Coherent Gluon Radiation in pA Collisions: beyond leading-log accuracy

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Program

- Reminder of Fully Coherent Energy Loss (FCEL) at leading-log

- Coherent radiation spectrum for $2 \to 2$ processes beyond leading-log
    - (color) matrix structure of the spectrum
    - an unusual effect: *fully coherent energy gain*

Talk based on :

G. Jackson, S.P., K. Watanabe JHEP 05 (2024) 207
(2312.11650 [hep-ph])

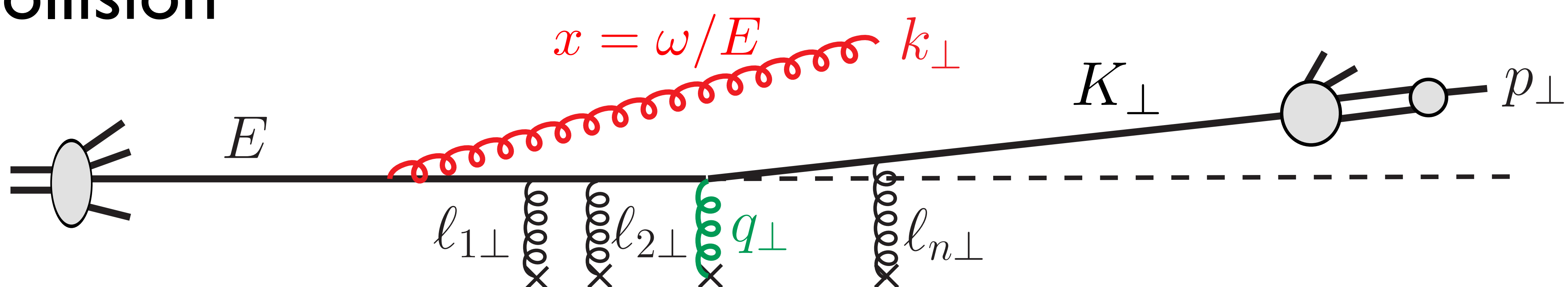
FCEL at leading-log

FCEL = induced radiative energy loss in parton small angle scattering

$2 \to 1$ processes

Arleo, S.P., Sami PRD 83 (2011)
S.P., Arleo, Kolevatov PRD 93 (2016)
Munier, S.P., Petreska PRD 95 (2017)

pA collision



(nucleus rest frame)

- *single* hard exchange $q_\perp \simeq K_\perp = p_\perp / z$
- soft rescatterings $\ell_\perp^2 = \left(\sum \vec{\ell}_{i\perp}\right)^2 \sim \underline{\hat{q}L \sim Q_s^2} \ll K_\perp^2$
- recoil parton is assumed to be soft

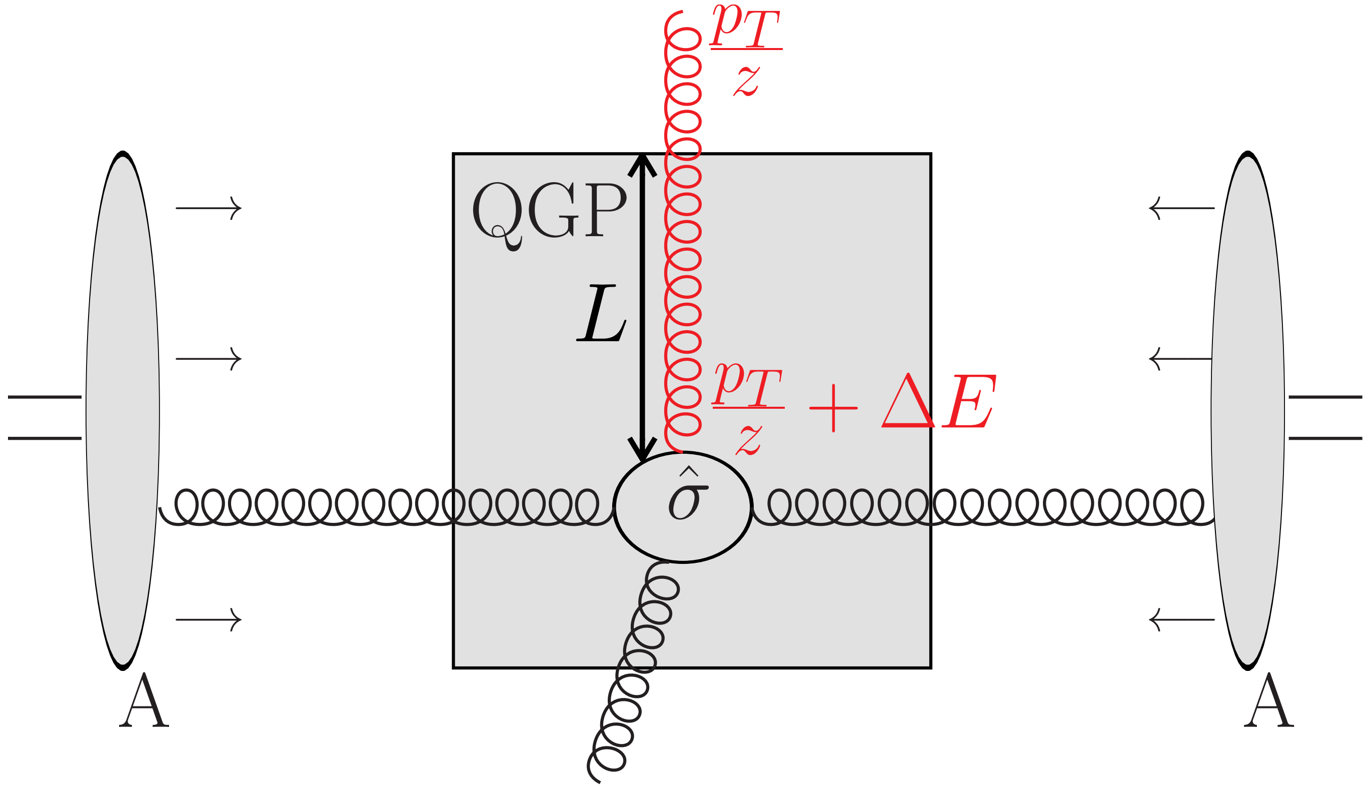
$$x \frac{\mathrm{d}I}{\mathrm{d}x}\bigg|_{2\to 1} = (C_1 + C_3 - C_t)\,\frac{\alpha_s}{\pi}\,\log\left(1 + \frac{\hat{q}L}{x^2 K_\perp^2}\right)$$

induced spectrum scales in x

$$\longrightarrow \quad \Delta E = E \int_0^1 \mathrm{d}x\, x \frac{\mathrm{d}I}{\mathrm{d}x} \propto \alpha_s \frac{\sqrt{\hat{q}L}}{K_\perp} E$$

to avoid confusion:

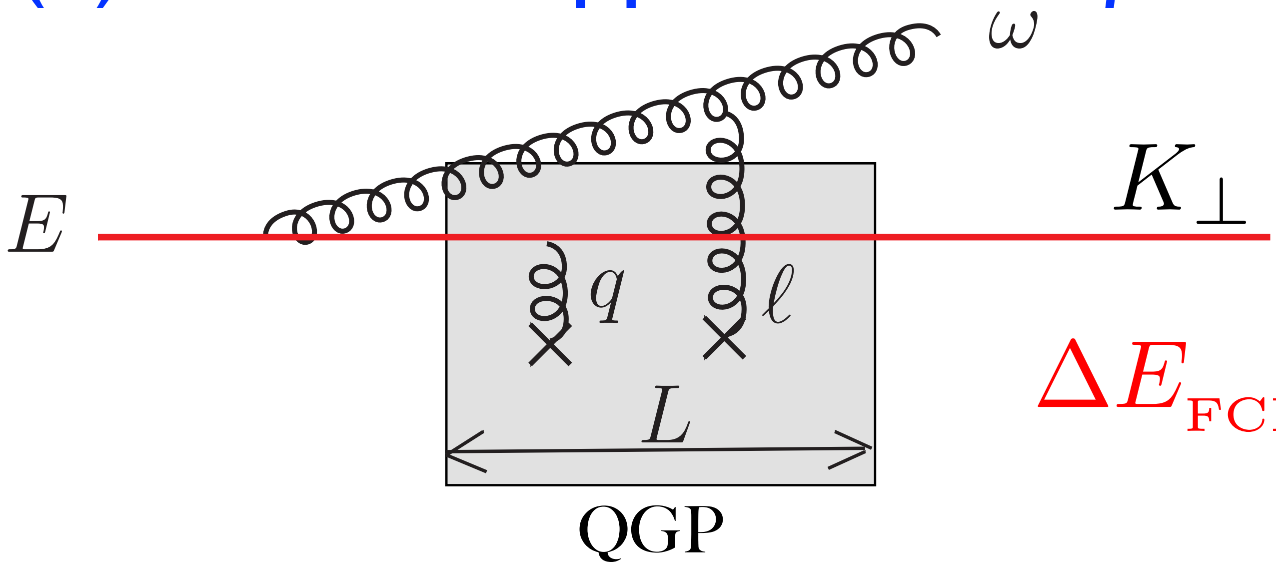
(i) jet-quenching in AA collisions



$\Delta E_{\text{LPM}} \propto \alpha_s \, \hat{q}_{\text{hot}} \, L^2$

independent of $E = p_\perp / z$

(ii) nuclear suppression of *forward* hadron production in AA

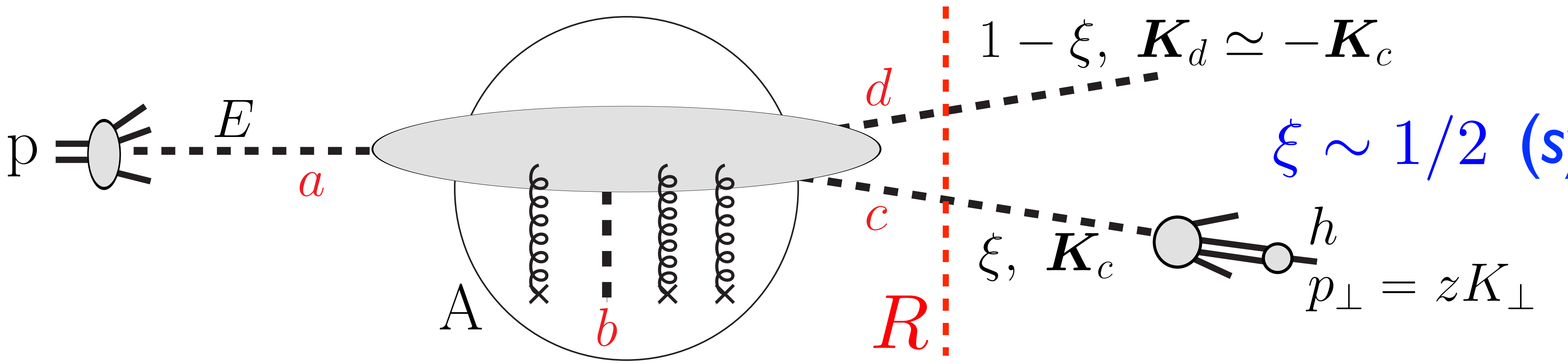


$$\Delta E_{\text{FCEL}} \propto \alpha_s \frac{\sqrt{\hat{q}_{\text{hot}} L}}{K_\perp} E \gg \Delta E_{\text{LPM}}$$

different ΔE arises from interferences ISR/FSR in case (ii)

$2 \to 2$ processes

Liu, Mueller PRD 89 (2014)
S.P., Kolevatov JHEP 01 (2015)



$\xi \sim 1/2$ *(symmetric dijet)*

for $\xi = 0.5$ *or to leading-log: radiated gluon does not probe the dijet*

→ effectively equivalent to $2 \to 1$

$$x \frac{\mathrm{d}I}{\mathrm{d}x}\bigg|_{2\to 2}^{\mathrm{LL}} = \sum_R \rho_R \left(C_a + C_R - C_b\right) \frac{\alpha_s}{\pi} \log\left(1 + \frac{\hat{q}L}{x^2 M^2_{\mathrm{dijet}}}\right)$$

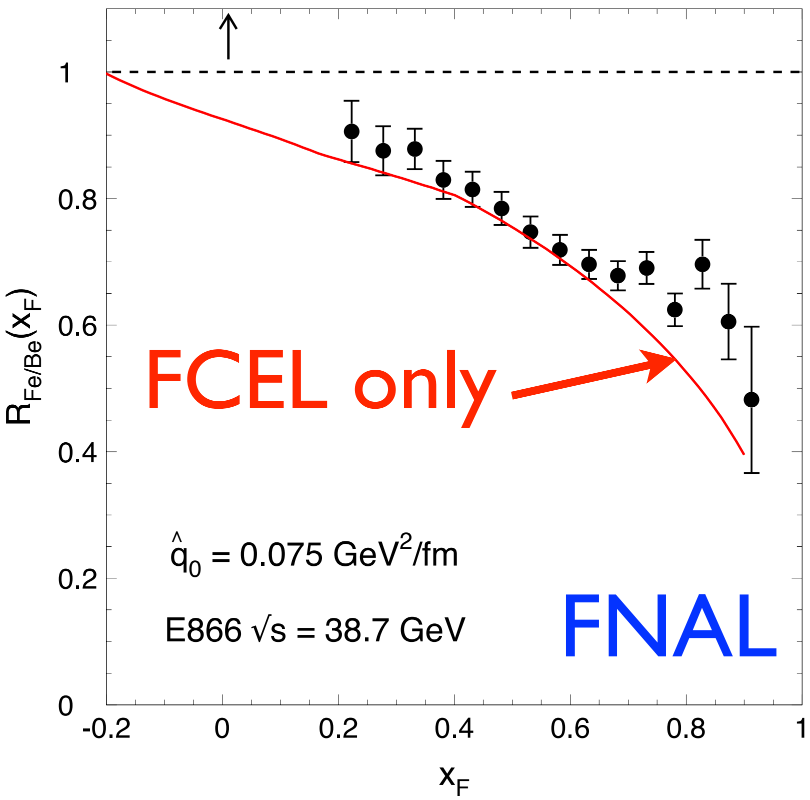
ρ_R proba for dijet to be produced in color state *R*

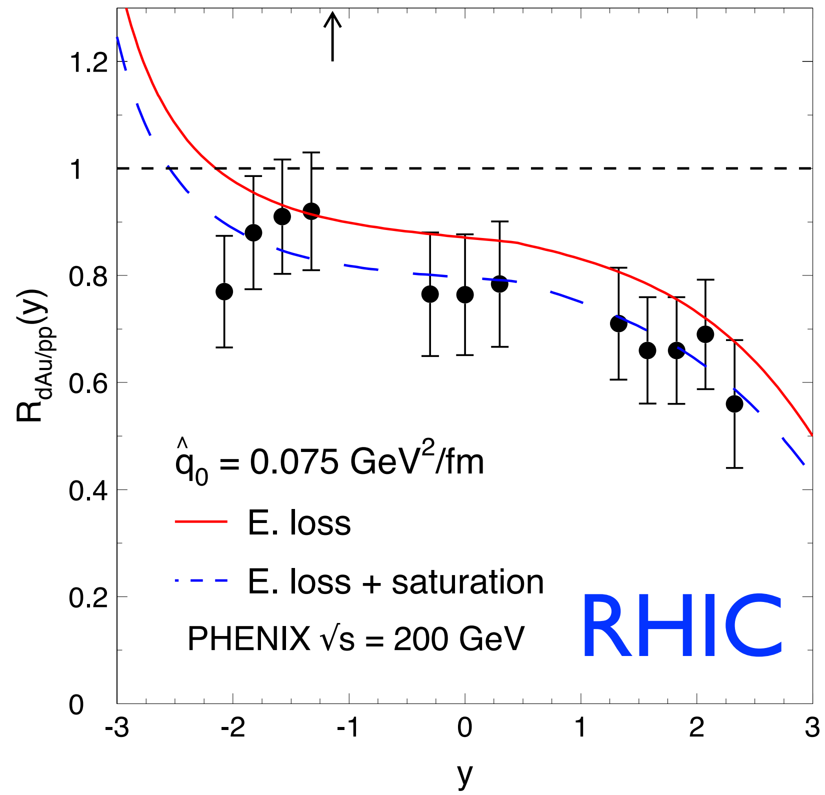
C_R global dijet color charge (Casimir) in state *R*

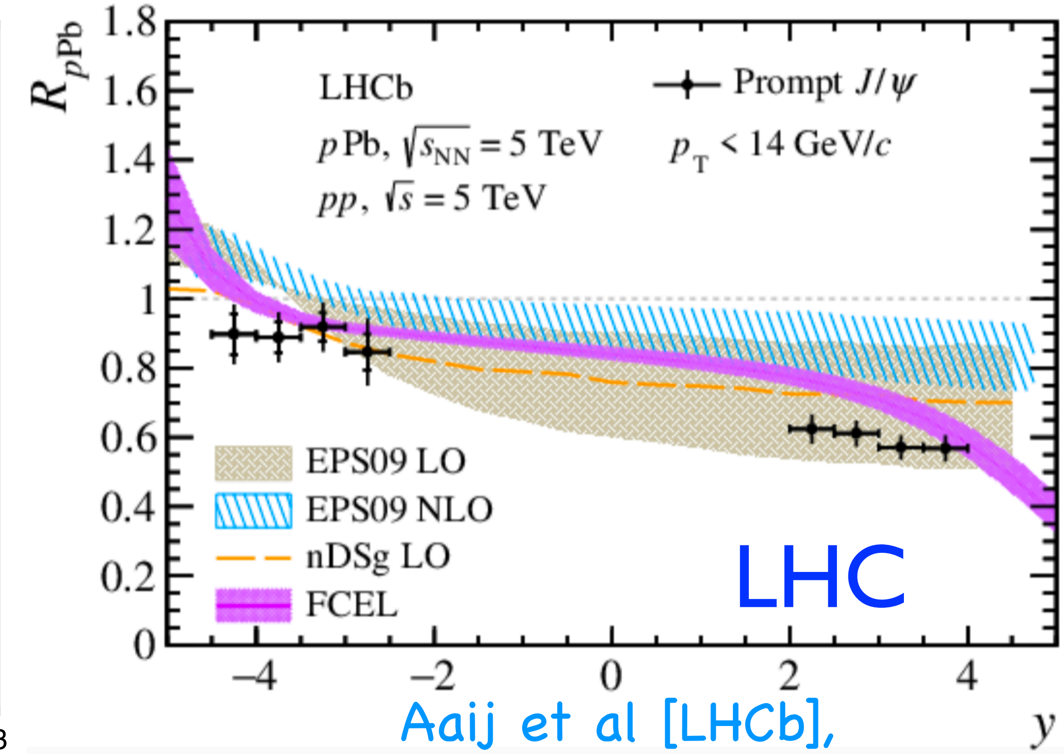
effect of FCEL (at LL) on hadron suppression in pA

FCEL in quarkonium production $(2 \to 1)$

Arleo and Peigne, PRL109, 122301 (2012), JHEP03, 122 (2013)

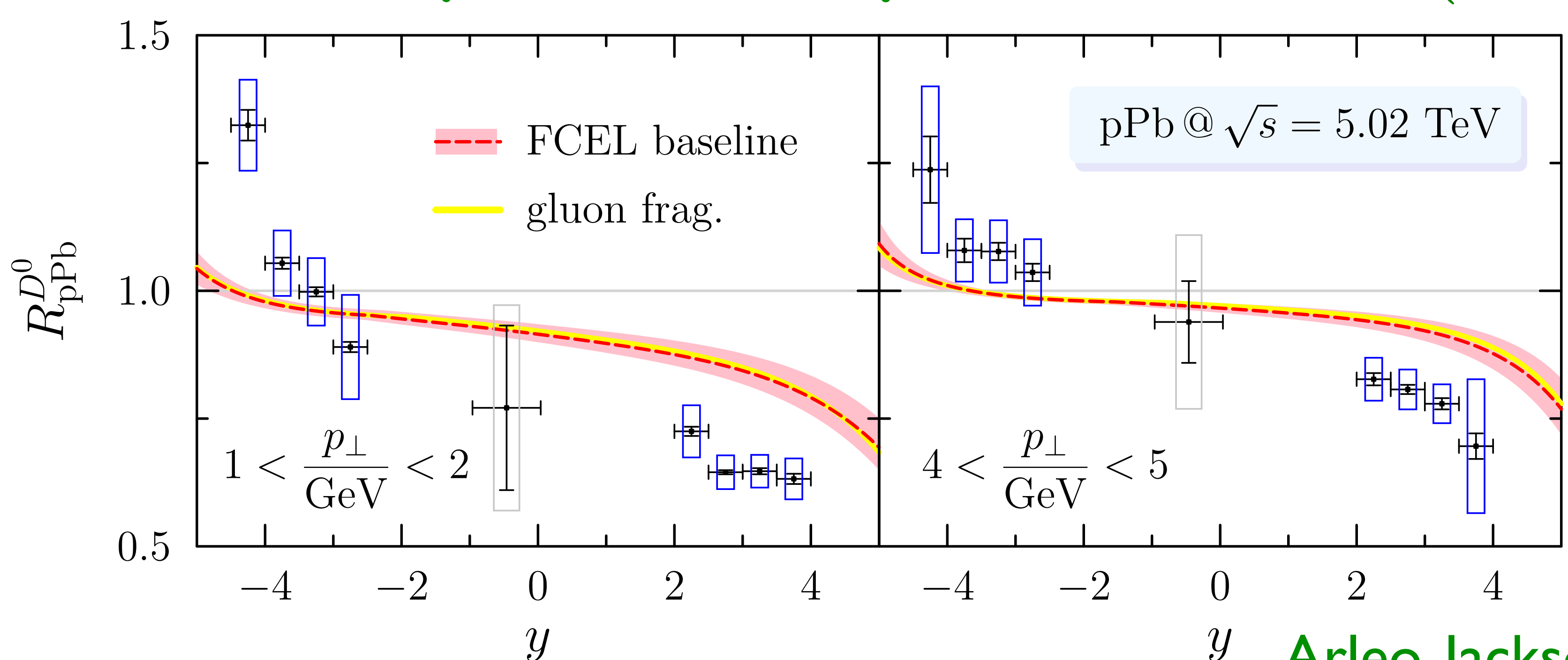






Aaij et al [LHCb], JHEP11, 181 (2021)

FCEL in heavy flavour production $(2 \to 2 \text{ and } \xi = 1/2)$



Aaij et al [**LHCb**], JHEP 10 (2017) 090

Abelev et al [**ALICE**], PRL 113 (2014) 232301

Arleo, Jackson, S.P. JHEP 01 (2022) 164

- FCEL contributes to *substantial* hadron suppression in pA

at least as much as other nuclear effects (nPDFs, saturation,...)

$\longrightarrow$ implement FCEL in pA hadron cross sections...

- FCEL = pQCD prediction, with small uncertainty

... *before* extracting nPDF sets (or estimating saturation)

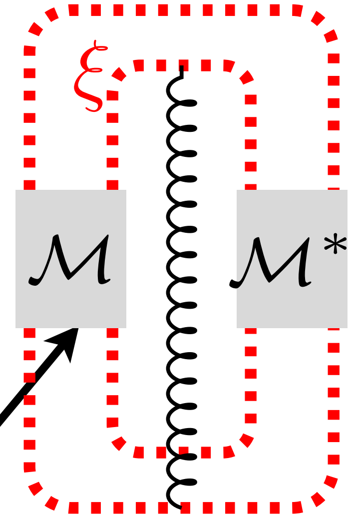
$\longrightarrow$ needs $x \left. \frac{\mathrm{d}I}{\mathrm{d}x} \right|_{2 \to 2}$ in full $\xi-\mathrm{range}: \quad 0 \leq \xi \leq 1$

Coherent radiation spectrum beyond leading-log

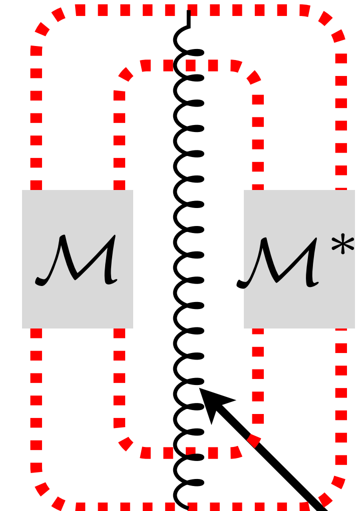
G. Jackson, S.P., K. Watanabe JHEP 05 (2024) 207

- structure of the spectrum for any $2 \to 2$ process :

$$x\frac{\mathrm{d}I}{\mathrm{d}x} = \frac{\alpha_s}{\pi}\,\frac{2}{|\mathcal{M}|^2}\left\{ \mathcal{L}_\xi \right.$$



$$+ \quad \mathcal{L}_{\bar{\xi}}$$



$$\left. \vphantom{\frac{1}{1}} \right\}$$

hard process

color only

soft rescatterings factorize into $\mathcal{L}_\xi \simeq \log\left(1+\xi^2\frac{Q_s^2}{x^2 m_\perp^2}\right) - \log\left(1+\xi^2\frac{Q_{s,\mathrm{p}}^2}{x^2 m_\perp^2}\right)$

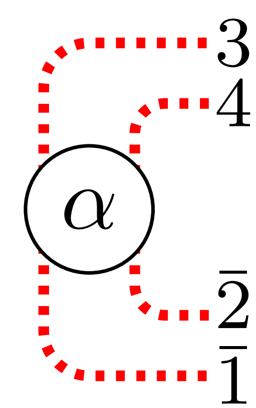
color decomposition of hard amplitude:

color indices only

$$\mathcal{M}_{12\to 34} = \sum_\alpha \nu_\alpha \, \langle \alpha |$$

kinematics, spin, flavour

$$\langle \alpha | = \frac{1}{\sqrt{K_\alpha}}$$



(*s*-channel basis)

color projectors

G. Jackson, S.P., K. Watanabe JHEP 05 (2024) 207

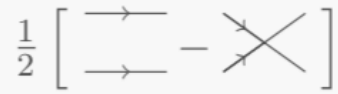
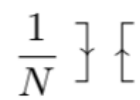
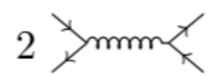
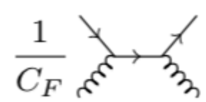
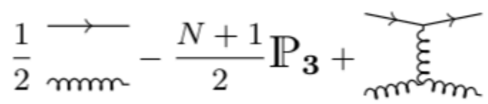
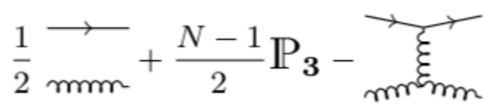
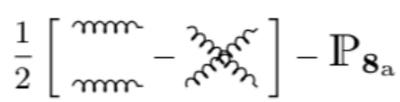
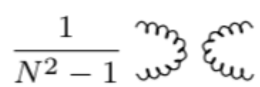
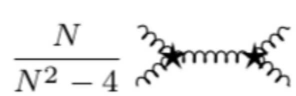
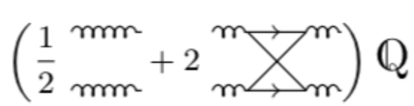
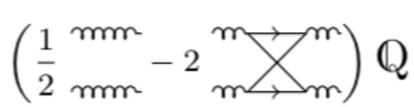
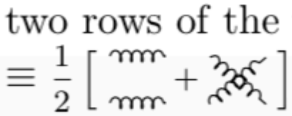
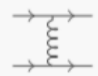
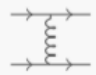
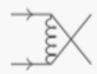
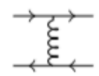
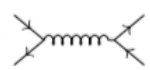
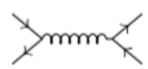
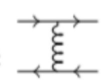
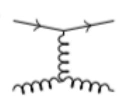
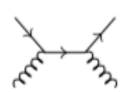
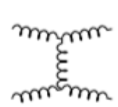
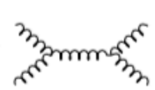
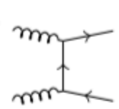
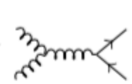
system	irrep α	projector $\mathbb{P}_\alpha$	dimension K_α	Casimir C_α
$\mathbf{3}\otimes\mathbf{3}$	$\bar{\mathbf{3}}$		$\frac{1}{2}N(N-1)$	$2C_F - \frac{N+1}{N}$
	$\mathbf{6}$		$\frac{1}{2}N(N+1)$	$2C_F + \frac{N-1}{N}$
$\mathbf{3}\otimes\bar{\mathbf{3}}$	$\mathbf{1}$		1	0
	$\mathbf{8}$		N^2-1	N
$\mathbf{3}\otimes\mathbf{8}$	$\mathbf{3}$		N	C_F
	$\bar{\mathbf{6}}$		$\frac{1}{2}N(N+1)(N-2)$	C_F+N-1
	$\mathbf{15}$		$\frac{1}{2}N(N-1)(N+2)$	C_F+N+1
$\mathbf{8}\otimes\mathbf{8}$	$\mathbf{8}_\mathrm{a}$		N^2-1	N
	$\mathbf{10}\oplus\overline{\mathbf{10}}$		$\frac{1}{2}(N^2-1)(N^2-4)$	$2N$
	$\mathbf{1}$		1	0
	$\mathbf{8}_\mathrm{s}$		N^2-1	N
	$\mathbf{27}$		$\frac{1}{4}N^2(N-1)(N+3)$	$2(N+1)$
	$\mathbf{0}$		$\frac{1}{4}N^2(N+1)(N-3)$	$2(N-1)$

Table 1. Projectors, dimensions and quadratic Casimirs of the SU(N) irreps associated to the qq, $q\bar{q}$, qg and gg systems. An SU(N) irrep is labelled according to its dimension for $N=3$. We express the arising color factors via N and $C_F=(N^2-1)/(2N)$. In the last two rows of the table, for the projectors $\mathbb{P}_{\mathbf{27}}$ and $\mathbb{P}_{\mathbf{0}}$ of a gg pair we use the shorthand notation $\mathbb{Q}\equiv$  $-\mathbb{P}_{\mathbf{8}_\mathrm{s}}-\mathbb{P}_{\mathbf{1}}$.

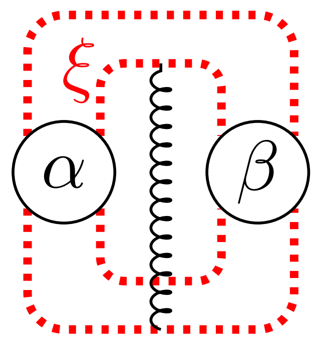
channel	$\mathcal{M}$	$\frac{\mathrm{tr_d tr_c}\lvert\mathcal{M}\rvert^2}{4g^4(N^2-1)}$	α	$\frac{\nu_\alpha}{\sqrt{K_\alpha}}$
$qq' \to qq'$	$\mathcal{A}$ 	$\frac{1+\bar\xi^2}{2\xi^2}$	$\bar{\mathbf{3}}$ $\mathbf{6}$	$\mathcal{A}\,\frac{N+1}{2N}$ $-\mathcal{A}\,\frac{N-1}{2N}$
$qq \to qq$	$\mathcal{B}_t$  $+\,\mathcal{B}_u$ 	$\frac{1+\xi^2}{2\bar\xi^2}+\frac{1+\bar\xi^2}{2\xi^2}-\frac{1}{N\xi\bar\xi}$	$\bar{\mathbf{3}}$ $\mathbf{6}$	$\frac{N+1}{4N}\,(\mathcal{B}_t-\mathcal{B}_u)$ $-\frac{N-1}{4N}\,(\mathcal{B}_t+\mathcal{B}_u)$
$q\bar q' \to q\bar q'$	$\mathcal{C}$ 	$\frac{1+\bar\xi^2}{2\xi^2}$	$\mathbf{1}$ $\mathbf{8}$	$C_F\,\mathcal{C}$ $-\frac{1}{2N}\,\mathcal{C}$
$q\bar q \to q'\bar q'$	$\mathcal{D}$ 	$\frac{\xi^2+\bar\xi^2}{2}$	$\mathbf{1}$ $\mathbf{8}$	0 $\frac{1}{2}\,\mathcal{D}$
$q\bar q \to q\bar q$	$\mathcal{E}_s$  $+\,\mathcal{E}_t$ 	$\frac{\xi^2+\bar\xi^2}{2}+\frac{1+\bar\xi^2}{2\xi^2}+\frac{\bar\xi^2}{N\xi}$	$\mathbf{1}$ $\mathbf{8}$	$C_F\,\mathcal{E}_t$ $\frac{1}{2}(\mathcal{E}_s-\frac{1}{N}\mathcal{E}_t)$
$qg \to qg$	$\mathcal{F}\,\Big[$  $-\,\xi$  $\Big]$	$(1+\bar\xi^2)\Big(\frac{N}{\xi^2}+\frac{C_F}{\bar\xi}\Big)$	$\mathbf{3}$ $\bar{\mathbf{6}}$ $\mathbf{15}$	$\big(\frac{1}{2N}+\bar\xi C_F\big)\,\mathcal{F}$ $\frac{1}{2}\,\mathcal{F}$ $-\frac{1}{2}\,\mathcal{F}$
$gg \to gg$	$\mathcal{G}\,\Big[$  $-\,\xi$  $\Big]$	$4N^2\frac{(1-\xi\bar\xi)^3}{\xi^2\bar\xi^2}$	$\mathbf{8}_\mathrm{a}$ $\mathbf{10}\ominus\overline{\mathbf{10}}$ $\mathbf{1}$ $\mathbf{8}_\mathrm{s}$ $\mathbf{27}$ $\mathbf{0}$	$\frac{N}{2}(\bar\xi-\xi)\,\mathcal{G}$ 0 $N\,\mathcal{G}$ $\frac{N}{2}\,\mathcal{G}$ $-\mathcal{G}$ $\mathcal{G}$
$gg \to q\bar q$	$\mathcal{H}\,\Big[$  $-\,\xi$  $\Big]$	$(\xi^2+\bar\xi^2)\Big(\frac{C_F}{\xi\bar\xi}-N\Big)$	$\mathbf{1}$ $\mathbf{8}_\mathrm{a}$ $\mathbf{8}_\mathrm{s}$	$\frac{\sqrt{N^2-1}}{2\sqrt{N}}\,\mathcal{H}$ $\frac{1}{2}\,(\bar\xi-\xi)\,\frac{\sqrt{N}}{\sqrt{2}}\,\mathcal{H}$ $\frac{\sqrt{N^2-4}}{2\sqrt{2N}}\,\mathcal{H}$

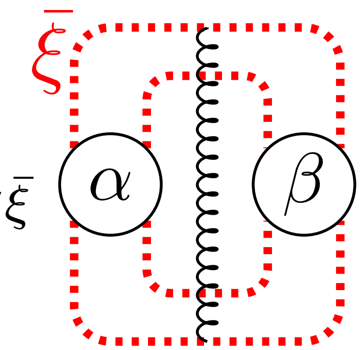
$$\frac{\mathrm{d}I}{\mathrm{d}x} = \Phi_{\alpha\beta}\, S(x)_{\alpha\beta} = \mathrm{Tr}\left\{\Phi \cdot S(x)\right\}$$

$$\Phi_{\alpha\beta} = \frac{\mathrm{tr}_{\mathrm{d}}\left(\nu_\alpha\, \nu^*_\beta\right)}{\mathrm{tr}_{\mathrm{d}}\, \mathrm{tr}_{\mathrm{c}}\, |\mathcal{M}|^2}$$ color density matrix

$\rho_\alpha \equiv \Phi_{\alpha\alpha}$

s-channel probability

$$S(x)_{\alpha\beta} \equiv \frac{\alpha_s}{\pi\, x} \frac{2}{\sqrt{K_\alpha K_\beta}} \Bigg(\mathcal{L}_\xi$$


$$+\, \mathcal{L}_{\bar{\xi}}$$


$$\Bigg)$$
soft radiation matrix

$\xi = \bar{\xi} = 1/2$ or $\mathcal{L}_\xi \simeq \mathcal{L}_{\bar{\xi}} \gg 1$

$$T_3 + T_4 = T_\alpha = T_1 + T_2$$

$$\langle \alpha | 2\, T_1\, T_\alpha | \beta \rangle = \langle \alpha | T_1^2 + T_\alpha^2 - T_2^2 | \beta \rangle = \left(C_1 + C_\alpha - C_2\right) \delta_{\alpha\beta}$$

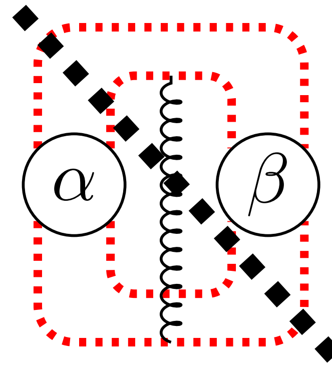
$$\left.\frac{\mathrm{d}I}{\mathrm{d}x}\right|_{\xi=\frac{1}{2}} = \mathrm{Tr}\left\{\Phi \cdot S(x)\right\} = \sum_\alpha \rho_\alpha \left(C_1 + C_\alpha - C_2\right) \frac{\alpha_s}{\pi\, x}\, \mathcal{L}_{1/2}$$

$\xi \neq 1/2$ and beyond leading-log

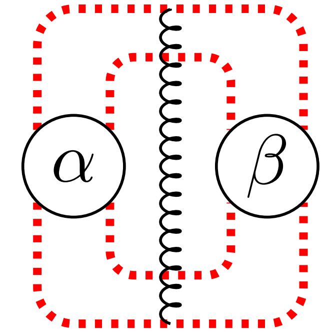
soft gluon radiation can induce color transitions

• limit $\xi \to 0$: matching with $2 \to 1$ processes

$$S(x)_{\alpha\beta} \equiv \frac{\alpha_s}{\pi\, x} \frac{2}{\sqrt{K_\alpha K_\beta}} \left(\mathcal{L}_\xi \right.$$



$$+ \mathcal{L}_{\bar{\xi}}$$



$$\left. \right)$$

$$2T_1T_3 = T_1^2 + T_3^2 - (T_1 - T_3)^2 = C_1 + C_3 - T_t^2$$

color exchange in *t*-channel

$$\mathcal{M} = \sum_\alpha \nu_\alpha \langle \alpha | = \sum_{\alpha^t} \nu_{\alpha^t} \langle \alpha^t |$$ independent of color-basis

$$\Rightarrow \frac{\mathrm{d}I}{\mathrm{d}x} = \mathrm{Tr}\left\{ \Phi \cdot S(x) \right\} = \Phi_{\alpha\beta}\, S(x)_{\alpha\beta} = \Phi^t_{\alpha^t\beta^t}\, S(x)^t_{\alpha^t\beta^t}$$

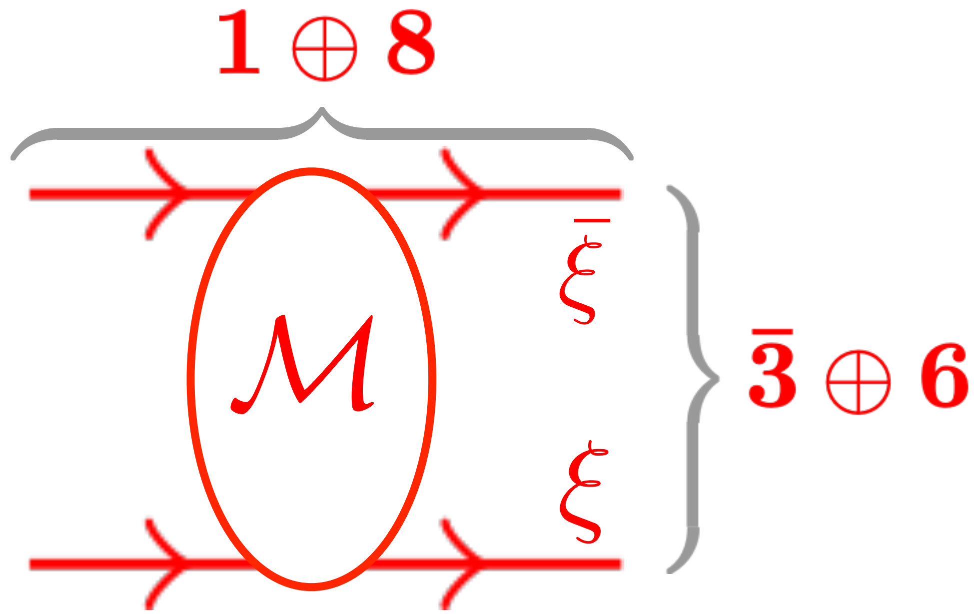
for interpretation purpose, better choose *t*-basis where *S(x)* is diagonal:

proba for *t*-channel pair to be in irrep α^t

$$\left.\frac{\mathrm{d}I}{\mathrm{d}x}\right|_{\xi\to 0} = \underbrace{\sum_{\alpha^t} \rho^t_{\alpha^t} \left(C_1 + C_3 - C_{\alpha^t} \right)}\, \frac{\alpha_s}{\pi\, x}\, \mathcal{L}_1$$

matches with $C_1 + C_3 - N$ for $2 \to 1$ processes studied previously (with purely octet *t*-channel exchange)

illustration: $qq' \to qq'$ and $qq \to qq$ processes

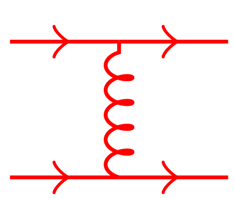
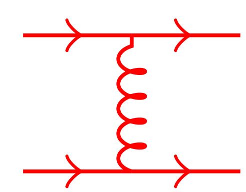
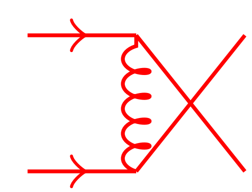


$$\left.\frac{\mathrm{d}I}{\mathrm{d}x}\right|_{\xi=\frac{1}{2}} = \frac{\alpha_s}{\pi\, x}\,\mathcal{L}_{1/2}\left[\,\rho_{\bar{\mathbf{3}}}C_{\bar{\mathbf{3}}} + \rho_{\mathbf{6}}C_{\mathbf{6}}\,\right]$$

$$\left.\frac{\mathrm{d}I}{\mathrm{d}x}\right|_{\xi\to 0} = \frac{\alpha_s}{\pi\, x}\,\mathcal{L}_{1}\left[\,\rho^t_{\mathbf{1}}(2C_F) + \rho^t_{\mathbf{8}}(2C_F - N)\,\right]$$

$$\left.\frac{\mathrm{d}I}{\mathrm{d}x}\right|_{\xi\to 1} = \frac{\alpha_s}{\pi\, x}\,\mathcal{L}_{1}\left[\,\rho^u_{\mathbf{1}}(2C_F) + \rho^u_{\mathbf{8}}(2C_F - N)\,\right]$$

probas depend on specific process:

$\mathcal{M}_{qq'\to qq'} \propto$  $\quad \mathcal{M}_{qq\to qq} \propto \mathcal{B}_t$  $+\, \mathcal{B}_u$ 

- an unusual effect: *fully coherent energy gain* (FCEG)

$$qq' \to qq',\ qq \to qq: \qquad 2C_F - N = -\frac{1}{N} < 0 \ !$$

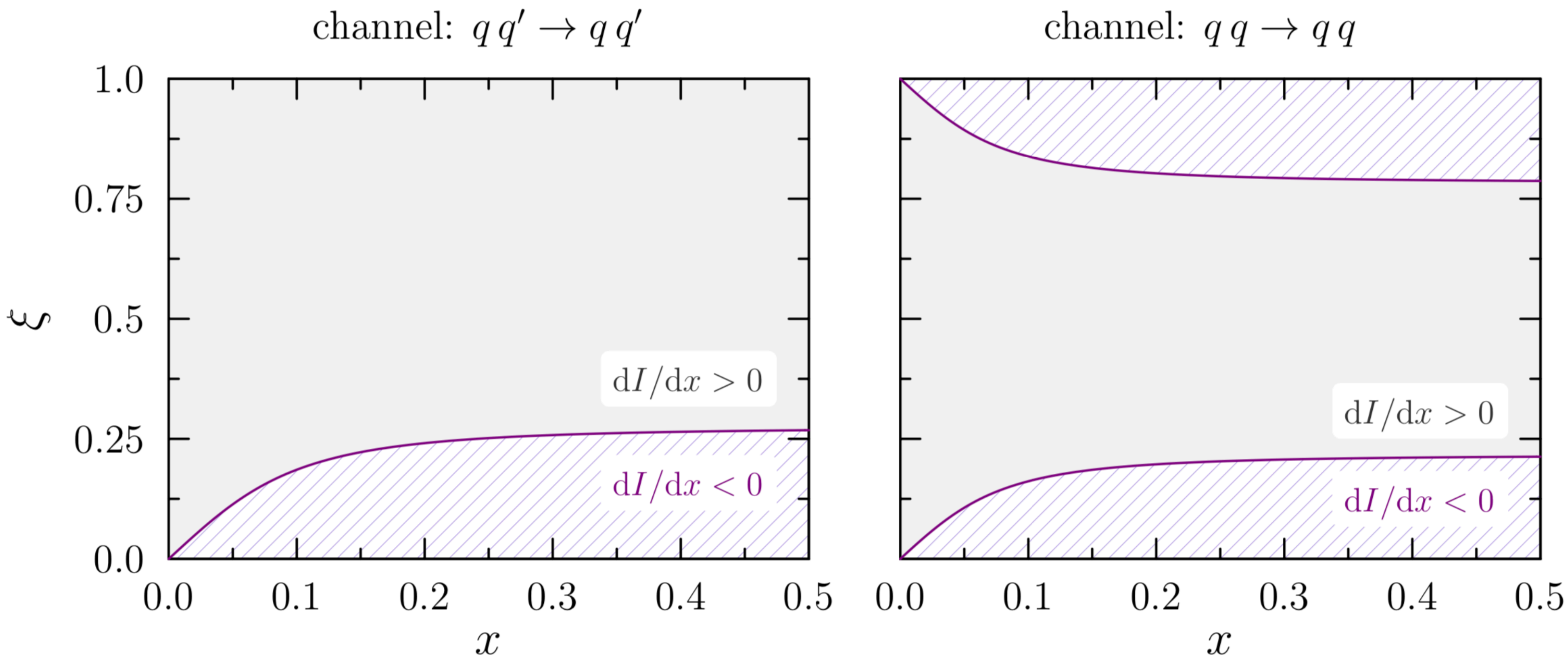


Figure 2. Regions in the (x, ξ)-plane, corresponding to energy-loss (solid gray) or energy-gain (hatched purple). In this figure, $N = 3$, $Q_{s,\mathrm{A}} = \frac{1}{4} m_\perp$ and $Q_{s,\mathrm{p}} = \frac{1}{10} m_\perp$.

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FCEG contributions appear in other channels:

e.g., $q\bar{q} \to q'\bar{q}'$

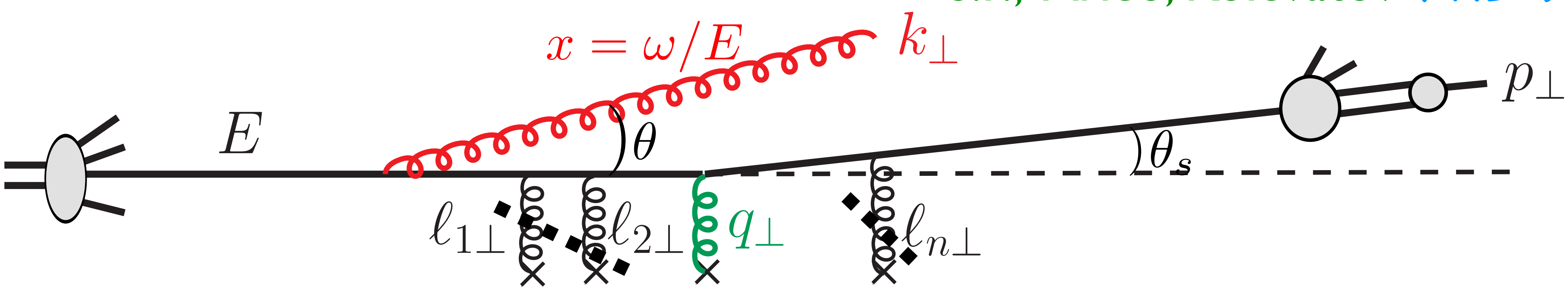
$\mathcal{M} \propto$

$\bar{\mathbf{3}} \oplus \mathbf{6}$

$$\left.\frac{\mathrm{d}I}{\mathrm{d}x}\right|_{\xi\to 0} = \frac{\alpha_s}{\pi\, x}\,\mathcal{L}_1 \left[\, \rho^t_{\bar{\mathbf{3}}} \underbrace{(2C_F - C_{\bar{\mathbf{3}}})}_{\frac{N+1}{N}} + \rho^t_{\mathbf{6}} \underbrace{(2C_F - C_{\mathbf{6}})}_{-\frac{N-1}{N} < 0} \,\right]$$

FCEG can be inferred *heuristically* from features of *total* spectrum $=$ spectrum at 0th order in opacity $(n=0)$

S.P., Arleo, Kolevatov PRD 93 (2016)



$qg \to q$

$$x\frac{\mathrm{d}I^{(0)}}{\mathrm{d}x} \simeq \frac{\alpha_s}{\pi}\Big[\overbrace{\underbrace{2C_R\log\left(\frac{x^2\boldsymbol{q}^2}{\Lambda_{\rm IR}^2}\right)}_{k_\perp \lesssim xq_\perp}}^{\theta<\theta_s\ \text{(abelian)}} + \overbrace{\underbrace{N_c\log\left(\frac{\Lambda_{\rm S}^2}{x^2\boldsymbol{q}^2}\right)}_{xq_\perp<k_\perp<\Lambda_{\rm S}} + \underbrace{N_c\log\left(\frac{\boldsymbol{q}^2}{\Lambda_{\rm S}^2}\right) + N_c\log\left(\frac{\boldsymbol{q}^2}{\Lambda_{\rm IR}^2}\right)}_{k_\perp>\Lambda_{\rm S}}}^{\theta>\theta_s\ \text{(non-abelian)}}\Big]$$

($\Lambda_{\rm S}$ arbitrary scale: $xq_\perp \ll \Lambda_{\rm S} \ll q_\perp$)

total spectrum *increases* when $q_\perp \nearrow$ with rate $\propto 2C_R + N_c$

additional radiation expected from rescattering $\ell_\perp \sim Q_s$?

- $k_\perp \lesssim Q_s \ll \Lambda_S \Rightarrow$ only $k_\perp < \Lambda_{\rm S}$ region is affected
- $\boldsymbol{q}^2 \to \boldsymbol{q}^2 + \mathcal{O}\left(Q_s^2\right) \Rightarrow \Delta\left(x\frac{\mathrm{d}I^{(0)}}{\mathrm{d}x}\right) \propto 2C_F - N < 0$

Summary

- coherent radiation spectrum beyond leading-log and for any ξ now available for $2 \to 2$ processes

  - encompasses leading-log results
  - FCEL beyond LL probes *off-diagonal* elements of hard process *color density matrix* Φ
  - some contributions to FCEL spectrum can be negative

- FCEL can now be systematically implemented in pA

Merci !