SRC studies with proton probes

Alexei Larionov

Joint Institute for Nuclear Research, Dubna, Moscow region, 141980 Russia

Based on:
AL, Yu.N. Uzikov, PRC 109, 064601 (2024) [arXiv:2311.06042]

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Outline:

- Introduction: short-range correlations (SRCs) in nuclear many-body theory, the role of the mean field and of the short-range interaction.

- Translationally-invariant shell model (TISM) and SRCs in p-shell nuclei. Initial- and final-state interactions (ISI/FSI). Charge exchange process (CEX).

- Comparison of model calculations with BM@N data for reactions $^{12}C(p,ppn_s)^{10}B$ and $^{12}C(p,ppp_s)^{10}Be$ in inverse kinematics: carbon beam at 48 GeV/c, proton target.

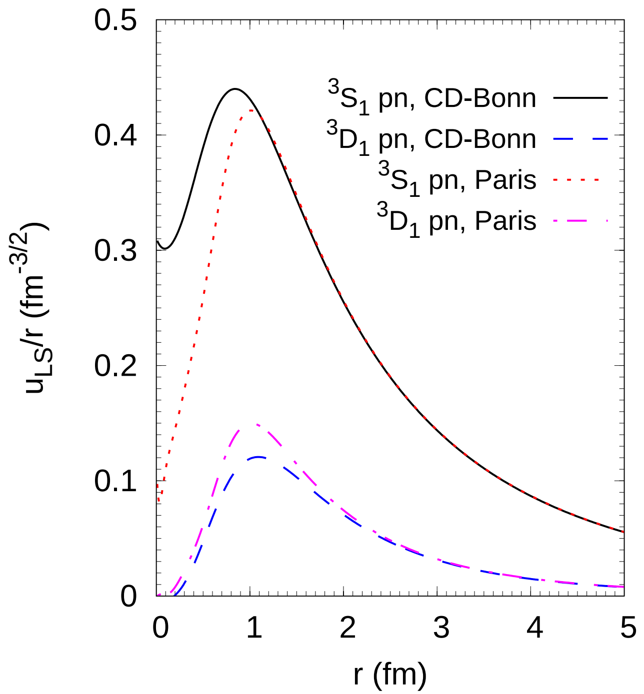
- Summary.

Introduction

- Solution of the Schroedinger equation with mean field potential yields the many-body wave function (Slater determinant), so that the relative NN distance distribution has a maximum near r=0

- ***But***: realistic NN potentials are (strongly) repulsive at r< 0.5 fm

- This is reflected in the dropping deuteron WFs at r< 1 fm (correlation hole). Maximum at r ≈1 fm is due to attractive tensor interaction in the S=1 channel.



Note: DWFs for AV14 and AV18 potentials are similar to Paris see ***R.B. Wiringa, V.G.J. Stocks, R. Schiavilla, PRC 51, 38 (1995)***

- Same short-distance behavior of the many-body WF in nuclei with A > 2 (supported by exact numerical solutions of many-body SE with realistic NN interactions), see ***C. Ciofi degli Atti, Phys. Rep. 590, 1 (2015)***

- For nuclei with A ≤ 12 the independent particle model (IPM) predicts the number of the NN pairs in different spin-isospin states ST with an accuracy of about 10-15% compared with calculation taking into account correlations, see
M. Piarulli, S. Pastore, R. B. Wiringa, S. Brusilow, R. Lim, PRC 107, 014314 (2023)

- Thus, amounts of NN SRCs and the WFs of their c.m. motion are governed by mean field. However, the internal WFs are influenced by short-range NN interaction.

- One can use a shell model to get amounts of NN pairs in different quantum states. The internal WF of the NN pair should be then replaced either by the DWF (for S=1, T=0) or by the zero energy solution of the two-body scattering problem (for S=0,T=1).

- The state of the outgoing nucleus may act as a “filter” for selecting the state of the valence NN pair in SRC reactions. See, e.g., the studies of ^{16}O(e,e’pp)^{14}C reaction at NIKHEF
C.J.G. Onderwater et al., PRL 78, 4893 (1997);
PRL 81, 2213 (1998); R. Starink et al., PLB 474, 33 (2000)
and MAMI
G. Rosner, Prog. Part. Nucl. Phys. 44, 99 (2000)

Translationally-invariant shell model (TISM)

- developed starting from 60's in Moscow State University

V. G. Neudatchin and Yu. F. Smirnov, Nuklonnye assotsiatsii v legkikh yadrakh (Nucleon Associations in Light Nuclei), Nauka, Moscow, 1969 [in Russian]

V.V. Balashov, A.N. Boyarkina, and I. Rotter, Nucl. Phys. 59, 417 (1964)

A. N. Boyarkina, Struktura yader 1p-obolochki (Structure of 1p-shell Nuclei), Moskovskij Gosudarstvennij Universitet (Moscow State University), 1973 [in Russian]

- based on the solution of many-body Schroedinger equation with harmonic oscillator (HO) potential

- precise separation of the spurious c.m. motion

- the obtained TISM set of the internal wave functions of the nucleus is used as a basis for the diagonalisation of the phenomenological Hamiltonian (***intermediate coupling model***)

single-particle Hamiltonian of the HO shell model

acts between 1p-state nucleons

$$\hat{H} = \sum_{i=1}^{A} \hat{H}_i + \sum_{i<j} \hat{V}_{ij} + a \sum_{i=1}^{A} \hat{\boldsymbol{l}}_i \hat{\boldsymbol{s}}_i \ , \quad a = -5 \text{ MeV}$$

$$\hat{V}_{12} = [W + M\hat{P}_x + B\hat{P}_\sigma + H\hat{P}_x\hat{P}_\sigma] V(r_{12}) \ , \quad V(r_{12}) = V_0 \exp[-(r_{12}/d)^2] \ ,$$

$$W = -0.13, \ M = 0.93, \ B = 0.46, \ H = -0.26 \qquad d = 1.6 \text{ fm}, \ V_0 = -56 \text{ MeV}.$$

"Rosenfeld variant" see D.R. Inglis, Rev. Mod. Phys. 25, 390 (1953)

- good description of the nuclear energy levels, magnetic dipole moments, probabilities for M1 gamma transitions and β-decay for nuclei with A=5-16.

The TISM internal WF of the nucleus A

$$
\begin{aligned}
&|AN[f](\lambda\mu)\alpha LSTM_LM_SM_T\rangle = \sum (L_BM_{L_B}\mathcal{L}M_{\mathcal{L}}|LM_L)\,(\Lambda M_\Lambda L_XM_{L_X}|\mathcal{L}M_{\mathcal{L}})\\
&\times (S_BM_{S_B}S_XM_{S_X}|SM_S)\,(T_BM_{T_B}T_XM_{T_X}|TM_T)\\
&\times\langle AN[f](\lambda\mu)\alpha LST|(A-b)N_B[f_B](\lambda_B\mu_B)\alpha_BL_BS_BT_B;n\Lambda,bN_X[f_X](\lambda_X\mu_X)\alpha_XL_XS_XT_X\{\mathcal{L}\}\rangle\\
&\times|(A-b)N_B[f_B](\lambda_B\mu_B)\alpha_BL_BS_BT_BM_{L_B}M_{S_B}M_{T_B}\rangle\\
&\times\psi_{n\Lambda}^{M_\Lambda}(\boldsymbol{R}_B-\boldsymbol{R}_X)\,|bN_X[f_X](\lambda_X\mu_X)\alpha_XL_XS_XT_XM_{L_X}M_{S_X}M_{T_X}\rangle\ ,
\end{aligned}
$$

Relative WF of B and X

Internal WF of the cluster X

Internal WF of the nucleus B

Fractional parentage coefficient (FPC)

N - number of oscillator quanta,

$[f]$ - Young scheme (permutation symmetry of the orbital part of WF),

$(\lambda\mu)$ - Elliott symbol (SU(3) symmetry due to interchange of the oscillator quanta),

α - additional (if any) quantum numbers needed for full characterization of the state

Oscillator sum rule: $N = N_B + n + N_X$

Spectroscopic amplitude of the virtual transition A → B + X:

$$
\begin{aligned}
S_A^{X,\mathrm{TISM}} &= \begin{pmatrix} A \\ b \end{pmatrix}^{1/2} \langle \Psi_B, \Psi_{n\Lambda}^{M_\Lambda}(\boldsymbol{R}_B-\boldsymbol{R}_X), \Psi_X|\Psi_A\rangle\\
&= \begin{pmatrix} A \\ b \end{pmatrix}^{1/2} \sum_{\mathcal{L}J_0M_0} \begin{Bmatrix} L_B & S_B & J_B \\ \mathcal{L} & S_X & J_0 \\ L & S & J \end{Bmatrix} \sqrt{(2L+1)(2S+1)(2J_B+1)(2J_0+1)}\\
&\quad\times\langle AN[f](\lambda\mu)\alpha LST|(A-b)N_B[f_B](\lambda_B\mu_B)\alpha_BL_BS_BT_B;n\Lambda,bN_X[f_X](\lambda_X\mu_X)\alpha_XL_XS_XT_X\{\mathcal{L}\}\rangle\\
&\quad\times U(\Lambda L_XJ_0S_X;\mathcal{L}J_X)(J_BM_BJ_0M_0|JM)\,(\Lambda M_\Lambda J_XM_X|J_0M_0)\,(T_BM_{T_B}T_XM_{T_X}|TM_T)\ .
\end{aligned}
$$

Y. F. Smirnov, Y. M. Tchuvil'sky, PRC 15, 84 (1977); Y. Uzikov, A. Uvarov, Phys. Part. Nucl. 53, 426 (2022)

The physical states of initial and final nuclei are superpositions of the TISM basis states:

$$\Psi_A^{J,T} = \sum_i \alpha^{A,JT}_{[f_i]L_iS_i} \, |A[f_i]L_iS_i(J)T\rangle \ , \quad \sum_i (\alpha^{A,JT}_{[f_i]L_iS_i})^2 = 1 \ ,$$

$$\Psi_B^{J_B,T_B} = \sum_j \alpha^{A-b,J_BT_B}_{[f_j]L_jS_j} \, |(A-b)[f_j]L_jS_j(J_B)T_B\rangle \ , \quad \sum_j (\alpha^{A-b,J_BT_B}_{[f_j]L_jS_j})^2 = 1 \ .$$

The spectroscopic amplitude for transition between physical states:

$$S_A^X(A_{\mathrm{phys}} \to B_{\mathrm{phys}} + X) = \sum_{i,j} \alpha^{A,J_AT_A}_{[f_i]L_iS_i} \, \alpha^{A-b,J_BT_B}_{[f_j]L_jS_j} \, S_A^{X,\mathrm{TISM}}(A_i \to B_j + X) \ .$$

The amplitudes ɑ are real-valued and taken from the book
A. N. Boyarkina, Struktura yader 1p-obolochki (Structure of 1p-shell Nuclei),
Moskovskij Gosudarstvennij Universitet (Moscow State University), 1973 [in Russian]

$^{12}C(p,ppn_s)^{10}B$ and $^{12}C(p,ppp_s)^{10}Be$ processes include spectroscopic amplitudes
$^{12}C \rightarrow {}^{10}B$ + <pn> and $^{12}C \rightarrow {}^{10}Be$ + <pp>

Table 1: Contributing $(1p)^8$ TISM states denoted as $[f]^{(2T+1)(2S+1)}L$ with corresponding coefficients $\alpha^{A,JT}_{[f]LS}$ for the ^{12}C ground state ($J = T = 0$).

$[44]^{11}S$	~~$[431]^{13}P$~~	~~$[422]^{11}S$~~	~~$[422]^{15}D$~~	~~$[332]^{13}P$~~
0.840	~~0.492~~	~~0.064~~	~~-0.200~~	~~0.086~~

Main component (replace 0.840 → 1)

Allowed transitions to the X(≡<NN>) state with minimum number of oscillator quanta N_X=0, $[f_X]$=[2], L_X=0 (***most compact cluster configuration***),
and the B state with minimum number of oscillator quanta N_B=6 $[f_B]$=[42]:

$^{(2T_B+1)\,(2S_B+1)}L_B$	$n\ \Lambda$	FPC
^{13}S	2 0	$-\sqrt{8/275}$
^{31}S	2 0	$\sqrt{8/275}$
$^{13}D_I$	2 2	$-\sqrt{3/550}$
$^{31}D_I$	2 2	$\sqrt{3/550}$
$^{13}D_{II}$	2 2	$-\sqrt{7/110}$
$^{31}D_{II}$	2 2	$\sqrt{7/110}$

Y. Uzikov, A. Uvarov, Phys. Part. Nucl. 53, 426 (2022)

Table 1: Experimental energy levels of ^{10}B with the partial amplitudes of the [42] TISM states.

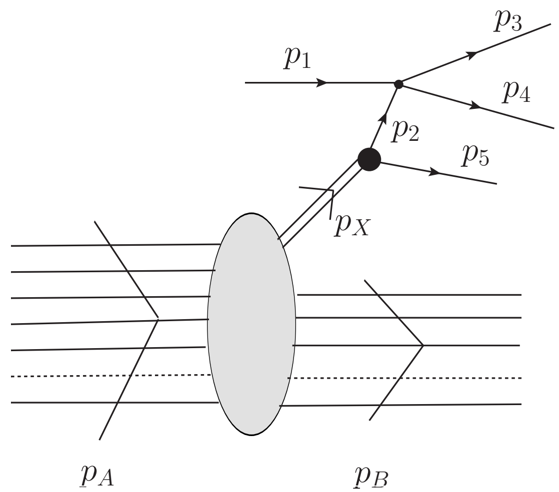
E^*, MeV	T J	TISM state	α
0	0 3	$^{13}D_I$	-0.418
		$^{13}D_{II}$	0.679
0.717	0 1	^{13}S	-0.351
		$^{13}D_I$	0.682
		$^{13}D_{II}$	0.541
2.15	0 1	^{13}S	0.885
		$^{13}D_I$	0.307
		$^{13}D_{II}$	0.224
3.58	0 2	$^{13}D_I$	0.401
		$^{13}D_{II}$	0.778
1.74	1 0	^{31}S	0.772
5.17	1 2	$^{31}D_I$	0.728
		$^{31}D_{II}$	0.209

Table 2: Same as in Table 1 but for ^{10}Be.

E^*, MeV	T J	TISM state	α
0	1 0	^{31}S	0.772
3.368	1 2	$^{31}D_I$	0.728
		$^{31}D_{II}$	0.209
5.96	1 2	$^{31}D_I$	-0.226
		$^{31}D_{II}$	0.892

A. N. Boyarkina, Struktura yader 1p-obolochki (The Structure of 1p-shell Nuclei), MSU, Moscow, 1973 [in Russian]

A(p,ppN)B process amplitude



p_i – four-momenta,
1 – incoming proton,
A – initial nucleus (^{12}C),
B – final nucleus (^{10}B or ^{10}Be),
X - <NN>-pair,
2 – struck proton,
3,4 – outgoing fast protons (in the r.f. of A),
5 - outgoing slow nucleon (in the r.f. of A)

$$M^{\mathrm{IA}} = M_{\mathrm{hard}}(p_3,p_4,p_1)\frac{i\Gamma_{X\to pN}(p_X,p_5)}{p_2^2-m^2+i\epsilon}\frac{i\Gamma_{A\to XB}(p_A,p_B)}{p_X^2-m_X^2+i\epsilon}\ ,$$

in the r.f. of A

$$\frac{i\Gamma_{A\to XB}(p_A,p_B)}{p_X^2-m_X^2+i\epsilon} = S_A^X\left(\frac{2E_B m_A}{p_X^0}\right)^{1/2}(2\pi)^{3/2}\psi_{n\Lambda}^{M_\Lambda}(-\boldsymbol{p}_X) \quad \text{for } p_B^2=m_B^2\ ,$$

$$\frac{i\Gamma_{X\to pN}(p_X,p_5)}{p_2^2-m^2+i\epsilon} = \left(\frac{2E_5 m_X}{p_2^0}\right)^{1/2}(2\pi)^{3/2}\sqrt{2}\psi_X(\boldsymbol{p}_2) \quad \text{for } p_5^2=m^2\ .$$

in the r.f. of X

from antisymmetrized
plane-wave-WF product
of decay nucleons 2 and 5

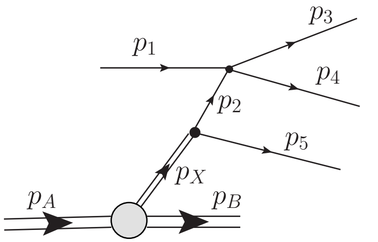
For derivation see
AL et al., PRC 98, 054611 (2018),
[arXiv:1807.05105]

Normalization of WFs:

$$\int d^3p|\psi_{n\Lambda}^{M_\Lambda}(\boldsymbol{p})|^2 = 1\ ,$$

$$\int d^3p|\psi_X(\boldsymbol{p})|^2 = 1\ .$$

S_A^X - spectroscopic amplitude of a given X-B state



Eikonal approximation:

$$\exp[i\boldsymbol{p}\boldsymbol{r}] \to \exp[i\boldsymbol{p}\boldsymbol{r} - \frac{i}{v}\int\limits_{-\infty}^{0} d\eta\, U(\boldsymbol{r}+\hat{\boldsymbol{p}}\eta)] \simeq \exp[i\boldsymbol{p}\boldsymbol{r} - \frac{1}{2}\sigma_{NN}\int\limits_{-\infty}^{0} d\eta\, \rho(\boldsymbol{r}+\hat{\boldsymbol{p}}\eta)]$$

$$U(r) \simeq i\,\mathrm{Im}U(r) \simeq -\frac{i}{2}\langle v\sigma_{NN}\rangle\rho$$ - optical potential

$$(2\pi)^{3/2}\psi_{n\Lambda}^{M_\Lambda}(-\boldsymbol{p}_X) = \int d^3r \mathrm{e}^{-i\boldsymbol{p}_X\boldsymbol{r}}\psi_{n\Lambda}^{M_\Lambda}(-\boldsymbol{r}) = \int d^3r \mathrm{e}^{i(\boldsymbol{p}_1-\boldsymbol{p}_3-\boldsymbol{p}_4-\boldsymbol{p}_5)\boldsymbol{r}}\psi_{n\Lambda}^{M_\Lambda}(-\boldsymbol{r})$$
$$\to \int d^3r \mathrm{e}^{-i\boldsymbol{p}_X\boldsymbol{r}}\psi_{n\Lambda}^{M_\Lambda}(-\boldsymbol{r})F_{\mathrm{abs}}(\boldsymbol{r})\ , \quad \boldsymbol{r} = \boldsymbol{R}_X - \boldsymbol{R}_B\ .$$

$$F_{\mathrm{abs}}(\boldsymbol{r}) \equiv F_1(\boldsymbol{r})F_3(\boldsymbol{r})F_4(\boldsymbol{r})F_5(\boldsymbol{r})$$ - absorption factor,

$$F_i(\boldsymbol{r}) = \exp\left(-\frac{1}{2}\sigma_{NN}(p_i)T_i(\boldsymbol{r})\right)\ ,$$

$$T_i(\boldsymbol{r}) = \begin{cases} \int\limits_{-\infty}^{0} d\eta\,\rho(\boldsymbol{r}+\hat{\boldsymbol{p}}_i\eta) \quad \text{for} \quad i=1 \\ \int\limits_{0}^{+\infty} d\eta\,\rho(\boldsymbol{r}+\hat{\boldsymbol{p}}_i\eta) \quad \text{for} \quad i=3,4,5\ . \end{cases}$$

- thickness functions of the nucleus B

$$M_{\rm tot} = \sum_{\lambda_2} M_{\rm hard}(p_3,p_4,p_1)\left[\left(\frac{2E_5 m_X}{p_2^0}\right)^{1/2}(2\pi)^{3/2}\sqrt{2}\sum_{M_X}\psi_X(\boldsymbol{p}_2)\right]_{\rm r.f.of\,X}$$
$$\times\sum_i \alpha_i \sum_{M_\Lambda} S^X_{A,i}\left(\frac{2E_B m_A}{p_X^0}\right)^{1/2}\int d^3r {\rm e}^{-i\boldsymbol{p}_X\boldsymbol{r}}\psi^{M_\Lambda}_{n_i\Lambda_i}(-\boldsymbol{r})F_{\rm abs}(\boldsymbol{r}) , \quad \boldsymbol{r}=\boldsymbol{R}_X-\boldsymbol{R}_B ,$$

sum over TISM states of the outgoing nucleus B,

$$F_{\rm abs}(\boldsymbol{r}) \equiv F_1(\boldsymbol{r})F_3(\boldsymbol{r})F_4(\boldsymbol{r})F_5(\boldsymbol{r}), \quad F_j(\boldsymbol{r}) = \exp\left(-\frac{1}{2}\sigma_{NN}(p_j)T_j(\boldsymbol{r})\right)$$ - absorption factors.

$$\overline{|M_{\rm tot}|^2} = \frac{1}{2}\sum_{\lambda_1,\lambda_3,\lambda_4,\lambda_5,M_B}|M_{\rm tot}|^2$$
$$\simeq (2J_B+1)\overline{|M_{\rm hard}(p_3,p_4,p_1)|^2}\left(\frac{2E_5 m_X}{p_2^0}\right)(2\pi)^3 2\overline{|\psi_X(\boldsymbol{p}_2)|^2}$$
$$\times\sum_{i,j}\alpha_i\alpha_j\delta_{\Lambda_i,\Lambda_j}S^{X0}_{A,i}S^{X0}_{A,j}\left(\frac{2E_B m_A}{p_X^0}\right)\int d^3R\, f_{n_i\Lambda_i}(-\boldsymbol{R},-\boldsymbol{p}_X)F^2_{\rm abs}(\boldsymbol{R}) ,$$

neglecting interference
between amplitudes
with different M_x
and spin correlations
between M_{hard} and ψ_x

$$S^{X0}_{A,i} \equiv \left(\begin{array}{c} A \\ 2 \end{array}\right)^{1/2}\frac{FPC_i}{\sqrt{(2L_B+1)(2S_B+1)(2T_B+1)}}$$

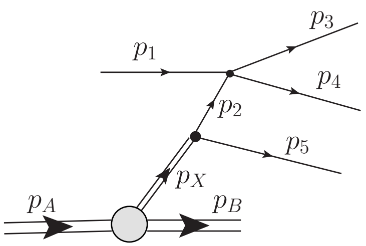
- reduced spectroscopic amplitude.

$$\overline{|\psi_X(\boldsymbol{p}_2)|^2} \equiv \frac{1}{2J_X+1}\sum_{M_X,\lambda_2,\lambda_5}|\psi_X(\boldsymbol{p}_2)|^2 ,$$

$$\overline{|M_{\rm hard}(p_3,p_4,p_1)|^2} \equiv \frac{1}{4}\sum_{\lambda_1,\lambda_2,\lambda_3,\lambda_4}|M_{\rm hard}(p_3,p_4,p_1)|^2 ,$$

$$f_{n_i\Lambda_i}(-\boldsymbol{R},-\boldsymbol{p}_X) \equiv \int d^3\xi {\rm e}^{-i\boldsymbol{p}_X\boldsymbol{\xi}}\frac{1}{2\Lambda_i+1}\sum_{M_\Lambda}\psi^{M_\Lambda}_{n_i\Lambda_i}(-\boldsymbol{R}-\boldsymbol{\xi}/2)\psi^{M_\Lambda\,*}_{n_i\Lambda_i}(-\boldsymbol{R}+\boldsymbol{\xi}/2)$$

- Wigner function, i.e. the distribution function in the phase space $(\boldsymbol{R}=\boldsymbol{R}_X-\boldsymbol{R}_B, \boldsymbol{p}_X=-\boldsymbol{p}_B)$



CEX processes (most important ones are from collision with pn SRC):

1) Slow spectator neutron (5) may be converted to proton.

2) Hard scattering may occur on the neutron (2), and then the fast knock-out neutron (3 or 4 with probability ½) may be converted to proton. Assume isospin-independent $\overline{|M_{\mathrm{hard}}|^2}$.

$$P_{n_5 \to p_5} = F^2_{\mathrm{abs}}(\boldsymbol{R})\sigma_{\mathrm{CEX}}(p_5)T_5(\boldsymbol{R})/2 \ ,$$
$$P_{n_4 \to p_4} = F^2_{\mathrm{abs}}(\boldsymbol{R})\sigma_{\mathrm{CEX}}(p_4)\underbrace{T_4(\boldsymbol{R})/2}_{\text{proton thickness function}} \ .$$

Total CEX probability = $P_{n_5 \to p_5} + P_{n_4 \to p_4} = F^2_{\mathrm{abs}}(\boldsymbol{R})[\sigma_{\mathrm{CEX}}(p_5)T_5(\boldsymbol{R}) + \sigma_{\mathrm{CEX}}(p_4)T_4(\boldsymbol{R})]/2$.

Note: similar expressions for CEX probabilities are used in GCF calculations
c.f. ***M. Duer et al., PRL 122, 172502 (2019)***

Wave functions

S=1, T=0 pn-pairs :

$$\overline{|\psi_{X,T=0}(\boldsymbol{p}_2)|^2} = \frac{1}{2}\,\frac{u^2(p_2)+w^2(p_2)}{4\pi}\ ,$$

$$\int dp\, p^2[u^2(p)+w^2(p)] = 1\ .$$

S- and D-wave components of the deuteron WF of CD-Bonn model
R. Machleidt, PRC 63, 024001 (2001)

S=0, T=1 pp (T_z=1) and pn (T_z=0) pairs :

$$\overline{|\psi_{X,T=1}(\boldsymbol{p}_2)|^2} = \frac{1}{2}(1+T_z)|\psi_s(p_2)|^2\ ,$$

$$\psi_s(p) = \kappa\frac{f(p,0;0)}{\alpha^2+p^2}\ , \qquad 4\pi\int dp\, p^2|\psi_s(p)|^2 = 1\ .$$

- zero-energy solution of NN scattering problem in the $^1S_0$ channel
Y. Uzikov, A. Uvarov, Phys. Part. Nucl. 53, 426 (2022)

$\alpha = 0.104\ \mathrm{fm}^{-1}$ - range parameter of the ``bound state WF" $(\propto \exp(-\alpha r)/r$ at large $r)$

corresponding to a virtual level with “binding energy” $\epsilon = \alpha^2/m = 0.45\ \mathrm{MeV}$

G. Faeldt, C. Wilkin, PLB 382, 209 (1996)

$f(p,k;k)$ - half-of-shell NN scattering amplitude in the $^1S_0$ channel

V. Lensky et al., EPJA 26, 107 (2005)

Squared WFs of NN-B relative motion:

$$|\psi_{20}(R)|^2 = \frac{3}{2R_0^3\pi^{3/2}}\left[1-\frac{2}{3}\left(\frac{R}{R_0}\right)^2\right]^2 \exp[-(R/R_0)^2] \ ,$$

$$\overline{|\psi_{22}(R)|^2} = \frac{4}{15R_0^3\pi^{3/2}}\left(\frac{R}{R_0}\right)^4 \exp[-(R/R_0)^2] \ ,$$

$$4\pi\int dR\, R^2\, \overline{|\psi_{n\Lambda}(R)|^2} = 1 \ .$$

$R_0 = r_0\sqrt{A/2(A-2)}$ - HO parameter of the NN-B relative motion,

$r_0 = 1.736 \ \mathrm{fm}$ - HO parameter of the standard shell model fit of momentum distributions of p-shell and s-shell nucleons for $^{12}C(e,e'p)^{11}B$ reaction

Y. Uzikov, Phys. Part. Nucl. 52, 652 (2021)

For the WFs in momentum space: $R \to p_X, \ R_0 \to 1/R_0.$

Simplified calculation:
(for comparison
with other models)

$$\overline{|\psi_{00}(R)|^2} = \frac{1}{R_0^3\pi^{3/2}}\exp[-(R/R_0)^2] \ , \quad R_0 = \sqrt{\alpha_{cm}}$$

$\alpha_{cm} = 1 \ \mathrm{fm}^2$ ***C. Ciofi degli Atti, S. Simula, PRC 53, 1689 (1996)***

$\sigma_{cm} = 1/\sqrt{2\alpha_{cm}} = 139.5 \ \mathrm{MeV/c}$

- consistent with $\sigma_{cm} = (156 \pm 27) \ \mathrm{MeV/c}$ from analysis of BM@N data

M. Patsyuk et al., Nature Phys. 17, 693 (2021)

- Exp. data for pp elastic differential cross section $d\sigma/d\Omega_{\mathrm{c.m.}}$ at $\Theta_{c.m.} = 30^\circ - 90^\circ$ for p_{lab}=4 GeV/c, ***R.C. Kammerud et al., PRD 4, 1309 (1971)***

$$\overline{|M_{\mathrm{hard}}(p_3,p_4,p_1)|^2} = 64\pi^2 s\, d\sigma/d\Omega_{\mathrm{c.m.}}\ ,$$

$$\Theta_{\mathrm{c.m.}} = \arccos[1+\max(t,u)/2(s/4-m^2)],\ \ s=(p_3+p_4)^2,\ t=(p_1-p_3)^2,\ u=(p_1-p_4)^2.$$

- Total pp and pn cross sections: parameterizations from ***J. Cugnon et al., NIM B 111, 215 (1996)***

- Resulting cross sections are weighted with proton and neutron numbers of the outgoing nucleus:

$$\begin{aligned} \sigma_{pN} &= [\sigma_{pp}Z + \sigma_{pn}(A-Z)]/A\ , \\ \sigma_{nN} &= [\sigma_{pn}Z + \sigma_{pp}(A-Z)]/A\ . \end{aligned}$$

- CEX cross section: integrated np → np differential cross section at large $\Theta_{\mathrm{c.m.}}$ (typically at $\Theta_{\mathrm{c.m}} > 90^o$)

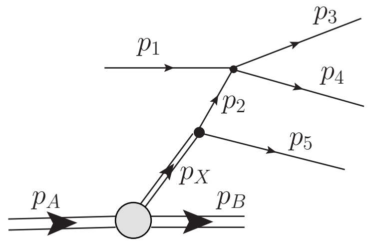
$$\sigma_{\mathrm{CEX}}(E) = \sigma_{\mathrm{CEX}}(800)\frac{s(800)}{s(E)}\ ,\quad s(E) = 2m(E+2m)$$

- scaling relation for 0.1 GeV/c < p_{lab} < 100 GeV/c
W.R. Gibbs, B. Loiseau, PRC 50, 2742 (1994)

$$\sigma_{\mathrm{CEX}}(800) = 4.25\ \mathrm{mb\ for}\ \Theta_{\mathrm{c.m.}} = 135^\circ - 180^\circ$$

- exp. data at E=800 MeV
M. Jain et al., PRC 30, 566 (1984)

Observables



$$d\sigma_{1A\to 345B} = \frac{(2\pi)^4 \overline{|M_{\rm tot}|^2}}{4I} d\Phi_4 \ ,$$

$$I = [(p_1 p_A)^2 - m^2 m_A^2]^{1/2} = p_{\rm beam} m$$ - Moeller flux factor,

in the rest frame of proton 1 (lab. frame)

$$d\Phi_4 = \delta^{(4)}(p_1 + p_A - p_3 - p_4 - p_5 - p_B) \frac{d^3 p_3}{(2\pi)^3 2E_3} \frac{d^3 p_4}{(2\pi)^3 2E_4} \frac{d^3 p_5}{(2\pi)^3 2E_5} \frac{d^3 p_B}{(2\pi)^3 2E_B}$$

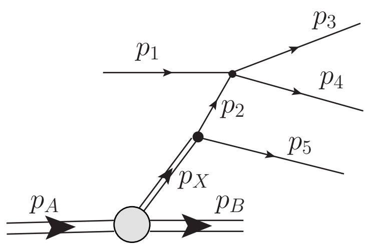
- inv. 4-body phase space,

$$\sigma_{1A\to 345B} = \frac{1}{64(2\pi)^8 p_{\rm beam} m} \int d\Omega_3 \underbrace{\int d^3 p_2 \int \frac{d^3 p_B}{E_5 E_B}}_{\text{in the r.f. of A}} \frac{\overline{|M_{\rm tot}|^2} p_3}{|E_3 + E_4 - E_3 \mathcal{P} \chi / p_3|} \ ,$$

$$\boldsymbol{\mathcal{P}} \equiv \boldsymbol{p}_3 + \boldsymbol{p}_4$$

$$\chi \equiv \boldsymbol{\mathcal{P}} \boldsymbol{p}_3 / \mathcal{P} p_3$$

BM@N acceptance cuts



Velocities of fast protons in the lab. frame: $0.8 < \beta_{3,4} < 0.96$.

Polar angles of fast protons in the lab. frame: $24° < \Theta_{3,4} < 37°$.

Azimuthal angles of fast protons in the lab. frame: $-14° < \phi_3 < 14°$, $-180° < \phi_4 < -166°$ and $166° < \phi_4 < 180°$.

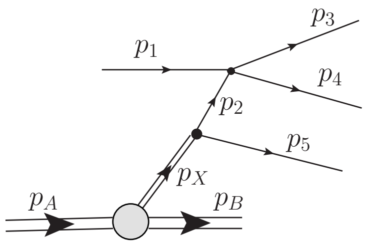
In-plane opening angle: $\Theta_3 + \Theta_4 > 63°$.

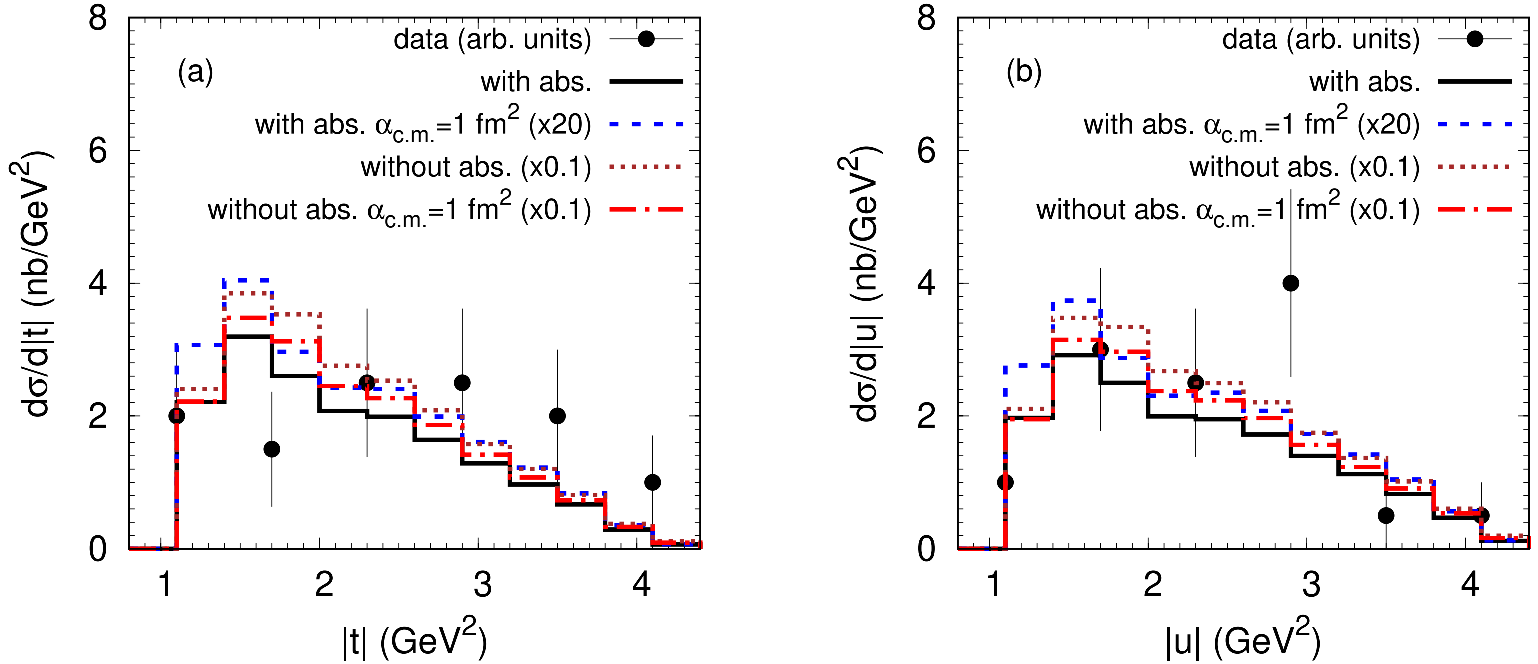
Missing momentum in the r.f. of ^{12}C: 0.350 GeV/c $< p_2 <$ 1.2 GeV/c.

Missing energy $E_{\rm miss} \equiv m - p_2^0$ in the r.f. of ^{12}C: -0.110 GeV $< E_{\rm miss} <$ 0.240 GeV.

M. Patsyuk et al., Nature Phys. 17, 693 (2021)

$^{12}C(p,ppn_s)^{10}B$



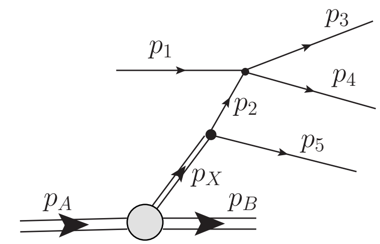


$$t = (p_1 - p_3)^2,\ u = (p_1 - p_4)^2$$

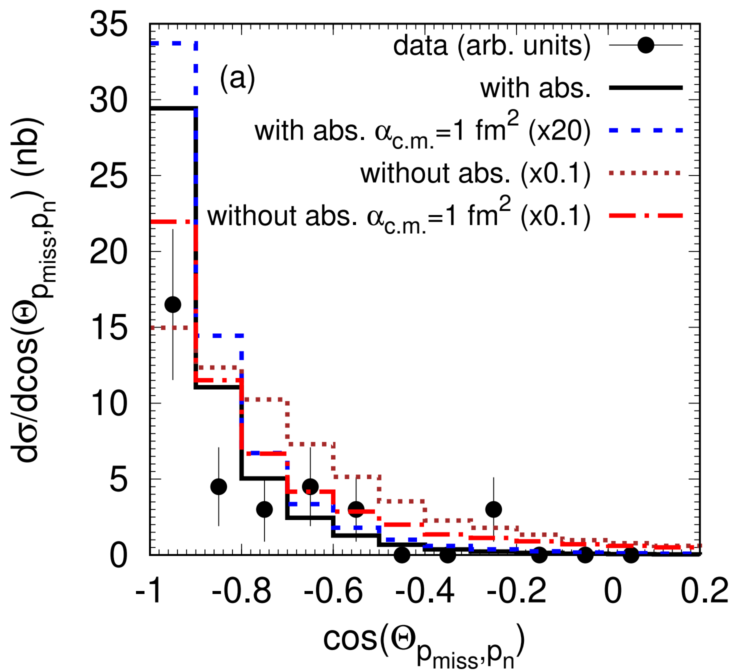
- ***distributions governed by hard pp→ pp scattering, no dependence on WFs and absorption***
- ***theory predicts peaks at |t|=|u|≈1.5 GeV^2***
- ***no peaks in the data (might be due to too large bins and low statistics)***

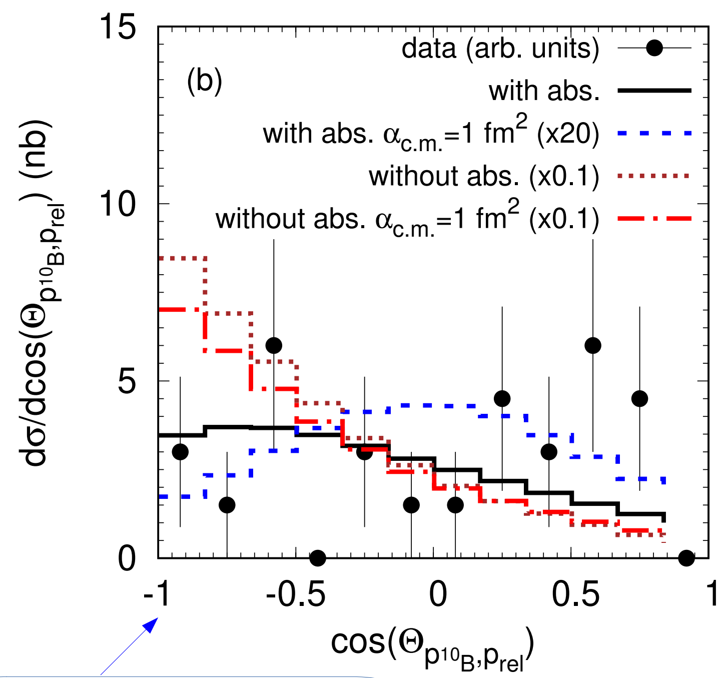
Data: ***M. Patsyuk et al., Nature Phys. 17, 693 (2021)***

$^{12}C(p,ppn_s)^{10}B$



$\boldsymbol{p}_{\rm miss} \equiv \boldsymbol{p}_2$
$\boldsymbol{p}_{\rm n} \equiv \boldsymbol{p}_5$





$\boldsymbol{p}_{\rm rel} \equiv (\boldsymbol{p}_{\rm miss} - \boldsymbol{p}_n)/2$

small

$$\boldsymbol{p}_{\rm miss} + \boldsymbol{p}_n + \boldsymbol{p}_{^{10}B} = 0$$

large, $|\boldsymbol{p}_{\rm miss}| > 0.35$ GeV/c

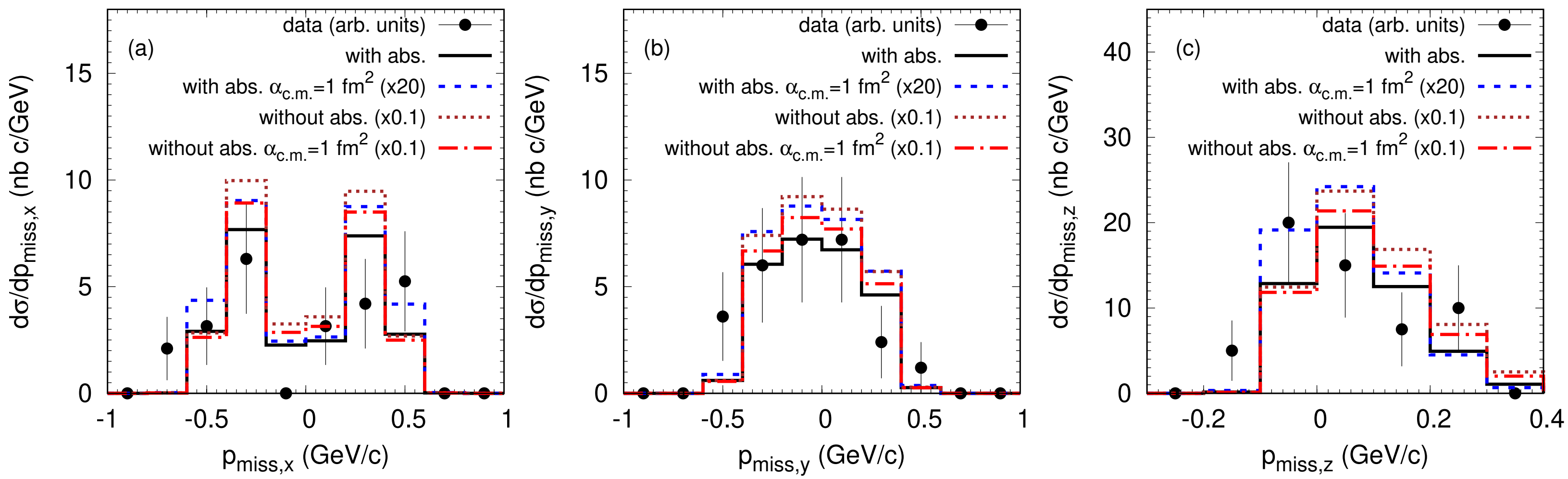
large

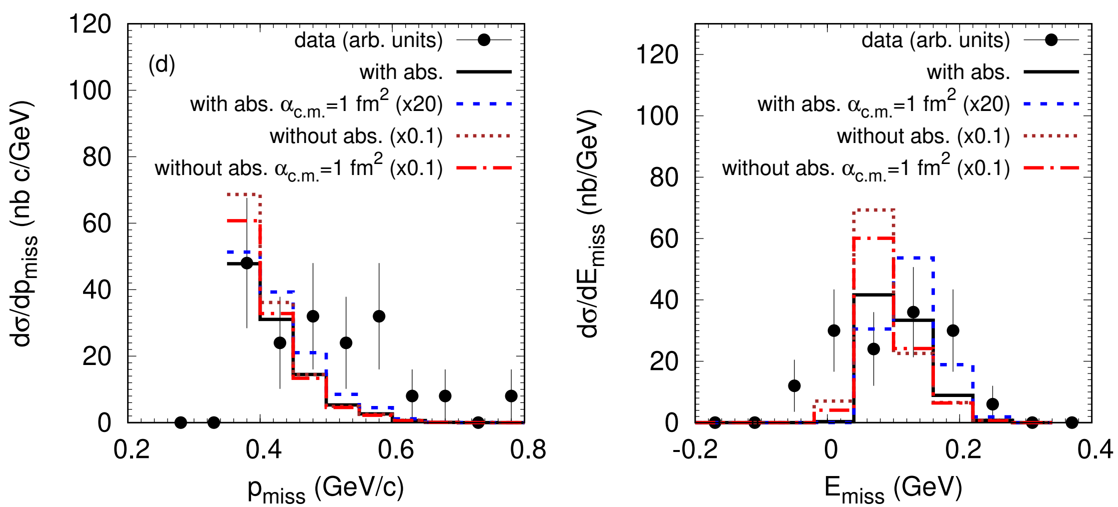
- Phenomenological WF gives a sharper back-to-back correlation due to narrower relative NN−B momentum distribution.

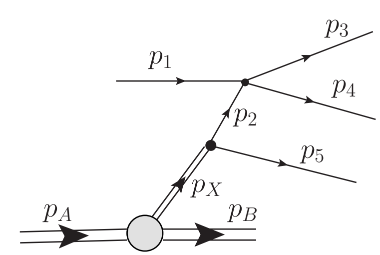
- Strong absorption of slow neutrons flattens-out angle distribution between nuclear fragment and relative NN momentum

Data: ***M. Patsyuk et al., Nature Phys. 17, 693 (2021)***

$^{12}C(p,ppn_s)^{10}B$







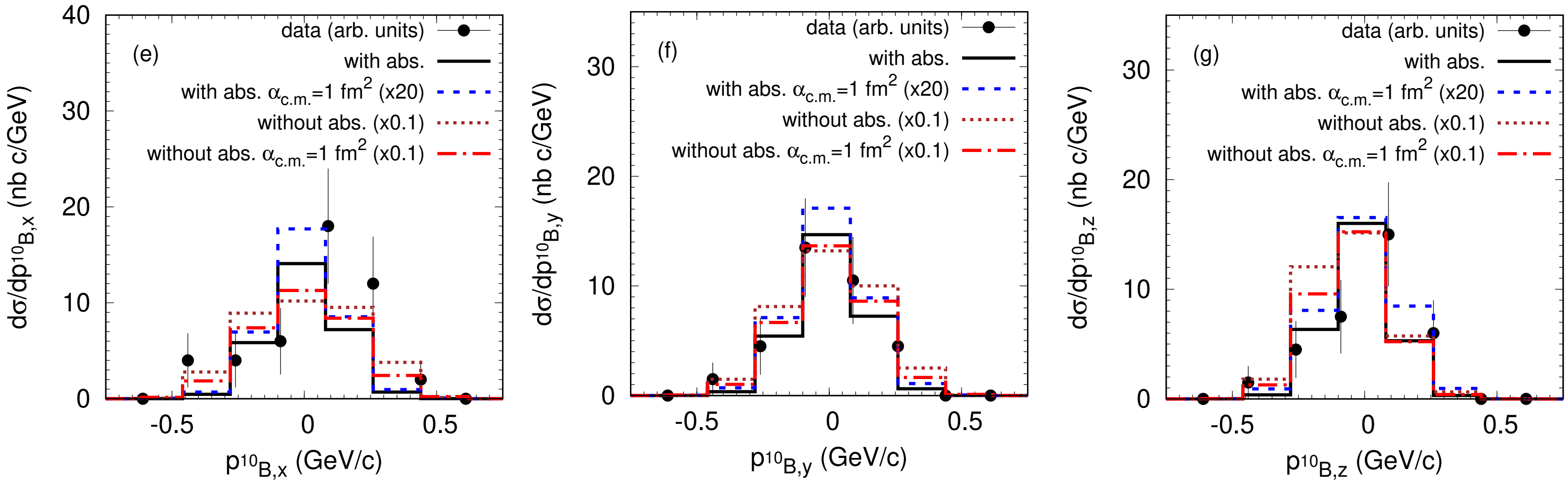
$$\left.\begin{array}{l}\boldsymbol{p}_{\text{miss}} \equiv \boldsymbol{p}_2 \\ E_{\text{miss}} \equiv m - p_2^0\end{array}\right\} \quad \text{in the r.f. of } {}^{12}\text{C}$$

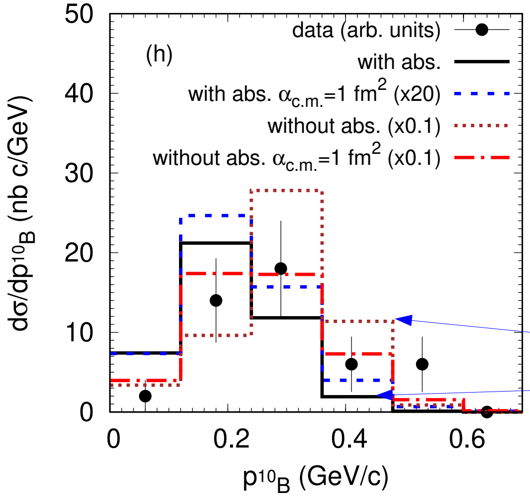
$$z \upuparrows \boldsymbol{p}_1$$

- ***Two-hump shape of $p_{miss,x}$-distribution due to two-arm spectrometer in the y=0 plane.***
- ***Far off-mass-shell struck proton (p_{miss} > 0.35 GeV/c, E_{miss} > 0)***
- ***Reasonable agreement with data. Quite weak sensitivity to absorption and WF of relative NN-B motion.***

Data: ***M. Patsyuk et al., Nature Phys. 17, 693 (2021)***

$^{12}C(p,ppn_s)^{10}B$



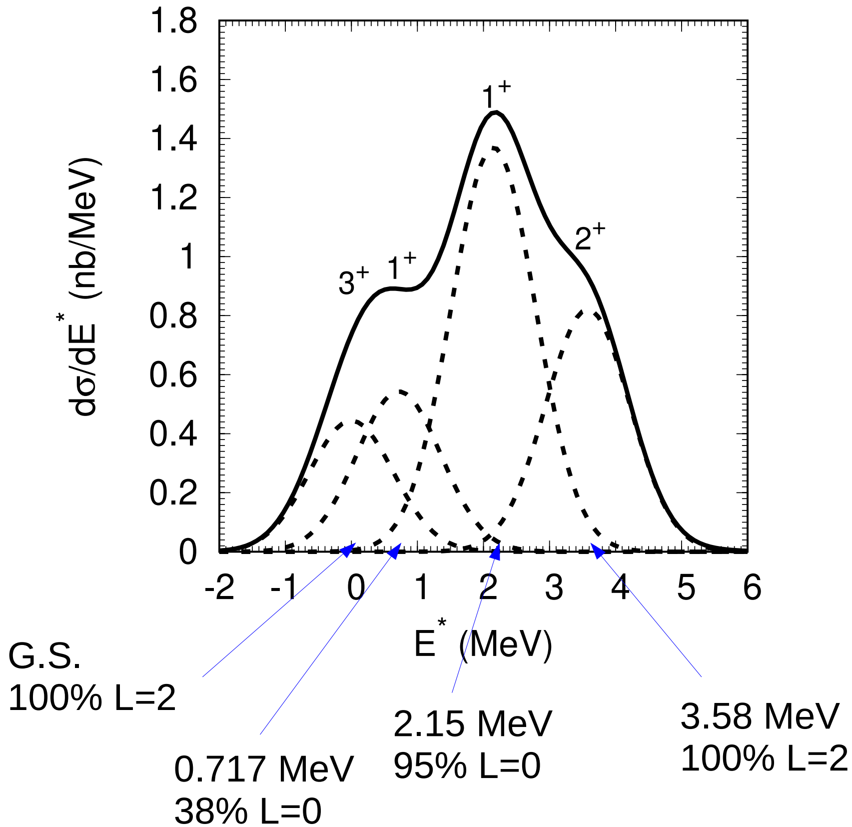


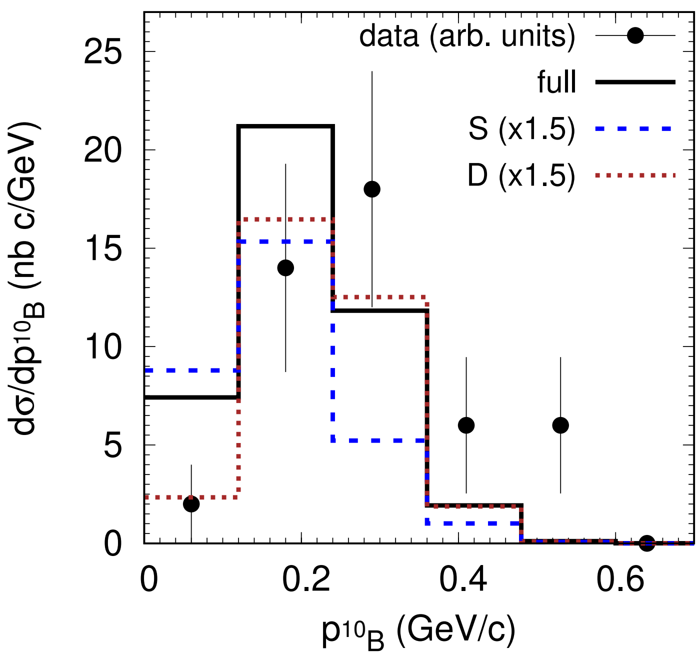
- ***Witrhout absorption: slightly broader x,y,z distributions for TISM Wfs.***
- ***With absorption: shapes are almost the same.***

Stronger absorption effect for large momenta of ^{10}B due to slow neutron

Data: ***M. Patsyuk et al., Nature Phys. 17, 693 (2021)***

$^{12}C(p,ppn_s)^{10}B$





- Data seem to indicate enhanced contribution of D-state ^{10}B production.

- Selecting certain windows in E* it is possible to restrict the orbital angular momentum of ^{10}B and, thus, the partial waves in the relative NN-B WF.

Data: ***M. Patsyuk et al., Nature Phys. 17, 693 (2021)***

Table 1: Integrated cross sections (in nb) in the kinematics of the BM@N experiment . Lowest row gives the ratio $R = \sigma[^{12}\mathrm{C}(p, 2pp_s)^{10}\mathrm{Be}]/\sigma[^{12}\mathrm{C}(p, 2pn_s)^{10}\mathrm{B}]$ (in %). Results obtained with phenomenological relative NN-B WFs are given in parentheses.

	IA	Abs	Abs+CEX
$^{12}\mathrm{C}(p, 2pn_s)^{10}\mathrm{B}$	63.7 (57.1)	5.1 (0.31)	5.0 (0.29)
$^{12}\mathrm{C}(p, 2pp_s)^{10}\mathrm{Be}$	4.2 (3.3)	0.13 (0.0058)	0.23 (0.030)
R	6.6 (5.8)	2.5 (1.8)	4.6 (10.4)

BM@N data: $R = 2/23 = (8.7 \pm 6)\%$

-Quite weak effect of WFs in the IA.

- Absorption reduces cross sections by an order of magnitude for TISM WFs and by 2 orders for phenomenological WF and also reduces R.

- CEX processes increase R. Especially strong effect and better agreement with data for phenomenological WF.

Summary

- TISM is applied to the hard proton knock-out reactions $^{12}C(p,ppn_s)^{10}B$ and $^{12}C(p,ppp_s)^{10}Be$ with an outgoing nucleus in the ground state or particle-stable excited states (up to E*~6 MeV). The ISI/FSI of absorptive- and CEX-type are included.

- TISM allows to calculate the spectroscopic amplitude of a NN-correlation and the WF of relative NN-B motion.

- The BM@N data (so far the shapes of distributions only) are described by TISM reasonably well. Strong effect of absorption on the absolute cross sections (reduction by 1-2 orders of magnitude), but only moderate effect on the shapes of distributions. CEX processes enhance the cross section ratio $^{12}C(p,ppp_s)^{10}Be/^{12}C(p,ppn_s)^{10}B$ by about 40% (for phenomenological relative NN-B WF - by 5 times).

Further steps

- Including other g.s. configurations of ^{12}C apart from [44] ^{11}S ([431] ^{13}P etc.).

- CEX FSI needs to be improved: so far suitable for inclusive yield of ^{10}Be (not for discrete states).

- Model application for (p,pd) reactions, cumulative processes ...

- More precise data on SRC in light nuclei needed to further test TISM (specific states of the outgoing nucleus, bigger statistics). Absolute cross sections are desirable from experimental side (not only distributions).

Thank you for your attention !

Backup

Total pN cross section $\sigma^{tot}_{pN} = [\sigma^{tot}_{pp} Z_B + \sigma^{tot}_{pn}(A_B - Z_B)]/A_B$,

- two options:

1) "***Vacuum***":

pp and pn cross section parameterizations from
J. Cugnon et al., NIM B 111, 215 (1996)

2) "***In-medium***":

at p < 1 GeV/c

$$\sigma_{NN}(E) = \frac{2m}{p}\int d^3r W(r,E)/A = \frac{2m}{p} J_W/A \ ,$$

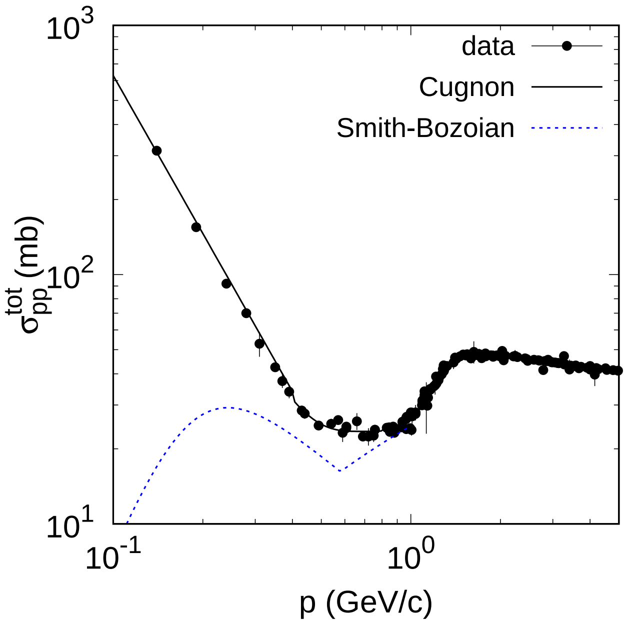
$$W(r,E) \equiv -\mathrm{Im}U(r,E) \ , \quad E = \sqrt{p^2+m^2} - m \ ,$$

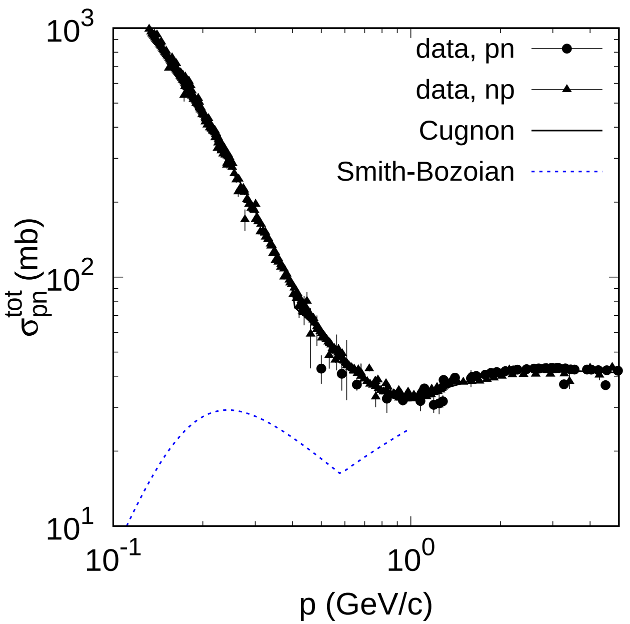
phenomenological optical potential

$$J_W/A = \begin{cases} 100E^2/(E^2+18^2) \text{ MeV fm}^3 & \text{for } E < 164\,\text{MeV} \\ 0.6E \text{ MeV fm}^3 & \text{for } E > 164\,\text{MeV} \end{cases}$$

– NN cross section parameterization from
R.D. Smith and M. Bozoian, PRC 39, 1751 (1989)
(effectively takes into account Pauli blocking)

at p > 1 GeV/c
– pp and pn cross section parameterizations from
J. Cugnon et al., NIM B 111, 215 (1996)





Exclusive $^{12}C(p,pp)^{11}B$ process

$$M = M_{\rm el}(p_3,p_4,p_1)\frac{i\Gamma_{A\to XB}(p_A,p_B)}{p_X^2-m^2+i\epsilon}\ ,$$

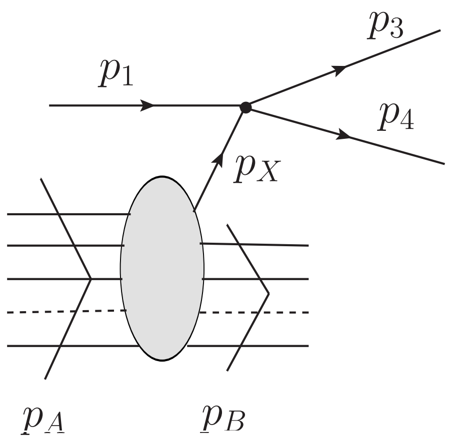
in the r.f. of A for B on the mass shell

$$\frac{i\Gamma_{A\to XB}(p_A,p_B)}{p_X^2-m^2+i\epsilon} = S_A^X\left(\frac{2E_Bm_A}{p_X^0}\right)^{1/2}(2\pi)^{3/2}\psi_{nl}^{m_l}(-\boldsymbol{p}_X)\ ,$$

n – main HO quantum number,

l – relative orbital momentum of B and X,

m_l – magnetic quantum number.



p_i – four-momenta,
1 – incoming proton,
A – initial nucleus (^{12}C),
B – final nucleus (^{11}B),
X – struck proton,
3,4 – outgoing protons (in the r.f. of A)

$$(2\pi)^{3/2}\psi_{nl}^{m_l}(-\boldsymbol{p}_X) = \int d^3r e^{i\boldsymbol{p}_X\boldsymbol{r}}\psi_{nl}^{m_l}(\boldsymbol{r})$$

Spectroscopic amplitude: $S_A^X = A^{1/2}\langle\Psi_B,\psi_{nl}^{m_l}(\boldsymbol{R}_B-\boldsymbol{R}_X)|\Psi_A\rangle\ .$

Normalization of WFs: $$\int d^3p_X|\psi_{nl}^{m_l}(-\boldsymbol{p}_X)|^2 = 1\ ,$$

$$\int d^3r|\psi_{nl}^{m_l}(\boldsymbol{r})|^2 = 1\ .$$

AL, Yu.N. Uzikov, PRC 110, 014610 (2024) [arXiv:2403.02812]

Table 1: Integrated cross sections of the process $^{12}\mathrm{C}(p,2p)^{11}\mathrm{B}$ with 400 MeV/nucleon ^{12}C beam for the ground state and two excited states of the residual nucleus ^{11}B. Listed are the results of full calculations (including absorption), calculations in the IA, and the theoretical spectroscopic factors. Experimental data are from Ref. [1]. Total errors are given in parentheses.

E^* (MeV)	J^π	σ_{exp} (mb)	σ_{full} (mb)	σ_{IA} (mb)	S
0.0 (G.S.)	$3/2^-$	15.8(18)	12.3	62.6	2.82
2.12	$1/2^-$	1.9(2)	2.9	14.9	0.67
5.02	$3/2^-$	1.5(2)	3.4	17.5	0.79
Total:		19.2(3)	18.6	95.0	4.28

[1] ***V. Panin et al., PLB 753, 204 (2016)***

- Good agreement with data for total cross section.

- Deviations for separate states.