李政道研究所
TSUNG-DAO LEE INSTITUTE

Phase Transitions, anomalous baryon number violation and electroweak multiplet dark matter

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Tsung-Dao Lee Institute / Shanghai Jiao Tong University

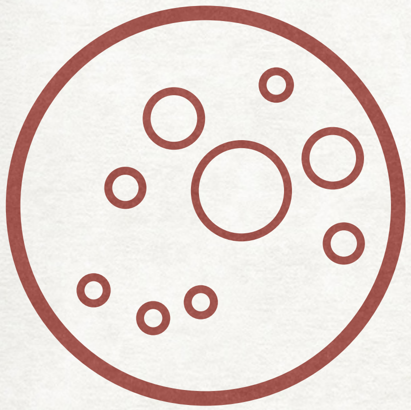
In cooperation with Michael Ramsey-Musolf and Wenxing Zhang

based on 2307.02187v1, v2 will appear soon

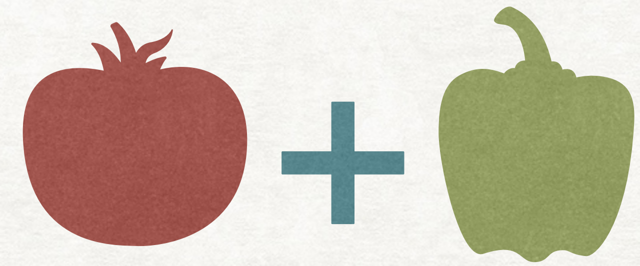
DPF-PHENO 2024

2024/5/14

Matter-antimatter asymmetry of our universe



Electroweak Baryogenesis via a
First order EWPT



Higgs

EW
Multiplet

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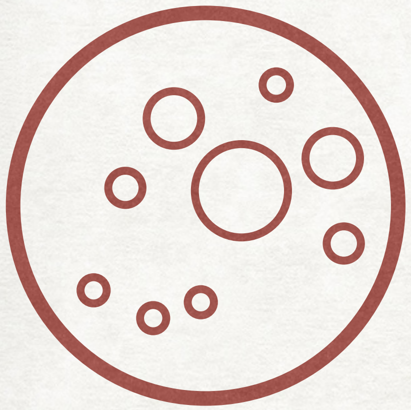

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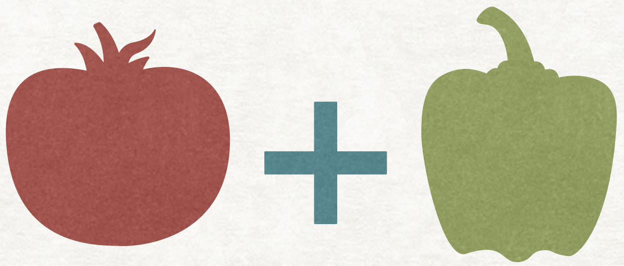

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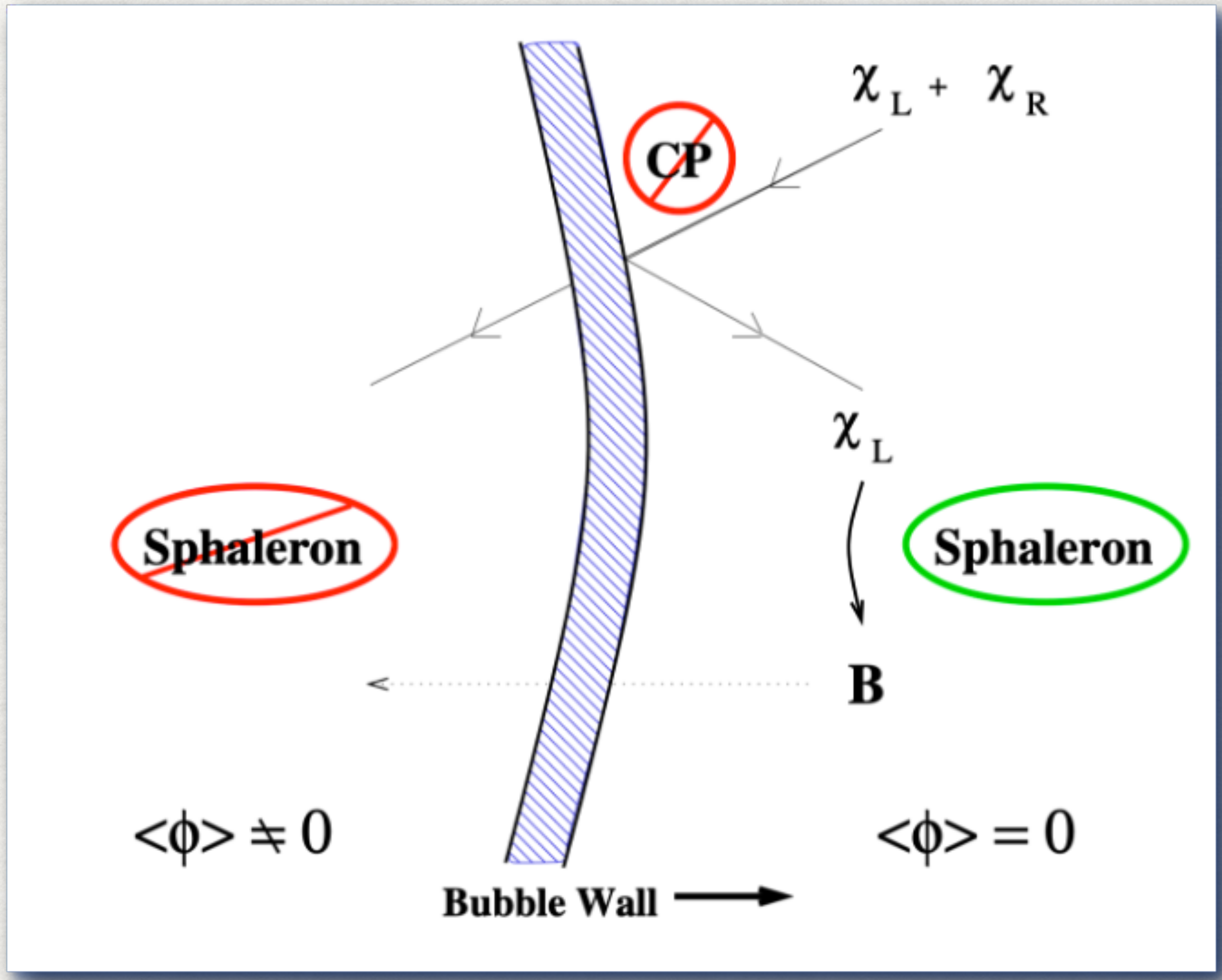

Morrissey, Ramsey-Musolf, New J. Phys. 2012

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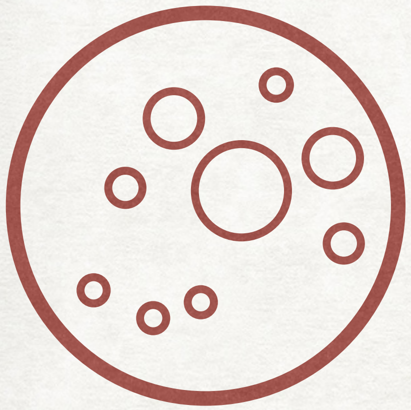

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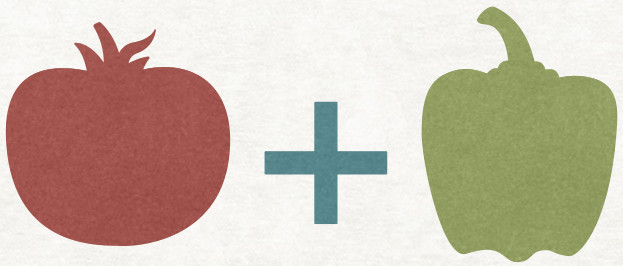
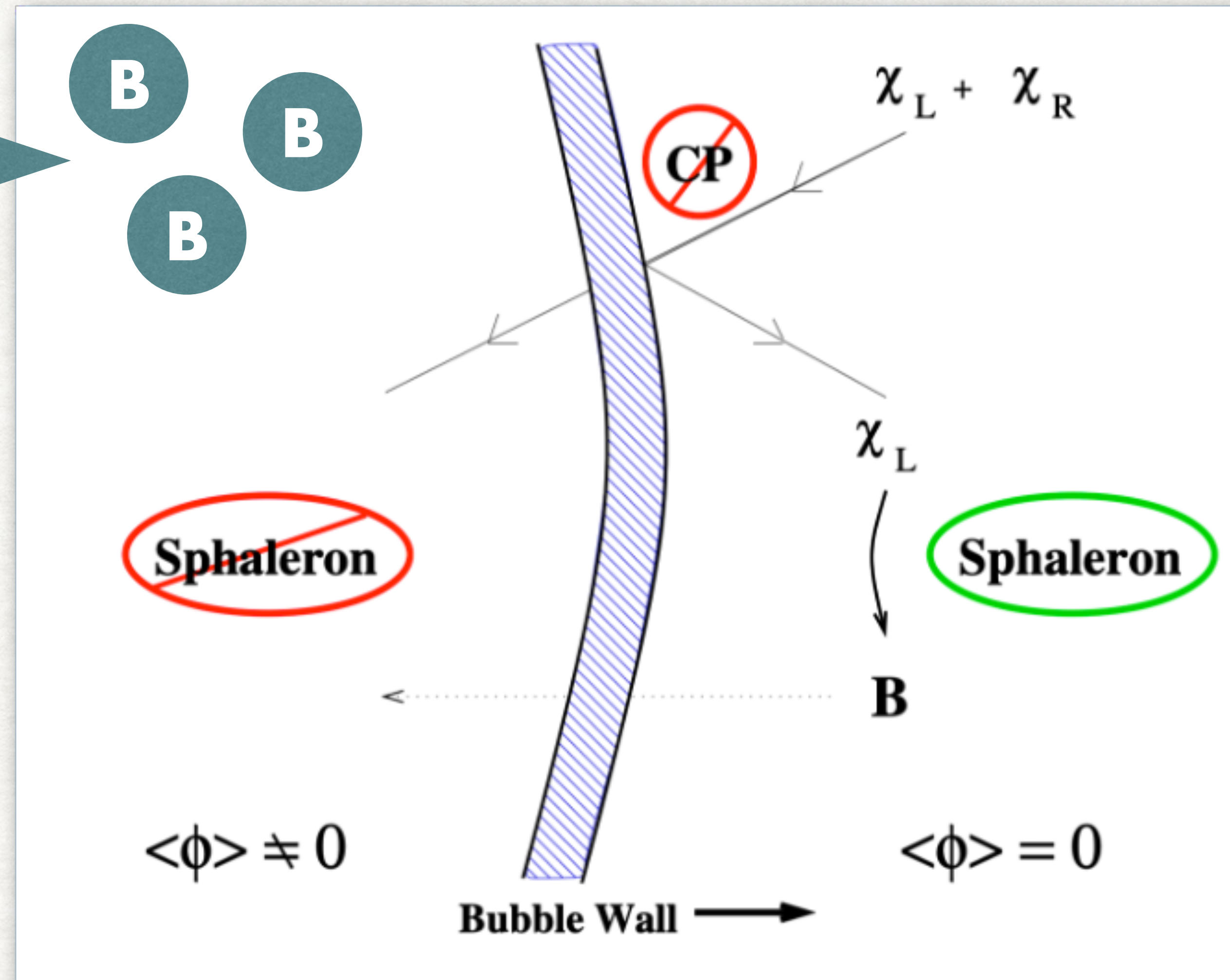


Higgs

EW Multiplet

CREATED NET BARYON NUMBER



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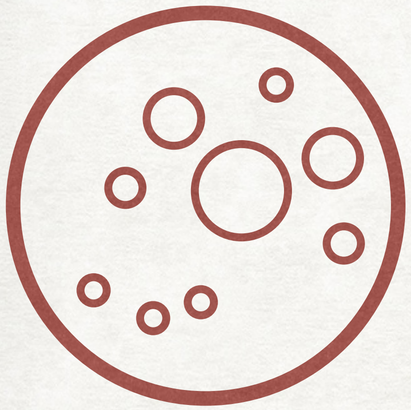

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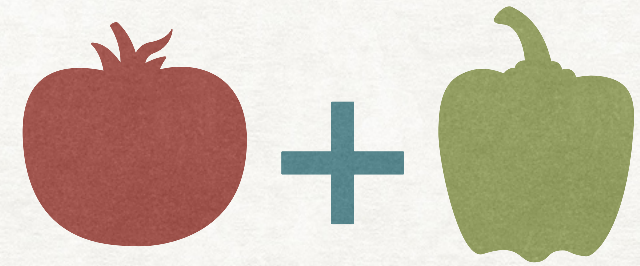

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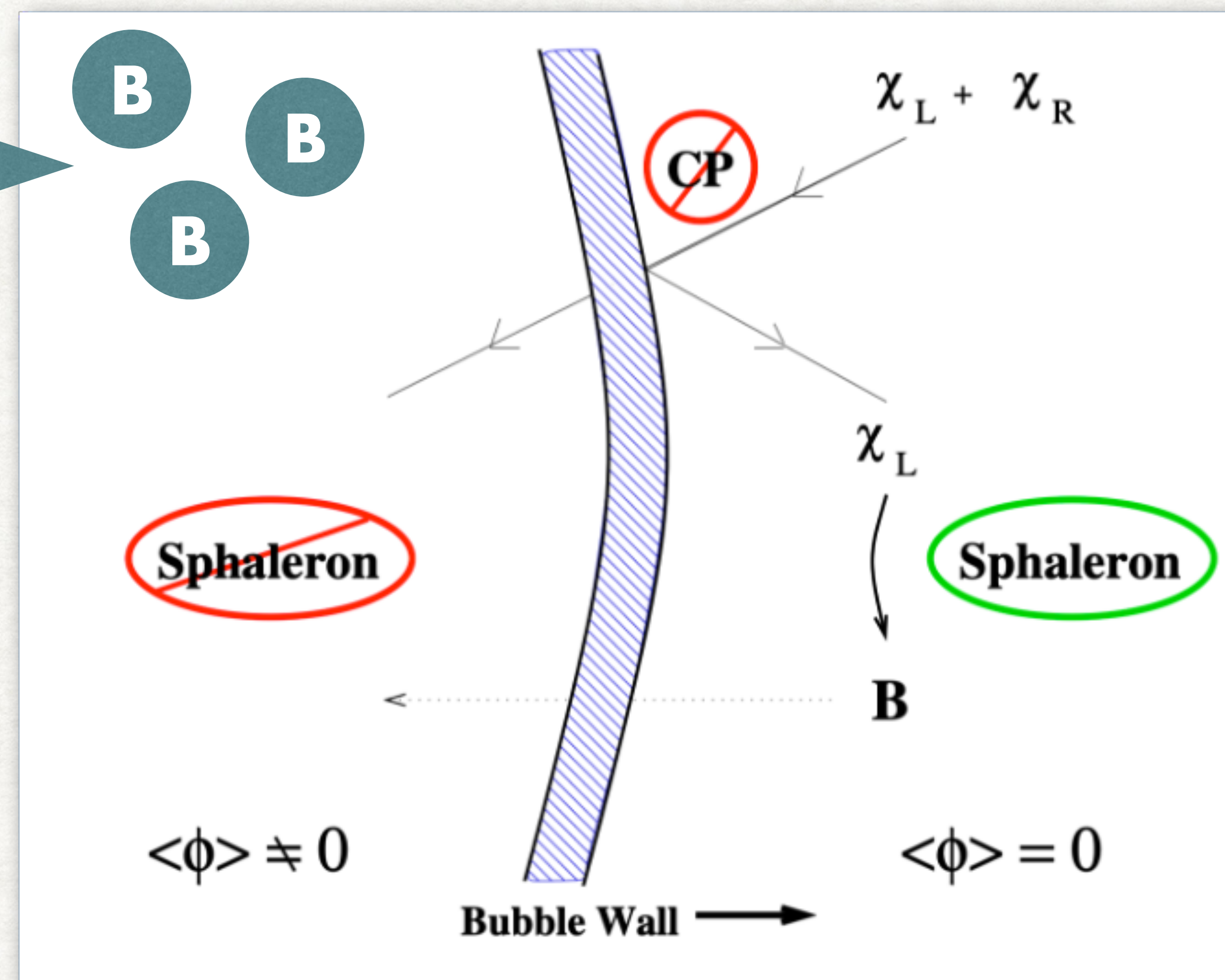
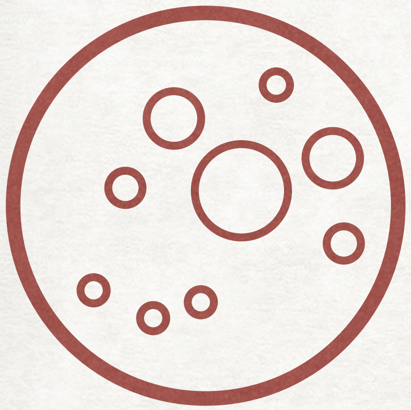


Morrissey, Ramsey-Musolf, New J. Phys. 2012

Can these baryon asymmetry be preserved to today?

Matter-antimatter asymmetry of our universe



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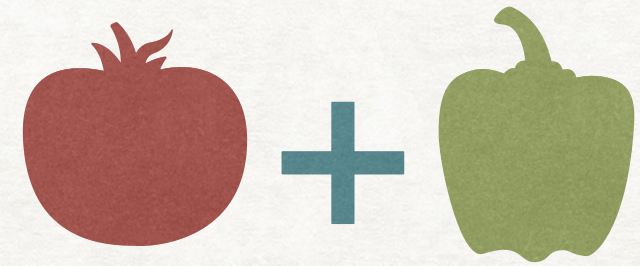

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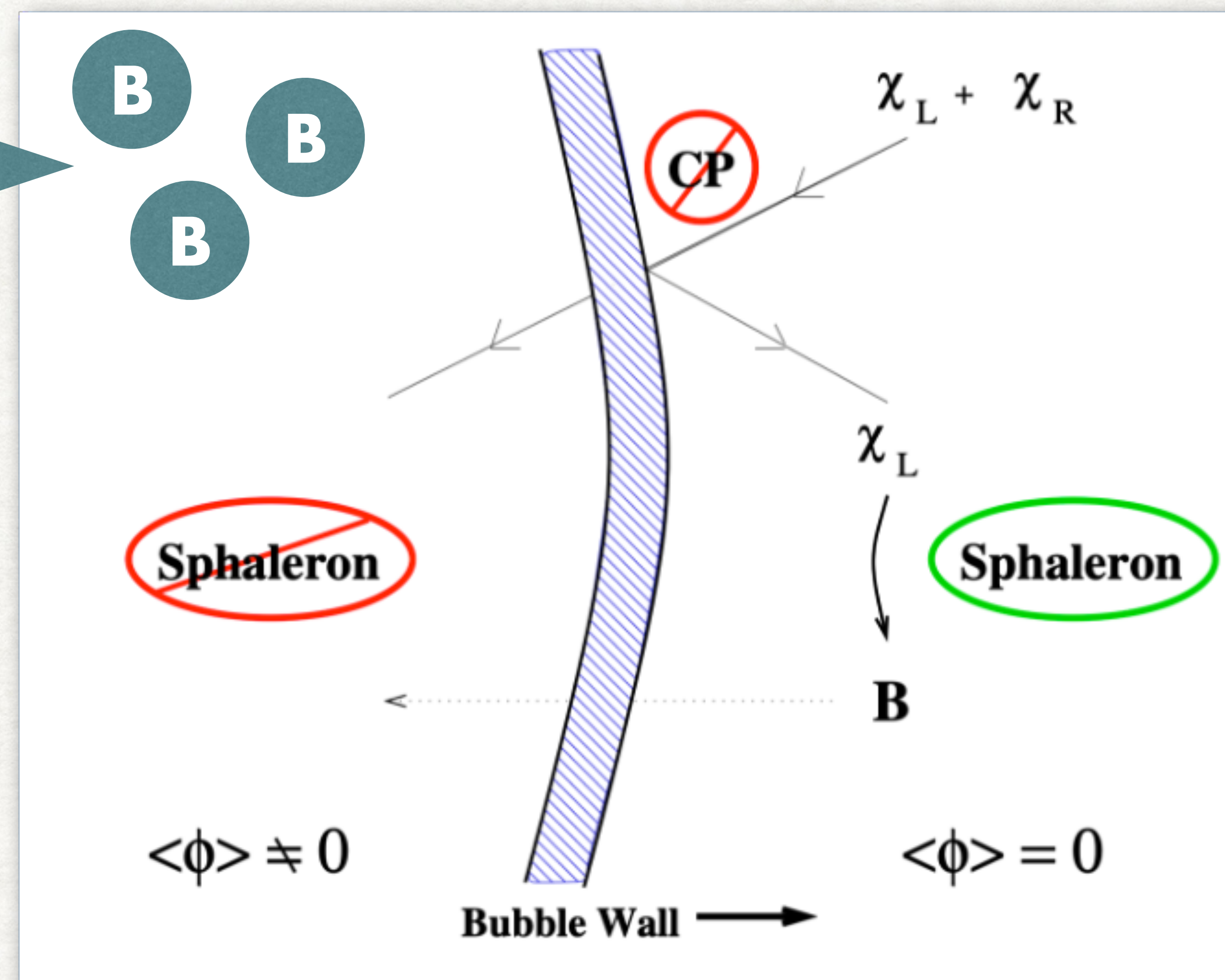

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Low Sphaleron rate in Broken phase

Matter-antimatter asymmetry of our universe

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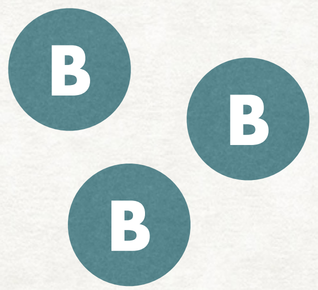

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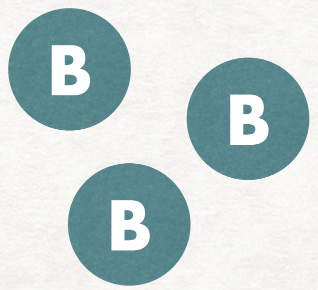

Low Sphaleron rate in Broken phase

Low monopole density in Broken phase

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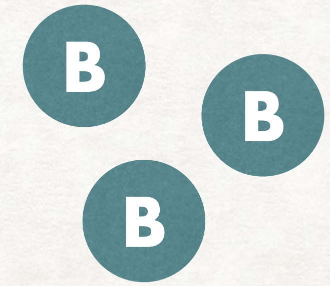
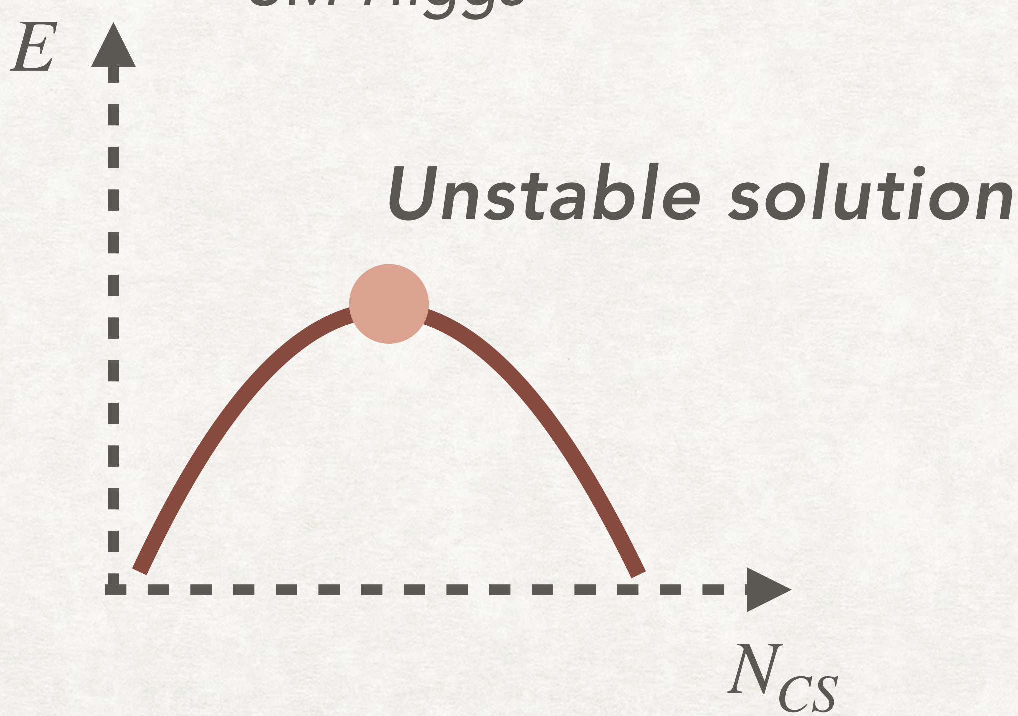
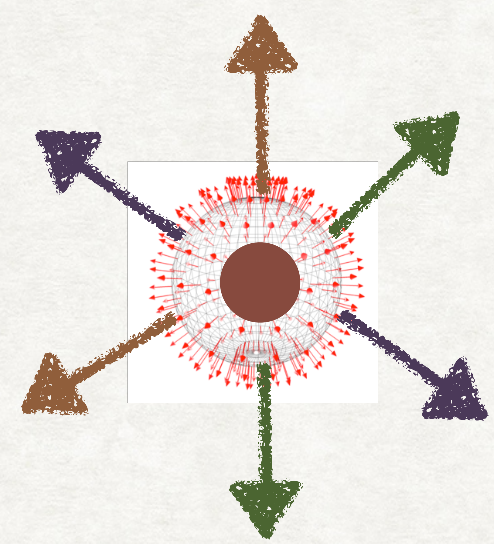


Low Sphaleron rate in Broken phase	Low monopole density in Broken phase
Manton sphaleron	*'t Hooft-Polyakov monopole*
SM Higgs	*real triplet*





$\nabla \cdot B^a \neq 0$ ***Stable solution***

Matter-antimatter asymmetry of our universe

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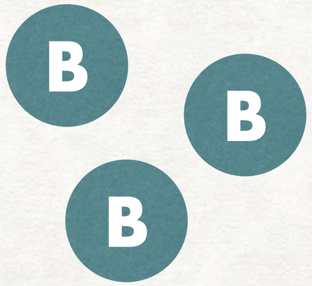
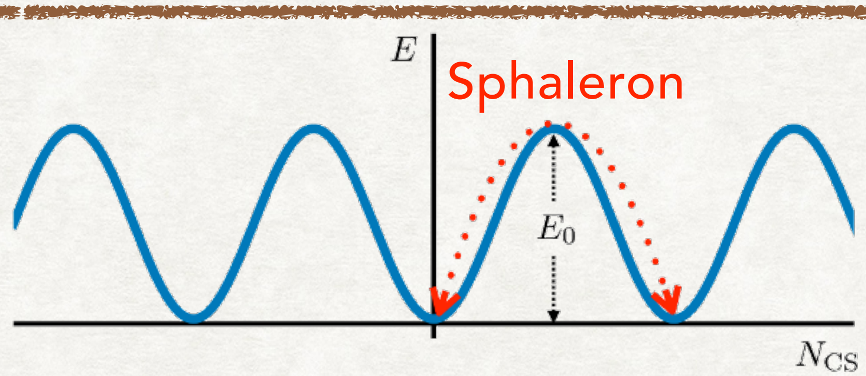


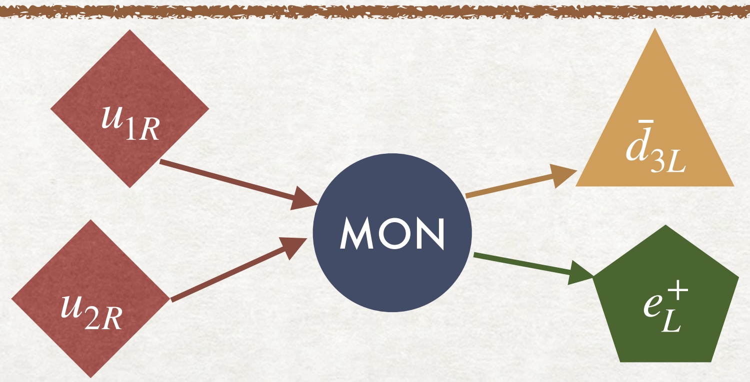
Low Sphaleron rate in Broken phase

Low monopole density in Broken phase

Both sphaleron and monopole can induce baryon number violation



$$N_{CS}(\text{sph}) = \frac{1}{2} \qquad \Gamma_{sph} = A(T)e^{-E_{sph}/T}$$

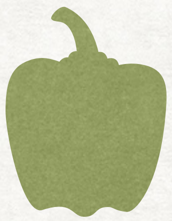


$$\Delta N_{CS}(\text{mon}) = 1 \qquad \Gamma_{mon} = B(T)e^{-m_M(T)/T}$$

Novel aspects of this study and its implications

Higgs

+



EW Multiplet Extension

- Complex doublet
- Triplet
- Quintuplet
- Septuplet
- ……

❖ Topological classification of field solutions (*sphaleron or monopole*) for general EW Multiplet during EWPT

❖ LO Baryon number violation rate computation (*sphaleron energy or monopole mass*) for EW multiplet extension to SM

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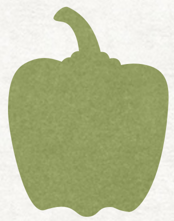

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➡ The topological perspective opens a new window for BSM research during EWPT, which also has implications for Dark Matter

Novel aspects of this study and its implications

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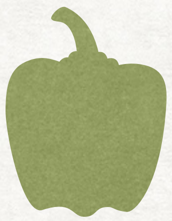
+



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➡ The topological perspective opens a new window for BSM research during EWPT, which also has implications for Dark Matter

➡ The baryon number violation rate computation can tell us if the created baryon asymmetry can be preserved, which is the condition for "Strong" first order EWPT

Why the topological analysis is important?

- In the literature, the **sphaleron** is regarded the main way for baryon number violation
- While **monopole** solution could appear in many BSM models, which can also lead to baryon number violation

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- In the literature, the **sphaleron** is regarded the main way for baryon number violation

- While **monopole** solution could appear in many BSM models, which can also lead to baryon number violation

Topological analysis can tell us whether sphaleron or monopole solution emerge during the EWPT.

Topological classification of field solution

Before EWSB: $G = SU(2)_L \times U(1)_Y$

After EWSB:

- If Multiplet Φ with $Y \neq 0$: $\quad H = U(1)_{em}$

$$\frac{G}{H} = \frac{SU(2)_L \times U(1)_Y}{U(1)_{em}} \simeq S^3$$

Sphaleron topology

i.e. SM Higgs

- If Multiplet Φ with $Y = 0$: $H = U(1)_{em} \times U(1)_Y$

$$\frac{G}{H} = \frac{SU(2)_L \times U(1)_Y}{U(1)_{em} \times U(1)_Y} \simeq S^2$$

Monopole topology

i.e. real triplet

$Y = 0$ multiplet can contribute to DM relic density

Complex septuplet ($Y = 0$) extension to the SM

- The Higgs field and septuplet field

1812.07829

$$V = V_0(H) + V_{portal}(H, \Phi) + V_{self}(\Phi)$$

$$H = \begin{pmatrix} \omega^+ \\ \frac{1}{\sqrt{2}}(v + h + i\pi) \end{pmatrix}$$

$$\Phi = \begin{pmatrix} \phi_{3,3} \\ \phi_{3,2} \\ \phi_{3,1} \\ \frac{1}{\sqrt{2}}(v_\phi + \phi + i\pi_\phi) \\ \phi_{3,-1} \\ \phi_{3,-2} \\ \phi_{3,-3} \end{pmatrix}$$

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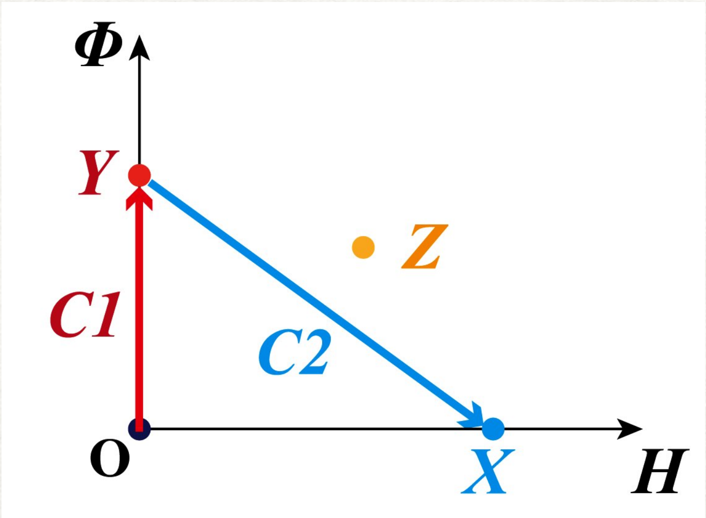
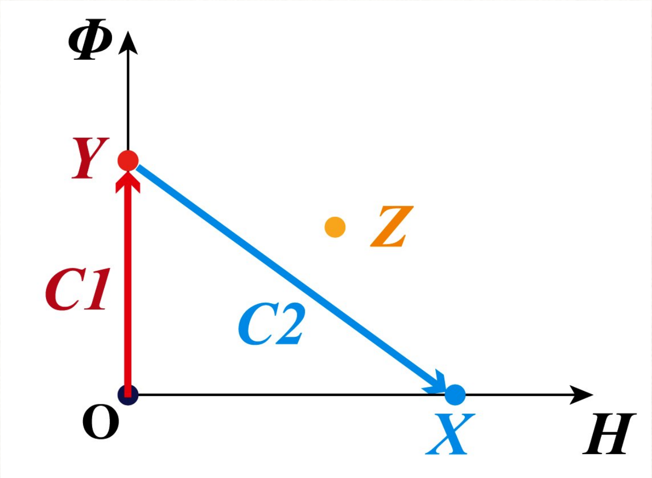


Baryon asymmetry could be made in $C1$, so it's important to know the Baryon number dilution rate (**monopole mass**) in broken phase at Y.

Complex septuplet ($Y = 0$) extension to the SM

- Monopole mass in the broken phase



λ_{13}: effective portal coupling

λ_s : effective self coupling

$$\Gamma_{BNV} \sim e^{-m/T}$$

$$m = B \times \frac{4\pi v}{g}$$

m :Sphaleron energy or monopole mass

Complex septuplet ($Y = 0$) extension to the SM

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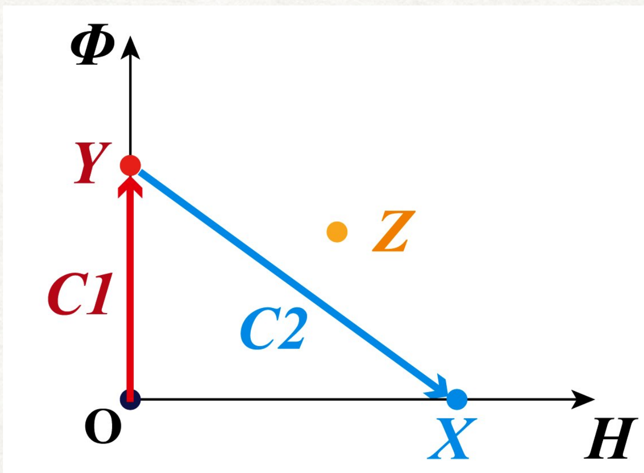

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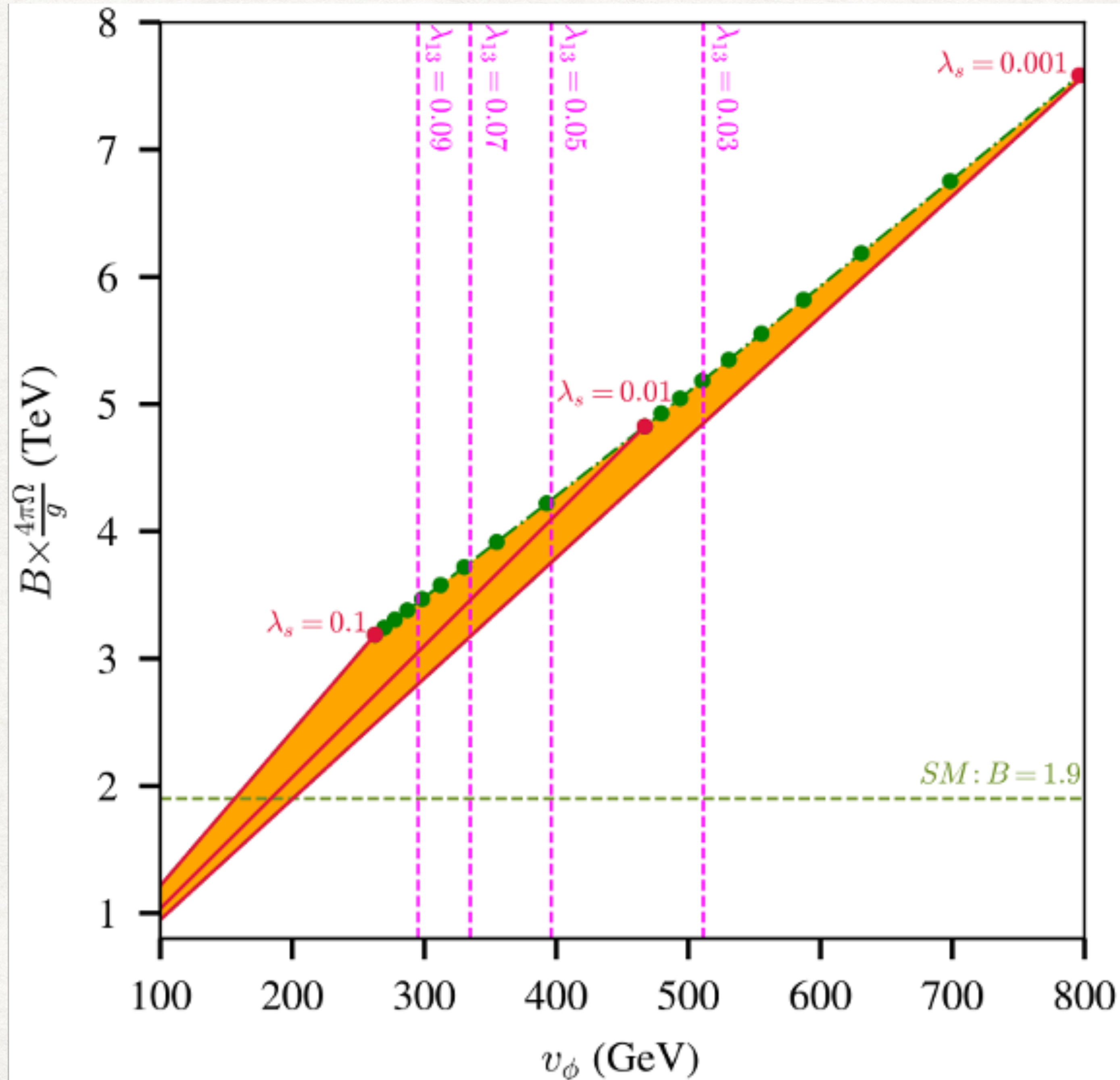

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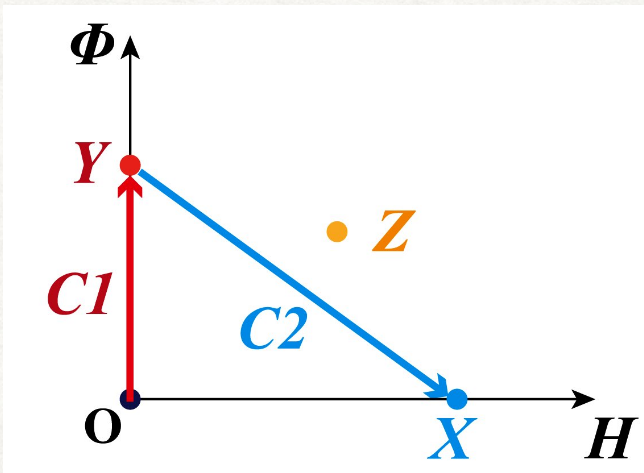
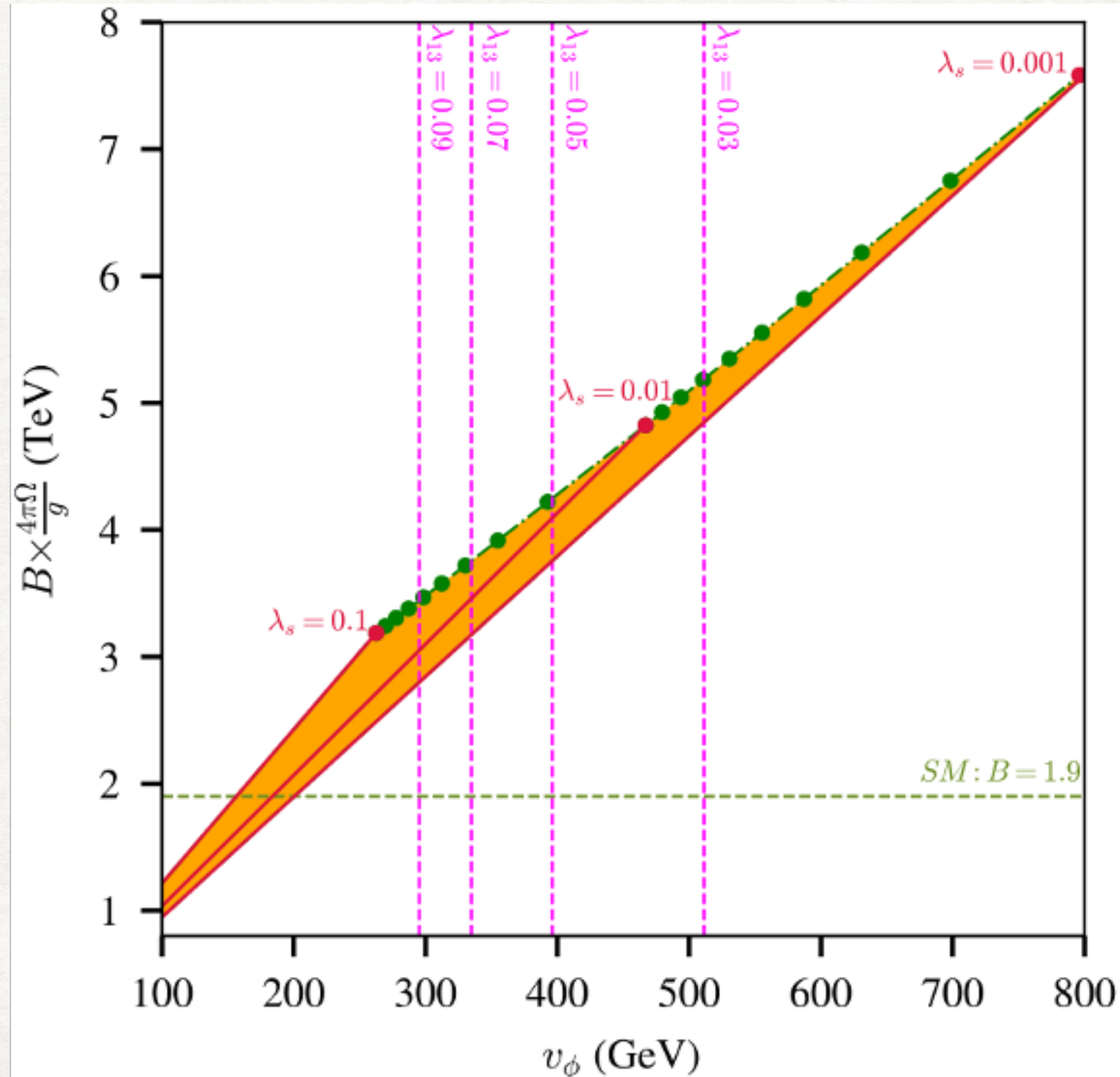


$$\frac{4\pi B}{g}\frac{\bar{v}\left(T_C\right)}{T_C} - 6\ln\frac{\bar{v}\left(T_C\right)}{T_C} > f(X, \frac{\Delta t_{EW}}{t_H}, \mathscr{Z}, \kappa)$$



BNPC CAN BE SATISFIED DURING THE FIRST BROKEN PHASE FOR LARGE MULTIPLET VEV

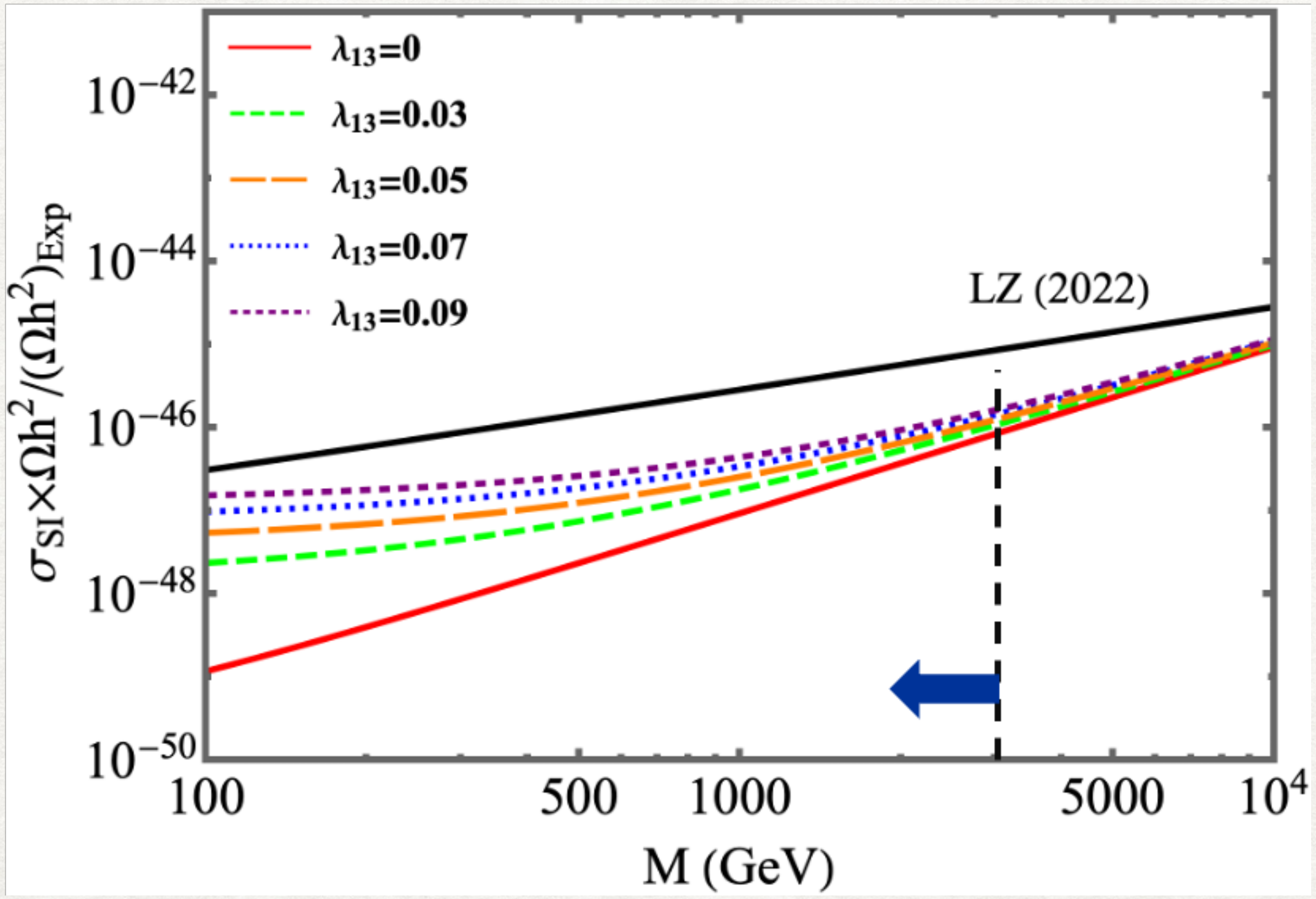
Patel, Ramsey-Musolf, JHEP 07 (2011) 029

Dark matter direct detection constrain

At the end of two-step EWPT, septuplet doesn't mix with Higgs field

Cross section: $\sigma_{SI} \sim g_{eff}^2$ $\quad g_{eff}^2 = f_N \frac{2\lambda_{13}}{m_h^2} + \frac{3}{4} f_T f_N^{PDF}$

Scaled cross section



Conclusion

- The **topological classification** of field solutions for EW multiplet Φ:
  - $Y = 0$: **Monopole** solution - Dark Matter
  - $Y \neq 0$: **Sphaleron** solution
- The **monopole mass** can reach a **substantial magnitudes** during the two-step EWPT, thereby facilitating the fulfillment of **BNPC**.
- The $Y = 0$ **complex septuplet** can contribute to the **dark matter** relic density.

Thanks!

Invariance property of the 1-form

- Electroweak scalar multiplet

$$E_{sph} = \frac{4\pi v}{g}\mathscr{F}(A_i^a, H, \Phi) \quad A_i^a T^a \sim f(\xi)(\partial_i U^\infty)U^{\infty -1} \quad i(U^{\infty -1})dU^\infty = \sum_{a=1}^{3} F_a T_a$$

Ahriche et.al. (2014) use the invariance property of F_a without proof

- Construction of general dimensional sphaleron unitary matrix

Express U^∞ as the multiplication of two Wigner-D matrices

$$U^\infty_{mn}(\mu, \theta, \phi) = \sum_{m'} D^J_{mm'}(\omega_-, -\theta, \mu) D^J_{m'n}(\mu, \theta, \omega_+),$$

$$\omega_\pm = -\mu \pm (\phi - \frac{\pi}{2}) \quad .$$

We demonstrate that F_a is invariant when $J = \left[\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3\right]$.

The general sphaleron energy expression

$$
\begin{aligned}
E_{sph} &= E(\mu = \frac{\pi}{2}) - E(\mu = -\frac{\pi}{2}) \\
&= \frac{4\pi\Omega}{g} \int d\xi \left[\frac{1}{4} F^a_{ij} F^a_{ij}(\xi, \mu = \frac{\pi}{2}) + \frac{1}{4} f_{ij} f_{ij}(\xi, \mu = \frac{\pi}{2}) + (D_i H)^\dagger (D_i H)(\xi, \mu = \frac{\pi}{2}) \right. \\
&\qquad \left. + (D_i \Phi)^\dagger (D_i \Phi)(\xi, \mu = \frac{\pi}{2}) + V(H, \Phi)(\xi, \mu = \frac{\pi}{2}) - V(H, \Phi)(\xi, \mu = -\frac{\pi}{2}) \right] .
\end{aligned}
$$

The field EOMs

$$f'' + \frac{2}{\xi^2}(1-f)\Big[f(f-2) + f_3\left(1+f_3\right)\Big] + (1-f)(\frac{v^2h^2}{4\Omega^2} + \alpha\phi^2) = 0,$$

$$f_3'' - \frac{2}{\xi^2}\Big[3f_3 + f(f-2)\big(1+2f_3\big)\Big] + (\frac{v^2}{4\Omega^2}h^2 + \beta\phi^2)(f_0 - f_3) = 0,$$

$$f_0'' + \frac{2}{\xi^2}\left(1-f_0\right) - \frac{g'^2}{g^2}(\frac{v^2}{4\Omega^2}h^2 + \beta\phi^2)(f_0 - f_3) = 0,$$

$$h'' + \frac{2}{\xi}h' - \frac{2}{3\xi^2}h[2(1-f)^2 + (f_0 - f_3)^2] - \frac{1}{g^2v^2\Omega^2}\frac{\partial V[h,\phi]}{\partial h} = 0,$$

$$\phi'' + \frac{2}{\xi}\phi' - \frac{8\Omega^2\phi}{3v_\phi^2\xi^2}[2\alpha(1-f)^2 + \beta(f_0 - f_3)^2] - \frac{1}{g^2v_\phi^2\Omega^2}\frac{\partial V[h,\phi]}{\partial \phi} = 0,$$

$$\alpha = \frac{[J(J+1) - J_3^2]v_\phi^2}{2\Omega^2}, \quad \beta = \frac{J_3^2 v_\phi^2}{\Omega^2}.$$