Understanding the large k_T behavior of the TMDs

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Based on

- Phenomenology of TMD parton distributions in Drell-Yan and Z^0 boson production in a hadron structure oriented approach

(ArXiv:2401.14266 accepted to PRD)

- (F. Aslan, M. Boglione, J. O. Gonzalez-Hernandez, T. Rainaldi, T. C. Rogers, A. Simonelli)

- The resolution to the problem of consistent large transverse momentum in TMDs

(PhysRevD.107.094029)

- (J. O. Gonzalez-Hernandez, T. Rainaldi, T. C. Rogers)

- Combining nonperturbative transverse momentum dependence with TMD evolution

(PhysRevD.106.034002)

- (J. O. Gonzalez-Hernandez, T. C. Rogers, N. Sato)

What we know

Drell-Yan

At small q_T << Q the cross section is determined solely by TMD factorization (TMD pdfs and/or TMD FFs)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\boldsymbol{q}_T\ldots} \overset{q_T \ll Q}{\sim} \sum_j H_{j\bar{j}} \int \mathrm{d}^2\boldsymbol{k}_{T,1}\mathrm{d}^2\boldsymbol{k}_{T,2} f_j(x, k_{T,1}; \mu, \zeta) f_{\bar{j}}(x, k_{T,1}; \mu, \zeta) \delta^{(2)}(\boldsymbol{q}_T - \boldsymbol{k}_{T,1} - \boldsymbol{k}_{T,2})$$

At large $q_T \sim$ Q the cross section is determined solely by fixed order collinear factorization (SIDIS, Drell-Yan, e^+e^- --> back-to-back hadrons,…)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\boldsymbol{q}_T\ldots} \overset{q_T \sim Q}{\sim} H(q_T) \otimes f \otimes f$$

What we know

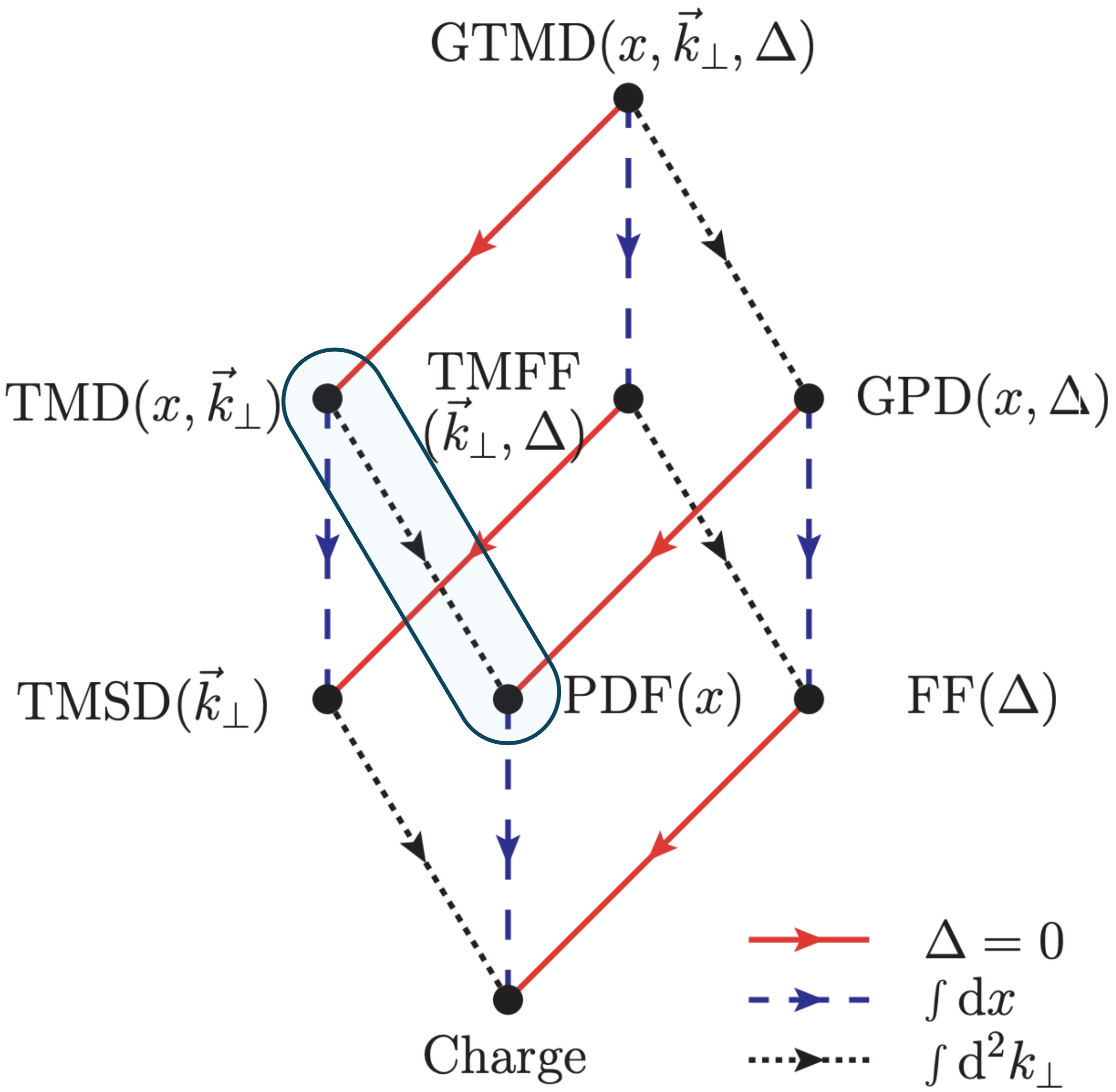
Similarly, at large TM (k_T)/ small b_T the TMDs are **uniquely determined** by an OPE expansion in terms of collinear PDFs/FFs

$$f_{i/H}(x, b_T; \mu, \zeta) = \widetilde{C}_{ij}(x, b_T; \mu, \zeta) \otimes f_{j/H}(x; \mu) + \mathcal{O}(m b_T)$$

Perturbatively calculable

Usual PDFs

What we know



Most of these integrals are divergent.
A more careful treatment is necessary

Credits: Lorcé, Pasquini and Vanderhaeghen

Integral relations are more complicated

$$\int_0^{\mu} \mathrm{d}^2\boldsymbol{k}_T\, f_{i/h}(x, k_T; \mu, \mu^2) = f_{i/h}(x;\mu) + C_{\Delta,ij} \otimes f_{j/h} + p.s.$$

Purely perturbative

Even for the polarized cases (Sivers, etc…)

Conventional (pheno) approach

Final parametrization of a TMD

$$\tilde{f}_{j/p}\left(x;\boldsymbol{b}_{\mathrm{T}};\mu_Q,Q\right)=\tilde{f}^{\mathrm{OPE}}_{j/p}\left(x;\boldsymbol{b}_*;\mu_{b_*},\mu_{b_*}\right)\times$$

$$\times\exp\left\{\int_{\mu_{b_*}}^{\mu_Q}\frac{\mathrm{d}\mu'}{\mu'}\left[\gamma\left(\alpha_S(\mu');1\right)-\ln\left(\frac{Q}{\mu'}\right)\gamma_K\left(\alpha_S(\mu')\right)\right]+\ln\left(\frac{Q}{\mu_{b_*}}\right)\tilde{K}\left(\boldsymbol{b}_*;\mu_{b_*}\right)\right\}$$

$$\times\exp\left\{-g_{j/p}\left(x,\boldsymbol{b}_{\mathrm{T}}\right)-g_K\left(\boldsymbol{b}_{\mathrm{T}}\right)\ln\left(\frac{Q}{Q_0}\right)\right\}$$

Nonperturbative

Perturbatively calculable

Drop this

$$\tilde{f}^{\mathrm{OPE}}_{j/p}\left(x,\boldsymbol{b}_*;\mu_{b_*},\mu_{b_*}\right)=\widetilde{C}_{j/j'}\left(x/\xi,\boldsymbol{b}_*;\mu_{b_*},\mu_{b_*}\right)\otimes\tilde{f}_{j'/p}\left(\xi;\mu_{b_*}\right)+\mathcal{O}\left(m^2 b^2_{\max}\right)$$

Same for FF

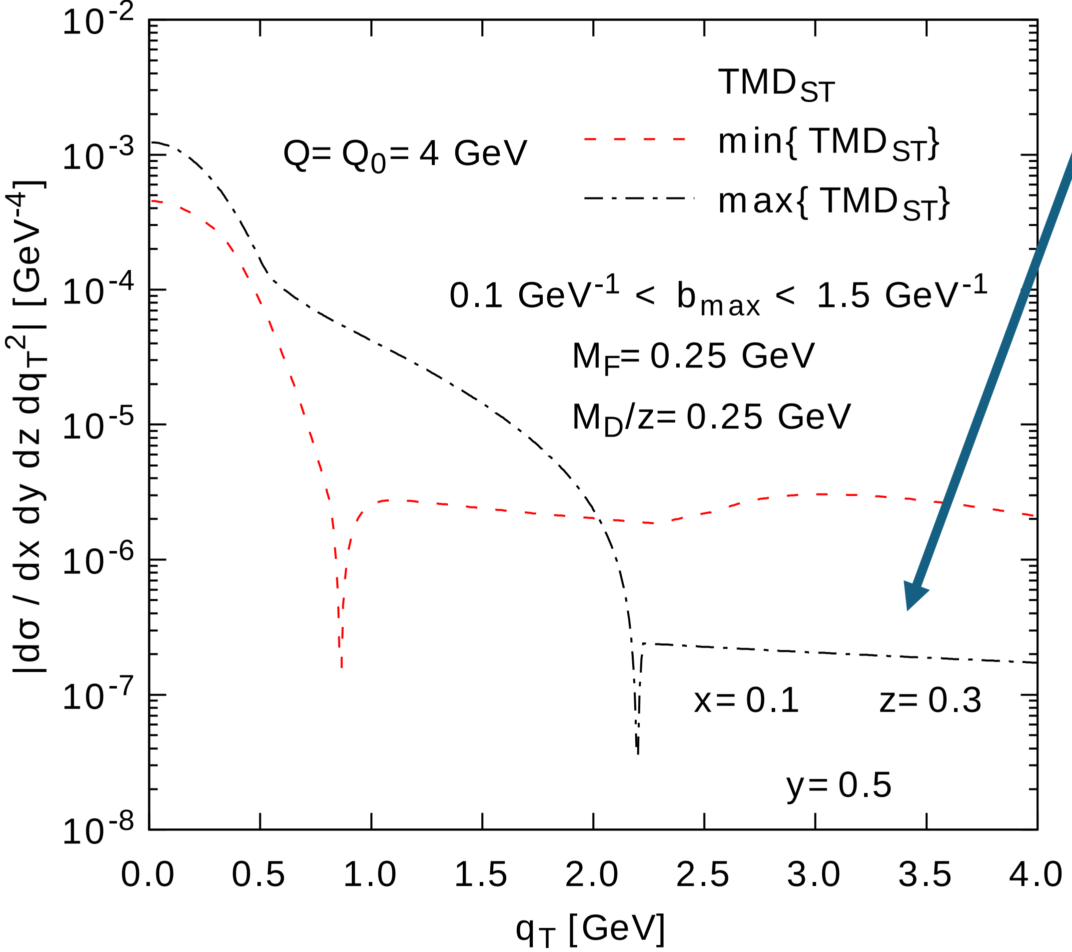
Fixed order collinear factorization

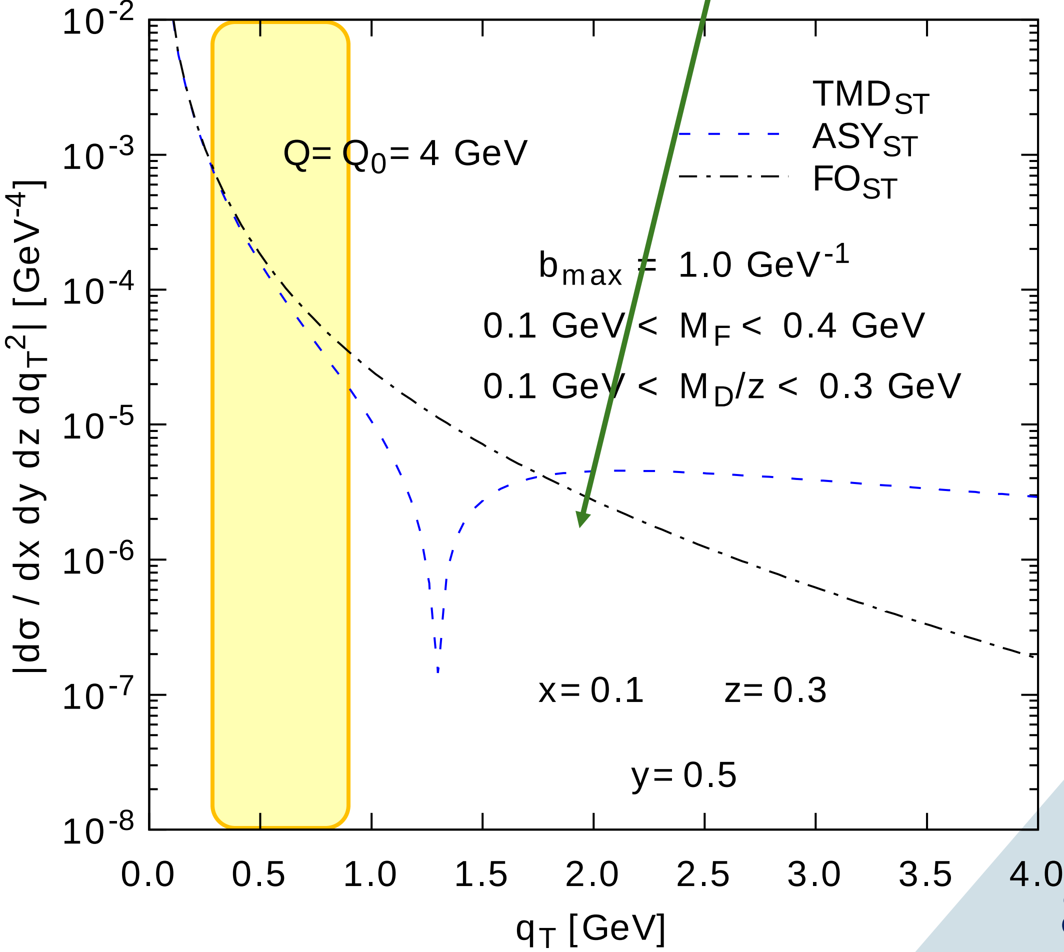
(Some) Issues with conventional approach

Large b_{max} dependence

What is going on?

Large q_T inconsistency

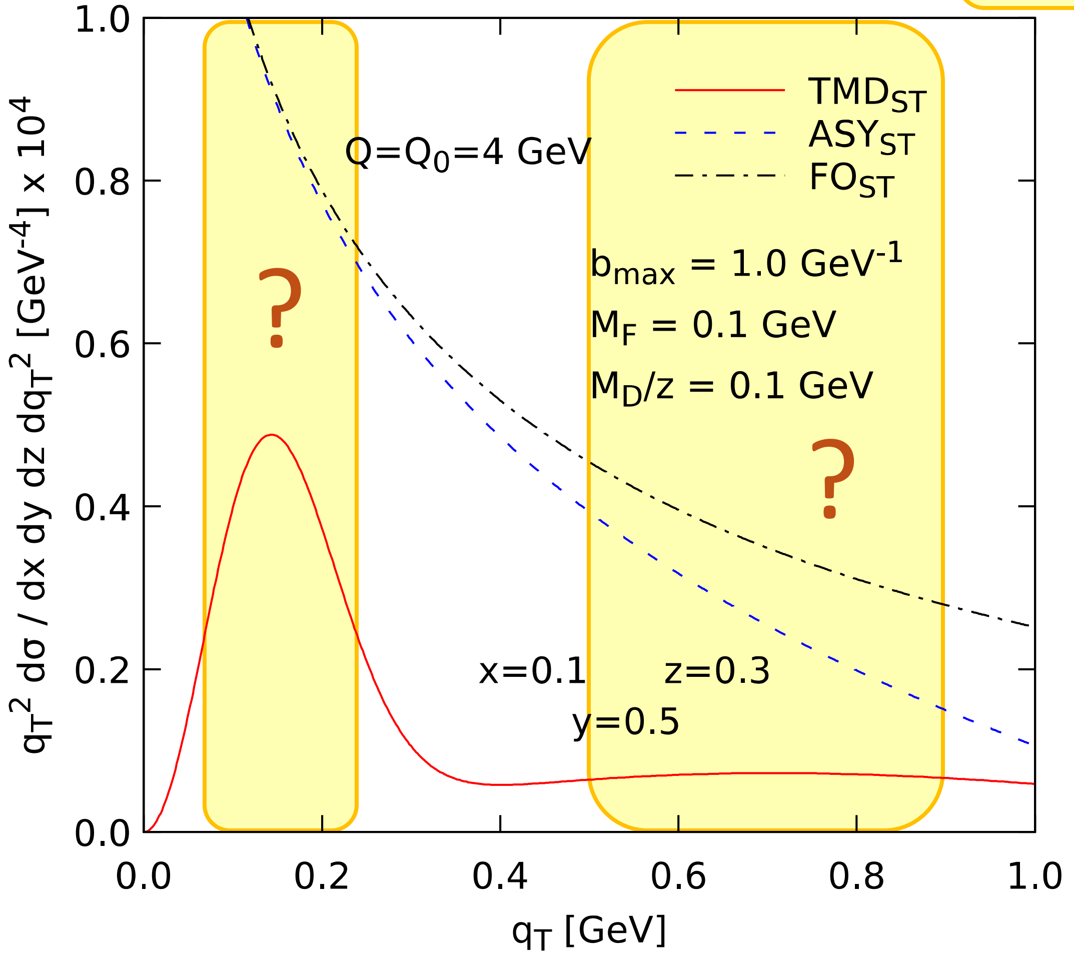




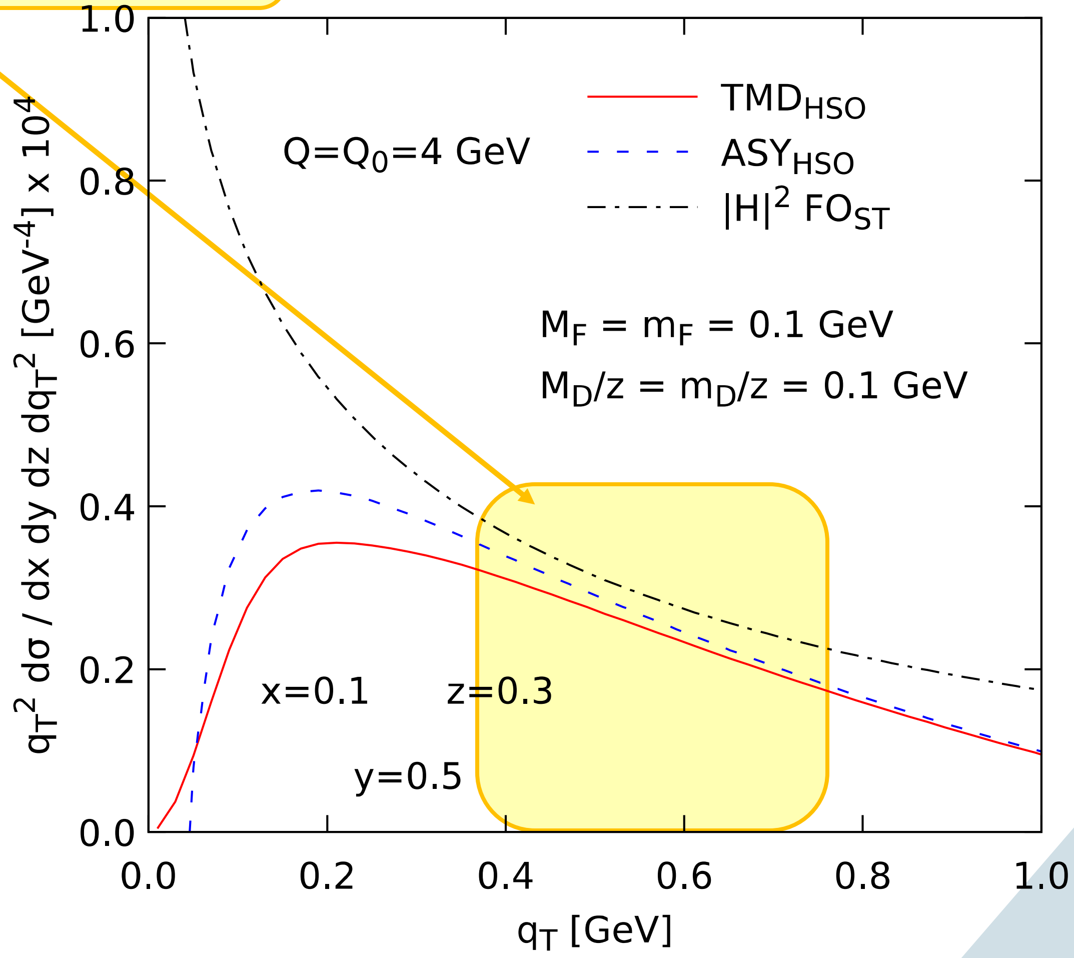
Conventional vs HSO - SIDIS cross section (not a fit)

Matching region !!!

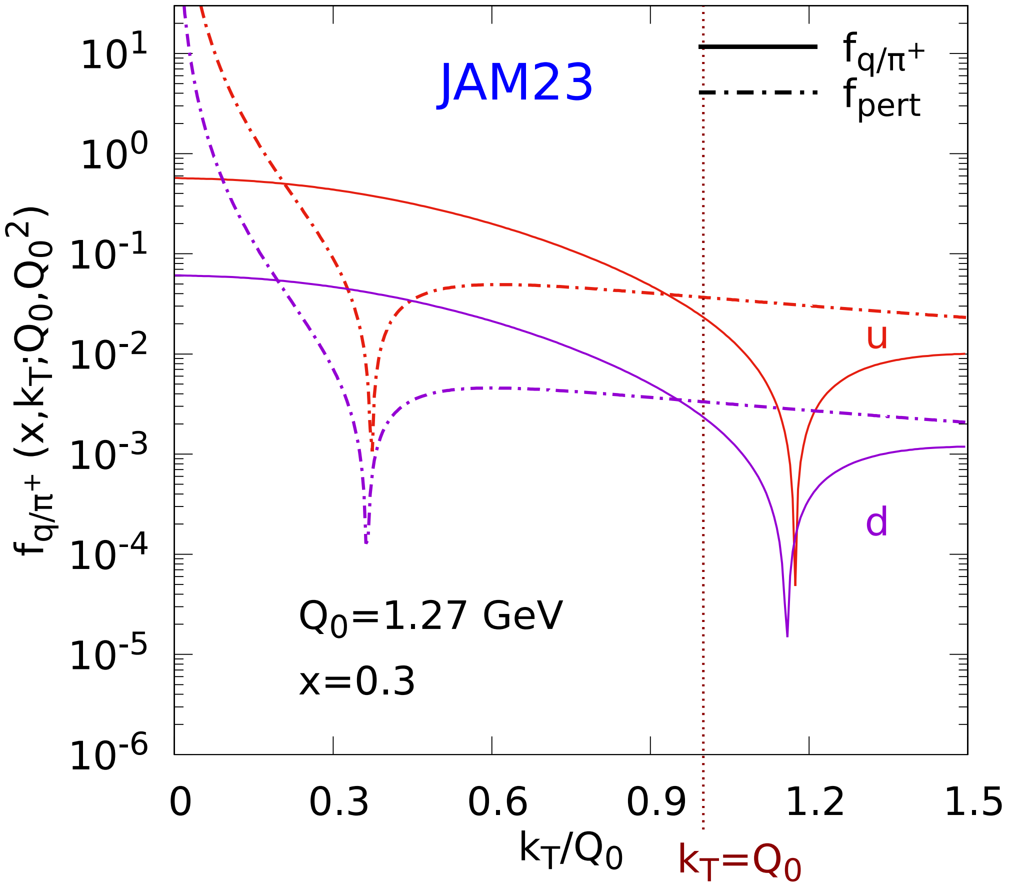
Conventional

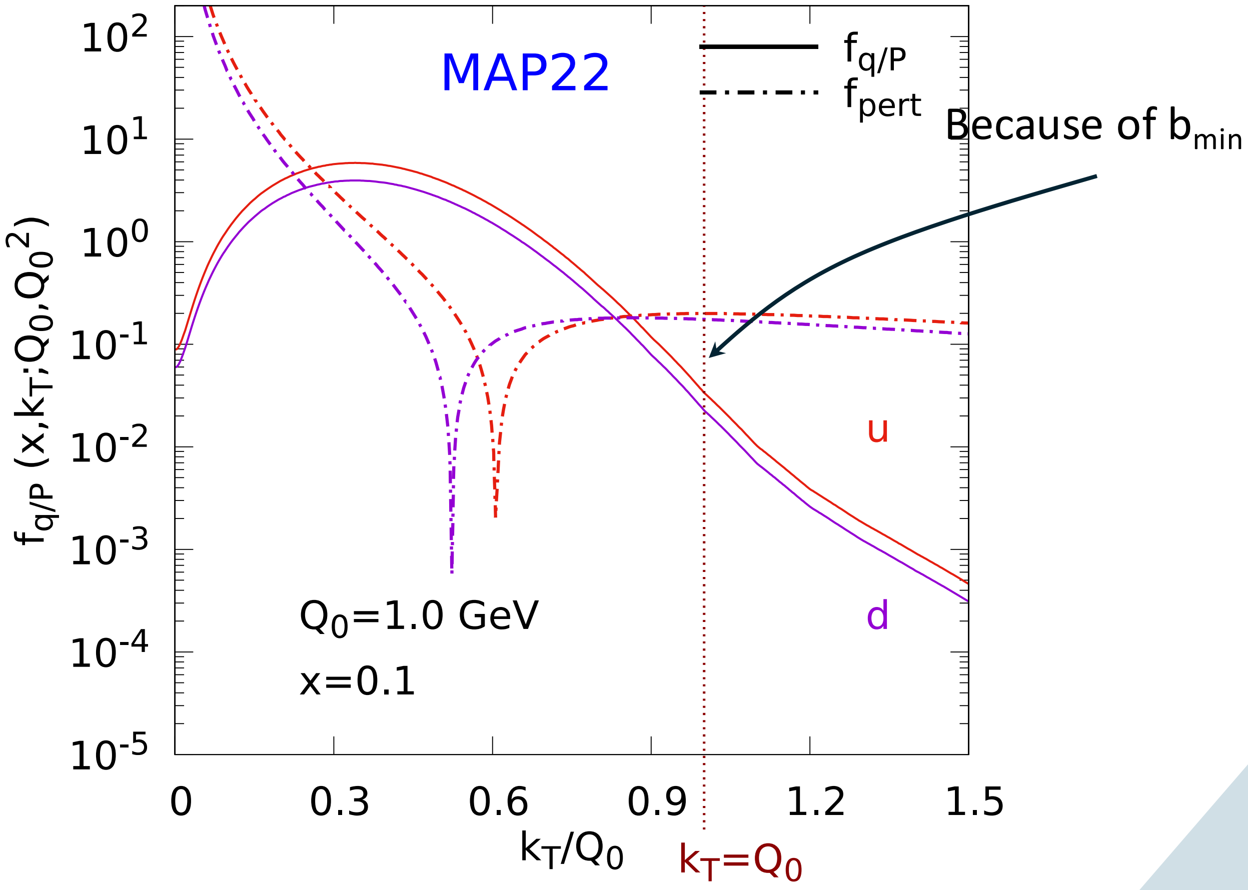


HSO (Gaussian)



Matching at and after $k_T/Q \sim 1$





(Some) Questions

- What do we mean by **perturbative and nonperturbative** contributions?
- How much **sensitivity to collinear functions** do the TMDs have?
- Can we test different models and our assumptions in a manageable manner?
- Can we **maximize the predictive power**?
- Do we have **control** over the theoretical/model errors?

Create a framework that facilitates the answers:

HSO approach

Hadron Structure Oriented approach

TMD PDF HSO parametrization at input scale

Integral relation/ OPE matching

No need of b_{max} or b_{min}

Smooth
Perturbative-Nonperturbative
interpolation

(regardless of the NP model !!)

TMD PDF HSO parametrization at input scale

Fixed order collinear factorization

$\mathcal{O}(\alpha_S)$

Large k_T OPE coefficients

$$f_{\mathrm{inpt,i/p}}(x,\boldsymbol{k}_T;\mu_{Q_0};Q_0^2) = \frac{1}{2\pi}\frac{1}{k_T^2+m^2}\left[A^f_{i/p}(x;\mu_{Q_0}) + B^f_{i/p}(x;\mu_{Q_0})\ln\frac{Q_0^2}{k_T^2+m^2}\right] + \frac{1}{2\pi}\frac{1}{k_T^2+m^2}A^{f,g}_{i/p}(x;\mu_{Q_0}) + C^f_{i/p} f_{\mathrm{core,i/p}}(x,\boldsymbol{k_T};Q_0^2)$$

Small k_T model

NP parameters

Such that

It is easily generalizable up to any orders in a_S

$$f^c_{j/p}(x;\mu_Q) \equiv 2\pi\int_0^{k_c} \mathrm{d}k_\mathrm{T}\, k_\mathrm{T} f_{j/p}\left(x,\boldsymbol{k}_\mathrm{T};\mu_Q,\sqrt{\zeta}\right) = f_{j/p}(x;\mu_Q) + \Delta_{j/p}(x;\mu_Q,k_c) + \mathrm{p.s.}$$

TMD PDF HSO parametrization at input scale

$f_{\text{inpt,i/p}}(x, \boldsymbol{k}_T; \mu_{Q_0}; Q_0^2) =$ pQCD and collinear factorization information

$+ C^f_{i/p}$ NP model

Such that

Integral relations/OPE expansion is satisfied

No need to forcibly divide space into two parts with b_{max}

Quick comparison

Conventional

HSO

Conventional		HSO
b_{max}: takes care of large logs at large b_T	→	Large logs taken care by the integral relation/OPE matching
b_{min}: takes care of integral relation (but changes OPE expansion)	→	Integral relation sastified by construction
$\mu_{b*}(b_T, b_{max})$: takes care of RG improvement	→	RG improvement with a functional of b_T only (no b_{max})

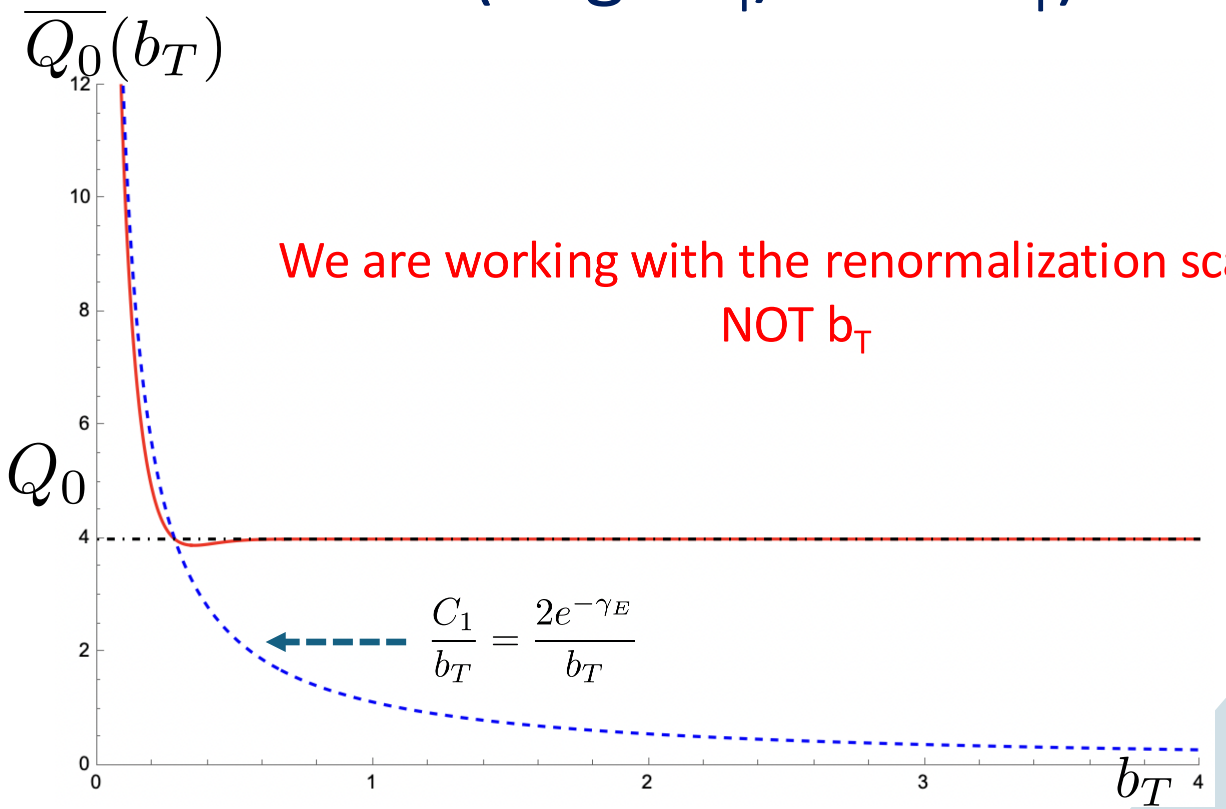
Evolution?

HSO Collins-Soper kernel at the input scale and RG improvement with $\overline{Q_0}(b_T)$ prescription.

Match small b_T/large k_T with OPE and assign "core" model (large b_T/small k_T)

We need to change scheme

$$\overline{Q_0}\,(b_T, a) = Q_0\left[1 - \left(1 - \frac{C_1}{Q_0 b_T}\right) e^{-a^2 b_T^2}\right]$$



We are working with the renormalization scale, NOT b_T

Phenomenology: test the HSO with Drell-Yan

Pheno strategy:

Data at different Q not on the same footing

Fit low-to-moderate Q → Evolve fits to higher Q

E288, E605 Drell-Yan experiments

Two different ”core” models:
Gaussian and Spectator-like

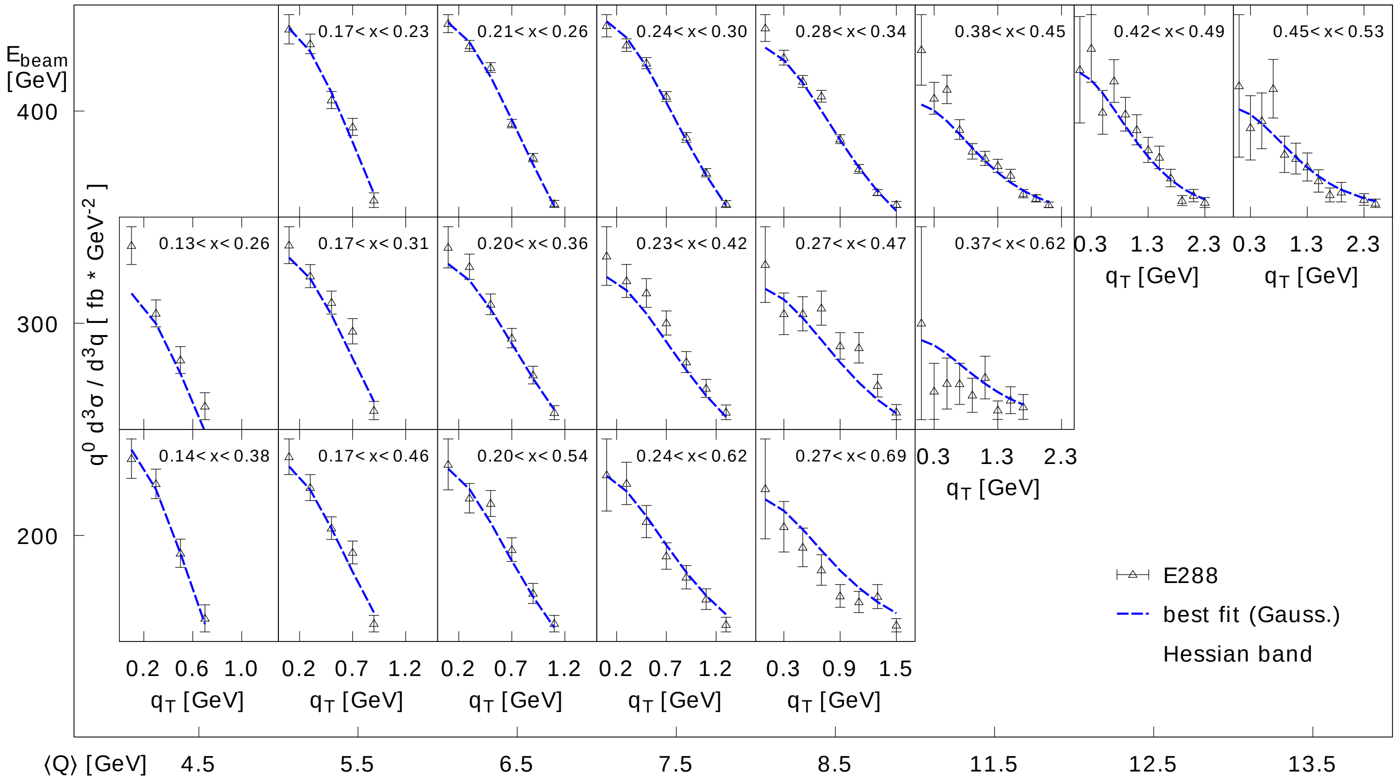
Postdiction

CDFI, CDFII, D0I

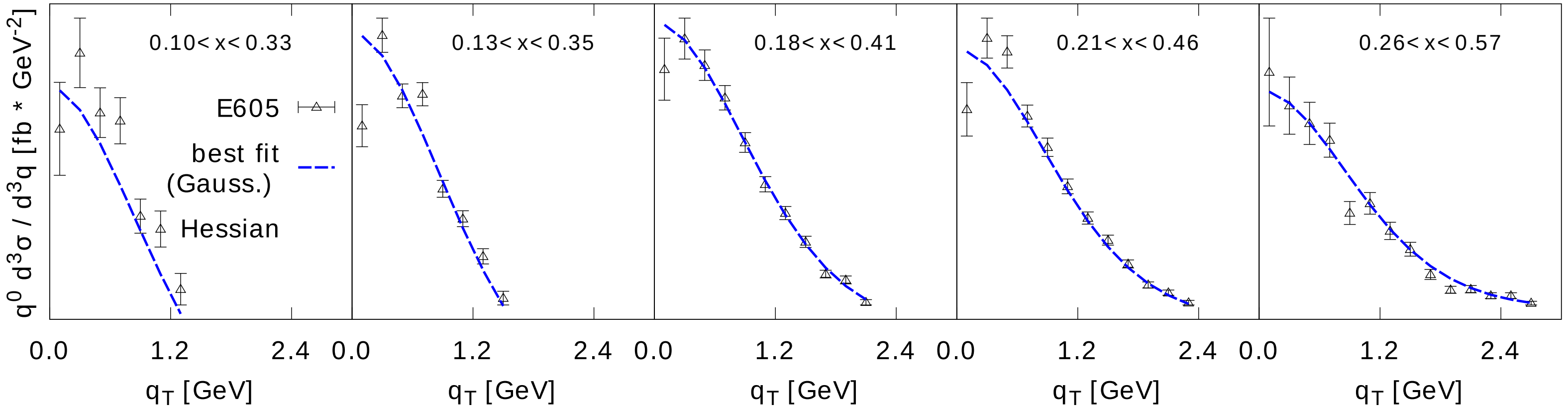
$$Q = M_Z$$

Low Q fit results

	Gaussian fits	
	E288 (130 pts.)	E605 (52 pts.)
χ^2_{dof}	1.04	1.68



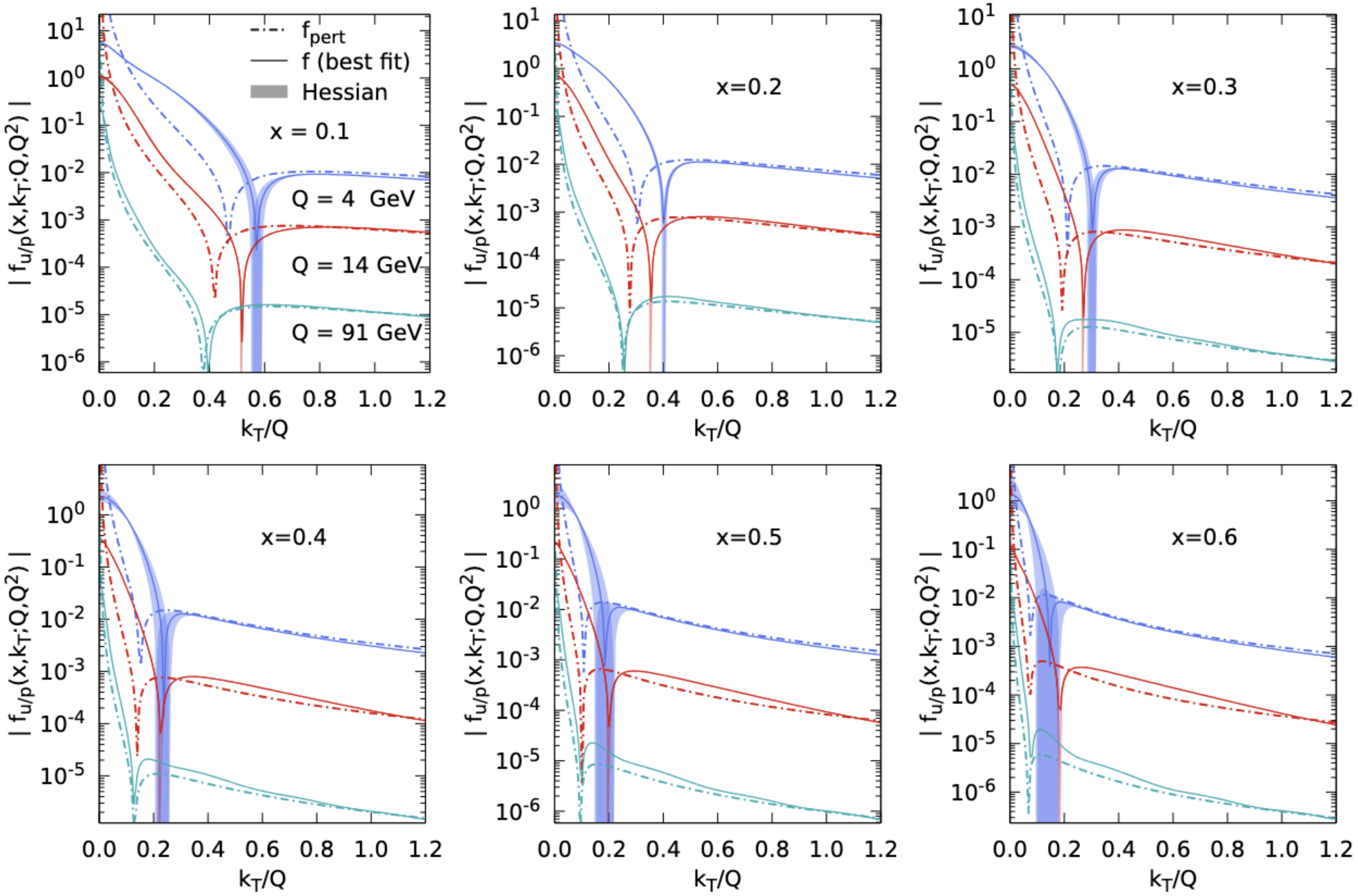
Just 4 parameters for now



Spectator model too:

	Spectator model fit
	E288 (130 pts.)
χ^2_{dof}	1.04

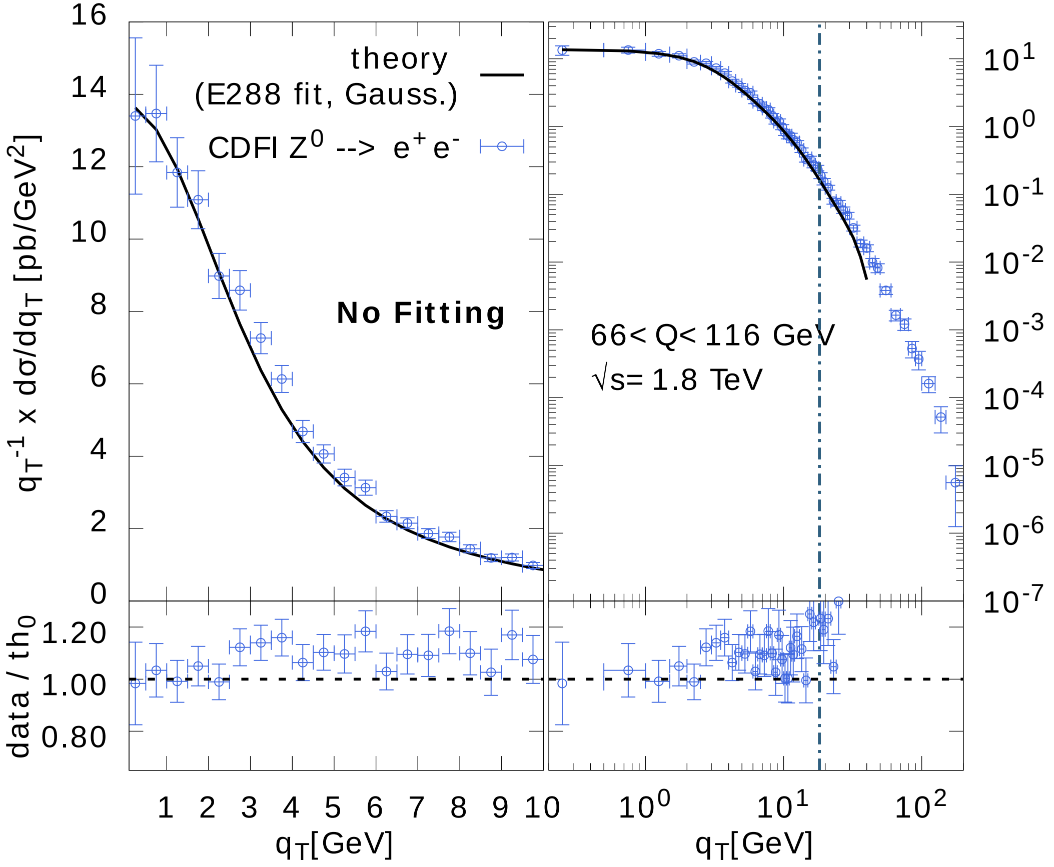
Extractions from E288

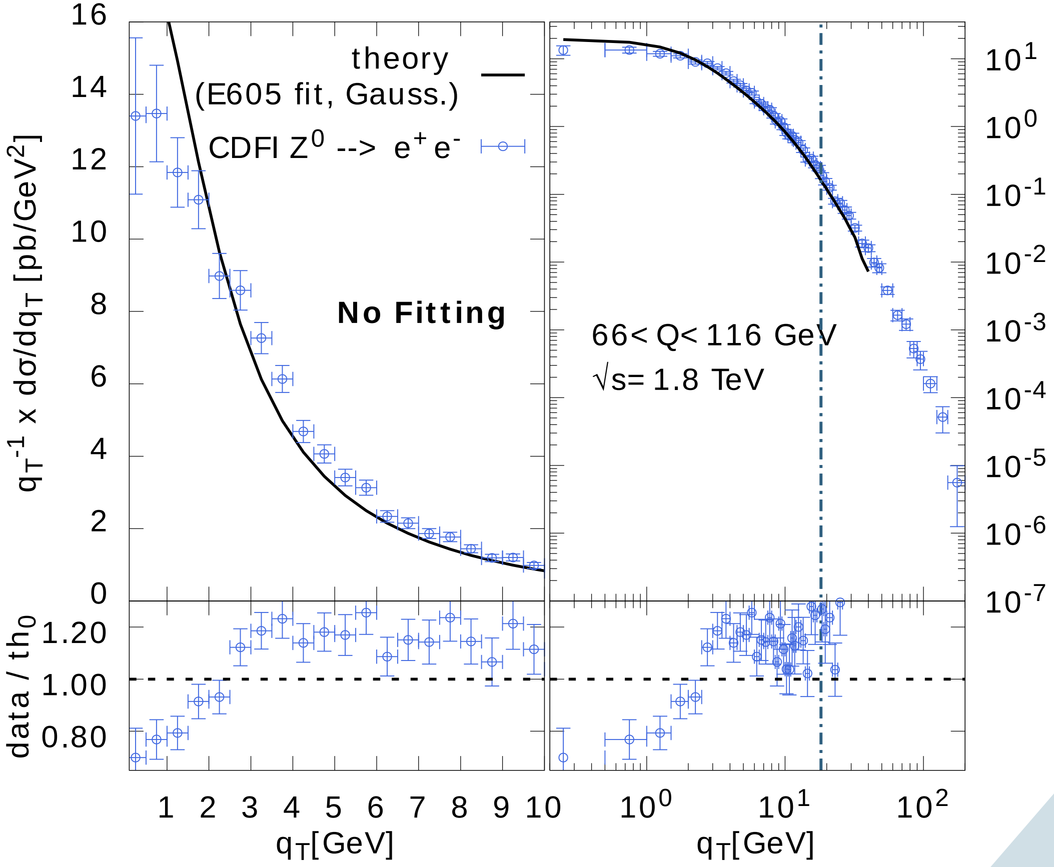


Higher Q postdictions: Testing the predictive power

A postdiction of CDFI with just E288 or E605 data

Just 3+1 parameters

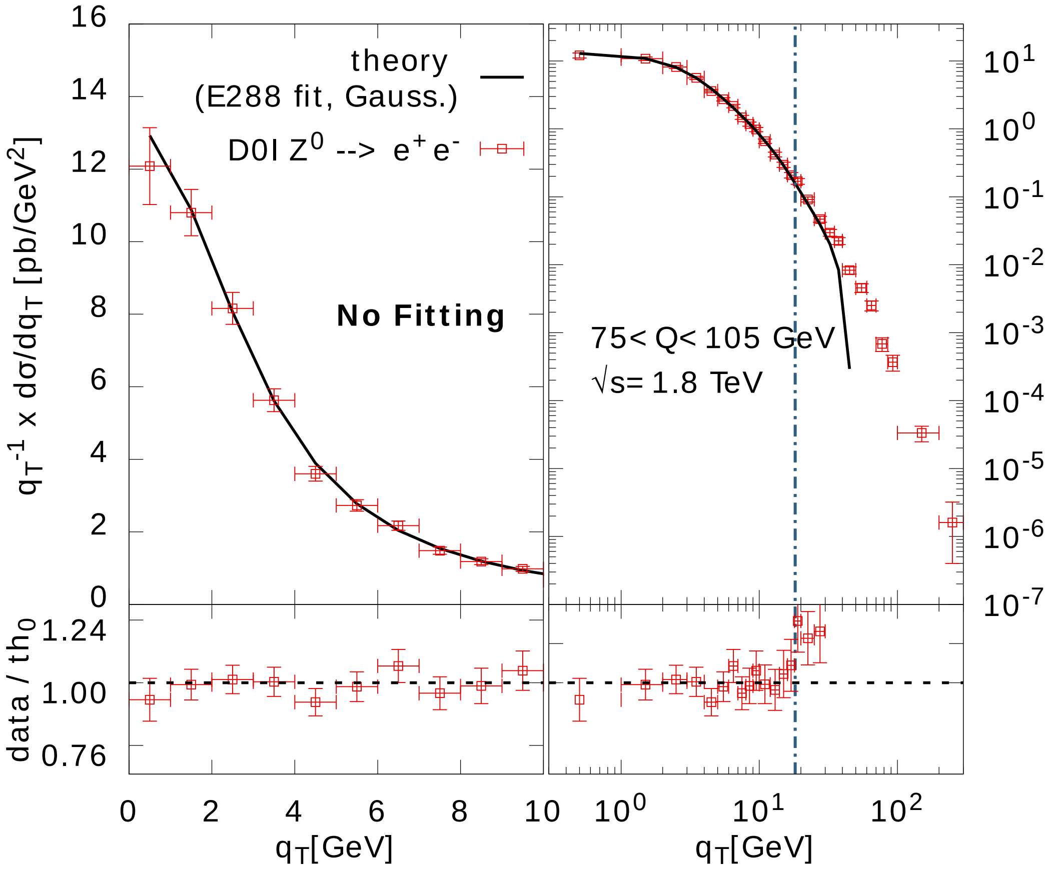


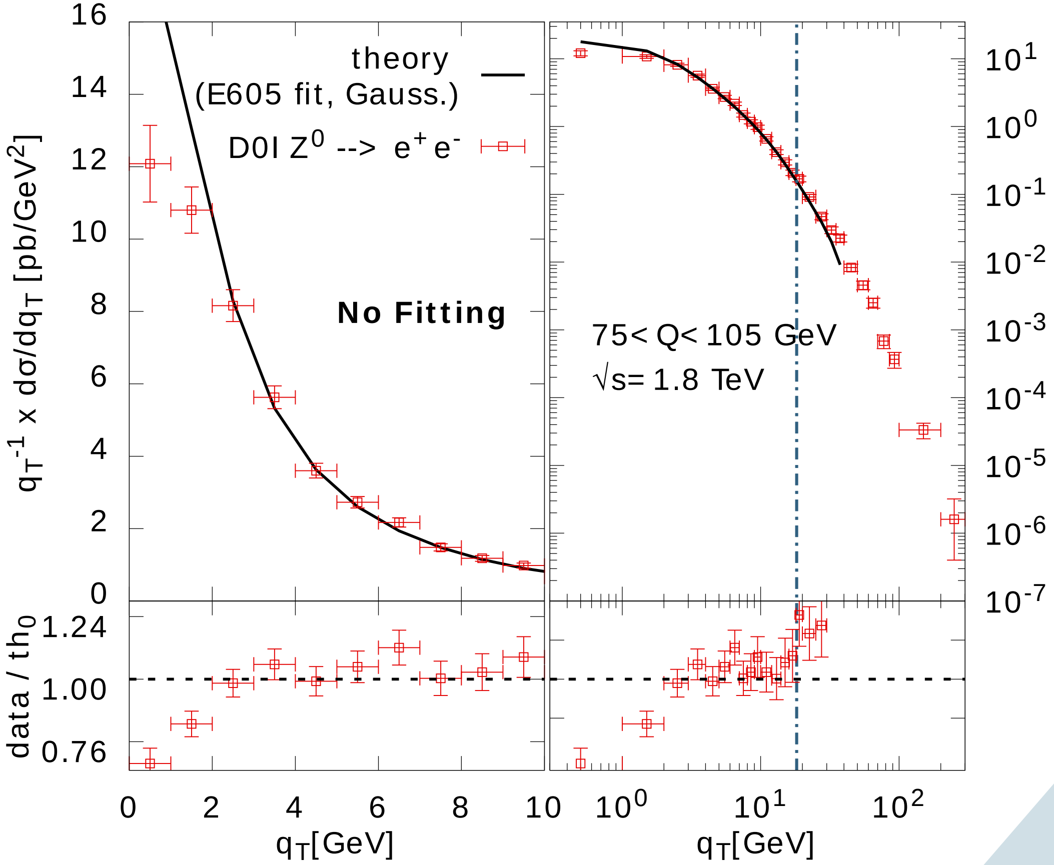


Higher Q postdictions: test different fits on the same experiment

A postdiction of D0I with just E288 or E605 data

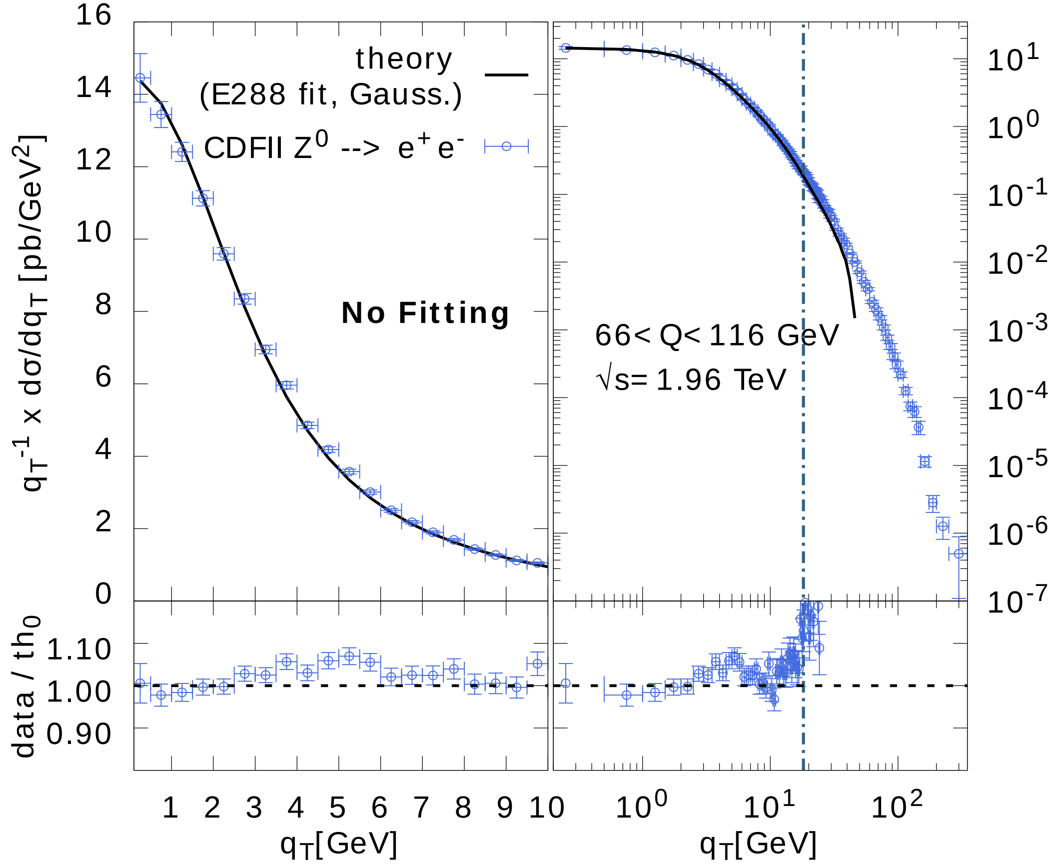
Just 3+1 parameters



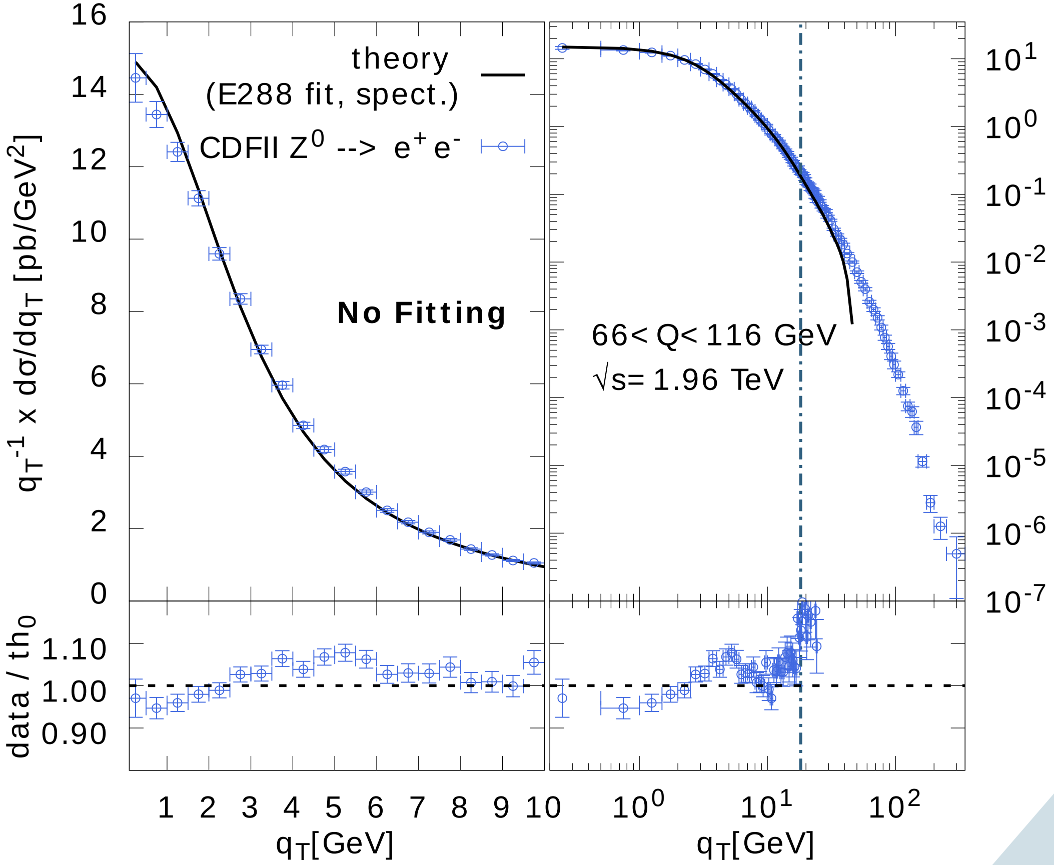


Higher Q postdictions: test different models on the same experiment

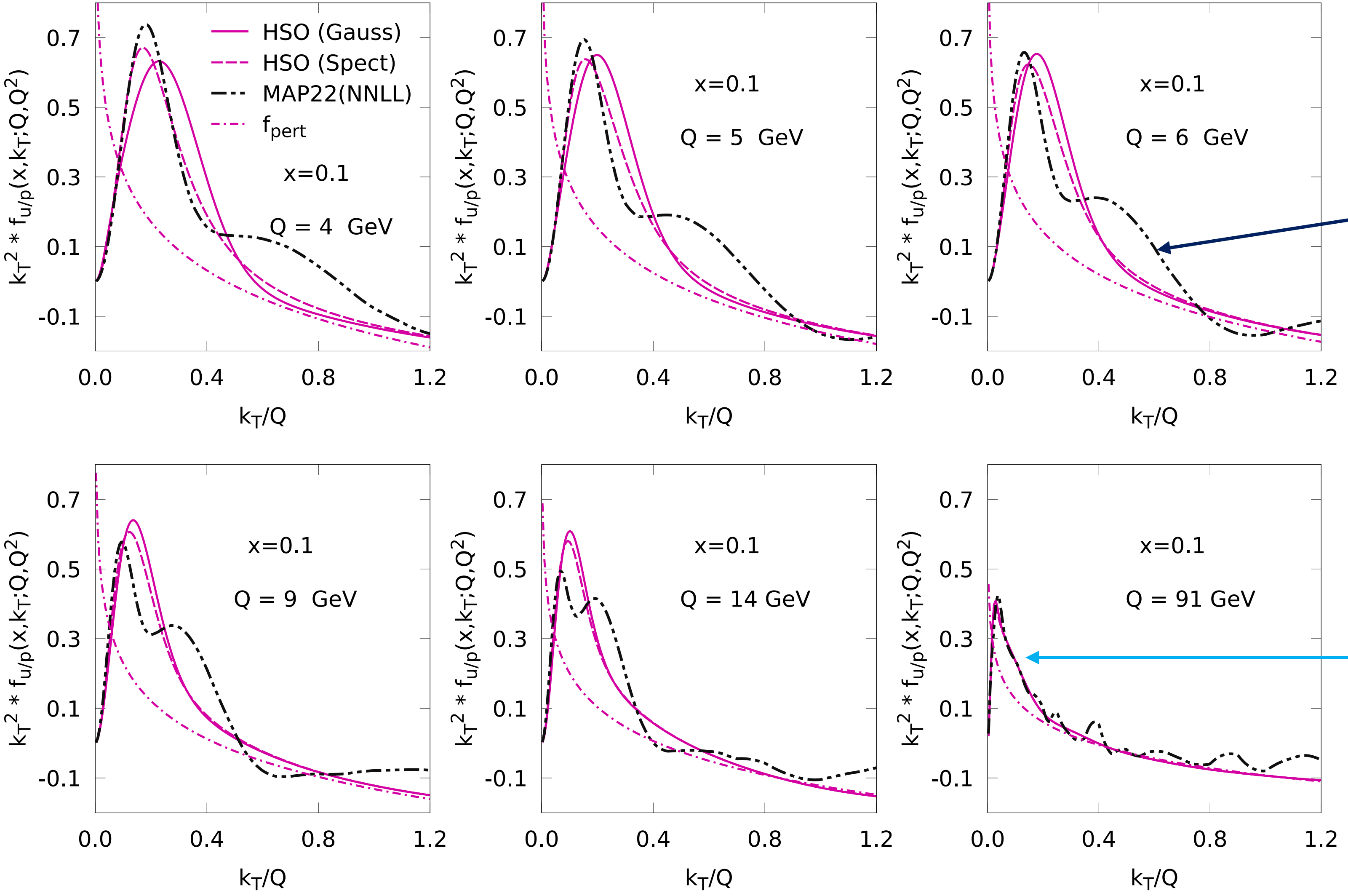
A postdiction of CDFII with E288 GAUSSIAN fit



A postdiction of CDFII with E288 SPECTATOR fit



Comparison with MAP22



Observations:

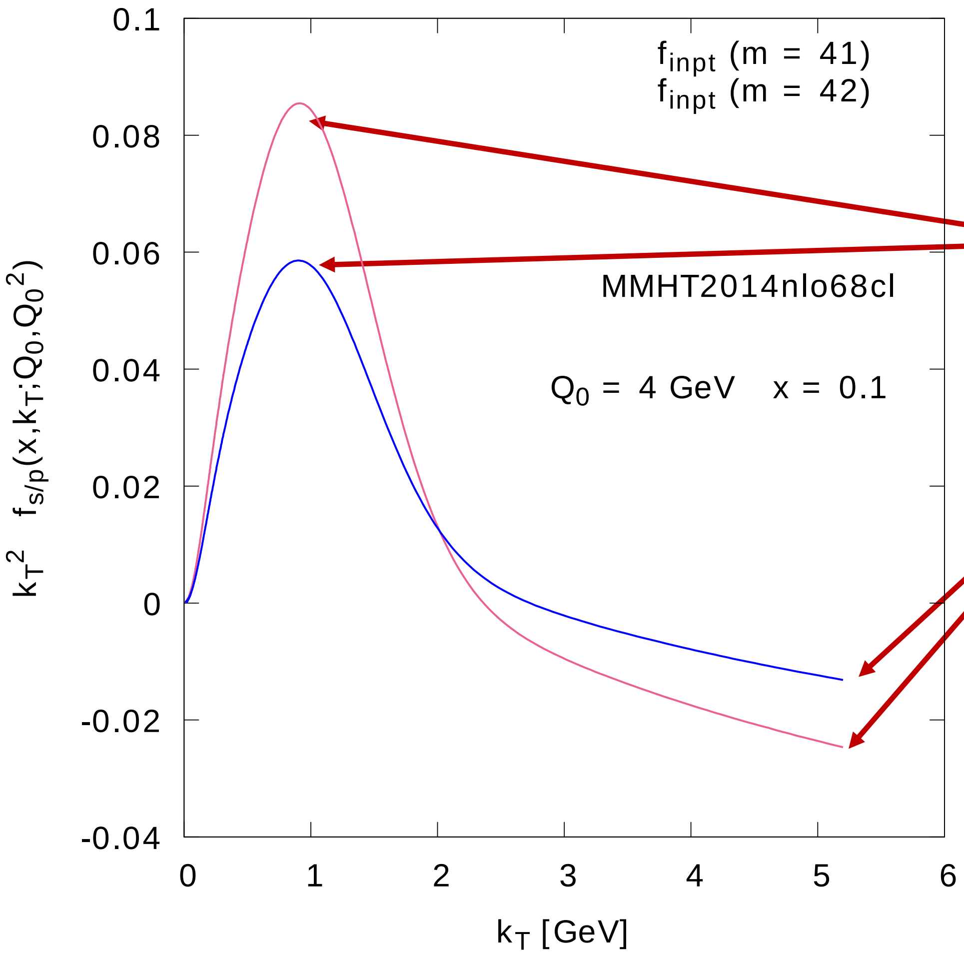
No tail matching for MAP

Different models can describe the small k_T region at low Q

Model dependence washes out at large Q

How do we choose?

TMDs are affected by collinear distributions



Example: take two pdfs associated with the same flavor (s here) and compute the input TMD

Maybe unexpected **different small k_T behavior** because of integral relation

Expected **different tails** because of the OPE expansion

<u>Changing the integral **necessarily** changes the integrand</u>

HSO: Can **quantify** these effects concretely

Next/Ongoing

Sivers TMD

Not the Qiu-Sterman function

$$\pi \int_0^{\mu^2} \mathrm{d}k_T^2\, k_T^2 f_{1T}^{\perp}(x, k_T; \mu, \mu^2) = M^2 f_{1T}^{\perp,(1)}(x; \mu)$$

Intrinsic transverse momentum and evolution in weighted spin asymmetries

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Similarly, just exploiting the properties of the Fourier transform and the OPE expansion:

$$\tilde{f}_{1T}^{\prime\perp}(x, b_T; \mu, \zeta) = \sum_{n=0}^{\infty} \sum_{k=0}^{2n} a_S^n b_T L_b^k \tilde{C}_{1T}^{\perp}(x; \mu, \zeta) + \mathcal{O}(\Lambda^2 b_T^2)$$

Ongoing calculations and extension of the HSO approach to Sivers

Summary

We have a framework that

1. Is consistent with the large k_T tail from theory (where it should)
2. Satisfies an integral relation: pseudo probabilistic interpretation
3. No b_{max} or b_{min} dependence: all errors are under control
4. NP (core) models are very easily swappable and testable

Pheno methodology: Fit low Q, test against higher Q (not mandatory)

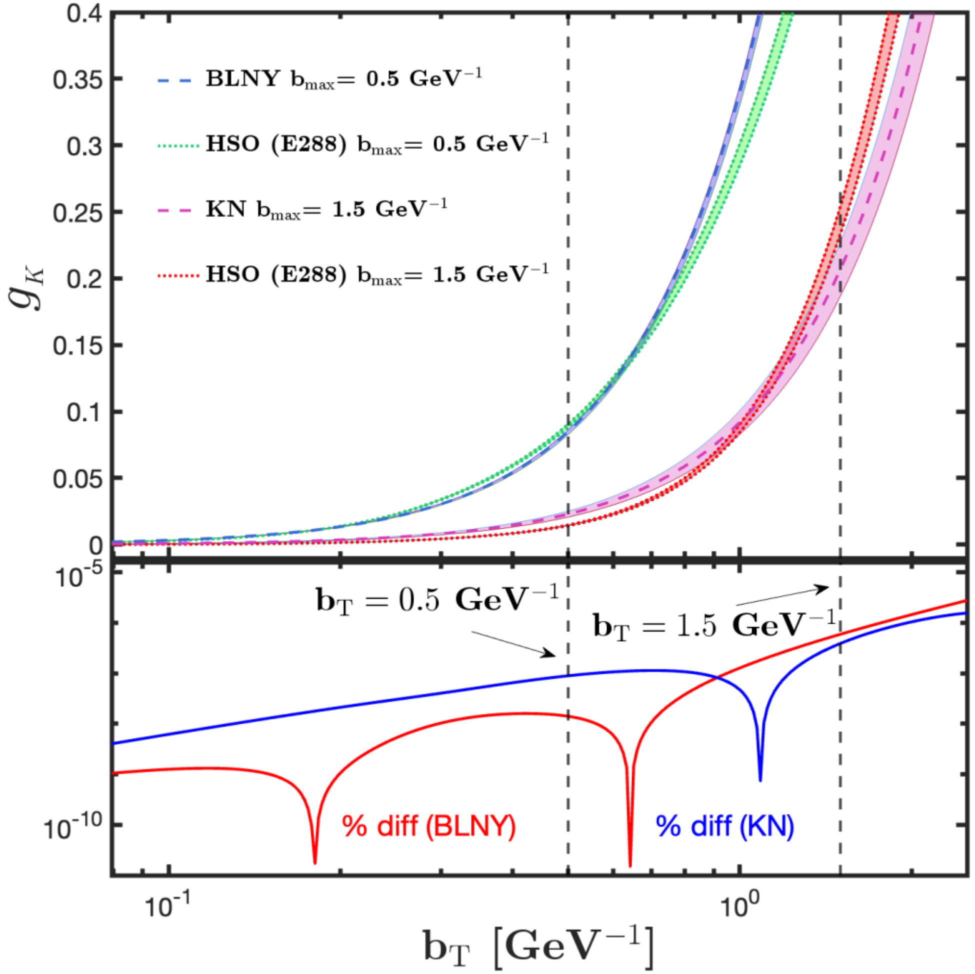
NEXT/SOON:

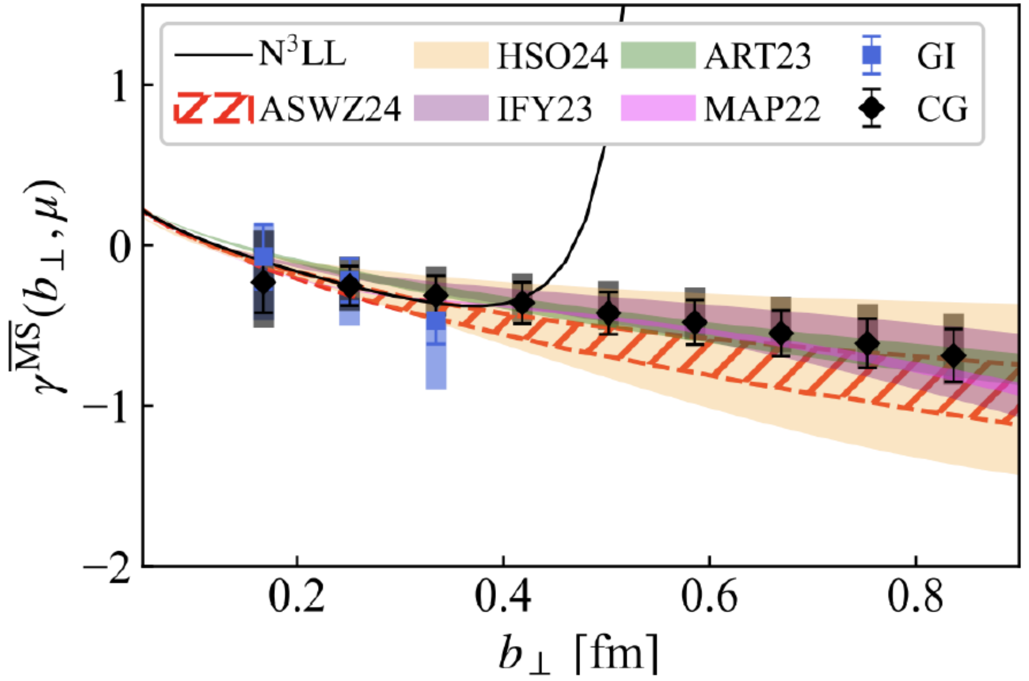
Sivers, SIDIS large q_T issue, more refined models, input from Lattice?, higher orders…

Thank you

Backup slides

The NP Collins-Soper kernel





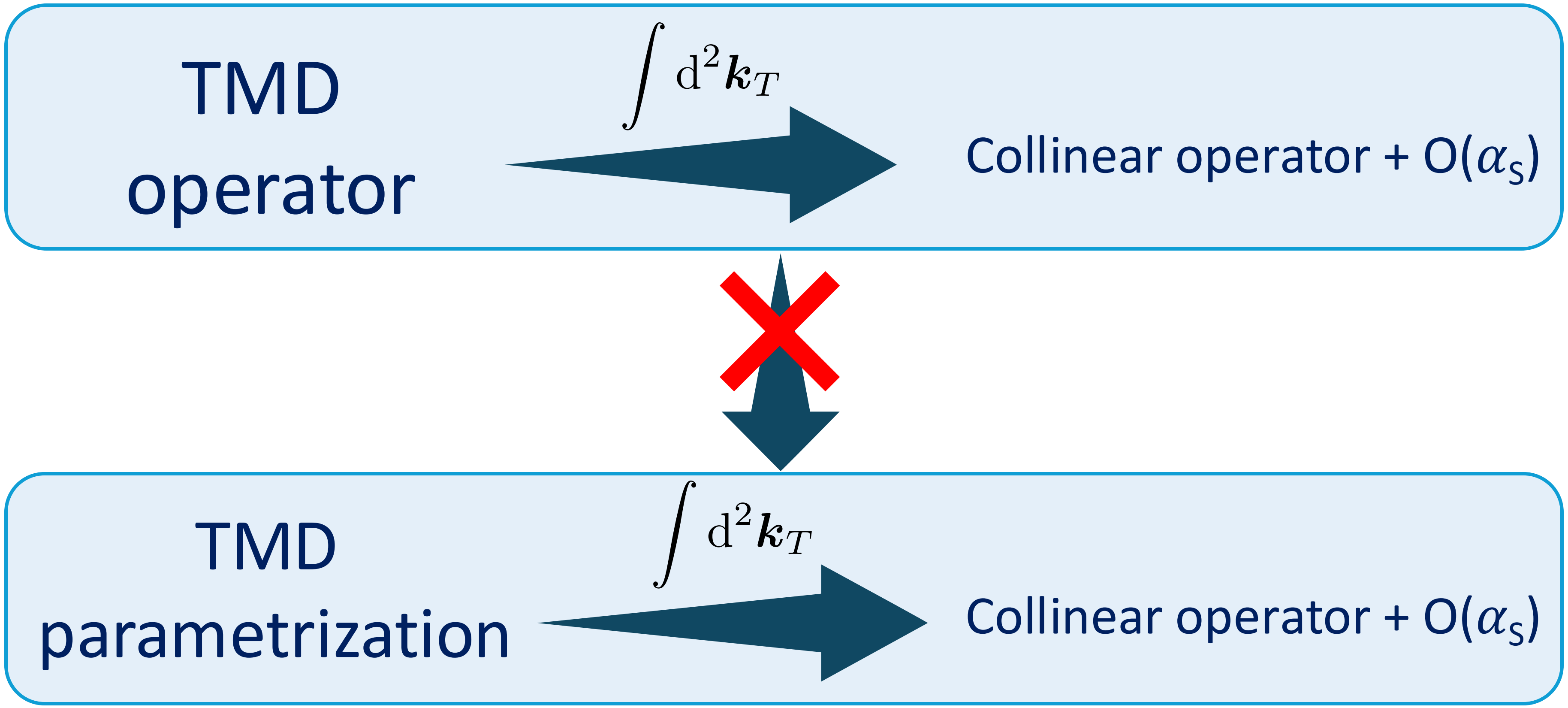
Lattice calculation from

Bollweg, Gao, Mukherjee, Zhao, (2024), 2403.00664 [hep-lat]

Why is this important?

- We can **quantitatively** and **conclusively** answer the question:

  How much collinear dependence do my TMD extractions carry?



Choose "core" models (examples)

$$f^{\text{Gauss}}_{\text{core},i/p}\left(x,\boldsymbol{k}_{\text{T}};Q_0^2\right) = \frac{e^{-k_{\text{T}}^2/M_F^2}}{\pi M_F^2}$$

Gaussian "core" models

Spectator-like "core" models

$$f^{\text{Spect}}_{\text{core},j/p}\left(x,\boldsymbol{k}_{\text{T}};Q_0^2\right) = \frac{6M_{0F}^6}{\pi\left(2M_F^2+M_{0F}^2\right)}\frac{M_F^2+k_{\text{T}}^2}{\left(M_{0F}^2+k_{\text{T}}^2\right)^4}$$