Radiative Corrections to SIDIS: Current Status and Perspectives

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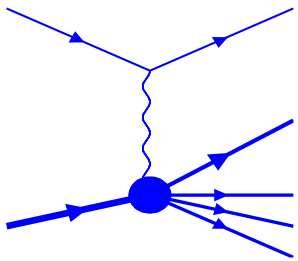
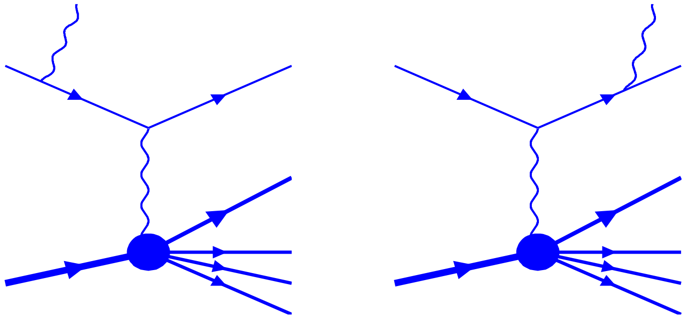
Contribution to RC in SIDIS

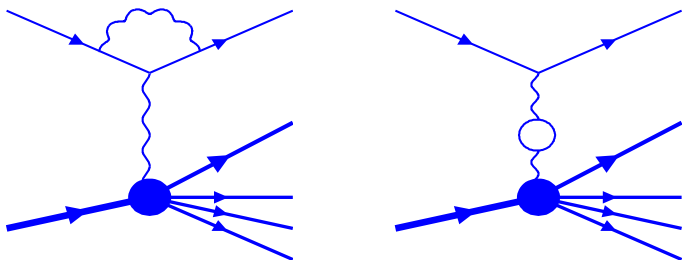
The Born cross section



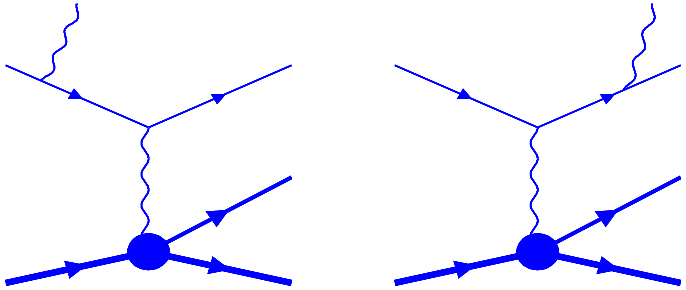
Emission of a radiated photon (semi-inclusive processes)



Loop diagrams



Emission of a radiated photon (exclusive processes)



Original studies of RC in SIDIS Completed by Our Group

- The original formalism, simple quark-parton model:
  - Soroko AV, Shumeiko NM (1989) Radiative effects in deep inelastic scattering of leptons on nucleons in a semi-inclusive experiment. *Soviet Journal of Nuclear Physics*, 49(5); p. 838-844
- Fortran code POLRAD 2.0, Patch SIRAD
  - Akushevich I, Ilyichev A, Shumeiko N, Soroko A, and Tolkachev A, (1997) POLRAD 2.0. FORTRAN code for the radiative corrections calculation to deep inelastic scattering of polarized particles. *Computer physics communications*, 104(1-3), pp.201-244.
- RC to unpolarized SIDIS cross section, angular structure
  - Akushevich I, Shumeiko N, and Soroko A (1999) Radiative effects in the processes of hadron electroproduction. *European Physical Journal*, C10(4), pp.681-687.
- Radiative tail from exclusive peak
  - Akushevich I, Ilyichev A, and Osipenko M (2009) Lowest order QED radiative corrections to five-fold differential cross section of hadron leptoproduction. *Physics Letters* B672(1), pp.35-44.
- General calculation of RC for polarized particles
  - Akushevich I and Ilyichev A (2019) Lowest order QED radiative effects in polarized SIDIS. *Physical Review* D100(3), p.033005.

Current Status of Our Calculations

- RC to unpolarized SIDIS cross section, angular structure
  - Akushevich I, Shumeiko N, and Soroko A (1999) Radiative effects in the processes of hadron electroproduction. *European Physical Journal*, C10(4), pp.681-687.
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- General calculation of RC for polarized particles
  - Akushevich I and Ilyichev A (2019) Lowest order QED radiative effects in polarized SIDIS. *Physical Review* D100(3), p.033005.
- Leading Log and the best approximation
  - Akushevich I, Srednyak S, and Ilyichev A (2024) Exact and leading order radiative effects in semi-inclusive deep inelastic scattering. *Physical Review* D109(7), p.076028.
- Radiative tail from exclusive peak
  - Akushevich I, Avakian H, Ilyichev A, Srednyak S (2023) Complete lowest order radiative corrections in semi-inclusive scattering of polarized particles. *The European Physical Journal* A59(10), 246.

Current Status of Our Calculations

- All analytic calculations were performed without any approximations and presented in our paper:
  - Akushevich, Igor, and Alexander Ilyichev. “Lowest order QED radiative effects in polarized SIDIS” Physical Review D 100, no. 3 (2019): 033005.
- Fortran code was created by Alexander Ilyichev
  - SIDIS SF were implemented as in (Wandzura-Wilczek-type approximation):
    - *Bastami S, Avakian H, Efremov AV, Kotzinian, Musch BU, Parsamyan B, Prokudin A et al. (2019) SemiA-inclusive deep-inelastic scattering in Wandzura-Wilczek-type approximation. Journal of High Energy Physics no. 6 p.7.*
    - *However, this model is not completely satisfactory: there are kinematical regions with unrealistic estimates of SFs, applicability of the model in entirely region necessary for RC has to be reviewed.*
  - Exclsive SFs were evaluated using the MAID2007 parameterization for the six amplitudes of exclusive processes:
    - *Hilt M, Lehnhart BC, Scherer S, and Tiator L. (2013) Pion photo-and electroproduction in relativistic baryon chiral perturbation theory and the chiral MAID interface. Physical Review C 88(5), 055207.*
- Monte Carlo Generator of Radiative Corrections:
  - Will be presented by Dr. Ilyichev (next presentation today)

Exact Formulae for RC in SIDIS

The total RC is calculated exactly in our paper (PR D100 (2019) 033005):

$$\sigma_{RC} = \frac{\alpha}{\pi}\exp(\delta_{inf})\left(\delta_{VR} + \delta^{l}_{vac} + \delta^{h}_{vac}\right)\sigma_B + \sigma^F_R + \sigma_{AMM} + \sigma^{ex}_R$$

Here δ_{VR} and $\delta^{l,h}_{vac}$ come from the radiation of soft photons and the effects of vacuum polarization, the correction δ_{VR} is infrared-free sum of factorized parts of real and virtual photon radiation, and σ^F_R and σ^{ex}_R are contributions from the process of emission of an additional real photon:

$$\begin{aligned}
\sigma_B &= \frac{\alpha^2 S_x^2}{4Q^4\sqrt{\lambda_Y}S}\sum_{i=1}^{9}\theta_i^B\mathcal{H}_i \\
\sigma^F_R &= -\frac{\alpha^3 S S_x^2}{32\pi^2\lambda_S\lambda_Y}\int\limits_{\tau_{\min}}^{\tau_{\max}} d\tau \int\limits_0^{2\pi} d\phi_k \int\limits_0^{R_{\max}} dR \sum_{i=1}^{9}\left[\sum_{j=1}^{k_i}\tilde{\mathcal{H}}_i\theta_{ij}\frac{R^{j-2}}{\tilde{Q}^4} - \frac{\theta_{i1}\mathcal{H}_i}{RQ^4}\right], \\
\sigma^{ex}_R &= -\frac{\alpha^3 S S_x^2}{2^8\pi^5\lambda_S\lambda_Y}\int\limits_{\tau_{\min}}^{\tau_{\max}} d\tau \int\limits_0^{2\pi} d\phi_k \sum_{i=1}^{9}\sum_{j=1}^{k_i}\frac{\tilde{\mathcal{H}}^{ex}_i\theta_{ij}R_{ex}^{j-2}}{(1+\tau-\mu)\tilde{Q}^4}.
\end{aligned}$$

Explicit expressions for the functions θ_{ij} are given in Appendix B of our paper.

Leading, Next-to-Leading, and Exact Contributions to RC

By “exactly” calculated RC we understand the estimation of the lowest order RC contribution with any predetermined accuracy. The structure of the dependence on the electron mass in RC cross section:

$$\sigma_{RC} = A \log \frac{Q^2}{m^2} + B + O(m^2/Q^2)$$

where A and B do not depend on the electron mass m.

- If only A is kept, this is the leading log approximation.
- If both contributions are kept (i.e., contained A and B), this is the calculation with the next-to-leading accuracy, practically equivalent to exact calculation.

Three approaches to extract the leading Log contribution for the SIDIS cross section (i.e., to calculate A)

- use our exact formulae, collect all terms that result in leading log after integration over photon angles, combine them into the final expression
- extract the poles that correspond to radiation collinear to initial and final electron, integrate over angles, and find the factozing form traditional for leading log calculations.
- use the method of the electron structure functions.

Leading Log: Exatraction from Exact Formulae

The exact expression for σ_R^F is:

$$\sigma_R^F = -\frac{\alpha^3 S S_x^2}{32\pi^2 \lambda_S \lambda_Y} \int\limits_{\tau_{\min}}^{\tau_{\max}} d\tau \int\limits_0^{2\pi} d\phi_k \int\limits_0^{R_{\max}} dR \sum_{i=1}^{9} \left[\sum_{j=1}^{k_i} \tilde{\mathcal{H}}_i \theta_{ij} \frac{R^{j-2}}{\tilde{Q}^4} - \frac{\theta_{i1} \mathcal{H}_i}{R Q^4} \right],$$

Analysis of the integrand and tracing the origin of the leading log allows us to extract the leading log terms from the exact formulae:

$$\int\limits_{\tau_{\min}}^{\tau_{\max}} d\tau \int\limits_0^{2\pi} d\phi_k \sum_{j=1}^{k_i} \theta_{ij}^s R^{j-2} = -4\pi l_m \frac{\sqrt{\lambda_Y}}{S} \frac{1+z_1^2}{z_1(1-z_1)} \theta_i^{Bs}$$

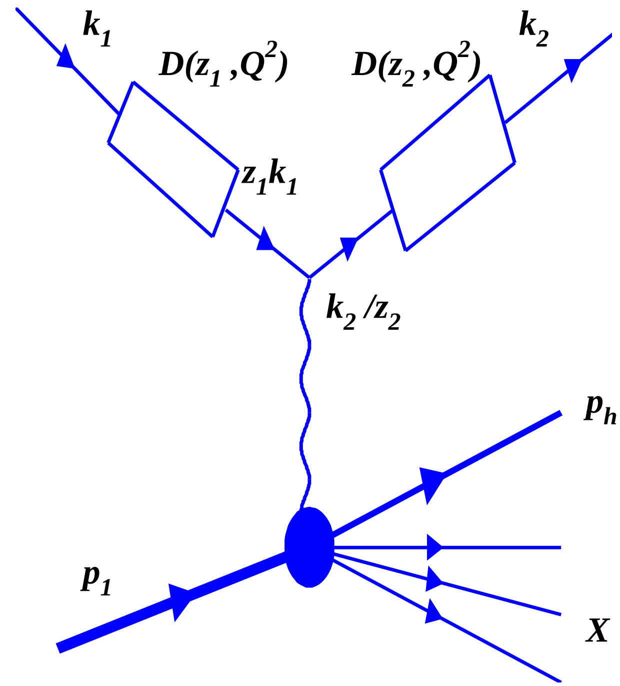
$$\int\limits_{\tau_{\min}}^{\tau_{\max}} d\tau \int\limits_0^{2\pi} d\phi_k \sum_{j=1}^{k_i} \theta_{ij}^p R^{j-2} = -4\pi l_m \frac{\sqrt{\lambda_Y}}{X} \frac{1+z_2^2}{1-z_2} \theta_i^{Bp}$$

Leading Log: Exatraction the Collinear Poles

- Born cross section: $\sigma_B(x, Q^2, z, t, \phi_h) = C L_{\mu\nu}(k_1, k_2) W_{\mu\nu}(p, q, p_h)$.
- RC cross section $\sigma_{RC}(x, Q^2, z, t, \phi_h) = \int C_p L^{rad}_{\mu\nu}(k_1, k_2, k) W_{\mu\nu}(p, q-k, p_h) d^3k/2k_0$.
- the pole for radiation collinear to the initial electron, $1/k.k_1$:
  $L^{rad}_{\mu\nu}(k_1, k_2, k) = A/k.k_1 + Bm^2/k.k_1^2 + \ldots$ and keep only the term $A/k.k_1$.
- Substitute $k = (1-z_1)k_1$ in A everywhere including $W_{\mu\nu}(p, q-k, p_h)$, neglect terms with m^2 in the numerator.
- Integrate $d^3k/k^0/k.k_1$ over angles (resulting in the leading log) and remaining integral over photon energy rewrite as the integral over z_1.
- Express the RC cross section as $\sigma_{RC}(x, Q^2, z, t, \phi_h) =$
  $\int_{z_{1\,min}}^{1} C_{pp} dz_1 (1+z_1^2)/(1-z_1) f(z_1) L_{\mu\nu}((1-z_1)k_1, k_2) W_{\mu\nu}(p, q-(1-z_1)k_1, p_h)$
- Write the final initial coliiner radiation as:
  $\sigma_{RC}(x, Q^2, z, t, \phi_h) = \int_{z_{1\,min}}^{1} C_{pp} dz_1 (1+z_1^2)/(1-z_1) f(z_1) \sigma_B(\tilde{x}, \tilde{Q}^2, \tilde{z}, \tilde{t}, \tilde{\phi}_h)$
- Obtain formulae for shifted variables and analyze shifted kinematics.
- Make similar calculation for the pole for radiation collinear to the final electron.

Leading Log: Using the Electron Structure Functions

- *The QED radiative corrections to the corresponding cross sections can be written as a contraction of two electron structure functions and the hard part of the cross section.*
- *Traditionally, these radiative corrections include effects caused by loop corrections and soft and hard collinear radiation of photons and e+e- pairs.*
- *This method can be improved by also including effects due to radiation of one noncollinear photon. The corresponding procedure results in a modification of the hard part of the cross section, which takes the lowest-order correction into account exactly and allows going beyond the leading approximation.*



$$\sigma^{in} = \int\limits_{z_1^m}^{1} dz_1 D(z_1, Q^2) \int\limits_{\hat{z}_2^m}^{1} \frac{dz_2}{z_2^2} D(z_2, Q^2) \sqrt{\frac{\hat{\lambda}_Y}{\lambda_Y} \frac{S_x^2}{\hat{S}_x^2}} \hat{\sigma}_t^B$$

with ESF $D(z_{1,2}, Q^2) = D^{\gamma}(z_{1,2}, Q^2) + D_N^{e+e-}(z_{1,2}, Q^2) + D_S^{e+e-}(z_{1,2}, Q^2)$ (see details in Afanasev, Akushevich, Merenkov (2004) Journal of Experimental and Theoretical Physics, 98(3) 403-416, and references therein).

The Cross Section in the Leading Log

The cross section of RC in the leading log approximation is:

$$\sigma_{RC}(x,Q^2,z,t,\phi_h) = \left[1+\frac{\alpha}{\pi}\delta_{\rm vac}\right]\sigma_B(x,Q^2,z,t,\phi_h)$$

$$+\frac{\alpha}{2\pi}l_m\int\limits_{z_1^m}^{1} dz_1\frac{1+z_1^2}{1-z_1}\left[\sqrt{\frac{\lambda_Y^s}{\lambda_Y}}\frac{S_x^2}{S_x^{s\,2}}\sigma_B(\tilde{x}_1,\tilde{Q}_1^2,\tilde{z}_1,\tilde{t}_1,\tilde{\phi}_{h1})-\sigma_B(x,Q^2,z,t,\phi_h)\right]$$

$$+\frac{\alpha}{2\pi}l_m\int\limits_{z_2^m}^{1} dz_2\frac{1+z_2^2}{1-z_2}\left[\frac{1}{z_2^2}\sqrt{\frac{\lambda_Y^p}{\lambda_Y}}\frac{S_x^2}{S_x^{p\,2}}\sigma_B(\tilde{x}_2,\tilde{Q}_2^2,\tilde{z}_2,\tilde{t}_2,\tilde{\phi}_{h2})-\sigma_B(x,Q^2,z,t,\phi_h)\right]$$

where quantities with tilde are calculated in the shifted kinematics.

Leading Log: Exclusive Radiative Tail

Exact contribution calculated inour 2019 paper is:

$$\sigma_R^{ex} \quad = \quad -\frac{\alpha^3 S S_x^2}{2^8\pi^5\lambda_S\lambda_Y}\int\limits_{\tau_{\min}}^{\tau_{\max}} d\tau \int\limits_0^{2\pi} d\phi_k \sum_{i=1}^{9}\sum_{j=1}^{k_i} \frac{\tilde{\mathcal{H}}_i^{ex}\theta_{ij} R_{ex}^{j-2}}{(1+\tau-\mu)\tilde{Q}^4}.$$

where $R_{ex} = \frac{p_x^2 - m_u^2}{1+\tau-\mu}$. The Born cross section of exclusive process is:

$$\sigma_{ex}^B = \frac{\alpha^2 S_x}{32\pi^3 Q^4 S\sqrt{\lambda_Y}} \sum_{i=1}^{9} \mathcal{H}_i^{ex}\theta_i^B$$

Therefore we have

$$\sigma_{sex}^1 = \frac{\alpha}{2\pi} l_m \sqrt{\frac{\lambda_Y^{se}}{\lambda_Y}} \frac{S_x^2}{S' S_x^{se}} \frac{1+z_{1ex}^2}{1-z_{1ex}} \sigma_{sex}^B,$$

$$\sigma_{pex}^1 = \frac{\alpha}{2\pi} l_m \sqrt{\frac{\lambda_Y^{pe}}{\lambda_Y}} \frac{S_x^2}{X' S_x^{pe}} \frac{1+z_{2ex}^2}{1-z_{2ex}} \sigma_{pex}^B,$$

where $z_{1ex} = 1 - (p_x^2 - m_u^2)/S'$ and $z_{2ex} = (1 + (p_x^2 - m_u^2)/X')^{-1}$.

The Best Approximation of RC to SIDIS

The idea is to update the definition of the non-radiated cross section for the ESF methods: from the Born cross section to the Born plus one-loop correction. To avoid double counting of the leading terms in the first order we need to exclude the leading terms from the definition of the non-radiated cross section:

$$\sigma_B + \sigma_{RC} = \sigma_B + A\log\frac{Q^2}{m^2} + B + O(m^2/Q^2) = A\log\frac{Q^2}{m^2} + \sigma_{hard}$$

where σ_{hard} is our new non-radiated cross section for ESF. The best approximation is:

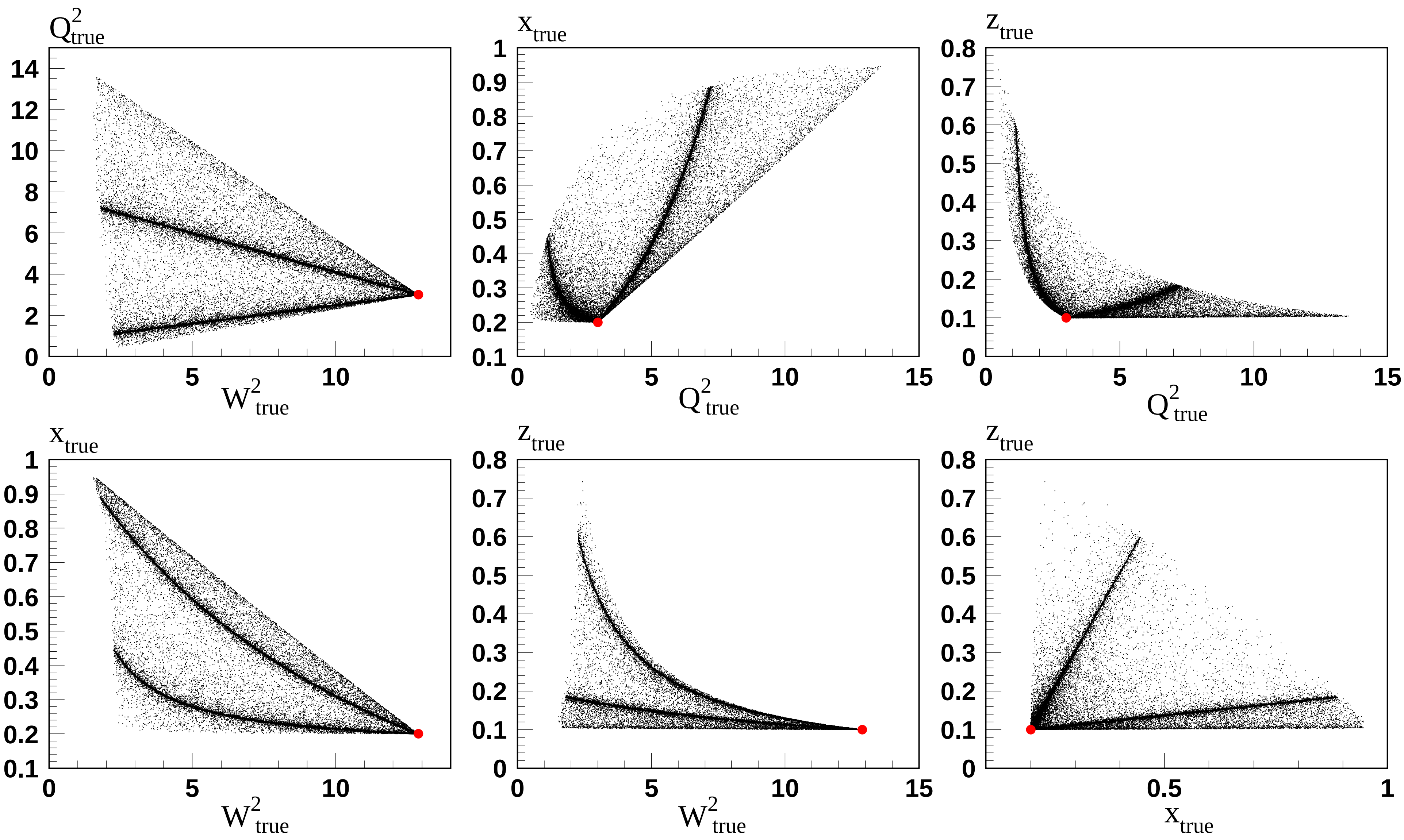
$$\sigma^{in}_{best} = \frac{S_x^2}{p_l}\int\limits_{z_{1i}}^{1} dz_1 D(z_1,Q^2)\int\limits_{\hat{z}_{2i}}^{1}\frac{dz_2}{z_2^2}D(z_2,Q^2)r^2\left(\frac{z_1}{z_2}Q^2\right)\frac{\hat{p}_l\sigma_{hard}(\hat{S},\hat{Q}^2,\hat{x},\hat{z},\hat{p}_t,\cos\hat{\phi}_h)}{(z_1S - X/z_2)^2},$$

where the coefficient r^2 in the integrand results from ressumation of the vacuum polarization by leptons and represented in the form of the running coupling constant.

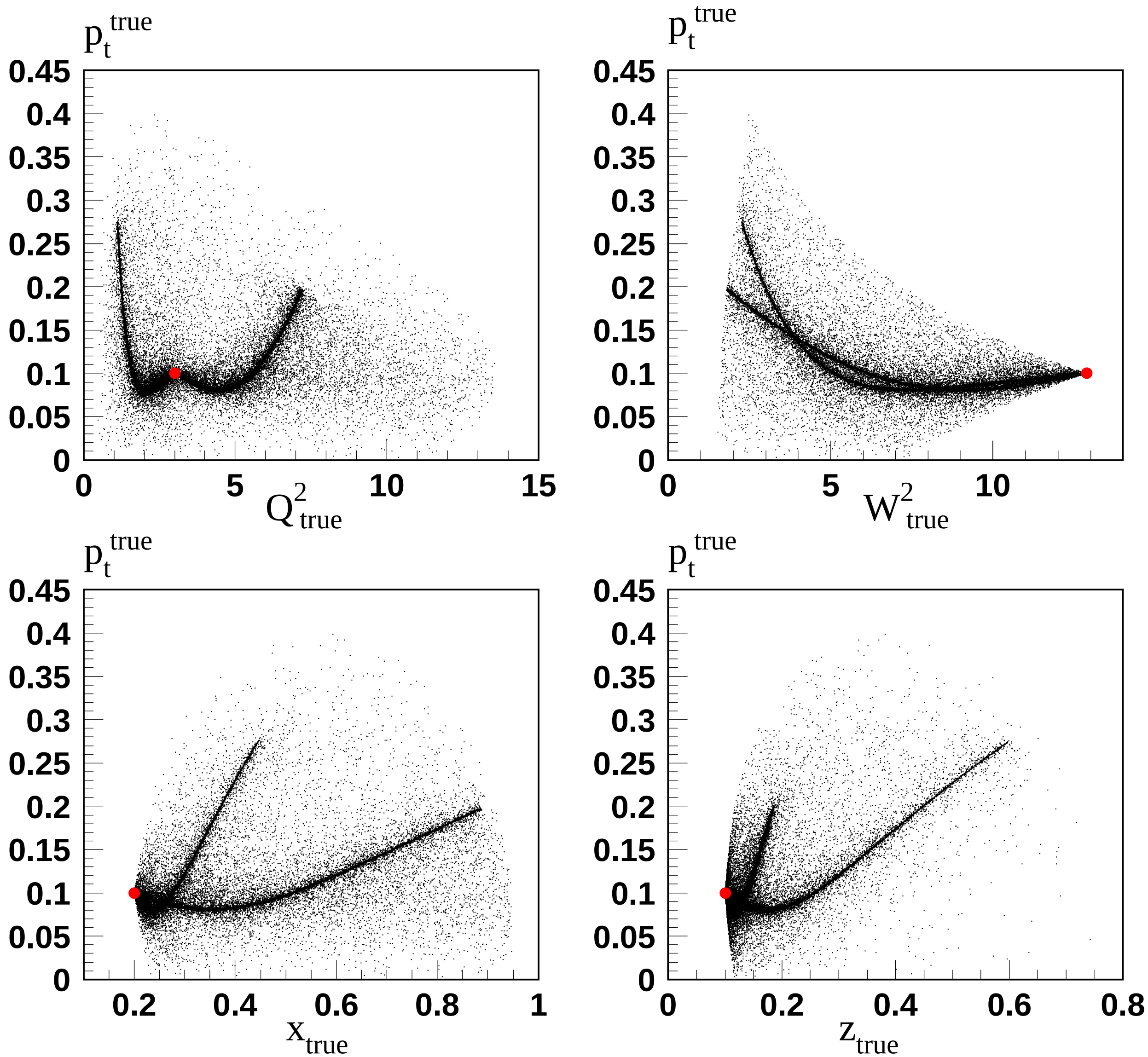
$$r(Q^2) = \sum_{i=0}^{\infty}\left(\frac{\alpha}{2\pi}\delta^l_{\rm vac}(Q^2)\right)^i = \left[1 - \frac{\alpha}{2\pi}\delta^l_{\rm vac}(Q^2)\right]^{-1},$$

Similarly the formulae for the exclusive radiative tail can be updated.

Kinematic Regions for SIDIS SFs (x,Q2,W2,z)



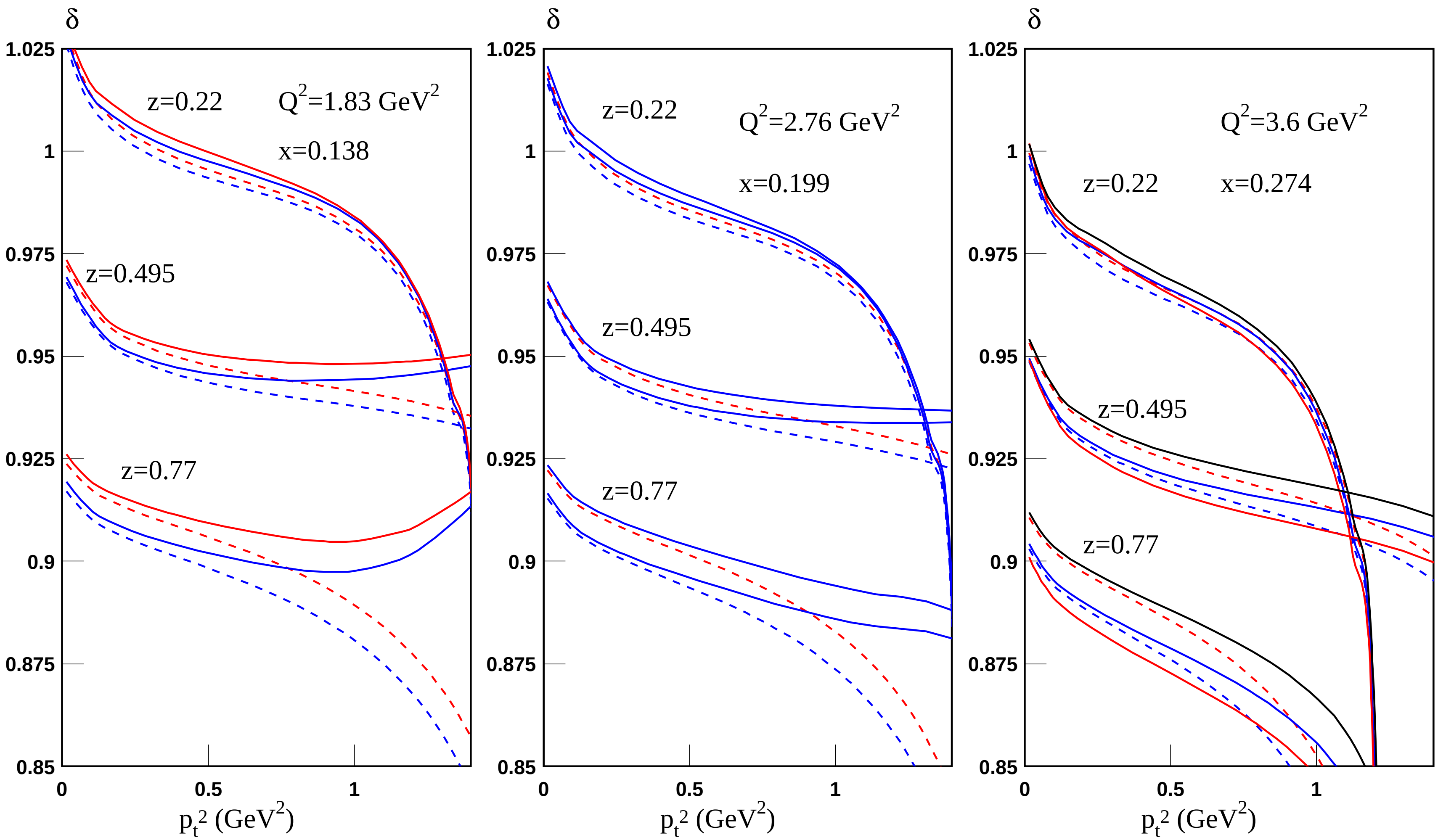
Kinematic Region for SIDIS SFs (pt)



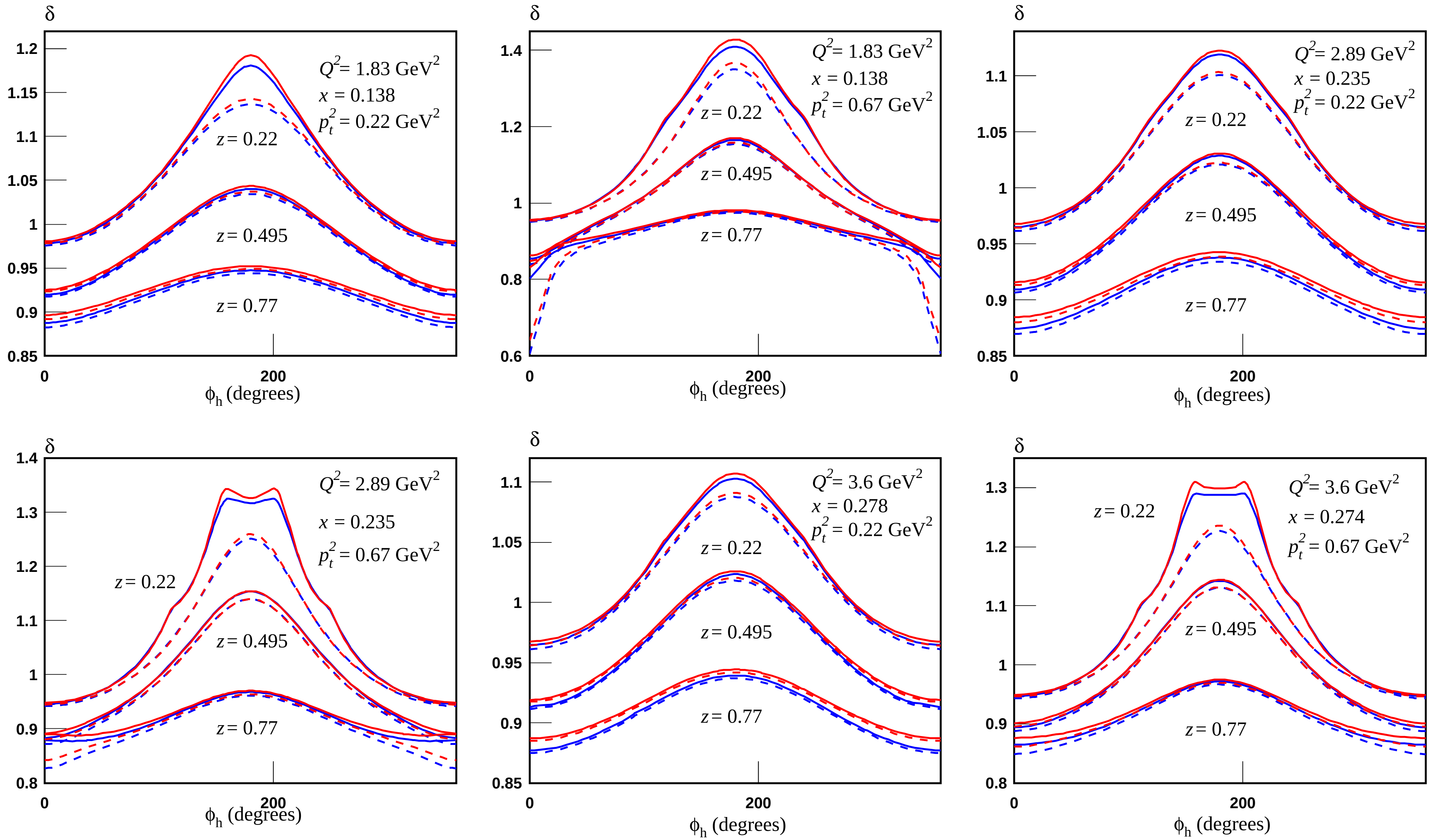
Conclusions from prior analyses

- RC to azimuthal asymmetry can reach 10-20%. We note that different input of SFs produce different corrections (even by sign).
- Exclusive radiative tail is important contribution to RC. there are kinematical regions (e.g., small M_X^2 where it is a dominant contribution.
- The RC may be very significant at large PT.
- There exist effects not observed at the level of the Born cross section (e.g., $< cos(3\phi) >$).
- RC to Sivers and Collins asymmetries can reach several dozens of persents and is expected to be sensitive to SF input.

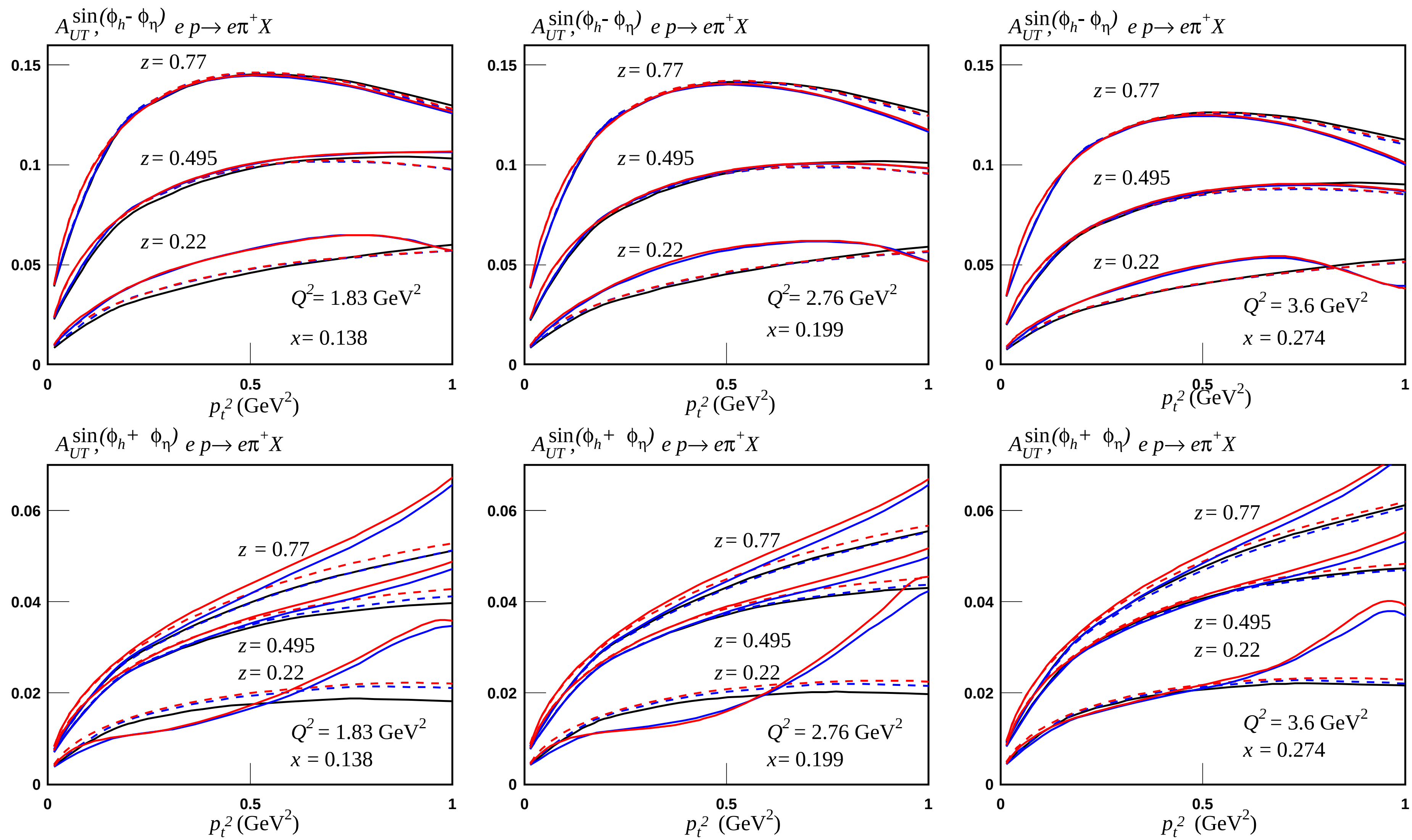
RC to Unpolarized Cross Section



RC to ϕ-dependence



RC to Collins and Sivers Asymmetries



Open Points

Exlusive SFs for polarized case *We are working on it, but we will need several models to test model-dependence. Any help will be appreciated.*

SIDIS SFs *We currently implemented WW approach, however, additional cross checks are required for specific regions. Bounds of applicability of WW are needed to be defined and discussed. Additional models would be helpful as well.*

Iteration procedure of RC *We need to decide whether iteration procedure of RC will be used in data analyses. We strongly recommend this (especially for asymmetry measurements) to avoid an additional bias due to RC procedure. The bias is proportional to the difference between the values of Born asymmetries in the given bin: i) finally extracted and ii) used in RC codes. These values coincide by definition when the iteration procedure is used.*

Comparison and agreement between teoretical calculations *We need to complete analyic and numeric comparison to leptonic RC calculated by Tianbo Liu, W. Melnitchouk, Jian-Wei Qiu, and N. Sato (Physical Review D, 104(9), 094033; Journal of High Energy Physics, 2021(11), 1).*

Hadronic corrections *Discuss and decide whether and how non-leptonic corrections (including box diagrams and emission by hadrons) will be calculated.*

Higher order corrections *We need to discuss approaches for higher order corrections, such as exponentiation, electron SFs, etc. Pay specific attention to the region with small* M_x^2*.*

Monte Carlo *Approaches to MC generators has to be discussed and implemented.*

Calculation of RC to SIDIS using Monte Carlo generator

Can we calculate the RC to SIDIS using generators like RADGEN or DJANGOH?

My answer is NO

DIS hadronic tensor (unpolarized):

$$W_{\mu\nu} = -g_{\mu\nu}F_1 + p_\mu p_\nu F_2$$

SIDIS hadronic tensor (unpolarized):

$$W_{\mu\nu} = -g_{\mu\nu}H_1 + p_\mu p_\nu H_2 + p_{h\mu}p_{h\nu}H_3 + (p_\mu p_{h\nu} + p_{h\mu}p_\nu)H_4$$

The DIS cross section:

$$\sigma = K_1(x,Q^2)F_1(x,Q^2) + K_2F_2(x,Q^2)$$

The SiDIS cross section:

$$\sigma = K_1\tilde{H}_1(x,z,p_T,Q^2) + K_2\tilde{H}_2(x,z,p_T,Q^2) + K_3\tilde{H}_3(x,z,p_T,Q^2)\cos^2\phi_h + K_4\tilde{H}_4(x,z,p_T,Q^2)\cos\phi_h$$

The contributions involving K_3 and K_4 cannot be reproduced using DIS generators.

Conclusion

- Newly achieved accuracies in Jlab and new physics studied at Jlab require paying renewed attention to RC calculations and their implementation in data analysis software.
- For SIDIS RC theoretical efforts are needed both for calculation of SIDIS RC in a bin and for generation of radiated events:
  - *Hadronic tensor for both SIDIS and exclusive cross sections in the covariant form is constructed and tested.*
  - *Exact calculation of RC for the complete SIDIS cross section containing 18 SFs is completed and coding is done, however, implementation of SFs still requires further developments.*
  - *We expect sensitivity of the results for RC to specific assumptions used for constructing SIDIS and exclusive SFs:*
  - *Broad discussion and efforts of theoreticians and experimentalists are required to complete the evaluation of all SIDIS SFs as well as SFs in resonance region and exclusive SFs.*
  - *Iteration procedure with fitting of measured SFs and joining with models beyond SIDIS measurements at each iteration step has to be designed, implemented and involved in data analysis.*
- Tools for generation of the radiated photon in DIS have to provide valid generation of the radiated events in SIDIS.
  - *Such generator can be constructed based on a code for RC in SIDIS in the same way of how RADGEN is constructed based on POLRAD 2.0 (Talk of Dr. Ilyichev at this Session).*