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Quantum error thresholds for gauge-redundant digitizations of lattice field theories

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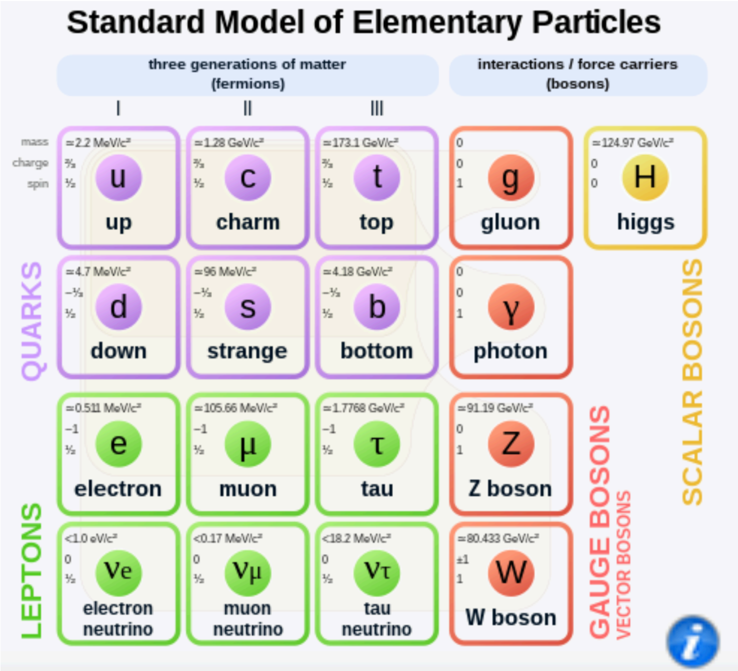
~~Quantum error thresholds for gauge-redundant digitizations of lattice field theories~~

Not the best title

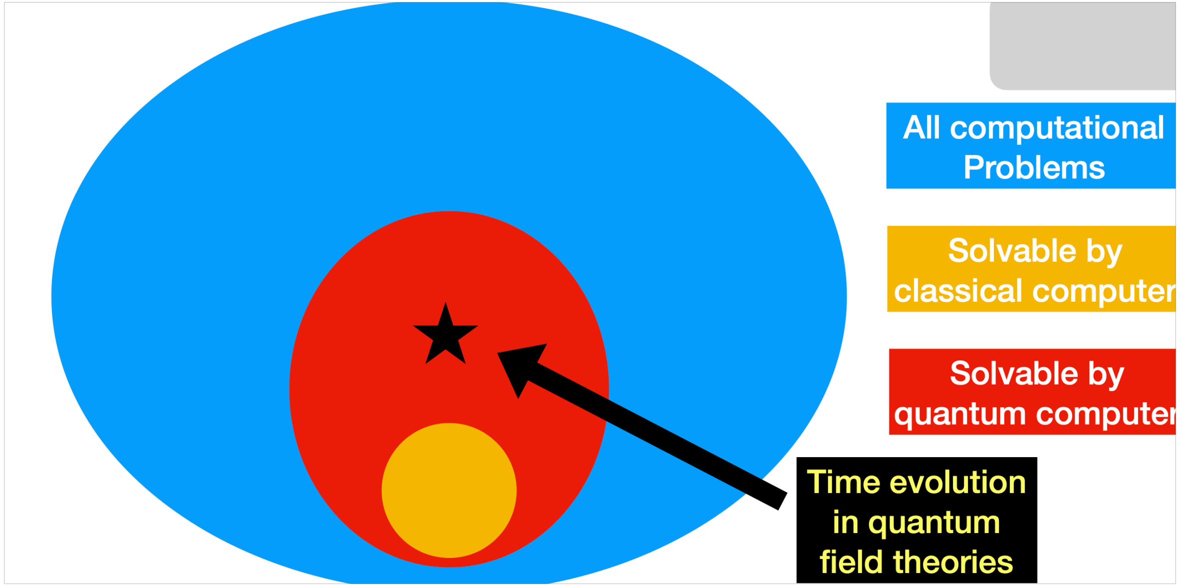
Gauge symmetry (redundancy) in quantum simulations: friend or foe?

Gauge symmetry: an elegant redundancy



- A_μ and $A'_\mu = A_\mu + \partial_\mu \alpha$ are redundant descriptions of the same physical state
- Redundancy keeps the theory elegant (local)
- Need to fix the gauge in perturbative calculations

In Christian Bauer's plenary talk Tuesday morning



Examples

- Shear viscosity
- Parton distribution function in hadron collisions
- Particle decay
- Neutrino oscillation in high density environment
- …

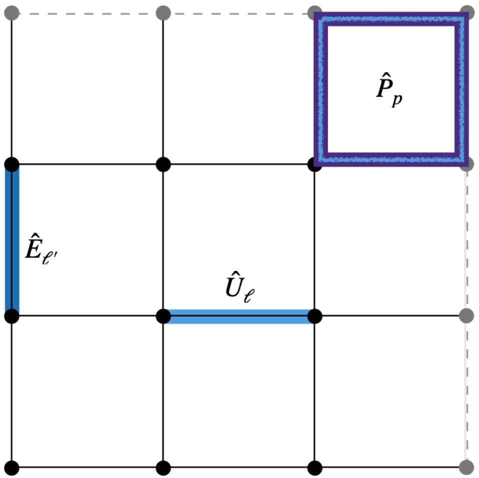
In Christian Bauer's plenary talk Tuesday morning

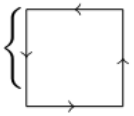
There are many different parts of the theory that need to be worked out when formulating a Hamiltonian lattice gauge theory

1. How to formulate a lattice theory that reproduces SU(3) in the limit of vanishing lattice spacing
   - Whether to add any additional expansions in the theory
2. What basis to choose for the Hilbert space
3. How to implement gauge invariance
4. How to truncate the theory (how to choose a discrete set of field values)

We know how to implement gauge invariance, in principle

$$U = e^{i\int d\mathbf{l}\cdot A}$$



$P_{\mu\nu}(x) = 1 - \frac{1}{N}\,\mathrm{Re\,Tr}\,\{$  $\}$

Kogut-Susskind Hamiltonian

$$\hat{H}_{KS}(a) = \frac{c(a)}{a}\left(g_H^2(a)\sum_l \hat{E}_L^2 + \frac{1}{g_H^2(a)}\sum_P \hat{P}_P\right) \equiv \hat{H}_E + \hat{H}_B$$

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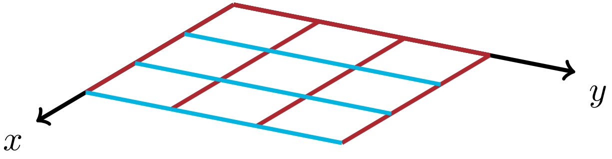
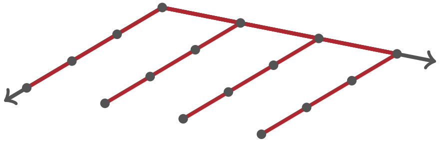
Gauge redundancy in the Hilbert space of LGT

d-dimension spatial lattice, N_L links, N_V vertices

$$\mathscr{H}_{\text{gauge}} = \text{span}\{|U\rangle, U \in G\}^{\otimes N_L}$$



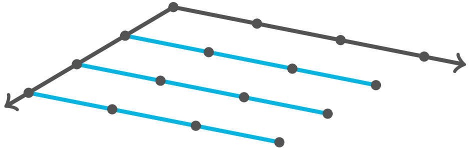
- Impose Gauss's Law:



N_T links in a maximal tree can be solved from the rest

$$N_T = N_V - 1$$

$$\mathscr{H}_{\text{inv}} \approx \text{span}\{|U\rangle, U \in G\}^{\otimes (N_L - N_T)}$$



Should quantum simulations keep the gauge redundancy?

	Keep the redundancy	Fix the gauge
Pros	**In arbitrary dimensions and groups:** • Hamiltonian is easy to derive • Local Hamiltonian	• Saves qubits by $\sim (1 - 1/d)$ • No symmetry breaking errors
Cons	• Redundant qubits • Errors can break symmetry • States are highly entangled	**In d>2 or non-Abelian groups** • Hamiltonian is hard to derive • Non-local Hamiltonian

When errors exist, Redundancy becomes Resources

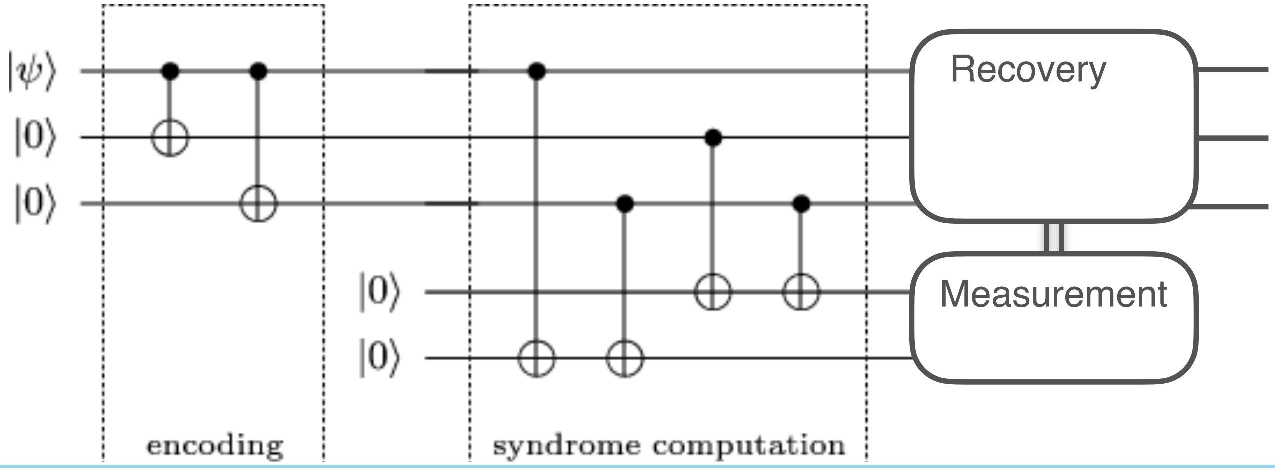
$\mathscr{H}_{\text{code}}$ $|111\rangle, |000\rangle$

Subspace invariant under the stabilizer group

$\mathscr{S} = \{I, Z_1Z_2, Z_2Z_3, Z_1Z_3\}$

$|001\rangle, |010\rangle, |100\rangle,$
$|011\rangle, |110\rangle, |101\rangle$

$\mathscr{H}_{\text{full}}$



3-qubit repetition code

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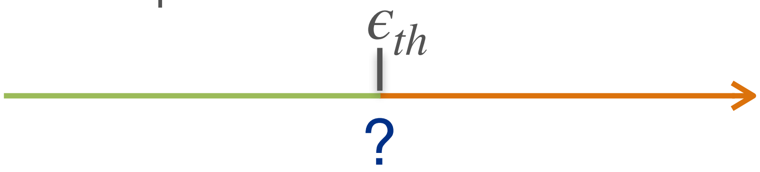
Use gauge redundancy as QEC:

This work: when is it worthwhile?

$\epsilon < \epsilon_{th}$: redundancy makes the code more error-proof

$\epsilon > \epsilon_{th}$: redundancy makes more errors than it can correct

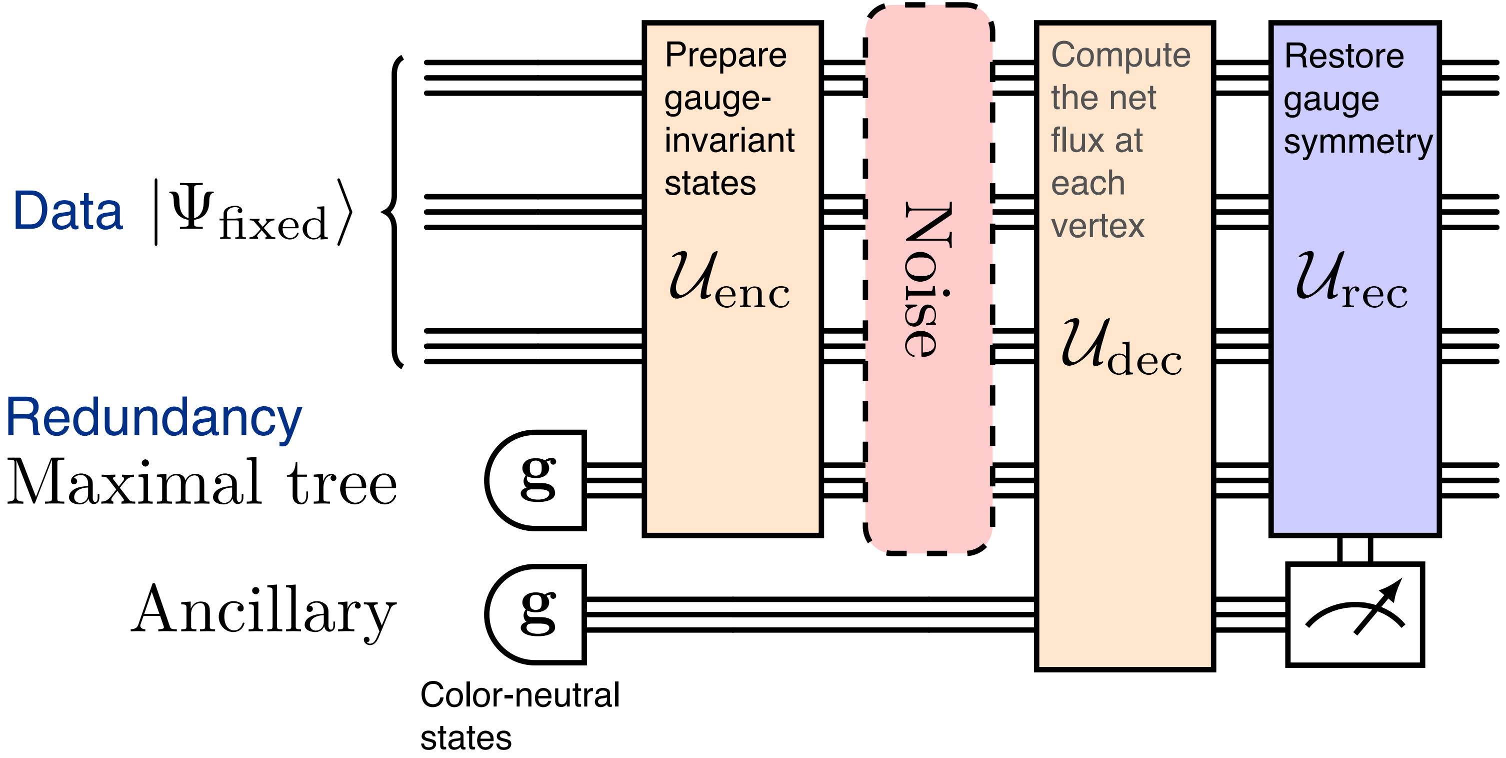


ϵ physical qubit error rate

Existing works

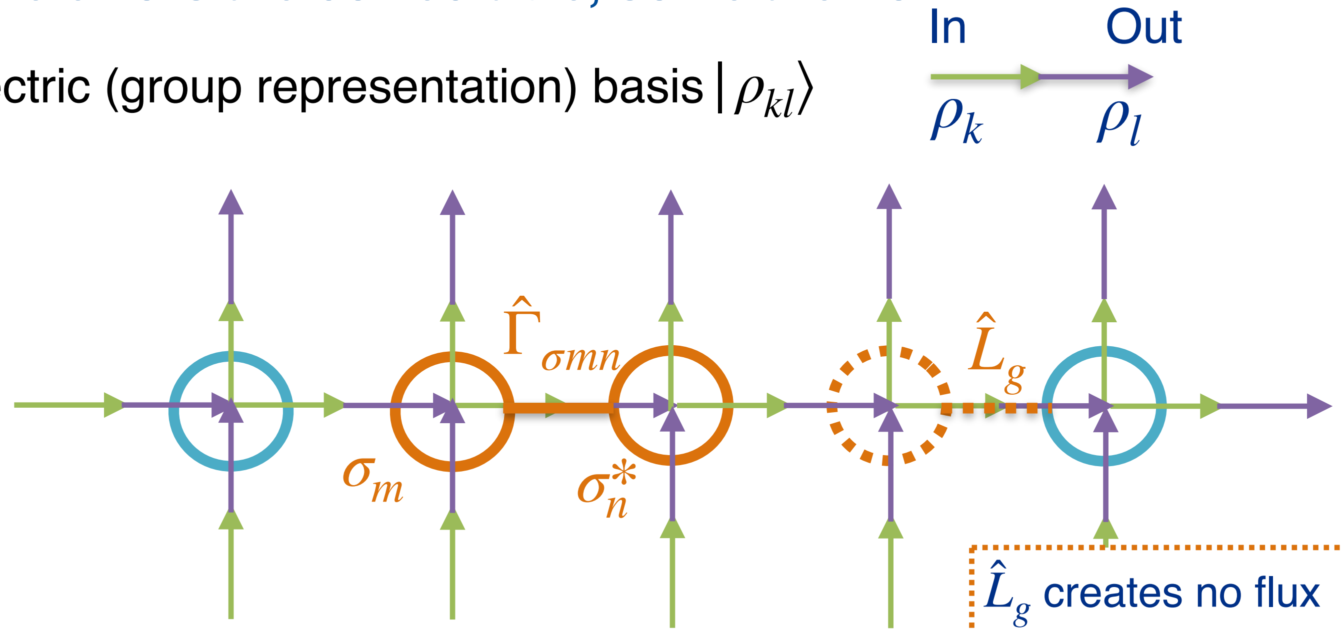
- For error mitigations (Quantum Zeno effect): Lamm et.al. 2020, Halimeh et.al. 2020, 2022, etc.
- For error corrections: Stryker 2019, Rajput et.al. 2023, Bao et.al. 2023
- Closely related to topological QEC

The circuits: encoding, detection and recovery



Some errors are correctable, some are not

Electric (group representation) basis $|\rho_{kl}\rangle$

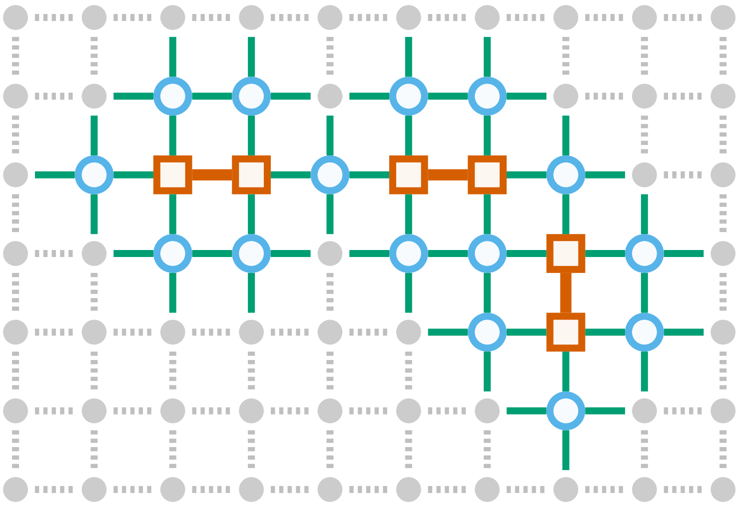


Gauss's law:
0 net flux

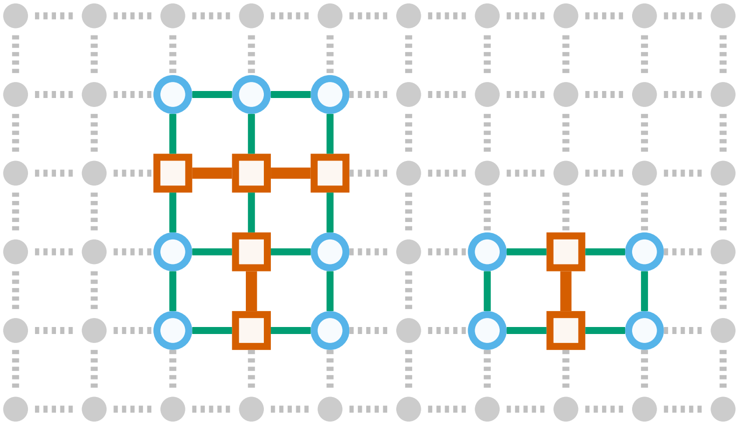
$\hat{\Gamma}_\sigma$ creates flux pair
Correctable

$\hat{L}_g$ creates no flux
(Abelian center)
or 1 flux (non-Abelian)
Not Correctable

Correctable $\hat{\Gamma}_\sigma$ -type errors on more than one link

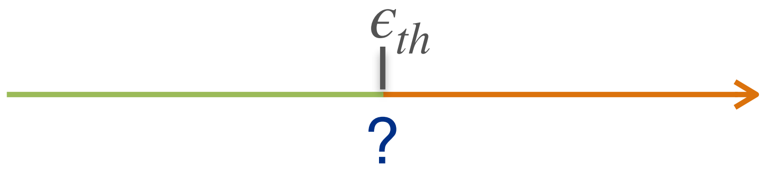


Minimal effort decoding condition:
"Isolated flux pairs"



KL condition (necessary and sufficient)
"$\lesssim$ 1 error / plaquette"
Knill & Laflamme (1997)

When does redundancy create more errors than it can correct?



- Total error rate ϵ per link independently.

Gauge-fixed

$\mathscr{H}_{\text{full}} = \mathscr{H}_{\text{code}} = \mathscr{H}_{\text{inv}}$

Quantum fidelity

$$F_{\text{fixed}} \geq (1-\epsilon)^{N_L - N_T}$$

Gauge-redundant

$\mathscr{H}_{\text{full}} = \mathscr{H}_{\text{gauge}}$

Quantum fidelity

$$F_{\text{redundant}} \geq (1-\epsilon)^{N_L}$$

→

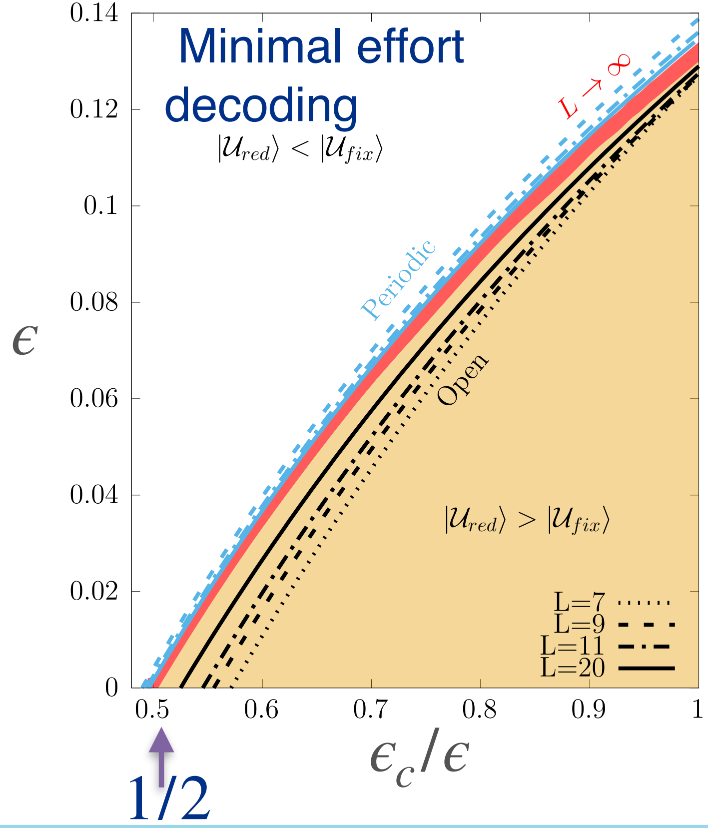
$\mathscr{H}_{\text{code}} = \mathscr{H}_{\text{inv}}$

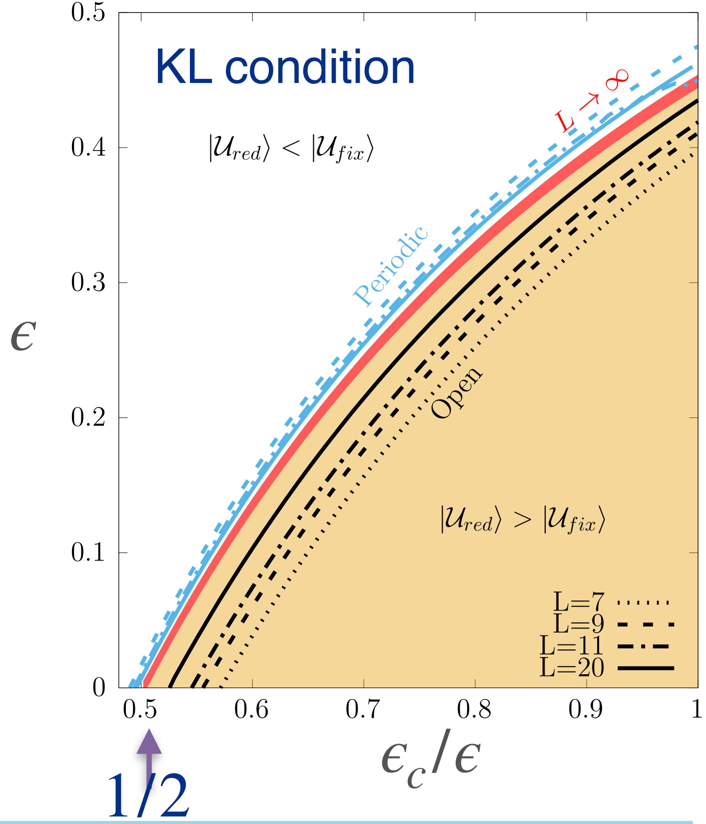
Restore gauge symmetry

$$F_{\text{restored}} \geq \sum_{n=0}^{N_L} Q_n \epsilon_c^n (1-\epsilon)^{N_L - n}$$

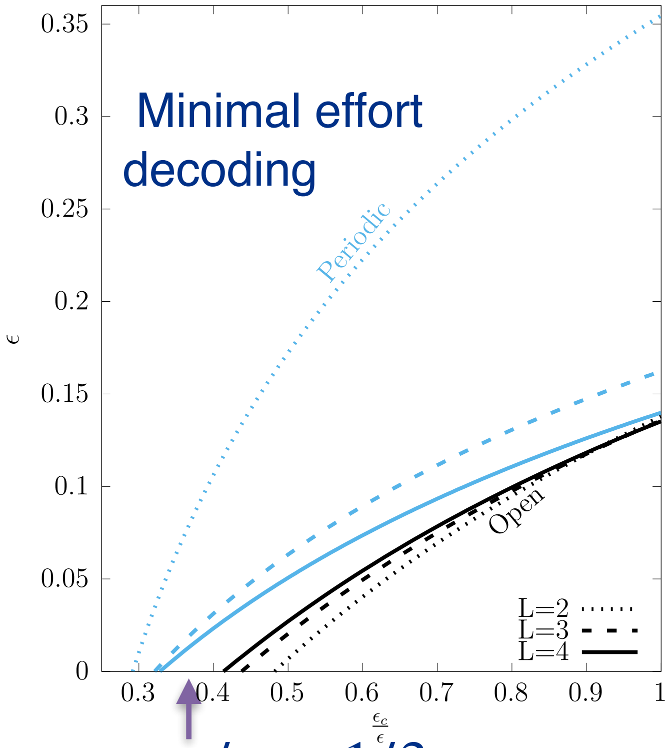
- Γ_σ-type (correctable type) error rate $\epsilon_c \leq \epsilon$.
- Q_n: # of ways to put n Γ_σ-type errors, s.t. they are still correctable.

The error thresholds: when lower bounds $F_{\text{restored}} \geq F_{\text{fixed}}$ (d=2)

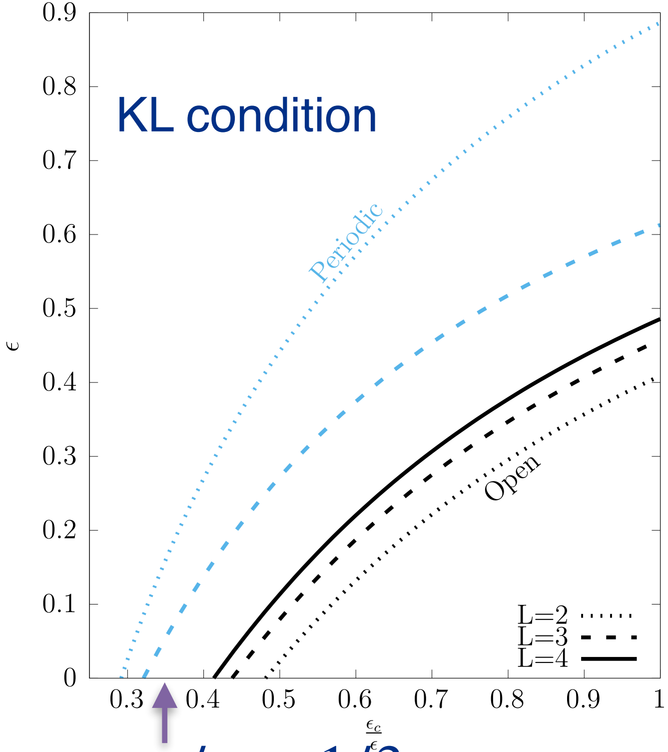




The error thresholds: when lower bounds $F_{\text{restored}} \geq F_{\text{fixed}}$ (d=3)



$\epsilon_c/\epsilon = 1/3$

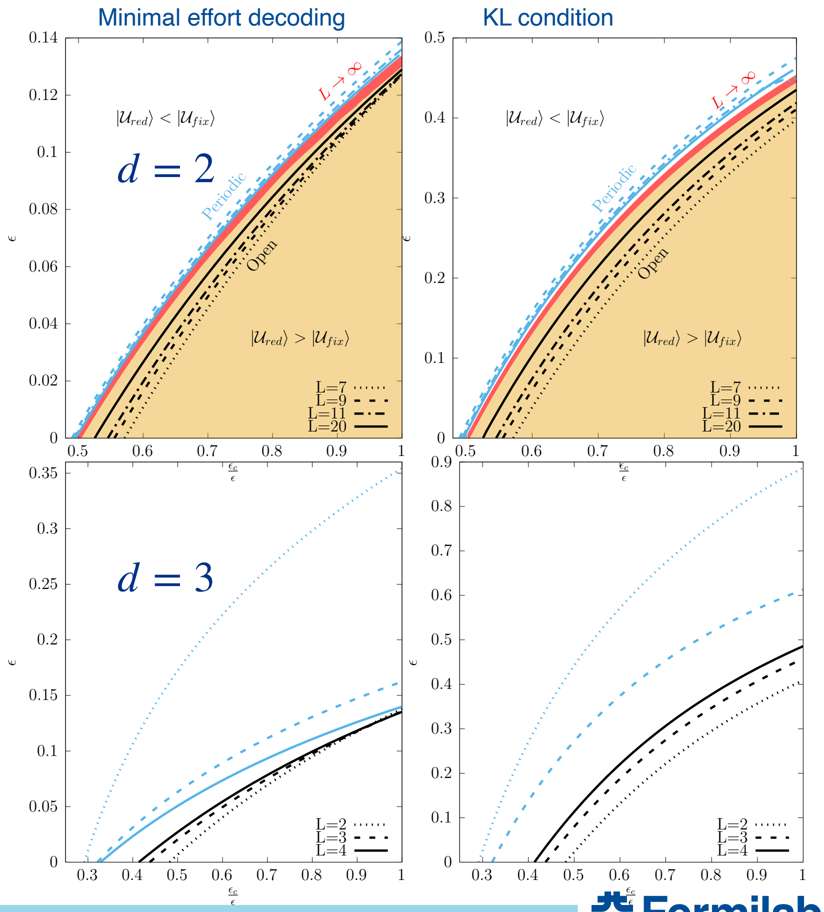


$\epsilon_c/\epsilon = 1/3$

Summary

$$\mathscr{H}_{\text{inv}} \, \mathscr{H}_{\text{gauge}} = \mathscr{H}_{\text{code}} \, \mathscr{H}_{\text{full}}$$

- When $\epsilon_c/\epsilon > 1/d$, any error rates "good enough" for quantum simulations easily guarantee $F_{\text{restored}} \geq F_{\text{fixed}}$.
- Detailed quantum circuits in the paper 2402.16780
- Lots of highly physics-inspired quantum algorithms designing works for theorists in the future!



Backup

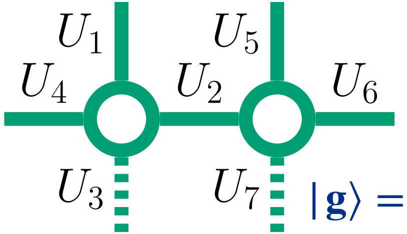
Correctable errors with gauge redundancy

» $\{\hat{L}_g \hat{\Gamma}_\sigma, g \in G, \sigma \in \hat{G}\}$ — a complete basis for operators in $\mathscr{H}_G = \mathrm{span}\{|U\rangle, U \in G\}$

$\mathbf{x}$ —— $\mathbf{x}+\mathbf{i}$

Type	Definition (group element basis)	Qubit counterpart $G = Z_2$	Gauge symmetry at $\mathbf{x}$, $\mathbf{x+i}$	Correctability with gauge symmetry
Group multiplication	$\hat{L}_g = \sum_{U \in G} \lvert gU\rangle\langle U \rvert$	X	Both preserved for g in the Abelian center; Otherwise broken only at $\mathbf{x}$	No
Representation matrix element	$\hat{\Gamma}_{\sigma,m,n} = \sum_{U \in G} \sqrt{d_\sigma}\Gamma^{(\sigma)}_{mn}(U)^* \lvert U\rangle\langle U \rvert$	Z	Both broken $\mathbf{x}$ —— $\mathbf{x}+\mathbf{i}$ σ_m σ_n^*	Yes $\hat{P}_{\mathrm{inv}}\hat{\Gamma}_\sigma{}^\dagger\hat{\Gamma}_{\sigma'}\hat{P}_{\mathrm{inv}} = \delta_{\sigma,\sigma'}\hat{P}_{\mathrm{inv}}$

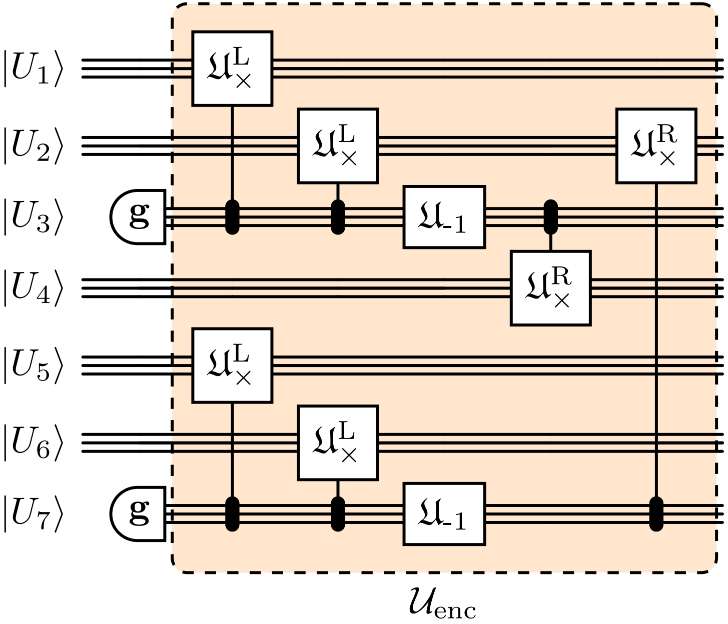
Encode: prepare states to satisfy Gauss's law (gauge invariance)



$$|\mathbf{g}\rangle = |G|^{-\frac{1}{2}} \sum_{h\in G} |h\rangle$$

Triple line: a quantum register for one group element (one lattice link)

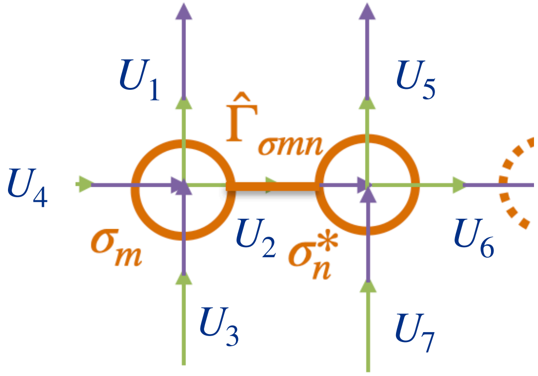
- Inverse gate: $\mathfrak{U}_{-1}|g\rangle = |g^{-1}\rangle$,
- Left and Right Multiplication gates: $\mathfrak{U}_\times^L|g\rangle|U\rangle = |g\rangle|gU\rangle$, $\mathfrak{U}_\times^R|g\rangle|U\rangle = |g\rangle|Ug\rangle$,



$$|U_1\rangle|U_2\rangle \sum_{h\in G} |h\rangle|U_4\rangle \xrightarrow{\mathcal{U}_{\mathrm{enc}}} \sum_{h\in G} |hU_1\rangle|hU_2\rangle|h^{-1}\rangle|U_4h^{-1}\rangle$$

Gauge invariant

Decode: compute the net flux at each vertex



$\hat{\Gamma}_\sigma$ creates flux pair
Correctable