Neutrino Mass and Gravitational Wave

Arnab Dasgupta, Tao Han, Tong Arthur Wu

Neutrino mass

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$$y_\nu L\tilde{H}\nu_R + M\nu_R S + \frac{1}{2}\mu SS + \text{h.c.} \rightarrow \frac{1}{2}\begin{pmatrix}\nu_L & \nu_R & S\end{pmatrix}\begin{pmatrix} 0 & \frac{y_\nu v}{\sqrt{2}} & 0 \\ \frac{y_\nu v}{\sqrt{2}} & 0 & M \\ 0 & M & \mu \end{pmatrix}\begin{pmatrix}\nu_L \\ \nu_R \\ S\end{pmatrix} + \text{h.c.}$$

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$\Rightarrow m_\nu = \frac{\mu}{2}\left(\frac{y_\nu v}{M}\right)^2 \Rightarrow$ Natural suppression!

Model

Scalar induced mass term:

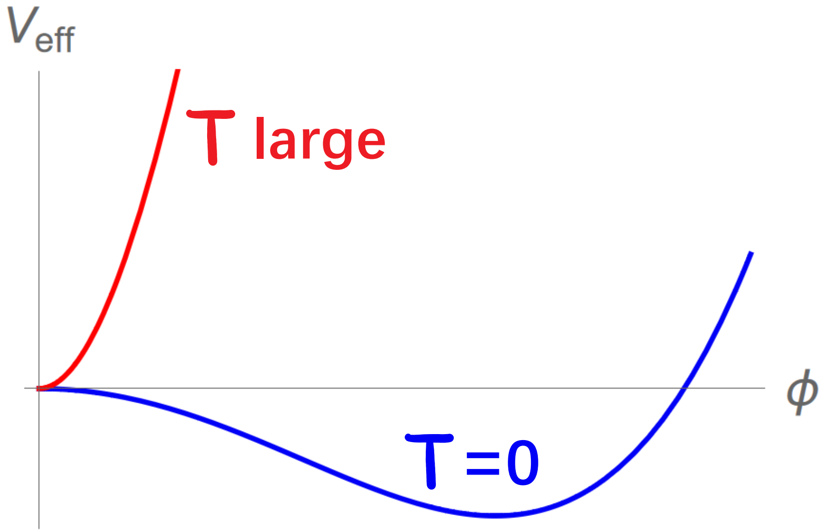
	(2s + 1)	$SU(2)_L \times U(1)_Y \times U(1)_D$
ν_R^i	2	$(1, 0, 0)$
S	2	$(1, 0, -1)$
$\bar{S}$	2	$(1, 0, 1)$
σ	1	(1,0,1)
ϕ	1	(1,0,2)

$$\mathcal{L}_{\rm Yukawa} \supset -\Big[y_\nu L\tilde{H}\nu_R + \big(y_\sigma\,\sigma\,\nu_R S + \bar{y}_\sigma\,\sigma^*\nu_R\bar{S}\big) + \big(y_\phi\,\phi SS + \bar{y}_\phi\,\phi^*\bar{S}\bar{S}\big) + \text{h.c.}\Big]$$

M μ

$$\Rightarrow m_\nu = \sqrt{2}\,y_\phi v_\phi\left(\frac{y_\nu v_H}{y_\sigma v_\sigma}\right)^2$$

Gravitational wave from phase transition



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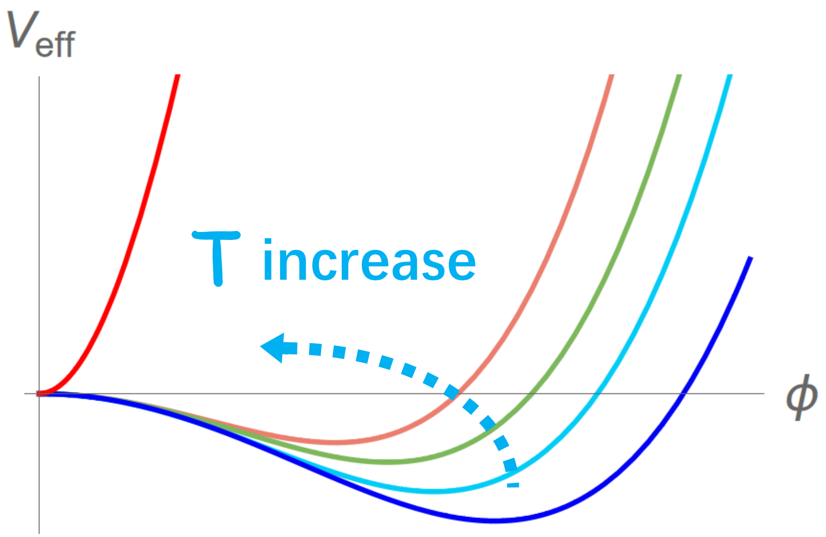

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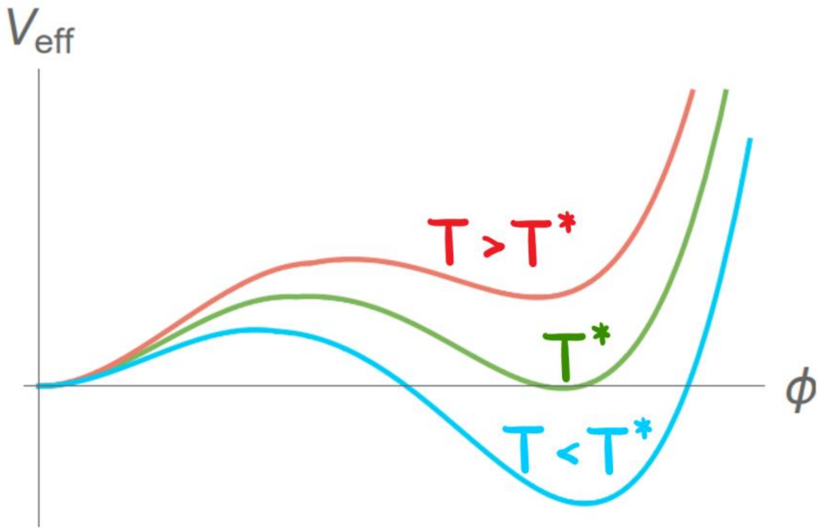

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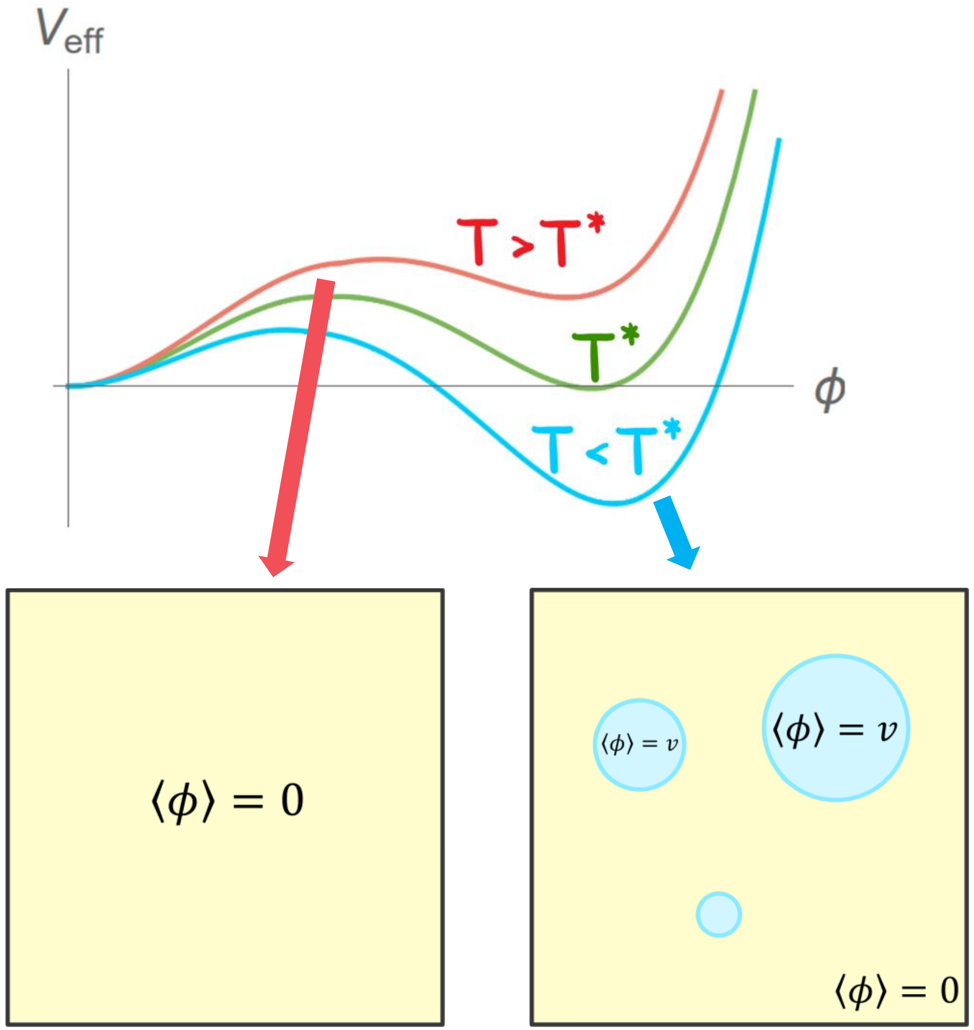

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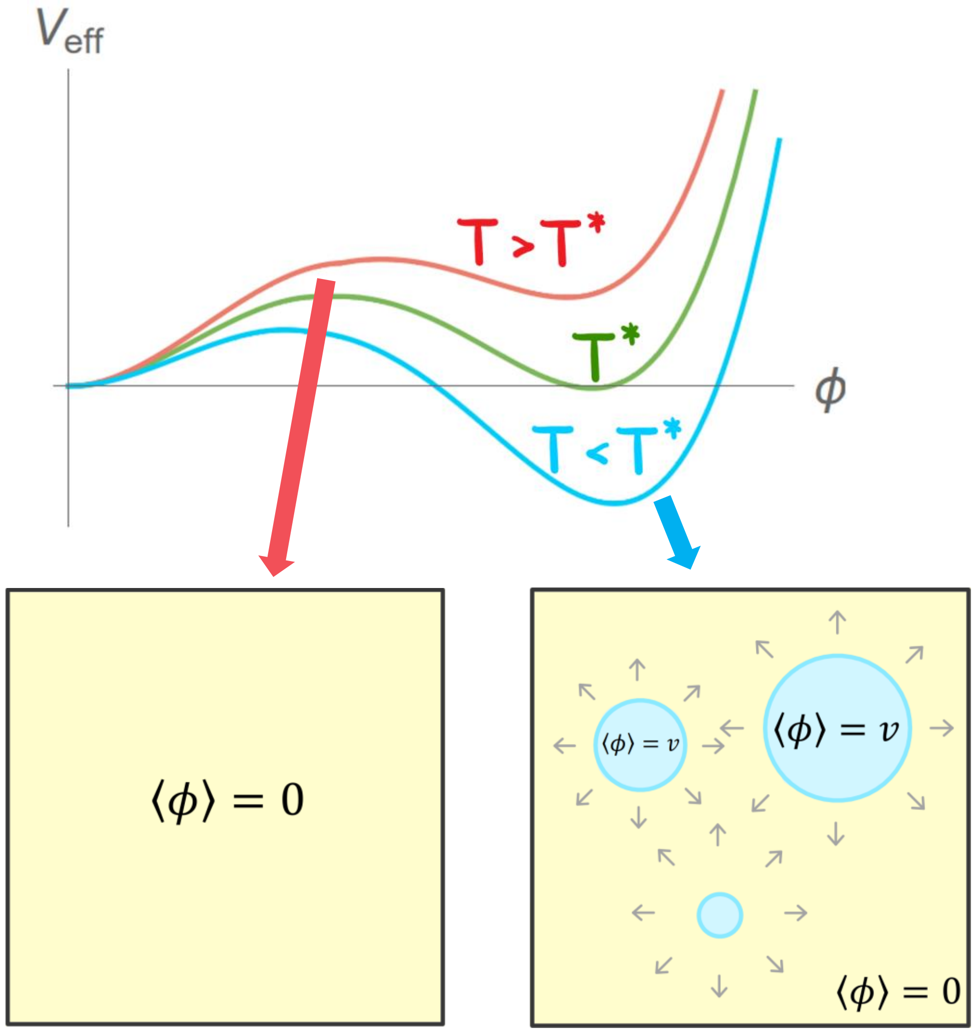

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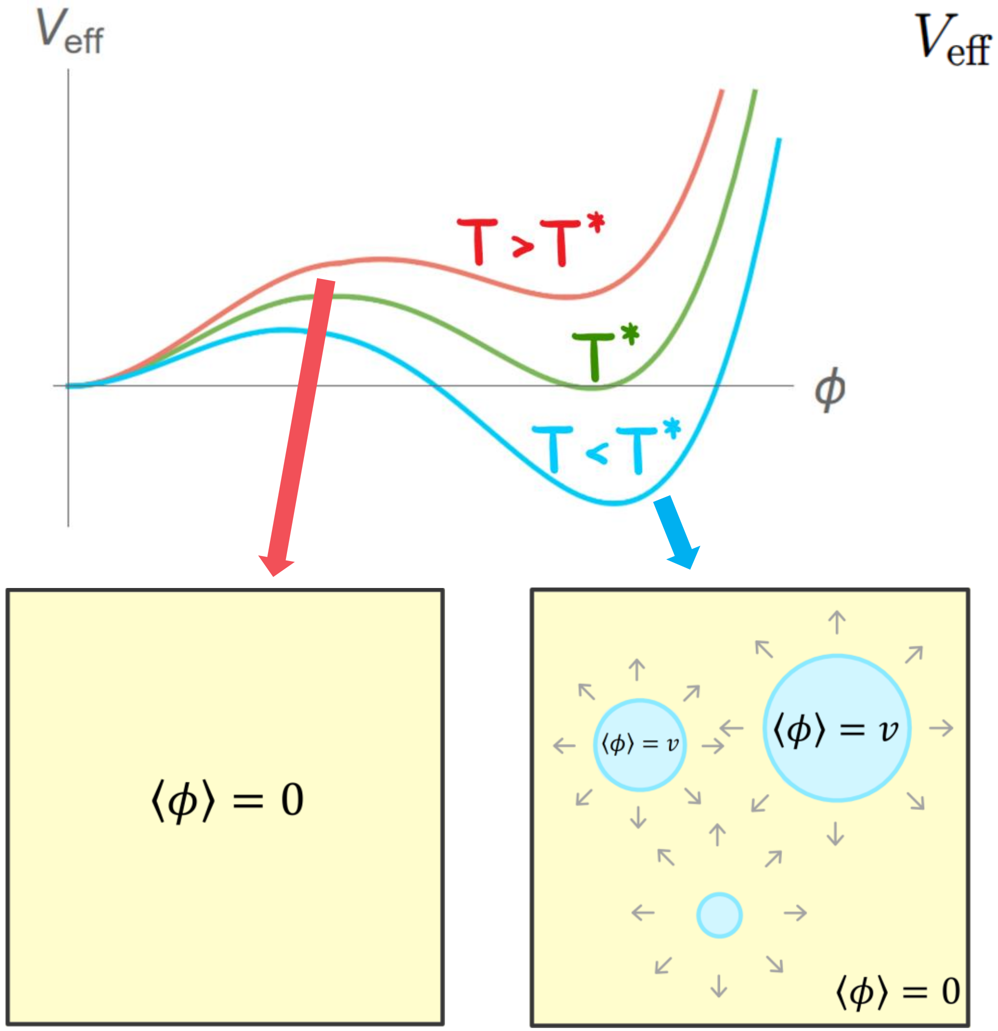

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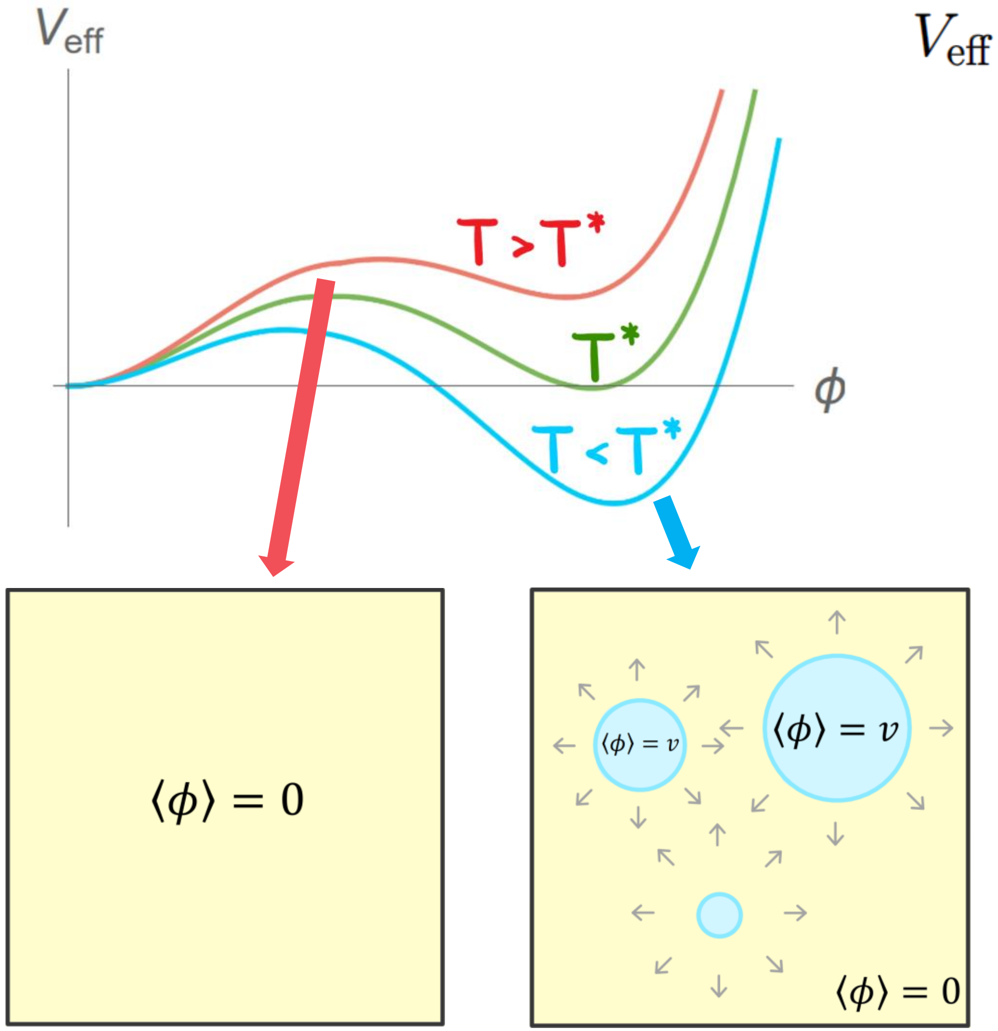

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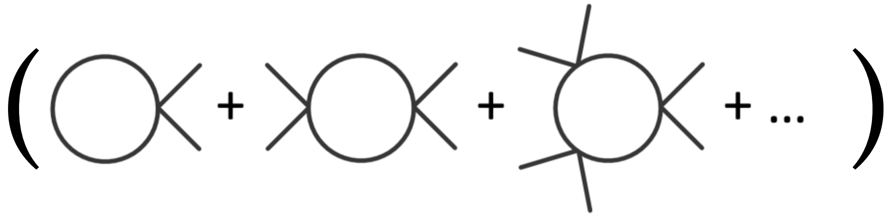

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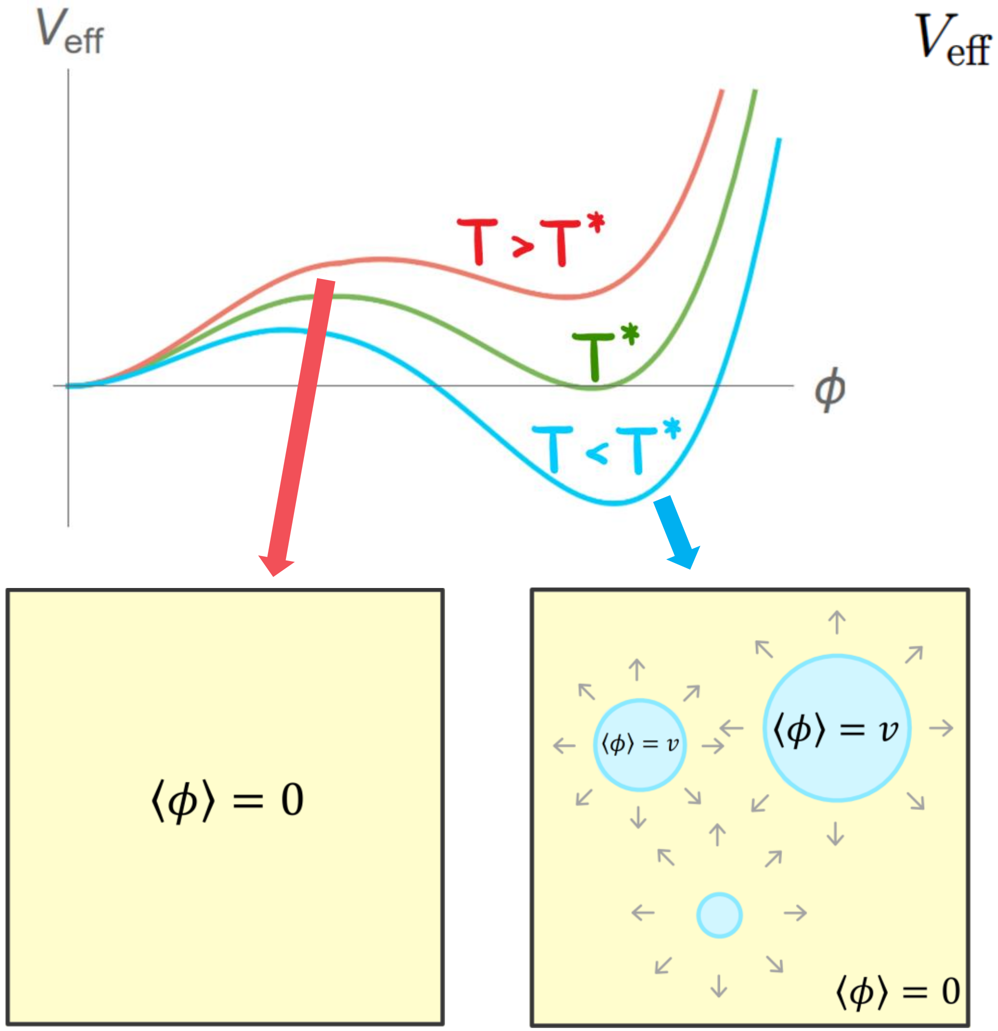

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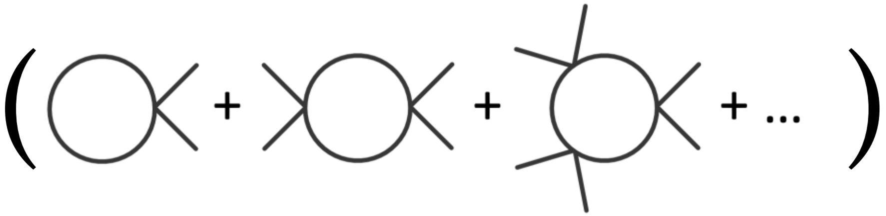

$$V_{\rm daisy}(\phi, T) = -\frac{T}{12\pi}\sum_i n_i^{\rm L}\left[\left(m^2(\phi) + \Pi(T)\right)_i^{3/2} - \left(m^2(\phi)\right)_i^{3/2}\right]$$

Relevant parameters

nucleation rate: $\boxed{\Gamma(T)} \sim T^4 \left(\frac{S_3(\phi,T)}{2\pi T}\right)^{3/2} e^{-S_3(\phi,T)/T}$

$$S_3(\phi,T) = \int \mathrm{d}^3x \left[\frac{1}{2}(\nabla\phi)^2 + V_{\rm eff}(\phi,T)\right]$$

nucleation temperature:

$\boxed{T_n}$ when $\left.\frac{\Gamma(T)}{H(T)^4}\right|_{T=T_n} = 1$ (one bubble per Hubble radius)

vacuum energy density:

$$\boxed{\alpha} = \frac{1}{\rho_{\rm rad}}\left(-1 + T\frac{\mathrm{d}}{\mathrm{d}T}\right)\Delta V_{\rm eff}\Big|_{T=T_n}$$

inverse timescale:

$$\boxed{\frac{\beta}{H}} = -\frac{T}{\Gamma}\frac{\mathrm{d}\Gamma}{\mathrm{d}T}\Big|_{T=T_n}$$

bubble-wall velocity:

$$\boxed{v_w} = \begin{cases} \sqrt{\frac{\Delta V_{\rm eff}}{\alpha\rho_{\rm rad}}} & \text{for } \frac{\Delta V_{\rm eff}}{\alpha\rho_{\rm rad}} < v_J \\ 1 & \text{for } \frac{\Delta V_{\rm eff}}{\alpha\rho_{\rm rad}} > v_J \end{cases}$$

Gravitational wave spectrum

$$h^2\Omega_{\rm GW}(f) = h^2\Omega_b(f) + h^2\Omega_s(f) + h^2\Omega_t(f)$$

	collisions of bubble walls	sound waves	turbulence
	=	=	=
magnitude	$1.67\times10^{-5}\left(\frac{v_w}{\beta/H}\right)^2\left(\frac{100}{g(T_n)}\right)^{1/3}\left(\frac{\kappa_b\alpha}{1+\alpha}\right)^2\left(\frac{0.11v_w}{0.42+v_w^2}\right)$	$2.65\times10^{-6}\left(\frac{v_w}{\beta/H}\right)\left(\frac{100}{g(T_n)}\right)^{1/3}\left(\frac{\kappa_s\alpha}{1+\alpha}\right)^2$	$3.35\times10^{-4}\left(\frac{v_w}{\beta/H}\right)\left(\frac{100}{g(T_n)}\right)^{1/3}\left(\frac{\kappa_t\alpha}{1+\alpha}\right)^{3/2}$
	×	×	×
shape function	$\left(\frac{f}{f_b}\right)^{2.8}\left(\frac{3.8}{1+2.8(f/f_b)^{3.8}}\right)$	$\left(\frac{f}{f_s}\right)^3\left(\frac{7}{4+3(f/f_s)^2}\right)^{7/2}$	$\left(\frac{f}{f_t}\right)^3\left(\frac{1}{1+(f/f_t)}\right)^{11/3}\left(\frac{1}{1+8\pi f/h_*}\right)$

- Obtained from simulation
- Parametrized by $T_n,\ \alpha,\ \beta,\ v_w$

Gravitational wave from seesaw model

$$\mathcal{L}_{\text{scalar}} \supset \mu_\sigma^2 |\sigma|^2 - \lambda_\sigma |\sigma|^4 + \mu_\phi^2 |\phi|^2 - \lambda_\phi |\phi|^4 - \lambda_{\sigma\phi} |\sigma|^2 |\phi|^2 - (\mu \phi^* \sigma^2 + \text{h.c.})$$

$$\mathcal{L}_{\text{Yukawa}} \supset -\left[y_\nu L \tilde{H} \nu_R + \left(y_\sigma \, \sigma \, \nu_R S + \bar{y}_\sigma \, \sigma^* \nu_R \bar{S} \right) + \left(y_\phi \, \phi S S + \bar{y}_\phi \, \phi^* \bar{S} \bar{S} \right) + \text{h.c.} \right]$$

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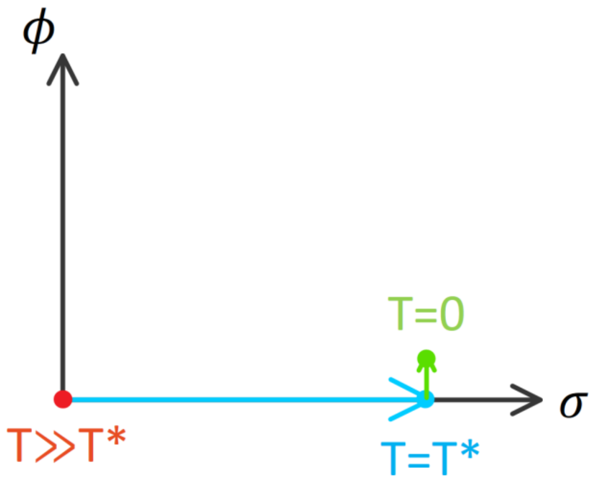
Phase transition:



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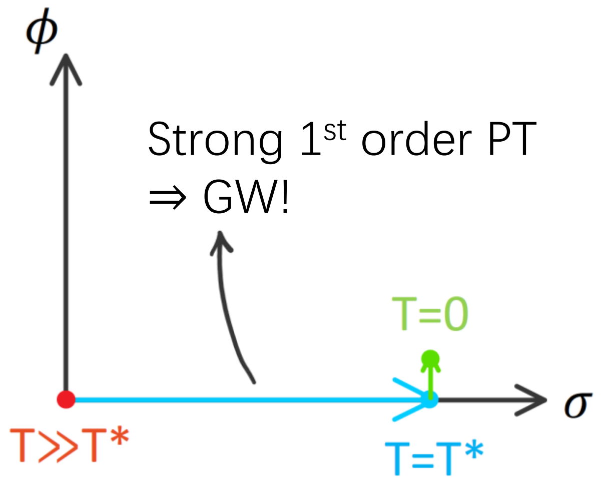

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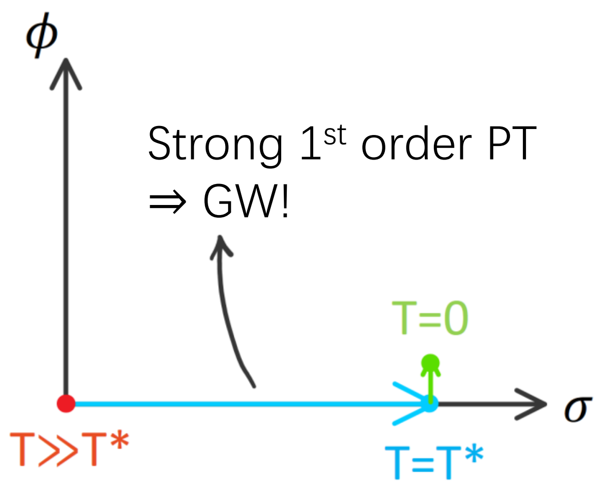

Benchmark parameters:

	v_σ/GeV	v_ϕ/GeV	λ_σ	λ_ϕ	$\lambda_{\sigma\phi}$	g_D	y_v	y_σ	y_ϕ
BM1	10000	1.4×10^{-5}	0.0003	0.2	0.02	0.6	0.04	0.1	0.1
BM2	10000	7.3×10^{-6}	0.0001	0.2	0.02	0.3	0.04	0.1	0.1
BM3	3000	2.5×10^{-5}	0.0001	1	0.02	0.6	0.04	0.05	0.4

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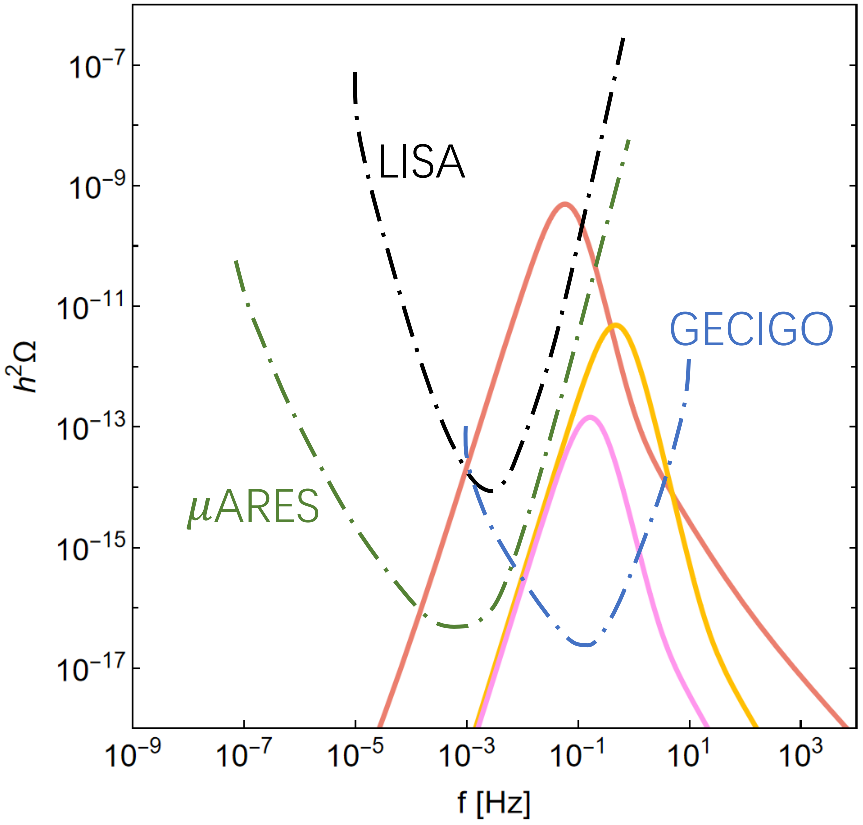

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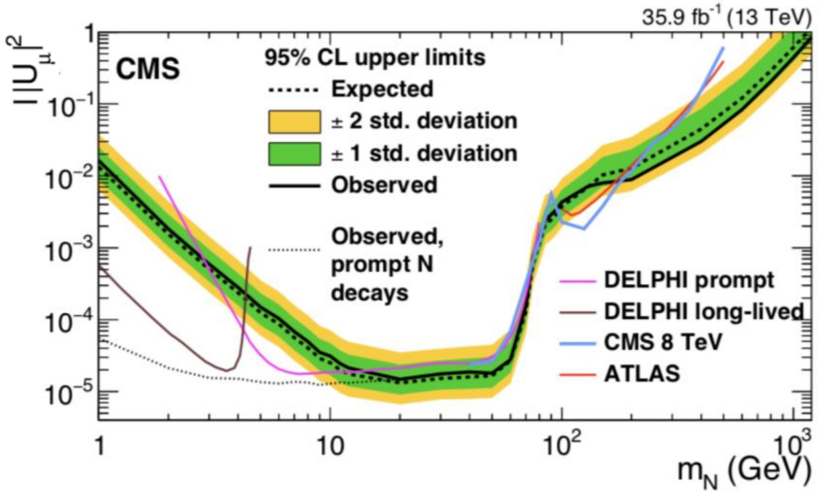
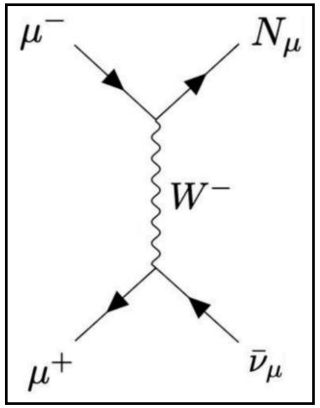
Collider signal from seesaw model

	m_ν/eV	m_N/GeV	m_σ/GeV	m_ϕ/GeV	U
BM1	0.09	707	526	812	0.007
BM2	0.05	707	160	985	0.007
BM3	0.06	849	255	234	0.006

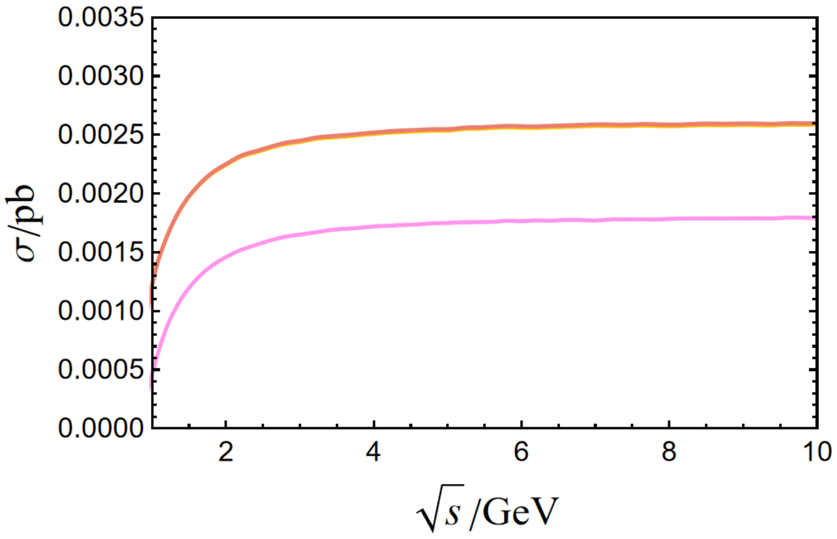
LHC:



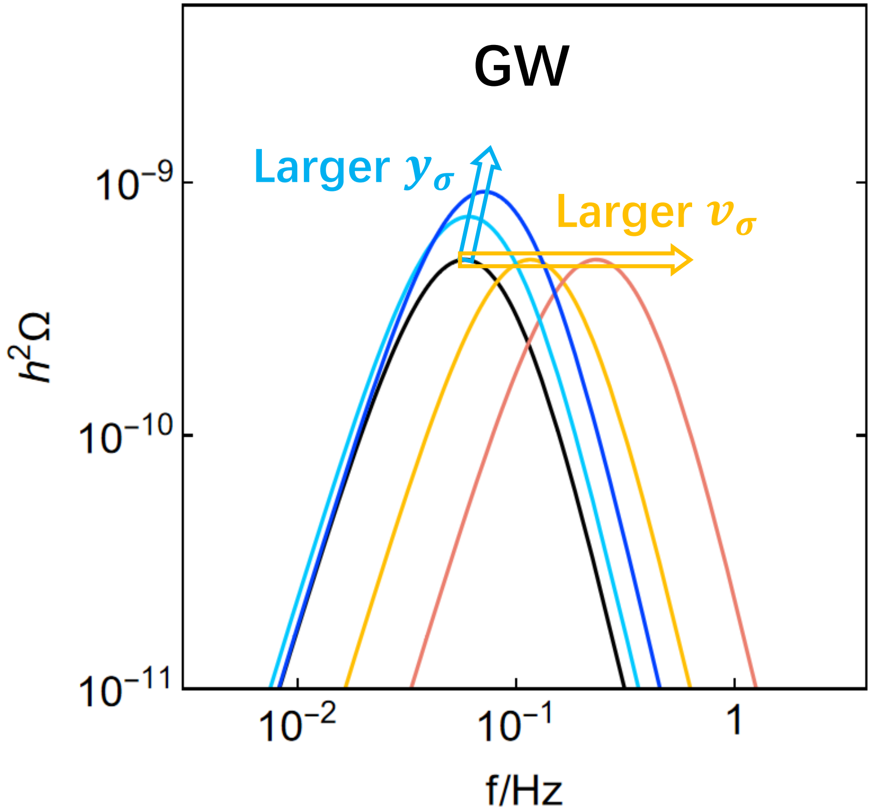
Muon collider:



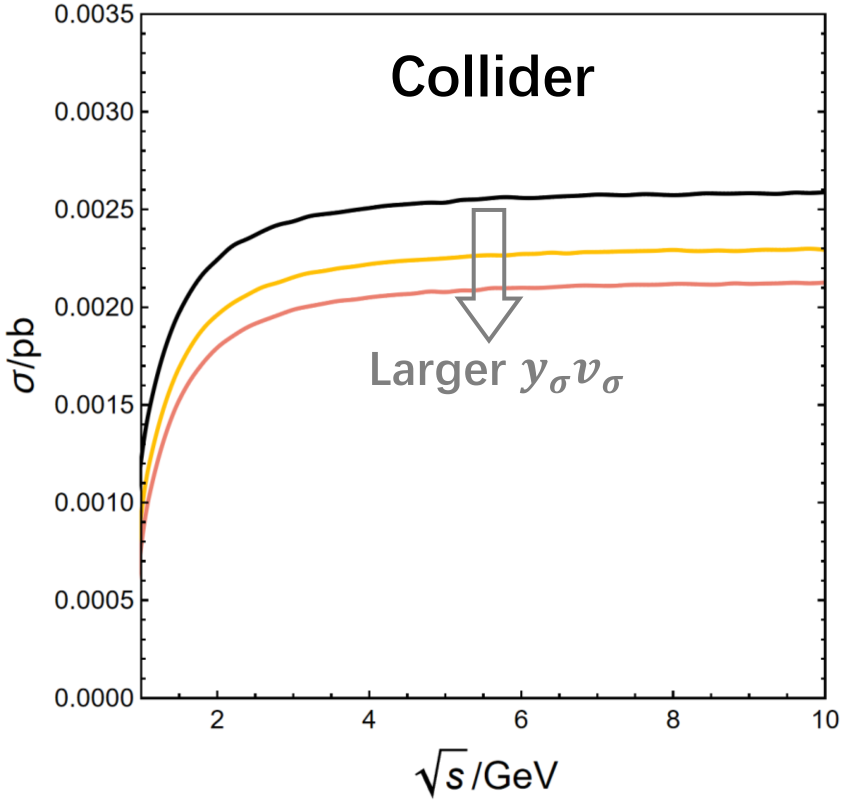
$\sim 10^{-3}\ pb$



Complementarity



v_σ, y_σ



$y_\sigma v_\sigma, y_\nu$

Conclusion

- Seesaw model + scalar fields ⇒ strong gravitational waves
- Gravitational wave observation is highly complementary to collider searches

Thank you!