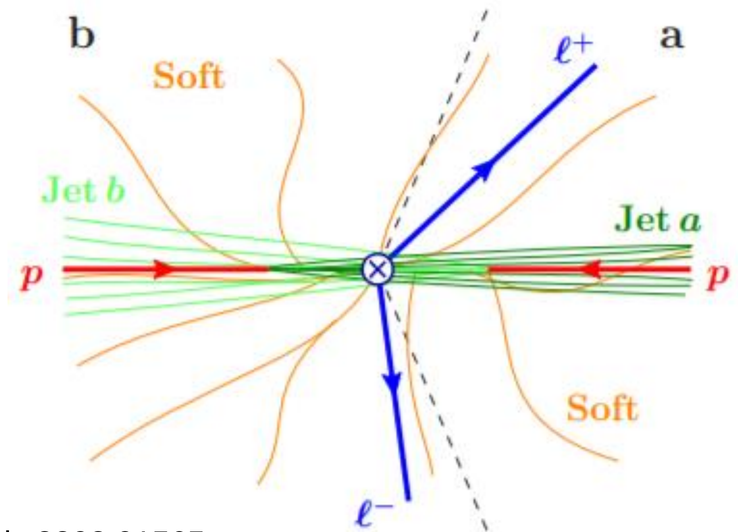
QED corrections to the Neutral Current Drell-Yan process using jettiness subtraction method

DPF – PHENO 2024, University of Pittsburg / Carnegie Mellon Unversity

Jiayang Xiao (Co-Author: Dr. Doreen Wackeroth)
May 15th, 2024

University at Buffalo The State University of New York

Motivation



Drell-Yan process is one of the best theoretically understood, and most precise experimentally studied process

- N3LO QCD corrections

X. Chen, et al. arxiv:2203.01565
T. Neumann, et al. arxiv:2207.07056
X. Chen, et al. arxiv:2107.09085

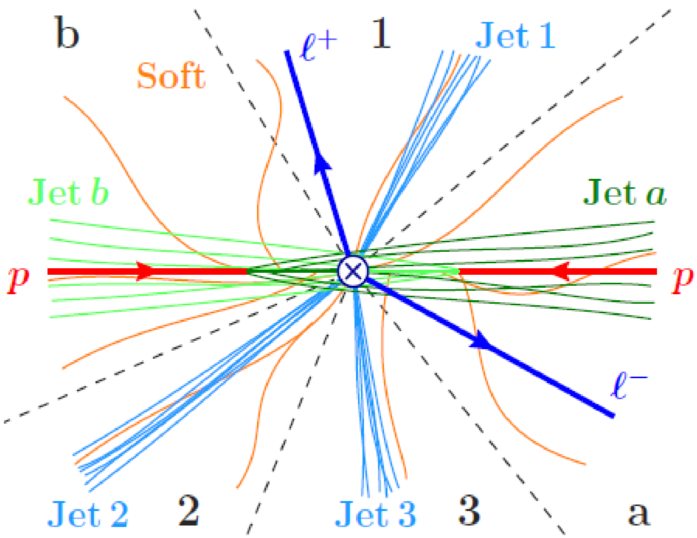
- NNLO EW×QCD corrections

Charged Current: T. Armadillo, et al. arxiv:2405.00612
Neutral Current: S. Dittmaier, et al. arxiv:2401.15682
(and references therein)

- Goal: NNLO mixed QED×QCD corrections with N-jettiness subtraction method
- First step: NLO QED corrections

N-jettiness and N-jettiness subtraction method

- Subtraction method for NNLO QCD, N-jettiness subtraction using n-jet resolution variable to handle IR divergence
- N-jettiness: A global inclusive event shape to distinguish between N-jet events and M-jet events (M > N).



$$\mathcal{T}_N = \sum_{m=1}^{M} min_i \left\{ \frac{2p_i \cdot q_m}{P_i} \right\}$$

Goal: apply this method to NNLO mixed QED×QCD corrections

- IR safe observable $\mathcal{T}_N(\Phi_{\geq N+1} \rightarrow \Phi_N) \rightarrow 0$

I. Stewart,et al. arXiv:1004.2489
R. Boughezal, et al. Phys. Rev. Lett. 115, 062002
J. Gaunt, et al. arXiv:1505.04794

N-jettiness and N-jettiness subtraction method

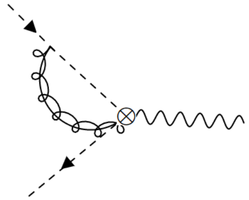
- Recycle lower order calculation

$$
\begin{aligned}
\sigma_{NNLO} = &\int d\Phi_N |\mathcal{M}_N|^2 \\
&+ \int_{\mathcal{T}_N^{cut}} d\Phi_{N+1} |\mathcal{M}_{N+1}|^2 + \int^{\mathcal{T}_N^{cut}} d\Phi_{N+1} |\mathcal{M}_{N+1}|^2 \\
&+ \int_{\mathcal{T}_N^{cut}} d\Phi_{N+2} |\mathcal{M}_{N+2}|^2 + \int^{\mathcal{T}_N^{cut}} d\Phi_{N+2} |\mathcal{M}_{N+2}|^2 \\
= &\sigma_{NNLO}(\mathcal{T}_N > \mathcal{T}_N^{cut}) + \sigma_{NNLO}(\mathcal{T}_N < \mathcal{T}_N^{cut})
\end{aligned}
$$

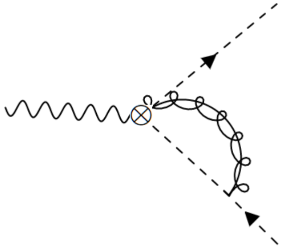
Factorization formula from Soft and Collinear effective field theory (SCET)

$$\sigma(\mathcal{T}_N < \mathcal{T}_N^{cut}) = \int B_a \otimes B_b \otimes S \otimes H \otimes \prod_{i=1}^{N} J_i + \mathcal{O}(\mathcal{T}_N^{cut})$$

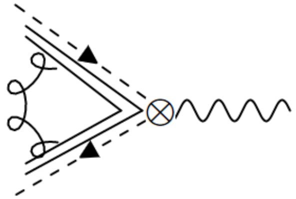
- $B_{\mathrm{a,b}}$: Beam function (contains initial state collinear emissions)



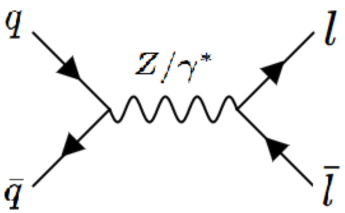
- J_{i} : Jet function (contains final state collinear emissions)



- H : Hard function (contains process specific hard interaction)
- S : Soft function (contains all soft emissions)



N-jettiness subtraction method In QED corrections

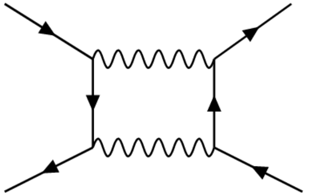


- From QCD to QED

Factorization formula from SCET

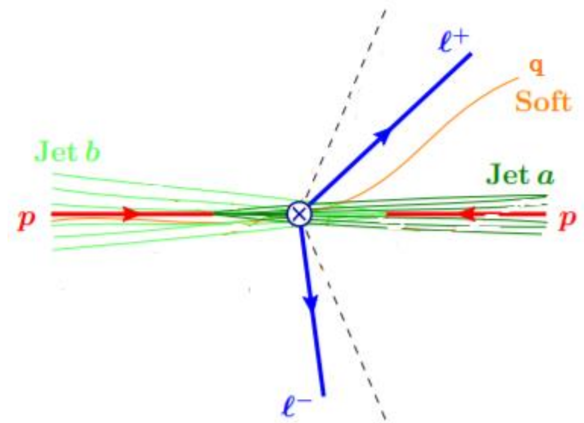
$$\sigma(\mathcal{T}_N < \mathcal{T}_N^{cut}) = \int B_a \otimes B_b \otimes S \otimes H \otimes \prod_{i=1}^{N} J_i + \mathcal{O}(\mathcal{T}_N^{cut})$$

- $B_{a,b}$: Beam function (adjust from QCD corrections)
- J_i : Jet function (adjust from QCD corrections)
- H : Hard function (adjust from QCD corrections and new contribution such as box diagram at NLO QED)
- S : Soft function (new calculation)



Soft function Calculation

- QCD case: 0-jettiness all coefficients are constant
- QED case: 2-jettiness all coefficients must be calculated for each phase space point



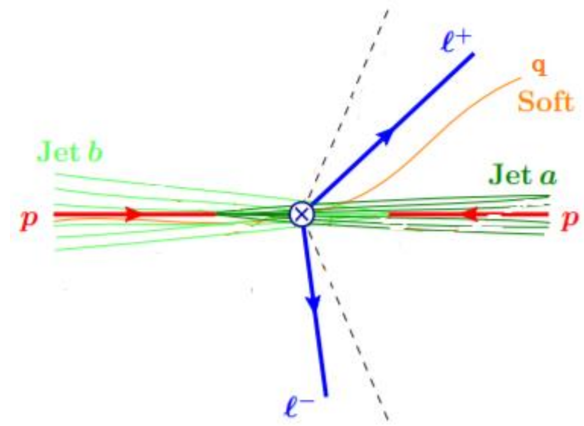
$$\frac{\alpha}{2\pi}S^{(1)} = |M_{ij}^{(1)}|^2\{PS^{(1)}(i,j)[F_i^{i,j} + F_j^{i,j}] + PS^{(1)}(k,i)[F_k^{i,j} + F_l^{i,j}]$$
$$= \frac{\alpha}{2\pi}[\mathcal{C}_{-1}\delta(\mathcal{T}_N) + \mathcal{C}_0\mathcal{L}_0(\mathcal{T}_N) + \mathcal{C}_1\mathcal{L}_1(\mathcal{T}_N)]$$

$$\int_0^1 dx\ \mathcal{L}_n(x)f(x) = \int_0^1 dx\frac{ln^n(x)}{x}[f(x) - f(0)]$$

J. Campbell, et al. arxiv:1711.09984
R. Boughezal, et al. arxiv:1504.02540
S. Jin, et al. arxiv:1901.10935

Soft function Calculation

Challenge: Calculation becomes more complex when final state leptons must be taken account!



Measurement Function

$$F = F_i + F_j + F_k + F_l$$

$$F_i = \delta(\mathcal{T}_N - q^i)\theta(q^j - q^i)\theta(q^k - q^i)\theta(q^l - q^i)$$

$$\mathcal{T}_2 = \min_i\left\{\frac{2p_i \cdot q}{P_i}\right\}$$
$$= \min_i\{q^i\}$$

F_i & F_j direction

$$|M_{i,j}^{(1)}|^2 PS^{(1)}(i,j) F_i^{i,j} =$$

$$-\frac{\alpha}{2\pi} Q_i Q_j \mathcal{T}_N^{-1-2\epsilon} e^{\gamma_E \epsilon} \mu^{\epsilon} \left(\frac{n_{ij}}{2}\right)^{\epsilon} \frac{2^{-2\epsilon}(-\epsilon)}{\Gamma[1-\epsilon]} \int_0^1 dx_1\, dx_2\, dx_3 (1-x_3)^{-\epsilon} \sin(\pi x_2)^{-2\epsilon} \left(\frac{\delta(x_1)}{\epsilon} + \mathcal{L}_0(x_1) + \epsilon \mathcal{L}_1(x_1)\right) \times \left(-\frac{\delta(x_3)}{\epsilon} + \mathcal{L}_0(x_3) - \epsilon \mathcal{L}_1(x_3)\right)$$

F_k & F_l direction

$$|M_{i,j}^{(1)}|^2 PS^{(1)}(k,i) F_k^{i,j} =$$

$$-\frac{\alpha}{2\pi} Q_i Q_j \mathcal{T}_N^{-1-2\epsilon} e^{\gamma_E \epsilon} \mu^{\epsilon} n_{ij} n_{ik}^{-1+\epsilon} \frac{2^{-3\epsilon}(-\epsilon)}{\Gamma[1-\epsilon]} \int_0^1 dx_1\, dx_2\, dx_3\, x_1^{\epsilon} (1-x_3)^{-\epsilon} \sin(\pi x_2)^{-2\epsilon} \times \left(-\frac{\delta(x_3)}{\epsilon} + \mathcal{L}(x_3) - \epsilon \mathcal{L}(x_3)\right)$$

$$\int_0^1 dx\, \mathcal{L}_n(x) f(x) = \int_0^1 dx \frac{ln^n(x)}{x} [f(x) - f(0)]$$

Calculation implementation

$$-\frac{\alpha}{2\pi}Q_iQ_j\mathcal{T}_N^{-1-2\epsilon}e^{\gamma_E\epsilon}\mu^{\epsilon}\left(\frac{n_{ij}}{2}\right)^{\epsilon}\frac{2^{-2\epsilon}(-\epsilon)}{\Gamma[1-\epsilon]}\int_0^1 dx_1\ dx_2\ dx_3(1-x_3)^{-\epsilon}\sin(\pi x_2)^{-2\epsilon}\left(\frac{\delta(x_1)}{\epsilon}+\mathcal{L}_0(x_1)+\epsilon\mathcal{L}_1(x_1)\right)$$
$$\times\left(-\frac{\delta(x_3)}{\epsilon}+\mathcal{L}_0(x_3)-\epsilon\mathcal{L}_1(x_3)\right)$$

- Already a 3-dimensional integration for each phase space point.
- Monte Carlo integration is too time consuming!

```
Time taken for soft function calculation: 20.07s
```

Calculation implementation

$$-\frac{\alpha}{2\pi}Q_iQ_j\mathcal{T}_N^{-1-2\epsilon}e^{\gamma_E\epsilon}\mu^\epsilon\left(\frac{n_{ij}}{2}\right)^\epsilon\frac{2^{-2\epsilon}(-\epsilon)}{\Gamma[1-\epsilon]}\int_0^1 dx_1\, dx_2\, dx_3(1-x_3)^{-\epsilon}\sin(\pi x_2)^{-2\epsilon}\left(\frac{\delta(x_1)}{\epsilon}+\mathcal{L}_0(x_1)+\epsilon\mathcal{L}_1(x_1)\right) \times\left(-\frac{\delta(x_3)}{\epsilon}+\mathcal{L}_0(x_3)-\epsilon\mathcal{L}_1(x_3)\right)$$

- Already a 3-dimensional integration for each phase space point.

- Monte Carlo integration is too time consuming!

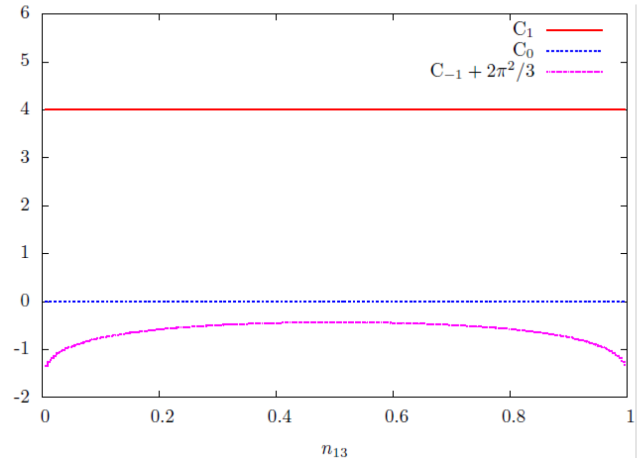
```
Time taken for soft function calculation: 20.07s
```

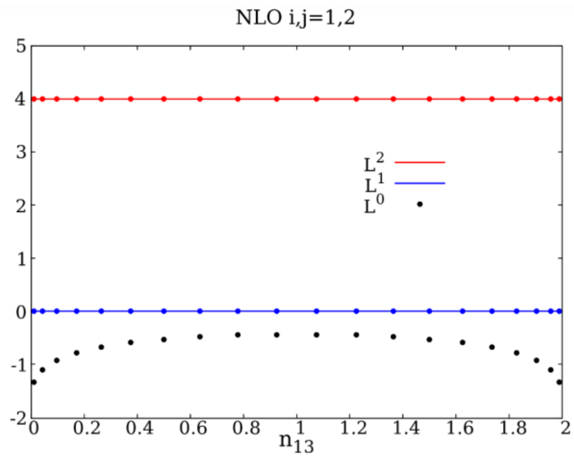
- Anaclitic simplification is applied to reduce to one dimensional integration, and Gauss integration is enough to achieve desired precision.

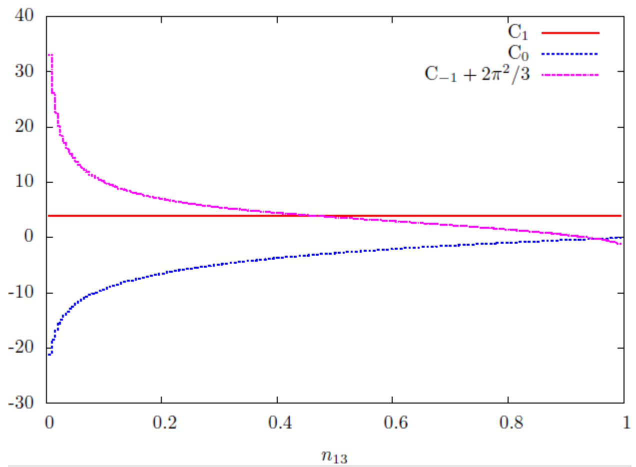
```
Time taken for soft function calculation:    1.63495541E-03 s
```

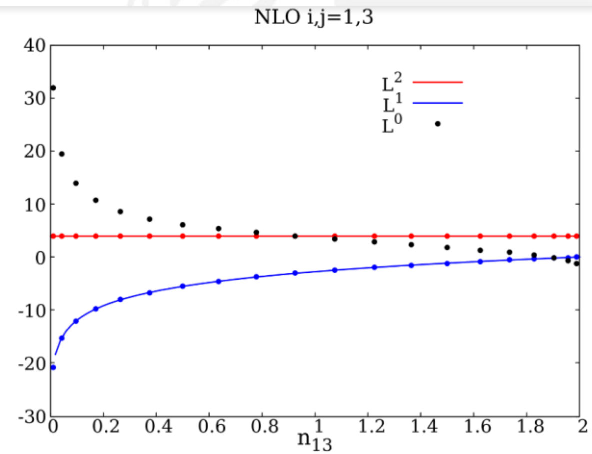
Results for NLO Soft Function

$$S^{(1)} = \mathcal{C}_{-1}\delta(\mathcal{T}_2) + \mathcal{C}_0\mathcal{L}_0(\mathcal{T}_2) + \mathcal{C}_1\mathcal{L}_1(\mathcal{T}_2)$$





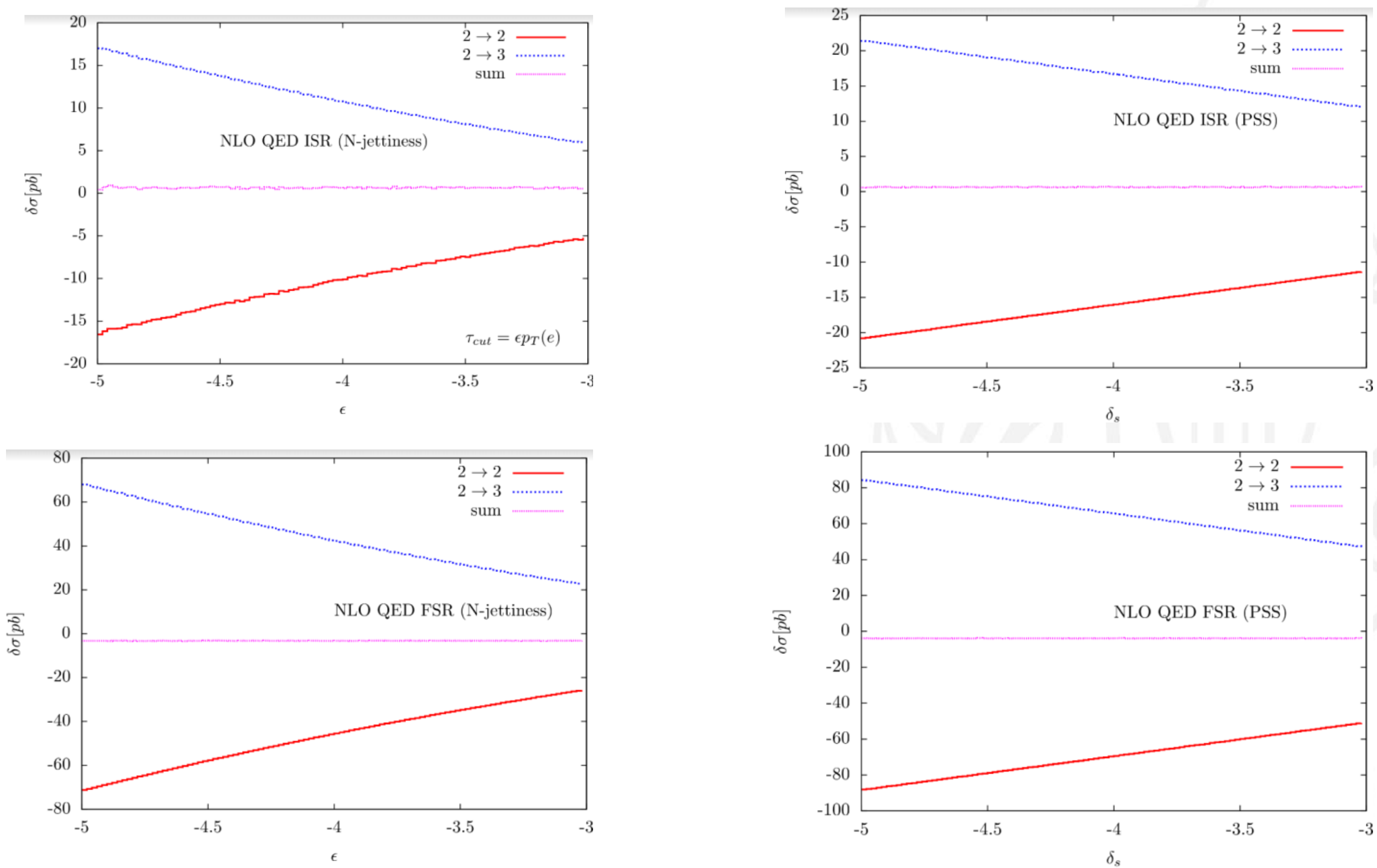




S. Jin, et al. arxiv:1901.10935

Prelimilary Results for NLO QED

- Final step, combining with B, J and H for implementation using ZGRAD Monte Carlo program.
- The N-jettiness subtraction method results are compared with Phase-space-slicing method implemented in ZGRAD as cross check.



Summary

- N-jettiness subtraction method for NLO QED calculation
- Universal Soft function can be applied to other similar processes
- Next step: NNLO QED$\times$QCD correction to Drell-Yan process

Thank You for Your Time and Attention!