Scalar scattering amplitudes: zero loci, factorization, and the double copy

Tonnis ter Veldhuis[1]

Macalester College

DPF-PHENO 2024, May 14, 2024

[1] Work in collaboration with Yang Li and Diederik Roest; arXiv:2403.12939 [hep-th].

Introduction

- It was recently discovered by Arkani-Hamed et al that the tree-level, color-ordered partial amplitudes of $\mathrm{Tr}(\Phi^3)$ vanish on specific hypersurfaces in the space of Mandelstam variables.
- These zero loci naturally arise as 'causal diamonds' in a two-dimensional kinematic mesh.
- The zero loci give rise to factorisation channels into lower-point amplitudes
- The zero loci of $\mathrm{Tr}(\Phi^3)$ carry over to two more color ordered theories: the chiral non-linear sigma model (NLSM), and scaffolded Yang-Mills.
- The zero loci include and extend the soft-limit behaviour (Adler zero) of the NLSM.

- We identify three non-color ordered theories that naturally generalise these results
  - flavor ordered: scaffolded General Relativity (GR).
  - flavor ordered: multi-Dirac-Born Infeld (DBI)
  - single scalar: special Galileon (SG).
- The six theories together naturally span the set of scalar field theories that are related via the double copy procedure.
- I will discuss two of the six theories as representative examples.
- The papers Cao et al, arXiv:2403.08855 [hep-th], and Bartsch et al. arXiv:2403.10594 [hep-th] contain related work with some overlap. Also see the plenary talk by Jaroslav Trnka at this conference, Wednesday at 11.00 am.

Color ordered theory: $\mathrm{Tr}(\Phi^3)$

- Real scalar transforming under the adjoint representation of $SU(N)$.
- Amplitudes at all-loop and of any multiplicity follow from compact geometric rules on surfaces.

Four point color ordered partial amplitude (canonical ordering):

- Zero locus: $s_{13} = t = 0$.
- Two poles available for exchange interactions: s- and u-channel.
- $A_4^{\Phi^3} \sim \frac{1}{s} + \frac{1}{t} \sim \frac{u}{st}$
- Partial amplitude is invariant under cyclic permutations.
- Other permutations transform this partial amplitude into another one, with corresponding zero locus.

Six point color ordered partial amplitude (canonical ordering):

- Zero loci:
  - "skinny rectangle": $s_{13} = s_{14} = s_{15} = 0$, and cyclic permutations.
  - "fat square": $s_{14} = s_{24} = s_{15} = s_{25} = 0$, and cyclic permutations.
- factorization near "skinny rectangle:"
  $A_6^{\Phi^3} \to (\frac{1}{X_T} + \frac{1}{X_B})A_3^{\Phi^3} A_5^{\Phi^3}$
- factorization near "fat square:"
  $A_6^{\Phi^3} \to (\frac{1}{X_T} + \frac{1}{X_B})A_4^{\Phi^3} A_4^{\Phi^3}$

Single scalar: Special Galileon

- Lagrangian invariant under the non-linear transformation $\delta\phi = s_{\mu\nu}x^\mu x^\nu + s_{\mu\nu}\partial^\mu\phi\partial^\nu\phi$.
- Maximal soft degree: $\sigma = 3$.

Four point full amplitude:

- Zero locus: $s_{13} = t = 0$, and any permutation, thus also $s_{12} = s = 0$ and $s_{14} = u = 0$.
- $A_4^{SG} \sim stu$
- Amplitude is invariant under any permutation.

Six point amplitude:

- Zero loci:
  - "skinny rectangle": $s_{13} = s_{14} = s_{15} = 0$, and any permutation.
  - "fat square": $s_{14} = s_{24} = s_{15} = s_{25} = 0$, and any permutation.
- Factorization near "skinny rectangle:"
  $A_6^{\mathrm{SG}} \to s^3 \times A_5^{\mathrm{ext\ SG}}$
- The extended 5 point amplitude can be generated from the Lagrangian: $\mathcal{L}^{\mathrm{ext}} = \sqrt{-g}(-\frac{1}{2}\partial^\mu\Phi\partial_\mu\Phi + \Phi^3)$ in terms of the special Galileon covariant metric $g_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu\partial_\rho\varphi\partial^\rho\partial_\nu\varphi$.
- factorization near "fat square:"
  $A_6^{\mathrm{SG}} \to \left(\frac{1}{X_B} + \frac{1}{X_T}\right) A_4^{\mathrm{up,SG}} \times A_4^{\mathrm{down,SG}}$
- Considering again the "skinny rectangle" loci, the amplitude vanishes when setting any three of the Mandelstam variables $s_{\alpha\beta}$ from the set $\alpha = (1)\,, \quad \beta = (\{2,3,4,5,6\})$ to zero. Accordingly, the amplitude must be trilinear in the above set. This implies that the amplitude scales with third power in the soft limit $p_1 \to 0$: $A_6^{\mathrm{SG}} \sim p_1^3$.

Double copy

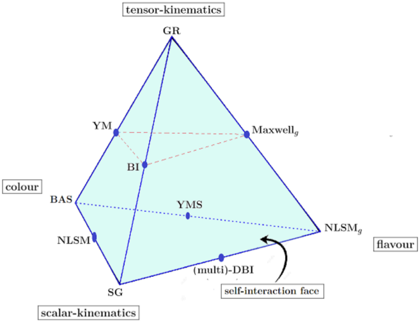

Figure: *Tetrahedron indicating BCJ relations among theories. Figure taken from De Neeling et al, HEP 10 (2022) 066, 2204.11629 [hep-th].*

Overview

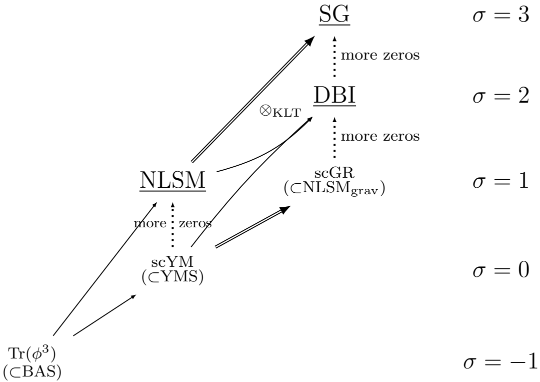

Figure: *The six theories with related zero loci, arranged in terms of their soft degree σ. The underlined theories have no odd vertices and hence the zeros imply their soft behaviours.*

Conclusions

- Three double copy theories inherit amplitude zeros from their single copies: scaffolded GR, multi-DBI and the special Galileon.
- For the two Goldstone theories, these zero loci underlie their enhanced soft behaviours.
- The extended Lagrangians that generate the mixed amplitudes from the near-zero factorization as covariantly coupled colorless Φ^3, where the minimal coupling is to different metrics for the three cases.
- The inherited zero loci of the double copy theories can be explained via the most general version of the KLT formula.