Radiative Corrections to low energy electron proton scattering

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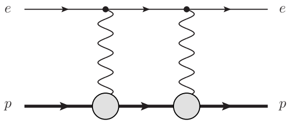
Overview

Having a handle on radiative corrections for lepton nucleon scattering, specifically that of electron (positron) scattering with protons has several applications [1]

- Extraction of electromagnetic form factors defining the protons charge distribution at low energies (Rosenbluth vs Polarization Transfer Method)
- Determination of the proton charge radius
- Testing of higher energy effects like resonances in diagrams associated with the process, most importantly the crossed and direct box diagrams, also known as Two Photon Exchange (TPE) diagrams.



[1] For a full overview of current progress see Afanasev, A et al. (2023). "Radiative Corrections: From Medium to High Energy Experiments.", arXiv:2306.14578 [hep-ph]

Overview

In this work we present a calculation of NLO QED corrections to electron proton scattering with the aim to perform a calculation without making some of the approximations used in the literature.

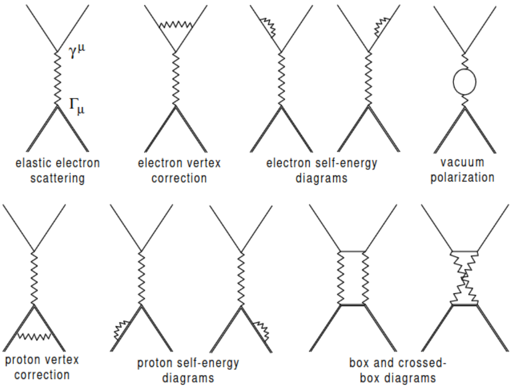


Figure: Maximon, L.C., Tjon, J.A. arXiv:nucl-th/0002058

In this calculation we use a proton-photon vertex function where the form factors of the proton depend on the photon momentum

$$\Gamma_\mu = F_1(q^2)\gamma_\mu + F_2(q^2)\frac{i\kappa}{2M}\sigma_{\mu\nu}q^\nu$$

Where in our case we take

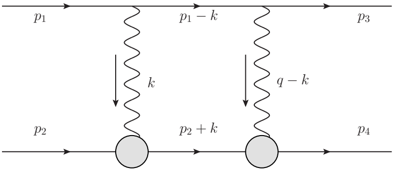
$$F_1(q^2) = F_2(q^2) = \left(\frac{-\Lambda^2}{q^2-\Lambda^2}\right)^n, \qquad n = \begin{cases} 1 & \text{monopole} \\ 2 & \text{dipole} \end{cases}$$

TPE in electron photon scattering is an active field

Our work is focused predominantly around the TPE calculation as it remains an important and complex part of any set of radiative corrections (RCs) to lepton nucleon scattering.
Some examples for discussions in the literature:

- Calculations involving form factors that depend on the loop momentum with approximations in the TPE diagrams performed by Maximon and Tjon (arXiv:nucl-th/0002058)
- Analysis concerning specifically corrections due to the TPE diagram for lepton proton scattering from Oleksander Tomalak (http://doi.org/10.25358/openscience-1037)
- Review on theoretical progress for TPE by Afanasev et al. (arXiv:1703.03874 [nucl-ex])

The TPE diagram



Corresponds to the following loop integral

$$(Ze^2)^2 \int_k \frac{\bar{u}_4 \Gamma_\nu (q-k)(\not{p}_2 + \not{k} + M)\Gamma_\mu (k) u_2 \bar{u}_3 \gamma^\nu (\not{p}_1 - \not{k} + m)\gamma^\mu u_1}{(k^2-\lambda^2)((q-k)^2-\lambda^2)((p1-k)^2-m^2)((p_2+k)^2-M^2)}$$

$$= (Ze^2)^2 \int_k \frac{\mathcal{N}(k)}{(k^2-\Lambda^2)^2((q-k)^2-\Lambda^2)^2(k^2-\lambda^2)((q-k)^2-\lambda^2)((p1-k)^2-m^2)((p_2+k)^2-M^2)}$$

The complication of the added denominators leads to higher-point scalar functions not readily available. So, one method we elected to do was to partial fraction the denominators. Using the below definitions:

$$D_0 = k^2 - \Lambda^2 \quad D_1 = (q-k)^2 - \Lambda^2 \quad D_2 = k^2 - \lambda^2$$
$$D_3 = (q-k)^2 - \lambda^2 \quad D_4 = (p_1 - k)^2 - m^2 \quad D_5 = (p_2 + k)^2 - M^2$$

The following result is derived:

$$\frac{1}{D_0^2 D_1^2 D_2 D_3 D_4 D_5} =$$

$$(\Lambda^{-2})^4 \left\{ \frac{1}{D_2 D_3} + \frac{1}{D_0 D_1} - \frac{1}{D_1 D_2} - \frac{1}{D_0 D_3} \right\} \frac{1}{D_4 D_5} \tag{1}$$

$$+ (\Lambda^{-2})^3 \left\{ \frac{1}{D_2 D_1^2} + \frac{1}{D_0^2 D_3} - \frac{1}{D_0 D_1^2} - \frac{1}{D_0^2 D_1} \right\} \frac{1}{D_4 D_5} \tag{2}$$

$$+ (\Lambda^{-2})^2 \left\{ \frac{1}{D_0^2 D_1^2} \right\} \frac{1}{D_4 D_5} \tag{3}$$

Furthermore we can rewrite the last two terms as derivatives:

$$(2) \to (\Lambda^{-2})^3 \frac{\partial}{\partial \Lambda^2} \left\{ \frac{1}{D_2 D_1} + \frac{1}{D_0 D_3} - \frac{1}{D_0 D_1} \right\} \frac{1}{D_4 D_5}$$

$$(3) \to (\Lambda^{-2})^2 \lim_{\Lambda_1^2, \Lambda_2^2 \to \Lambda^2} \frac{\partial^2}{\partial \Lambda_1^2 \partial \Lambda_2^2} \left\{ \frac{1}{D_0(\Lambda_1^2) D_1(\Lambda_2^2)} \right\} \frac{1}{D_4 D_5}$$

This greatly reduces the complexity of the work down to to calculating four point functions and their derivatives.

At this point we can turn to automatic methods for the calculations:

- Reduction to scalar functions through FeynCalc (Most current version: V. Shtabovenko et al. arXiv:2312.14089 [hep-ph])
- Supplying analytic expressions for scalar functions through Package-X (Patel, H. arXiv:1612.00009 [hep-ph])
- Implementation of the full set of RCs (including hard photon radiation) in a Monte Carlo program to obtain kinematic distributions

Independent checks:

- In-house code based on PV reduction and derivatives of IR finite scalar C, D functions provided by S.M.Hasan.
- Interpretation of form factor denominators as propagators and rearrangement of terms does not need partial fractions and derivatives.

Overview

Preliminary Results

We present results for δ defined as:

$$\sigma = \sigma_{\text{Born}}(1 + \delta)$$

TPE diagrams

	$\epsilon_1 = 4.4$ GeV $Q^2 = 6$ (GeV/c)^2		$\epsilon_1 = 12$ GeV $Q^2 = 16$ (GeV/c)^2		$\epsilon_1 = 21.5$ GeV $Q^2 = 31.3$ (GeV/c)^2	
	box	crossed box	box	crossed box	box	crossed box
dipole	-0.25619	0.242445	-0.768732	0.771787	-1.46347	1.47169
monopole	0.181968	-0.1940709	0.165889	-0.1711059	0.154493	-0.156826
	sum		sum		sum	
dipole	-0.013745		0.003055		0.008226	
monopole	-0.012103		-0.005216		-0.002333	

Figure: Contribution of the IR finite part of the TPE diagrams (direct and crossed boxes) to δ for dipole and monopole form factors (same setup used as in the Maximon and Tjon paper, Z=1)

Preliminary Results

Soft + Virtual Corrections

	$\epsilon_1 = 4.4$ GeV $Q^2 = 6$ (GeV/c)^2		$\epsilon_1 = 12$ GeV $Q^2 = 16$ (GeV/c)^2		$\epsilon_1 = 21.5$ GeV $Q^2 = 31.3$ (GeV/c)^2	
	CHW	MTj	CHW	MTj	CHW	MTj
Z^0	-0.2187	-0.2187	-0.2330	-0.2330	-0.2323	-0.2323
Z^1	-0.0598	-0.0569	-0.0327	-0.0517	-0.0304	-0.0625
$Z^2 + \delta_{el}^{(1)}$	-0.0146	-0.0174	-0.0185	-0.0243	-0.0202	-0.0267
$\delta_{el}^{(1)}$	$+0.0096$	$+0.0068$	$+0.0174$	$+0.0116$	$+0.0250$	$+0.0185$
δ	-0.2902	-0.2930	-0.3032	-0.3090	-0.3150	-0.3214

Figure: Comparison of Maximon and Tjon (MTj)[?] to our work, Crowe, Hasan, Wackeroth (CHW)

Preliminary Results

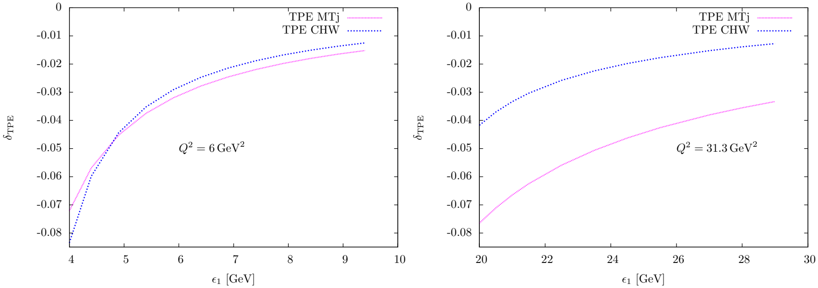


Figure: Comparison of TPE contribution to δ (virtual+soft) by MTj to our result for different values of the electron beam energy ϵ_1 and Q^2

Future Steps

- Comparison of our TPE calculation to existing work on Two Photon exchange corrections
- Predictions for the ratio of electron-proton and positron-proton cross sections
- The next phase will be to add onto this core machinery the NLO QED corrections to neutrino-Nucleon scattering
- Possibility to study the impacts of RCS in the transition from a lower energy regime to higher energy DIS regime using in house EW corrections to neutrino-Nucleon scattering [2]

[2] Kwangwoo P., Baur U., Wackeroth D. , "Electroweak radiative corrections to neutrino–nucleon scattering at NuTeV," arxiv.org/abs/0910.5013

Overview

References

- Maximon, L.C., Tjon, J.A. (2000). “Radiative Corrections to Electron-Proton Scattering.”, 35. arXiv:nucl-th/0002058
- Afanasev, A et al. (2023). “Radiative Corrections: From Medium to High Energy Experiments.”, arXiv:2306.14578 [hep-ph]
- V. Shtabovenko, R. Mertig and F. Orellana, "FeynCalc 10: Do multiloop integrals dream of computer codes?", arXiv:2312.14089 [hep-ph].
- V. Shtabovenko, R. Mertig and F. Orellana, "FeynCalc 9.3: New features and improvements", arXiv:2001.04407 [hep-ph]
- V. Shtabovenko, R. Mertig and F. Orellana, "New Developments in FeynCalc 9.0", Comput. Phys. Commun., 207, 432-444, 2016, arXiv:1601.01167.
- R. Mertig, M. Böhm, and A. Denner, "Feyn Calc - Computer-algebraic calculation of Feynman amplitudes", Comput. Phys. Commun., 64, 345-359, 1991.
- Patel, H. H. (2017). Package -X 2.0: A Mathematica package for the analytic calculation of one-loop integrals. arXiv:1612.00009 [hep-ph]