RG Improvement of the effective potential in Finite Temperature Quantum Field Theory

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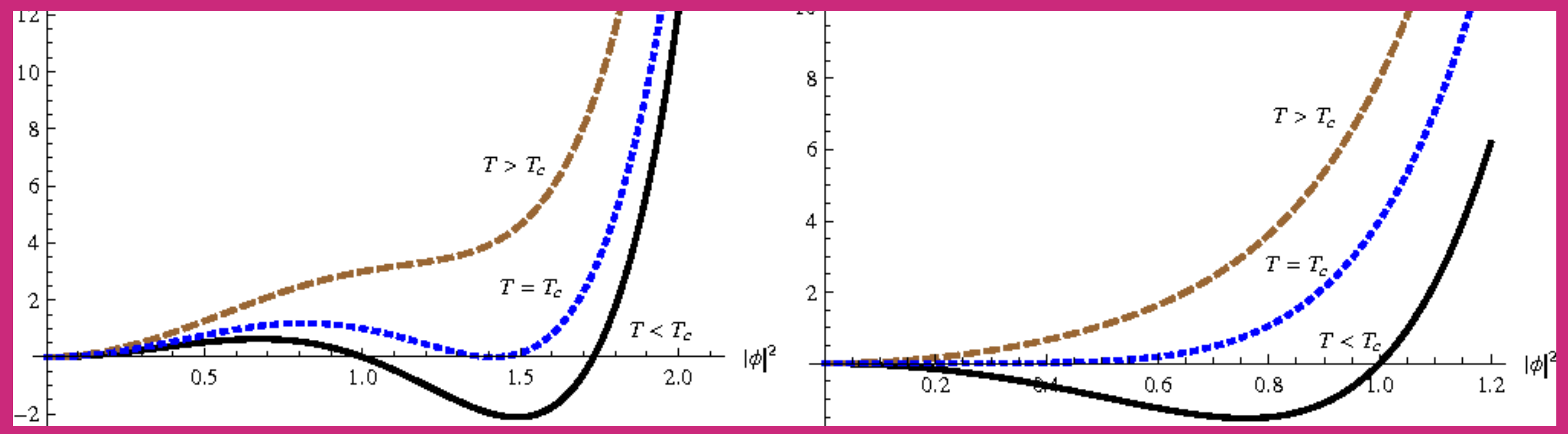
Outline

- Finite Temperature QFT - why do we care?
- IR Problem of FTQFT; DR and Resummation
- Optimized Partial Dressing - The *"Correct"* way to do resummation
- RG improvement of the scalar potential at $T = 0$ and $T \neq 0$
- Results
- Outlook and conclusions

FTQFT

Cosmological PT

Electro-weak PT



1st order

2nd order/

Increasing m_h

How does BSM physics modify this picture?
Electro-weak Baryogenesis?

Dark Sector PT

Almost any non-minimal DM model could have a phase transition
arx: 1407.0688, 1612.00466, 2403.09558, 1702.02117

If LISA and future GW detectors detect a signal, could cosmological PT be responsible?
What can we learn about dark sectors from such GW signal

$$\Omega_{GW} \sim T^{-18}$$

We need FTQFT framework that is:

1) Theoretically tractable, ie. it is relatively simple to implement new BSM physics into the calculation

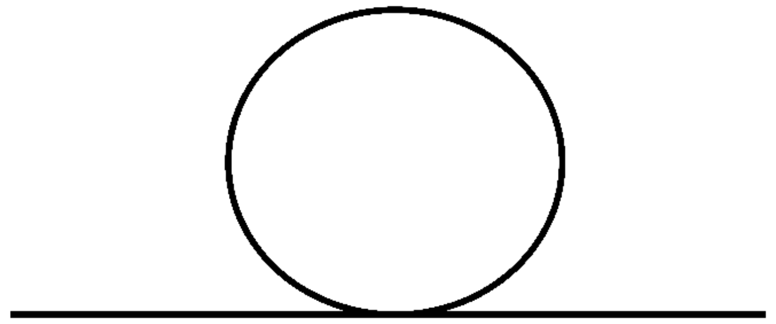
2) Stable under theoretical uncertainties (e.g. GW signal strength has strong dependence on the temperature of nucleation of bubbles)

Breakdown of perturbative expansion in FTQFT

Arx: 9901312

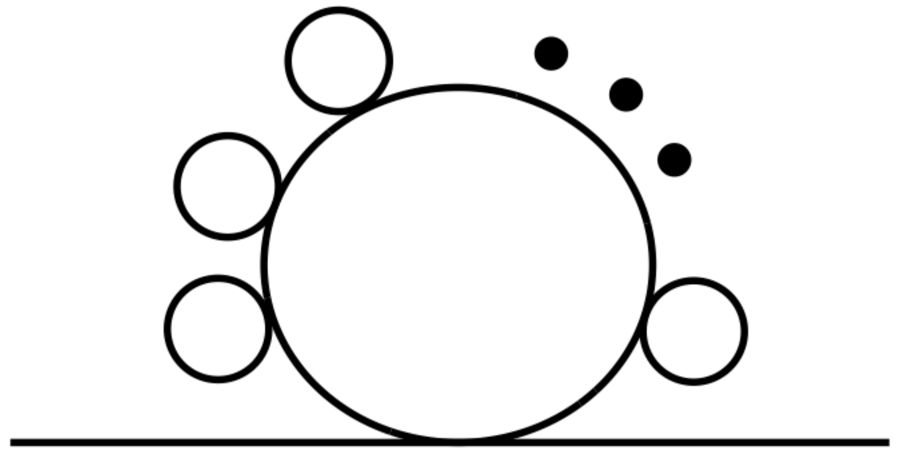
Consider ϕ^4 theory: $$V = \frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4}\phi^4$$

One-loop mass correction:



$$\sim \lambda T^2$$

Diasy diagram:



$$\sim \lambda^{n+1}\frac{T^{2n+1}}{m^{2n-1}} = \lambda^2\frac{T^3}{m}\left(\lambda\frac{T^2}{m^2}\right)^{2n-1}$$

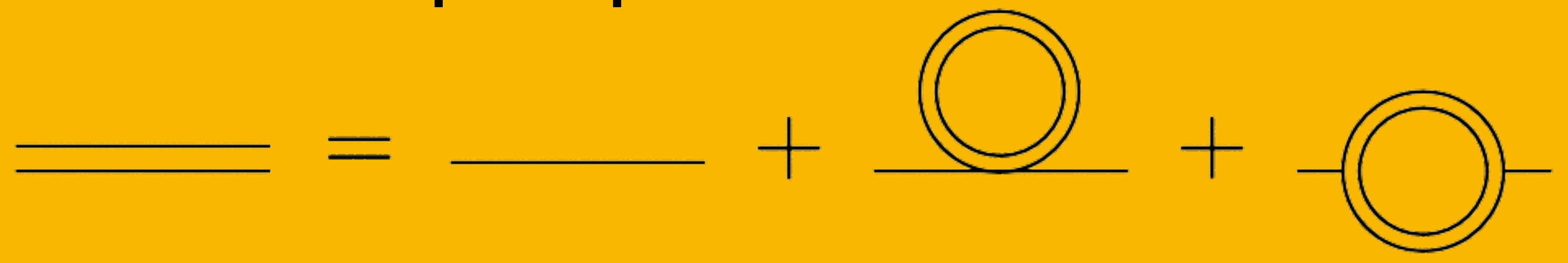
$$\alpha = \lambda\frac{T^2}{m^2} \ll 1$$

But near T_c, $\alpha \sim 1$, perturbative expansion breaks down.

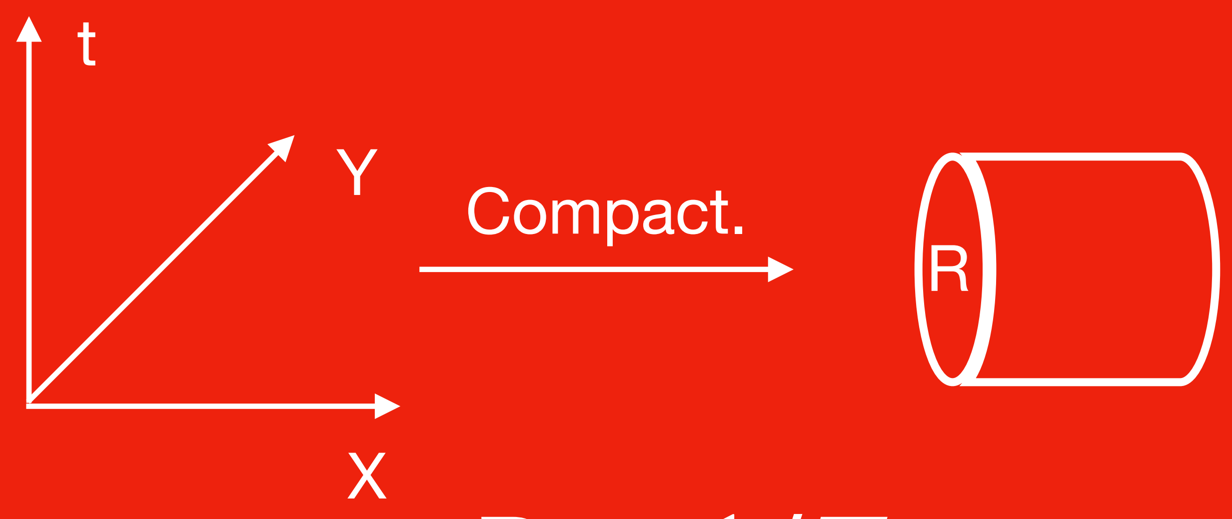
We need to resum diagrams in the order of highest IR importance!!!

Calculational methods

Gap Equation

$M^2 =$ 

Dimensional Reduction



$$R = 1/T$$

Pros:
Theoretically robust EFT.

Cons:
Depends on the hierarchy $T \gg m$ and on high-T expansion. Many of the BSM models escape this regime. Very technically involved and not easy to extend to different models.

Full Dressing

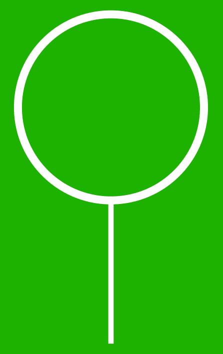
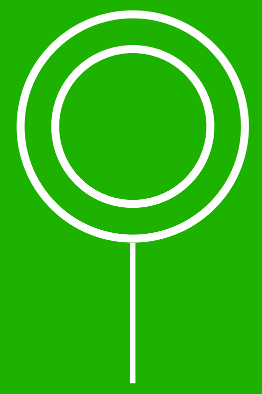
$V_{eff}(m^2) =$  $\longrightarrow V_{eff}(M^2) =$ 

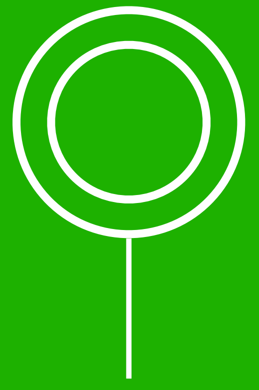
Pros:
Easy to do, resums most of the daisy diagrams and super-daisy diagrams. In princple easily applied to BSM models

Cons:
Its been shown that it miscounts some daisy and super-daisy diagrams
Misses sunset diagrams

arx:9204216
9212235

Partial Dressing

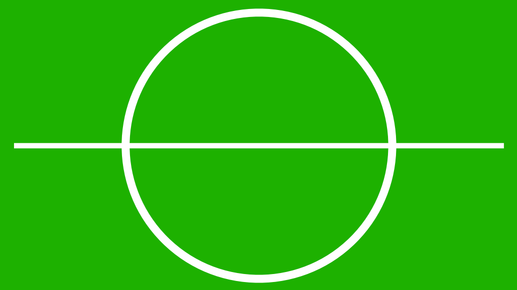
$\dfrac{\partial V_{eff}}{\partial \phi} =$  $\longrightarrow \dfrac{\partial V_{eff}}{\partial \phi}\Big|_{m^2 \to M^2} =$ 

$V_{eff} = \int d\phi$ 

Pros:
Easy to do, resums all of the daisy diagrams and super-daisy diagrams. “Simple” to adapt for any BSM model.

Cons:
Misses sunset diagrams

Arx: 9304254



Optimized Partial Dressing

Based on arx: 2211.08218 and 1612.00466

We present OPD as proposed in above papers:

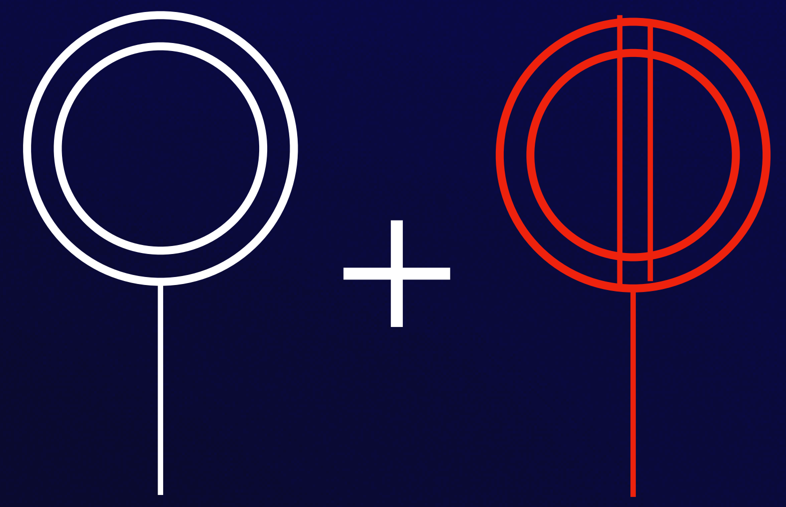
1) We numerically solve gap equation (we explicitly add sunset diagrams to the potential):

$$V_{OPD} =$$

$$M^2 = \frac{\partial^2 V_{OPD}}{\partial \phi^2} =$$

2) We plug it in V'_{OPD} and integrate wrt. ϕ to obtain potential

$$V_{\mathrm{OPD}} = \int d\phi V'_{\mathrm{OPD}}\,|_{m^2 \to M^2} = \int$$



$$d\phi$$

In the later paper its been shown that this method is comparable with DR in the high-T limit, and therefore demonstrates that OPD is advantageous for BSM models.

Renormalization group improvement

Renormalization group improvement in zero temperature QFT

Let us remind ourselves how the RG improvement works in massless ϕ^4 theory

$$V = \frac{1}{4!}\lambda\phi^4$$

With $\bar{\lambda}$ solution of RG equation RG:

$$\bar{\lambda}(\mu) \simeq \lambda(\mu_0)\left(1 + 3\frac{\lambda(\mu_0)}{32\pi^2}\log\frac{\mu_R^2}{\mu_0^2}\right)$$

We can see that we can capture large log part of the CW potential by plugging above into the potential and set $\mu_R^2 = m^2(\phi) = 3\lambda\phi^2$:

$$V \sim \lambda\phi^4 + m^4(\phi)\log\left(\frac{m^2(\phi)}{\mu_0^2}\right)$$

General statement about RG improvement: Putting solutions of one-loop RG equations into V_{eff} computed to n-loop fixed order, one should capture behaviour of large logs from n+1-loop fixed order.

RG improvement of OPD procedure

- Guided by the lesson from tree level procedure, we can do the same in finite temperature. We have our standard MS-bar RG solutions, we put them into the potential, and then set $\mu_R^2 \to M^2(\phi, T) = m^2(\phi) + \delta m^2(\phi, T)$

- It turns out this procedure seemingly greatly reduces the scale dependence of V_{OPD} (in comparison to arx: 2211.08218 for two real scalar theory):

$$V = -\frac{\mu_1^2}{2}\phi_1^2 + \frac{\mu_2^2}{2}\phi_2^2 + \frac{\lambda_1}{4}\phi_1^4 \frac{\lambda_2}{4}\phi_2^4 + \frac{\lambda_{12}}{2}\phi_1^2\phi_2^2$$

- In the case of the two fields, we do the replacement for whatever field has a large thermal mass (in our case it will be the field that does not get VEV)

Results

Results

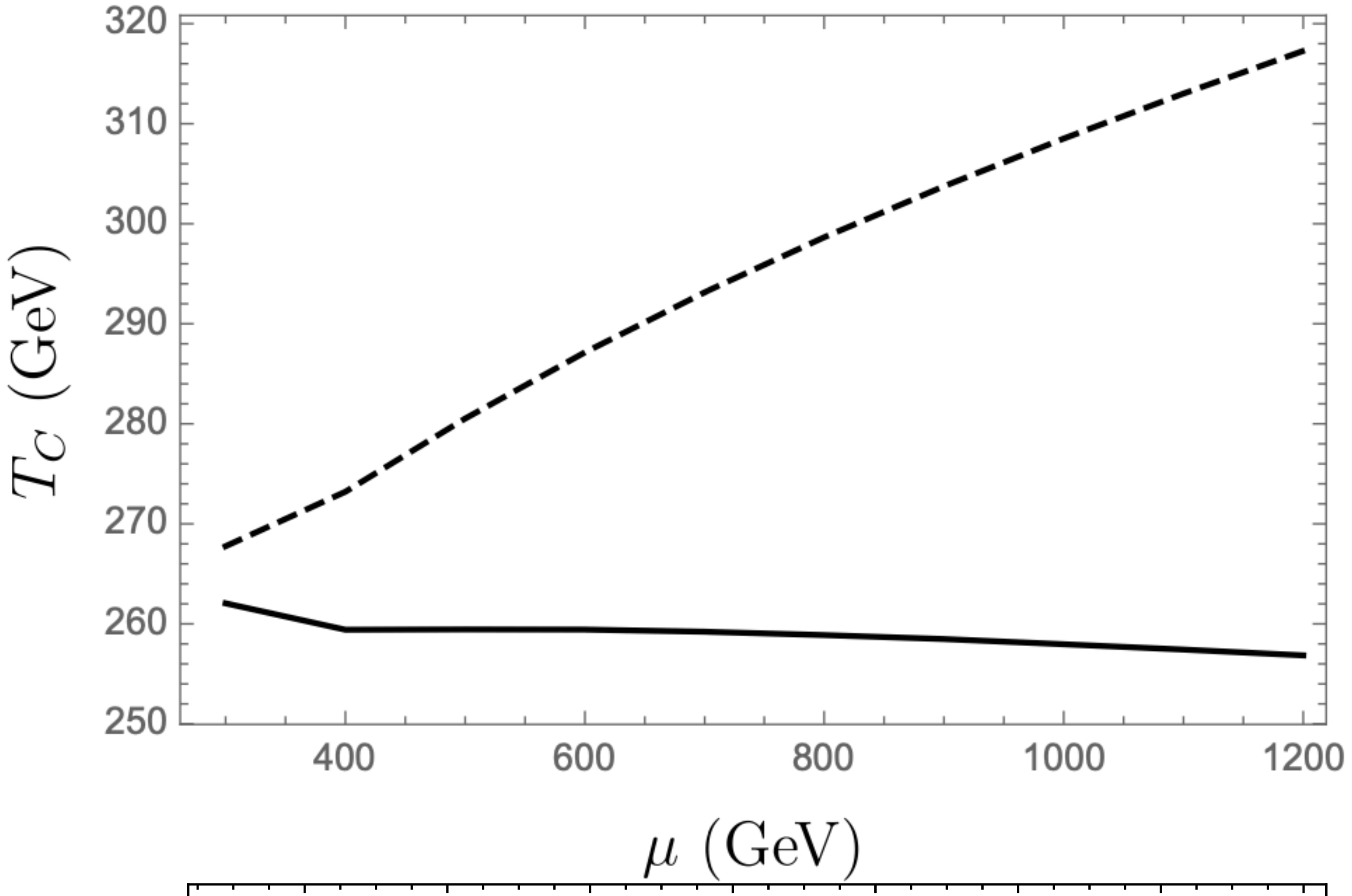
$M_1^{phys} = 125$ GeV

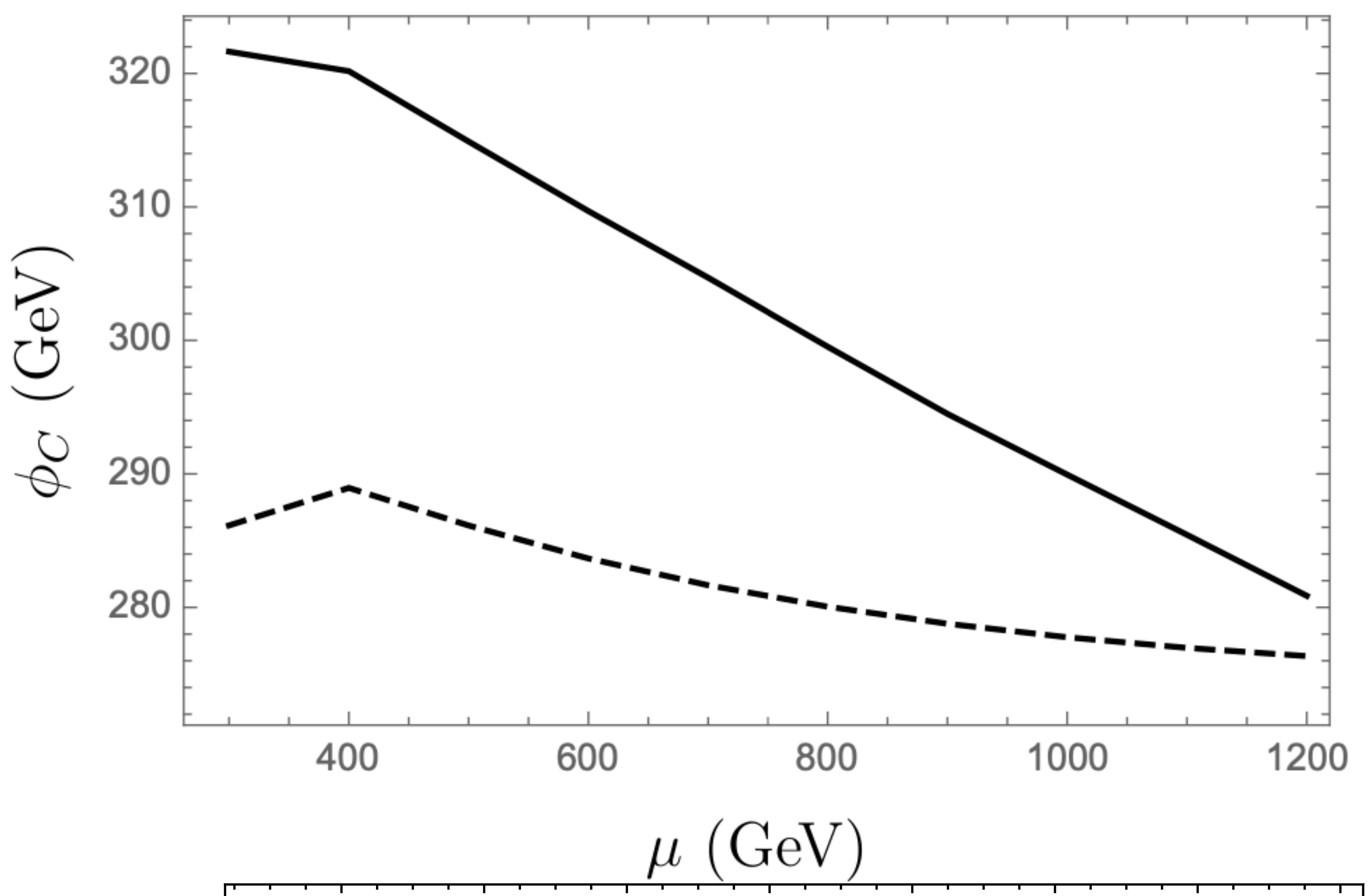
$M_2^{phys} = 600$ GeV

$v = 400$ GeV

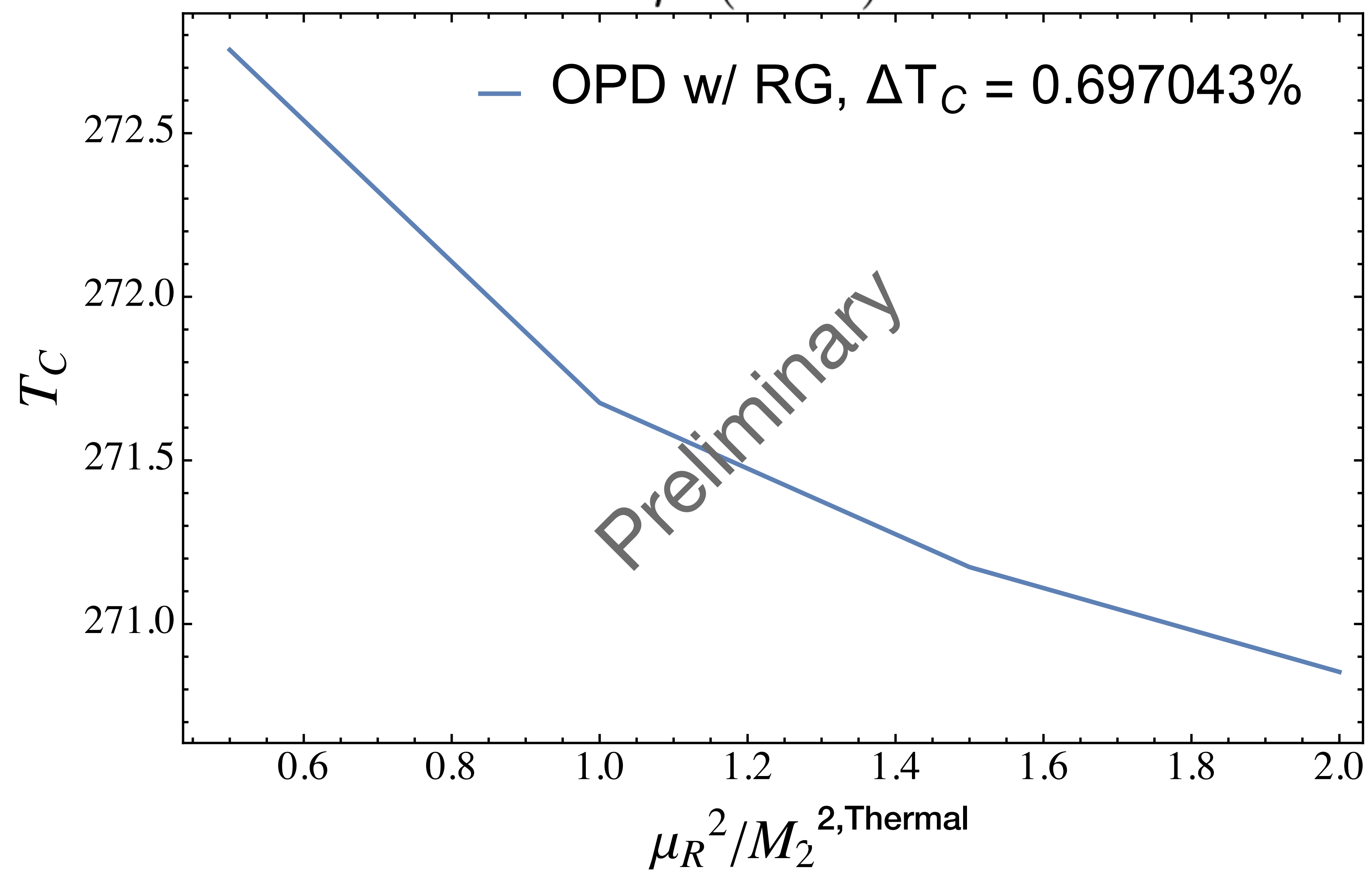
$\lambda_{12}^{phys} = 2.2$

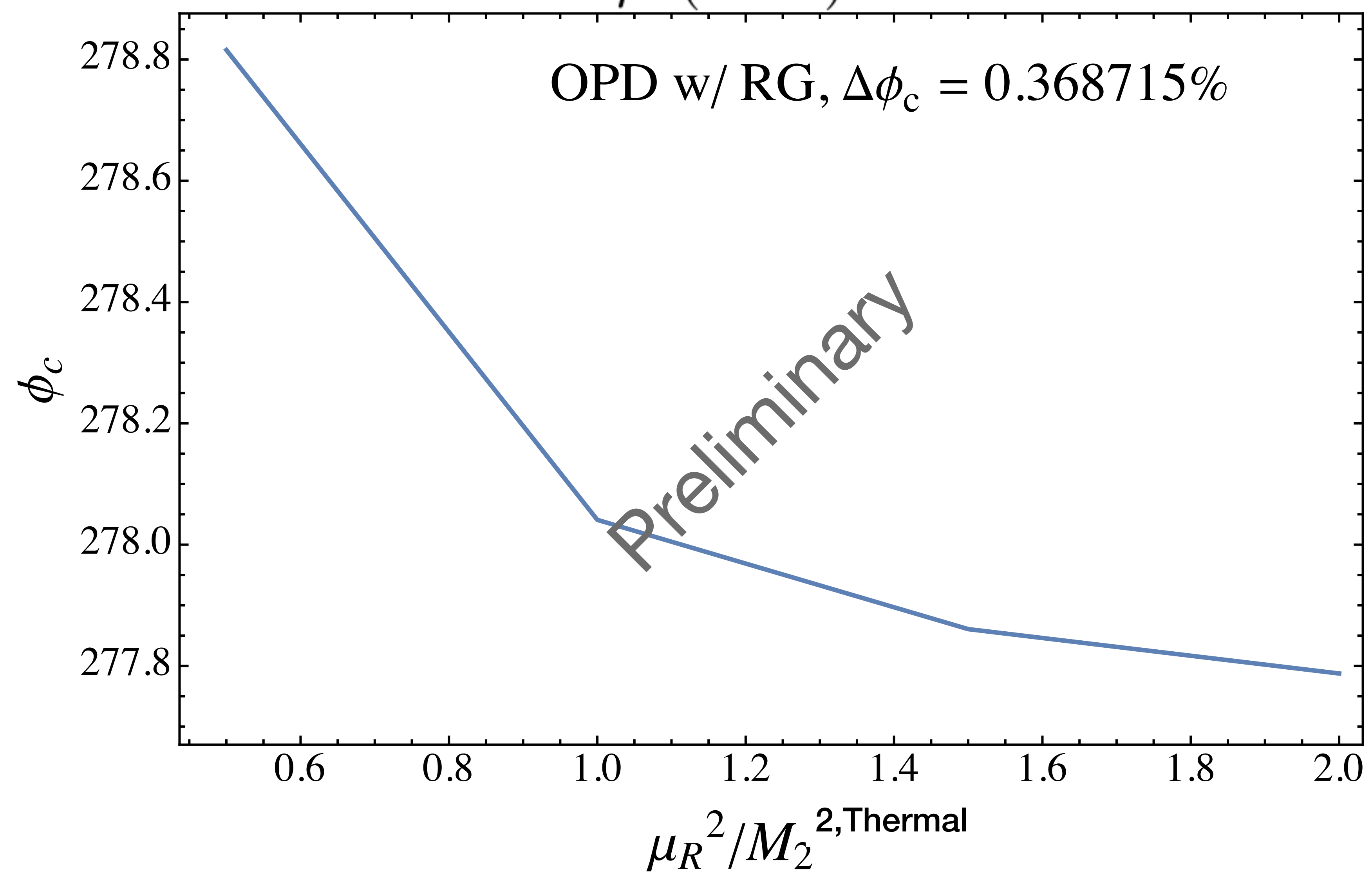
$\lambda_2^{phys} = 0.2$





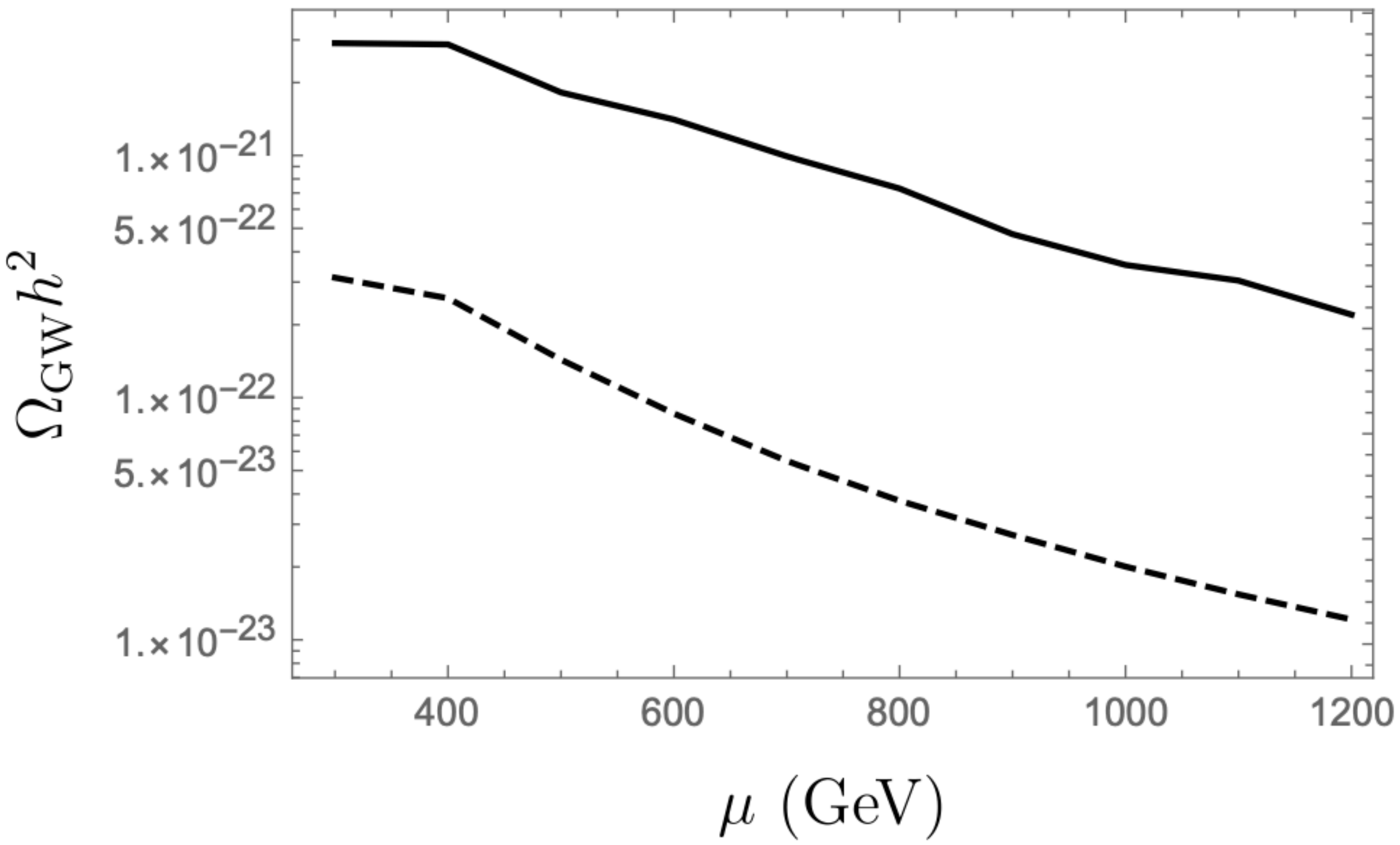
Arx:2211.08218

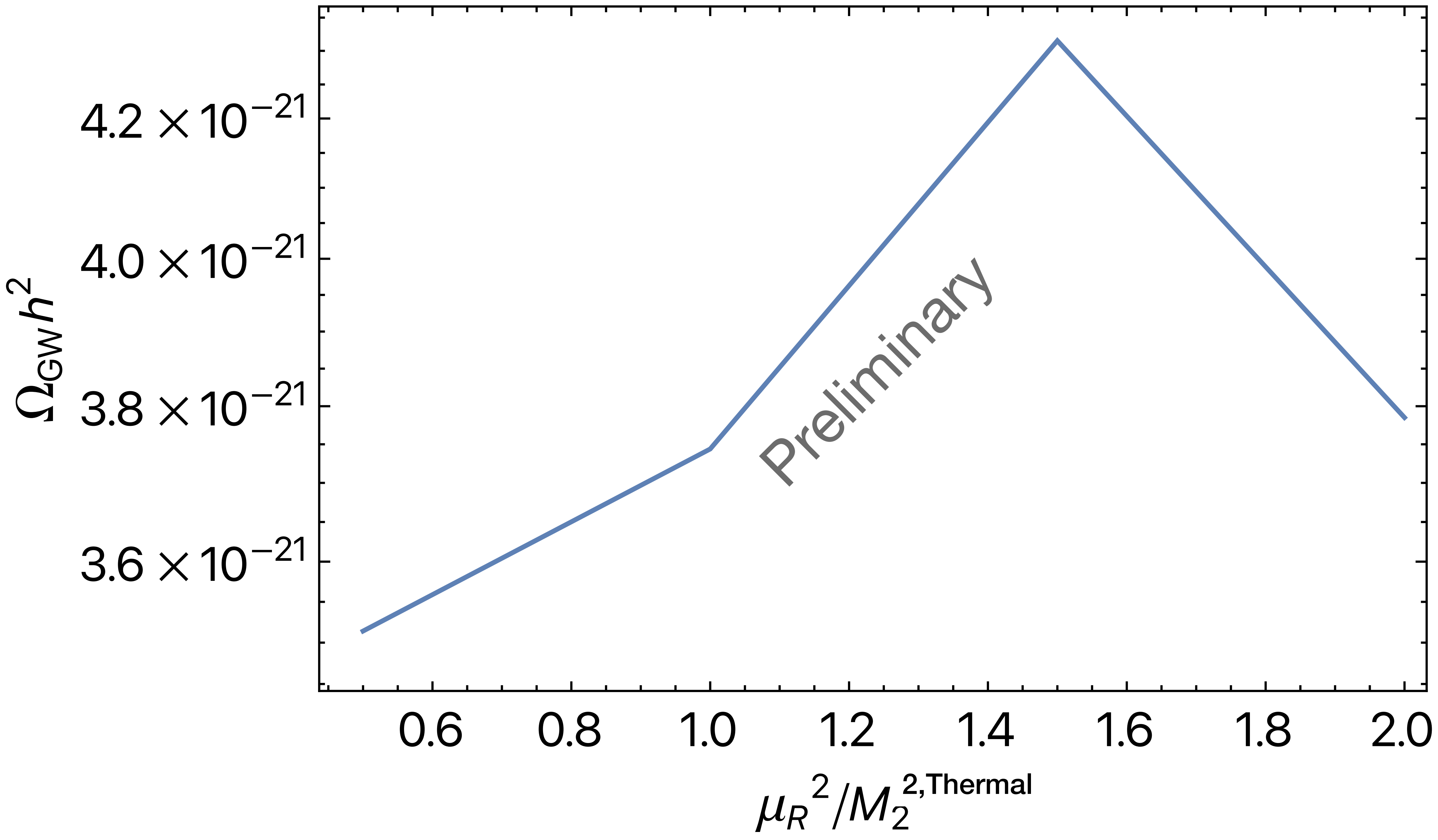




Results

Arx:2211.08218





Outlook and conclusions

Final steps

- We are working on analytically proving that indeed replacement $\mu_R^2 \to M^2(\phi, T) = m^2(\phi) + \delta m^2(\phi, T)$ indeed captures all the higher order effects
- Moreover, there is additional ambiguity how exactly one needs to RG improve at finite temperature, as OPD treats m^2 as a parameter, but we only have RGE for Lagrangian parameter μ^2.
- Can RG improvement be captured by just gap equation?

Conclusions

- We presented and demonstrated large reduction in scale dependence of the all observables
- The OPD framework is much more tractable and easier to do than currently developed methods, especially in comparison to DR.
- OPD has been proven to correctly count all the daisy and super-daisy diagrams, in comparison to other methods
- This work hopes to become the standard for doing calculations for any BSM model with phase transitions.

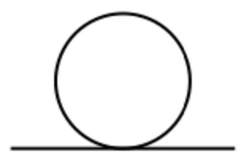
Thank you!

Questions?

BACK UP

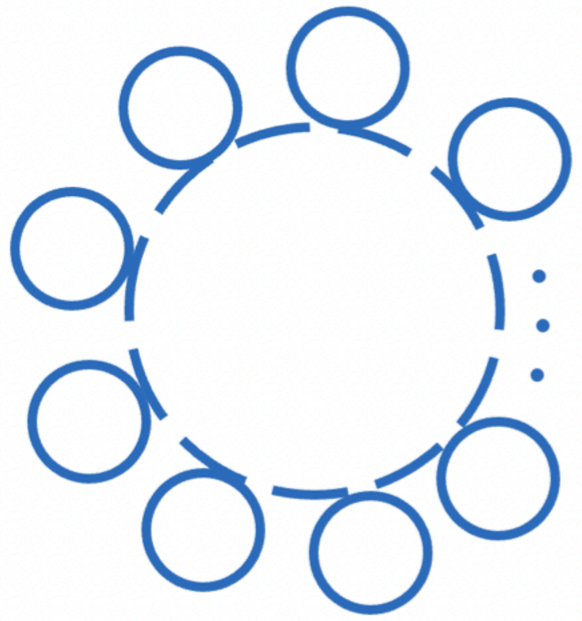
FTQFT and breakdown of perturbative expansion

Taken from arx:2404.12439



$$= \frac{\lambda}{2}\mathcal{I}[m] \equiv \frac{\lambda}{2} T \sum_{\omega_n} \int \frac{d^3k}{(2\pi)^3} \frac{1}{K^2 + m^2}$$

$$\mathcal{I}[m] = \underbrace{T \int \frac{d^3k}{(2\pi)^3} \frac{1}{\vec{k}^2 + m^2}}_{\mathcal{I}_{\text{soft}}[m]} + \underbrace{T \sum_{n\neq 0} \int \frac{d^3k}{(2\pi)^3} \frac{1}{\omega_n^2 + \vec{k}^2 + m^2}}_{\mathcal{I}_{\text{hard}}[m]} .$$



$$V_N^{\text{daisy}} \sim \left(T \int \frac{d^3k}{(2\pi)^3} \frac{1}{(\vec{k}^2 + m^2)^N}\right) \left(\lambda T \sum_{n\neq 0} \int \frac{d^3k}{(2\pi)^3} \frac{1}{\omega_n^2 + \vec{k}^2 + m^2}\right)^N$$

$$V_N^{\text{daisy}} \sim \left(m^{3-2N} T\right)\left(\lambda^N T^{2N}\right) = m^3 T \left(\frac{\lambda T^2}{m^2}\right)^N .$$

$$\alpha \equiv \frac{V_{N+1}^{\text{daisy}}}{V_N^{\text{daisy}}} = \frac{\lambda T^2}{m^2}$$

But near T_c, $\alpha \sim 1$, perturbative expansion breaks down.