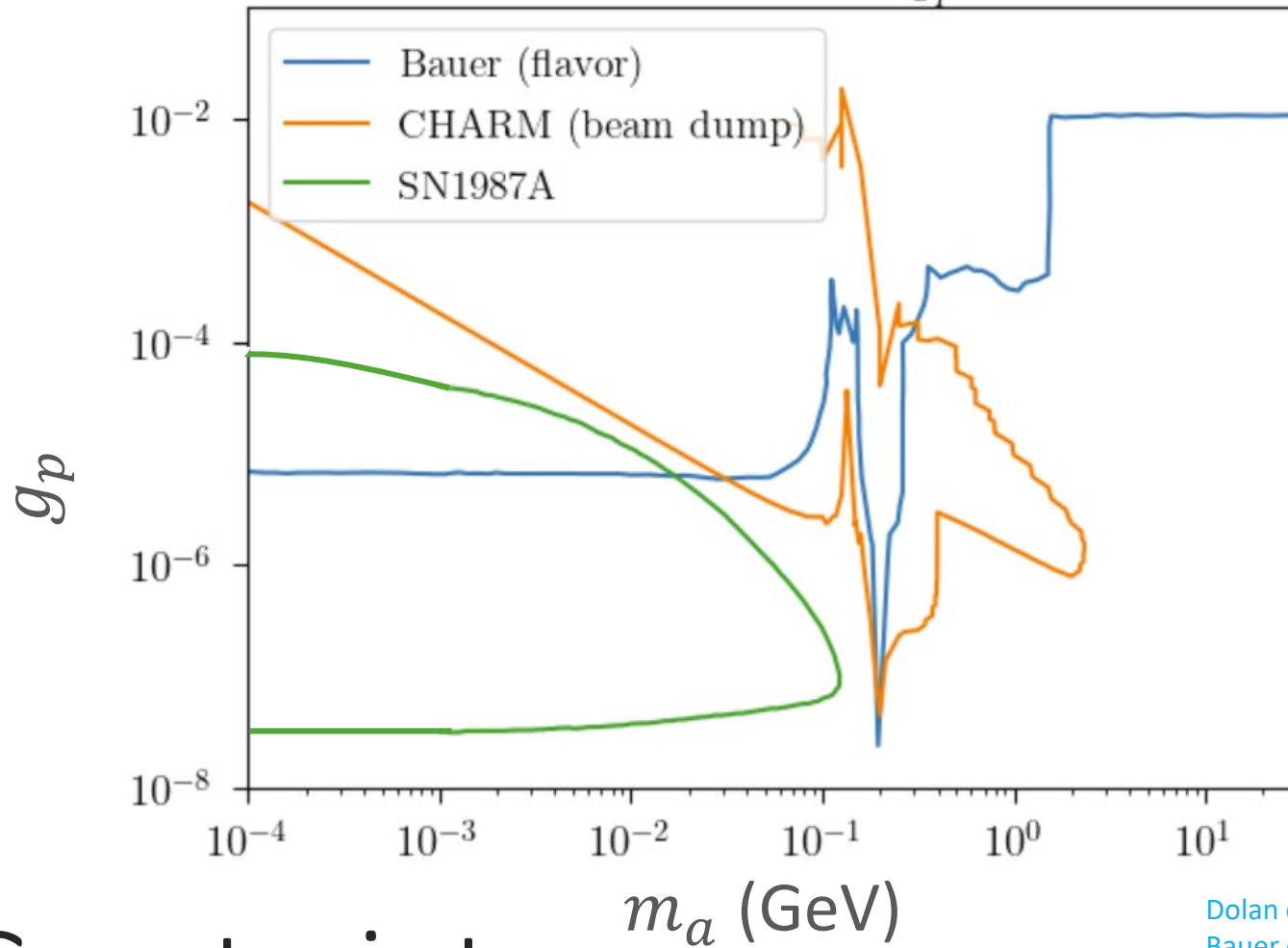
## Pseudovector mediator $A'_\mu$

$$\mathcal{L} \supset g_\chi A'_\mu \bar{\chi}\gamma^\mu\gamma^5\chi + g_p A'_\mu \bar{N}\gamma^\mu\gamma^5 N$$
$$\mathcal{L}_{NR} \sim (S_N \cdot S_\chi)$$

$$s_{A'_\mu} \sim \langle S_\chi^2 \rangle \left( \overline{f_d^2 \langle S_d^2 \rangle} \right)$$

Constraints on $g_p$

Bauer (flavor)
CHARM (beam dump)
SN1987A

$g_p$

$m_a$ (GeV)
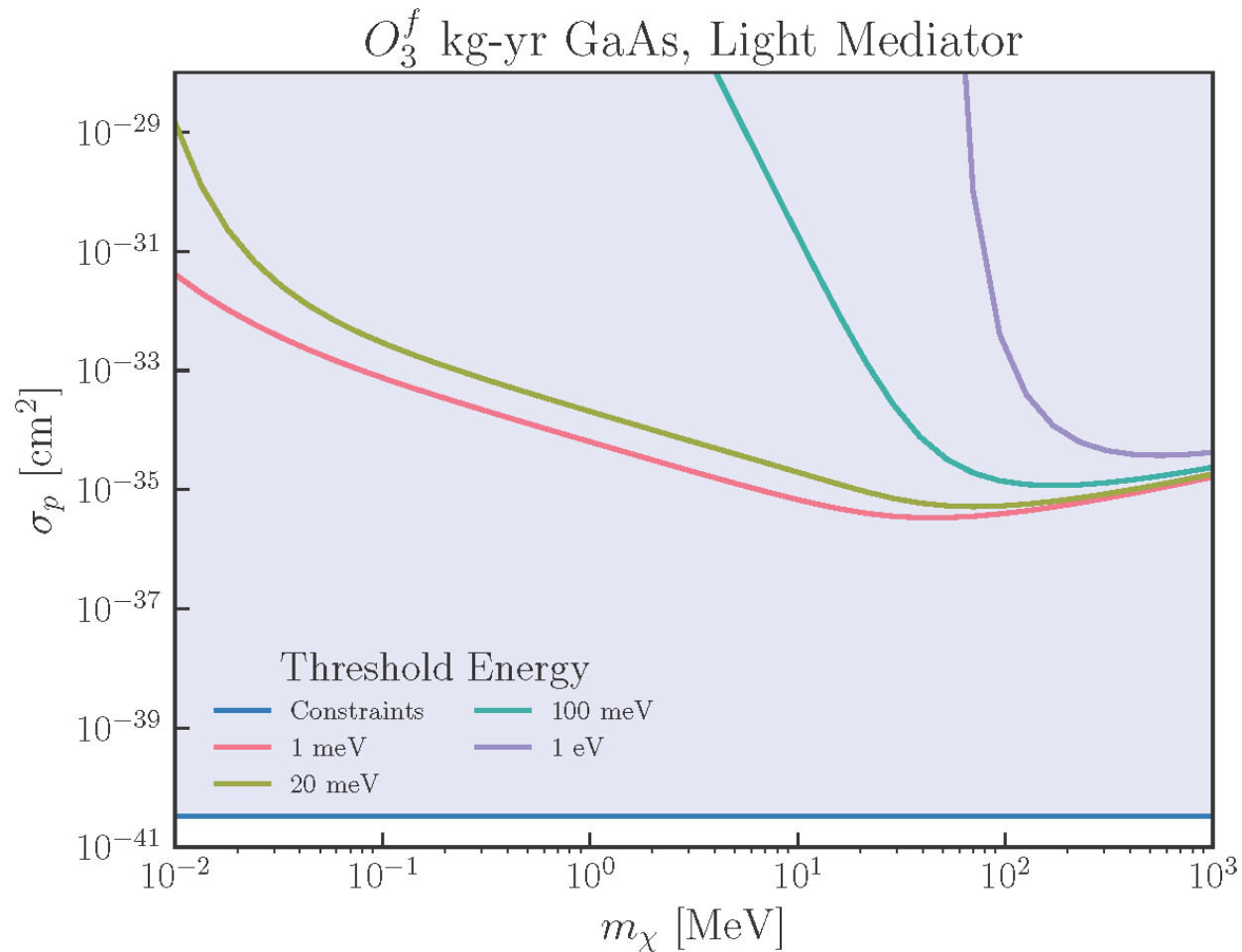
# Constraints

Dolan et. al: 1412.5174
Bauer et. al: 2110.10698
Chang et. al: 1803.00993

# Results

- Most optimistic reach curves: $O_3^f$ (scalar mediator)

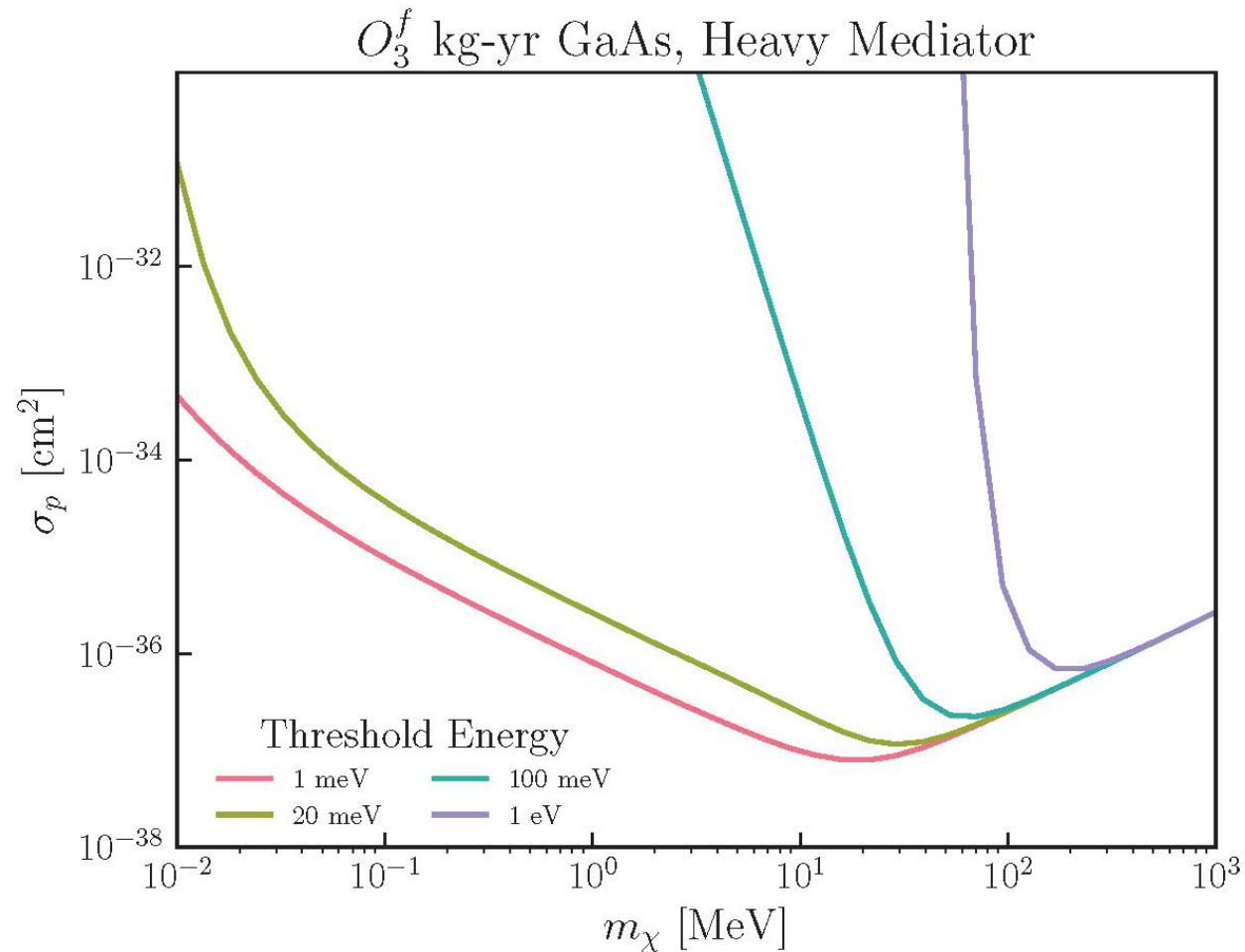- Beaten by supernovae constraints at light mediator masses (see shaded in area)

-



$O_3^f$ kg-yr GaAs, Light Mediator

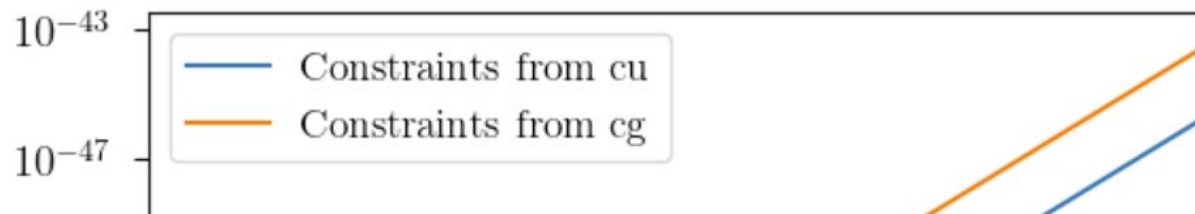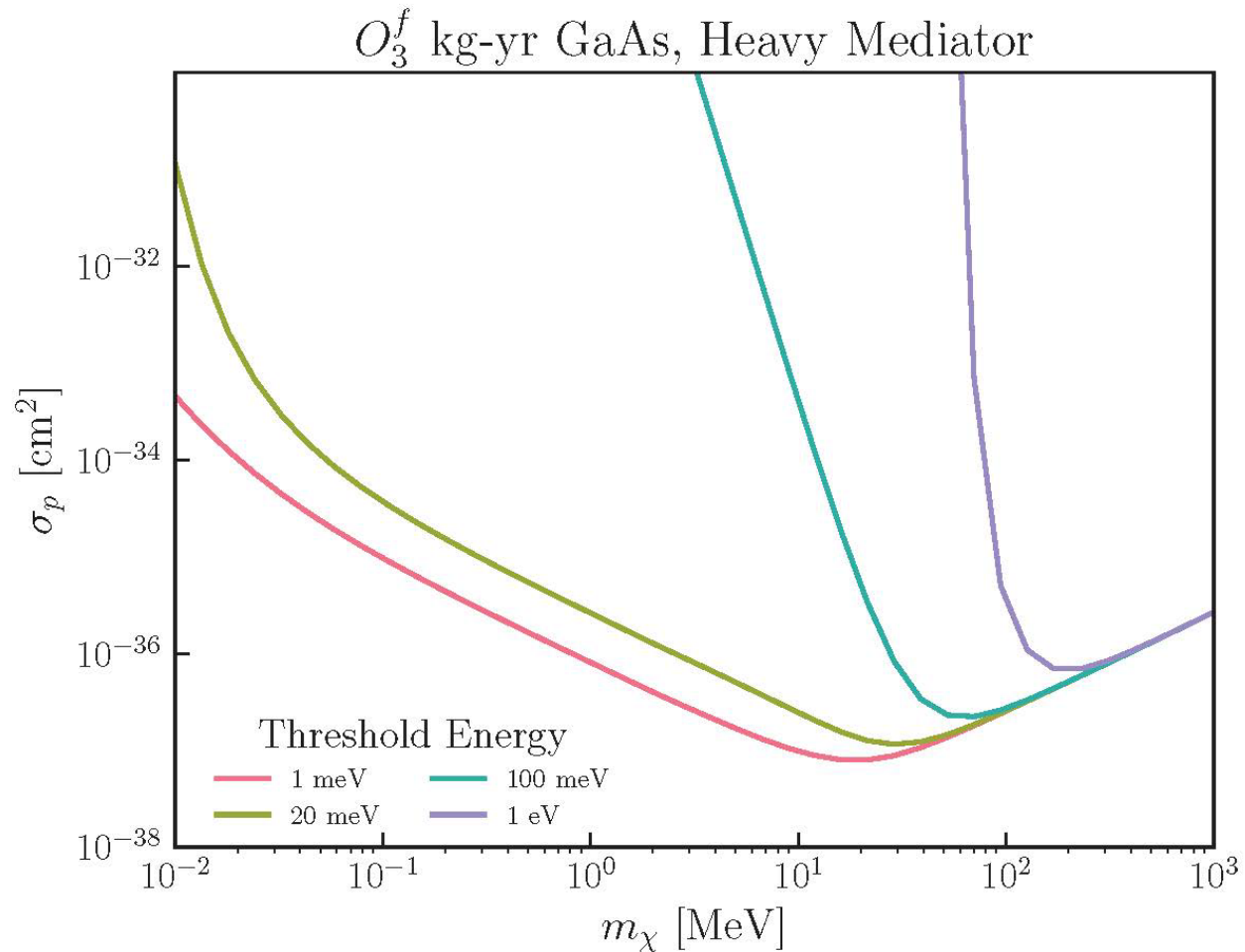Threshold Energy

Constraints
1 meV
20 meV
100 meV
1 eV

Cross sections needed for 3 events/kg-year rate for GaAs using several threshold energies
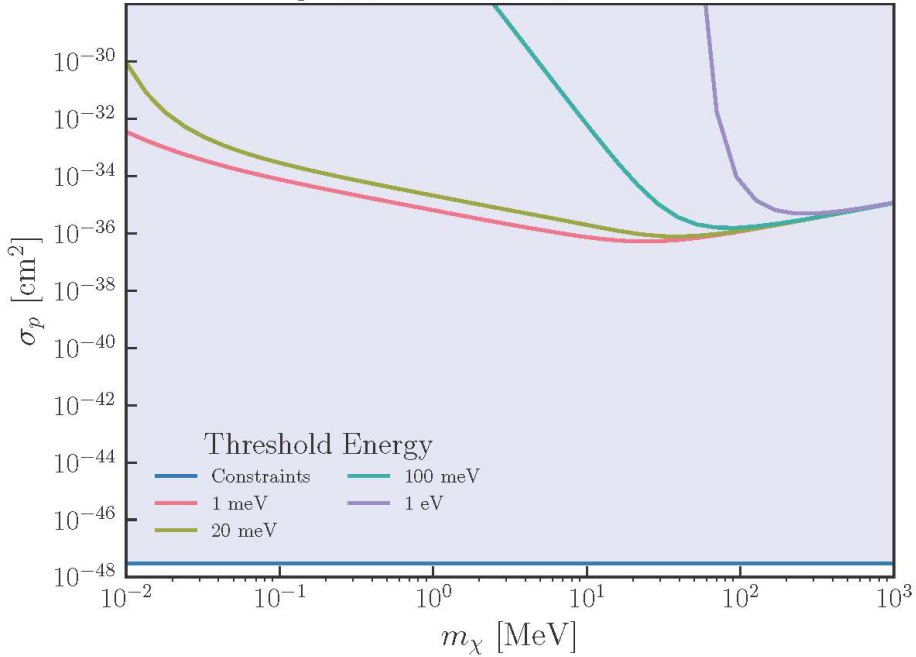
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# Results

• Most optimistic reach curves: $O_3^f$ (scalar mediator)

• Beaten by supernovae constraints at light mediator masses (see shaded in area)

• Beaten by flavor constraints at heavy mediator masses

$$\sigma_p \sim 10^{-63} - 10^{-43} \, cm^2$$



$O_3^f$ kg-yr GaAs, Heavy Mediator

Threshold Energy
— 1 meV — 100 meV
— 20 meV — 1 eV

Cross sections needed for 3 events/kg-year rate for GaAs using several threshold energies

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# Results

- Most optimistic reach curves: $O_3^f$ (scalar mediator)

- Beaten by supernovae constraints at light mediator masses (see shaded in area)

- Beaten by flavor constraints at heavy mediator masses

$$\sigma_p \sim 10^{-63} - 10^{-43} \; cm^2$$



$O_3^f$ kg-yr GaAs, Heavy Mediator

Threshold Energy
— 1 meV    — 100 meV
— 20 meV   — 1 eV

$\sigma_p \; [cm^2]$

$m_\chi \; [MeV]$

— Constraints from cu
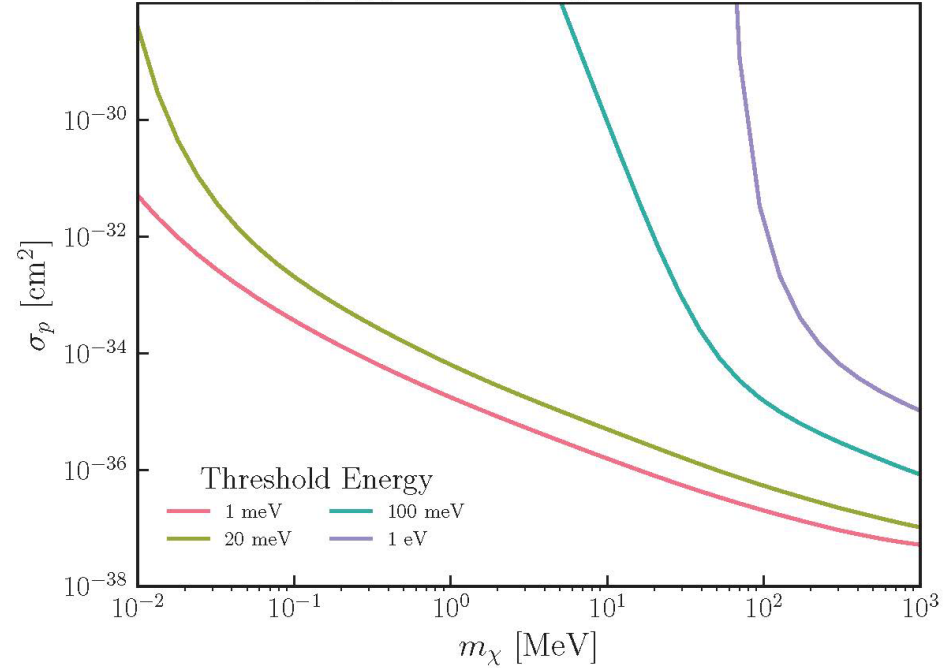— Constraints from cg

$O_4^f$ kg-yr GaAs, Light Mediator

Light pseudoscalar mediator
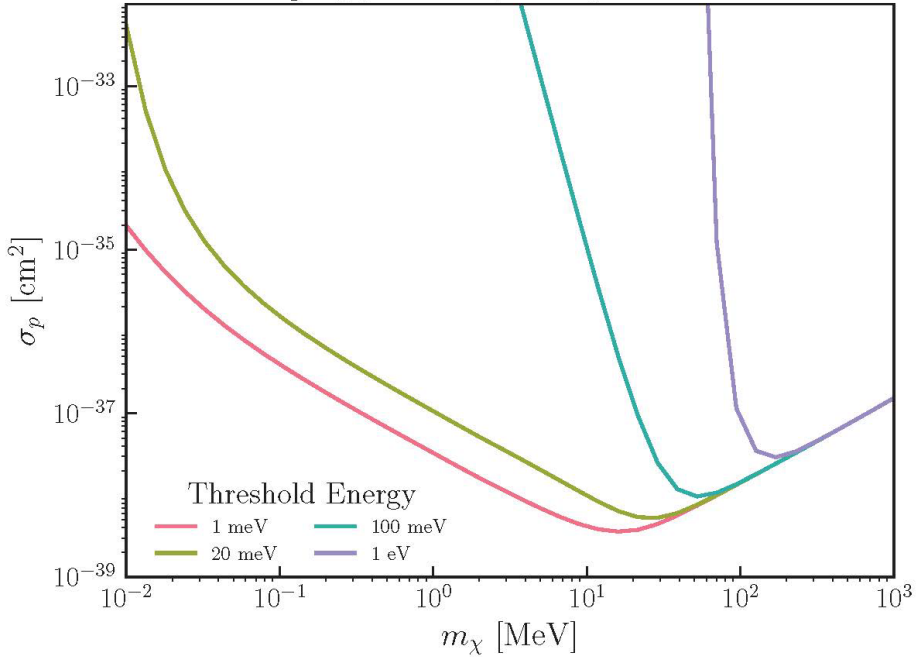


$O_8^f$ kg-yr GaAs, Light Mediator

Light pseudovector mediator

# Results

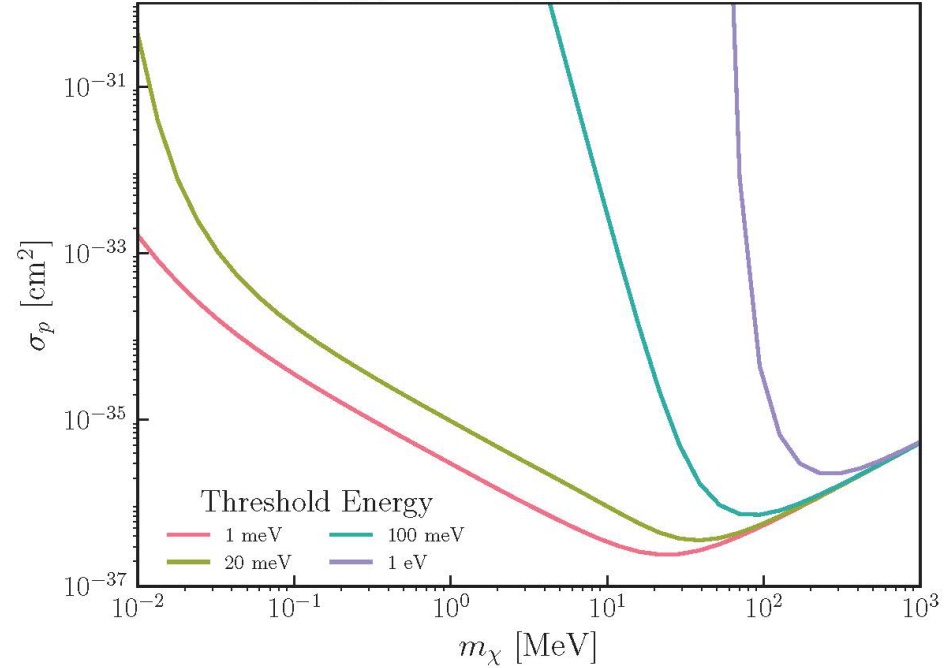Cross sections needed for 3 events/kg-year rate for GaAs using several threshold energies

$O_4^f$ kg-yr GaAs, Heavy Mediator

$O_8^f$ kg-yr GaAs, Heavy Mediator

Heavy pseudoscalar mediator

Heavy pseudovector mediator

# Results

Cross sections needed for 3 events/kg-year rate for GaAs using several threshold energies

# Summary

Multiphonon – DM interactions cover intermediate DM mass ranges between nuclear recoil and single phonon detection

Spin dependent multiphonon scattering utilizes similar methods as spin independent when crystal spins are randomly distributed

The reach of multiphonon experiments falls well short of current limits on spin dependent DM interactions (SN, mesons, etc)

Thus, repurposing a spin independent multiphonon experiment (like Tesseract) is not the best way to reach new spin dependent parameter space.  Sorry Experimentalists!

Questions?

# Backup Slides

# Phonons

Very complicated system of coupled harmonic oscillators

## Acoustic Phonons

- Coherent motion of the lattice atoms
- Wavelength → 0 corresponds to displacement of the crystal

## Optical Phonons

- Out of phase motion of the lattice atoms
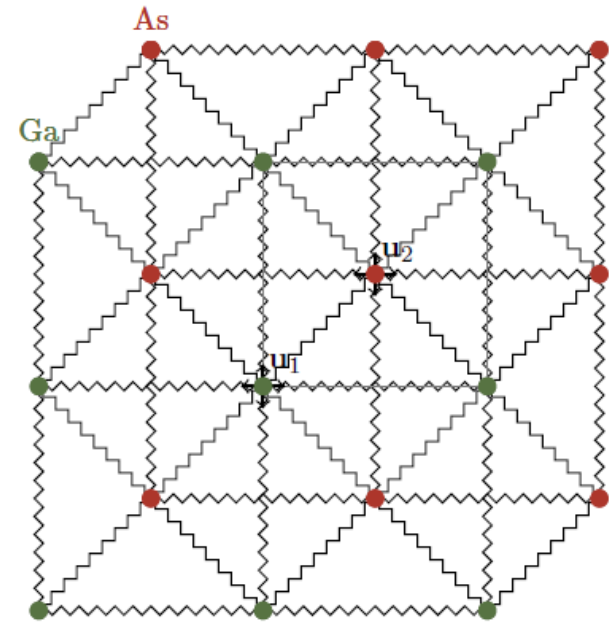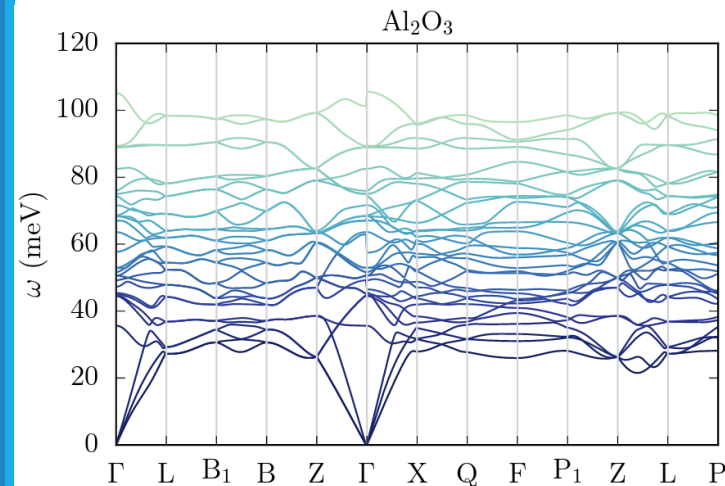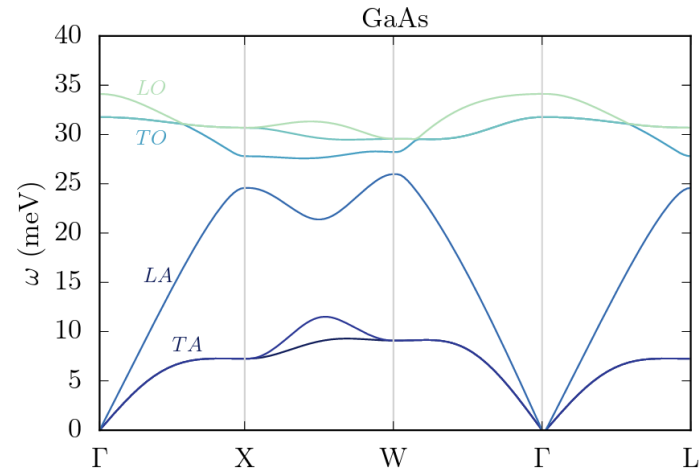- If material is polar, couples to EM field



Figure from S. Knapen

# Acoustic vs Optical

- Low momentum transfer = only excite acoustic phonon in 1st Brillouin zone

$$\omega = c_s |q| \approx 2 c_s v m_X$$
$$\sim 7\ meV \times \frac{m_X}{100 keV}$$

- Best threshold is in $10 - 100$ meV range, so difficult or impossible to detect
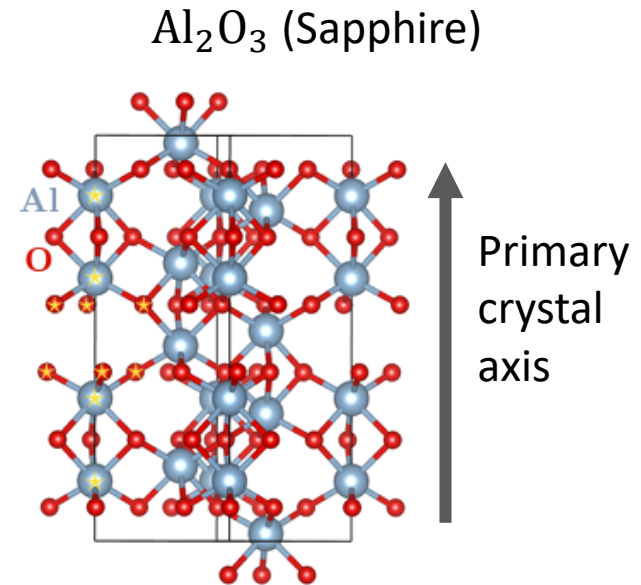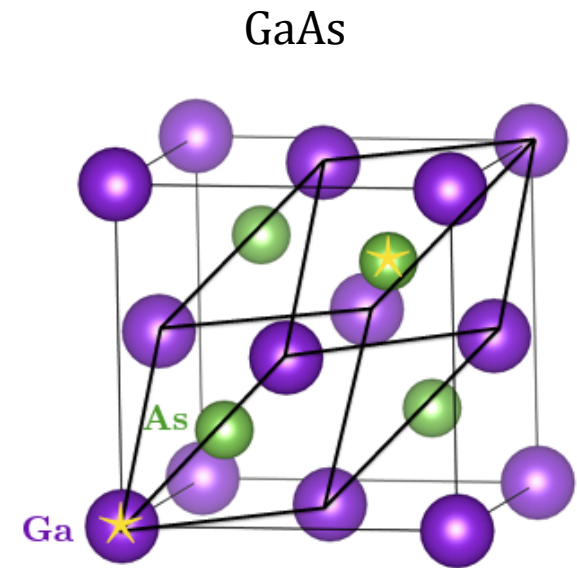
- Optical modes don't have this scaling:
$$\omega \sim 30\ meV \text{ as } |q| \to 0$$

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# Polar Materials

- At least two different atoms with **different** effective charges

- Each unit cell forms an electric dipole

- E field or dark photon causes vibrations
  - → Optical phonons

- GaAs
  - 2 atoms in unit cell
  - 3 acoustic phonons, 3 optical phonons

- Sapphire
  - 10 atoms in unit cell
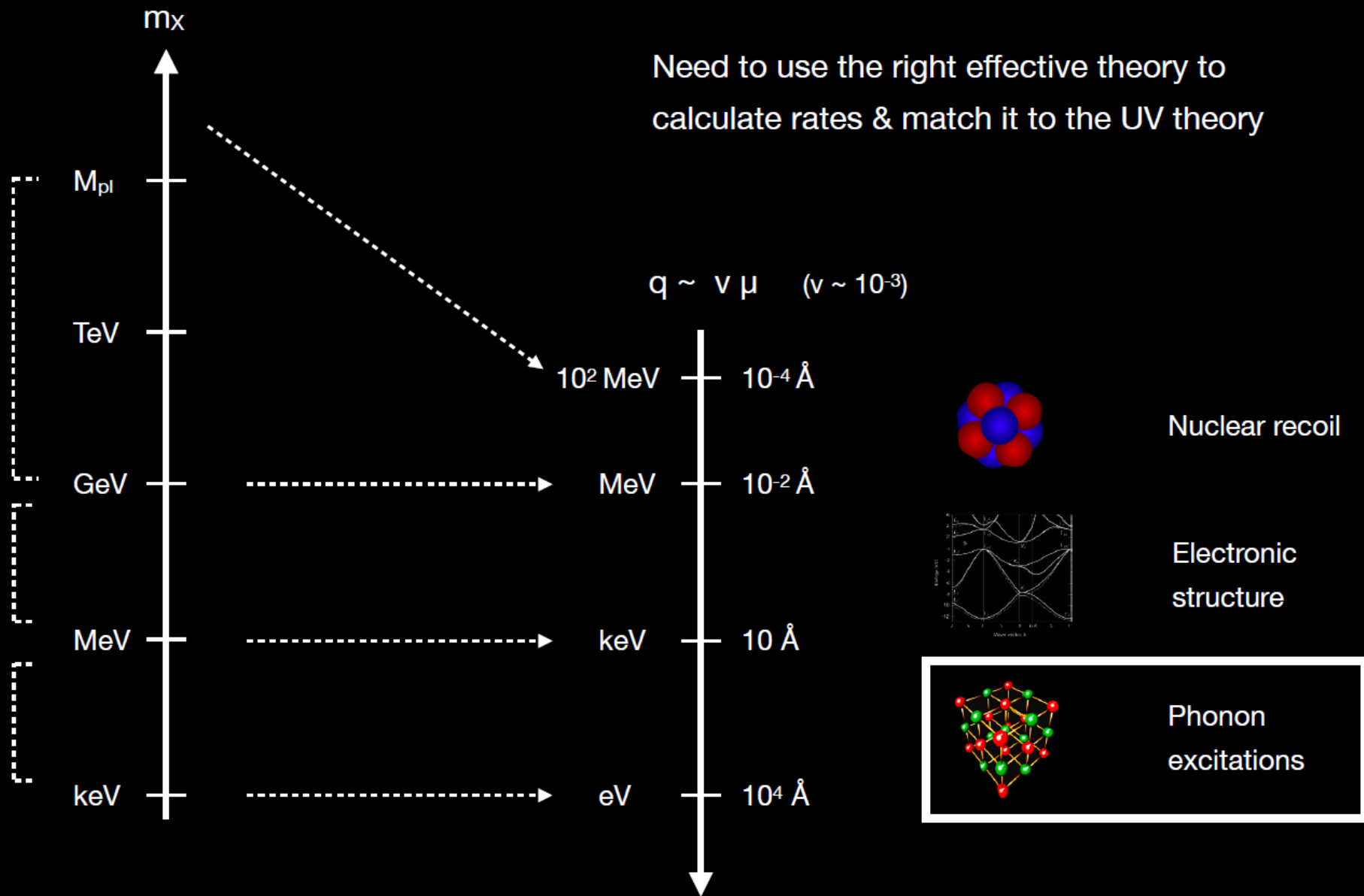  - 3 acoustic phonons, 27 optical phonons

GaAs



$Al_2O_3$ (Sapphire)



Primary crystal axis

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# Benefits of Polar Materials

- Gapped dispersion of optical phonons
  - Single or multiphonon

- Anisotropic crystal structures
  - Daily modulation in rate

- Low screening
  - Required: few free electrons, high polarizability
  - Gap for electronic excitations $\sim O(1 - 10 \; eV)$
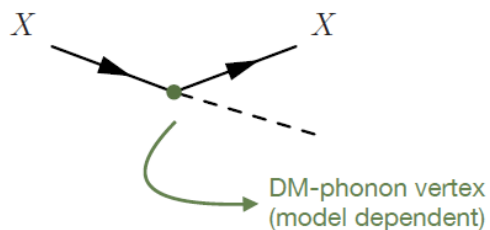  - Kinetic mixing with dark photon couples to dipole moment

- Easy to fabricate

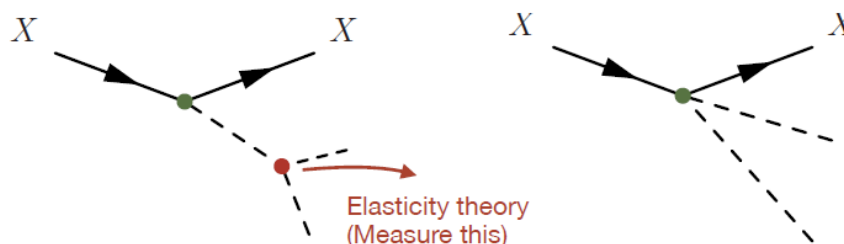S. Griffin, S. Knapen, T. Lin, M. Pyle, K. Zurek: 1807.10291

# The need for theory

$m_X$

Need to use the right effective theory to calculate rates & match it to the UV theory

$q \sim v \, \mu \quad (v \sim 10^{-3})$

| | | |
|---|---|---|
| $M_{pl}$ | | |
| TeV | | |
| | $10^2$ MeV | $10^{-4}$ Å |
| GeV | MeV | $10^{-2}$ Å |
| MeV | keV | $10$ Å |
| keV | eV | $10^4$ Å |

Nuclear recoil

Electronic structure

Phonon excitations

# DM-Multiphonon Expansion

## LO



DM-phonon vertex
(model dependent)

$\mathcal{O}(q)$, $\mathcal{O}(q^2)$ or $\mathcal{O}(q^4)$

(Depends on DM model & phonon branch)

## NLO



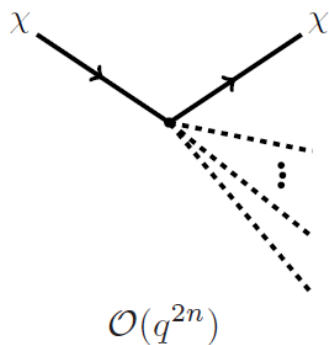Elasticity theory
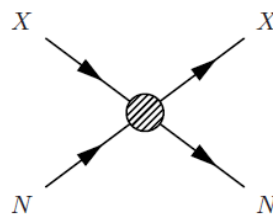(Measure this)

$\mathcal{O}(q^4)$

B. Campbell-Deem, P. Cox, S. Knapen, T. Lin, T. Melia: 1911.0348

## N$^n$LO



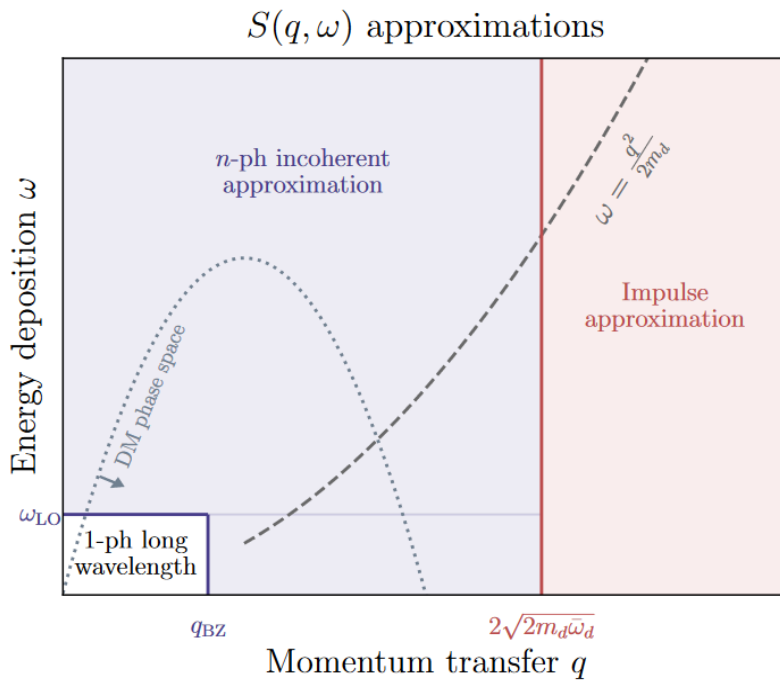$\mathcal{O}(q^{2n})$

## N$^\infty$LO = nuclear recoil



$$\sim \delta\left(\omega - \frac{q^2}{2m_N}\right)$$

B. Campbell-Deem, S. Knapen, T. Lin, E. Villarama: 2205.02250

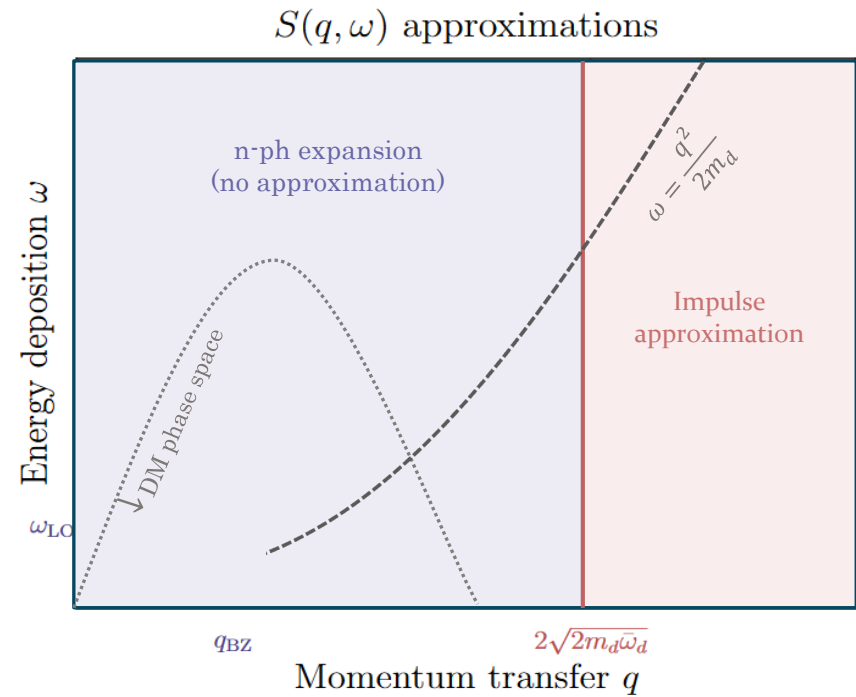Figures from S. Knapen
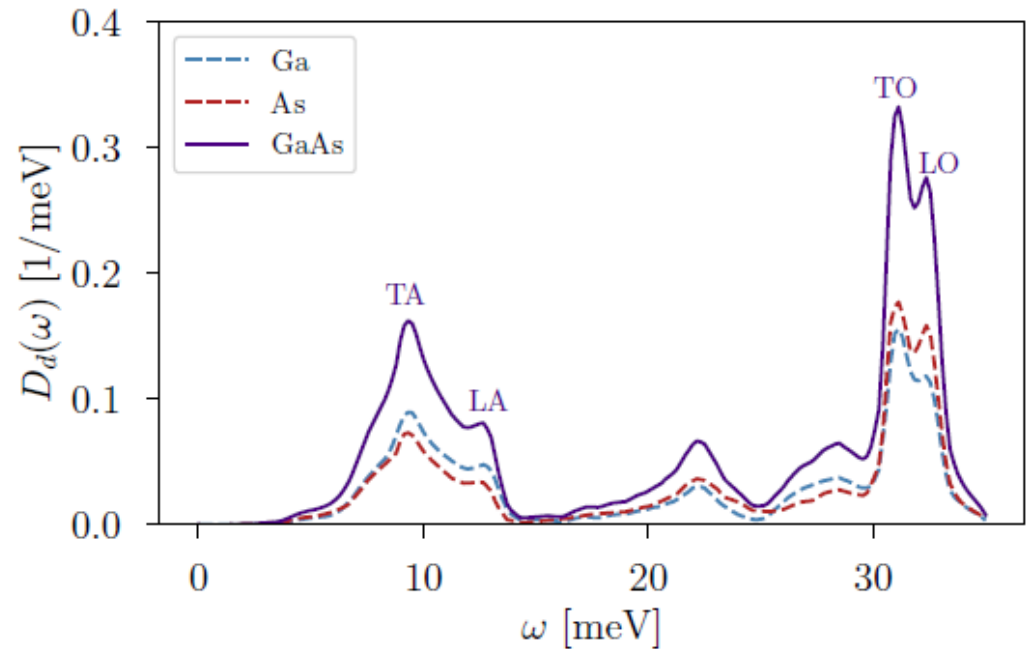
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# Approximations

## Spin Independent

$S(q, \omega)$ approximations

Energy deposition $\omega$

$n$-ph incoherent approximation

$\omega = \frac{q^2}{2m_d}$

DM phase space

Impulse approximation

$\omega_{LO}$

1-ph long wavelength

$q_{BZ}$          $2\sqrt{2m_d\bar{\omega}_d}$

Momentum transfer $q$

## Spin Dependent

$S(q, \omega)$ approximations

Energy deposition $\omega$

$n$-ph expansion (no approximation)

$\omega = \frac{q^2}{2m_d}$

DM phase space

Impulse approximation

$\omega_{LO}$

$q_{BZ}$          $2\sqrt{2m_d\bar{\omega}_d}$

Momentum transfer $q$

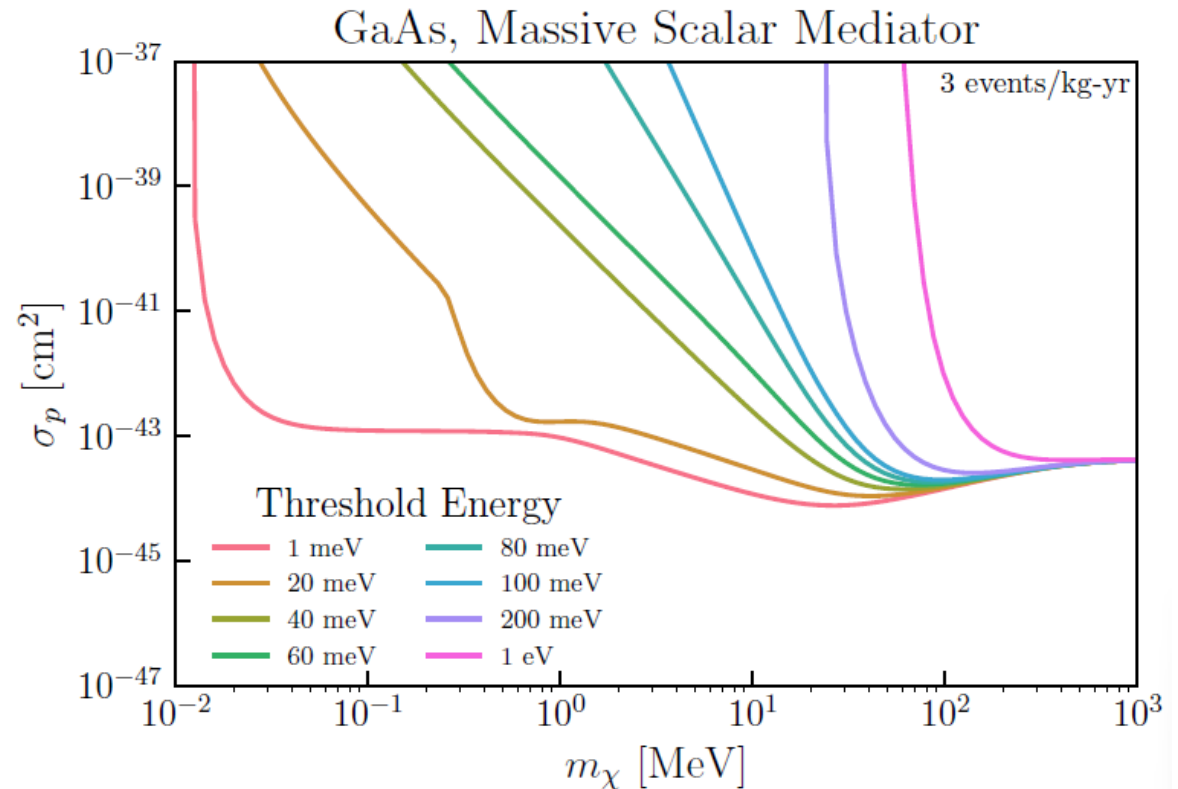B. Campbell-Deem, S. Knapen, T. Lin, E. Villarama: 2205.02250

# Density of States

- The peaks from each phonon are visible

- The partial density of states are calculated using Density Effective Theory where data is not yet taken

- Many materials' density of states can be taken from the Material Project data



A. Jain, Et. Al. https://doi.org/10.1063/1.4812323

# Spin Independent Results

- Assumes a coupling $\sim A_d$

- Isotropic approximation

- Anharmonic corrections around $m_\chi \sim 1 - 10\ MeV$: 2309.10839



GaAs, Massive Scalar Mediator

3 events/kg-yr

Threshold Energy
- 1 meV
- 20 meV
- 40 meV
- 60 meV
- 80 meV
- 100 meV
- 200 meV
- 1 eV

B. Campbell-Deem, S. Knapen, T. Lin, E. Villarama: 2205.02250

# Multiphonon Rate (SI)

$$\frac{d\sigma}{d^3q\,d\omega} \sim \sum_d^N f_d^2 e^{-2W_d(q)} \Sigma \left(\frac{q^2}{2m_d}\right)^n \frac{1}{n!} \left(\prod_{i=1}^n \int d\omega_i \frac{D_d(\omega_i)}{\omega_i}\right) \delta\left(\sum_j \omega_j - \omega\right)$$
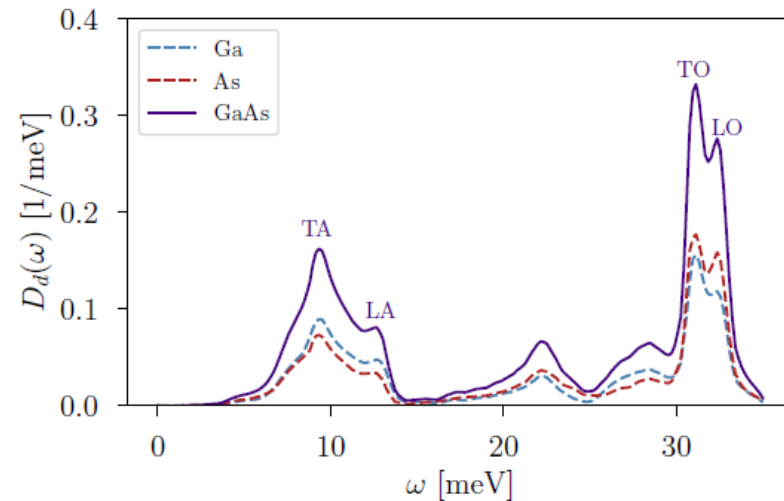
$q \gg \sqrt{2\omega m_d}$    Impulse Approximation    Partial density of States

$$\frac{d\sigma}{d^3q\,d\omega} \sim \sum_d^N f_d^2 \sqrt{\frac{2\pi}{\Delta_d^2}} \exp\left(-\frac{\left(\omega - \frac{q^2}{2m_d}\right)^2}{2\Delta_d^2}\right)$$

$q \ggg \sqrt{2\omega m_d}$

$$\frac{d\sigma}{d^3q\,d\omega} \sim \sum_d^N f_d^2 \times \delta\left(\omega - \frac{q^2}{2m_d}\right)$$

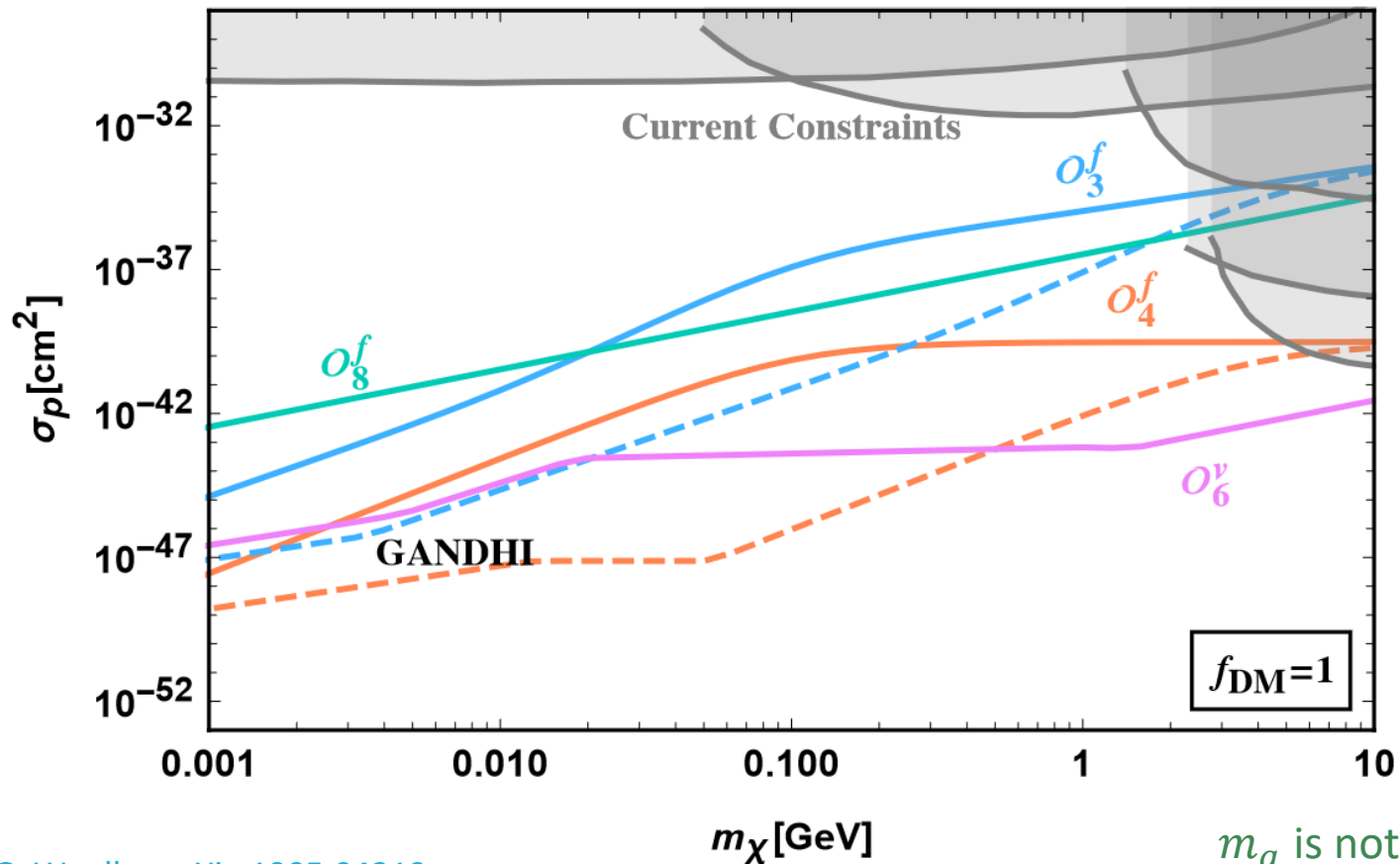Free nuclear recoil limit!



B. Campbell-Deem, S. Knapen, T. Lin, E. Villarama: 2205.02250
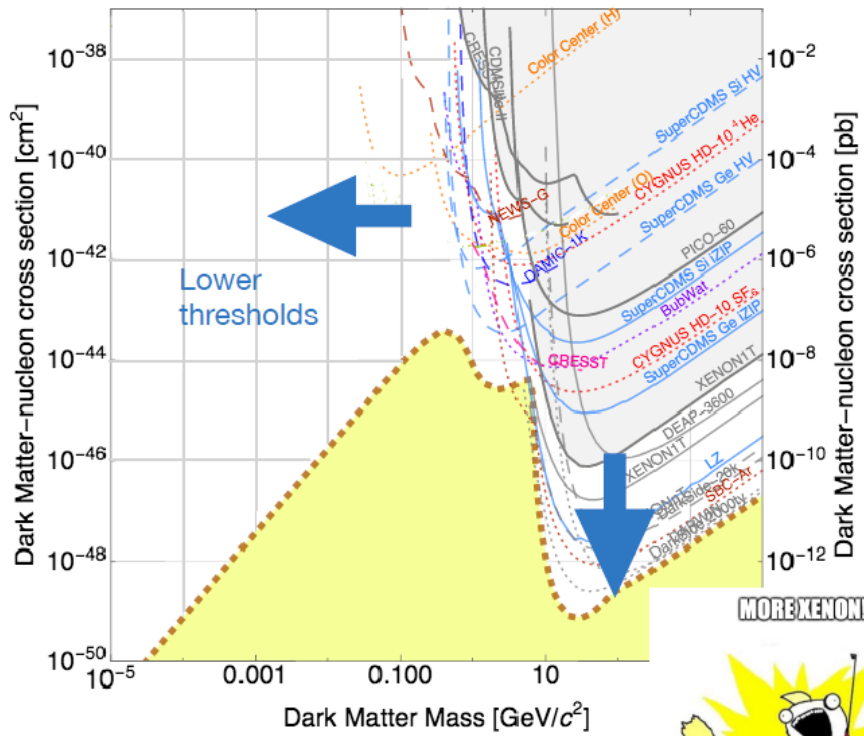
# Spin Dependent Constraints

$m_a$ is not fixed

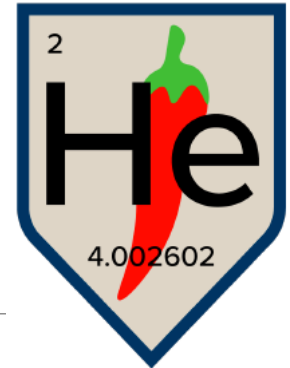# Light Dark Matter Direct Detection

## What do we need?



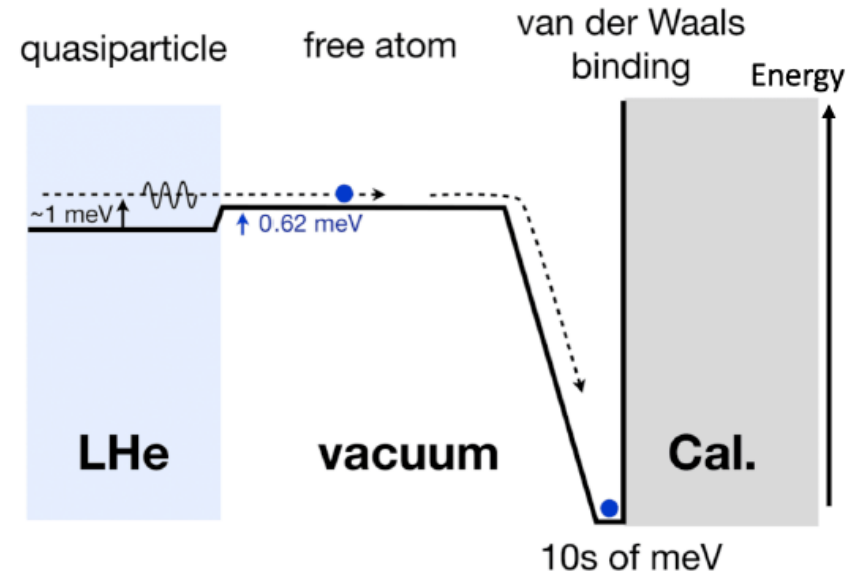Ultra-low threshold calorimeters: Transition Edge Sensors
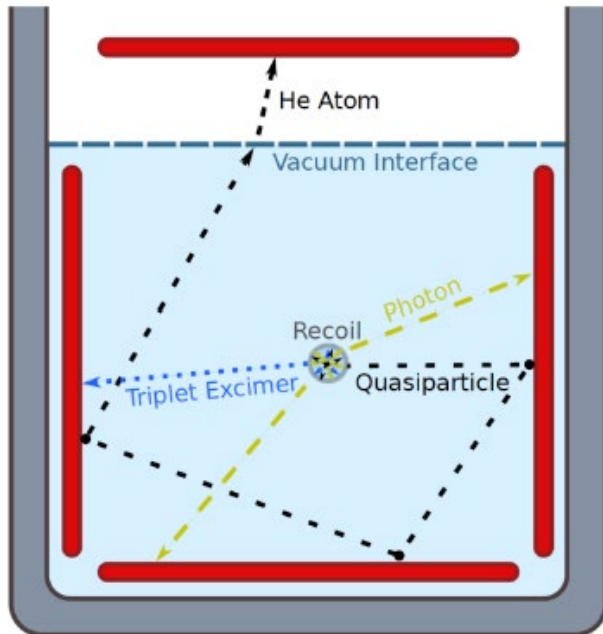
M. Battaglieri: 1707.04591
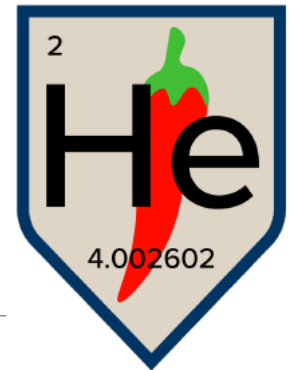Meme credit: S. Knapen

# Phonon Detector: HeRALD

SPICE / HeRALD
TESSERACT



- Calorimeters with TES readout

- Quantum evaporation of He atoms

# Phonon Detector: SPICE



TES and QP collection antennas (W)

Athermal Phonon Collection Fins (Al)

$1cm^3$ Polar Crystal

SPICE / HeRALD
TESSERACT

3'' sapphire detector



- Polar Materials: GaAs or Sapphire

- Scintillation & phonons
  - Background discrimination!

- Low energy TES

~ 10 meV threshhold

Figure from M. Pyle
Picture from TESSERACT Website

# Transition Edge Sensors (TES)

- Superconducting film acting at the phase "transition edge"

- Large change in resistance with tiny shifts in temperature

- Smaller band widths → lower threshold energies