Anomalies in Hadronic B Decays

[2311.18011]

Université de Montréal

An analysis of $SU(3)_F$ in the $B \to PP$ system $(P = \{\pi, K\})$

May 16th 2024

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Outline

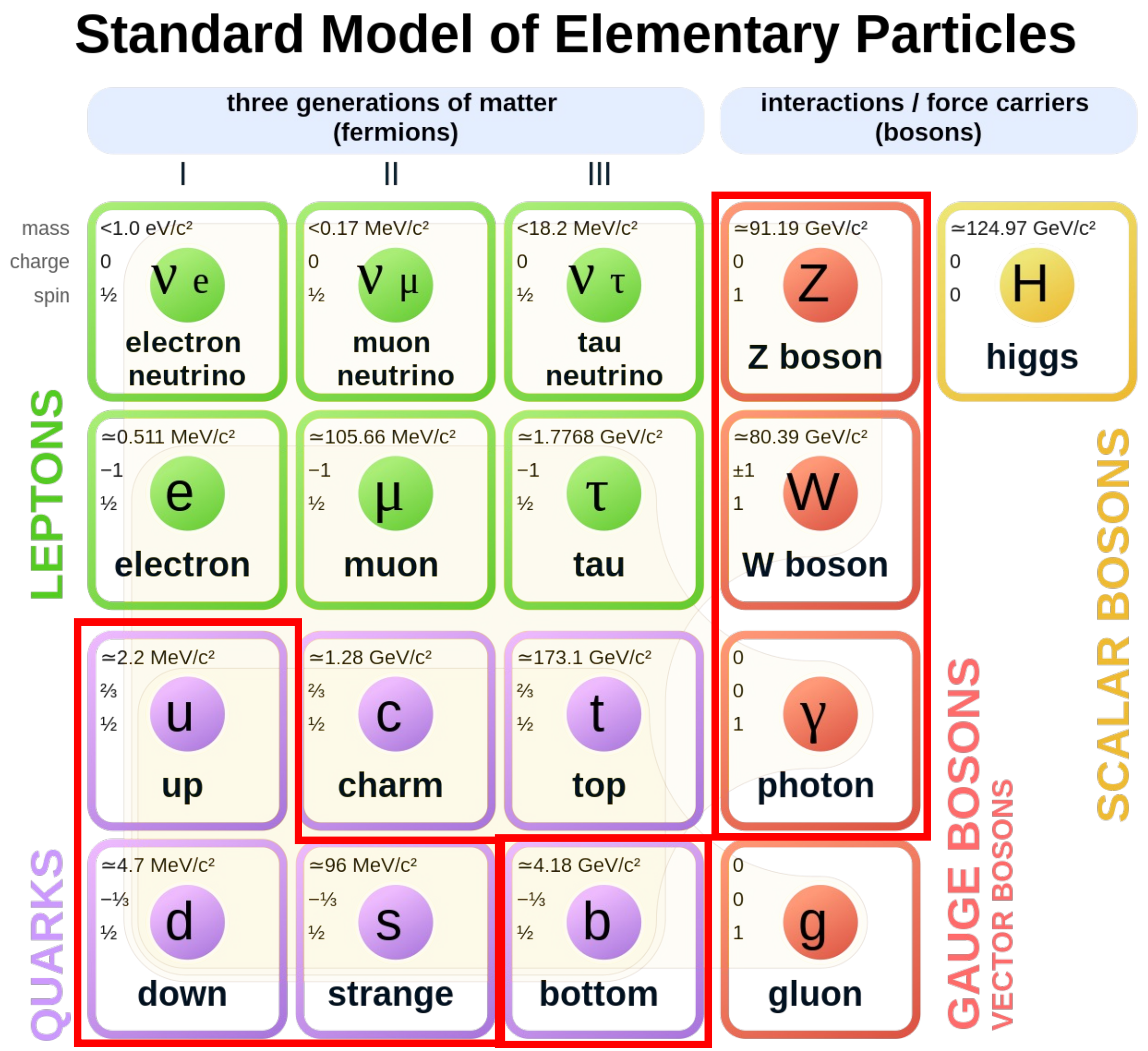
- $SU(3)_F$ symmetry
- Diagrams
- Data
- Fit results

16 decays $B \to PP$
where $B = \{B^+, B^0, B_s^0\}, P = \{\pi, K\}$

$\Delta s = 0$	$\Delta s = 1$
$B^+ \to \overline{K^0} K^+$	$B^+ \to \pi^+ K^0$
$B^+ \to \pi^0 \pi^+$	$B^+ \to \pi^0 K^+$
$B^0 \to K^0 \overline{K^0}$	$B^0 \to \pi^- K^+$
$B^0 \to \pi^+ \pi^-$	$B^0 \to \pi^0 K^0$
$B^0 \to \pi^0 \pi^0$	$B_s^0 \to K^+ K^-$
$B^0 \to K^+ K^-$	$B_s^0 \to K^0 \overline{K^0}$
$B_s^0 \to \pi^+ K^-$	$B_s^0 \to \pi^+ \pi^-$
$B_s^0 \to \pi^0 \overline{K^0}$	$B_s^0 \to \pi^0 \pi^0$

Introduction

- We are interested in:
  - b quark
  - Three lightest quarks $(SU(3)_F)$
  - Electroweak interaction

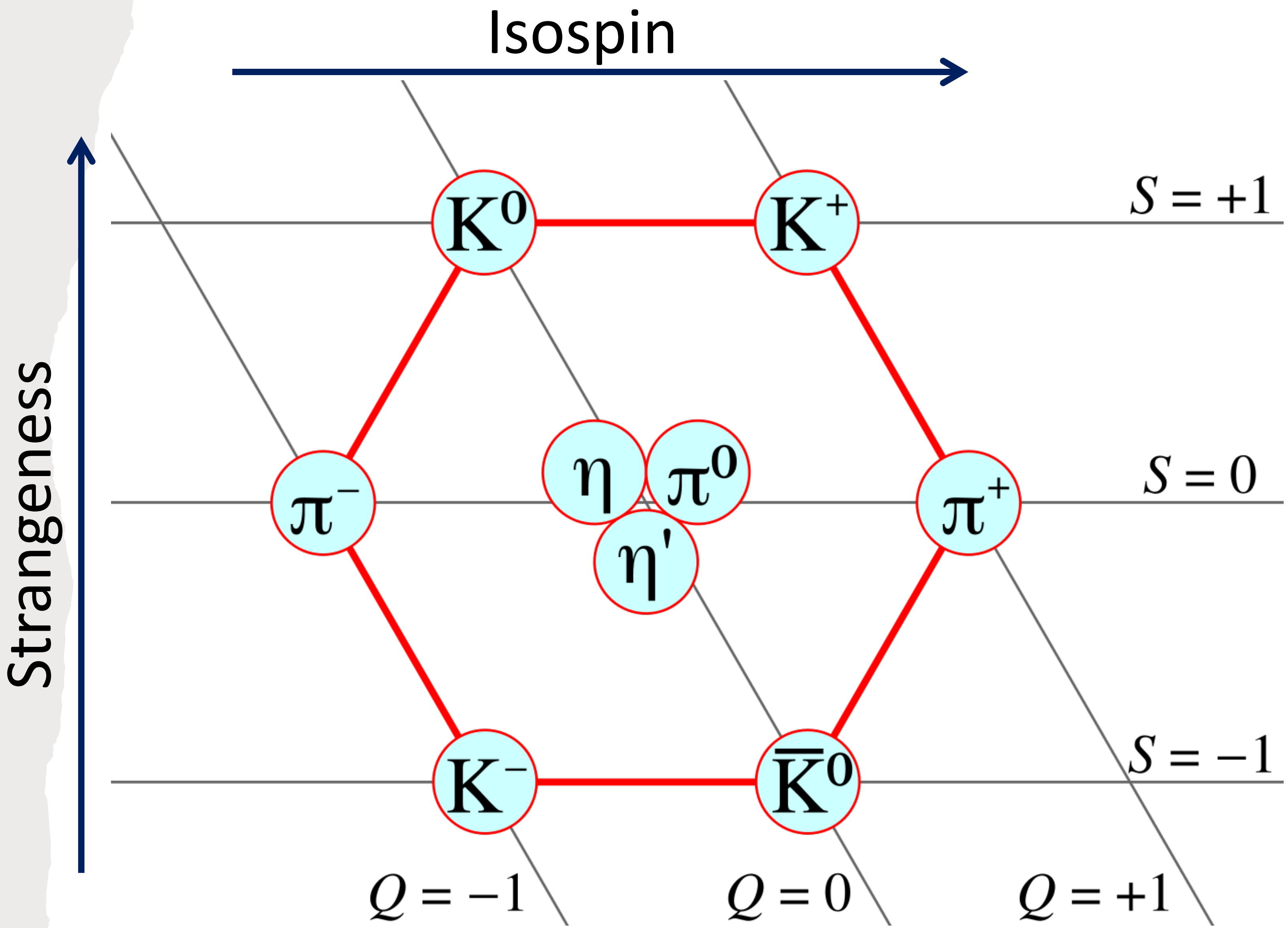


https://commons.wikimedia.org/wiki/File:Standard_Model_of_Elementary_Particles_edit.svg

$SU(3)_F$

$$\{B^+, B^0, B_s^0\}$$

$$P = \{\pi, K\}$$



https://en.wikipedia.org/wiki/Meson#/media/File:Meson_nonet_-_spin_0.svg

$SU(3)_F$

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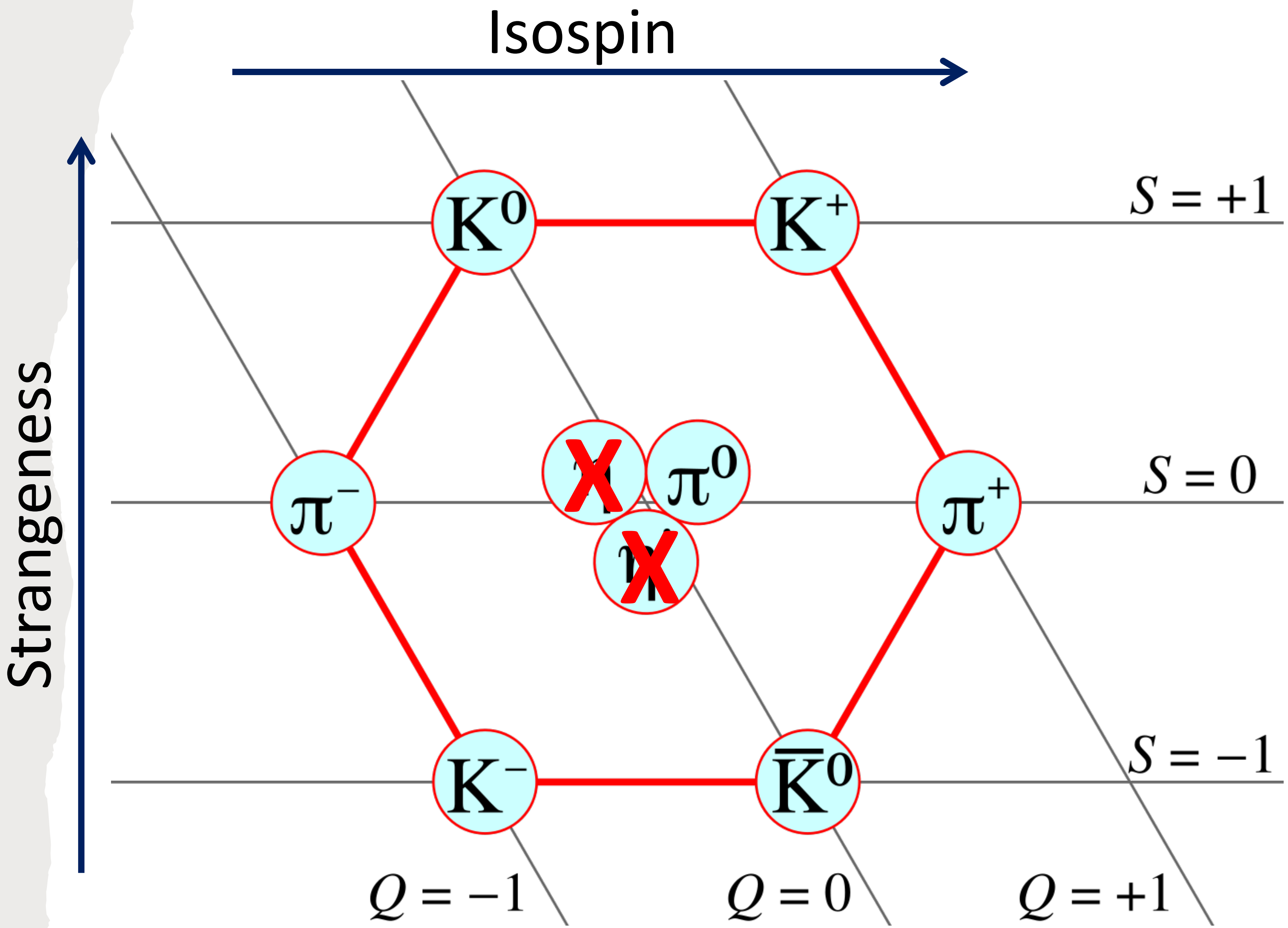

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CKM Matrix

$$V_{CKM} = \begin{matrix} & d & s & b \\ u & |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ c & -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ t & |V_{td}|e^{-i\beta} & -|V_{ts}|e^{-i\beta_s} & |V_{tb}| \end{matrix}$$

$$\sum_j V_{ij}V_{kj}^* = \delta_{ik} + \mathcal{O}(\lambda^4)$$

CKM Matrix

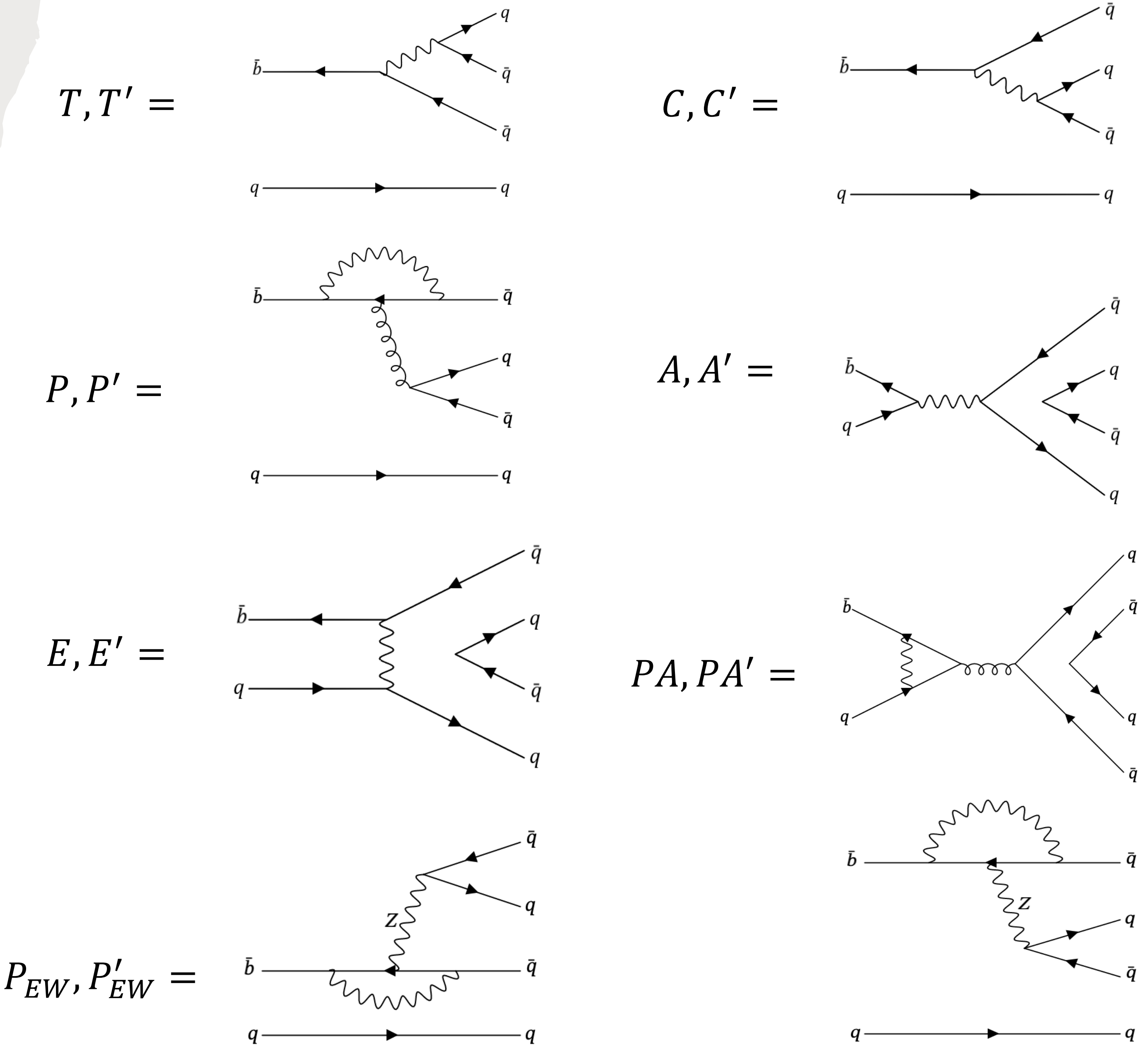
$$V_{CKM} = \begin{array}{c} \\ u \\ c \\ t \end{array} \begin{array}{c} \begin{array}{ccc} d & s & b \end{array} \\ \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}|e^{-i\beta_s} & |V_{tb}| \end{pmatrix} \end{array}$$

$$\sum_j V_{ij} V^*_{kj} = \delta_{ik} + \mathcal{O}(\lambda^4)$$

Diagrams

- Used to express the amplitudes
- Rigorous group theoretical objects
- Allow for a ‘physical’ interpretation

[arXiv: 9504326, 9504327]

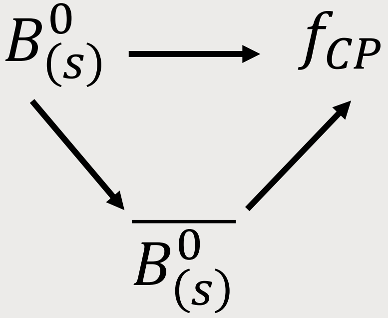


Observables

Branching ratio

Direct CP asymmetry

Indirect CP asymmetry



15 $\Delta s = 0$ observables
15 $\Delta s = 1$ observables

13 theoretical parameters

Decay	$\mathcal{B}_{CP}$ $(\times 10^{-6})$	A_{CP}	S_{CP}
$B^+ \to K^+\overline{K}^0$	1.31 ± 0.14	$0.04\pm0.14^\dagger$	
$B^+ \to \pi^+\pi^0$	5.59 ± 0.31	0.008 ± 0.035	
$B^0 \to K^0\overline{K}^0$	$1.21\pm0.16^\dagger$	0.06 ± 0.26	-1.08 ± 0.49
$B^0 \to \pi^+\pi^-$	5.15 ± 0.19	0.311 ± 0.030	-0.666 ± 0.029
$B^0 \to \pi^0\pi^0$	1.55 ± 0.16	0.30 ± 0.20	
$B^0 \to K^+K^-$	0.080 ± 0.015		
$B^0_s \to \pi^+K^-$	$5.90^{+0.87}_{-0.76}$	0.225 ± 0.012	
$B^0_s \to \pi^0\overline{K}^0$			

Decay	$\mathcal{B}_{CP}$ $(\times 10^{-6})$	A_{CP}	S_{CP}
$B^+ \to \pi^+K^0$	23.52 ± 0.72	-0.016 ± 0.015	
$B^+ \to \pi^0K^+$	13.20 ± 0.46	0.029 ± 0.012	
$B^0 \to \pi^-K^+$	19.46 ± 0.46	-0.0836 ± 0.0032	
$B^0 \to \pi^0K^0$	10.06 ± 0.43	-0.01 ± 0.13	$-0.58\pm0.17^\dagger$
$B^0_s \to K^+K^-$	$26.6^{+3.2}_{-2.7}$	-0.17 ± 0.03	0.14 ± 0.03
$B^0_s \to K^0\overline{K}^0$	17.4 ± 3.1		
$B^0_s \to \pi^+\pi^-$	$0.72^{+0.11}_{-0.10}$		
$B^0_s \to \pi^0\pi^0$	$2.8\pm2.8^*$		

The data has been taken from HFLAV and PDG

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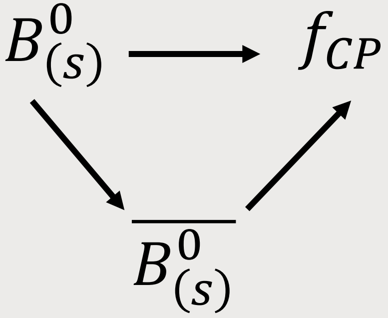

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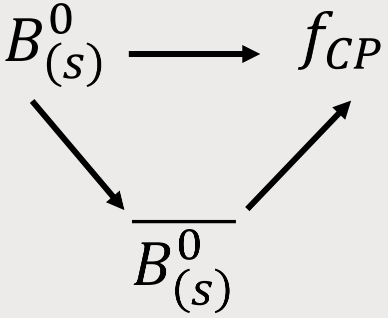

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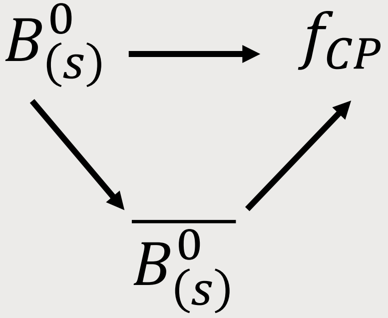

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Fits

- 15 observables
- 13 parameters
- 2 degrees of freedom
- Both fits are good

$\Delta s = 0$

$$\frac{\chi^2_{min}}{d.o.f} = 0.35/2$$

p-value of 0.84

0.20σ

$\Delta s = 1$

$$\frac{\chi^2_{min}}{d.o.f} = 1.8/2$$

p-value of 0.40

0.84σ

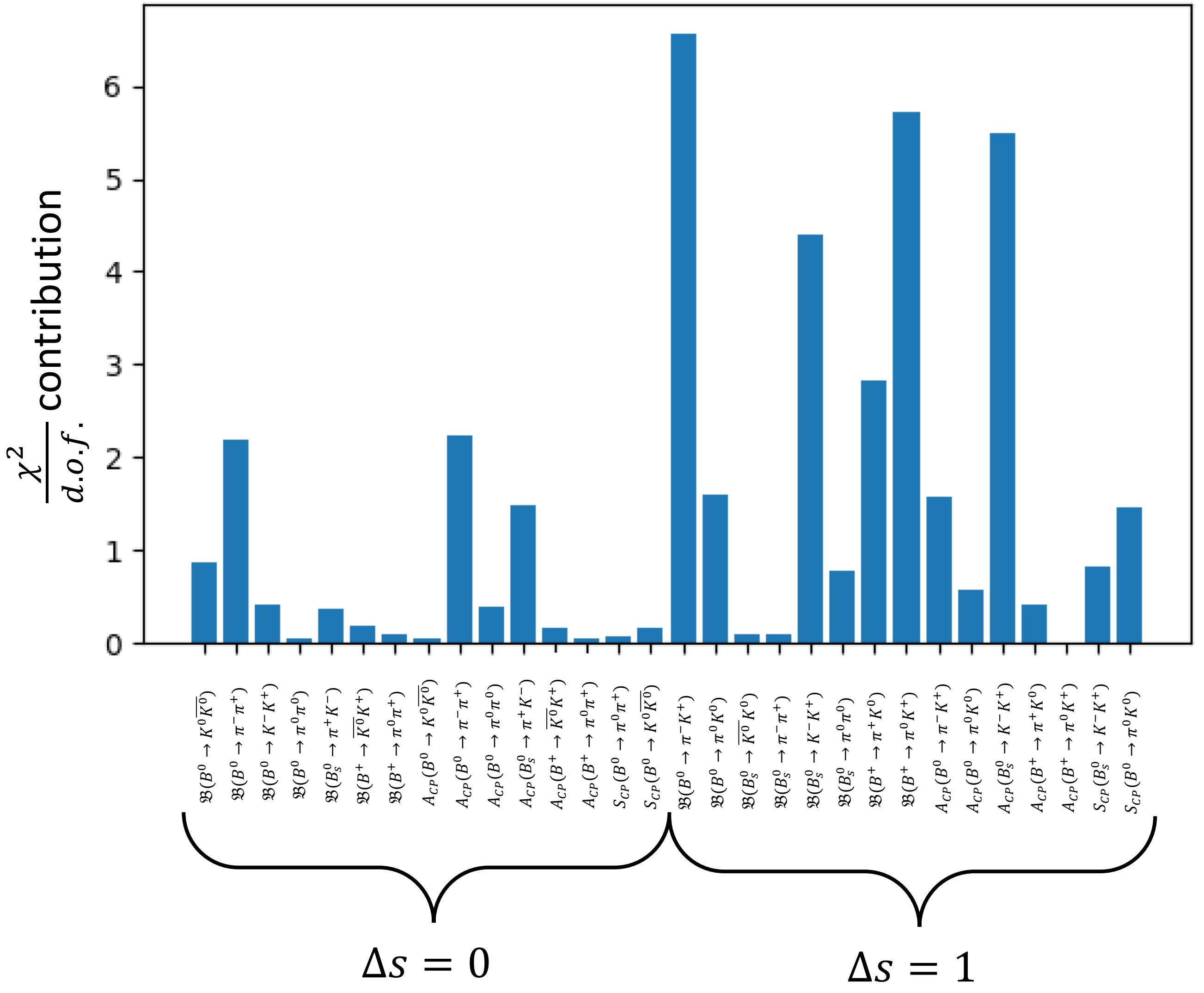
Complete $SU(3)_F$ fit

$SU(3)_F$

$$\frac{\chi^2_{min}}{d.o.f} = 43.8/17$$

p-value of 3.6×10^{-4}

3.6σ



$SU(3)_F$ breaking

$\Delta s = 0$	$\lvert\tilde{T}\rvert$	$\lvert\tilde{C}\rvert$	$\lvert\widetilde{P_{uc}}\rvert$	$\lvert\tilde{A}\rvert$	$\lvert\widetilde{PA_{uc}}\rvert$	$\lvert\widetilde{P_{tc}}\rvert$	$\lvert\widetilde{PA_{tc}}\rvert$
	4.0 ± 0.5	6.6 ± 0.7	3 ± 4	6 ± 5	0.7 ± 0.8	0.8 ± 0.4	0.2 ± 0.4
$\Delta s = 1$	$\lvert\widetilde{T'}\rvert$	$\lvert\widetilde{C'}\rvert$	$\lvert\widetilde{P'_{uc}}\rvert$	$\lvert\widetilde{A'}\rvert$	$\lvert\widetilde{PA'_{uc}}\rvert$	$\lvert\widetilde{P'_{tc}}\rvert$	$\lvert\widetilde{PA'_{tc}}\rvert$
	49 ± 13	42 ± 14	48 ± 14	84 ± 28	7 ± 4	0.76 ± 0.16	0.24 ± 0.04

$\left\lvert\frac{D'}{D}\right\rvert$	$\left\lvert\frac{\widetilde{T'}}{\tilde{T}}\right\rvert$	$\left\lvert\frac{\widetilde{C'}}{\tilde{C}}\right\rvert$	$\left\lvert\frac{\widetilde{P'_{uc}}}{\widetilde{P_{uc}}}\right\rvert$	$\left\lvert\frac{\widetilde{A'}}{\tilde{A}}\right\rvert$	$\left\lvert\frac{\widetilde{PA'_{uc}}}{\widetilde{PA_{uc}}}\right\rvert$	$\left\lvert\frac{\widetilde{P'_{tc}}}{\widetilde{P_{tc}}}\right\rvert$	$\left\lvert\frac{\widetilde{PA'_{tc}}}{\widetilde{PA_{tc}}}\right\rvert$
	12.3 ± 3.6	6.4 ± 2.2	16 ± 22	14 ± 13	10 ± 13	0.95 ± 0.52	1.2 ± 2.4

$$\frac{f_K}{f_\pi} - 1 = \sim 20\%$$

$SU(3)_F$ breaking

$\Delta s = 0$	$\lvert\tilde{T}\rvert$	$\lvert\tilde{C}\rvert$	$\lvert\widetilde{P_{uc}}\rvert$	$\lvert\tilde{A}\rvert$	$\lvert\widetilde{PA_{uc}}\rvert$	$\lvert\widetilde{P_{tc}}\rvert$	$\lvert\widetilde{PA_{tc}}\rvert$
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	12.3 ± 3.6	6.4 ± 2.2	16 ± 22	14 ± 13	10 ± 13	0.95 ± 0.52	1.2 ± 2.4

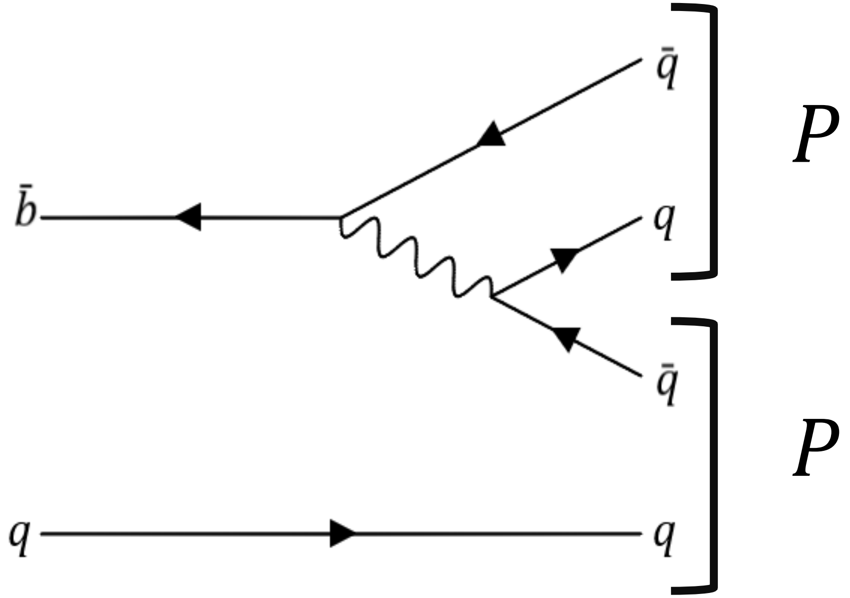
$$\frac{f_K}{f_\pi} - 1 \sim 20\% \quad \rightarrow 1000\%!$$

$\left|\frac{C}{T}\right|$ ratio

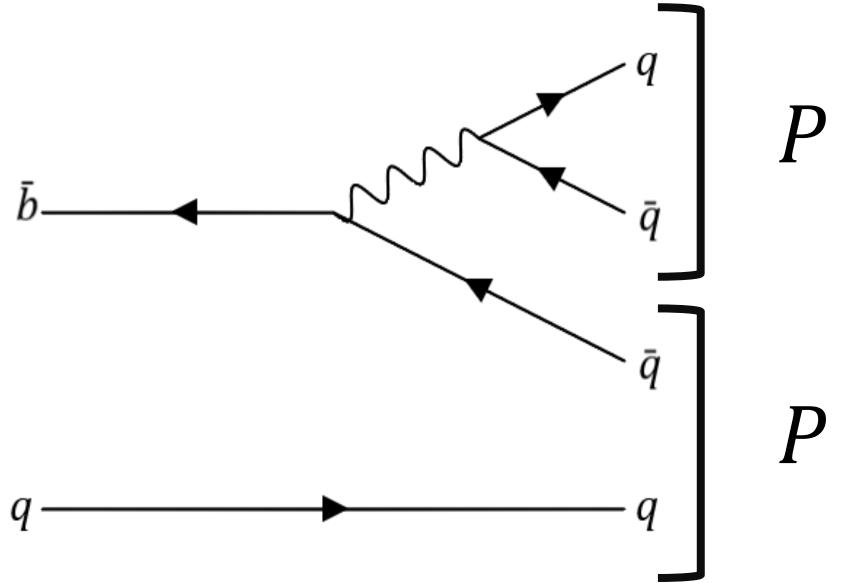
Naively we expect $\left|\frac{C}{T}\right| = \frac{1}{3}$

QCD factorization → $\left|\frac{C}{T}\right| \approx 0.2$

$C, C' =$



$T, T' =$



$\left|\frac{C}{T}\right|$ ratio

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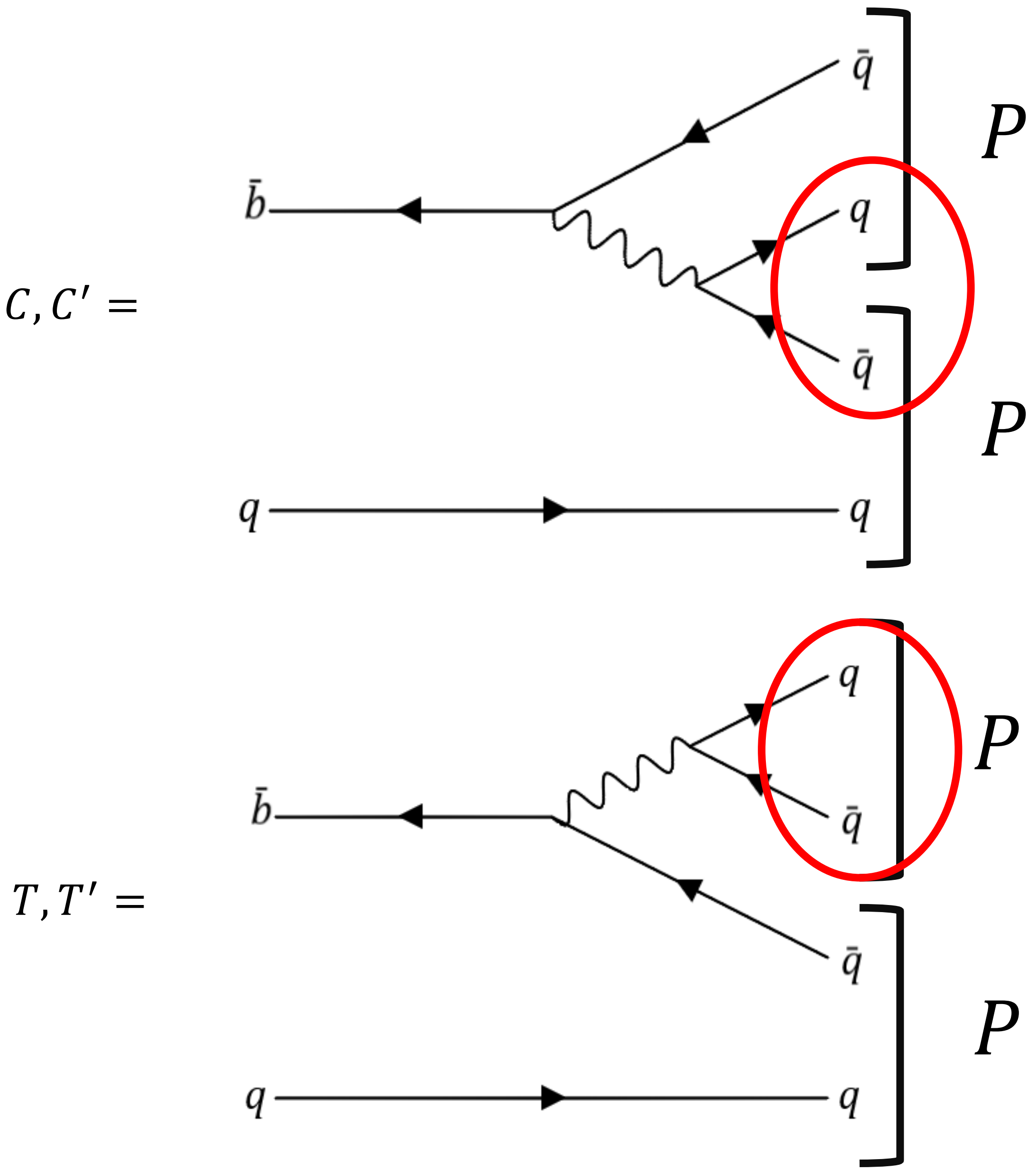
QCD factorization $\rightarrow \left|\frac{C}{T}\right| \approx 0.2$



Fits when imposing $\left|\frac{C}{T}\right| = 0.2$

$\Delta s = 0$ $\quad \frac{\chi^2_{min}}{d.o.f.} = 18.8/3$ $\quad$ p-value of 3.1×10^{-4}
3.6σ

$\Delta s = 1$ $\quad \frac{\chi^2_{min}}{d.o.f.} = 6.3/3$ $\quad$ p-value of 0.10

$SU(3)_F$ $\quad \frac{\chi^2_{min}}{d.o.f.} = 55.5/18$ $\quad$ p-value of 1.1×10^{-5}
4.4σ

Conclusion

- Poor fit with the data
- High symmetry breaking required
- New physics?

$$\left|\frac{\tilde{C}}{\tilde{T}}\right| = 0.2$$

$\Delta s = 0$ $\quad \frac{\chi^2_{min}}{d.o.f} = 0.35/2 \longrightarrow \frac{\chi^2_{min}}{d.o.f.} = 18.8/3$

$\Delta s = 1$ $\quad \frac{\chi^2_{min}}{d.o.f} = 1.8/2 \longrightarrow \frac{\chi^2_{min}}{d.o.f.} = 6.3/3$

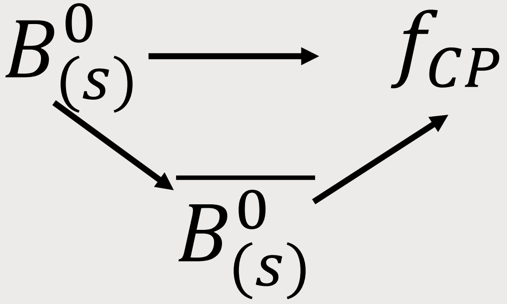
$SU(3)_F$ $\quad \frac{\chi^2_{min}}{d.o.f} = 43.8/17 \longrightarrow \frac{\chi^2_{min}}{d.o.f.} = 55.5/18$

Thank you!

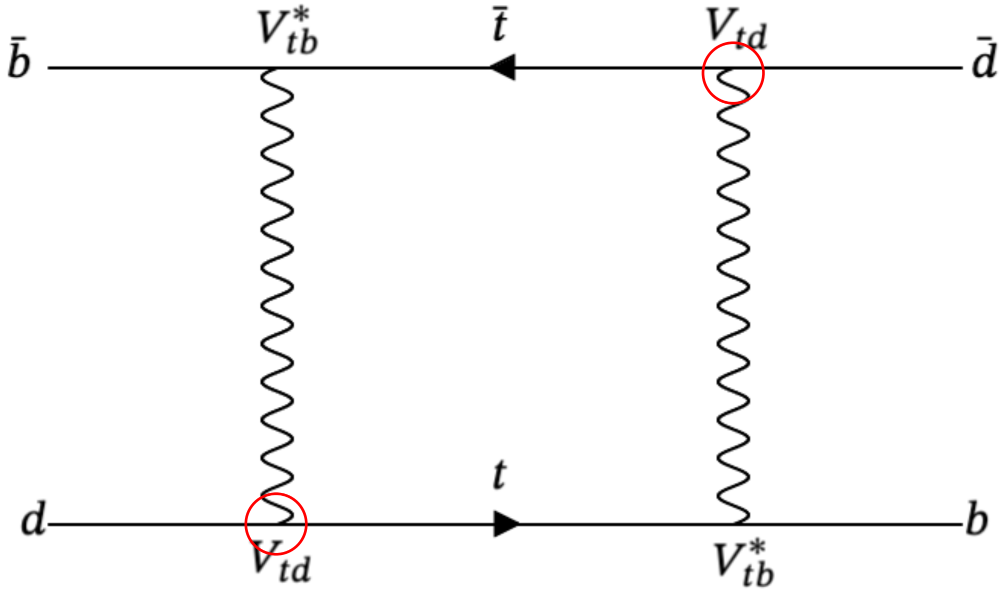
Supplemental material

Weak Phase Mixing

Tagging of $B^0_{(s)} - \overline{B^0_{(s)}}$



-2β phase



Wigner-Eckart Theorem

The Wigner-Eckart theorem contains 3 parts

$$\left\langle j, m \left| T_Q^{(K)} \right| j', m' \right\rangle = \left\langle j \left|\left| T^{(K)} \right|\right| j' \right\rangle \langle K, j'; Q, m' | j, m \rangle$$

The transition

The reduced matrix element

$$\left\langle G, Y, I, I_3 \left| T^{(G")}_{Y",I",I_3"} \right| G', Y', I', I_3' \right\rangle = \langle G \left|\left| T^{G"} \right|\right| G' \rangle \langle Y", Y', I", I'; I_3", I_3' | Y, I, I_3 \rangle$$

The Clebsch-Gordan coefficient

Reduced Matrix Elements (RMEs)

$$(8 \otimes 8)_s = (1 \oplus 8 \oplus 27)_s$$

$$A_1 = \langle 3|\overline{3_1}|1\rangle$$
$$A_8 = \langle 3|\overline{3_1}|8\rangle$$
$$R_8 = \langle 3|6|8\rangle$$
$$P_8 = \left\langle 3\middle|\overline{15}\middle|8\right\rangle$$
$$P_{27} = \langle 3|\overline{15}|27\rangle$$
$$B_1 = \langle 3|\overline{3_2}|1\rangle$$
$$B_8 = \langle 3|\overline{3_2}|8\rangle$$

Advantage of using diagrams

- RMEs' values don't have clear physical interpretation
- RMEs' involve calculation of $SU(3)$ Clebsch-Gordan coefficients

↓

- Diagrams have some physical interpretation
- Diagrams do not involve calculation

But, we have a problem…

Reduced Matrix Elements (RMEs)

$$A_1 = \langle 3|\overline{3_1}|1\rangle$$
$$A_8 = \langle 3|\overline{3_1}|8\rangle$$
$$R_8 = \langle 3|6|8\rangle$$
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$$P_{27} = \langle 3|\overline{15}|27\rangle$$
$$B_1 = \langle 3|\overline{3_2}|1\rangle$$
$$B_8 = \langle 3|\overline{3_2}|8\rangle$$

$$T, C, P_{uc}, A, E, PA_{uc}, P_{tc}, PA_{tc}, P_{EW}, P_{EW}^{C}$$

Absorb E

$$\tilde{T}, \tilde{C}, \tilde{P}_{uc}, \tilde{A}, \widetilde{PA}_{uc}, P_{tc}, PA_{tc}, P_{EW}, P_{EW}^{C}$$

We still have 2 more diagrams than RMEs!

P_{EW} tree relations

$$P_{EW} = -\frac{3}{4}\left(\frac{c_9 + c_{10}}{c_1 + c_2}\left(\tilde{T} + \tilde{C} + \tilde{A}\right) + \frac{c_9 - c_{10}}{c_1 - c_2}\left(\tilde{T} - \tilde{C} - \tilde{A}\right)\right)$$

$$P_{EW}^{c} = -\frac{3}{4}\left(\frac{c_9 + c_{10}}{c_1 + c_2}\left(\tilde{T} + \tilde{C} - \tilde{A}\right) - \frac{c_9 - c_{10}}{c_1 - c_2}\left(\tilde{T} - \tilde{C} - \tilde{A}\right)\right)$$

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$$\tilde{T}$$
$$\tilde{C}$$
$$\tilde{P}_{uc}$$
$$\tilde{A}$$
$$\widetilde{PA}_{uc}$$
$$P_{tc}$$
$$PA_{tc}$$

Equivalence between RMEs and diagrams

- Describe the decays:
  - In terms of RMEs
  - In terms of diagrams
- Make an augmented matrix
- Row reduce

$$A_1 = \frac{1}{2\sqrt{3}}\left(-3\tilde{T} + \tilde{C} - 8\tilde{P}_{uc} - 12\widetilde{PA}_{uc}\right)$$

$$A_8 = \frac{1}{8}\sqrt{\frac{5}{3}}\left(-3\tilde{T} + \tilde{C} - 8\tilde{P}_{uc} - 3\tilde{A}\right)$$

$$R_8 = \frac{\sqrt{5}}{4}\left(\tilde{T} - \tilde{C} - \tilde{A}\right)$$

$$P_8 = \frac{1}{8\sqrt{3}}\left(\tilde{T} + \tilde{C} + 5\tilde{A}\right)$$

$$P_{27} = -\frac{1}{2\sqrt{3}}\left(\tilde{T} + \tilde{C}\right)$$

$$B_1 = -\frac{4}{\sqrt{3}}\left(\frac{3}{2}PA_{tc} + P_{tc}\right)$$

$$B_8 = -\sqrt{\frac{5}{3}}P_{tc}$$