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ARCANE Reweighting:
A Solution to the Negative Weights Problem in Collider Monte Carlo

Prasanth Shyamsundar, Fermilab Quantum Division

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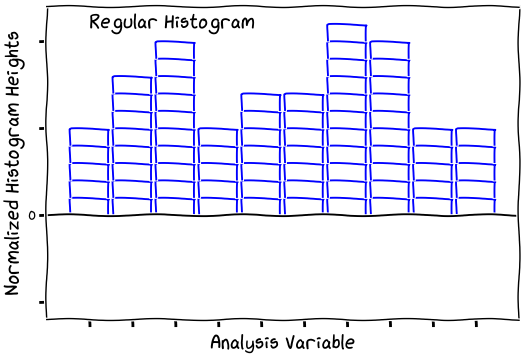
University of Pittsburgh and Carnegie Mellon University, Pittsburgh

What is the Negative Weights Problem in Collider Monte Carlo?



Histogram height:

$$\widehat{F}_B(b) = \frac{1}{N}\sum_{i=1}^{N} W_i\,\delta_K(B_i, b)$$

Histogram error estimate:

$$\widehat{\sigma}_B(b) = \frac{1}{\sqrt{N}}\sqrt{\frac{1}{N}\sum_{i=1}^{N} W_i^2\,\delta_K(B_i, b)}$$

$$\frac{\widehat{\sigma}_B(b)}{\widehat{F}_B(b)} = \frac{\sqrt{\sum_{i=1}^{N} W_i^2\,\delta_K(B_i, b)}}{\sum_{i=1}^{N} W_i\,\delta_K(B_i, b)}$$

- Individual events can have negative weights—artifact of our theory formalisms.
- The distribution of *any* physical (IR-safe) observable will be non-negative.
- Weight variance is bad. Variance from negative weights cannot be unweighted away.

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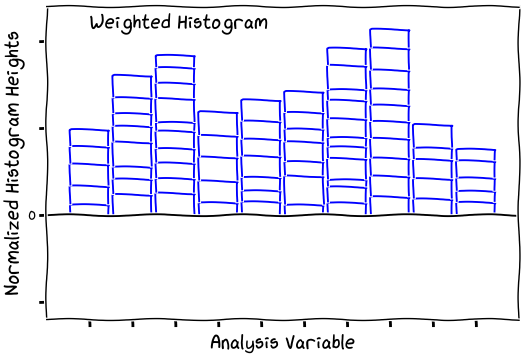

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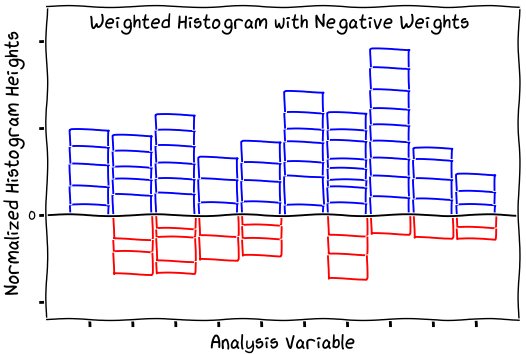

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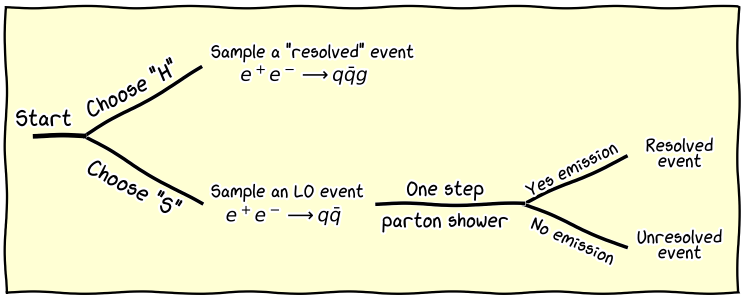
Origin of the Negative Weights Problem

- Different pathways within an event generator can lead to practically identical events (for the purposes of subsequent simulation steps and analyses).
- These different pathways can provide positive or negative contributions.
- Example: $\mathbb{H}$- and $\mathbb{S}$-type events in MC@NLO.



Generator flowchart for MC@NLO $e^+e^- \to q\bar{q} + 1\,\text{jet}$ events

Phrasing the Problem in Terms of Latent Variables

$$F_V^{\mathrm{ph}}(v) = \int_{\mathcal{H}} \mathrm{d}h \, F_{(V,H)}^{\mathrm{ph}}(v, h)$$

- V: Event attributes that are "visible" to the subsequent simulation stages.
- H: Latent or hidden event attributes that are not used by subsequent stages. Example: A variable indicating whether the event was of $\mathbb{H}$- or $\mathbb{S}$- type.
- Integration wrt h represents an aggregation or marginalization over the different pathways that lead to the same V.
- Even if $F_V^{\mathrm{ph}}(v)$ is guaranteed to be non-negative, $F_{(V,H)}^{\mathrm{ph}}(v, h)$ could be negative.
- Sidenote: In practice, $F_V^{\mathrm{ph}}(v)$ itself could be negative for some values of v.

- **Traditional solution approach:** Create an alternative $F_{(V,H')}^{\mathrm{ph}'}$ with less negative weights problem. **But this is difficult in practice.**

ARCANE Reweighting

- Idea: Modify $F^{\text{ph}}_{(V,H)}$ as follows:
$$F^{\text{arcane}}_{(V,H)}(v,h) \equiv F^{\text{ph}}_{(V,H)}(v,h) + G_{(V,H)}(v,h)$$
where $G_{(V,H)}(v,h)$ integrates wrt h to 0.
- Question: How to sample weighted events from $F^{\text{arcane}}_{(V,H)}$?
- An option: Sample some events from $F^{\text{ph}}_{(V,H)}$ and some events from $G_{(V,H)}$.
But, this won't reduce the negative weights problem.
- ARCANE's approach: Incorporate $G_{(V,H)}$ with an additive reweighting:
$$\boxed{W^{\text{arcane}} \equiv W + \frac{G_{(V,H)}(V,H)}{P^{\text{mc}}_{(V,H)}(V,H)}}$$
This is the core idea of ARCANE reweighting.

ARCANE Reweighting

$$F^{\text{arcane}}_{(V,H)}(v,h) \equiv F^{\text{ph}}_{(V,H)}(v,h) + G_{(V,H)}(v,h)$$
$$W^{\text{arcane}} \equiv W + \frac{G_{(V,H)}(V,H)}{P^{\text{mc}}_{(V,H)}(V,H)}$$

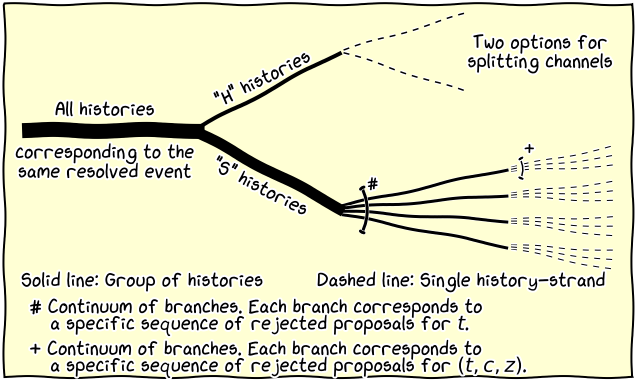
Ingredients of ARCANE Reweighting

- For each event, in addition to V, one needs to track and record H.
- One needs the ability to compute $P^{\text{mc}}_{(V,H)}$ for the different event records.
- **One needs to construct an appropriate redistribution function $G_{(V,H)}$.** $G_{(V,H)}$ can either be engineered or (machine) learned.

"So wait, you haven't actually solved anything yet?"

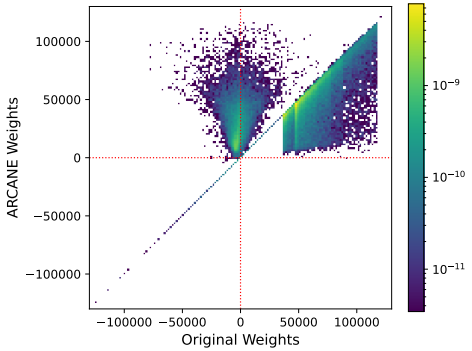
- **My claim:** It is way easier to construct a good $G_{(V,H)}$ than to construct a good alternative $F^{\text{ph}'}_{(V,H')}$ (that is suitable for performing Monte Carlo sampling with).

Sketch of Implementation for MC@NLO $(e^+e^- \longrightarrow q\bar{q} + 1\,\mathrm{jet})$ Events



- **Strategy:** Model $G_{(V,H)}$ as a chain of functions that systematically redistributes 0 across the different histories of a given "visible" event.
- My implementation of $G_{(V,H)}$ just involves performing two 1-dim polynomial fits (and an exploration of some alternative MC histories of each event).

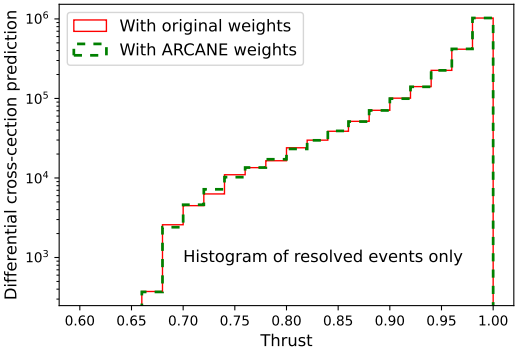
Some Results With MC@NLO $(e^+e^- \longrightarrow q\bar{q} + 1\,\mathrm{jet})$ Events



Weights of resolved events

Negative weights problem is reduced.

Note: Events along diagonal don't have contributions from both $\mathbb{H}$ and $\mathbb{S}$



Histogram of an example observable

Distributions of all "visible" observables will be unaffected.

Summary, What's Next?

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- ARCANE Reweighting Can Avoid Negative Events.
- The technique does not introduce any biases.
- The technique does not require any changes to the matching and merging prescriptions used.
- Events produced by this technique can be treated as being iid.

What's Next?

- Take on more complex and important processes like $(\mathrm{pp} \longrightarrow \mathrm{W} + \mathrm{jets})$ and $(\mathrm{pp} \longrightarrow \mathrm{t\bar{t}} + \mathrm{jets})$.

Thank you!

Acknowledgments

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