

Quasi-Elastic Lepton Nucleus Scattering and the Correlated Fermi Gas Model (arXiv: 2405.05342)

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Outline

- 1 Introduction
- 2 Quasielastic Lepton Scattering
- 3 Nucleon form factor
- 4 Models for the Nuclear Effect
 - The RFG Model
 - The CFG Model
 - Nucleon distribution
 - Differential cross-section
- 5 Results
 - Plots
- 6 Summary

Lepton-Nucleus QE Scattering

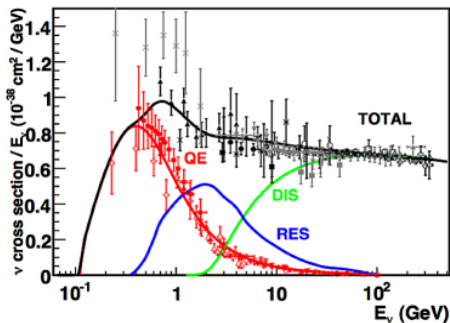


Figure: Total ν per nucleon CC cross sections
(Formaggio and Zeller, Rev. Mod. Phys. **84**, 1307 (2012))

- CCQE dominates cross-section at energies of interest for oscillation experiments.

Lepton-Nucleus QE Scattering

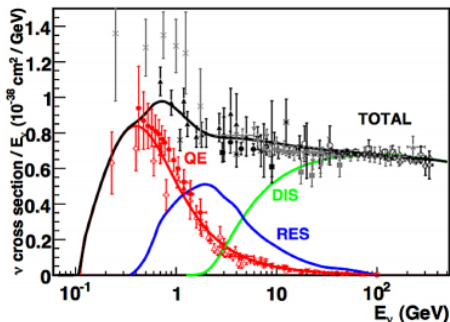


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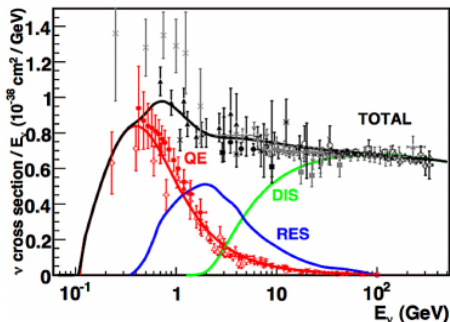


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($\nu_{\mu} n \rightarrow \mu^{-} p$)

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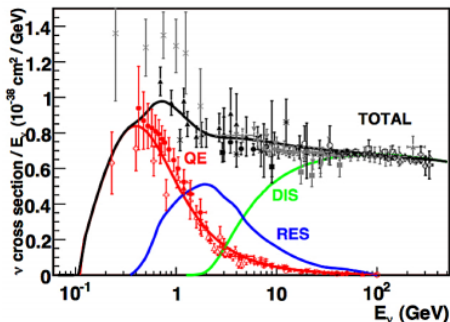


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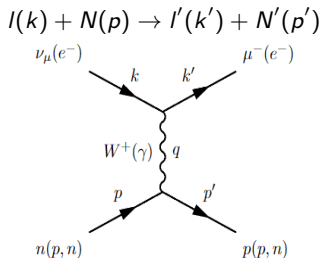
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 - ① **nucleon form factors**: lepton-quark interaction \Rightarrow lepton-nucleon interaction
($\nu_\mu n \rightarrow \mu^- p$)
 - ② **nuclear model**: nucleon \Rightarrow nuclear level ($\nu_\mu X_Z^A \rightarrow \mu^- p X_Z^{A-1}$)

Lepton-nucleon scattering process

Cross-section of Lepton-nucleon scattering

Bhattacharya, Hill and Paz, Phys. Rev. D, **84**, 073006 (2011)

For the process:



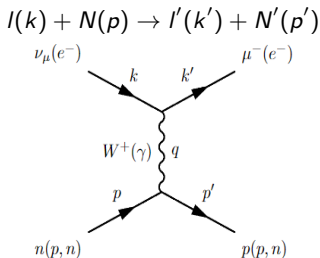
- For the Weak/Electromagnetic interaction, the Lagrangian is:

$$\mathcal{L} = C_{Weak,EM} \bar{l} \gamma^\mu (1 - \gamma_5) l \bar{q}_i \gamma_\mu (1 - \gamma_5) q_j, \left\{ C_{Weak,EM} = \frac{G_F}{\sqrt{2}} V_{ij}, \frac{e^2}{q^2} \right\}$$

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- The cross-section for the lepton-nucleon scattering:

$$\sigma_{nucleon} = \frac{\bar{C}_{EM,Weak}^2}{4|k \cdot p|} \int \frac{d^3 k'}{(2\pi)^3 2E_{k'}} L^{\mu\nu} \int \frac{d^3 p'}{(2\pi)^3 2E_{p'}} (2\pi)^4 \delta^4(p - p' + q) H_{\mu\nu}$$

Free nucleon cross-section

- nucleon tensor

$$H_{\mu\nu} = \text{Tr}[(\not{p}' + m_N)\Gamma_\mu(q)(\not{p} + m_N)\bar{\Gamma}_\nu(q)]$$

with the following decomposition:

$$H_{\mu\nu} = -g_{\mu\nu}H_1 + \frac{p_\mu p_\nu}{m_N^2}H_2 - i\frac{\epsilon_{\mu\nu\rho\sigma}}{2m_N^2}p^\rho q^\sigma H_3 + \frac{q_\mu q_\nu}{m_N^2}H_4 + \frac{(p_\mu q_\nu + q_\mu p_\nu)}{2m_N^2}H_5.$$

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- $\Gamma_\mu(q)$ defined via the matrix element of the EM or weak current as:

$$\langle N(p') | \bar{q}_i \gamma_\mu (1 - \gamma_5) q_j | N(p) \rangle = \bar{u}(p') \Gamma_\mu(q) u(p)$$

where $q = k - k' = p' - p$

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where $q = k - k' = p' - p$

- The vertex function can be expressed in terms of the form factors as:

$$\Gamma_\mu(q) = \gamma_\mu F_1(q^2) + \frac{i}{2m_N} \sigma_{\mu\nu} q^\nu F_2(q^2) + \gamma_\mu \gamma_5 F_A(q^2) + \frac{q_\mu}{m_N} \gamma_5 F_P(q^2),$$

Vector form factor (F_1 , F_2)

- ① **BBBA**: The functional form for this parameterization is given by (Bradford, Bodek, Budd and Arrington, Nucl. Phys. B Proc. Suppl., **159**:127–132 (2006))

$$G_{E/M}^{p,n} = \frac{\sum_{k=0}^2 a_k \tau^k}{1 + \sum_{k=1}^4 b_k \tau^k} \quad (\text{where } \tau = \frac{-q^2}{4m_N^2})$$

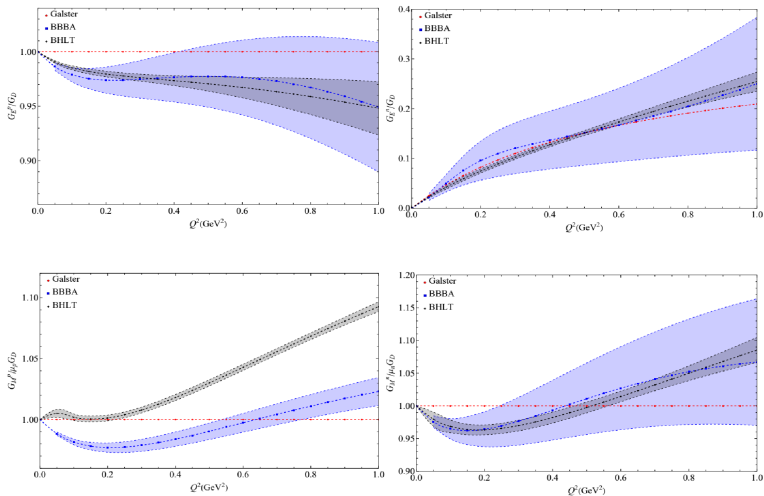
- ② **BHLT**: Determined from a global fit to electron scattering data. Expressed as convergent expansion variable $z(q^2)$ (Borah, Hill, Lee and Tomalak, Phys. Rev. D, **102**(7):074012, (2020))

$$G_E^{p,n} = \sum_{k=0}^{k_{\max}} a_k z(q^2)^k,$$

$$G_M^{p,n} = G_M^{p,n}(0) \sum_{k=0}^{k_{\max}} b_k z(q^2)^k,$$

$$z(q^2) = \frac{\sqrt{t_{\text{cut}} - q^2} - \sqrt{t_{\text{cut}} - t_0}}{\sqrt{t_{\text{cut}} - q^2} + \sqrt{t_{\text{cut}} - t_0}}$$

where, $t_{\text{cut}} = 4m_\pi^2$ and $t_0 = -0.21 \text{ GeV}^2$.

Vector form factor (F_1 , F_2)

Axial-Vector form factor (F_A, F_P)

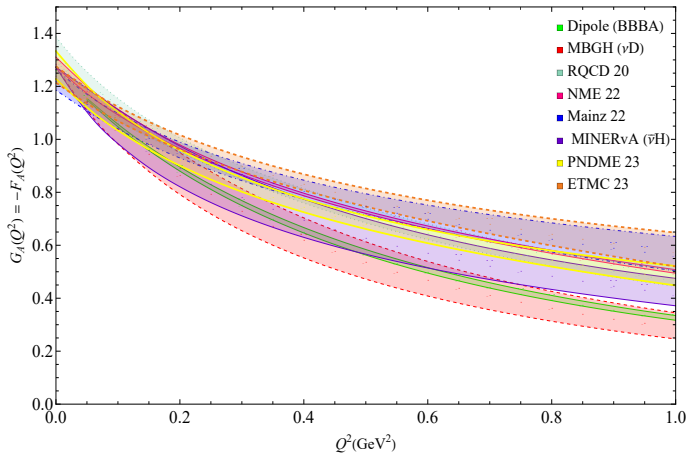
- The various axial-vector form factor models were extracted by using the z-expansion formalism. While, F_P is related to F_A by PCAC.

Scattering data:

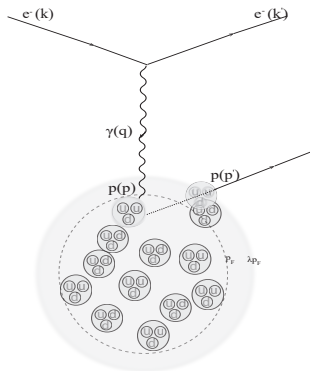
- 1 MBGH: Determined from charged-current neutrino-deuterium scattering data (Meyer, Betancourt, Gran, and Hill, *Phys. Rev. D*, **93**(11):113015, (2016)).
- 2 MINERvA: Extracted from the $\bar{\nu}$ -hydrogen scattering using the plastic scintillator target of MINERvA experiment (Cai, *et al.*, *Nature*, **614**(7946):48–53, (2023)).

Lattice QCD:

- 3 NME 22: (Park, *et al.*, *Phys. Rev. D*, **105**(5):054505, (2022))
- 4 Mainz 22: (Djukanovic, *et al.*, *Phys. Rev. D*, **106**(7):074503, (2022))
- 5 PNDME 23: (Jang, *et al.*, *Phys. Rev. D*, **109**(1):014503, (2024))
- 6 ETMC 23: (Alexandrou, *et al.*, *Phys. Rev. D*, **109**(3):034503, (2024))
- 7 RQCD 20: (Bali, *et al.*, *JHEP*, **05**:126, (2020))

Axial-Vector form factor (F_A , F_P)

Lepton-Nuclear Scattering Process



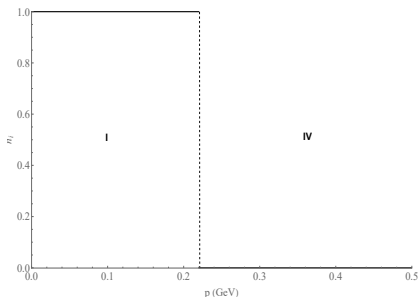
- Cross-section for lepton-nuclear scattering: convolution of free cross-section with nuclear distributions:

$$\sigma_{\text{nuclear}} = n_i(\mathbf{p}) \otimes \sigma_{\text{nucleon}}(\mathbf{p} \rightarrow \mathbf{p}') \otimes [1 - n_f(\mathbf{p}')],$$

$$d\sigma_{\text{nuclear}} = \frac{C_{\text{Weak,EM}}^2}{|k \cdot p|} \frac{d^3 k'}{2E_{k'}} L^{\mu\nu} W_{\mu\nu}$$

Relativistic Fermi Gas

Relativistic Fermi Gas



- Distribution of neutrons and protons in the model:

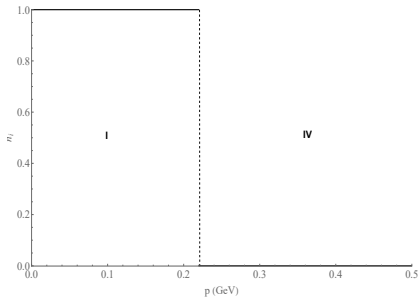
$$n^{RFG}(\mathbf{p}) = \theta(p_F - |\mathbf{p}|).$$

- Commonly used; nucleons: non-interacting Fermi gas in nucleus.

Figure: RFG momentum distribution

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Relativistic Fermi Gas



- Distribution of neutrons and protons in the model:

$$n^{RFG}(\mathbf{p}) = \theta(p_F - |\mathbf{p}|).$$

- Commonly used; nucleons: non-interacting Fermi gas in nucleus.
- Nucleons occupy all available energy states up to the maximum one, the Fermi energy E_F .

Figure: RFG momentum distribution

Correlated Fermi Gas

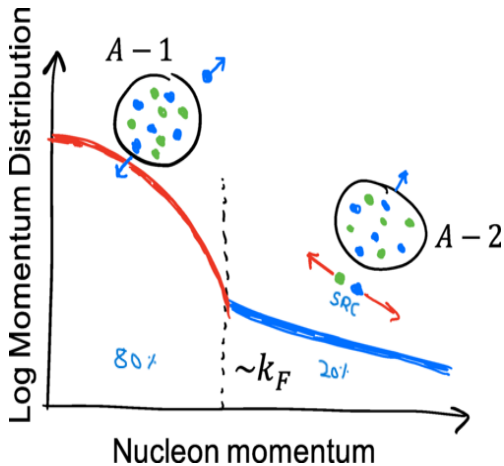


Figure: Inclusive and Exclusive electron scattering experiment observation of SRC pairs
 (Tropiano, Bogner and Furnstahl, Phys. Rev. C **104**, 034311 (2021))

Correlated Fermi Gas

Hen, Li, Guo, Weinstein and Piasetzky, Phys. Rev. C, **91**, 025803 (2015):

$$n^{\text{CFG}}(\mathbf{p}) = \begin{cases} 1 - \left(1 - \frac{1}{\lambda}\right) \frac{c_0}{\pi^2} \equiv \alpha_0 & |\mathbf{p}| \leq p_F \\ \frac{c_0}{3\pi^2} \left(\frac{p_F}{|\mathbf{p}|}\right)^4 \equiv \frac{\alpha_1}{|\mathbf{p}|^4} & p_F \leq |\mathbf{p}| \leq \lambda p_F \\ 0 & |\mathbf{p}| \geq \lambda p_F. \end{cases}$$

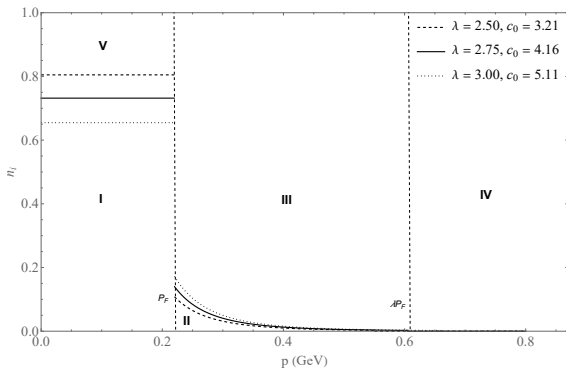
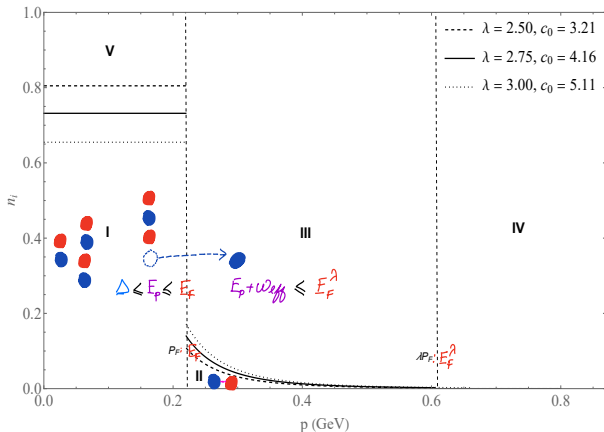


Figure: CFG momentum distributions

Limits of integration: CFG (I \rightarrow III)

$$I \rightarrow III: E_{\text{low}} = \max(\Delta, E_F - \omega_{\text{eff}}), \quad E_{\text{high}} = \min(E_F, E_F^\lambda - \omega_{\text{eff}}).$$

Differential cross-section

- In the rest frame of the nucleon, let $E_l (= |P_l|$ for $m_{e,\nu} \approx 0$) be the energy of the outgoing charged lepton, and θ_l be the angle between incoming and outgoing lepton.

$$\frac{d\sigma_{nuclear}^e}{d\Omega_\ell dE_\ell} = \frac{\alpha^2 E_\ell^2}{4\pi Q^4 m_T} [2W_1(1 - \cos\theta_\ell) + W_2(1 + \cos\theta_\ell)]$$

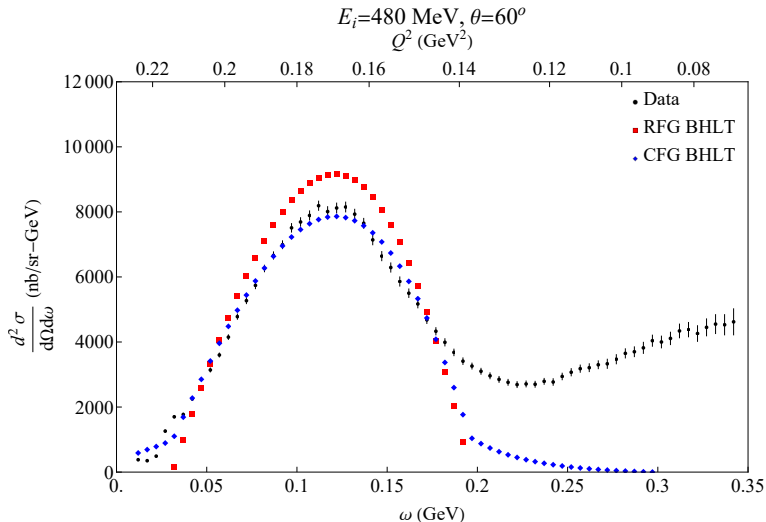
$$\begin{aligned} \frac{d\sigma_{nuclear}^\nu}{d\Omega_\ell dE_\ell} = & \frac{G_F^2 |\vec{P}_\ell|}{32\pi^3 m_T} \left\{ 2(E_\ell - |\vec{P}_\ell| \cos\theta_\ell) W_1 + (E_\ell + |\vec{P}_\ell| \cos\theta_\ell) W_2 \right. \\ & \left. \pm \frac{1}{m_T} \left[(E_\ell - |\vec{P}_\ell| \cos\theta_\ell)(E_\nu + E_\ell) - m_\ell^2 \right] W_3 + \frac{m_\ell^2}{m_T^2} (E_\ell - |\vec{P}_\ell| \cos\theta_\ell) W_4 - \frac{m_\ell^2}{m_T} W_5 \right\}, \end{aligned}$$

- To calculate the cross-section we add all the possible transitions, namely,

$$d\sigma = d\sigma_{I \rightarrow III} + d\sigma_{I \rightarrow IV} + d\sigma_{I \rightarrow V} + d\sigma_{II \rightarrow III} + d\sigma_{II \rightarrow IV} + d\sigma_{II \rightarrow V}.$$

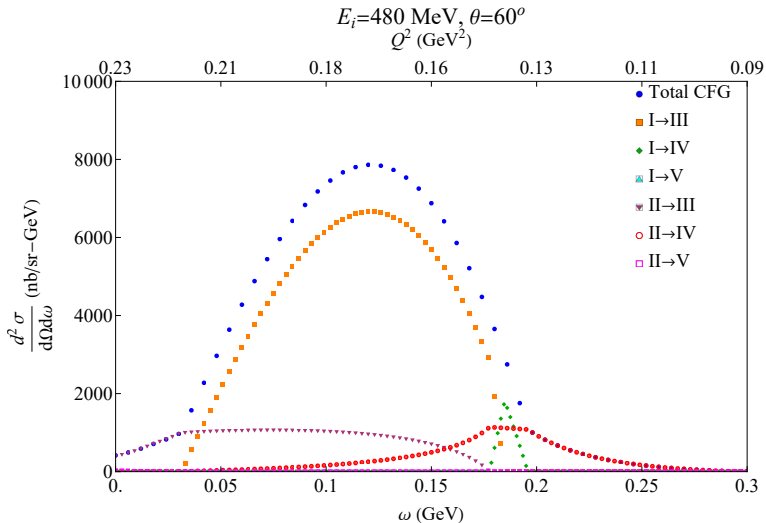
Results

Electron Scattering: 480 MeV; 60°

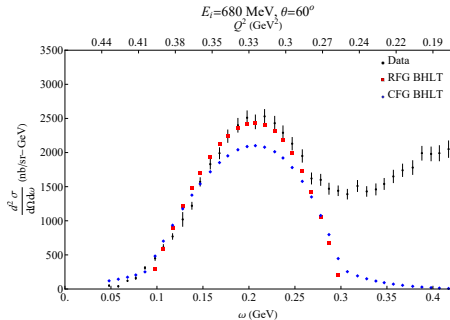
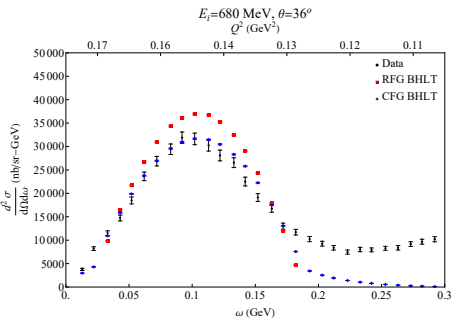


- **Fix vector form factor and vary nuclear models** in comparison with carbon data (<http://discovery.phys.virginia.edu/research/groups/qes-archive>).

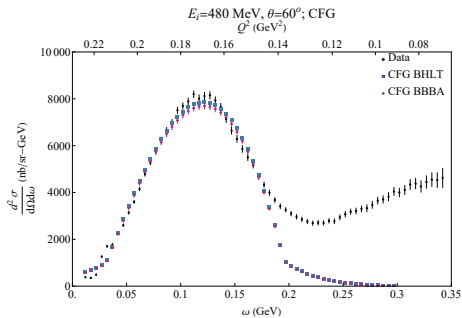
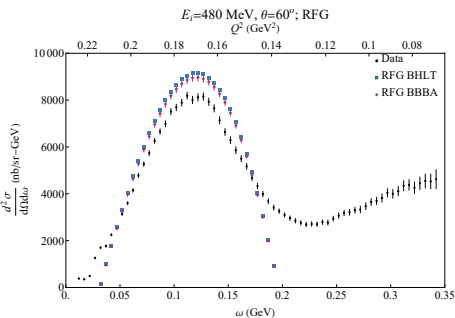
Electron Scattering: 480 MeV; 60° (by transitions)



Electron Scattering: 680 MeV; 36°, 60°

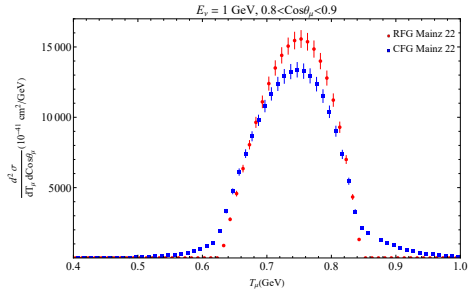
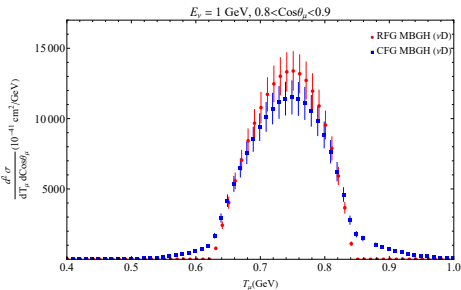


Electron Scattering: Form factor comparison



- **Fix nuclear model** and **vary form factor** models: BBBA and BHLT.
- Observe: differences from the form factors models small compared to differences between RFG and CFG.

Neutrino Scattering: Before flux averaging



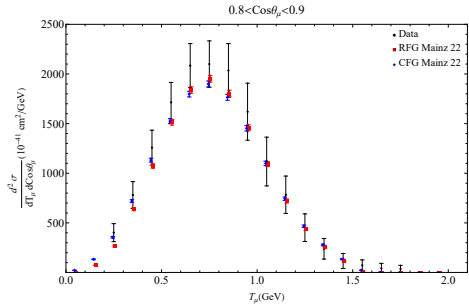
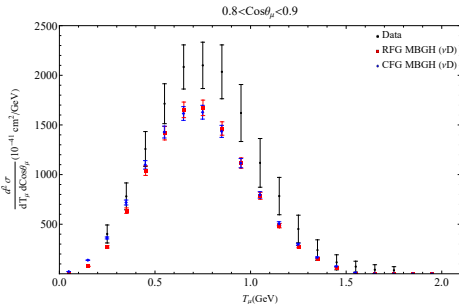
Where, $T_\mu = E_\nu - m_\mu - \omega$

- Hypothetical scenario: fixed neutrino energy.
- F_1 and F_2 : BHLT parameterization, F_A : MBGH and Mainz22.

Neutrino Scattering: After flux averaging

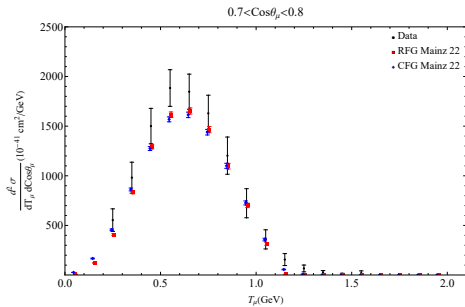
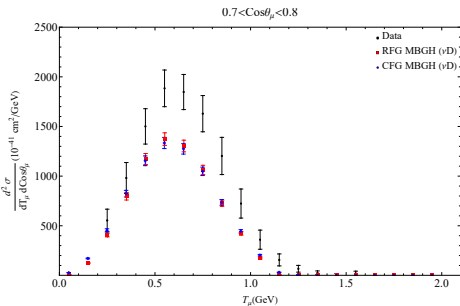
The neutrino cross-section is convoluted with the neutrino flux distribution:

$$\frac{d\sigma_{\text{carbon, per nucleon, avg.}}}{dE_\ell d \cos \theta_\ell} = \int dE_\nu f(E_\nu) \frac{d\sigma_{\text{carbon, per nucleon}}}{dE_\ell d \cos \theta_\ell}.$$

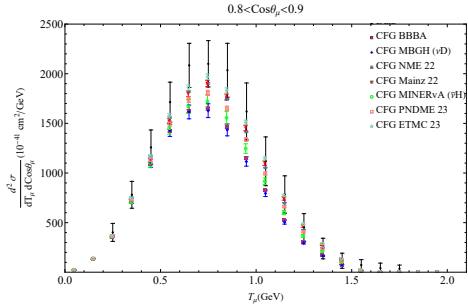
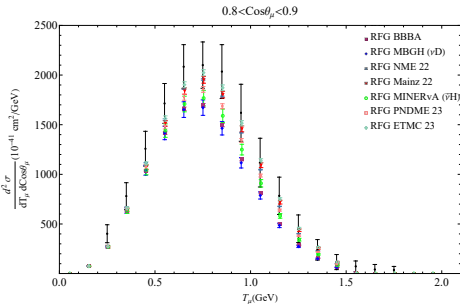


- **Fix axial form factor and vary nuclear models** in comparison to MiniBooNE data (Aguilar-Arevalo, *et al.*, Phys. Rev. D, 81:092005, (2010)).
- flux averaging \Rightarrow indistinguishable RFG, CFG cross-section data.

Neutrino Scattering: After flux averaging

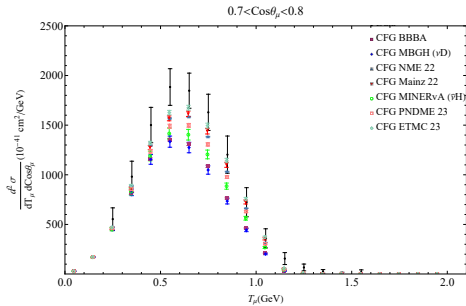
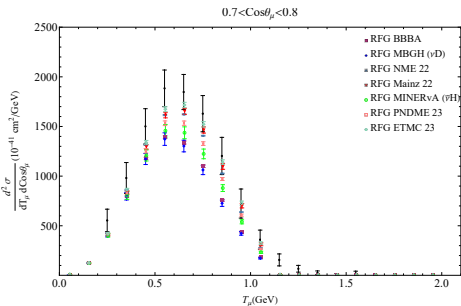


Neutrino Scattering: Axial form factor



- **Vary axial form factor and fix nuclear model.**
- Continuous spread from F_A parameterizations for both nuclear models.

Neutrino Scattering: Axial form factor



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- **Electron scattering**: distinguish two nuclear models; CFG model for small and large ω has a “tail”. At the peak CFG prediction smaller than of RFG; difference between form factors small.
- **Neutrino scattering**: indistinguishable - two nuclear models using either MBGH or Mainz22; Mainz22 ($\approx m_A^{\text{dipole, MiniBooNE}}$) fits data much better than MBGH; almost continuous “spread” using F_A parameterizations for both RFG and CFG nuclear models.

Thank You!

Backup Slides

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Lepton and Hadron tensor

- The leptonic tensor is:

$$L_{EM}^{\mu\nu} = 4(k^\mu k'^\nu + k^\nu k'^\mu - g^{\mu\nu} k \cdot k')$$

$$L_{Weak}^{\mu\nu} = 8(k^\mu k'^\nu + k^\nu k'^\mu - g^{\mu\nu} k \cdot k' - i\epsilon^{\mu\nu\rho\sigma} k_\rho k'_\sigma)$$

- The H_i 's are expressed in terms of the form factors F_i as,

$$H_1 = 8m_N^2 F_A^2 - 2q^2 \left[(F_1 + F_2)^2 + F_A^2 \right]$$

$$H_2 = H_5 = 8m_N^2 (F_1^2 + F_A^2) - 2q^2 F_2^2,$$

$$H_3 = -16m_N^2 F_A (F_1 + F_2),$$

$$H_4 = -\frac{q^2}{2} (F_2^2 + 4F_P^2) - 2m_N^2 F_2^2 - 4m_N^2 (F_1 F_2 + 2F_A F_P).$$

- The pseudo-scalar form factor related to F_A by Partial Conservation of Axial Current is:

$$F_P(q^2) = \frac{2m_N^2}{m_\pi^2 - q^2} F_A(q^2)$$

Correlated Fermi Gas

- We use the model by O. Hen, *et al.*, Phys. Rev. C, **91**, 025803:

$$n^{CFG}(\mathbf{p}) = \begin{cases} A_0 & |\mathbf{p}| \leq p_F \\ c_0 p_F / p^4 & p_F \leq |\mathbf{p}| \leq \lambda p_F \\ 0 & |\mathbf{p}| \geq \lambda p_F, \end{cases}$$

where λ is the high momentum cut-off, p_F is Fermi momentum.

- The factor A_0 is determined by normalization:

$$1 = 2 \int \frac{d^3 p}{(2\pi)^3} n^{CFG}(\mathbf{p}),$$

$$\implies A_0 = \frac{3\pi^2}{p_F^3} \left[1 - \left(1 - \frac{1}{\lambda} \right) \frac{c_0}{\pi^2} \right].$$

Phase space integrals

- The nuclear structure function:

$$W_{\mu\nu} = \int d^3p f(\mathbf{p}, q^0, \mathbf{q}) H_{\mu\nu}(\epsilon_p, \mathbf{p}; q^0, \mathbf{q}),$$

expanded similar to hadronic tensor,

$$W_{\mu\nu} = -g_{\mu\nu} W_1 + \frac{p_\mu^T p_\nu^T}{m_T^2} W_2 - \frac{i\epsilon_{\mu\nu\rho\sigma} p_T^\rho p_T^\sigma}{2m_T^2} W_3 + \frac{q_\mu q_\nu}{m_T^2} W_4 + \frac{p_\mu^T q_\nu + q_\mu p_\nu^T}{2m_T^2} W_5$$

- The W_i 's are written in terms of the H_i 's and the phase-space integrals a_i 's,

$$W_1 = a_1 H_1 + \frac{1}{2}(a_2 - a_3) H_2,$$

$$W_2 = \left[a_4 + \frac{\omega^2}{|\mathbf{q}|^2} a_3 - 2 \frac{\omega}{|\mathbf{q}|} a_5 + \frac{1}{2} \left(1 - \frac{\omega^2}{|\mathbf{q}|^2} \right) (a_2 - a_3) \right] H_2,$$

$$W_3 = \frac{m_T}{m_N} \left(a_7 - \frac{\omega}{|\mathbf{q}|} a_6 \right) H_3,$$

$$W_4 = \frac{m_T^2}{m_N^2} \left[a_1 H_4 + \frac{m_N}{|\mathbf{q}|} a_6 H_5 + \frac{m_N^2}{2|\mathbf{q}|^2} (3a_3 - a_2) H_2 \right],$$

$$W_5 = \frac{m_T}{m_N} \left(a_7 - \frac{\omega}{|\mathbf{q}|} a_6 \right) H_5 + \frac{m_T}{|\mathbf{q}|} \left[2a_5 + \frac{\omega}{|\mathbf{q}|} (a_2 - 3a_3) \right] H_2, \quad (1)$$

Phase space integrals

- The a_i 's are written in terms of the distribution function:

$$a_1 = \int d^3\mathbf{p} f(\mathbf{p}, q^0, \mathbf{q}),$$

$$a_2 = \int d^3\mathbf{p} f(\mathbf{p}, q^0, \mathbf{q}) \frac{|\mathbf{p}|^2}{m_N^2},$$

$$a_3 = \int d^3\mathbf{p} f(\mathbf{p}, q^0, \mathbf{q}) \frac{p_z^2}{m_N^2},$$

$$a_4 = \int d^3\mathbf{p} f(\mathbf{p}, q^0, \mathbf{q}) \frac{\epsilon_{\mathbf{p}}^2}{m_N^2},$$

$$a_5 = \int d^3\mathbf{p} f(\mathbf{p}, q^0, \mathbf{q}) \frac{\epsilon_{\mathbf{p}} p_z}{m_N^2}.$$

$$a_6 = \int d^3\mathbf{p} f(\mathbf{p}, q^0, \mathbf{q}) \frac{p_z}{m_N}.$$

$$a_7 = \int d^3\mathbf{p} f(\mathbf{p}, q^0, \mathbf{q}) \frac{\epsilon_{\mathbf{p}}}{m_N^2}.$$

Limits of integration: RFG

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- The definite integrals above have the limits $E_{\text{low}} \leq E_p \leq E_{\text{high}}$
- only one transition possible (I \rightarrow IV)
- first condition arises from the constraints on the scattering angle:

$$-1 \leq \cos \theta_{pq} \leq 1 \quad \Rightarrow \quad -1 \leq \frac{\omega_{\text{eff}}^2 - |\mathbf{q}|^2 + 2\omega_{\text{eff}} E_p}{2|\mathbf{q}|\sqrt{E_p^2 - m_N^2}} \leq 1,$$

- The latter condition can be expressed as

$$\left(\frac{E_p}{m_N} - \frac{cd + \sqrt{1 - c^2 + d^2}}{1 - c^2} \right) \left(\frac{E_p}{m_N} - \frac{cd - \sqrt{1 - c^2 + d^2}}{1 - c^2} \right) \geq 0.$$

Defining $\Delta \equiv m_N(cd + \sqrt{1 - c^2 + d^2})/(1 - c^2)$ this implies $\Delta \leq E_p$.

- Therefore the limits for the RFG transition are:

$$E_{lo} = \max(E_F - \omega_{\text{eff}}, \Delta), E_{hi} = E_F.$$

Limits of integration: CFG

- **I** → **III**:

$$E_{\text{low}} = \max(\Delta, E_F - \omega_{\text{eff}}), \quad E_{\text{high}} = \min(E_F, E_F^\lambda - \omega_{\text{eff}}).$$

- **I** → **IV**:

$$E_{\text{low}} = \max(\Delta, E_F^\lambda - \omega_{\text{eff}}), \quad E_{\text{high}} = E_F.$$

- **I** → **V**:

$$E_{\text{low}} = \max(\Delta, -\omega_{\text{eff}}), \quad E_{\text{high}} = \min(E_F, E_F - \omega_{\text{eff}}).$$

- **II** → **III**:

$$E_{\text{low}} = \max(\Delta, E_F, E_F - \omega_{\text{eff}}), \quad E_{\text{high}} = \min(E_F^\lambda, E_F^\lambda - \omega_{\text{eff}})$$

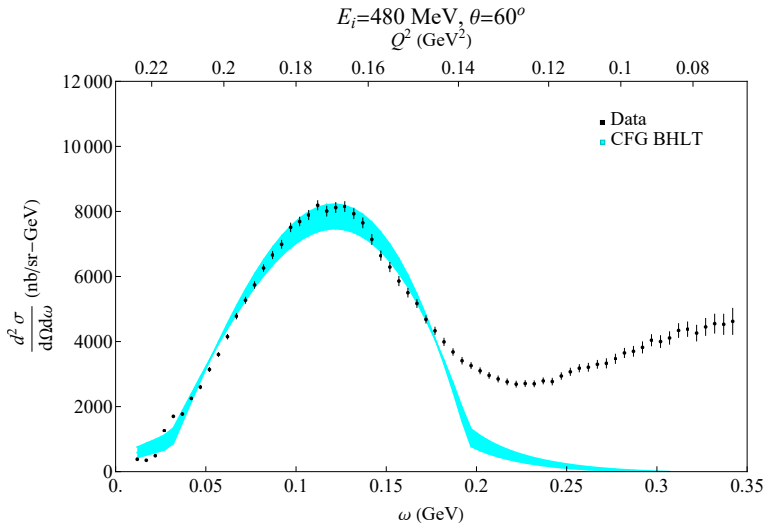
- **II** → **IV**:

$$E_{\text{low}} = \max(\Delta, E_F, E_F^\lambda - \omega_{\text{eff}}), \quad E_{\text{high}} = E_F^\lambda.$$

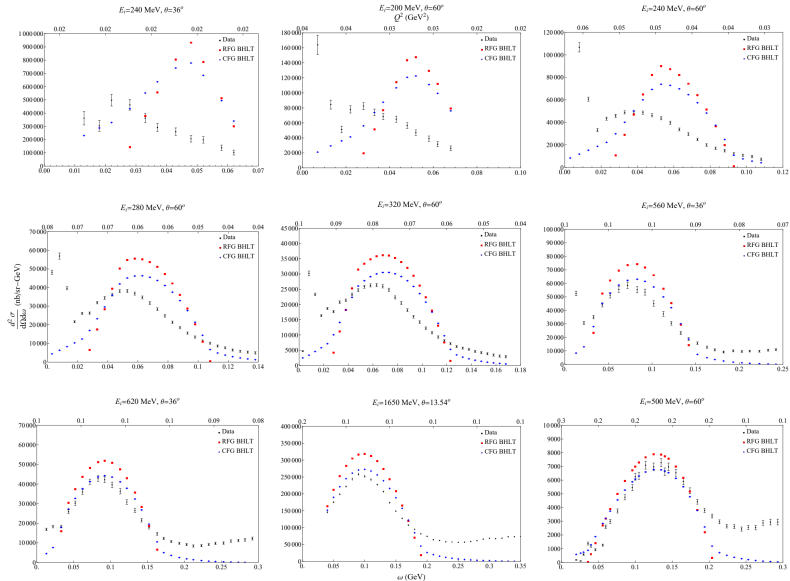
- **II** → **V**:

$$E_{\text{low}} = \max(\Delta, E_F, -\omega_{\text{eff}}), \quad E_{\text{high}} = \min(E_F^\lambda, E_F - \omega_{\text{eff}}).$$

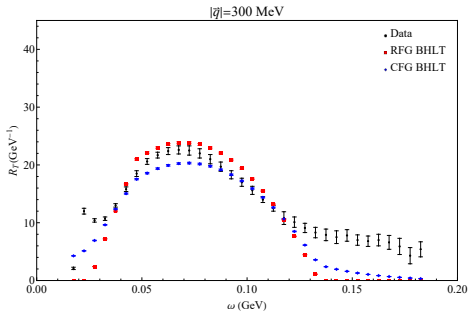
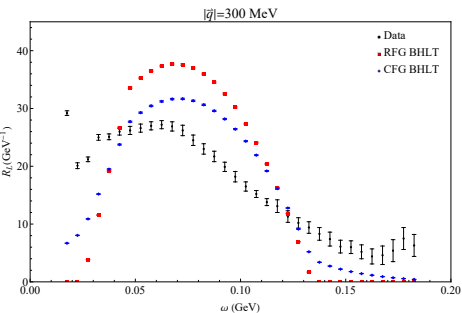
More plots



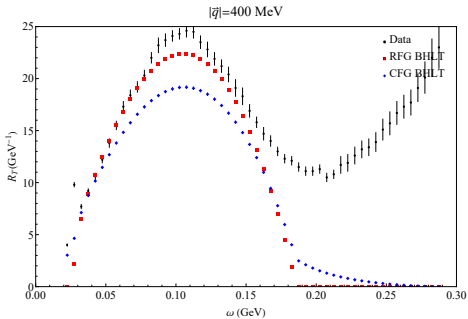
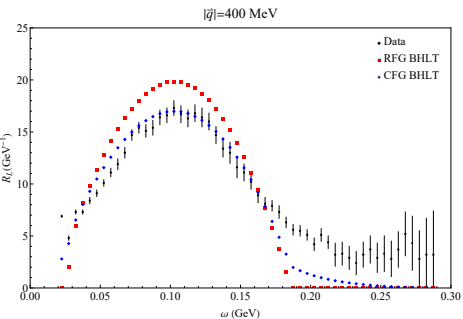
More plots



More plots: Responses ($\vec{q} = 300 \text{ MeV}$)



More plots: Responses ($\vec{q} = 400 \text{ MeV}$)



More plots: Responses ($\vec{q} = 550 \text{ MeV}$)

