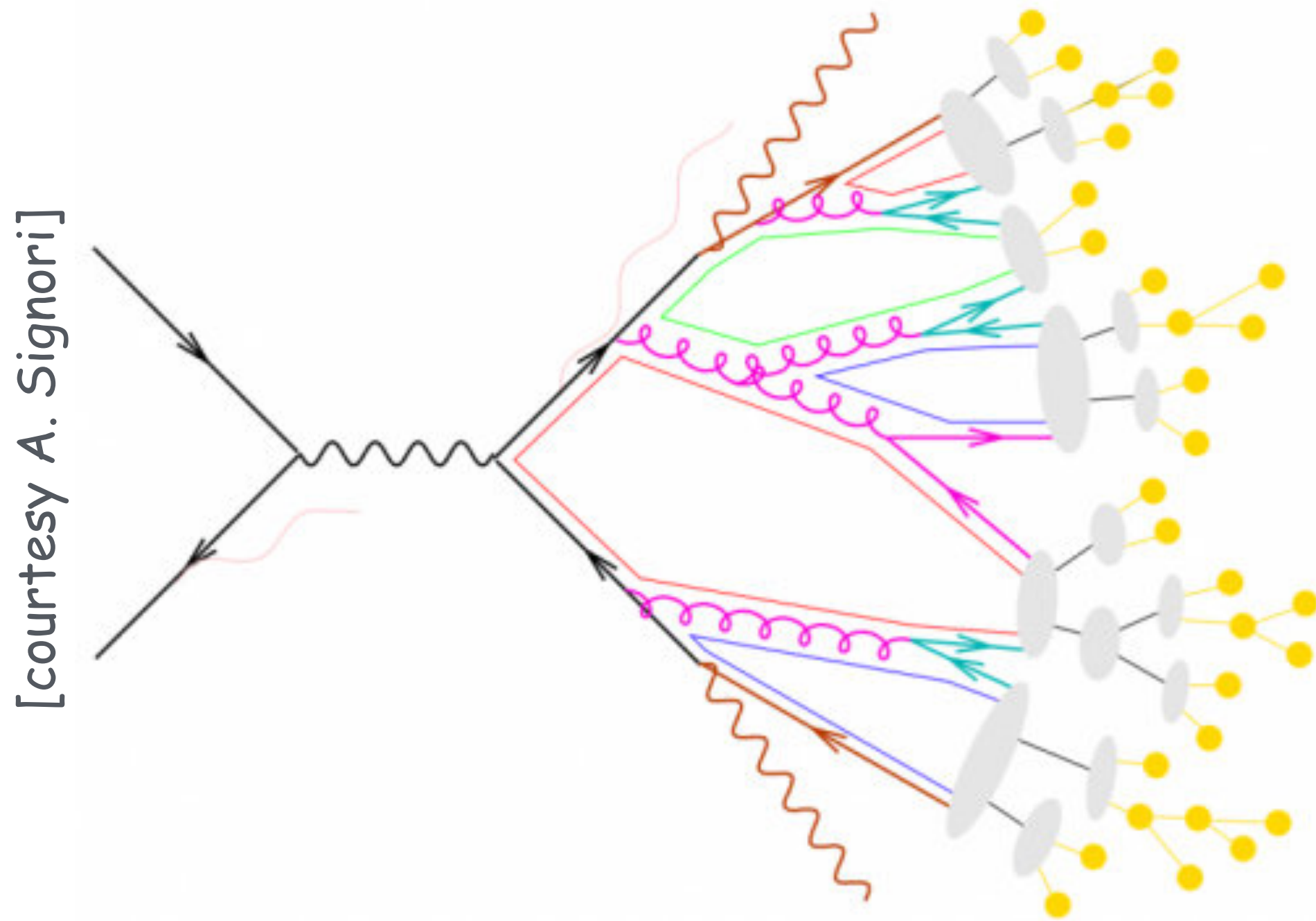
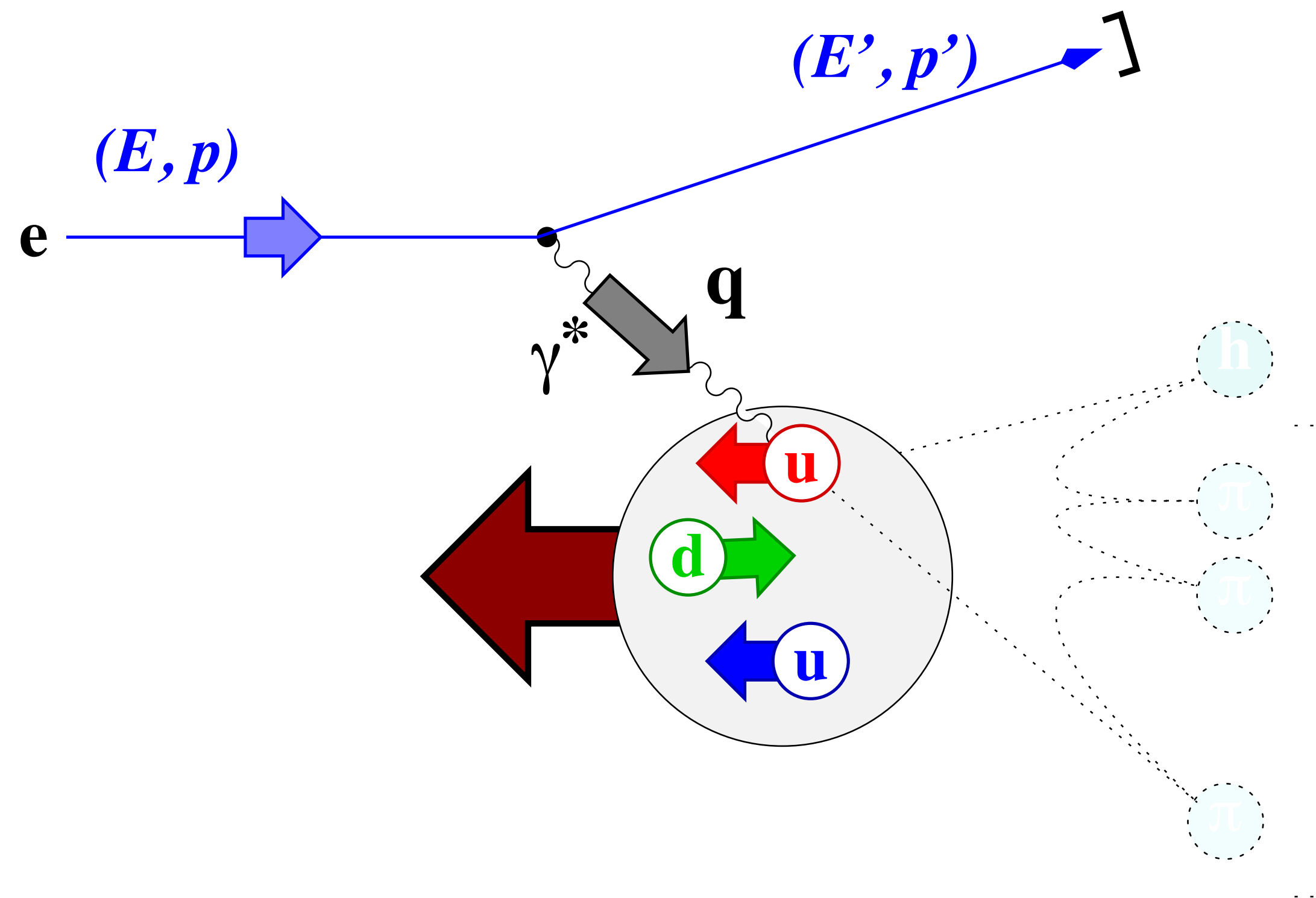
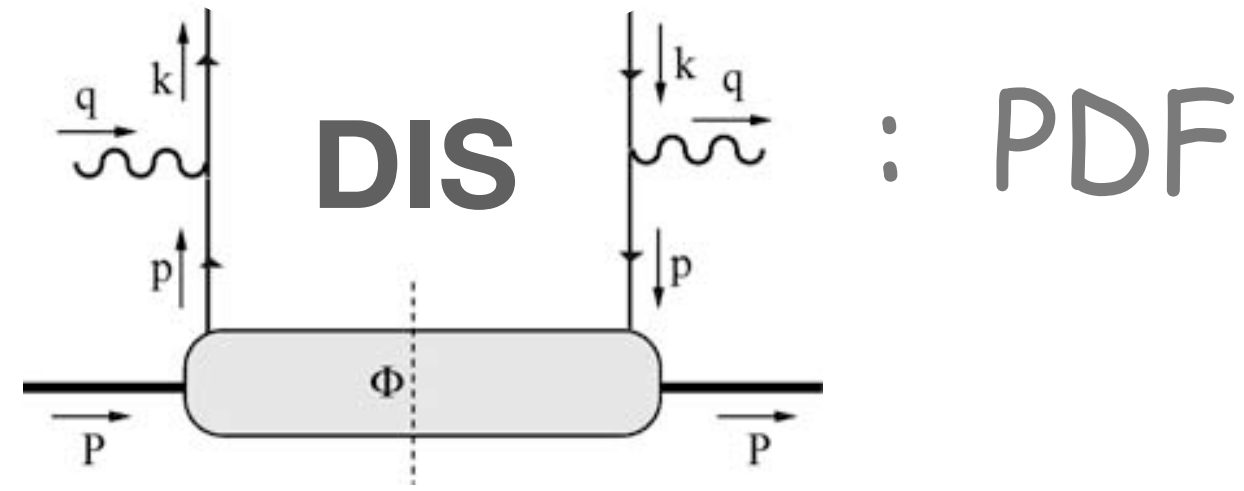


20th International Workshop on Hadron Structure and Spectroscopy & 5th Workshop on Correlations in Partonic and Hadronic Interactions

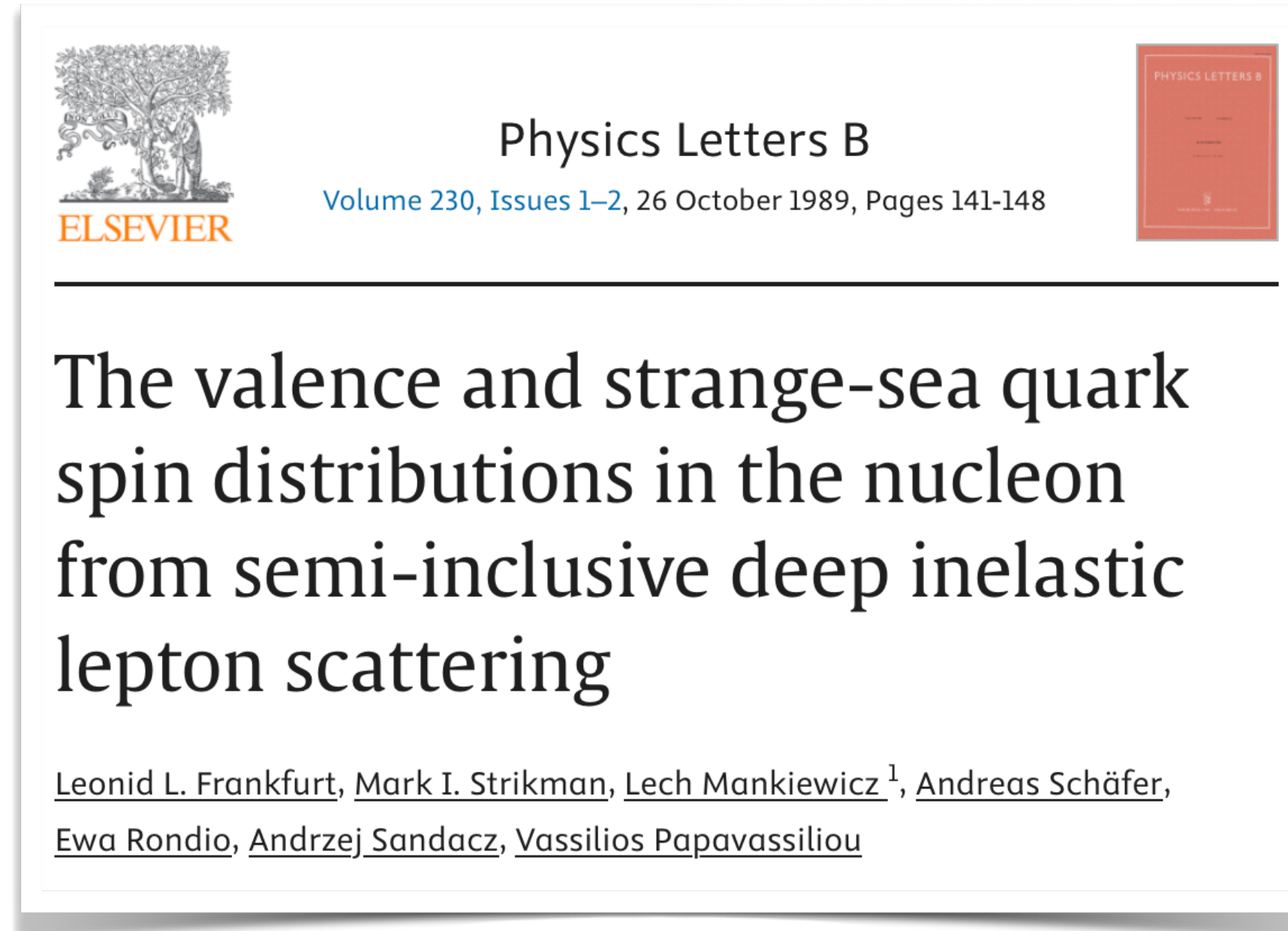
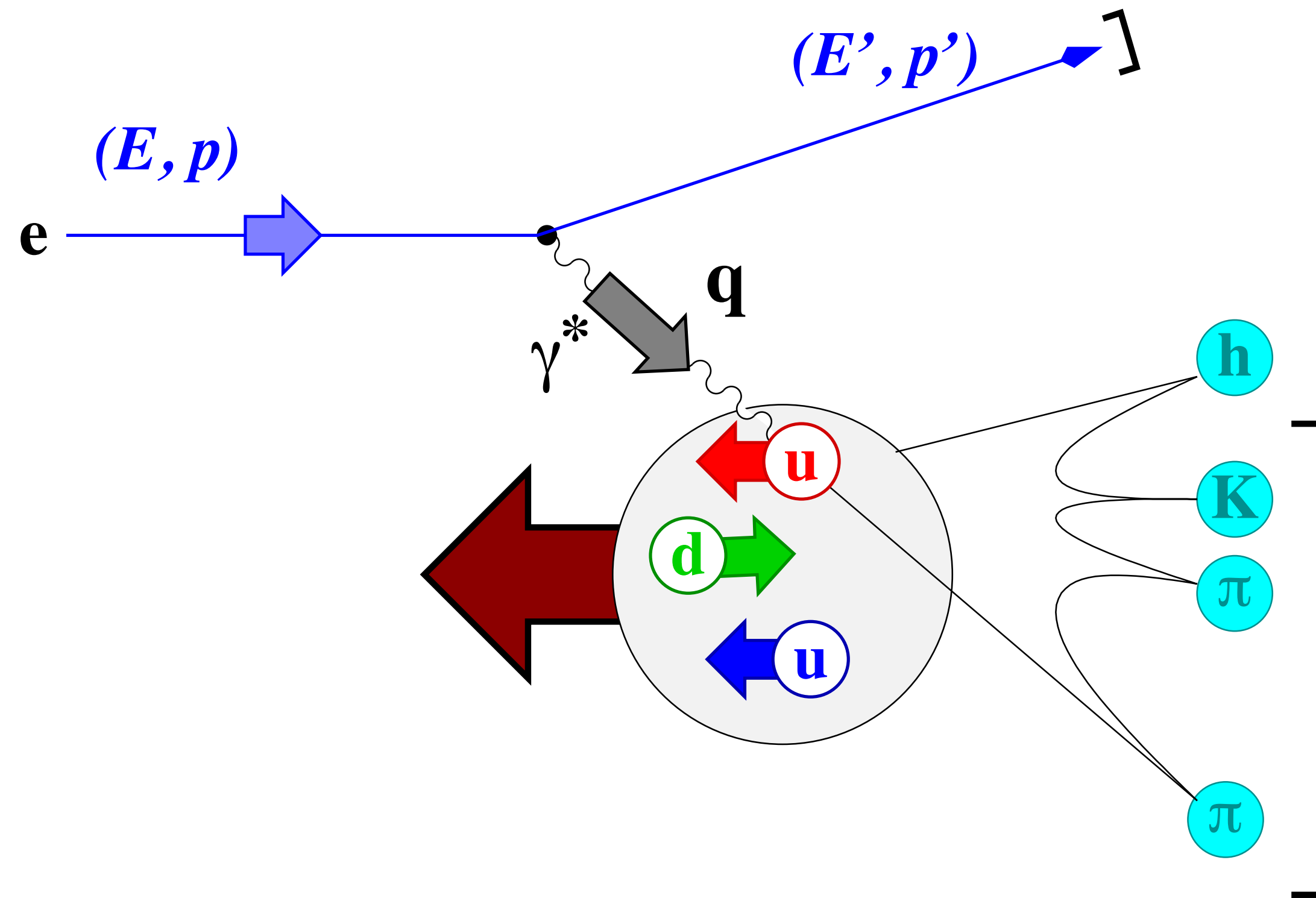
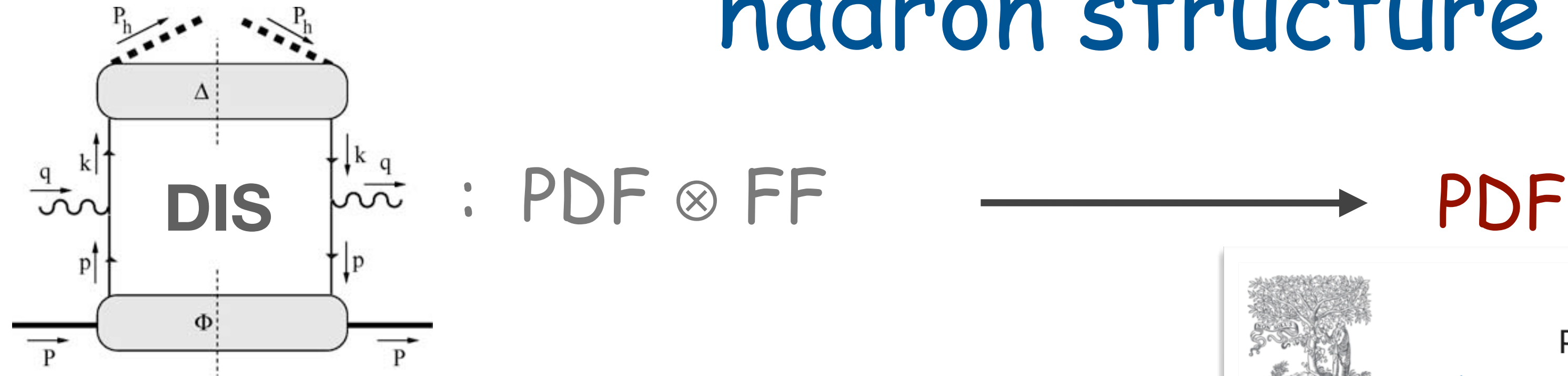


Hadronization in e^+e^- annihilation

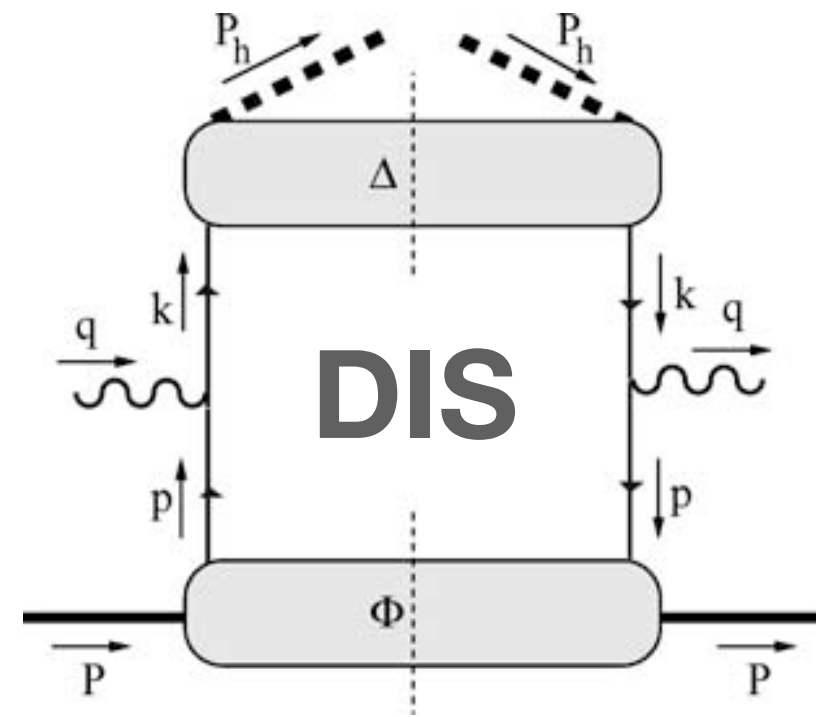
hadron structure - a global approach



hadron structure - a global approach



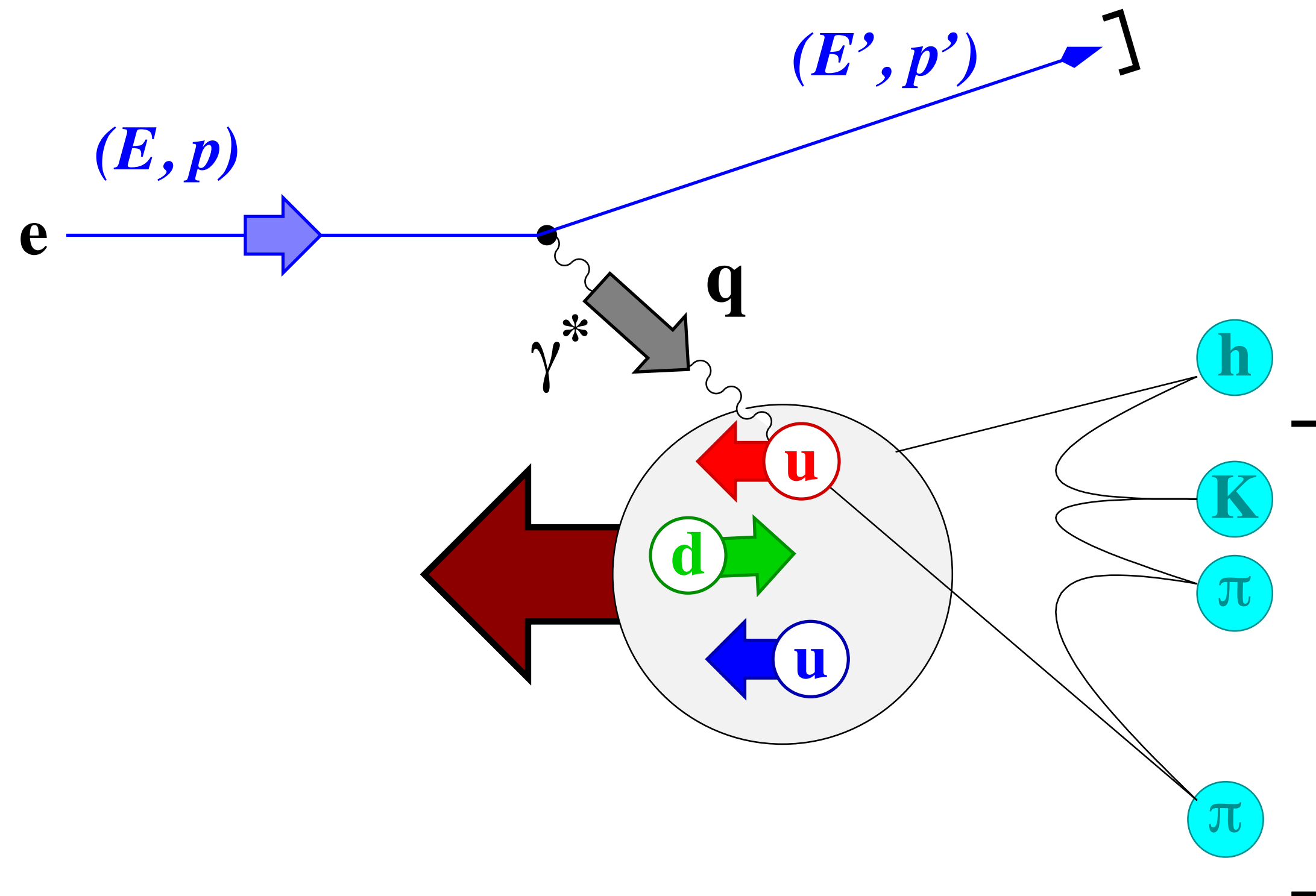
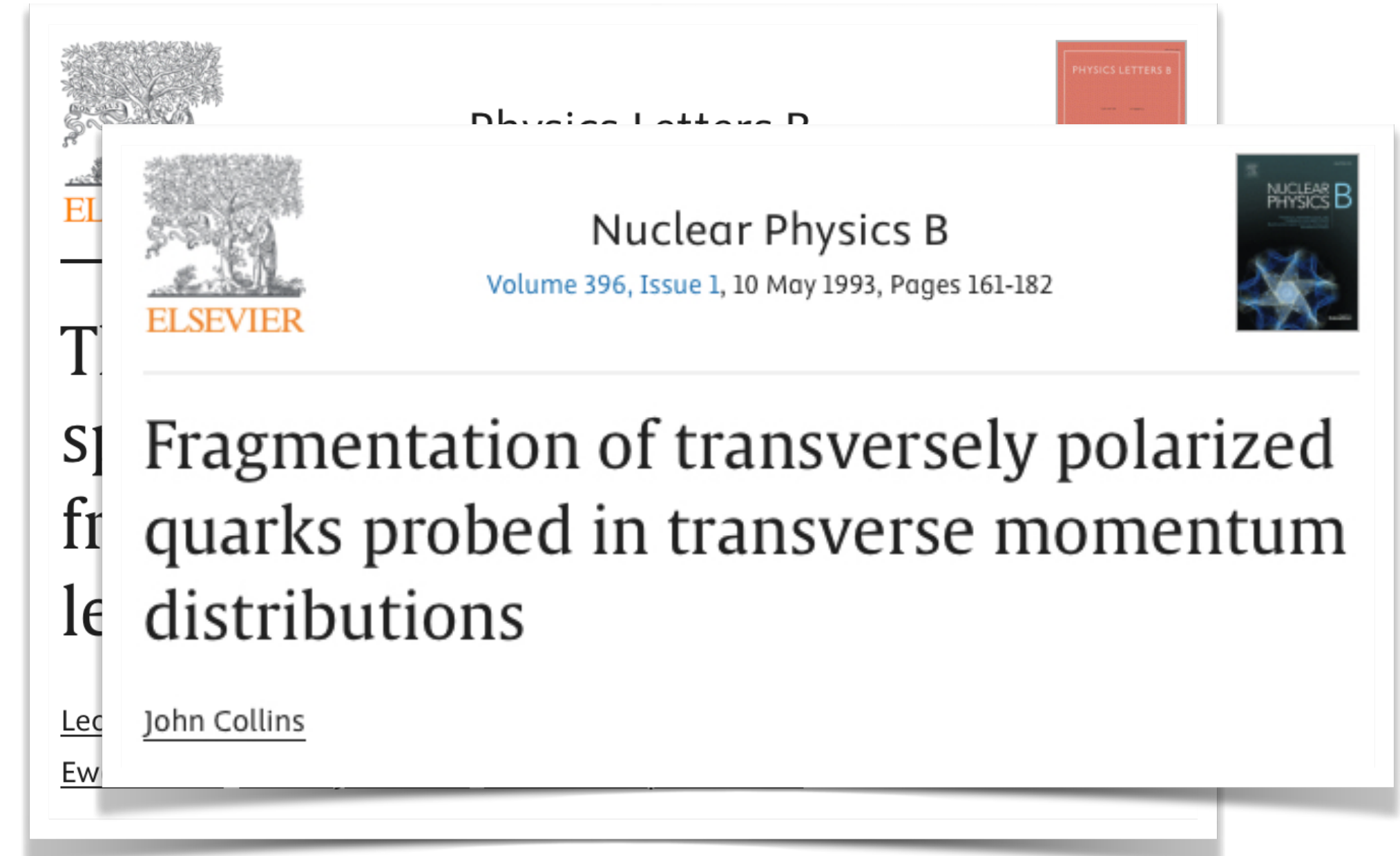
hadron structure - a global approach



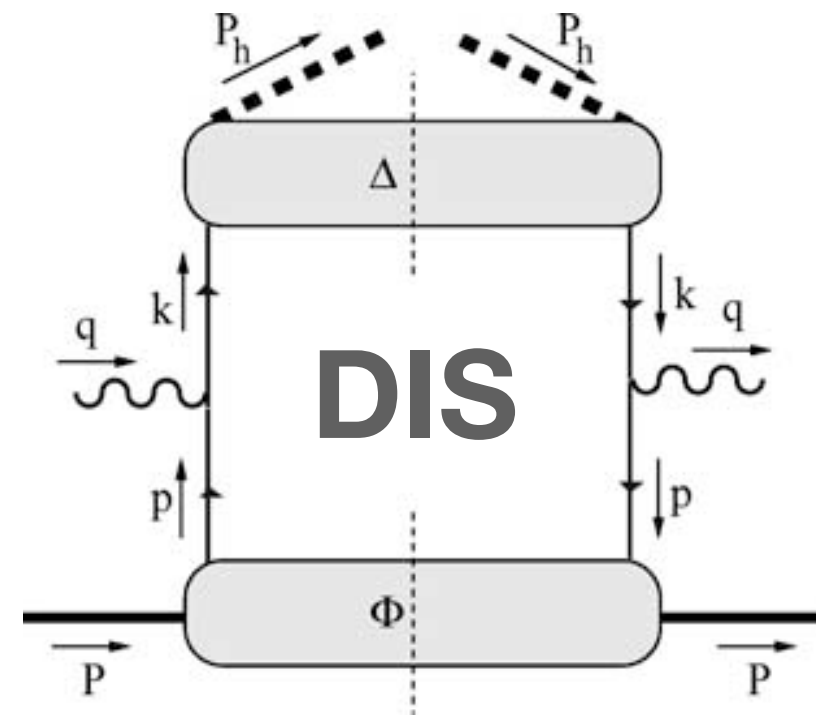
: PDF \otimes FF



PDF



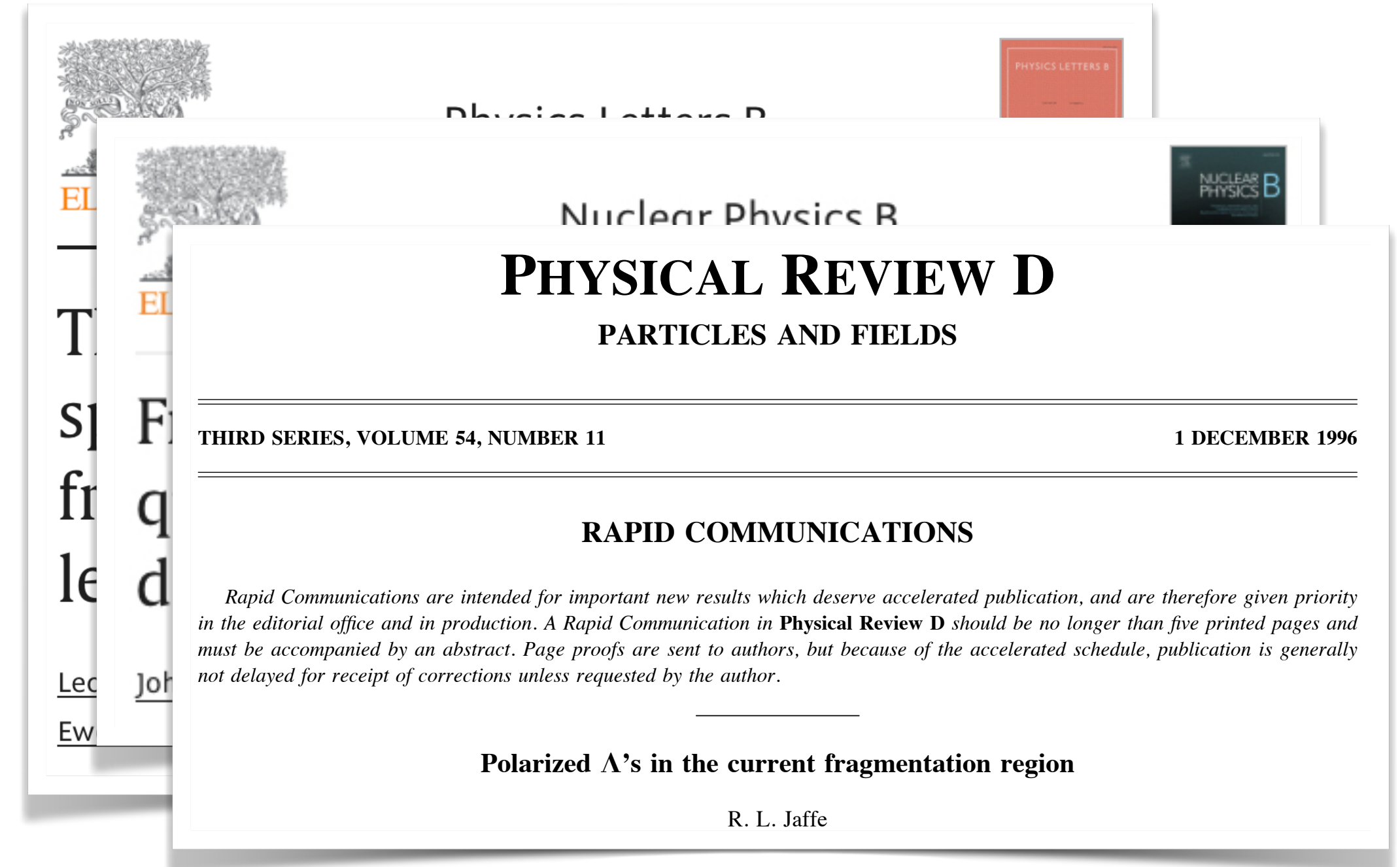
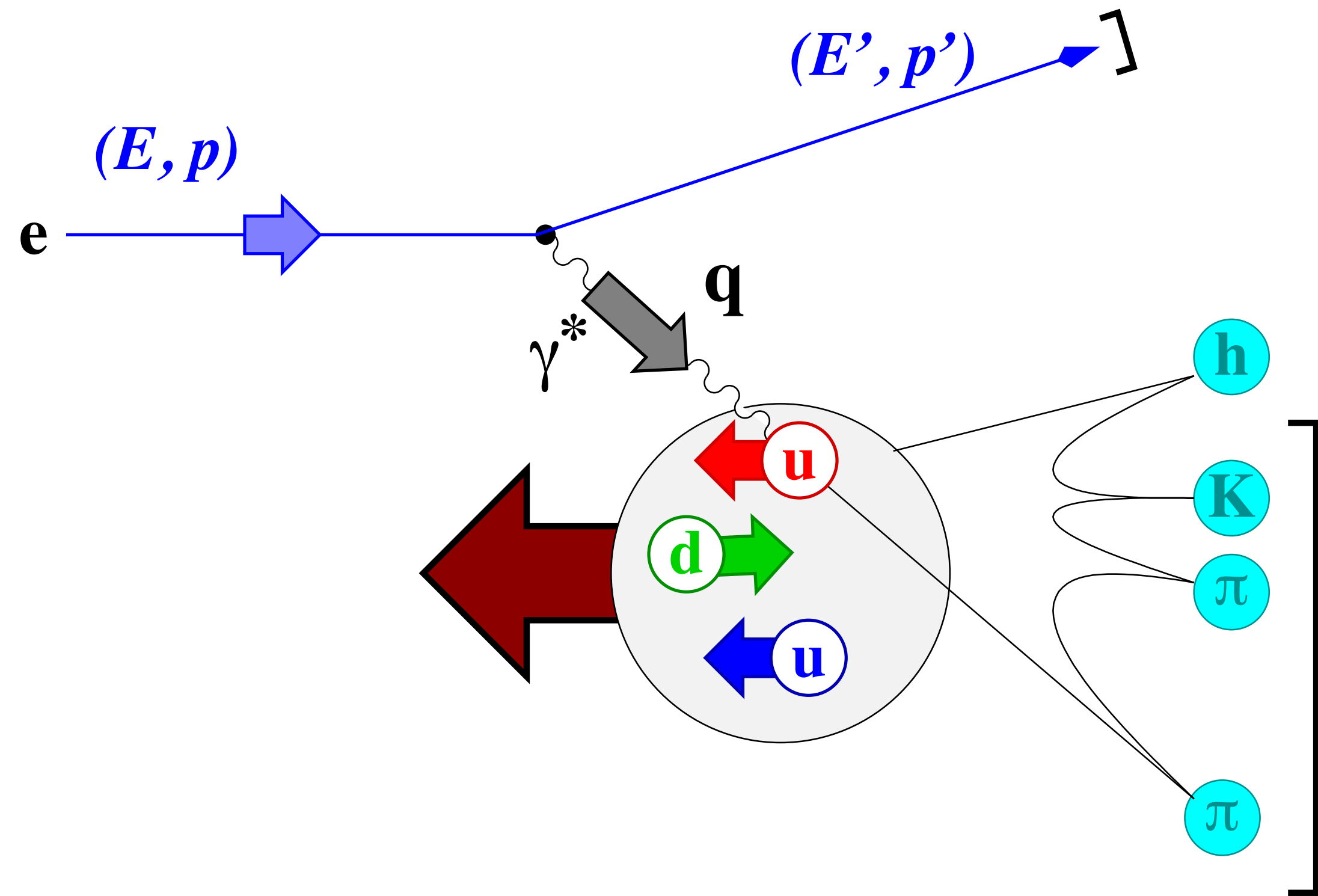
hadron structure - a global approach



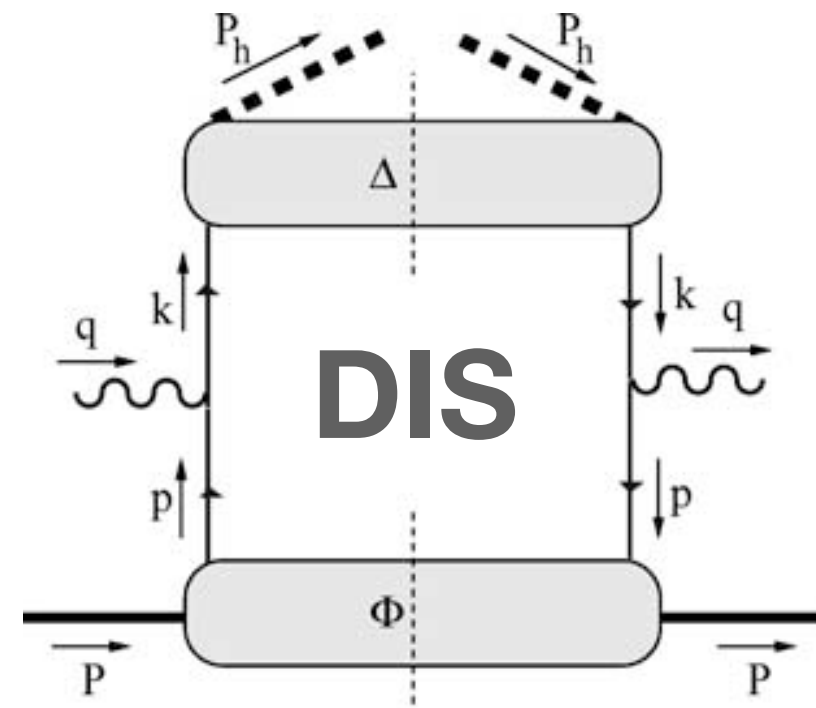
$: PDF \otimes FF$



PDF

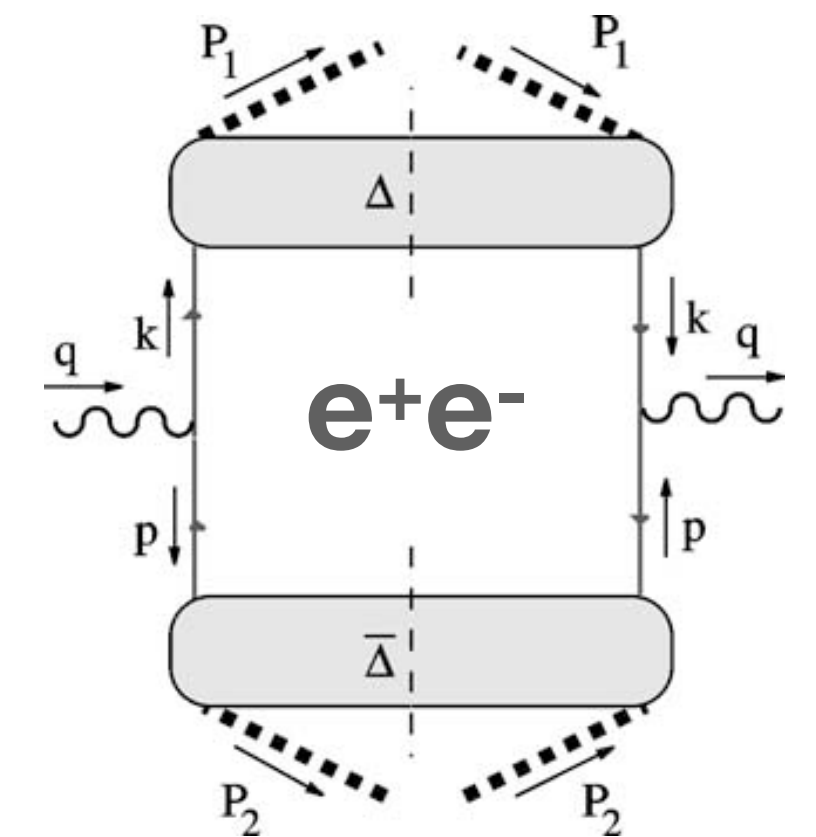


hadron structure - a global approach

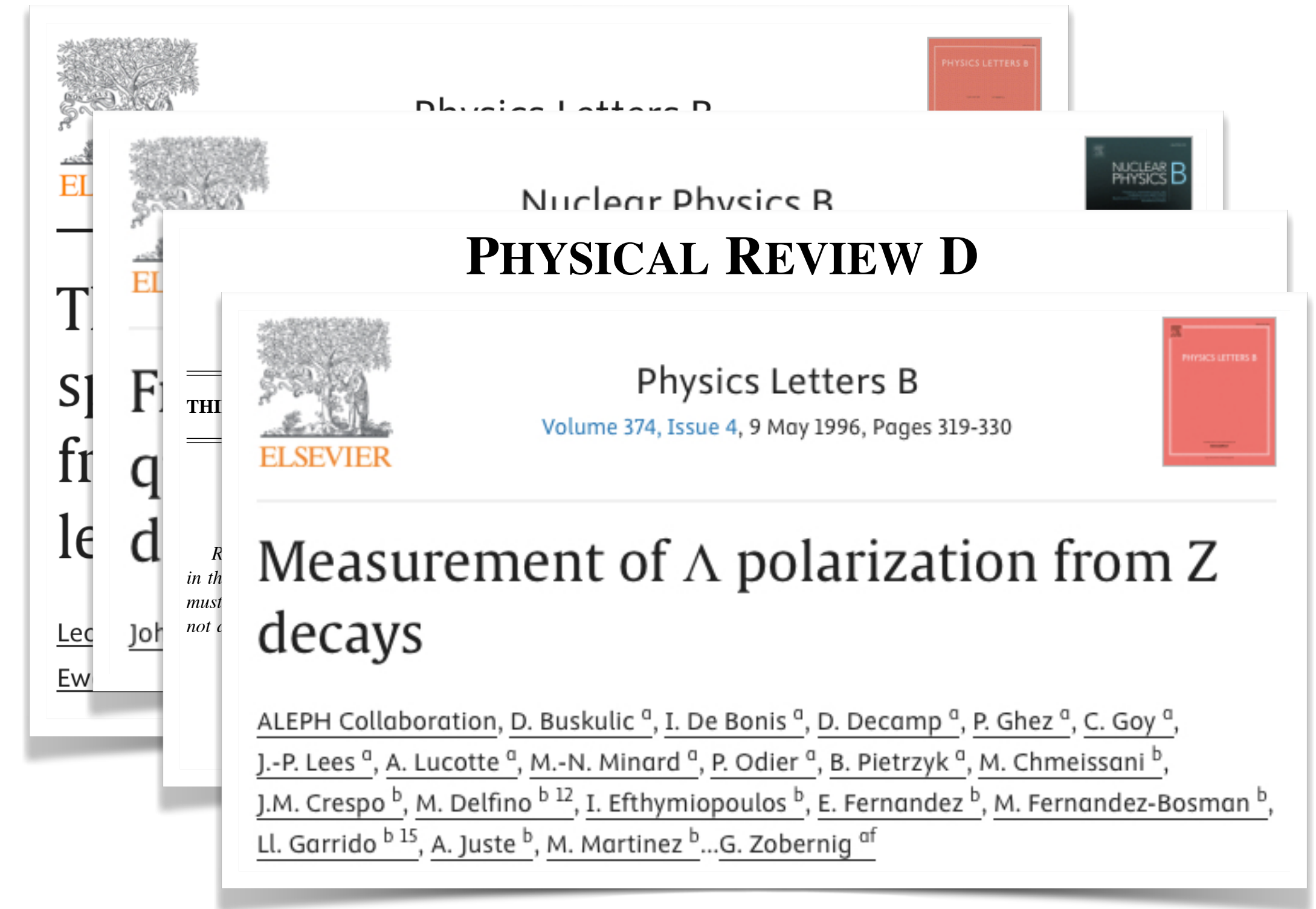


: PDF \otimes FF

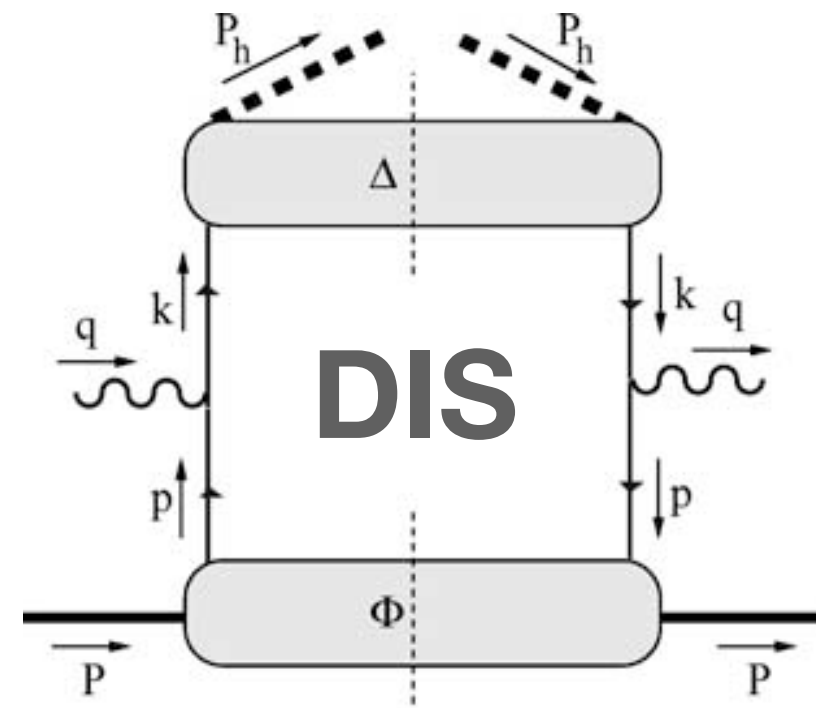
PDF



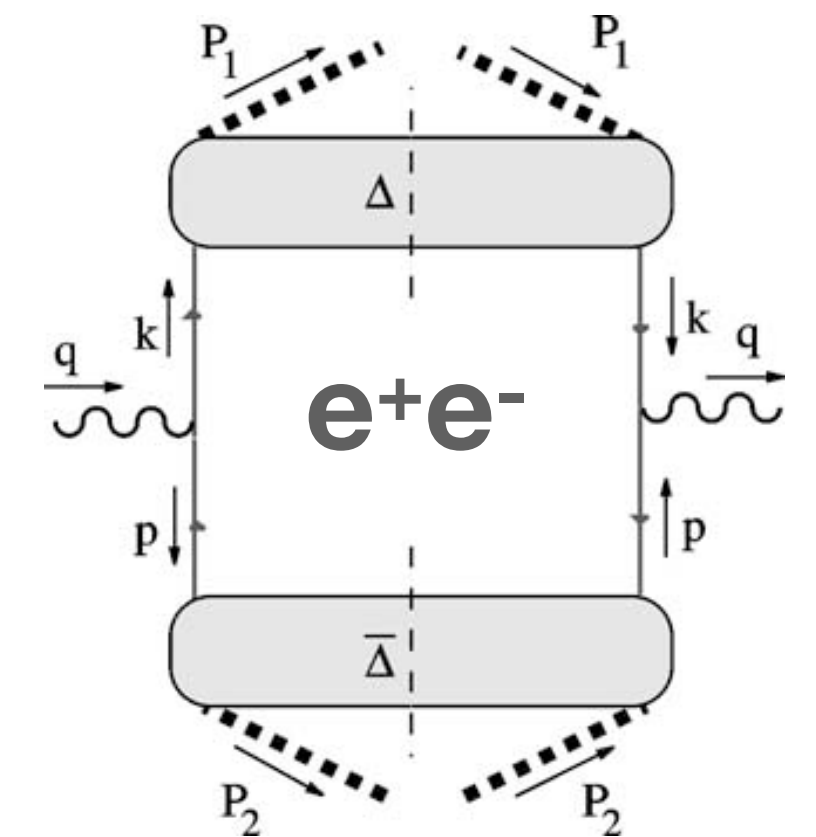
: FF



hadron structure - a global approach

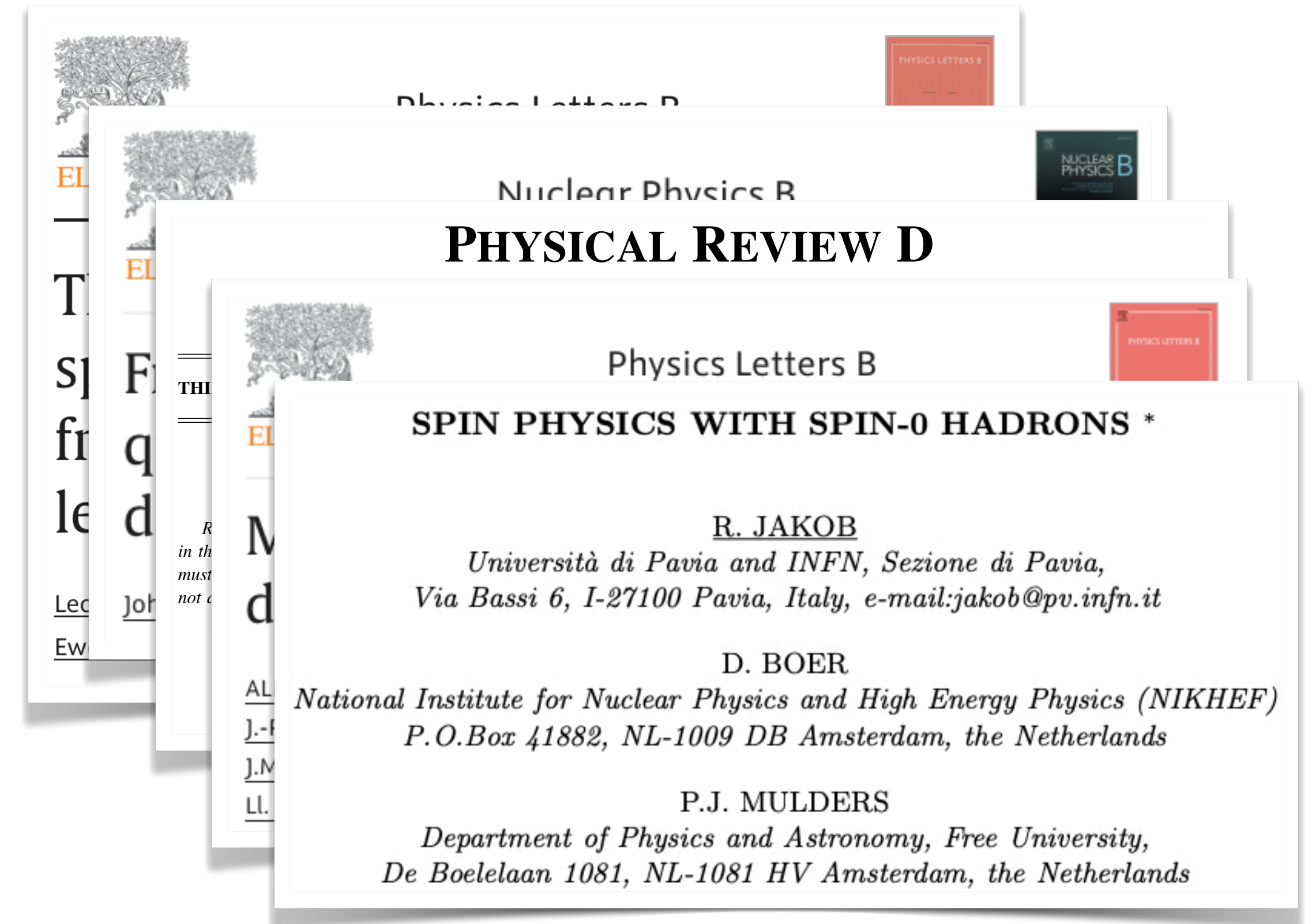


: PDF \otimes FF

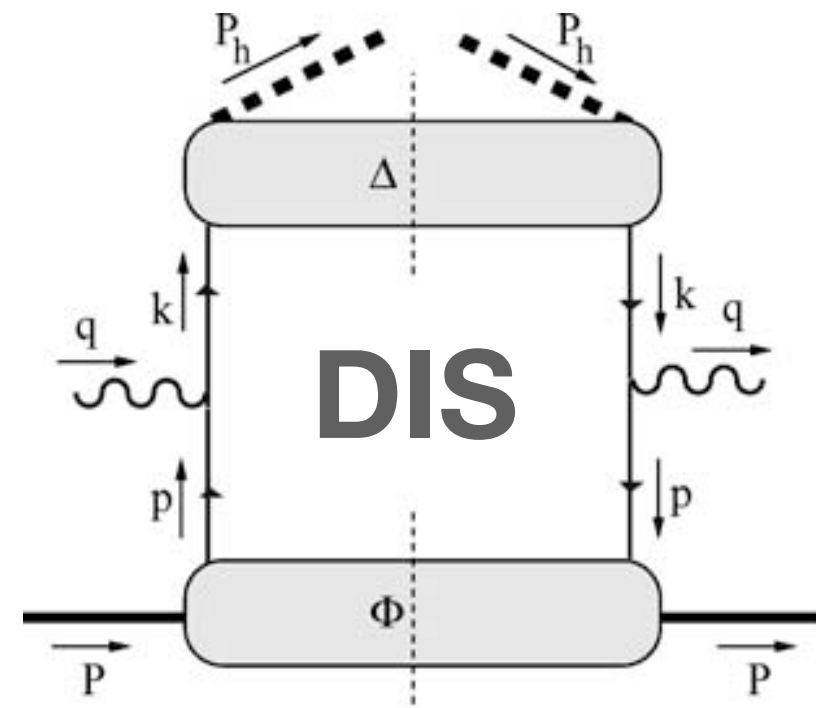


: FF

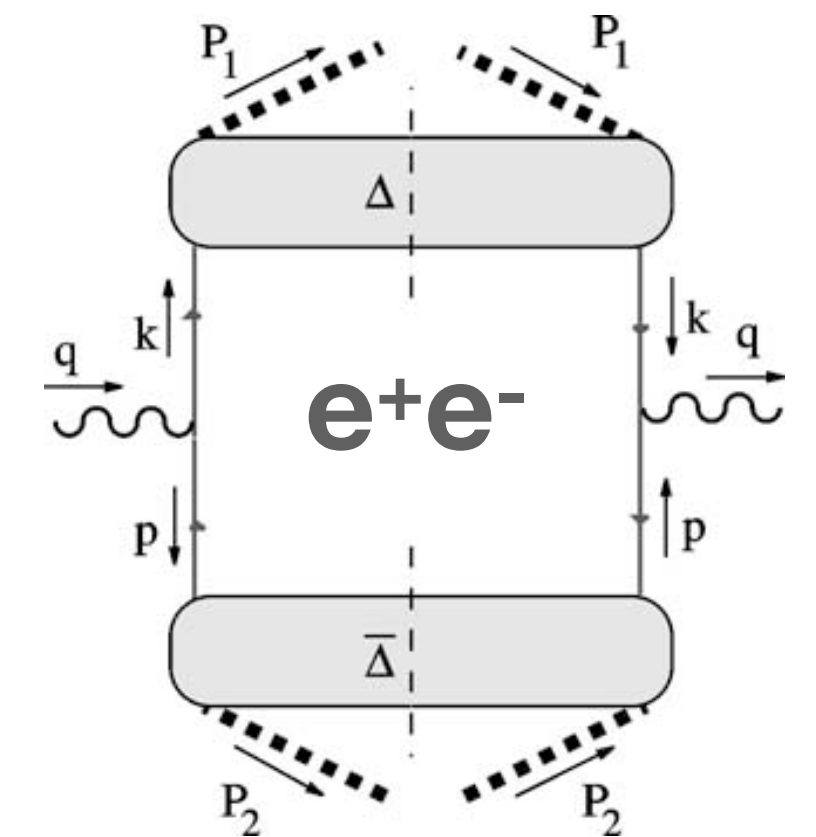
PDF



hadron structure - a global approach

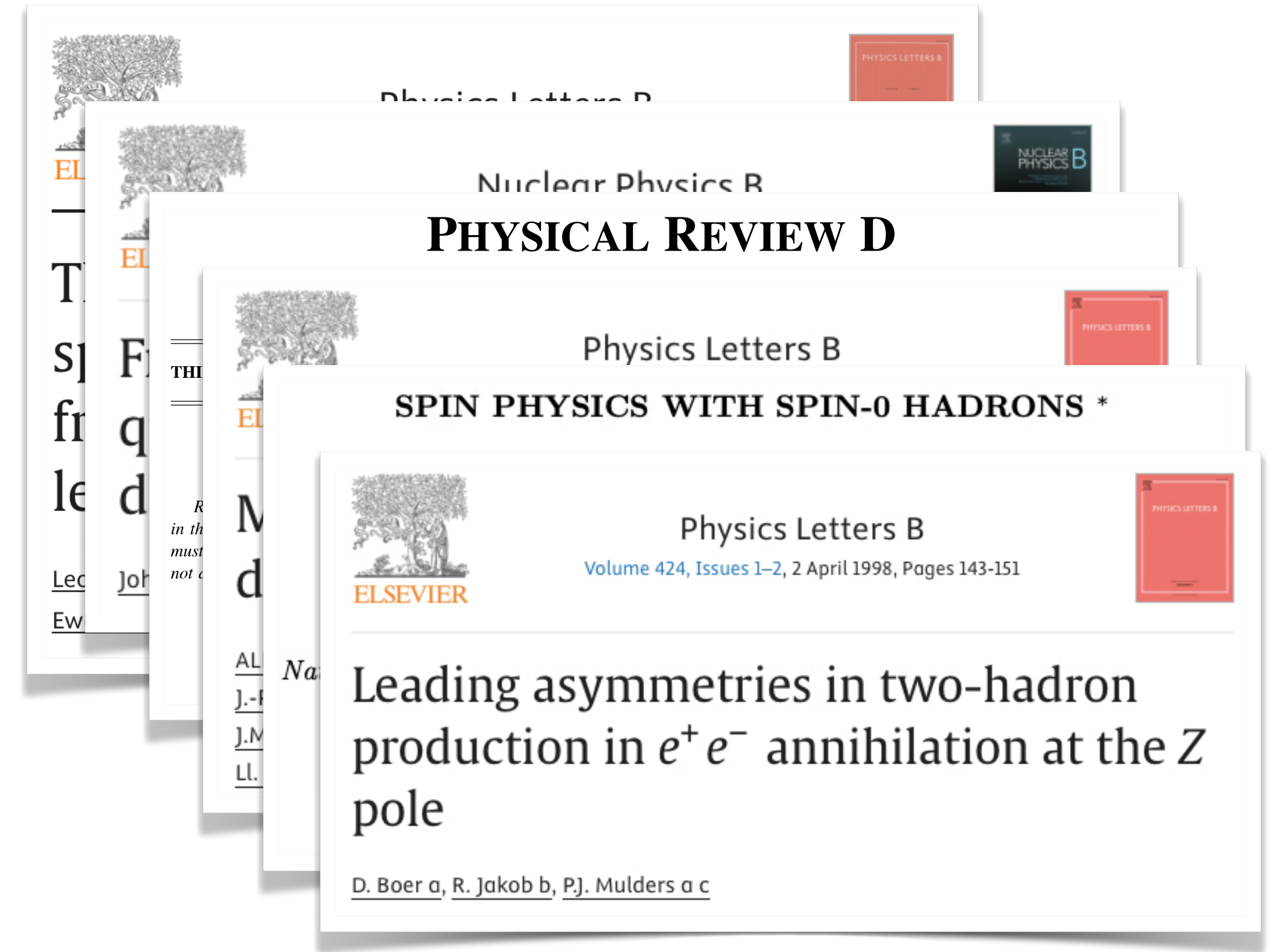


: PDF \otimes FF

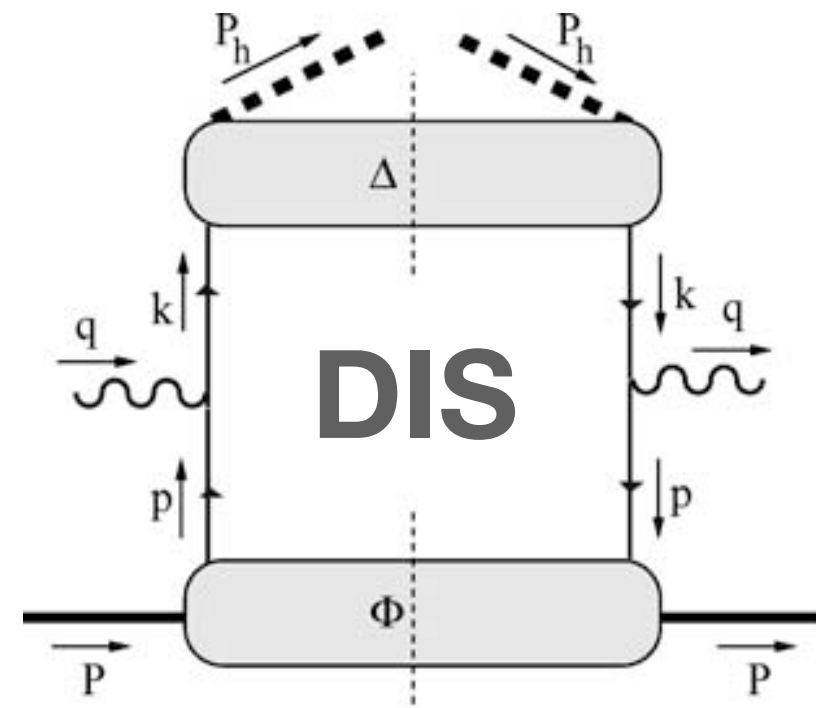


: FF

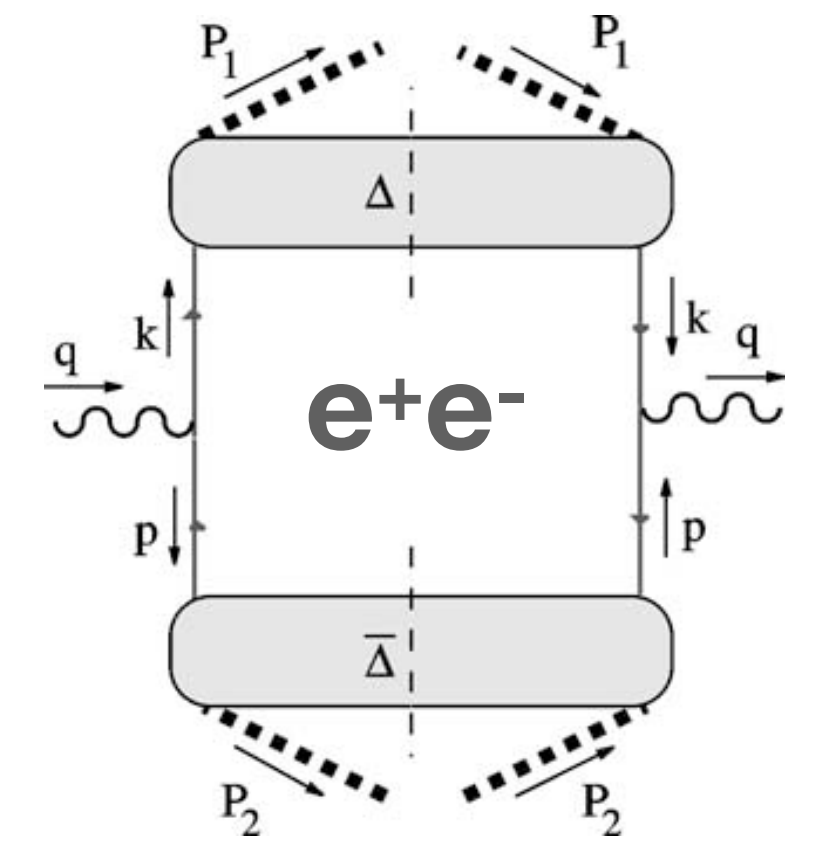
PDF



hadron structure - a global approach

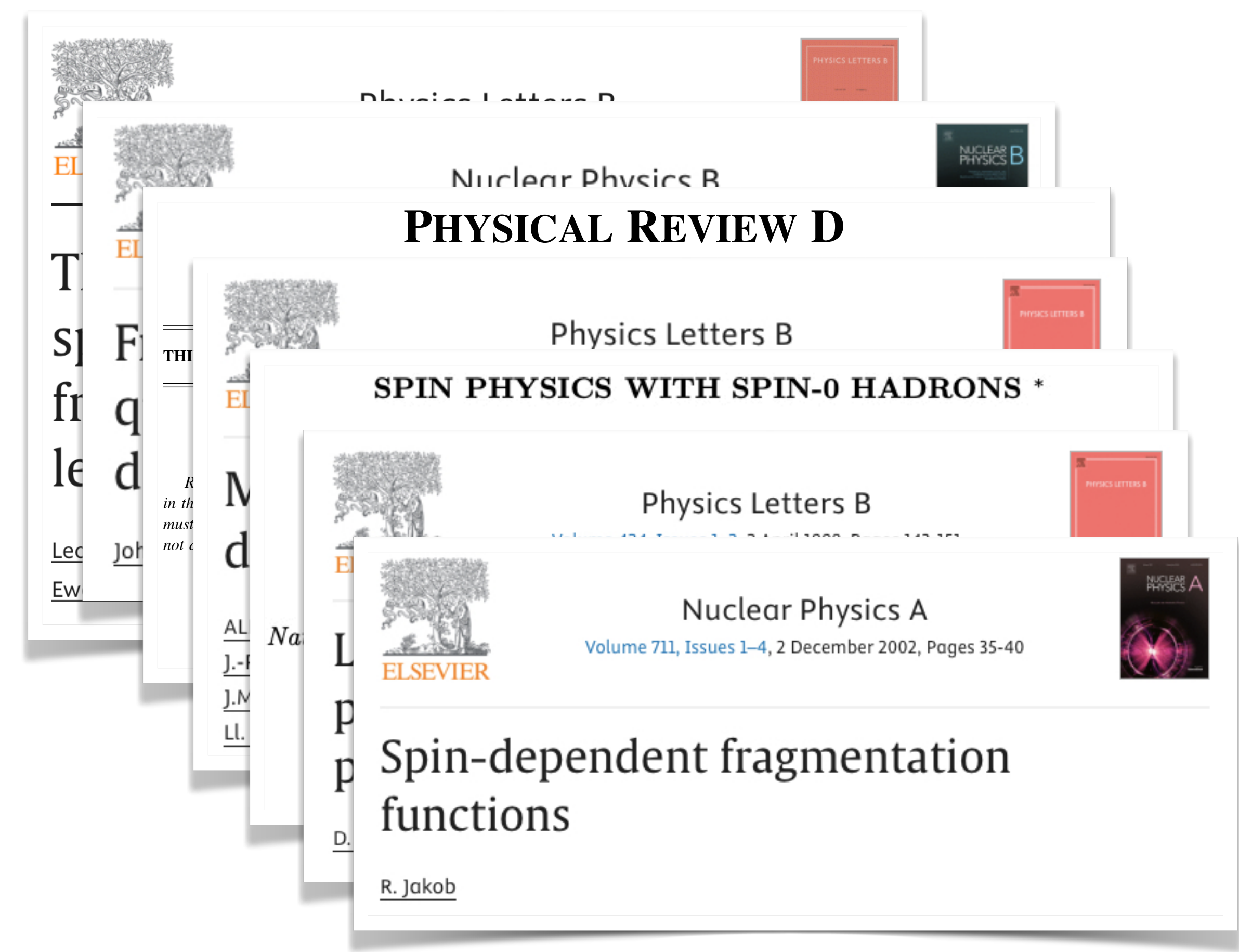
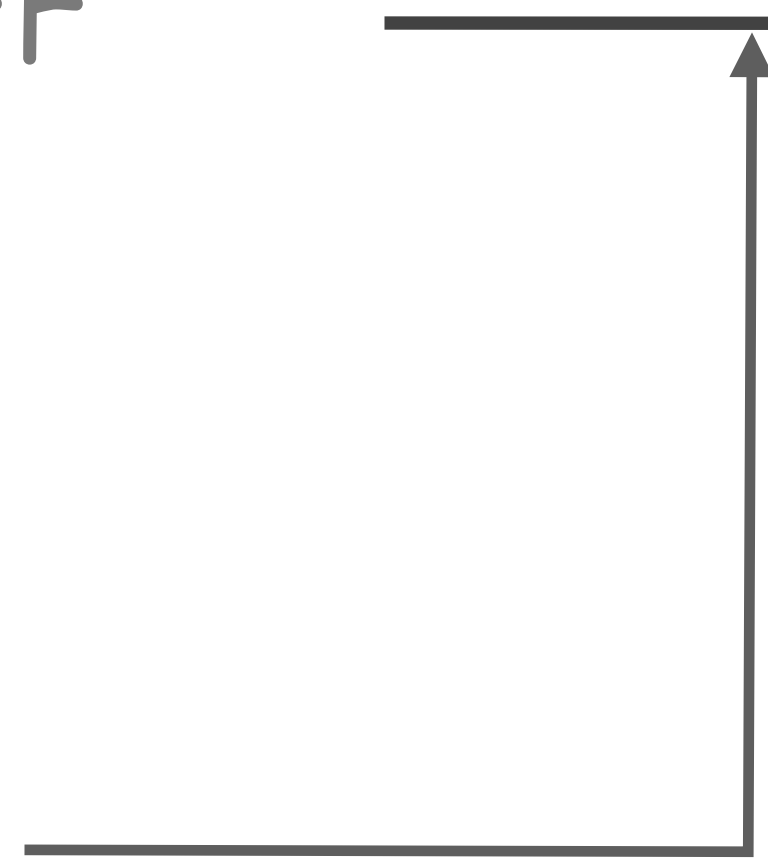


: PDF \otimes FF

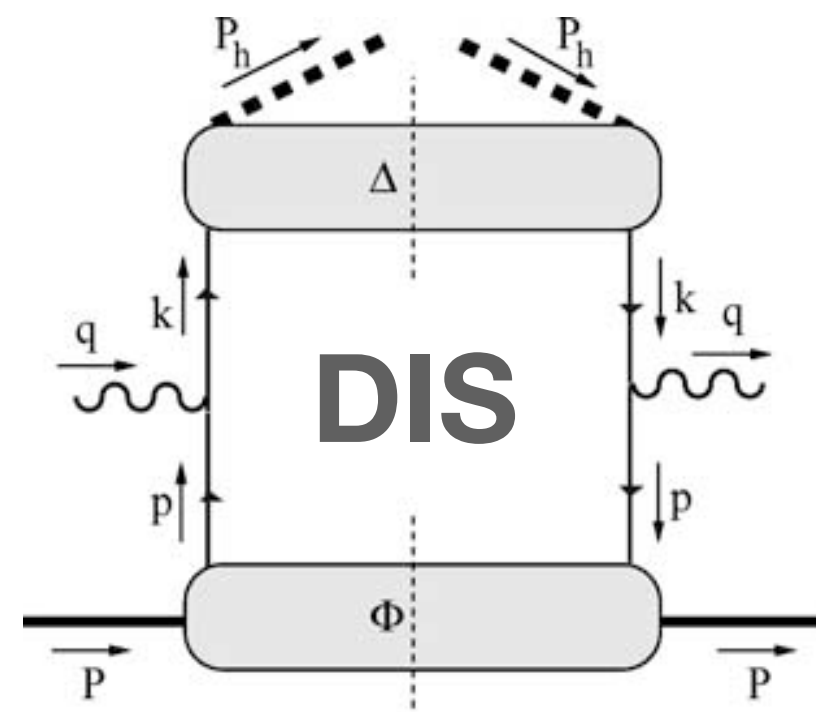


: FF

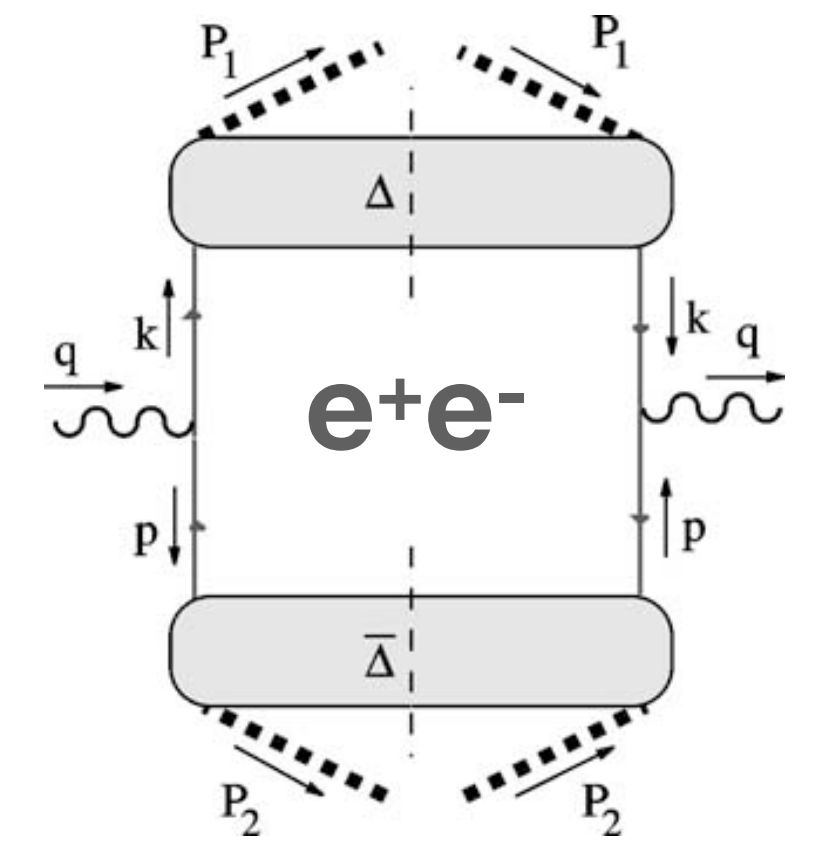
PDF



hadron structure - a global approach

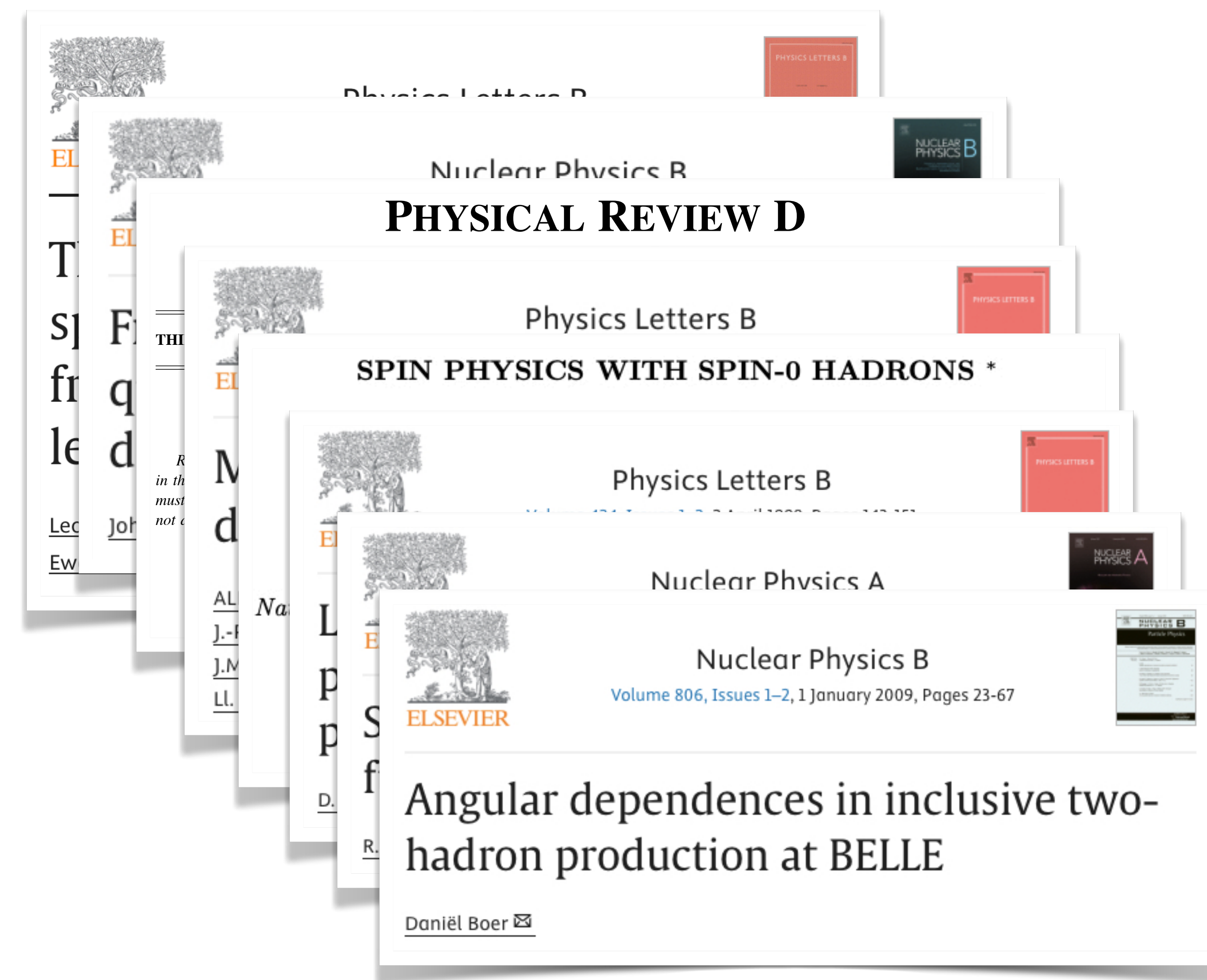


: PDF \otimes FF

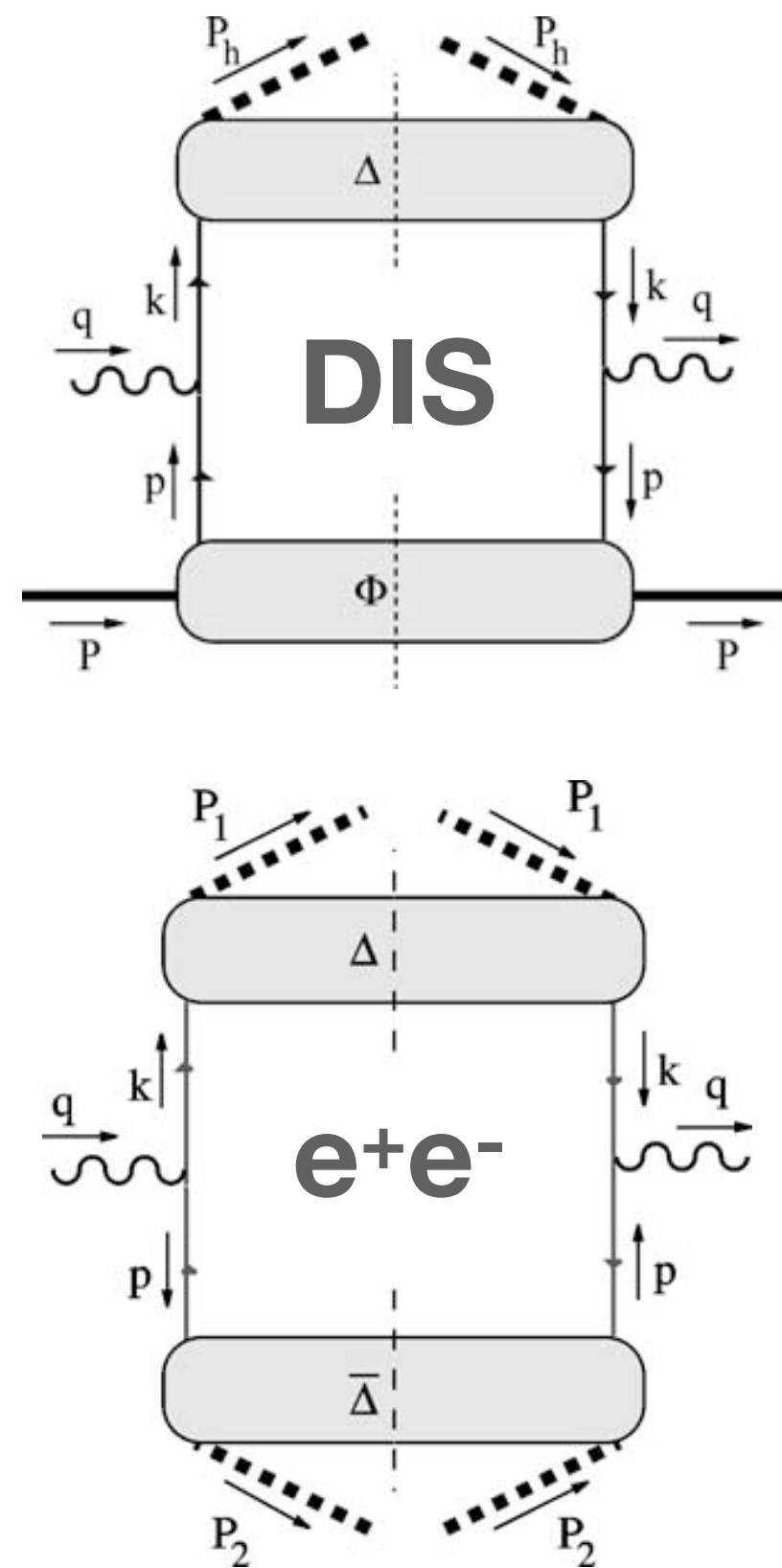


: FF

PDF



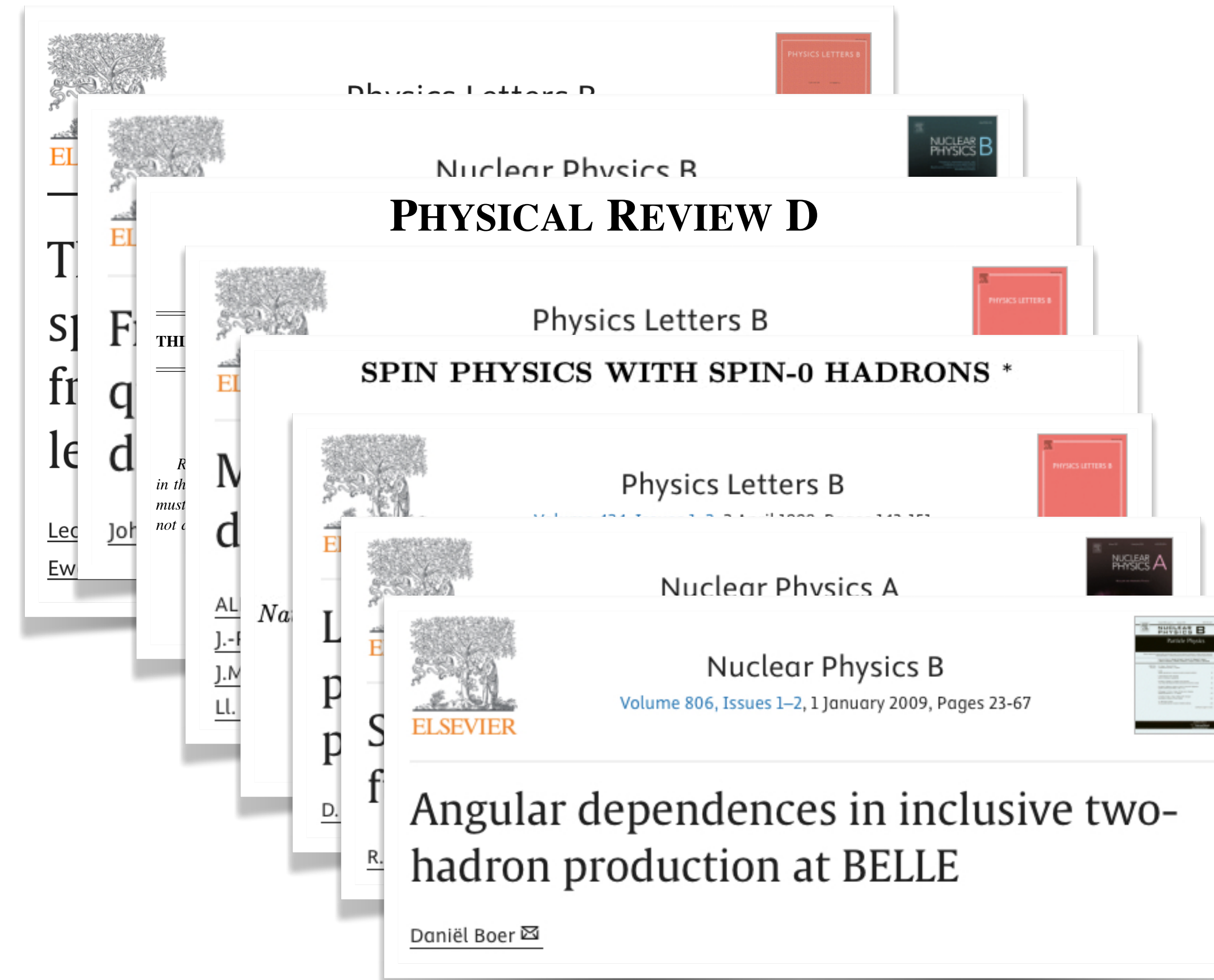
hadron structure - a global approach



: PDF \otimes FF

: FF

PDF



- FFs needed/interesting for many other studies! (confinement, heavy-ion physics, etc.)
- FFs not calculable from first principles
- hadronization very active field (Monte Carlo, hadrons in jets, event shapes, etc.)

👉 talks by Albi, Gevorg, Valerio, Xiao-Rui, Xuanbo, ...

single-hadron*) (TMD**) fragmentation functions

*) complemented by rich world of di-hadrons

***) transverse-momentum dependent

quark pol.

	U	L	T
hadron pol.	U	D_1	H_1^\perp
	L		G_1
	T	D_{1T}^\perp	G_{1T}^\perp

single-hadron*) (TMD**) fragmentation functions

*) complemented by rich world of di-hadrons

***) transverse-momentum dependent

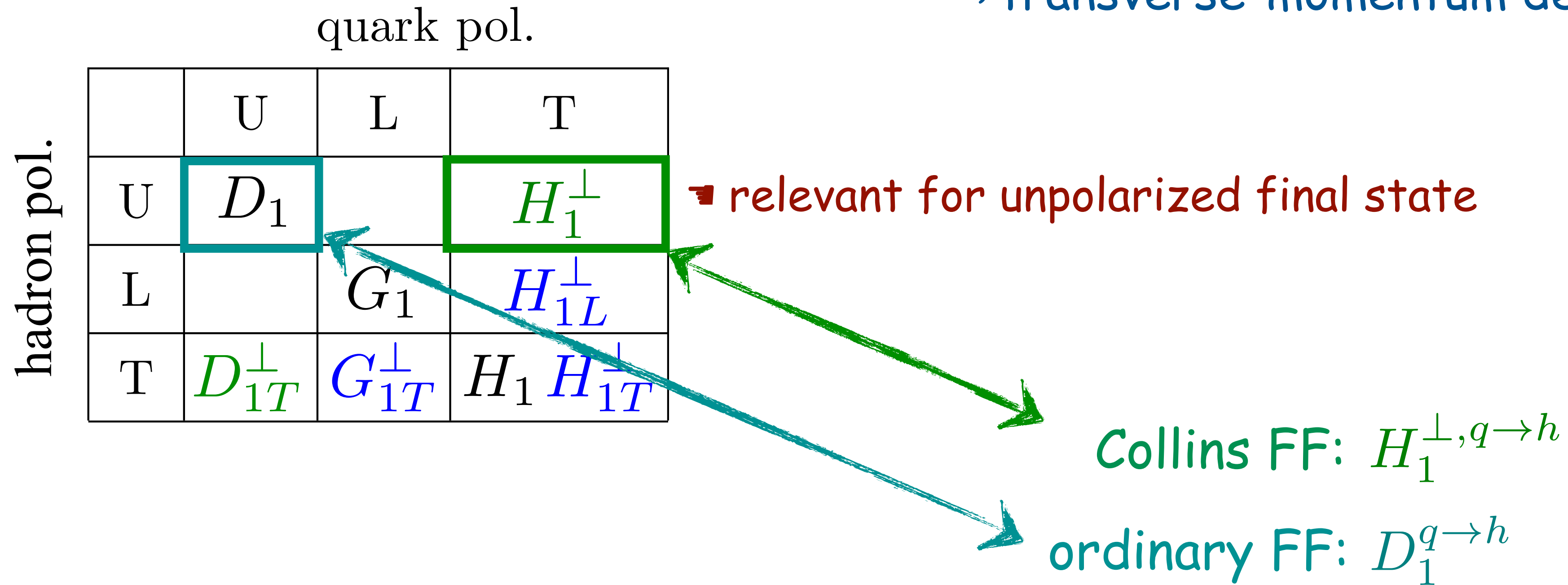
		quark pol.		
		U	L	T
hadron pol.	U	D_1		H_1^\perp
	L		G_1	H_{1L}^\perp
	T	D_{1T}^\perp	G_{1T}^\perp	$H_1 H_{1T}^\perp$

relevant for unpolarized final state

single-hadron*) (TMD**) fragmentation functions

*) complemented by rich world of di-hadrons

***) transverse-momentum dependent



FF ... fragmentation function

single-hadron*) (TMD**) fragmentation functions


*) complemented by rich world of di-hadrons

***) transverse-momentum dependent

quark pol.

	U	L	T
hadron pol.	U		
U	D_1		H_1^\perp
L		G_1	H_{1L}^\perp
T	D_{1T}^\perp	G_{1T}^\perp	$H_1 \ H_{1T}^\perp$

▣ relevant for unpolarized final state
} polarized final-state hadrons


 polarizing FF

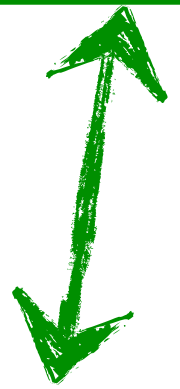
FF ... fragmentation function

single-hadron*) (TMD**) fragmentation functions

*) complemented by rich world of di-hadrons

***) transverse-momentum dependent

		quark pol.			
		U	L	T	
hadron pol.	U	D_1		H_1^\perp	} relevant for unpolarized final state } polarized final-state hadrons
	L		G_1	H_{1L}^\perp	
	T	D_{1T}^\perp	G_{1T}^\perp	H_1 H_{1T}^\perp	



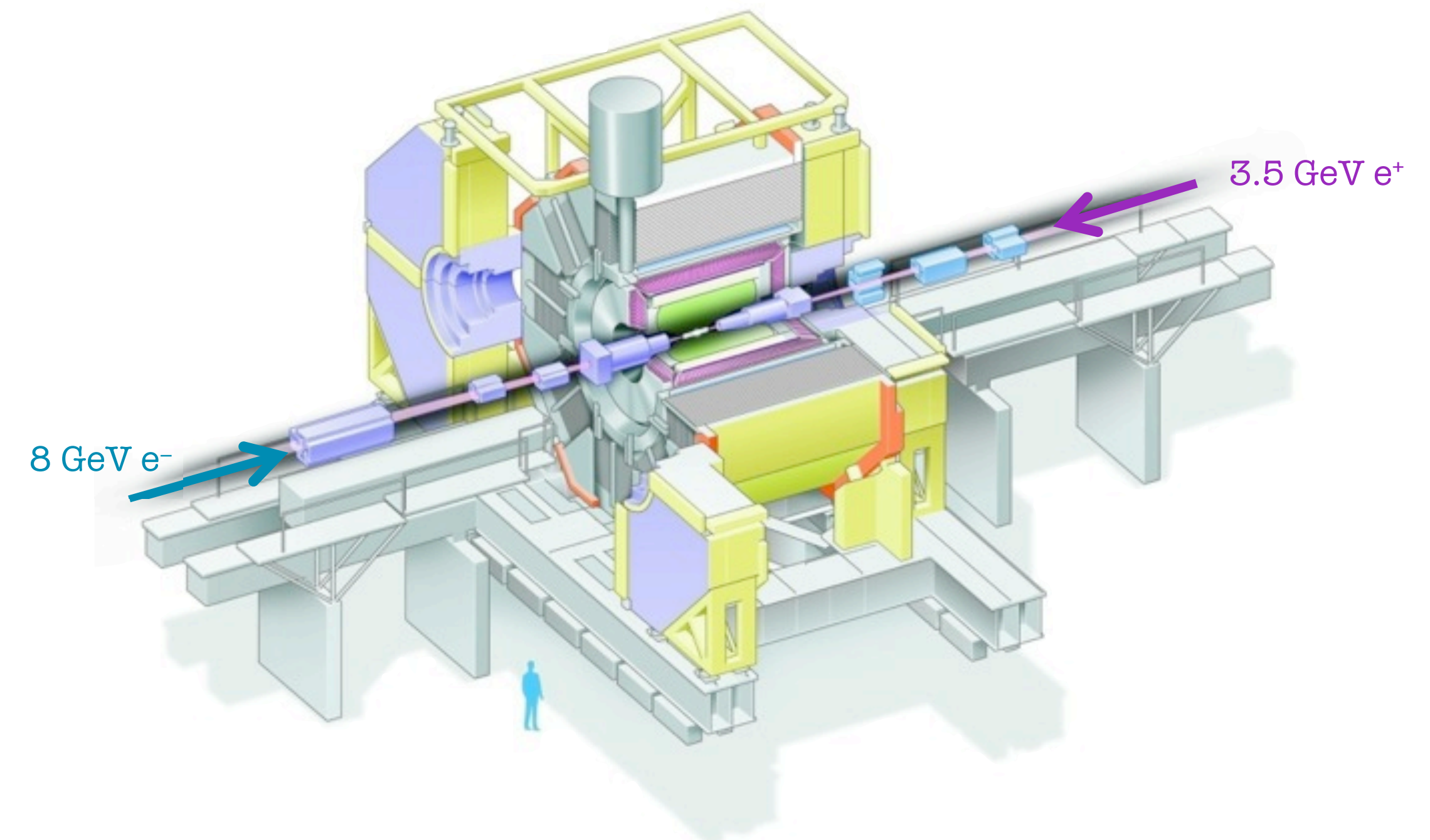
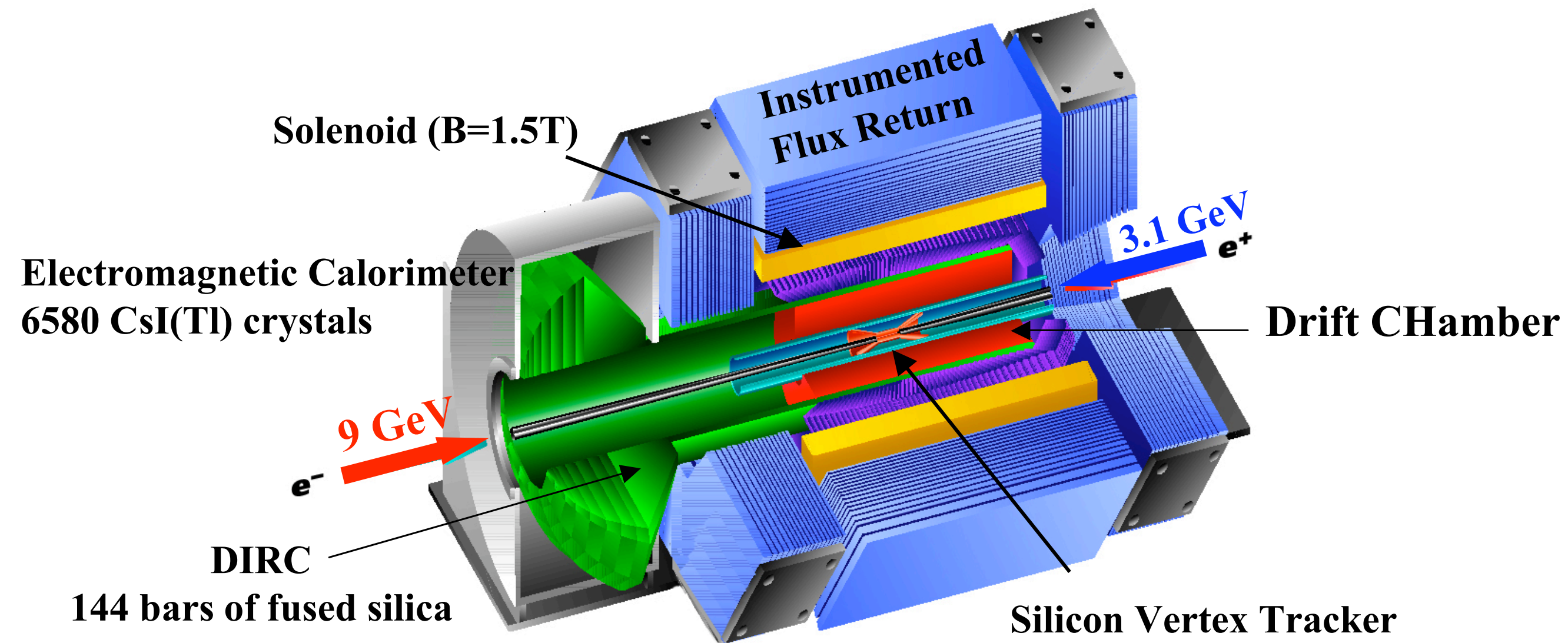
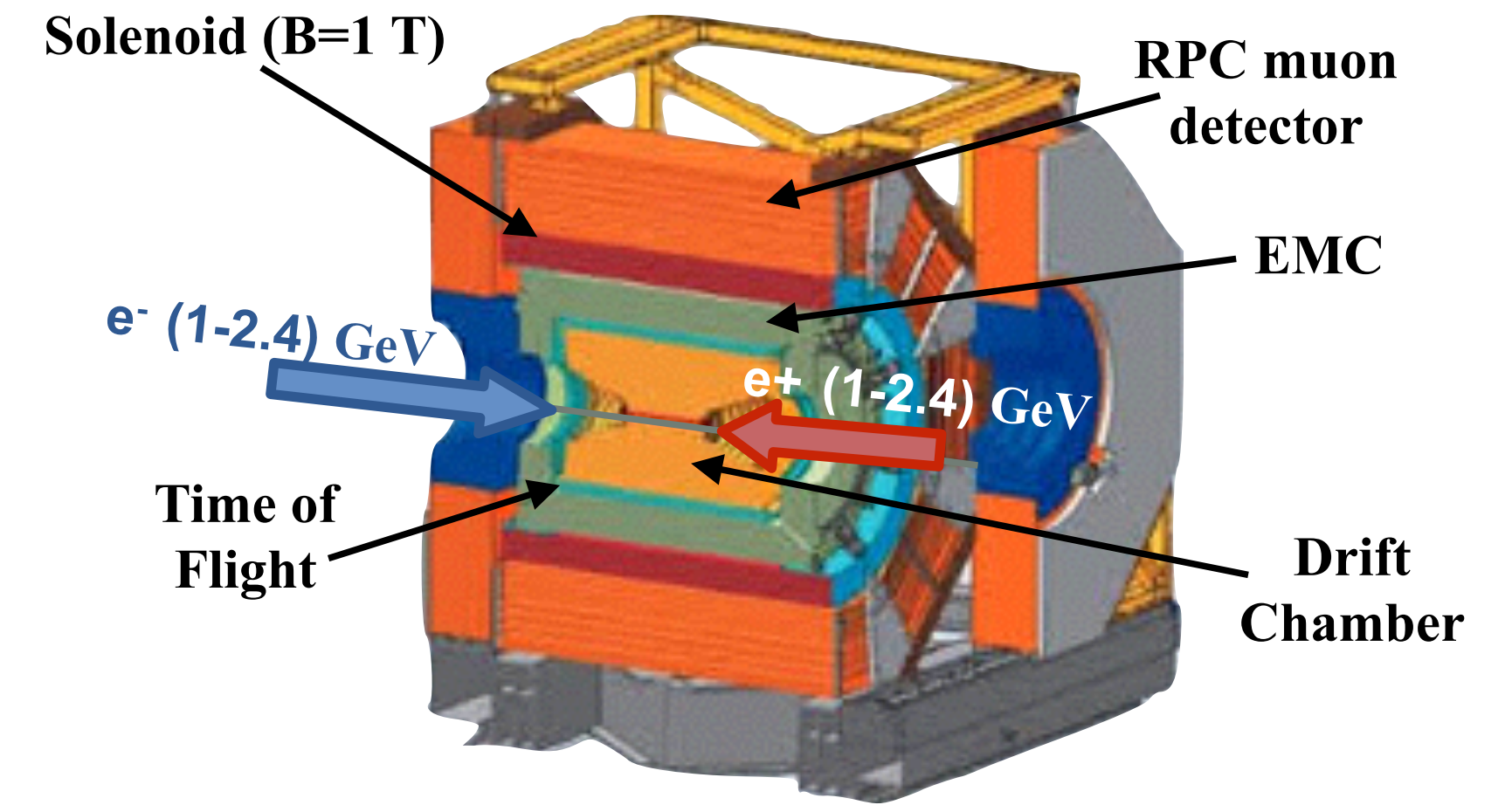
 polarizing FF

⇒ FFs act as quark flavor-tagger and polarimeter

FF ... fragmentation function

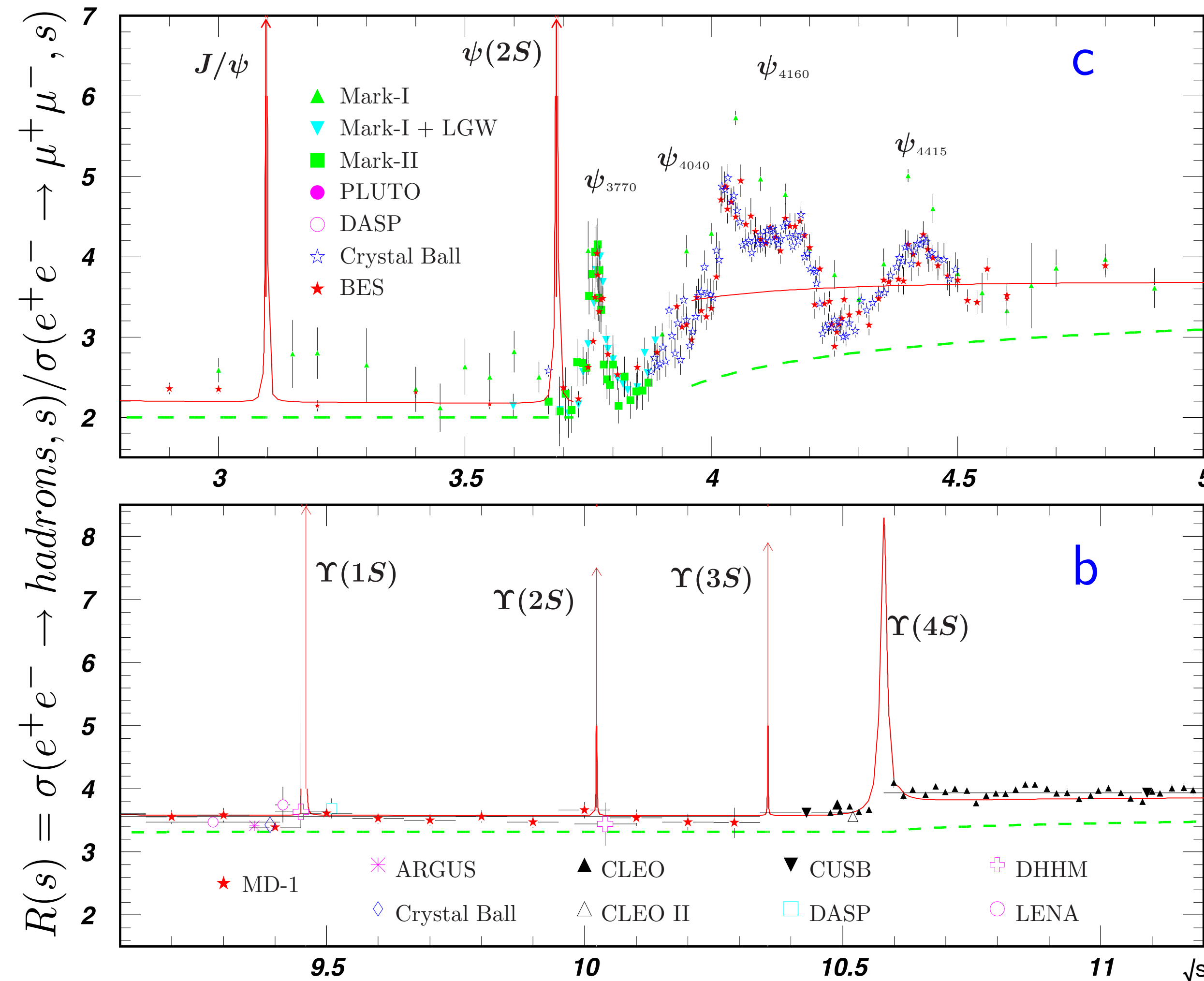
e^+e^- annihilation at BESIII, BaBar & Belle

- BESIII: symmetric collider ($E_e=1\dots 2.4$ GeV)
- BaBar/Belle: asymmetric beam-energy e^+e^- collider near/at $\Upsilon(4S)$ resonance



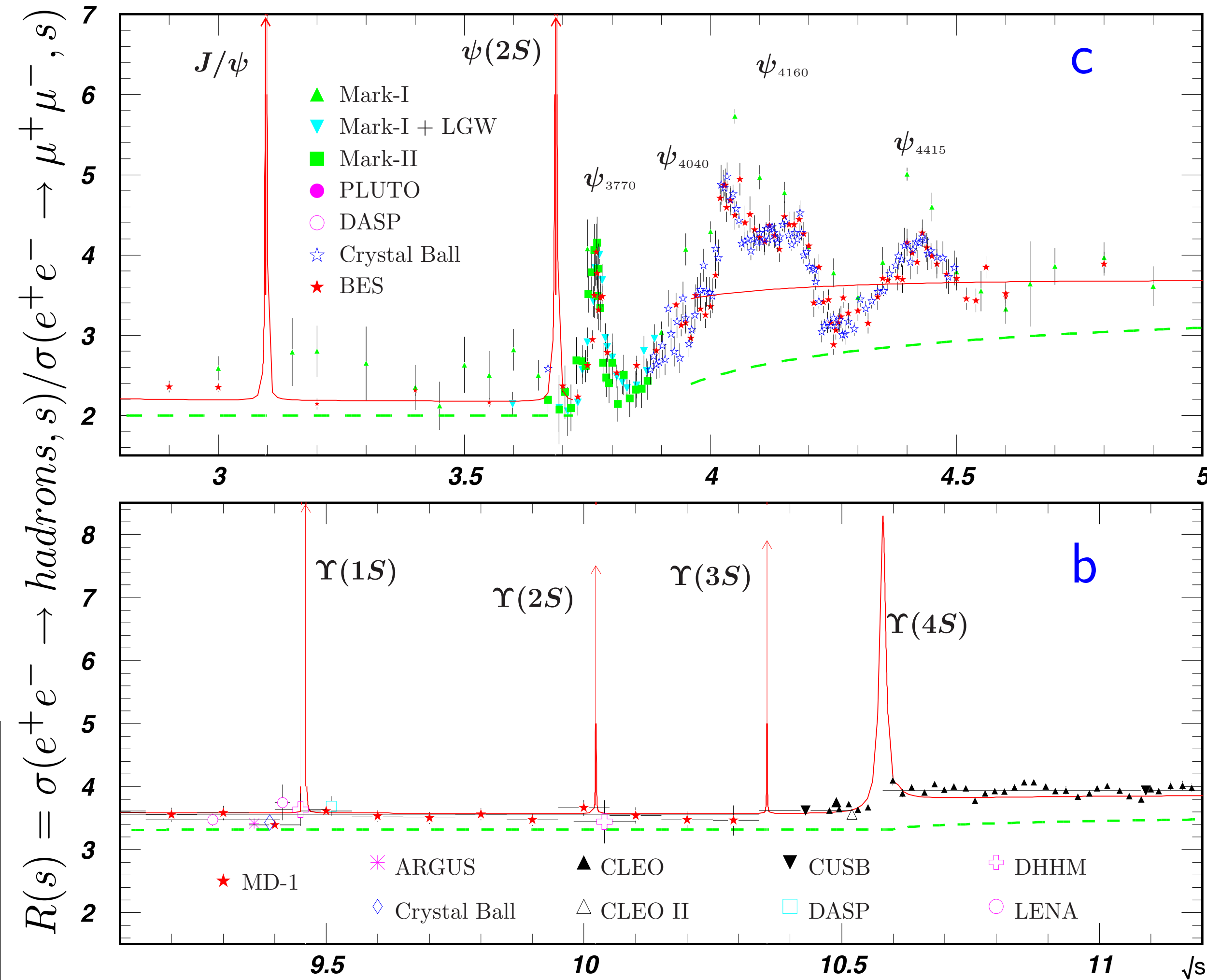
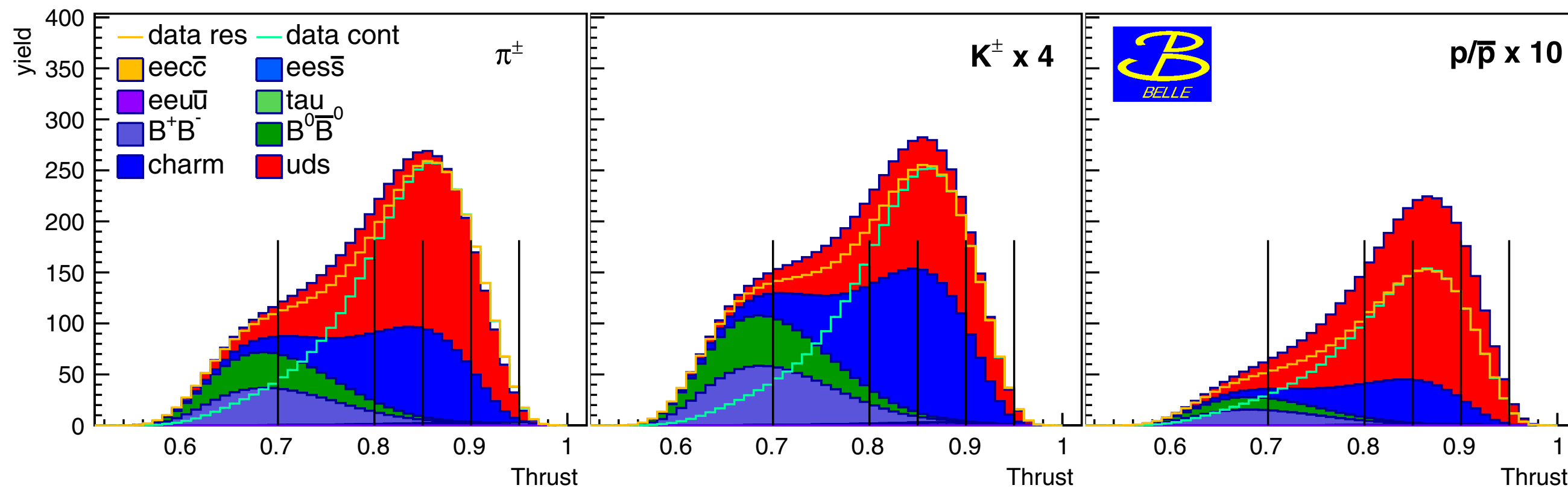
e^+e^- annihilation at BESIII, BaBar & Belle

- BESIII: symmetric collider ($E_e=1\dots 2.4$ GeV)
- BaBar/Belle: asymmetric beam-energy e^+e^- collider near/at $\Upsilon(4S)$ resonance
- different scales (\Rightarrow QCD evolution) and sensitivities to quark flavor
- BESIII below charm threshold closer to typical SIDIS case



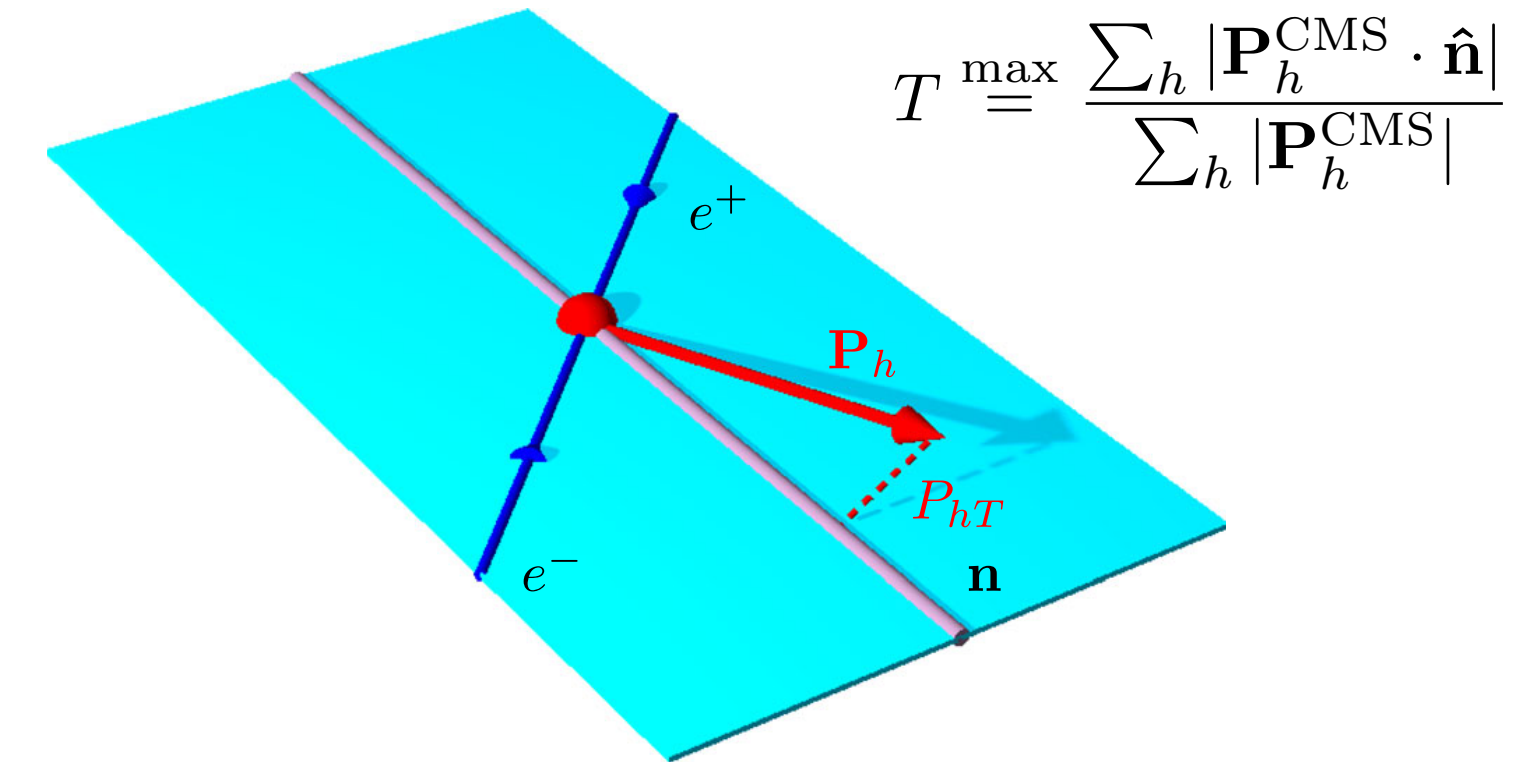
e^+e^- annihilation at BESIII, BaBar & Belle

- BESIII: symmetric collider ($E_e=1\dots 2.4$ GeV)
- BaBar/Belle: asymmetric beam-energy e^+e^- collider near/at $\Upsilon(4S)$ resonance
- different scales (\Rightarrow QCD evolution) and sensitivities to quark flavor
- BESIII below charm threshold closer to typical SIDIS case



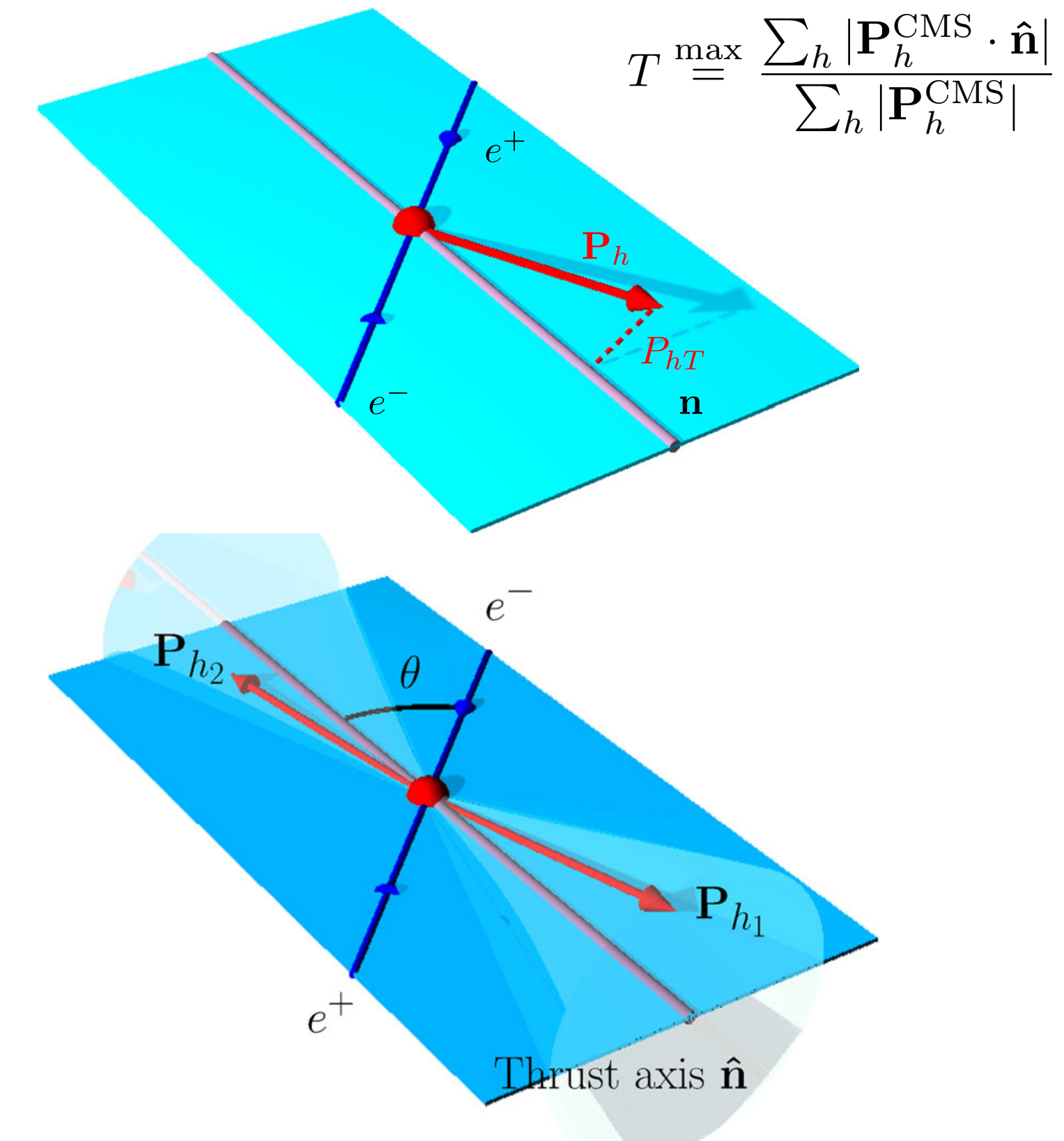
fragmentation in e^+e^- annihilation

- single-inclusive hadron production, $e^+e^- \rightarrow hX$
- D_1 fragmentation function
- (D_{1T^\perp} spontaneous transv. polarization)



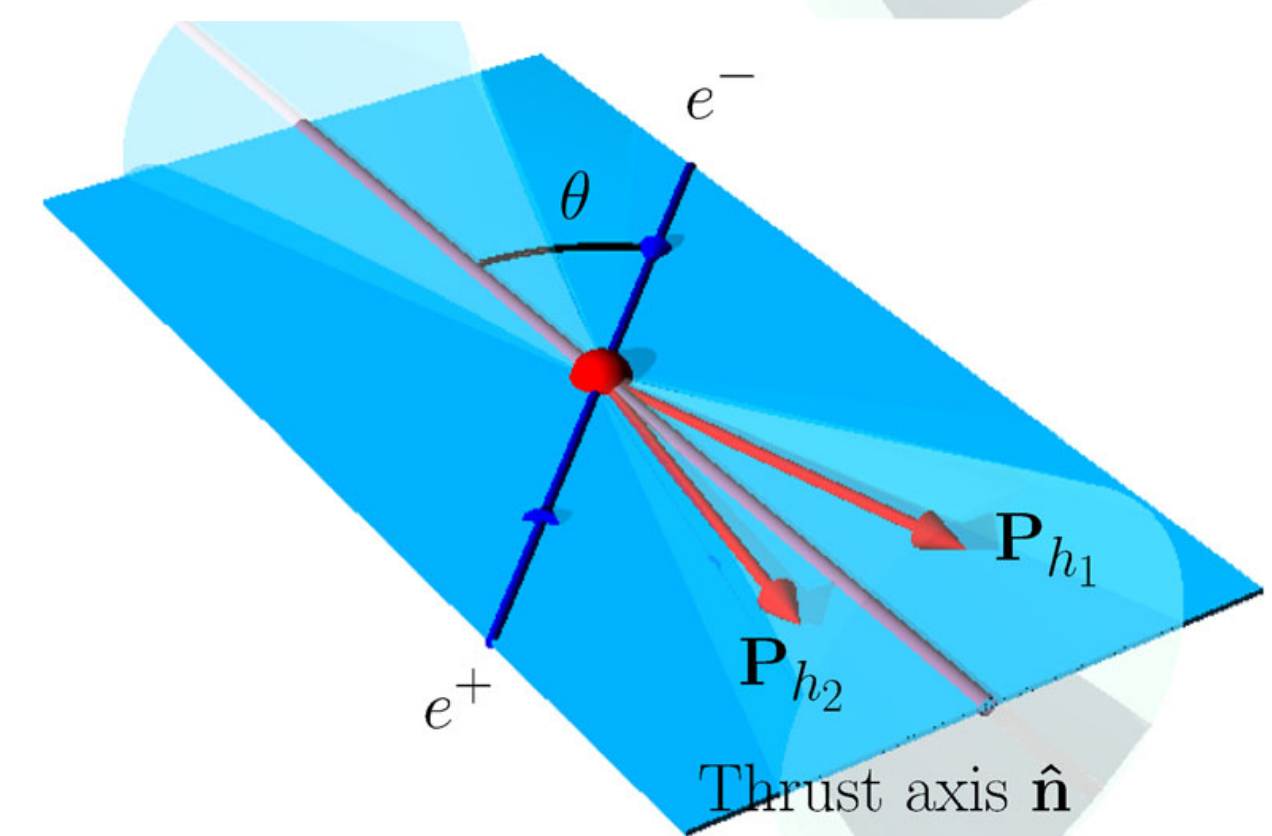
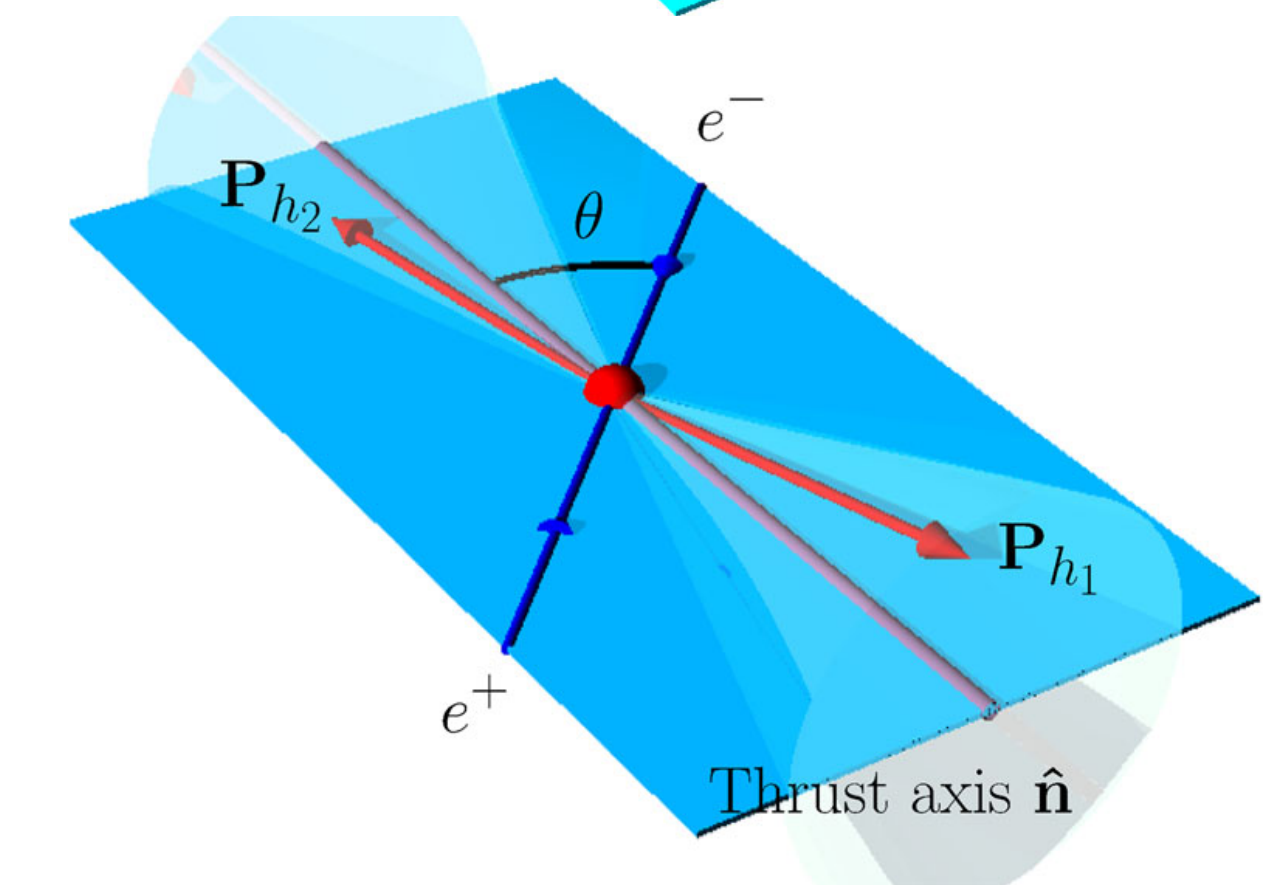
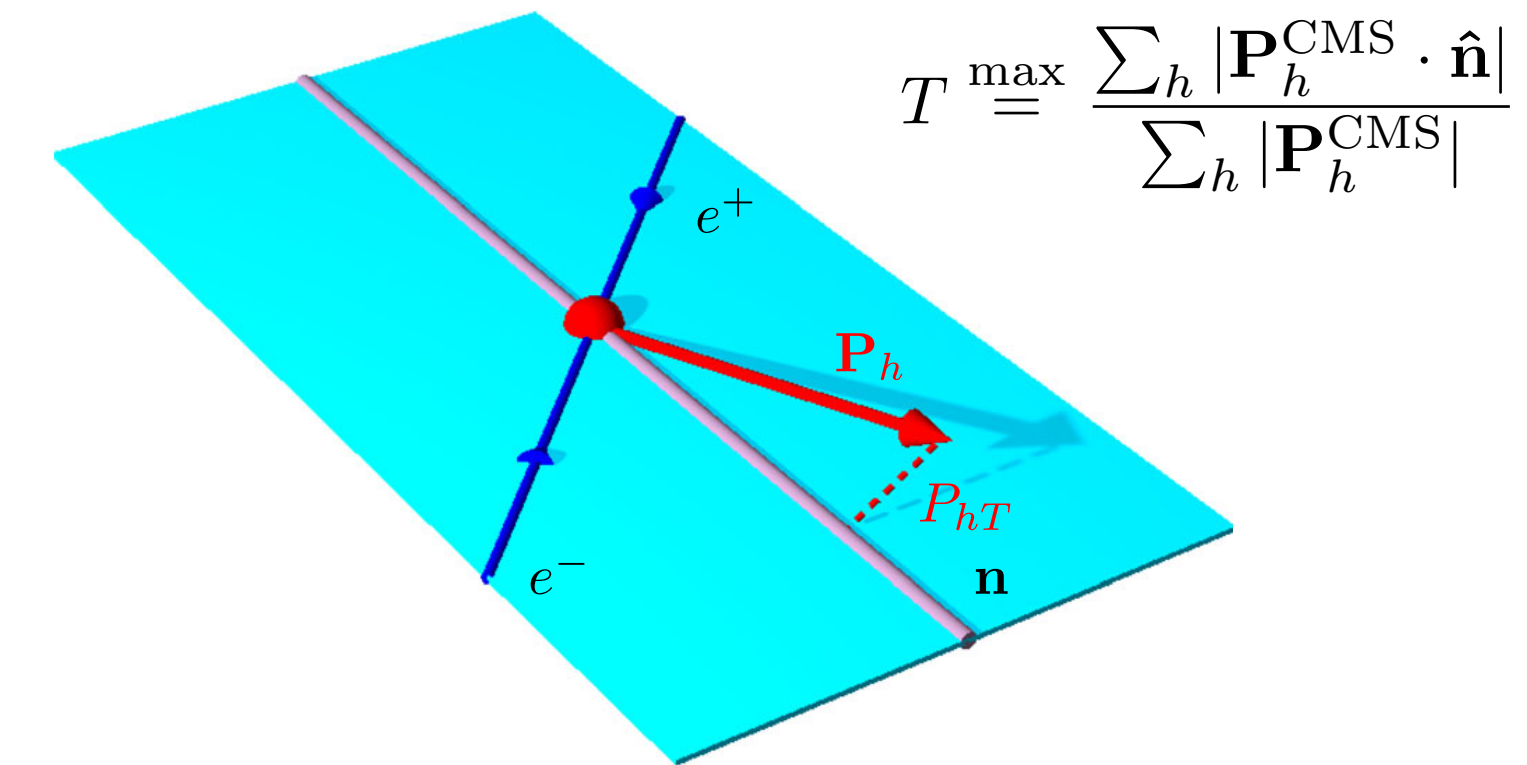
fragmentation in e^+e^- annihilation

- single-inclusive hadron production, $e^+e^- \rightarrow hX$
 - D_1 fragmentation function
 - (D_{1T^\perp} spontaneous transv. polarization)
- inclusive "back-to-back" hadron pairs, $e^+e^- \rightarrow h_1h_2X$
 - product of fragmentation functions
 - allows for **tagging** of flavor, transverse-momentum, and/or polarization



fragmentation in e^+e^- annihilation

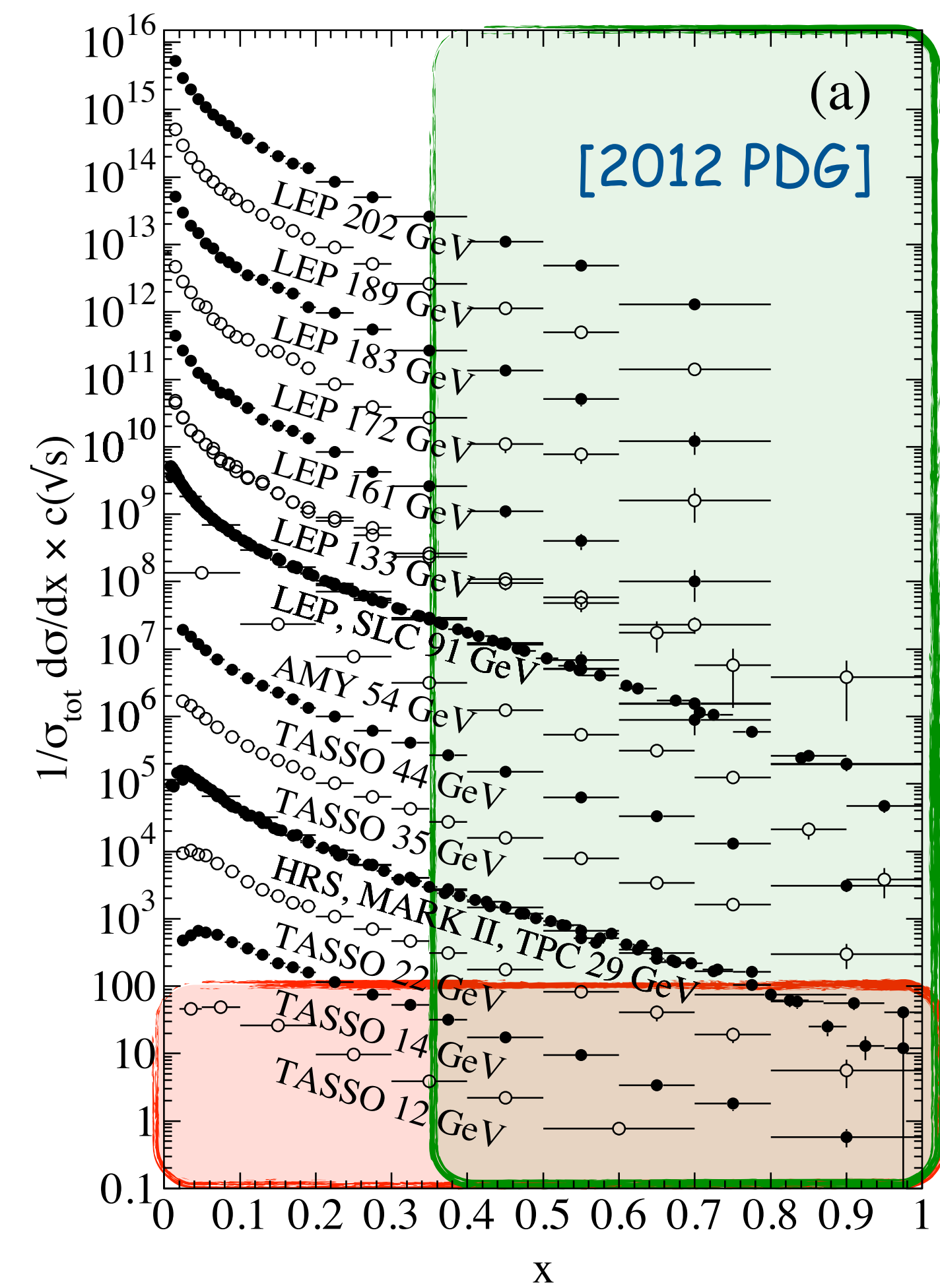
- single-inclusive hadron production, $e^+e^- \rightarrow hX$
 - D_1 fragmentation function
 - (D_{1T^\perp} spontaneous transv. polarization)
- inclusive "back-to-back" hadron pairs, $e^+e^- \rightarrow h_1h_2X$
 - product of fragmentation functions
 - allows for **tagging** of flavor, transverse-momentum, and/or polarization
- inclusive same-hemisphere hadron pairs, $e^+e^- \rightarrow h_1h_2X$
 - di-hadron fragmentation



the collinear case

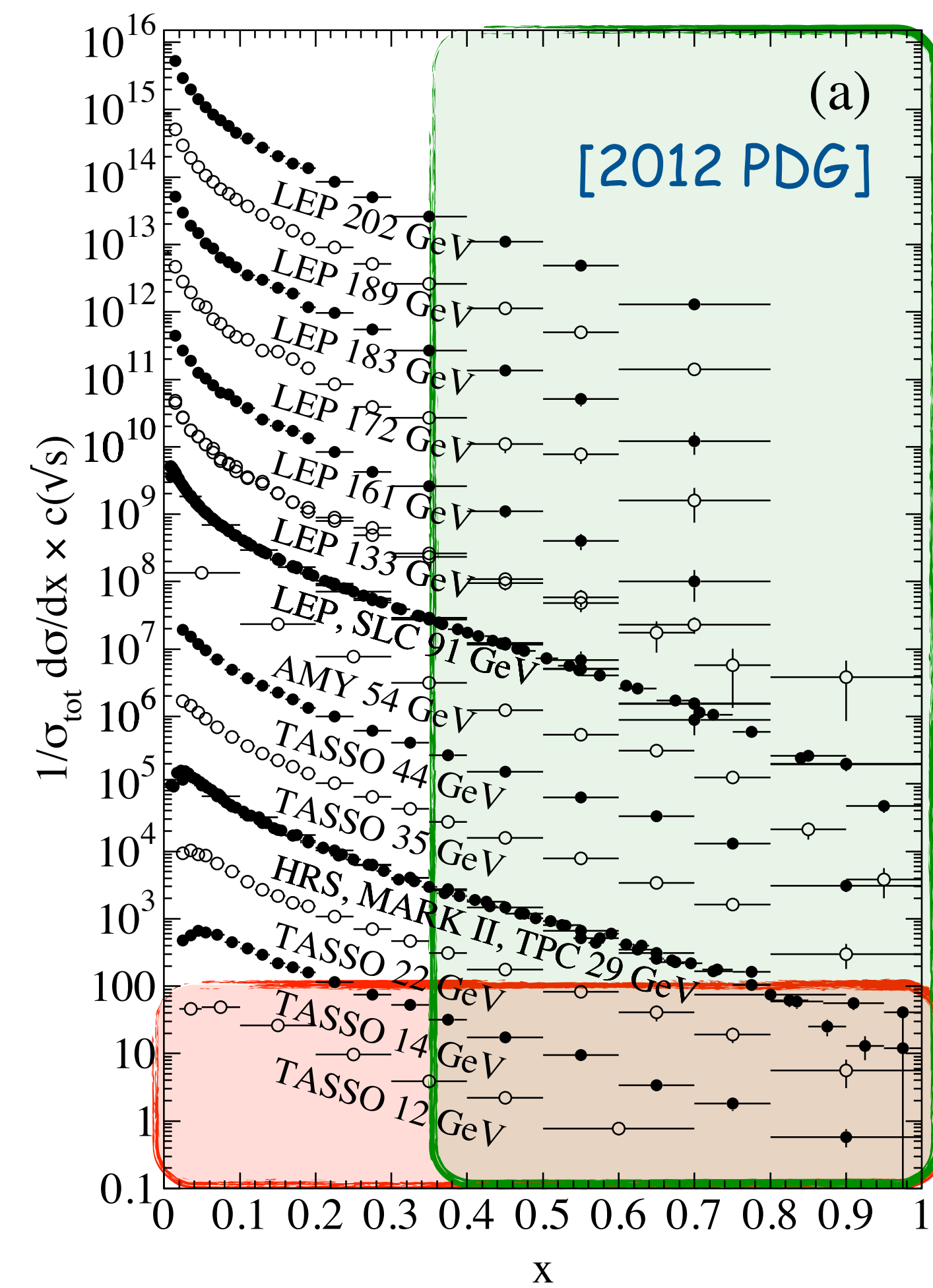
single-hadron production

- before 2013: lack of precision data at (moderately) high z and low \sqrt{s}
- limits analysis of evolution and gluon fragmentation
- limited information in kinematic region often used in semi-inclusive DIS



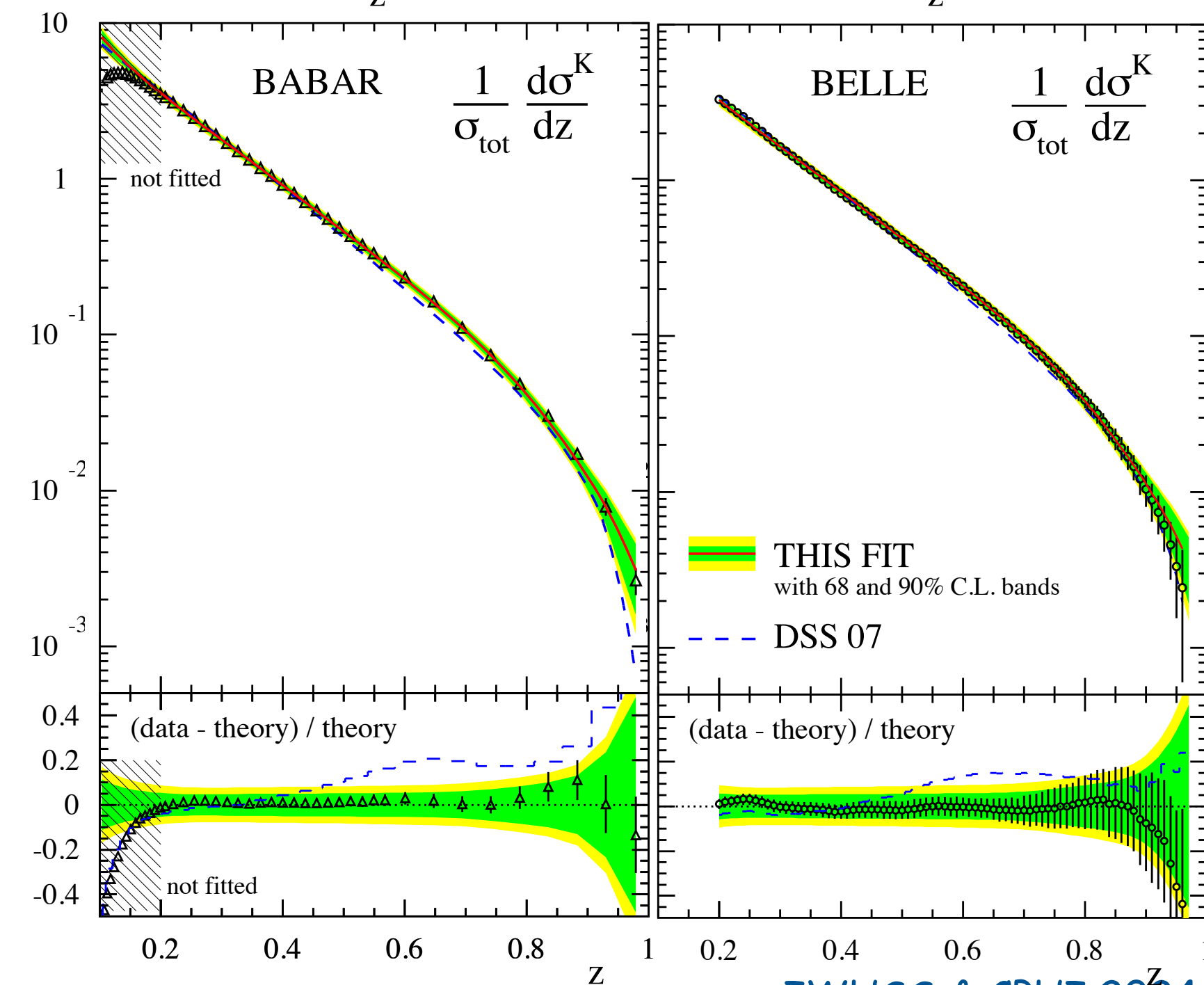
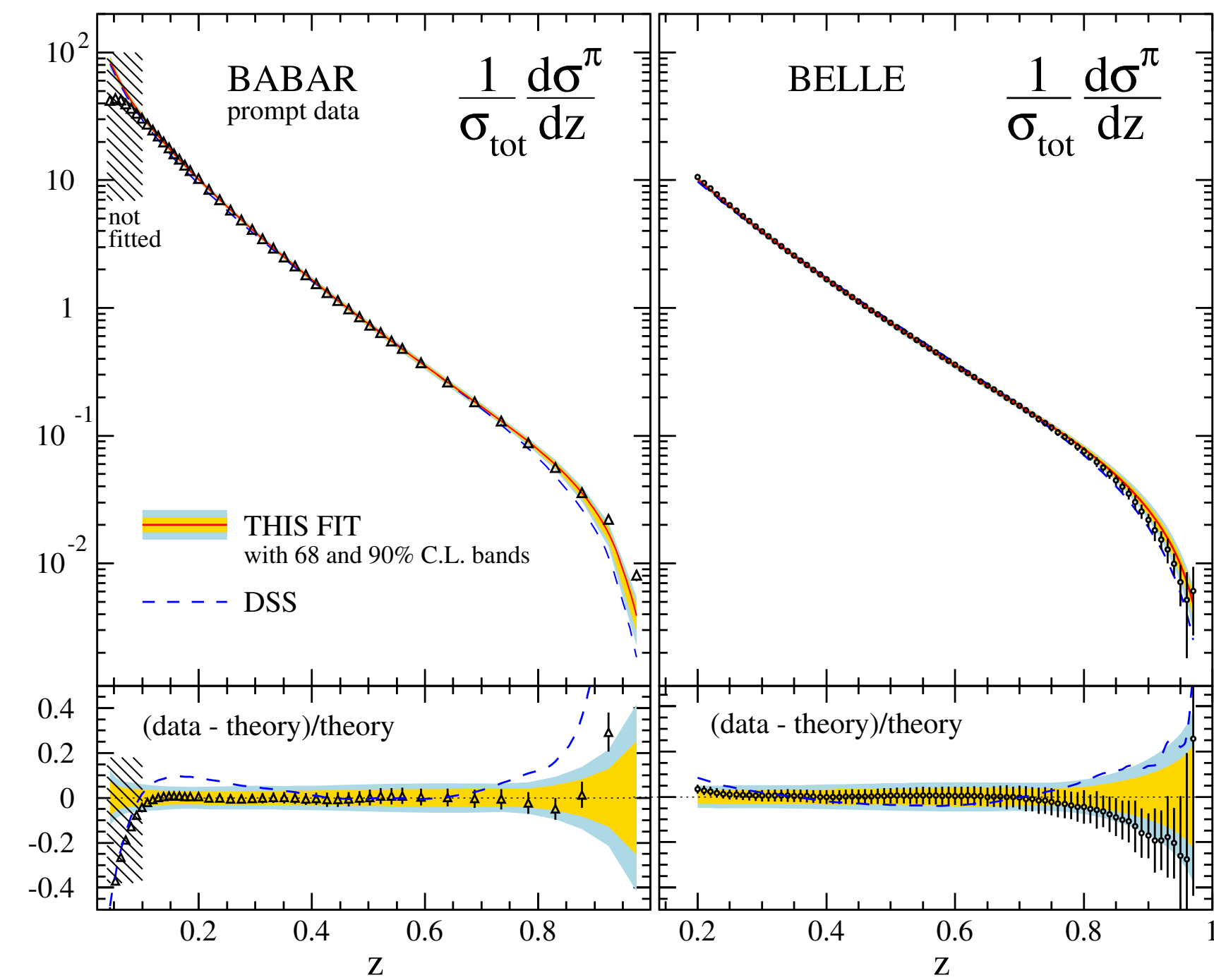
single-hadron production

- before 2013: lack of precision data at (moderately) high z and low \sqrt{s}
- limits analysis of evolution and gluon fragmentation
- limited information in kinematic region often used in semi-inclusive DIS
- by now also results from BaBar, Belle, and BESIII:
 - BaBar Collaboration, PRD 88 (2013) 032011: $\pi^\pm, K^\pm, p+\bar{p}$
 - Belle Collaboration, PRL 111 (2013) 062002: π^\pm, K^\pm
 - Belle Collaboration, PRD 92 (2015) 092007 & 101 (2020) 092004: $\pi^\pm, K^\pm, p+\bar{p}$
 - **NEW:** BESIII Collaboration, PRL 130 (2023) 231901 & 133 (2024) 021901 : π^0, K_S^0 & η



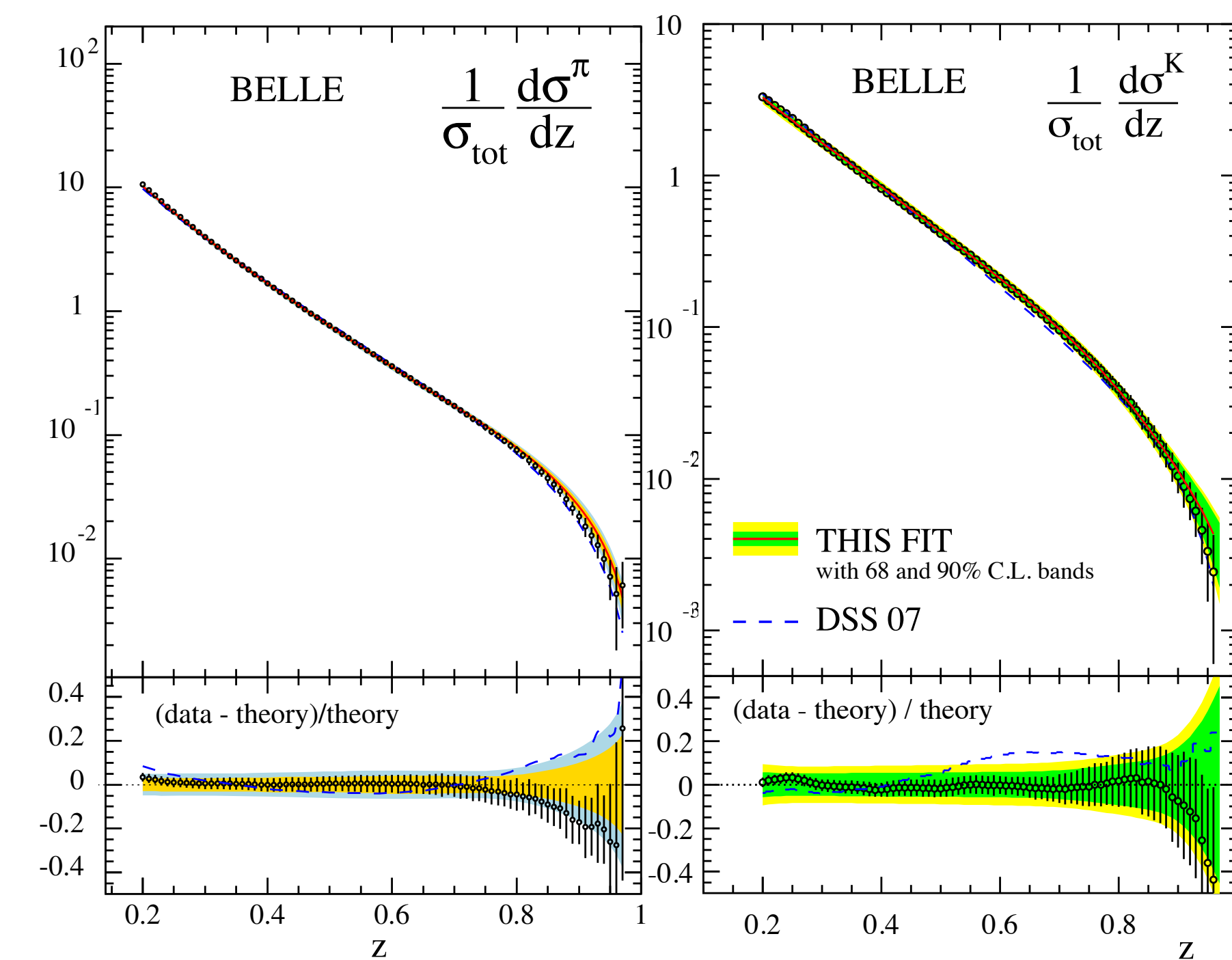
single-hadron production

- very precise data for charged pions and kaons
- Belle data available up to very large z ($z < 0.98$)
- DEHSS fits [e.g., PRD91 (2015) 014035]
- slight tension at low- z for BaBar and high- z for Belle



single-hadron production

- very precise data for charged pions and kaons
- Belle data available up to very large z ($z < 0.98$)
- DEHSS fits [e.g., PRD91 (2015) 014035]
- slight tension at low- z for BaBar and high- z for Belle
- Belle radiative corrections generally “undone” in FF fits

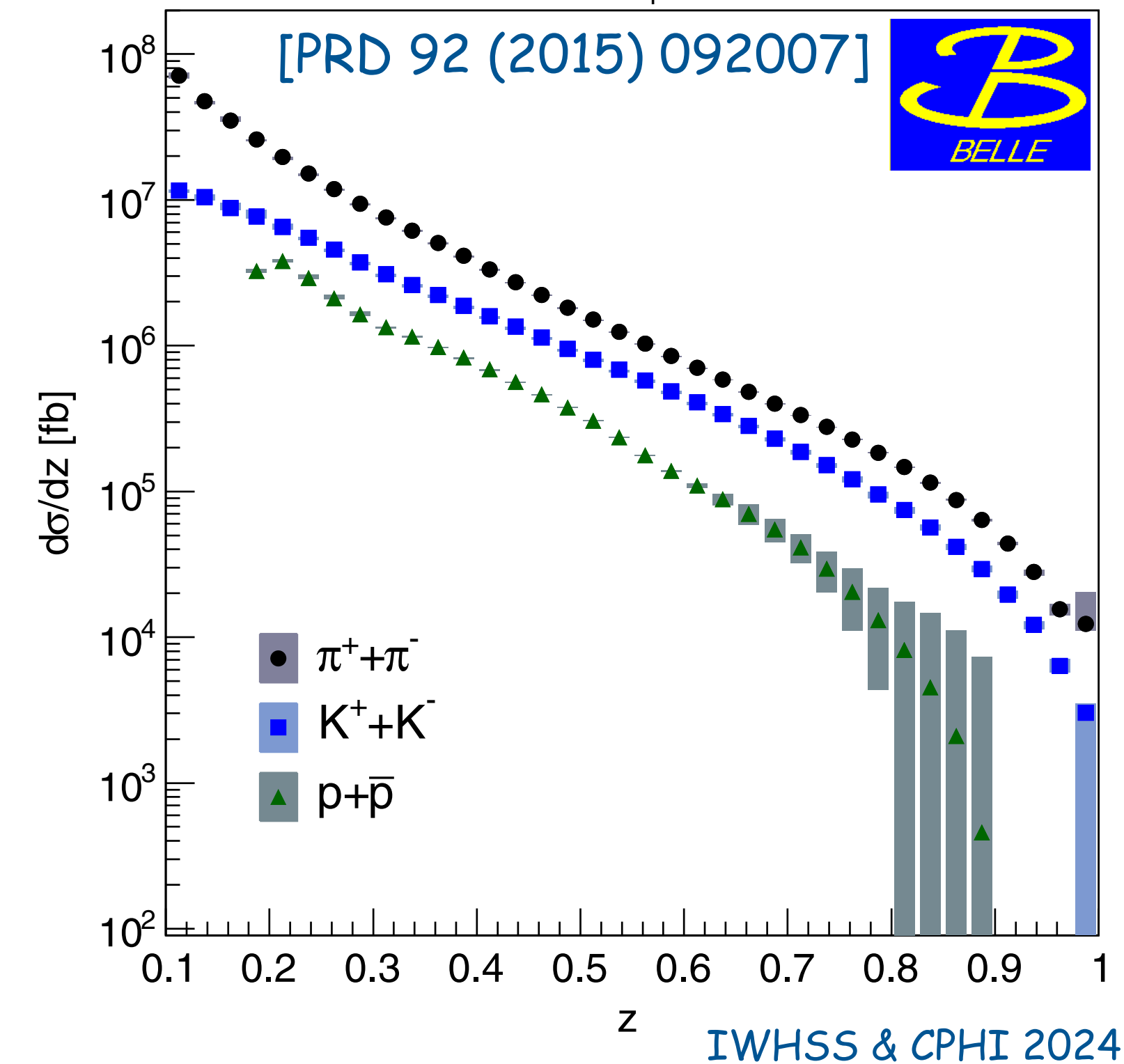
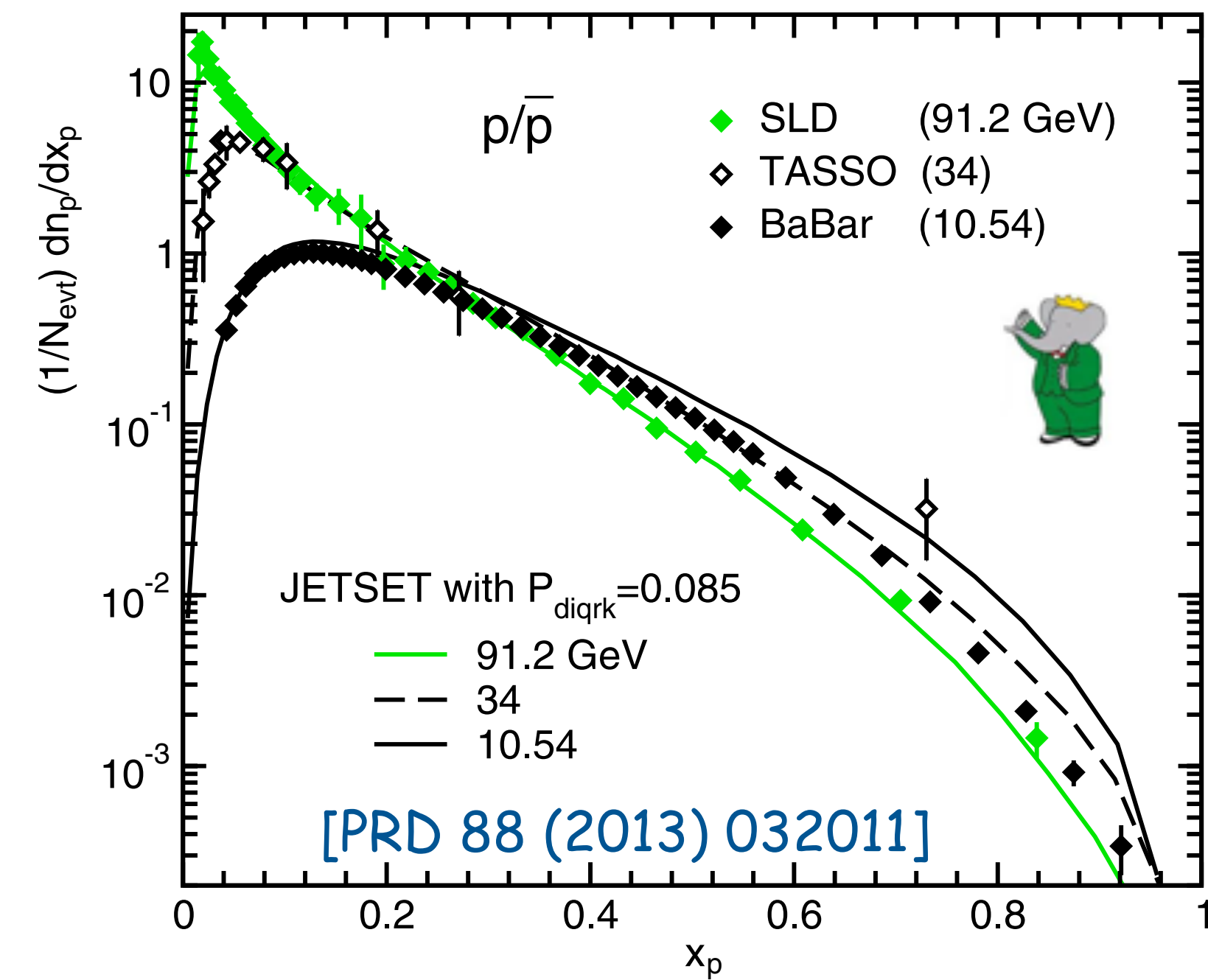


[EPJC 77 (2017) 516, NNFF1.0]

In the case of the BELLE experiment we multiply all data points by a factor $1/c$, with $c = 0.65$ for charged pions and kaons [69] and with c a function of z for protons/antiprotons [53]. This correction is required in order to treat the BELLE data consistently with all the other SIA measurements included in NNFF1.0. The reason is that a kinematic cut on radiative photon events was applied to the BELLE data sample in the original analysis instead of unfolding the radiative QED effects. Specifically, the energy scales

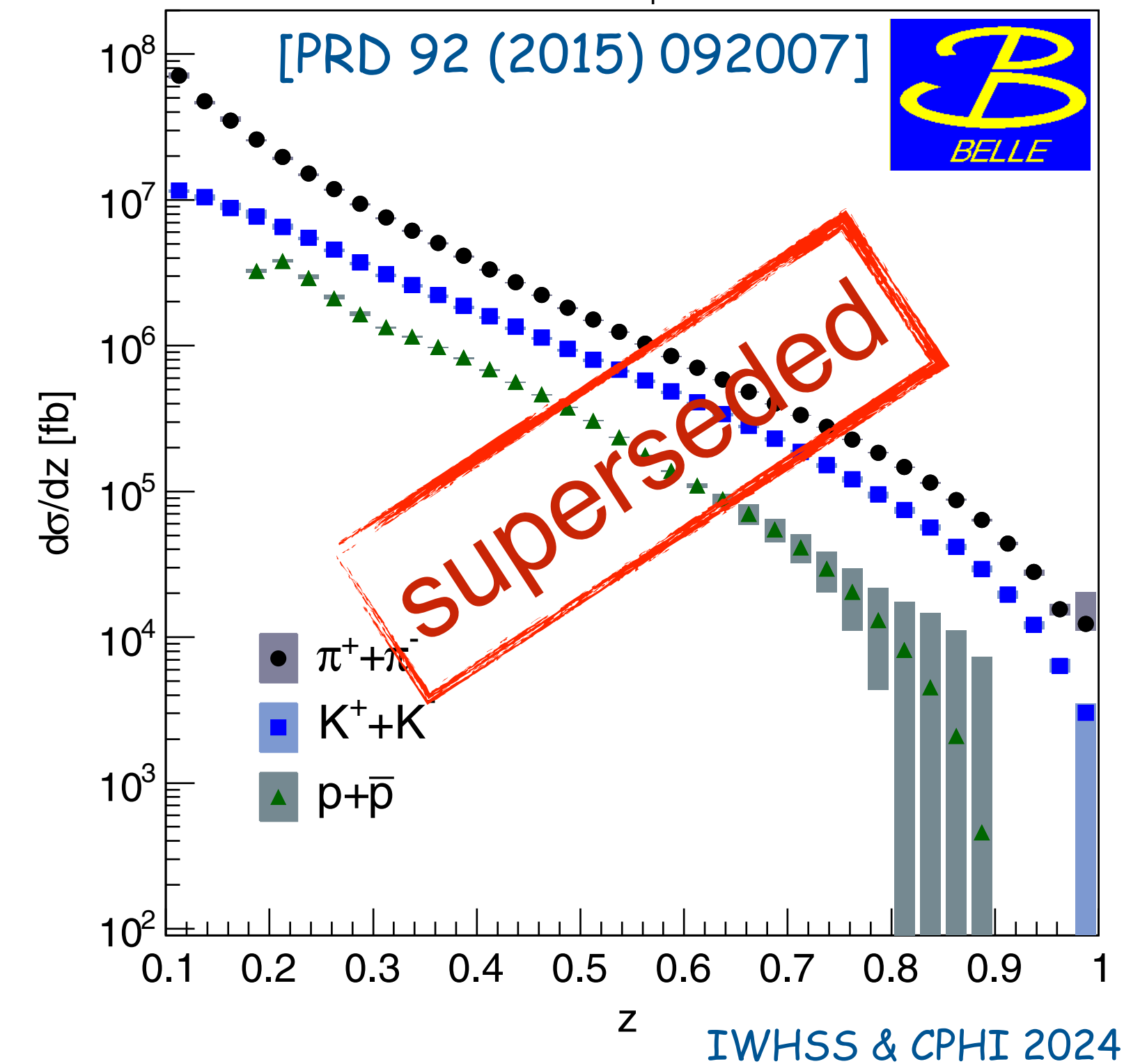
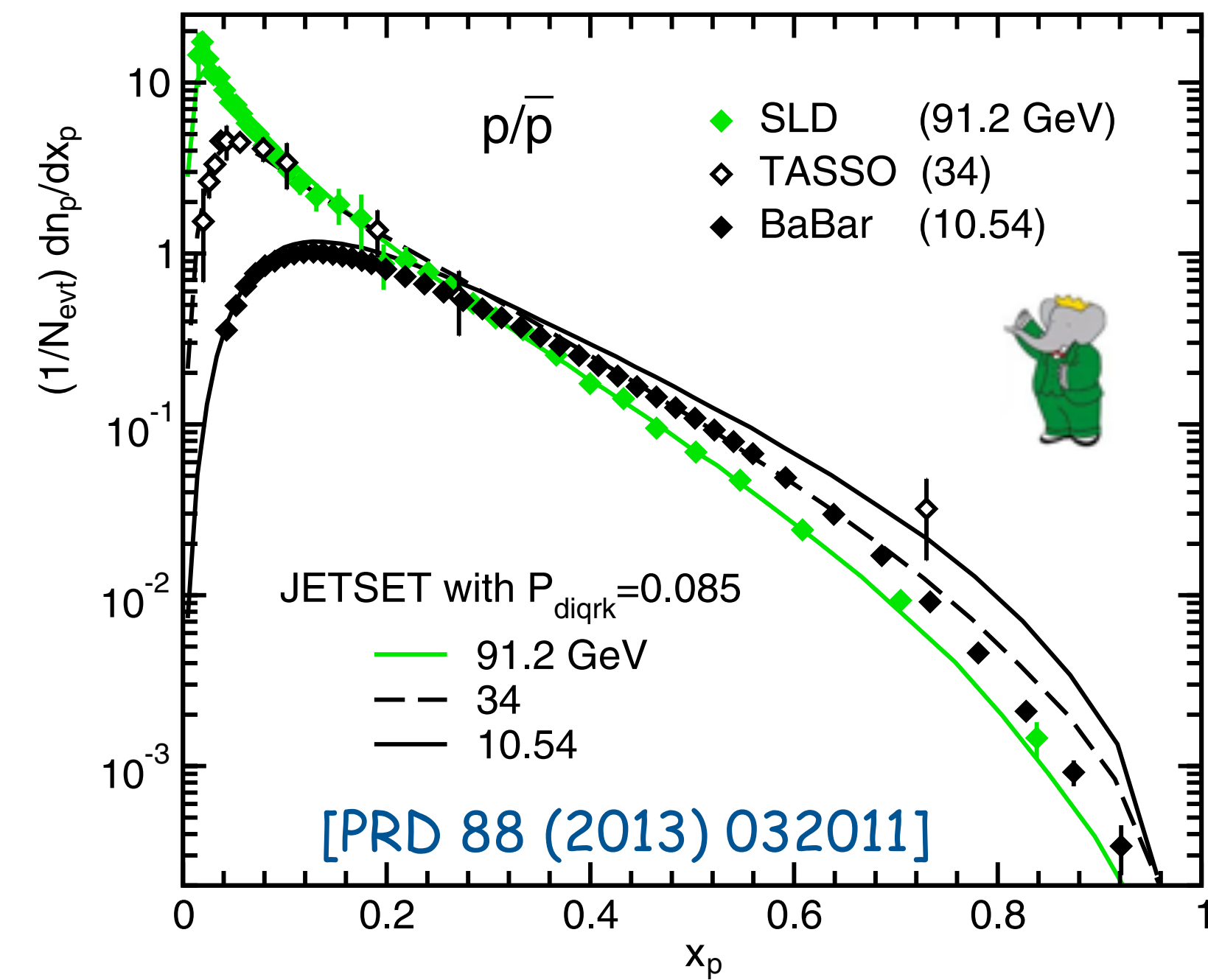
single-hadron production

- very precise data for charged pions and kaons
- Belle data available up to very large z ($z < 0.98$)
- 2015 DEHSS fits [e.g., PRD91 (2015) 014035]
 - slight tension at low- z for BaBar and high- z for Belle
- Belle radiative corrections generally “undone” in FF fits
- data available for (anti)protons
 - similar z dependence as pions
 - about $\sim 1/5$ of pion cross sections



single-hadron production

- very precise data for charged pions and kaons
- Belle data available up to very large z ($z < 0.98$)
- 2015 DEHSS fits [e.g., PRD91 (2015) 014035]
 - slight tension at low- z for BaBar and high- z for Belle
- Belle radiative corrections generally “undone” in FF fits
- data available for (anti)protons
 - similar z dependence as pions
 - about $\sim 1/5$ of pion cross sections
- Belle re-analysis presented in PRD 101 (2020) 092004



interlude
about counting

- cross sections are basically count rates
- “how to count?” sounds like a simple question, but the devil is in the details
 - what to do with hadrons that have (somewhere!) an ISR photon
 - in general, how to deal with events that are assigned to “wrong” kinematic bin due to instrumental effects [e.g., measured and true momentum might differ]
 - book-keeping of assigning event's contribution to bin's statistical uncertainty

- cross sections are basically count rates
- “how to count?” sounds like a simple question, but the devil is in the details
 - what to do with hadrons that have (somewhere!) an ISR photon
 - in general, how to deal with events that are assigned to “wrong” kinematic bin due to instrumental effects [e.g., measured and true momentum might differ]
 - book-keeping of assigning event's contribution to bin's statistical uncertainty
- hadron yields also undergo series of other corrections:
 - particle (mis)identification [e.g., not every identified pion was a pion]
 - non-qq processes [e.g., two-photon processes, $\Upsilon \rightarrow BB, \dots$]
 - “ 4π ” correction [e.g., selection criteria and limited geometric acceptance]
 - “optional”: weak-decay removal [e.g., “prompt fragmentation”]

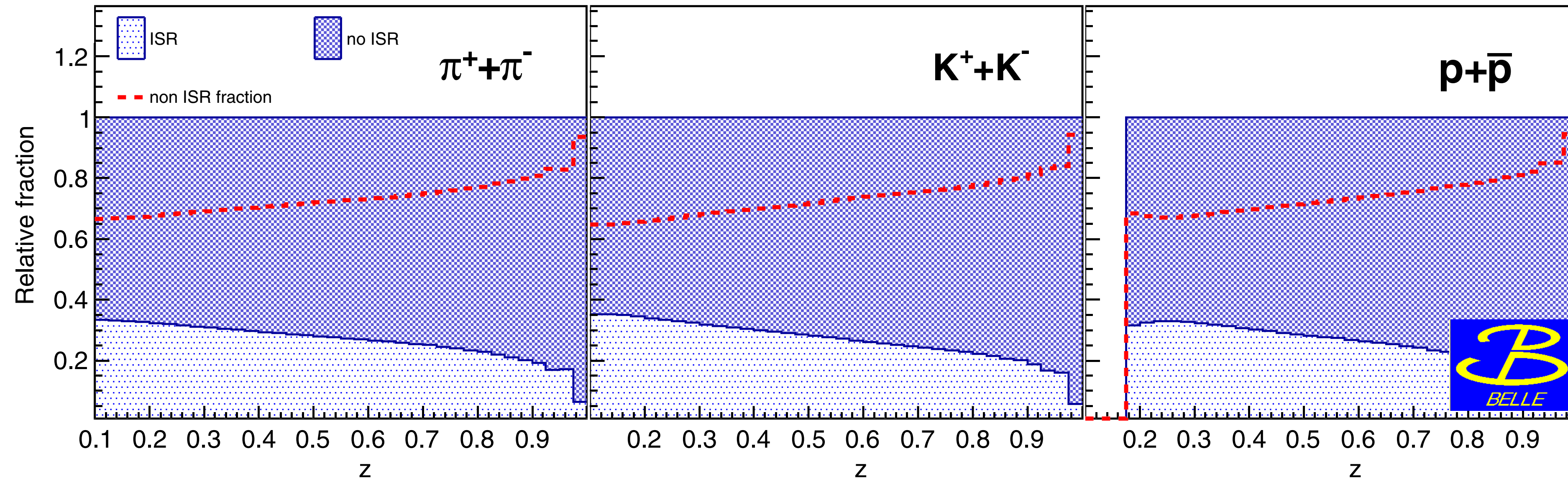
- what to do with hadrons that have (somewhere) an ISR photon
 - nothing! — leave it to phenomenology to deal with QED corrections
 - however, (uncorrected/corrected) yields are ISR & detector dependent
 - reject all events that have an isolated photon?
 - detectors almost never fully hermetic, many ISR photons travel down the beam pipe
 - still fully inclusive reaction?
 - use some Monte Carlo to estimate event fraction with an ISR photon that carries away more than $x\%$ of total available energy (e.g., 0.5% as in earlier Belle analyses)
 - what is a reasonable choice for x ?
 - ISR treatment model dependent, indeed depends on annihilation cross section
 - use some Monte Carlo to estimate ratio of hadrons produced in absence of ISR vs. full QED+QCD simulation
 - again model dependent: number of hadrons produced at given z for different s depends on differential cross section (e.g., from evolution)

- what to do with hadrons that have (somewhere) an ISR photon
 - nothing! — leave it to phenomenology to deal with QED corrections
 - however, (uncorrected/corrected) yields are ISR & detector dependent
 - reject all events that have an isolated photon?
 - detectors almost never fully hermetic, many ISR photons travel down the beam pipe
 - still fully inclusive reaction?
- use some Monte Carlo to estimate event fraction with an ISR photon that carries away more than $x\%$ of total available energy (e.g., 0.5% as in earlier Belle analyses)
 - what is a reasonable choice for x ?
 - ISR treatment model dependent, indeed depends on annihilation cross section
- use some Monte Carlo to estimate ratio of hadrons produced in absence of ISR vs. full QED+QCD simulation
 - again model dependent: number of hadrons produced at given z for different s depends on differential cross section (e.g., from evolution)

- what to do with hadrons that have (somewhere) an ISR photon
 - nothing! — leave it to phenomenology to deal with QED corrections
 - however, (uncorrected/corrected) yields are ISR & detector dependent
 - reject all events that have an isolated photon?
 - detectors almost never fully hermetic, many ISR photons travel down the beam pipe
 - still fully inclusive reaction?
 - use some Monte Carlo to estimate event fraction with an ISR photon that carries away more than $x\%$ of total available energy (e.g., 0.5% as in earlier Belle analyses)
 - what is a reasonable choice for x ?
 - ISR treatment model dependent, indeed depends on annihilation cross section
 - use some Monte Carlo to estimate ratio of hadrons produced in absence of ISR vs. full QED+QCD simulation
 - again model dependent: number of hadrons produced at given z for different s depends on differential cross section (e.g., from evolution)

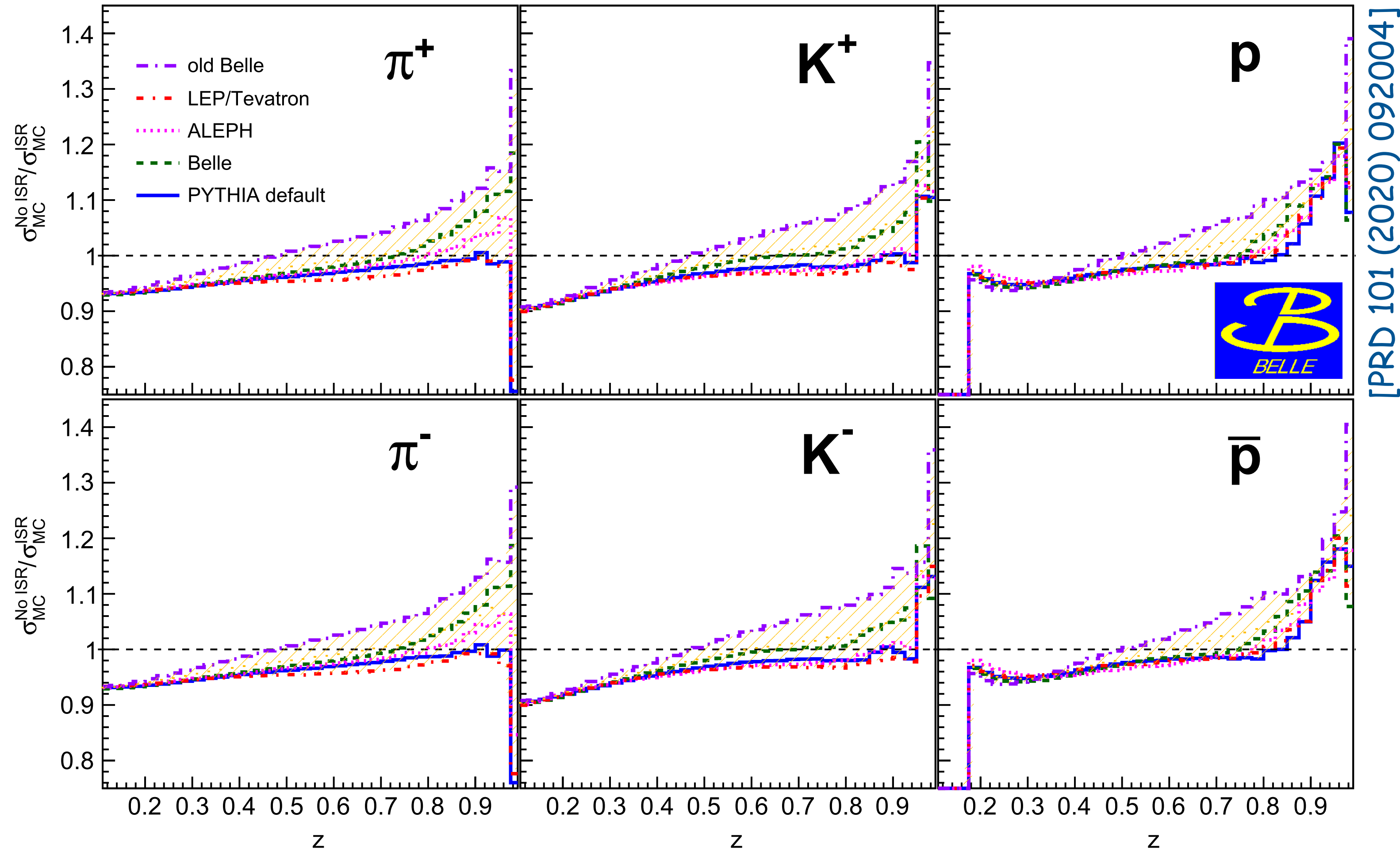
- what to do with hadrons that have (somewhere) an ISR photon
 - nothing! — leave it to phenomenology to deal with QED corrections
 - however, (uncorrected/corrected) yields are ISR & detector dependent
 - reject all events that have an isolated photon?
 - detectors almost never fully hermetic, many ISR photons travel down the beam pipe
 - still fully inclusive reaction?
- use some Monte Carlo to estimate event fraction with an ISR photon that carries away more than $x\%$ of total available energy (e.g., 0.5% as in earlier Belle analyses)
 - what is a reasonable choice for x ?
 - ISR treatment model dependent, indeed depends on annihilation cross section
- use some Monte Carlo to estimate ratio of hadrons produced in absence of ISR vs. full QED+QCD simulation
 - again model dependent: number of hadrons produced at given z for different s depends on differential cross section (e.g., from evolution)

ISR corrections - PRD 92 (2015) 092007



- relative fractions of hadrons as a function of z originating from ISR or non-ISR events (\equiv energy loss less than 0.5%)
- large non-ISR fraction at large z , as otherwise not kinematically reachable (remember $z = E_h / 0.5\sqrt{s_{\text{nominal}}}$)
- keep only fraction of the events \rightarrow strictly speaking not single-inclusive annihilation
- currently used constant 0.65 correction to undo ISR correction is not a constant vs. z

ISR corrections - PRD 101 (2020) 092004

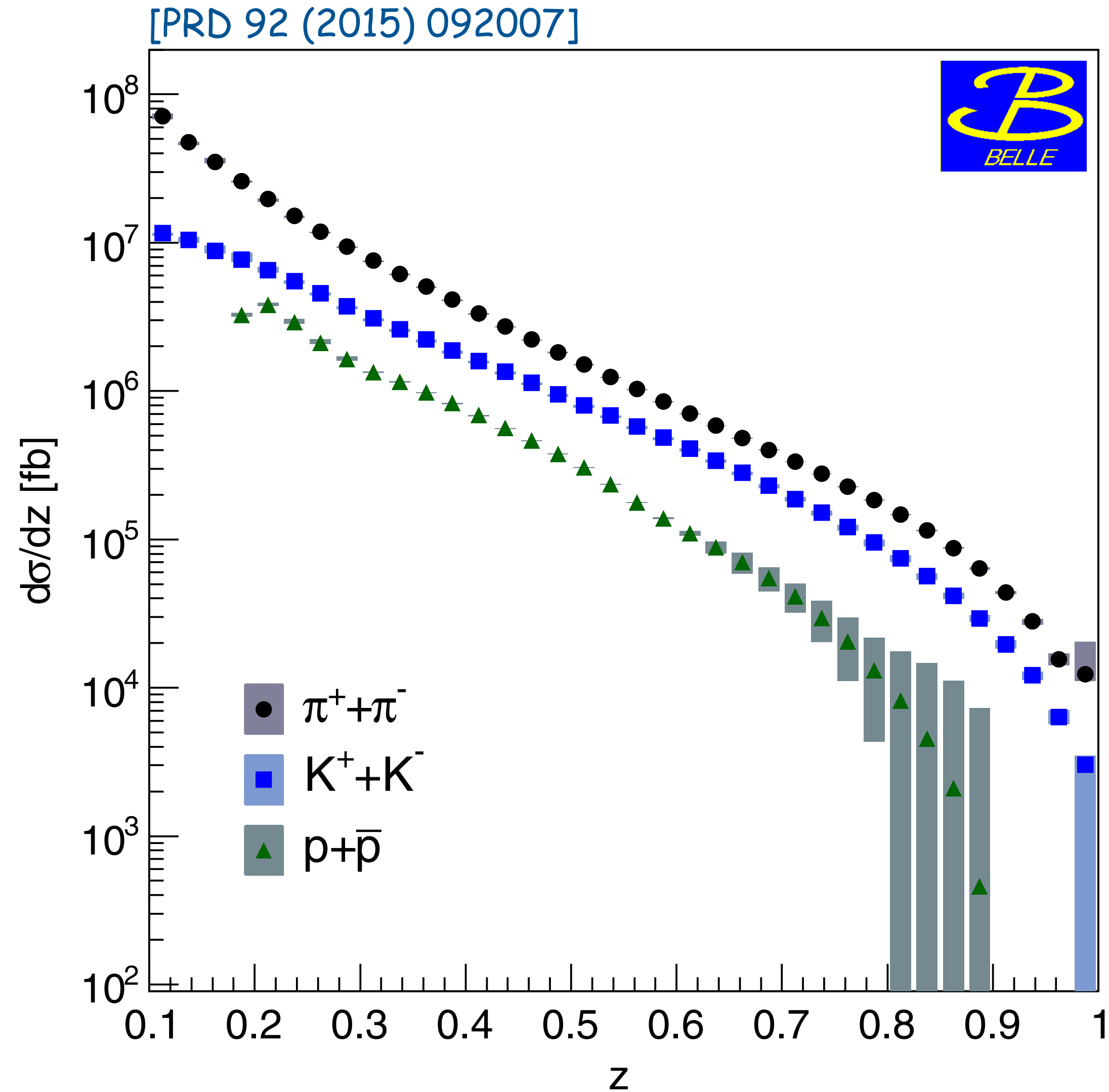


[PRD 101 (2020) 092004]

- non-ISR / ISR fractions based on PYTHIA switch MSTP(11)
- PYTHIA model dependence; absorbed in systematics by variation of tunes

comparison old&new Belle single-hadron cross sections

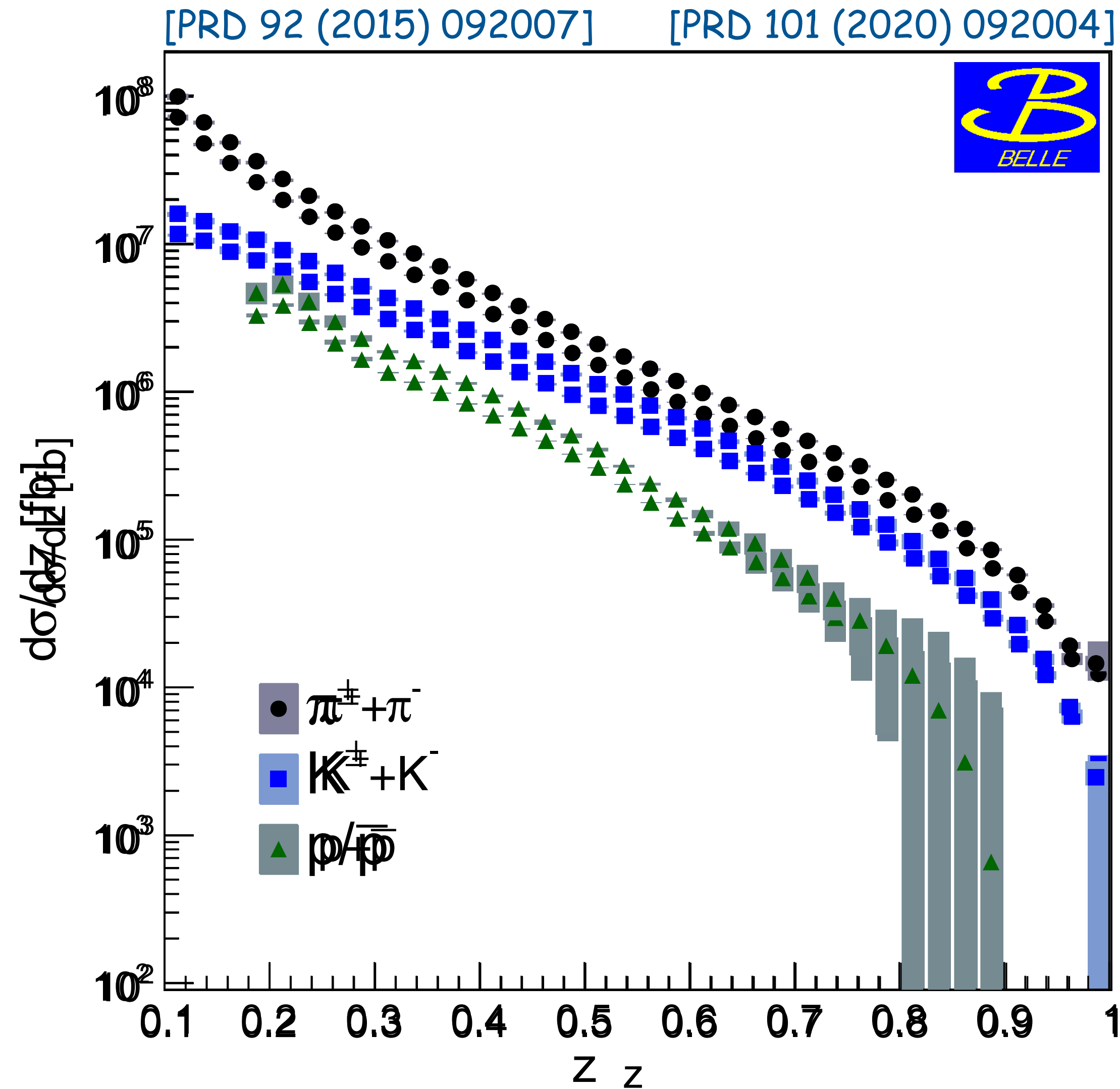
● previous analysis



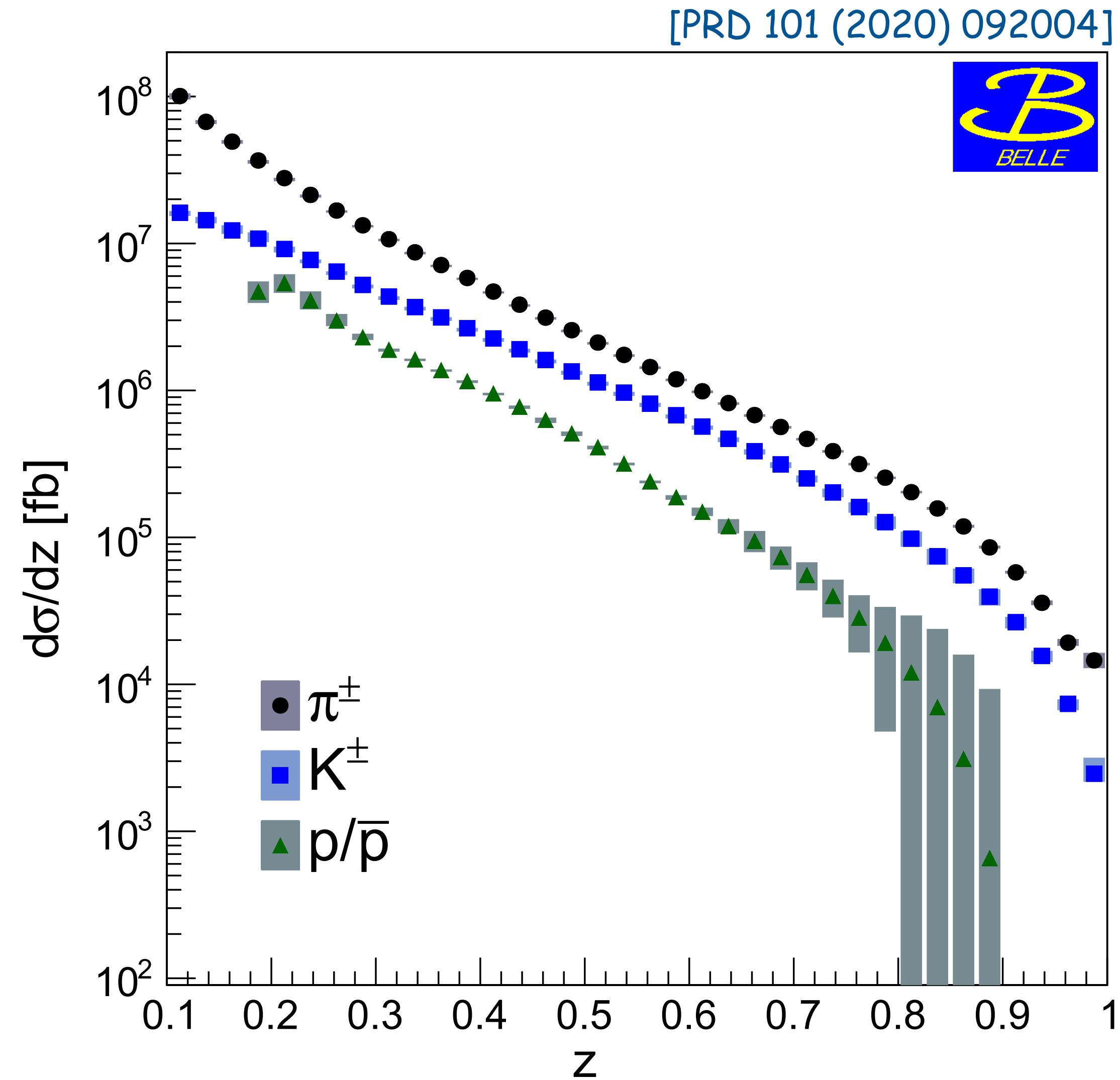
comparison old&new Belle single-hadron cross sections

● previous analysis

● updated analysis

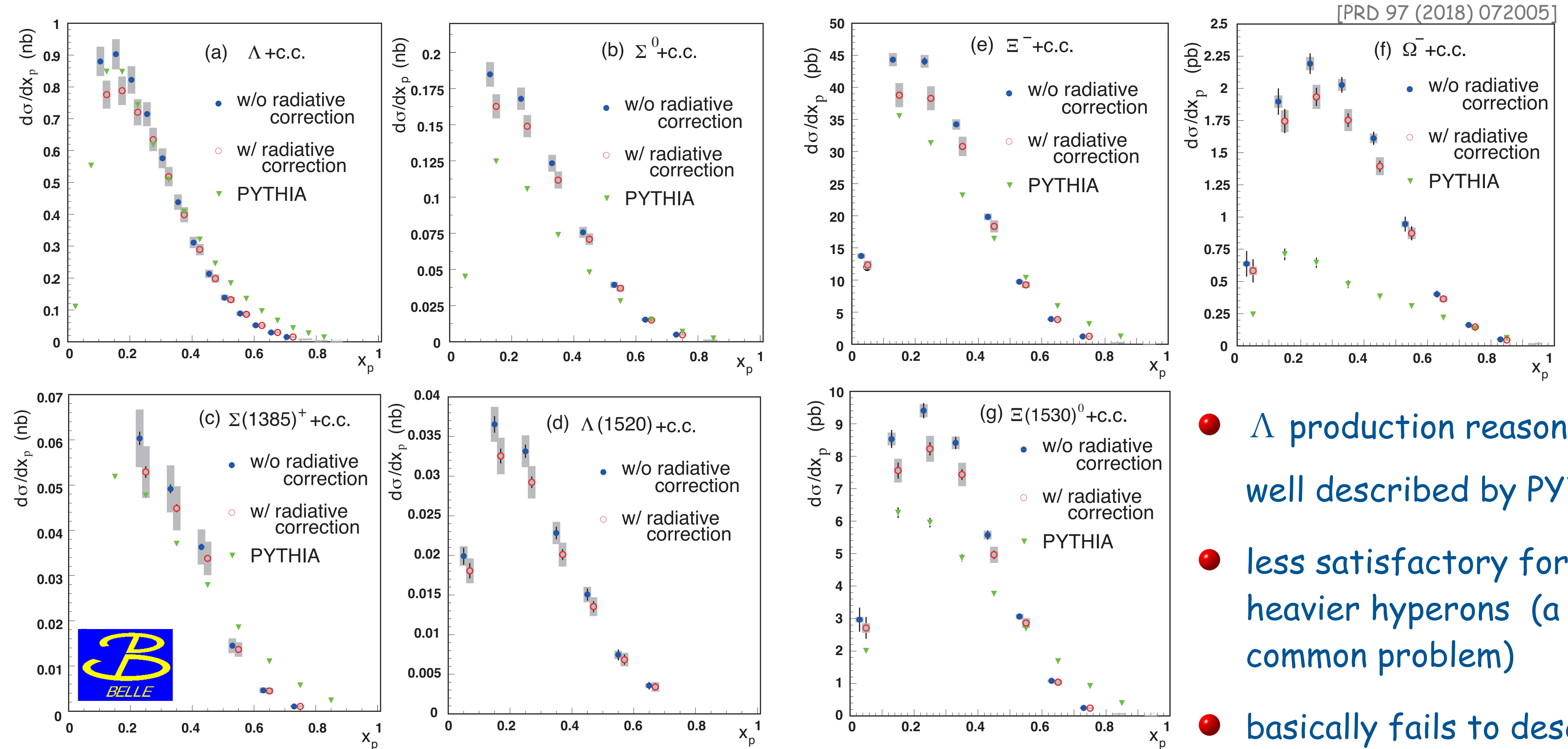


comparison old&new Belle single-hadron cross sections



● updated analysis

single-hadron production: hyperons

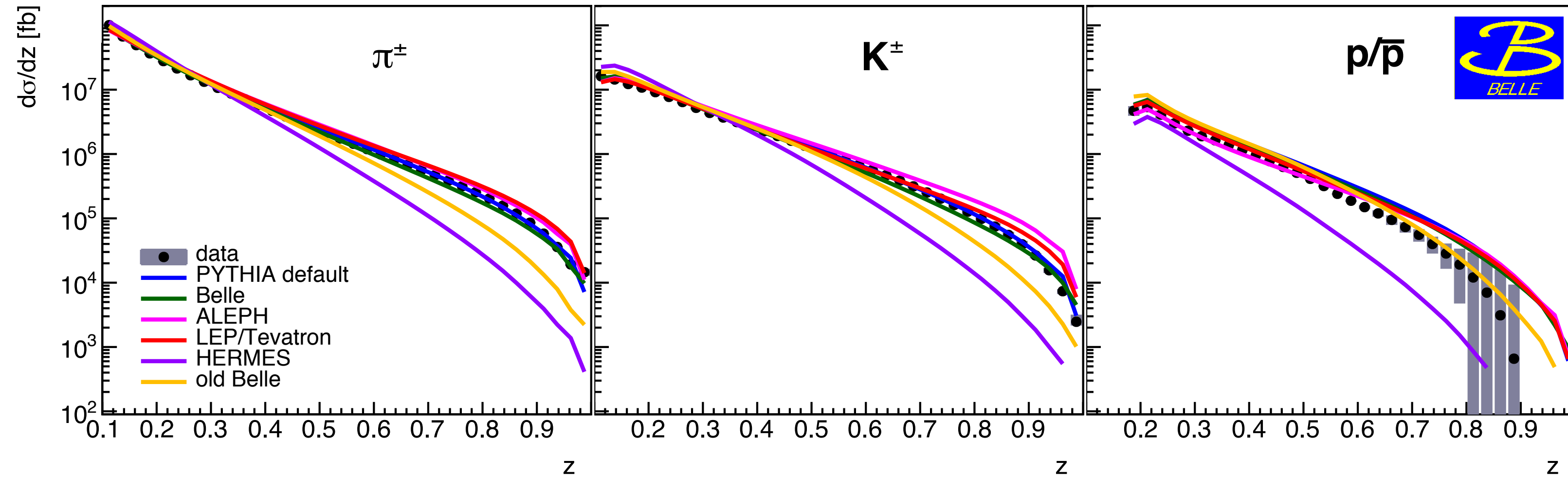


- Λ production reasonably well described by PYTHIA
- less satisfactory for heavier hyperons (a quite common problem)
- basically fails to describe Ω^- production

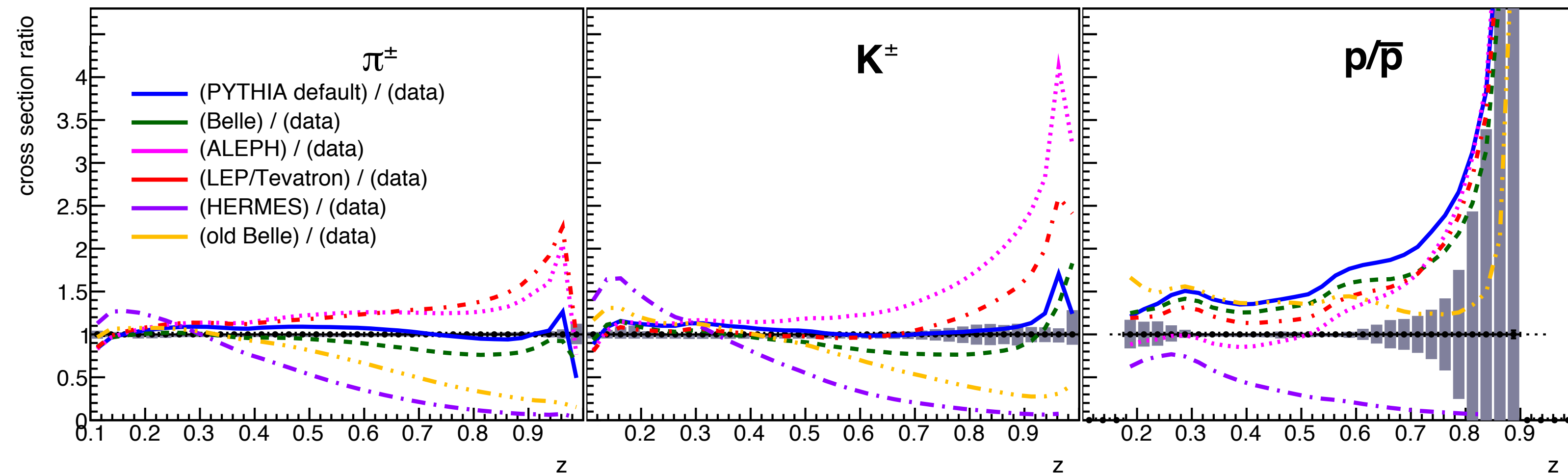
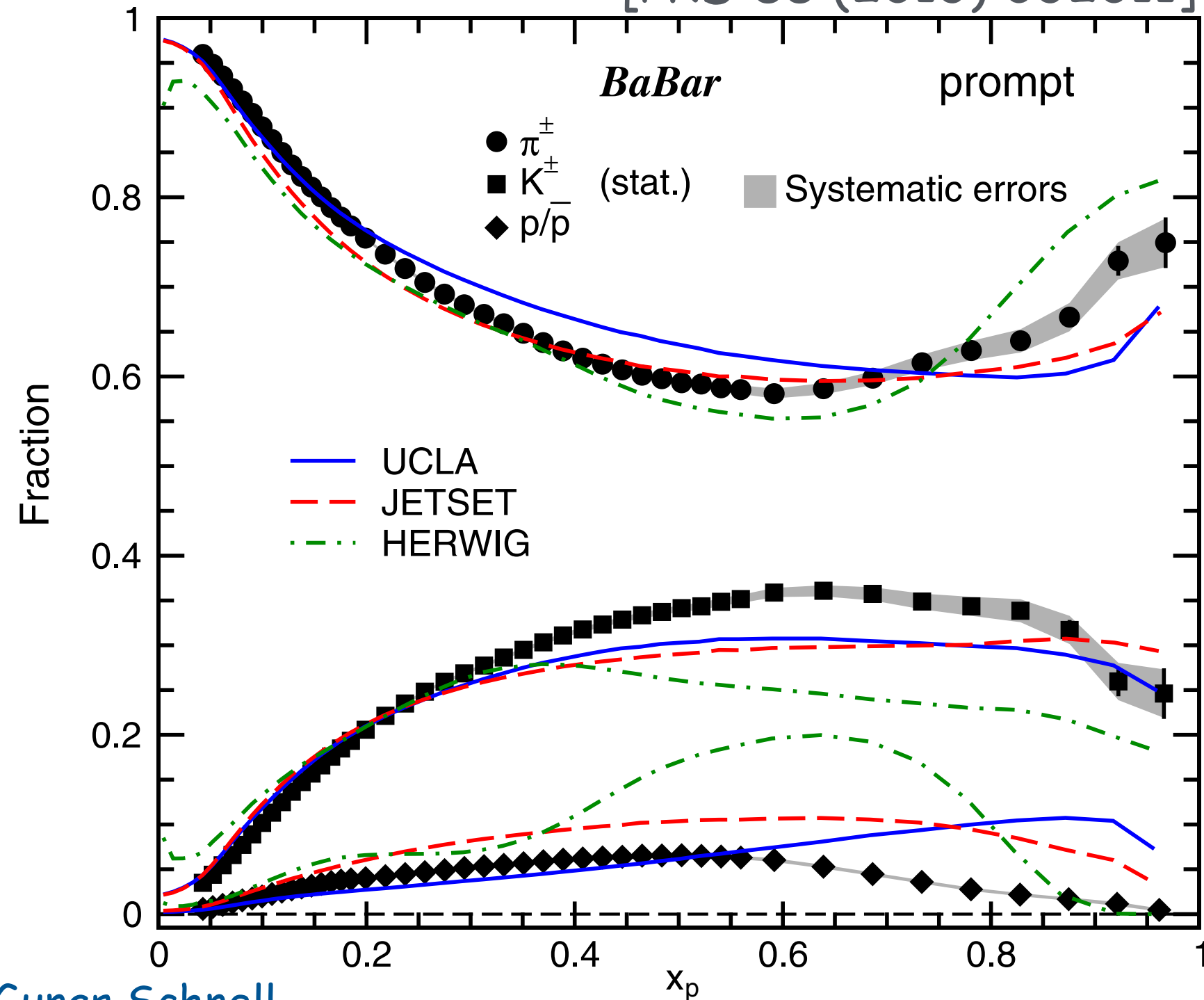
single-hadron production: data-MC comparison

- pion and(?) kaon data reasonably well described by Jetset
- protons difficult to reproduce, especially at large z
- MC overshoots data

[PRD 101 (2020) 092004]



[PRD 88 (2013) 032011]

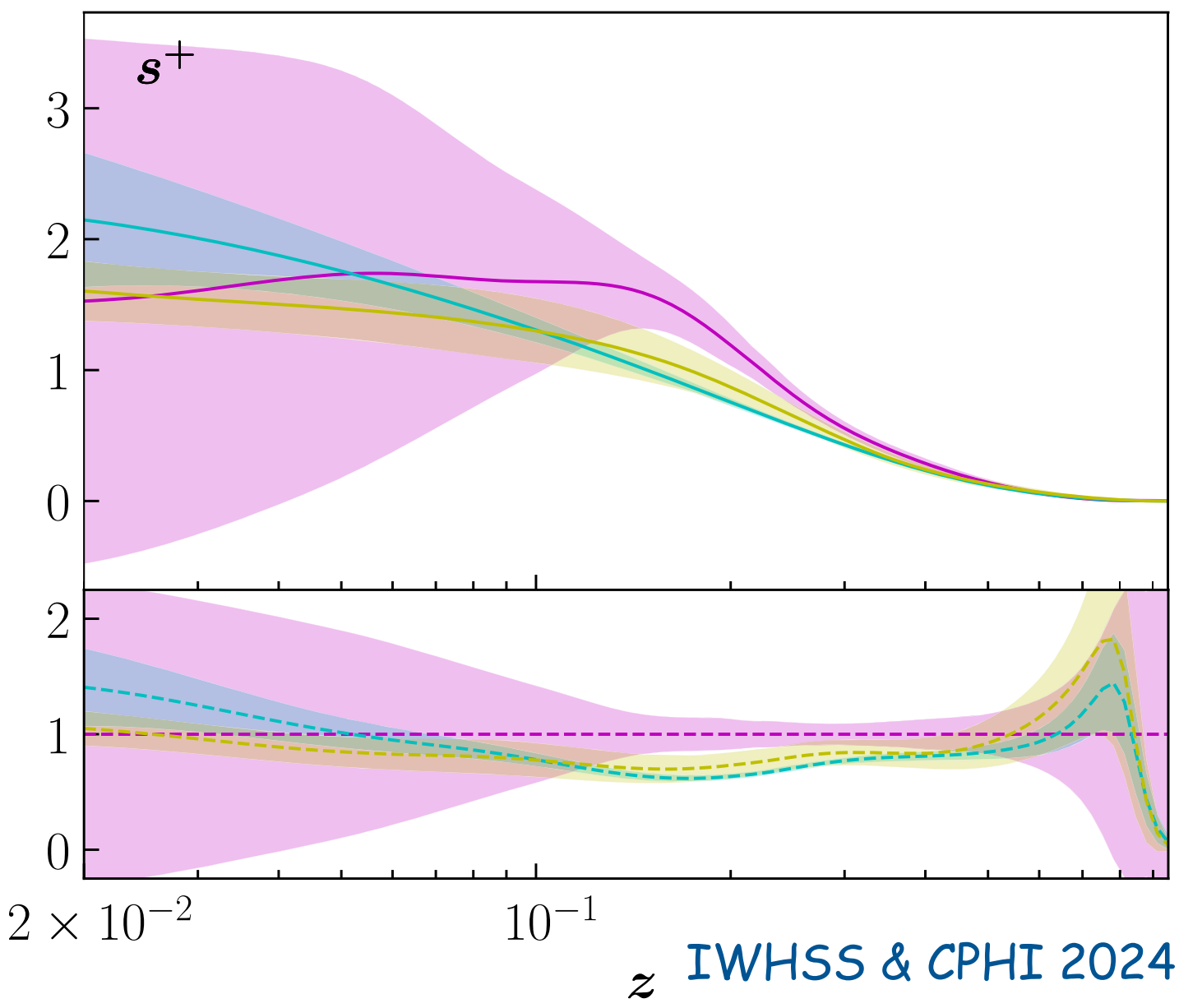
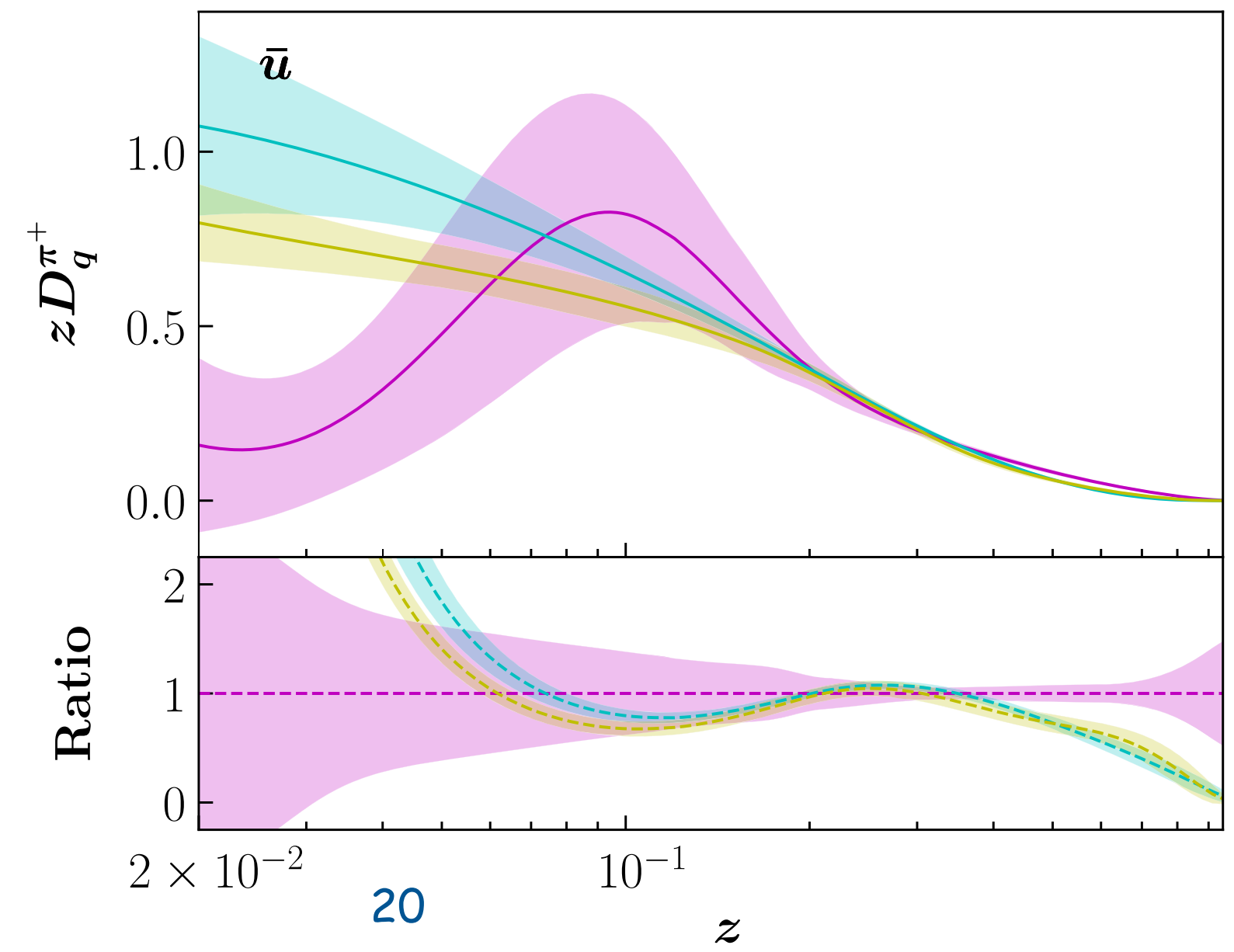
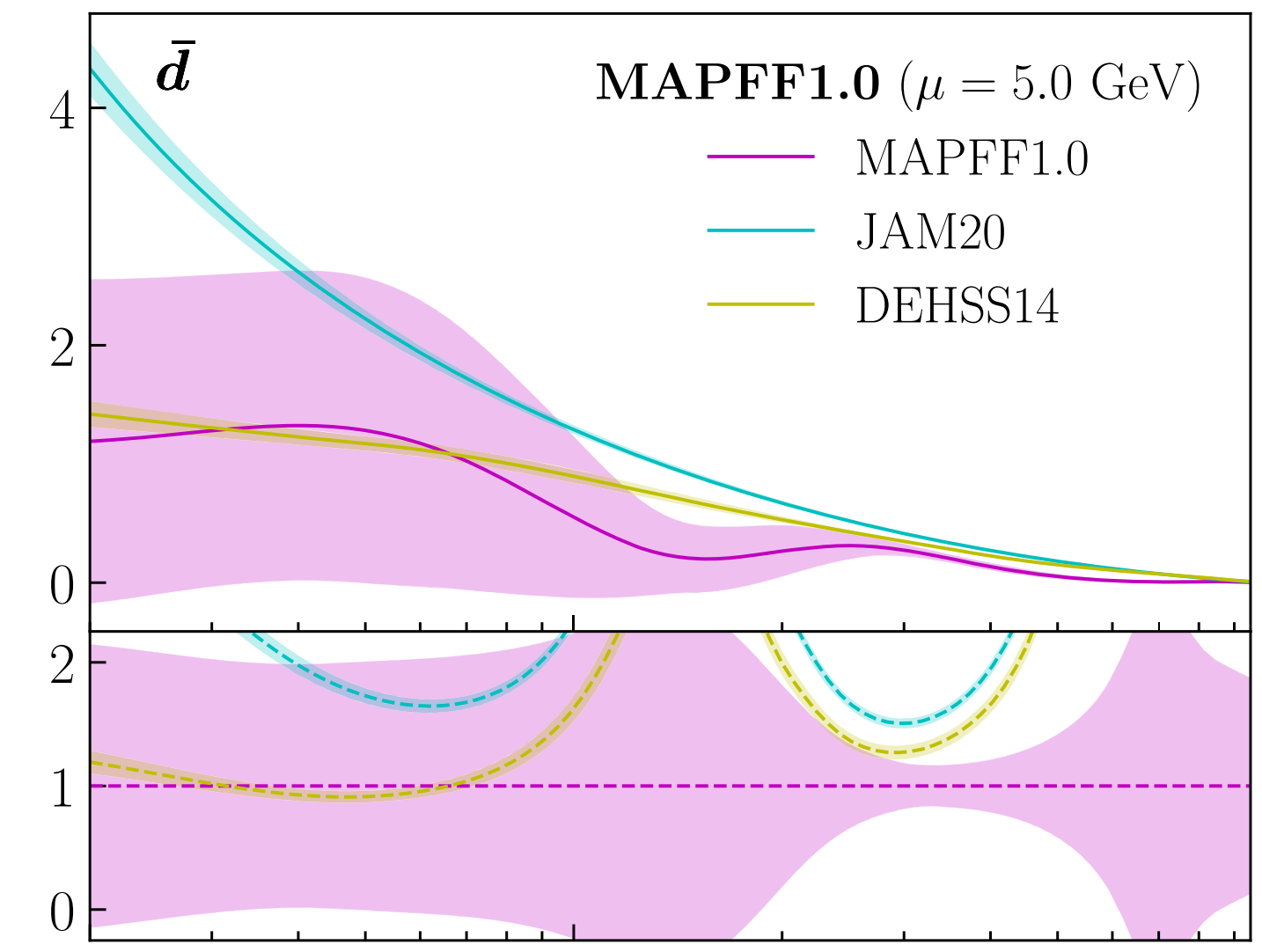
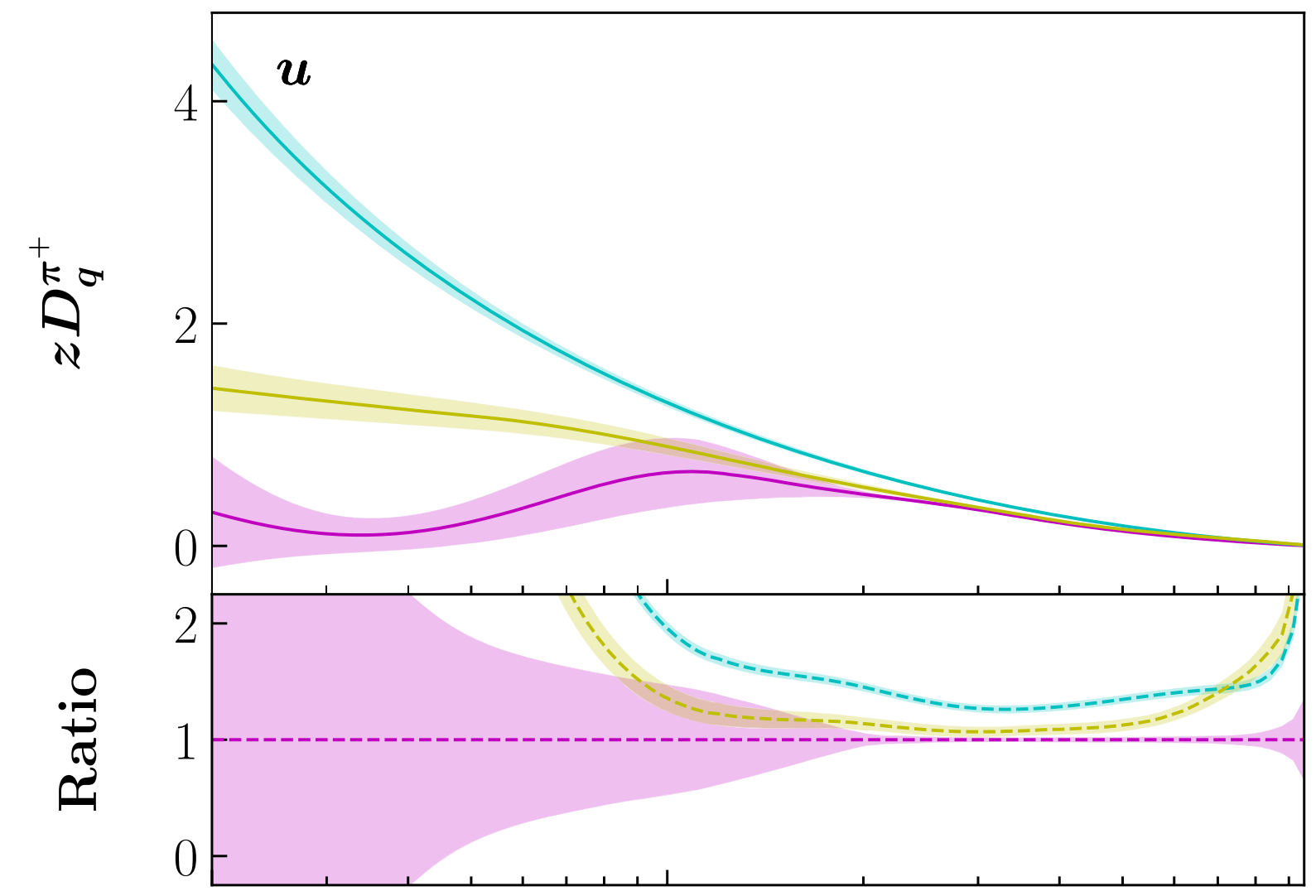
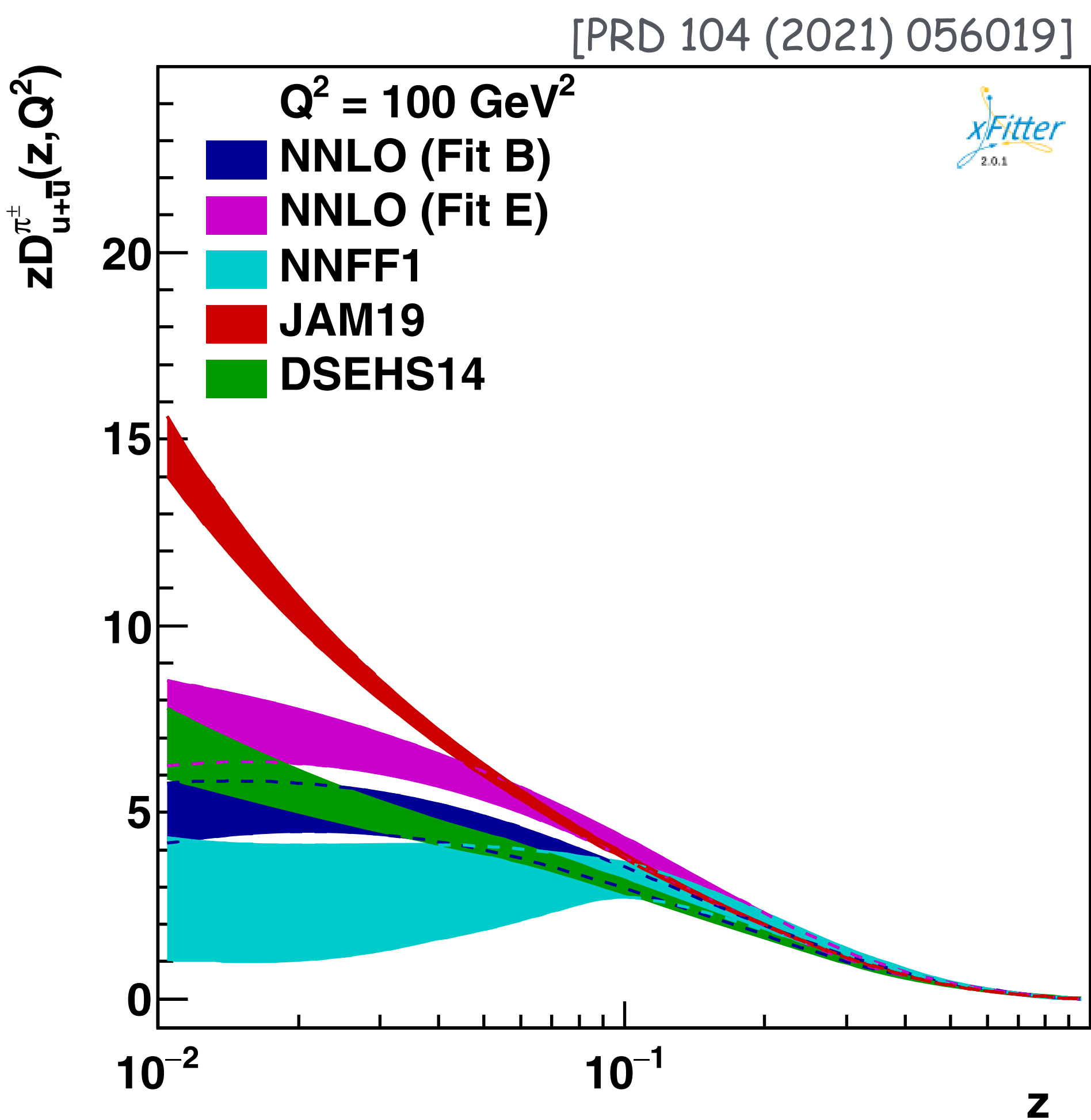


pion fragmentation functions: fit comparisons

● still large differences in FF fits

👉 talk by Valerio

[PRD 104 (2021) 034007]

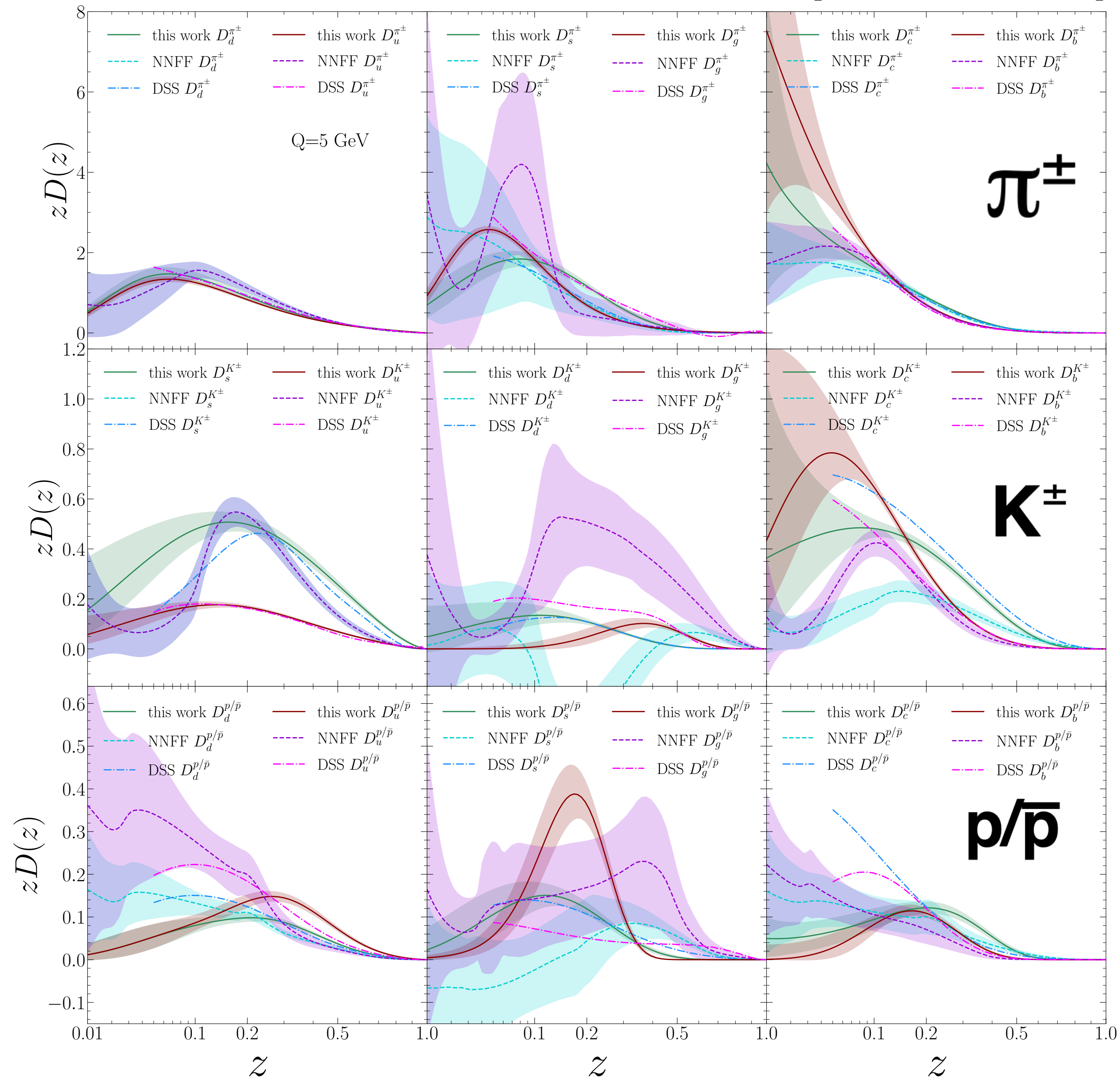
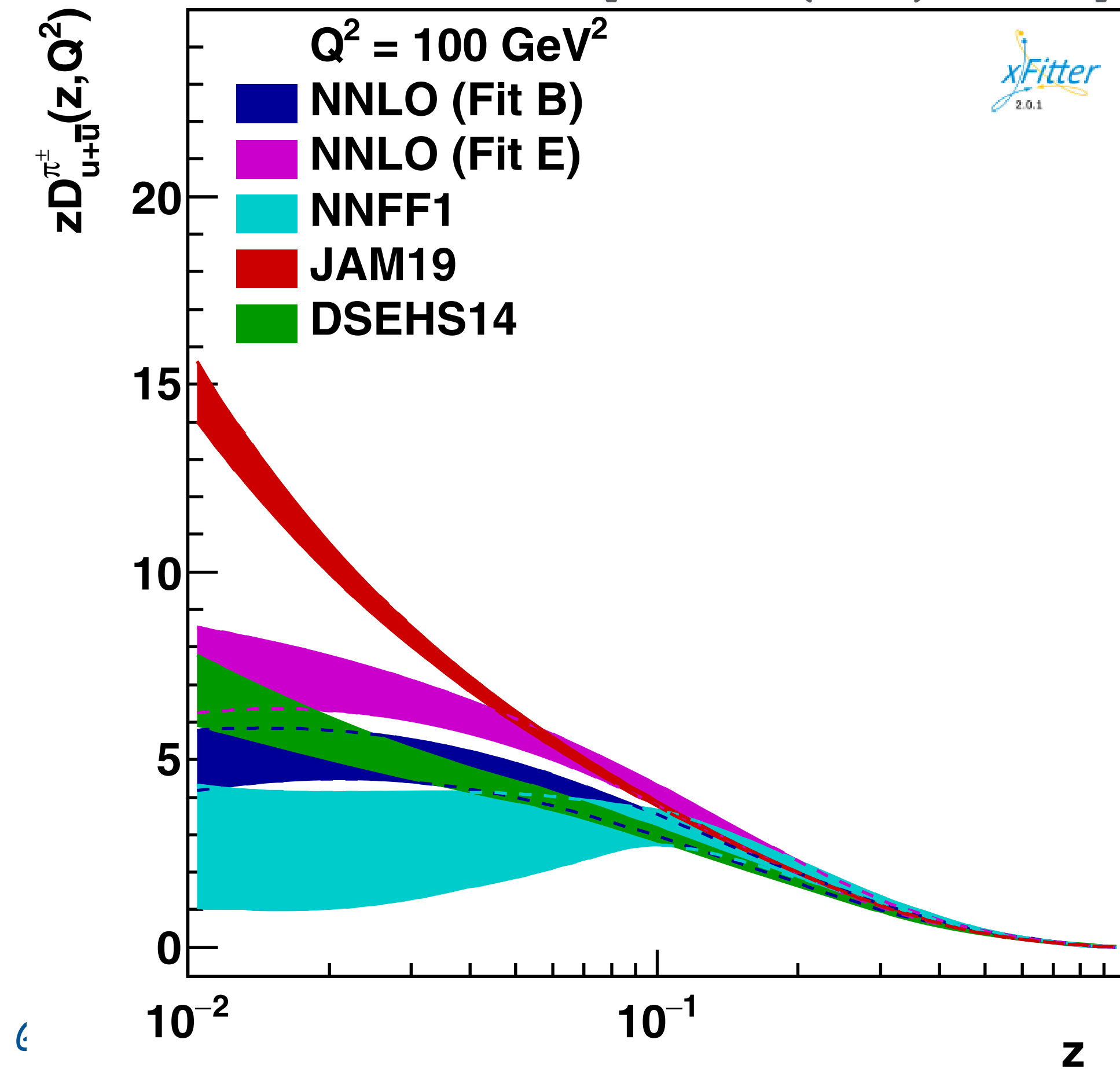


pion fragmentation functions: fit comparisons

[arXiv:2407.04422]

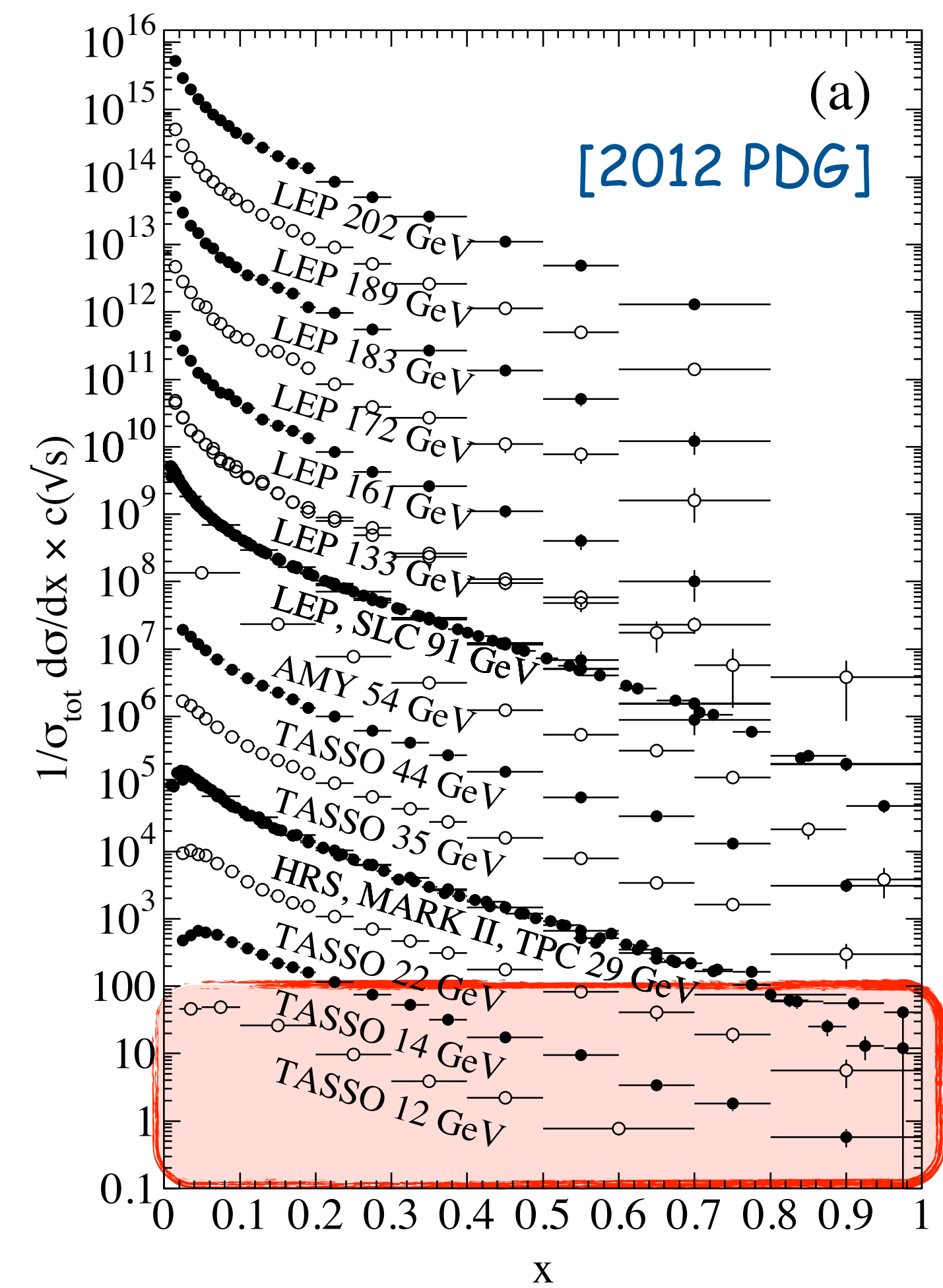
- still large differences in FF fits
- also in "SIDIS" region, where needed as flavor tagger

[PRD 104 (2021) 056019]



single-hadron production

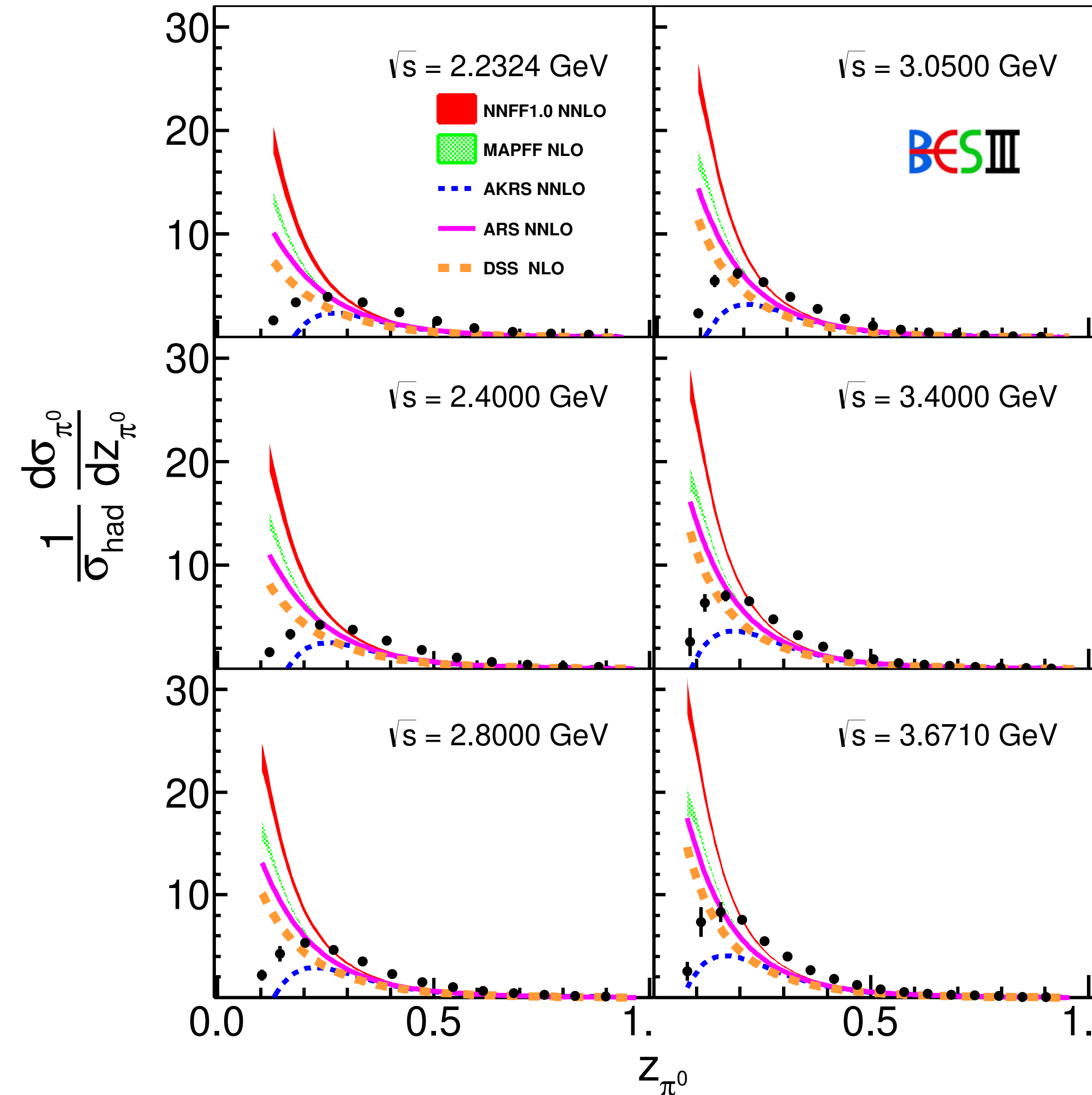
- before 2022: lack of precision data at low \sqrt{s}
- even B factories somewhat troublesome due to large charm contribution



single-hadron production

[PRL 130 (2023) 231901]

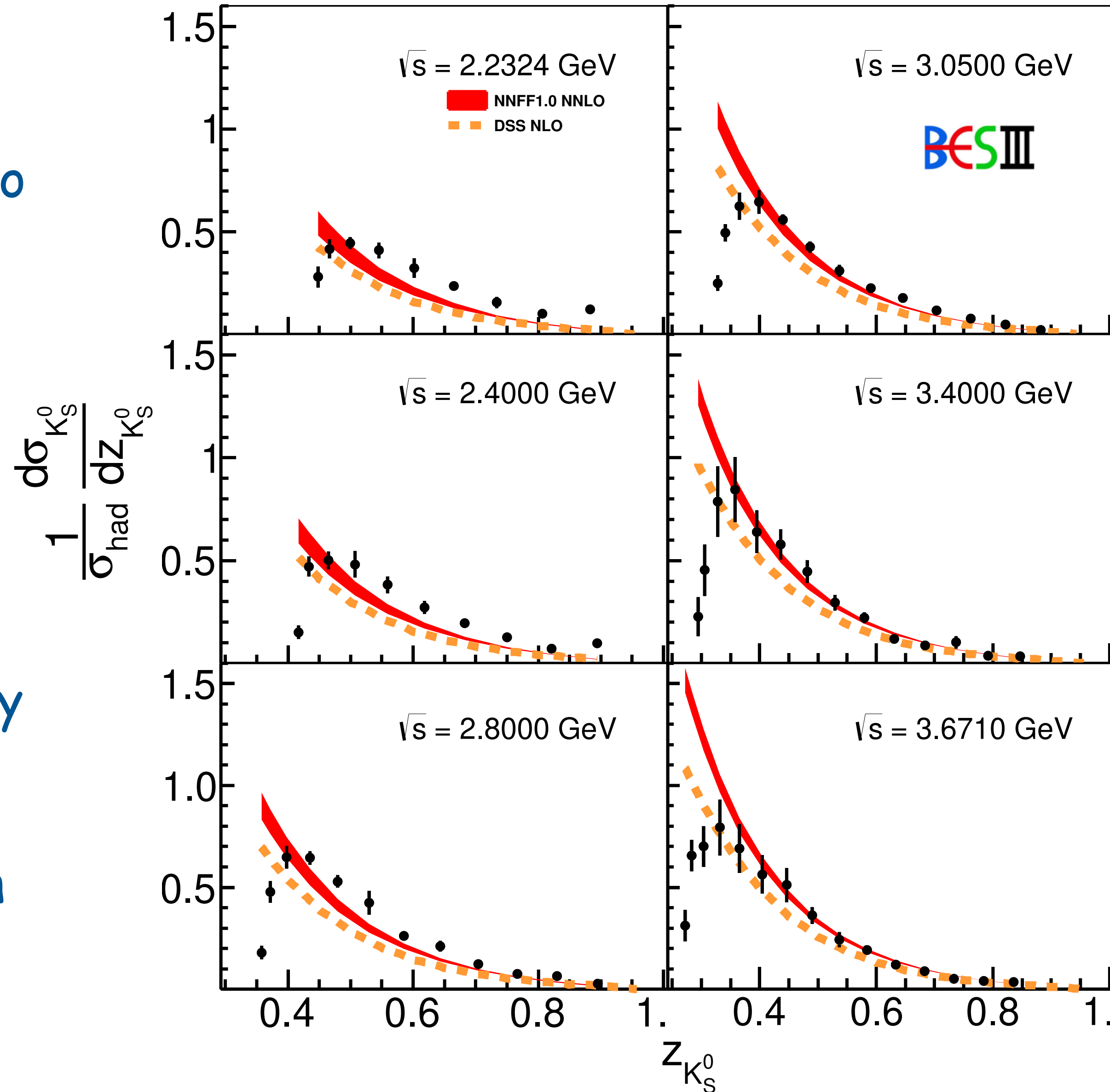
- before 2022: lack of precision data at low \sqrt{s}
- even B factories somewhat troublesome due to large charm contribution
- by now also results from BESIII
- [PRL 130 \(2023\) 231901](#) & [133 \(2024\) 021901](#)
- “challenge” to current FF parametrizations
- somewhat surprising for neutral pions as easily related to charge-pion FFs



single-hadron production

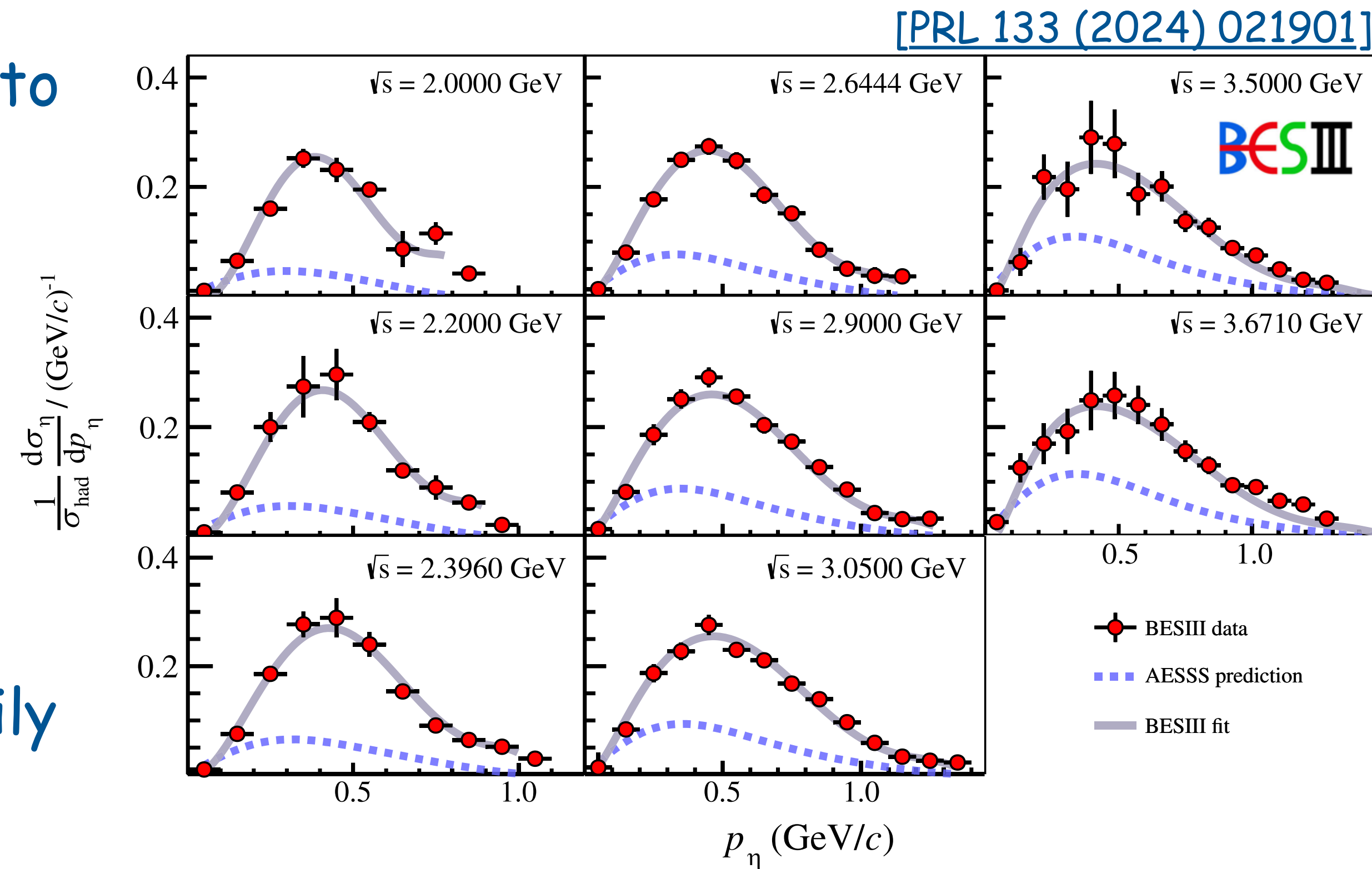
[PRL 130 (2023) 231901]

- before 2022: lack of precision data at low \sqrt{s}
- even B factories somewhat troublesome due to large charm contribution
- by now also results from BESIII
- [PRL 130 \(2023\) 231901](#) & [133 \(2024\) 021901](#)
- “challenge” to current FF parametrizations
- somewhat surprising for neutral pions as easily related to charge-pion FFs
- neutral-kaon FF related here to charged-kaon FFs as charge average



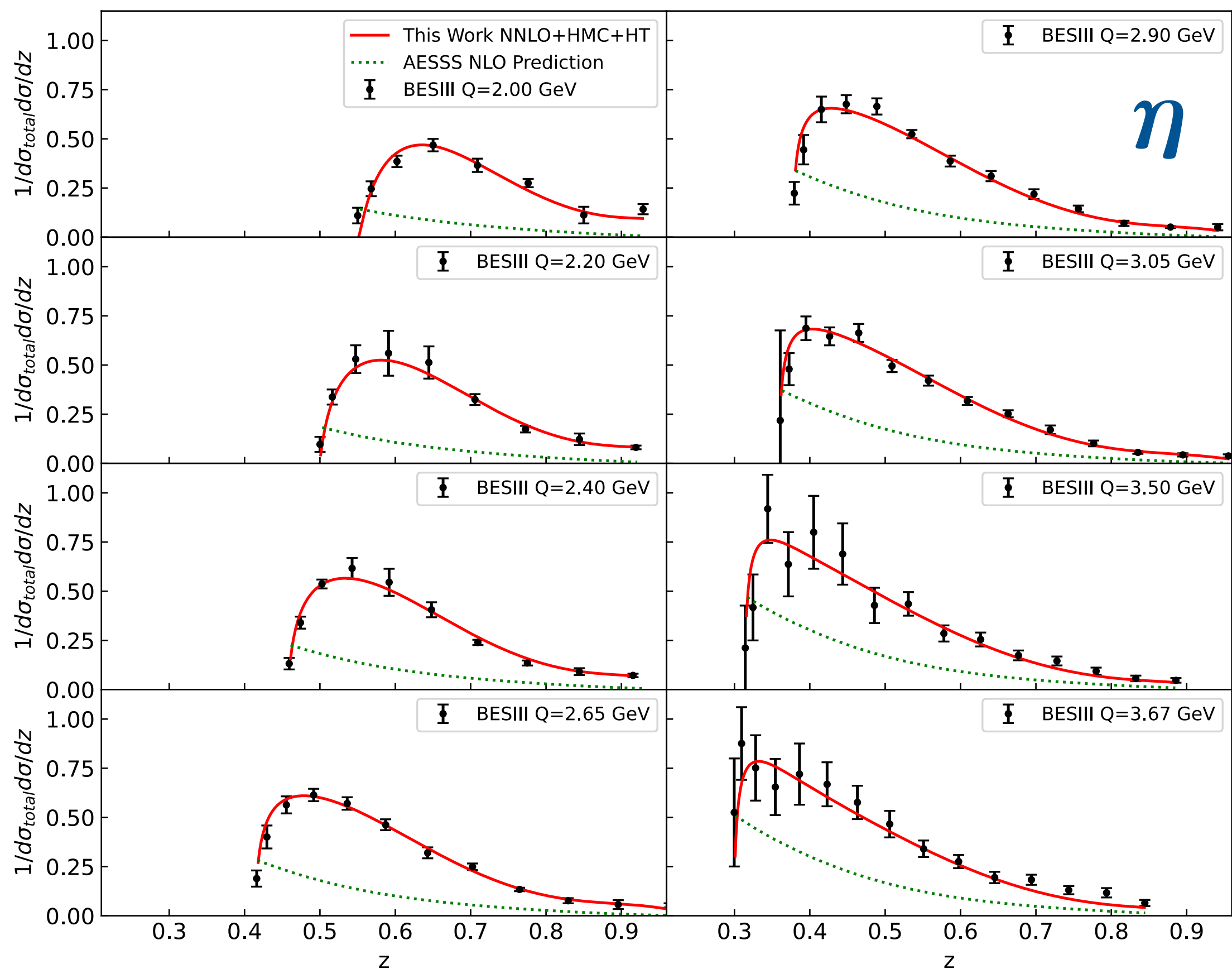
single-hadron production

- before 2022: lack of precision data at **low \sqrt{s}**
- even B factories somewhat troublesome due to large charm contribution
- by now also results from BESIII
- [PRL 130 \(2023\) 231901](#) & [133 \(2024\) 021901](#)
- “challenge” to current FF parametrizations
- somewhat surprising for neutral pions as easily related to charge-pion FFs
- neutral-kaon FF related here to charged-kaon FFs as charge average
- previous **eta** FF fit severely below BESIII data

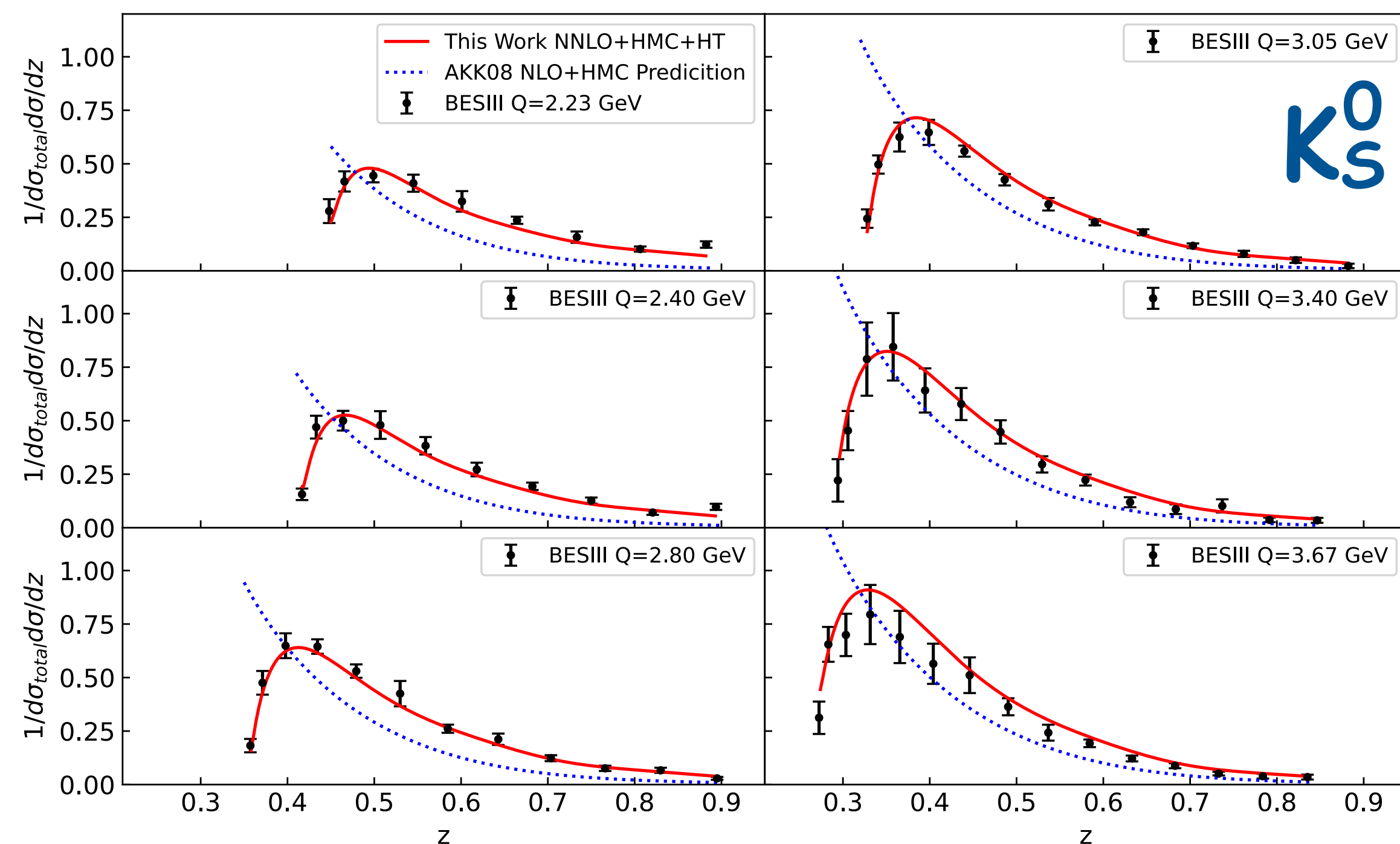
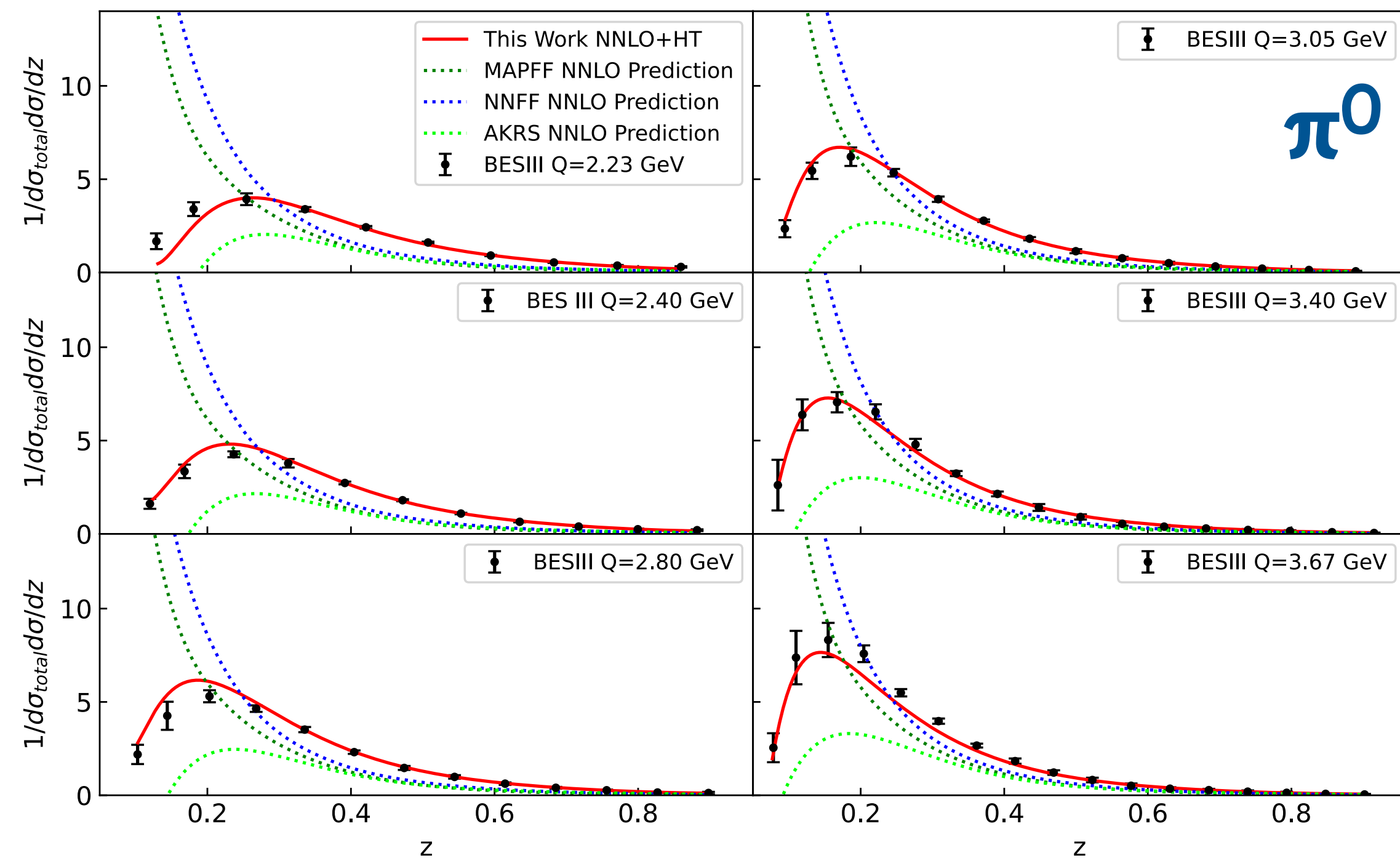


FFs including higher twist

- in view of poor description of BESIII data, include **higher-twist** in phenomenology
- much better description, but only SIA data



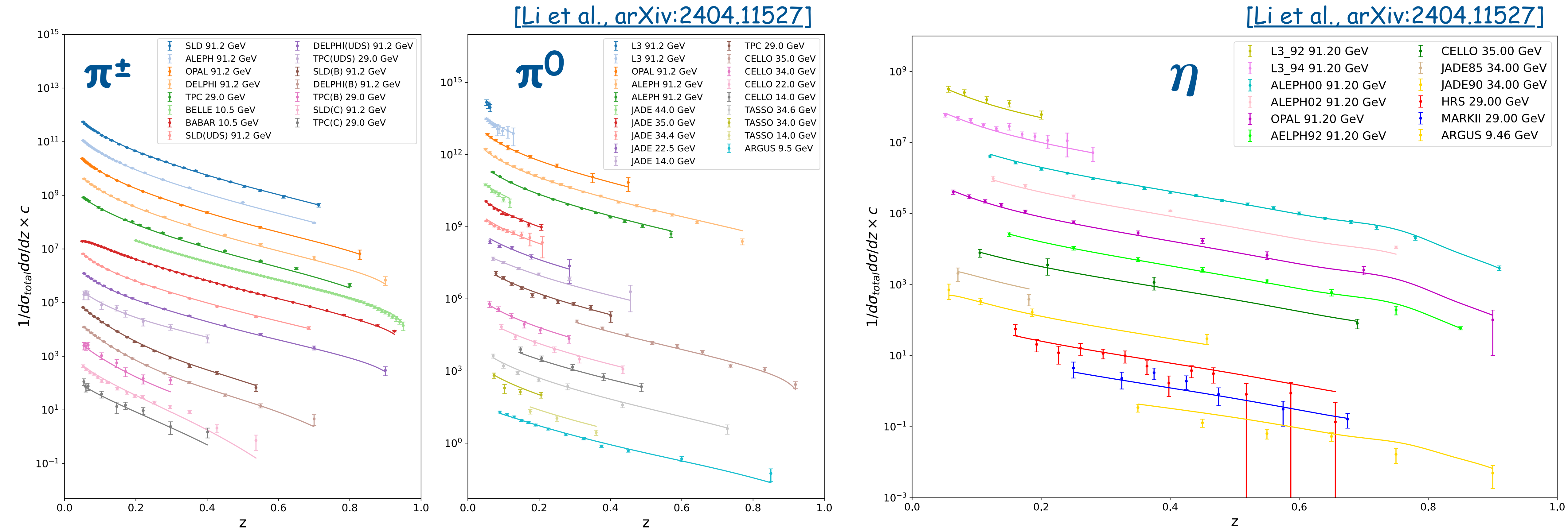
[Li et al., arXiv:2404.11527]



(see also M. Soleymaninia et al. PRD 110 (2024) 014019)

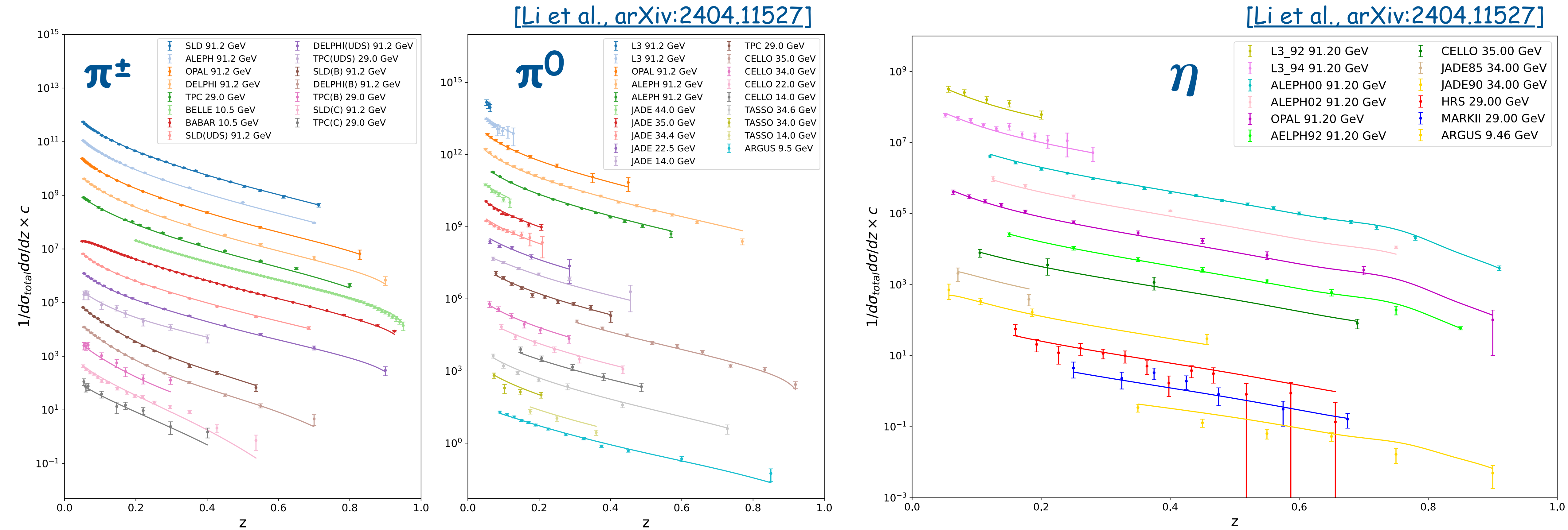
light-meson SIA data

- currently rather limited collection of SIA (and other!) data on eta production



light-meson SIA data

- currently rather limited collection of SIA (and other!) data on eta production

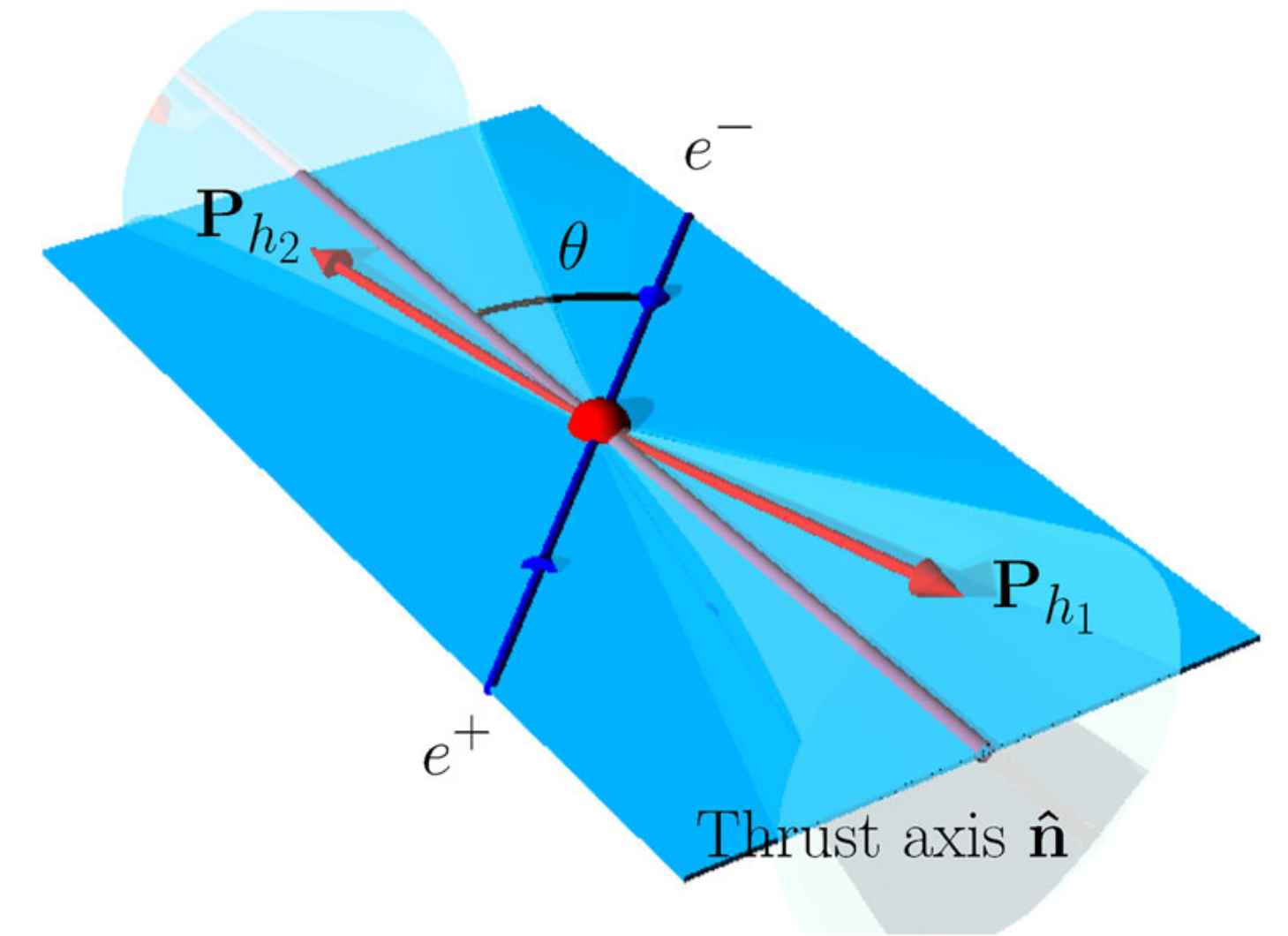


- new data from Belle to come out soon

hadron-pair production

- single-hadron production has low discriminating power for parton flavor
- can use 2nd hadron in opposite hemisphere to “tag” flavor, transverse momentum, as well as polarization
- mainly sensitive to product of single-hadron FFs, e.g.,

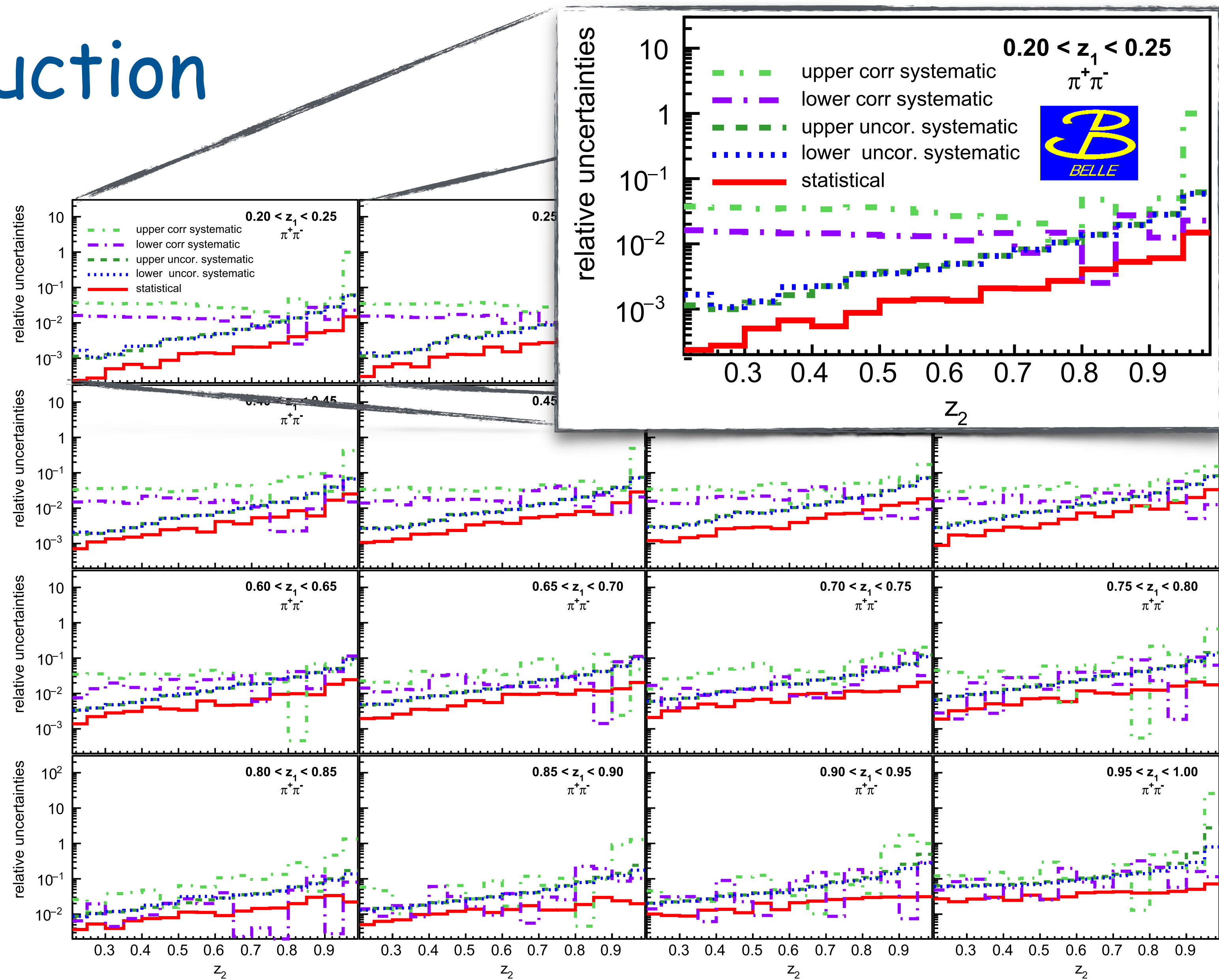
$$\sigma^{e^+e^- \rightarrow h_1 h_2 X} \propto \sum_q e_q^2 \left(D_1^{q \rightarrow h_1} D_1^{\bar{q} \rightarrow h_2} + D_1^{\bar{q} \rightarrow h_1} D_1^{q \rightarrow h_2} \right)$$



$$T \stackrel{\text{max}}{=} \frac{\sum_h |\mathbf{P}_h^{\text{CMS}} \cdot \hat{\mathbf{n}}|}{\sum_h |\mathbf{P}_h^{\text{CMS}}|}$$

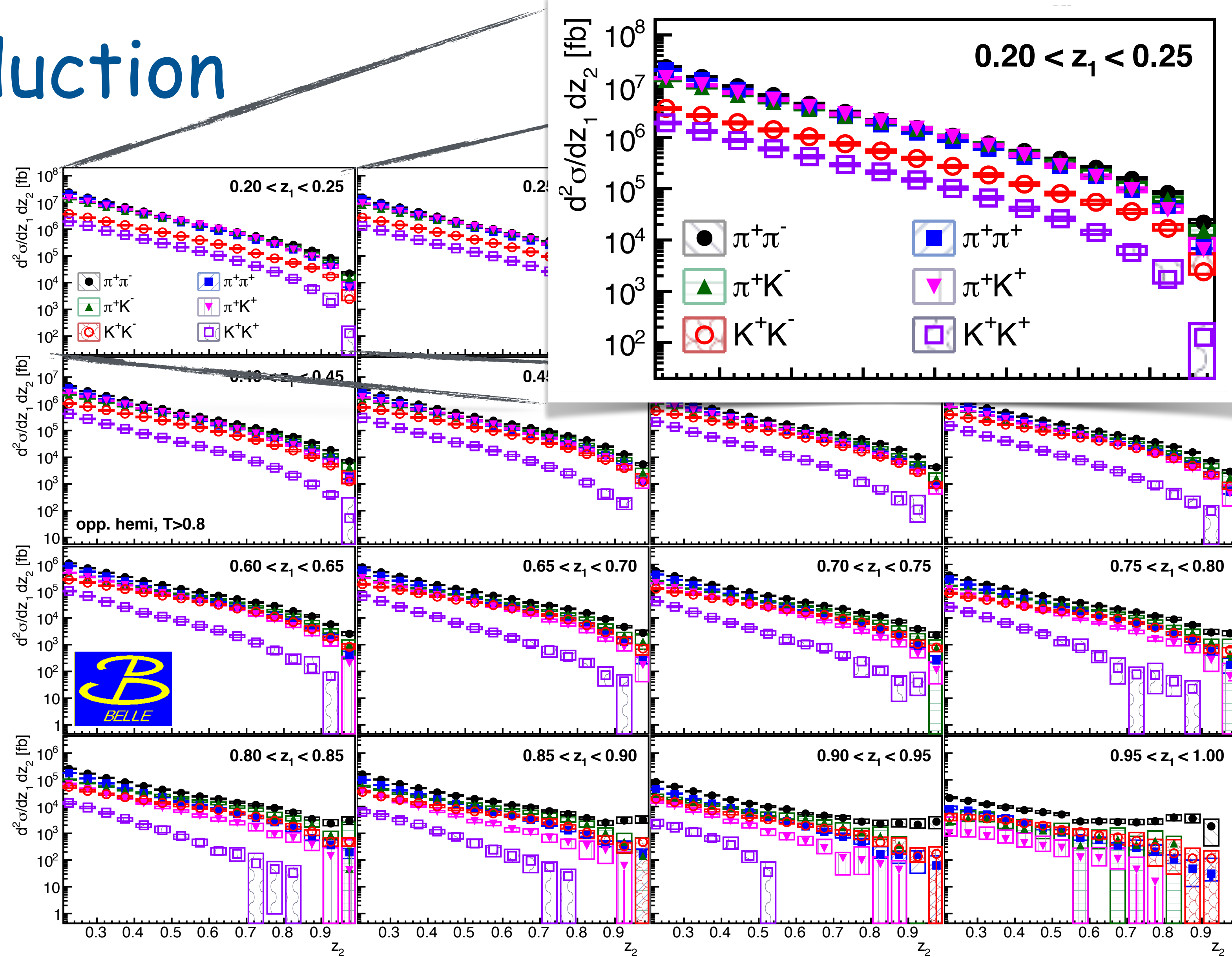
light-meson pair production

- systematics-dominated over entire kinematic range
- strongly asymmetric systematics
- main contribution from Monte Carlo tune dependence



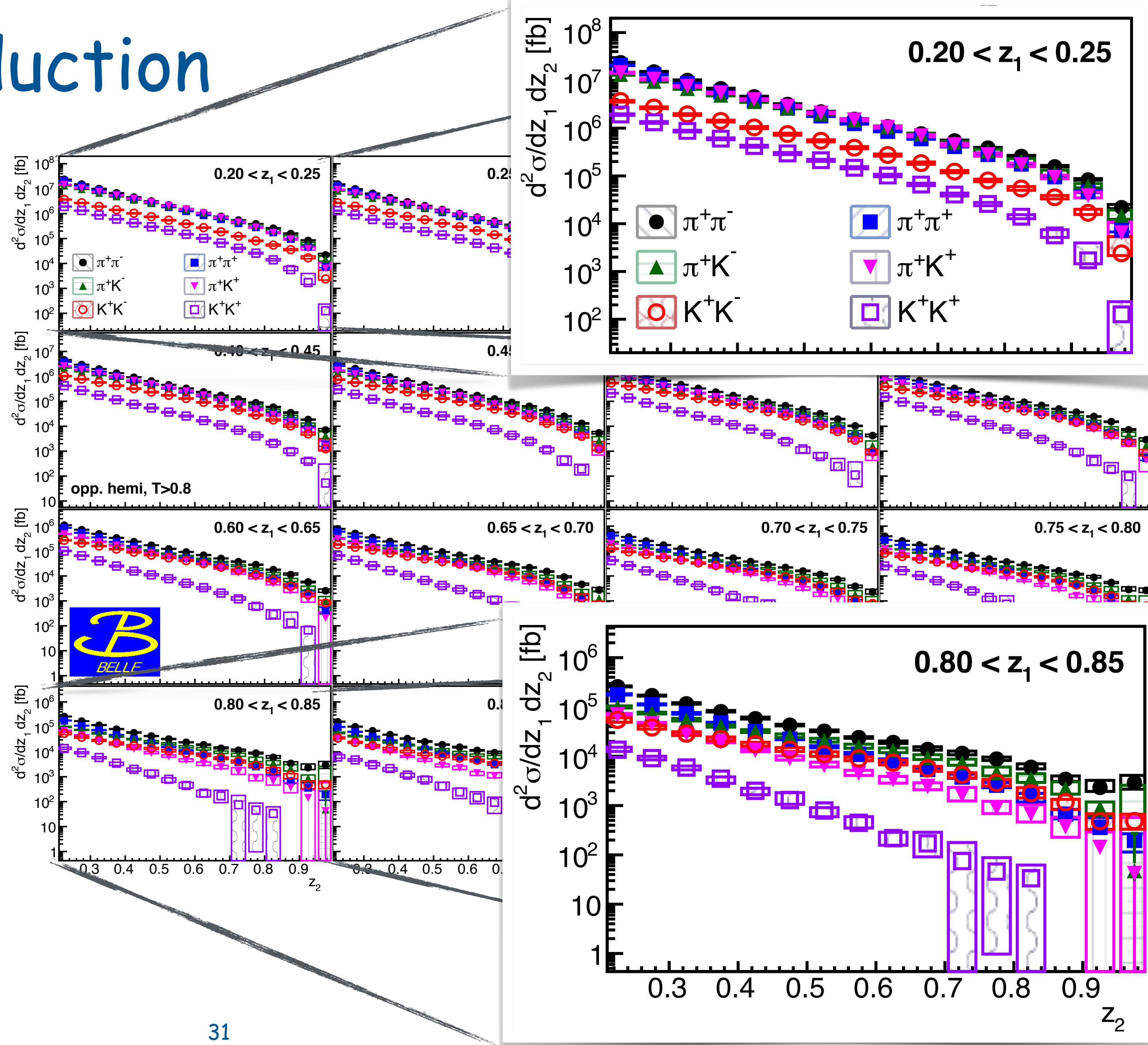
light-meson pair production

- systematics-dominated over entire kinematic range
- clear flavor dependence
- suppression of kaons
- suppression of like-sign pairs
- more pronounced at large z (stronger flavor sensitivity)



light-meson pair production

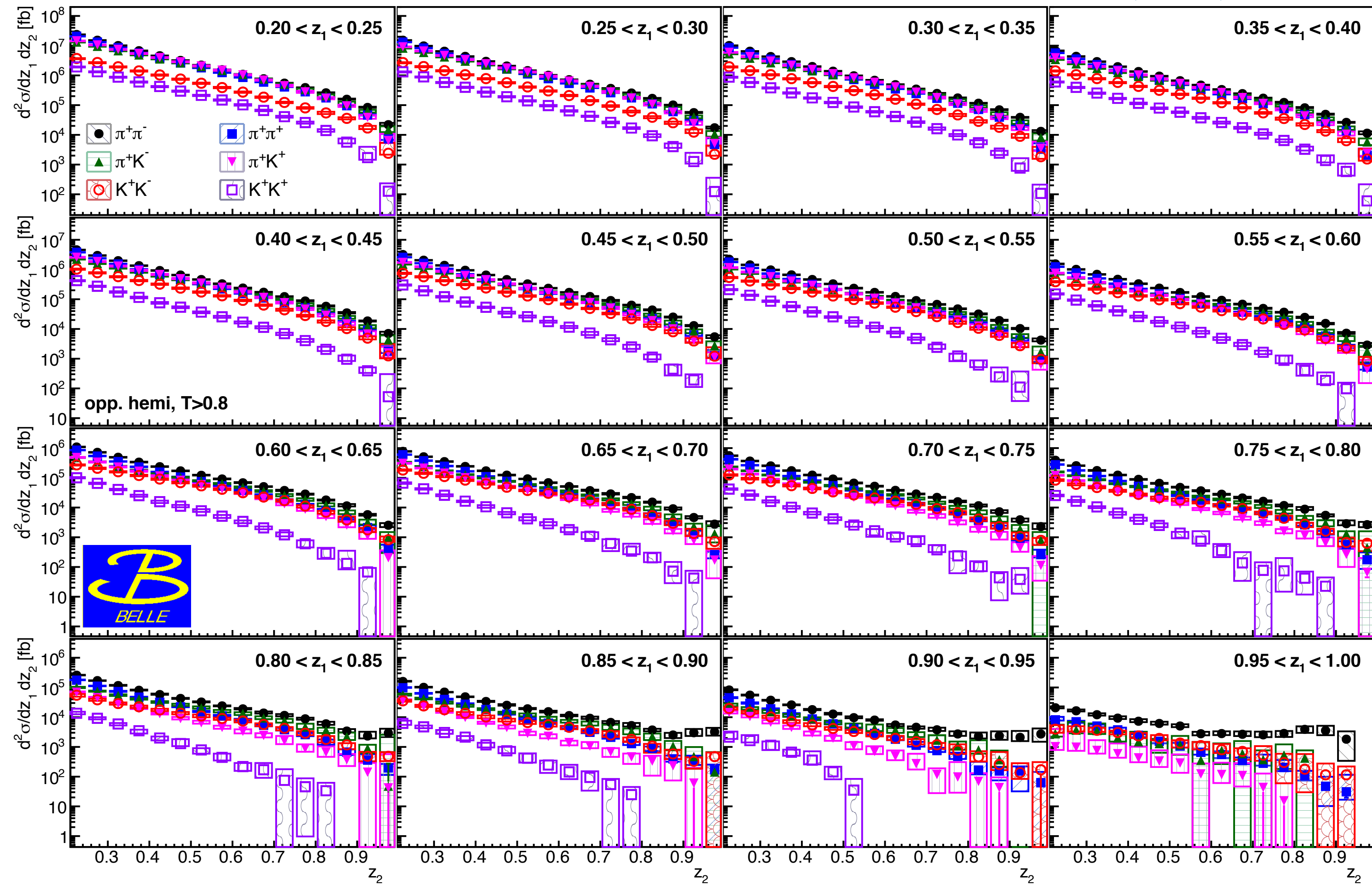
- systematics-dominated over entire kinematic range
- clear flavor dependence
- suppression of kaons
- suppression of like-sign pairs
- more pronounced at large z (stronger flavor sensitivity)



light-meson pair production

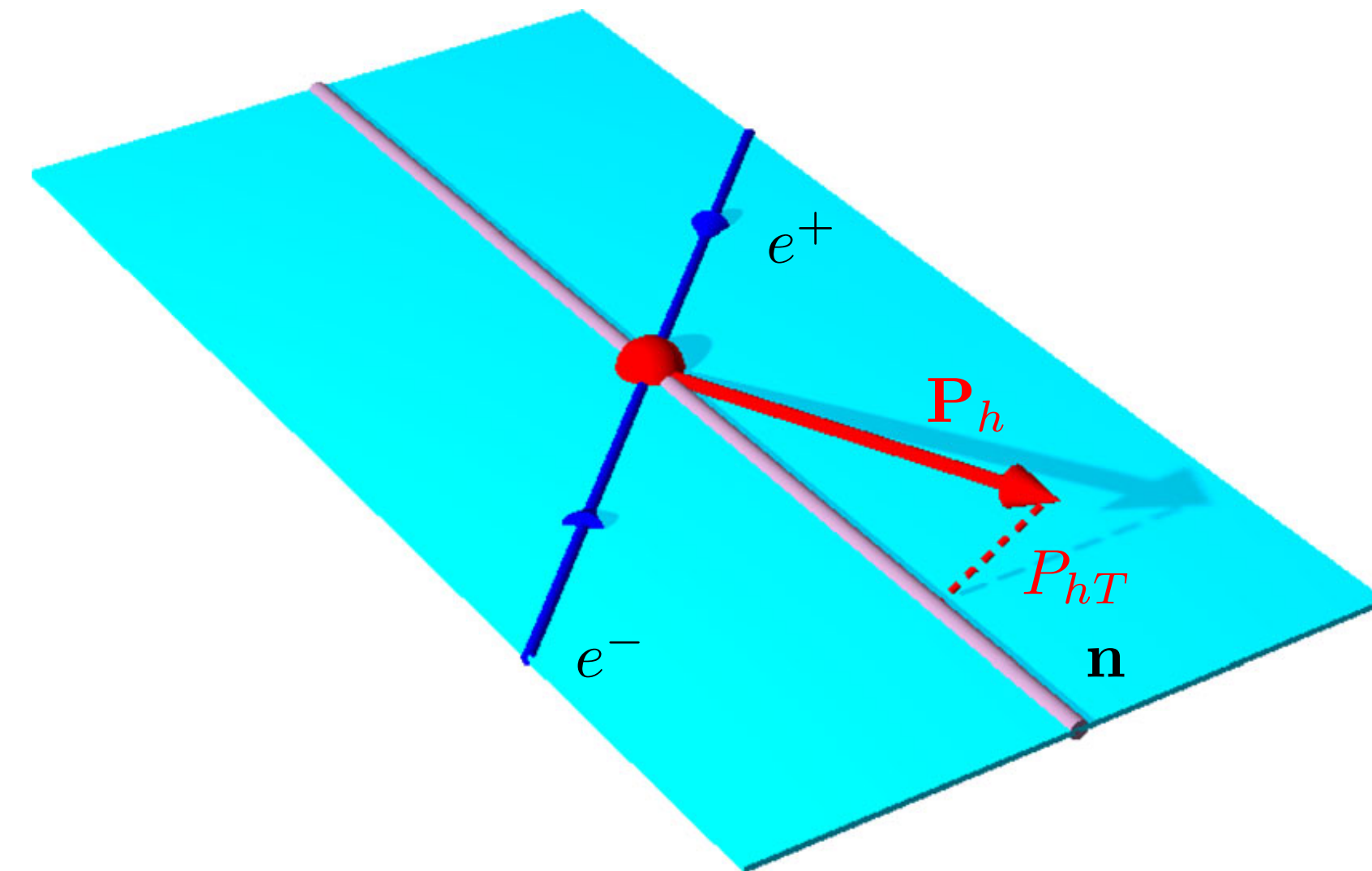
[PRD 101 (2020) 092004]

- systematics-dominated over entire kinematic range
- clear flavor dependence
- suppression of kaons
- suppression of like-sign pairs
- more pronounced at large z (stronger flavor sensitivity)
- rich but currently still mainly unexplored set of data on flavor-dependence of FFs



inclusive hadrons - transverse momentum

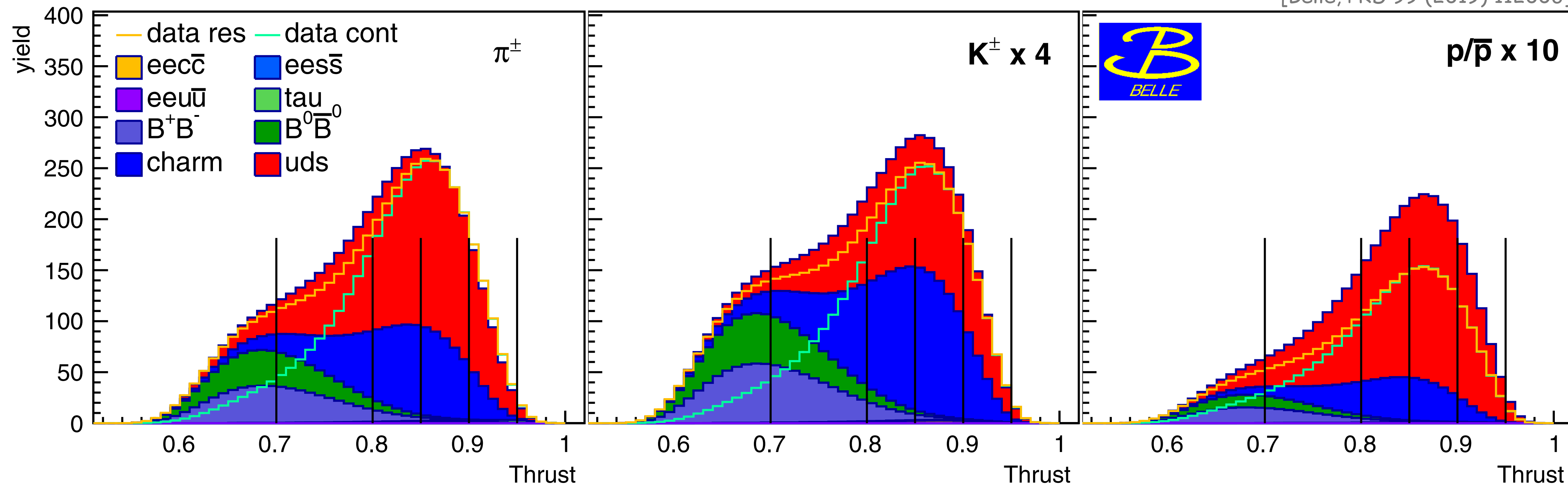
- quasi-inclusive hadron production gives access to transverse momentum in fragmentation
- transverse momentum measured with respect to thrust axis \mathbf{n}
- analysis performed differential in z & P_{hT} , in various slices in thrust T (\Rightarrow 18x20x6 bins)
- correction steps similar as for P_{hT} -integrated cross sections
- Gaussian fits to transverse-momentum distribution provided for all hadrons in (z, T) -bins



$$T \stackrel{\text{max}}{=} \frac{\sum_h |\mathbf{P}_h^{\text{CMS}} \cdot \hat{\mathbf{n}}|}{\sum_h |\mathbf{P}_h^{\text{CMS}}|}$$

thrust distribution: process contributions

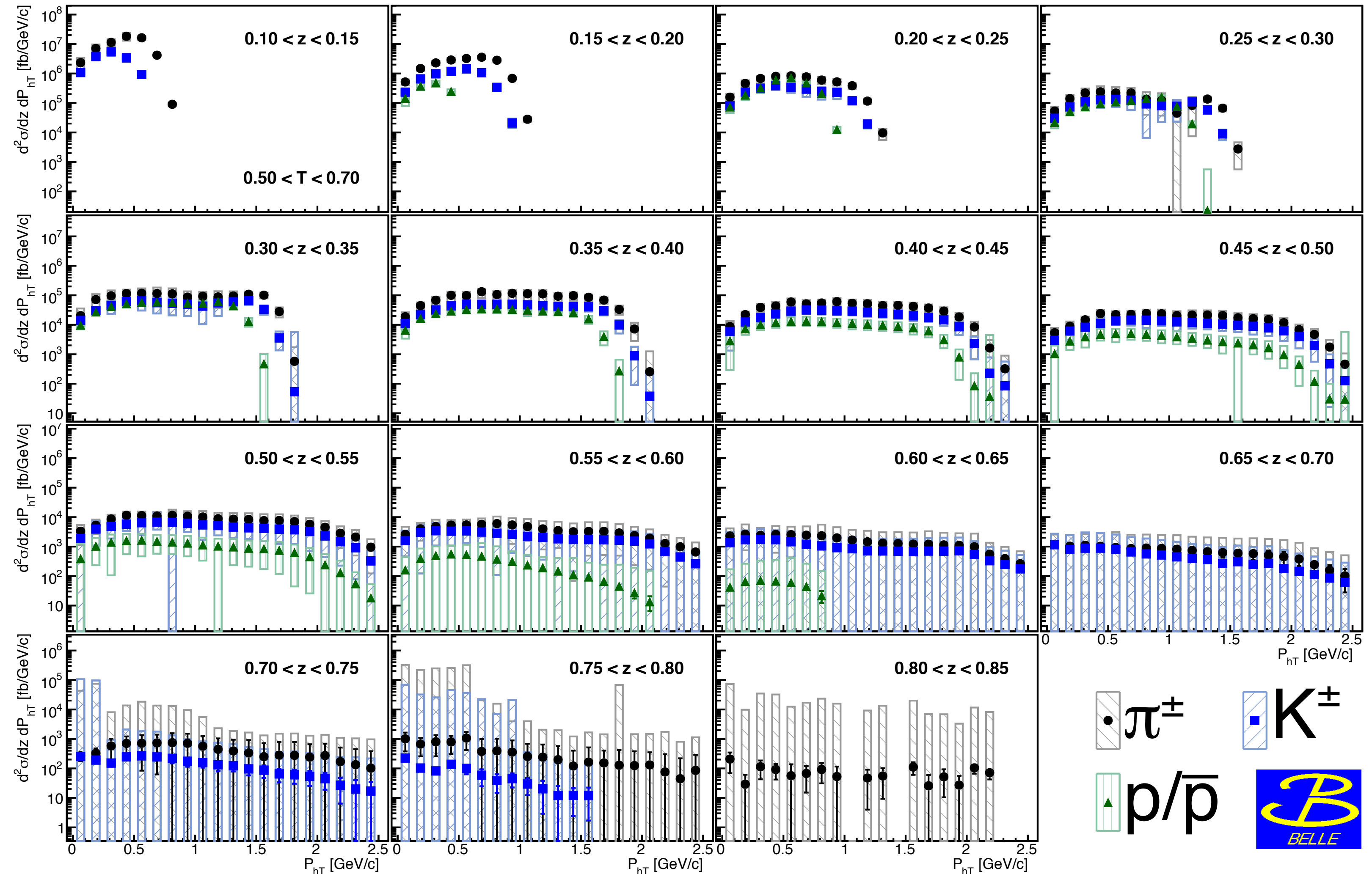
[Belle, PRD 99 (2019) 112006]



- large contribution from $B\bar{B}$ at lower thrust
- large thrust dominated by uds and charm fragmentation
(at very large T significant τ contribution for pions, not visible here)

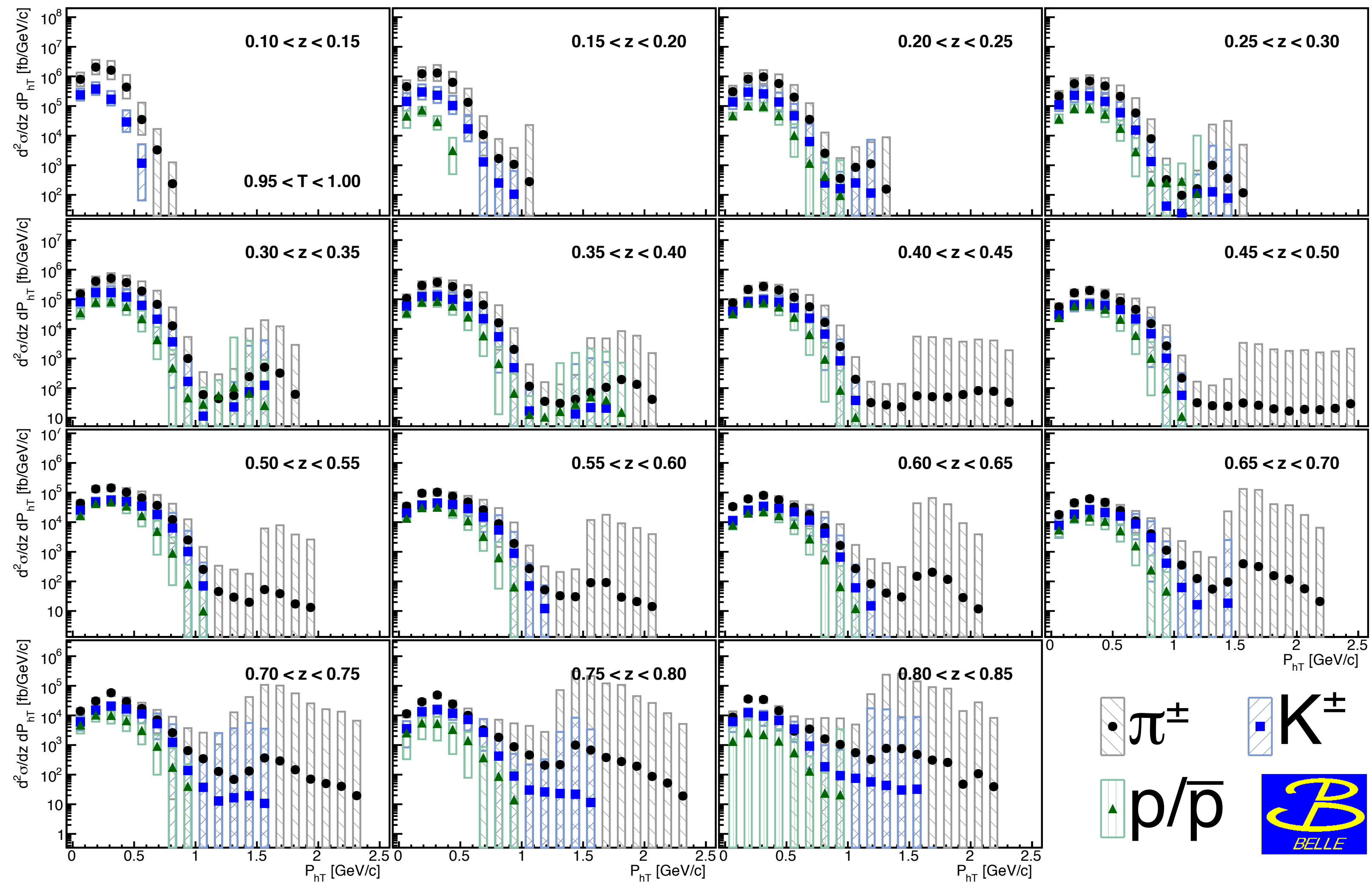
transverse-momentum distributions

- lowest T bin
→ rather spherical events
- transverse momenta almost uniformly distributed in medium-z bins
- faster drop for heavier hadrons



transverse-momentum distributions

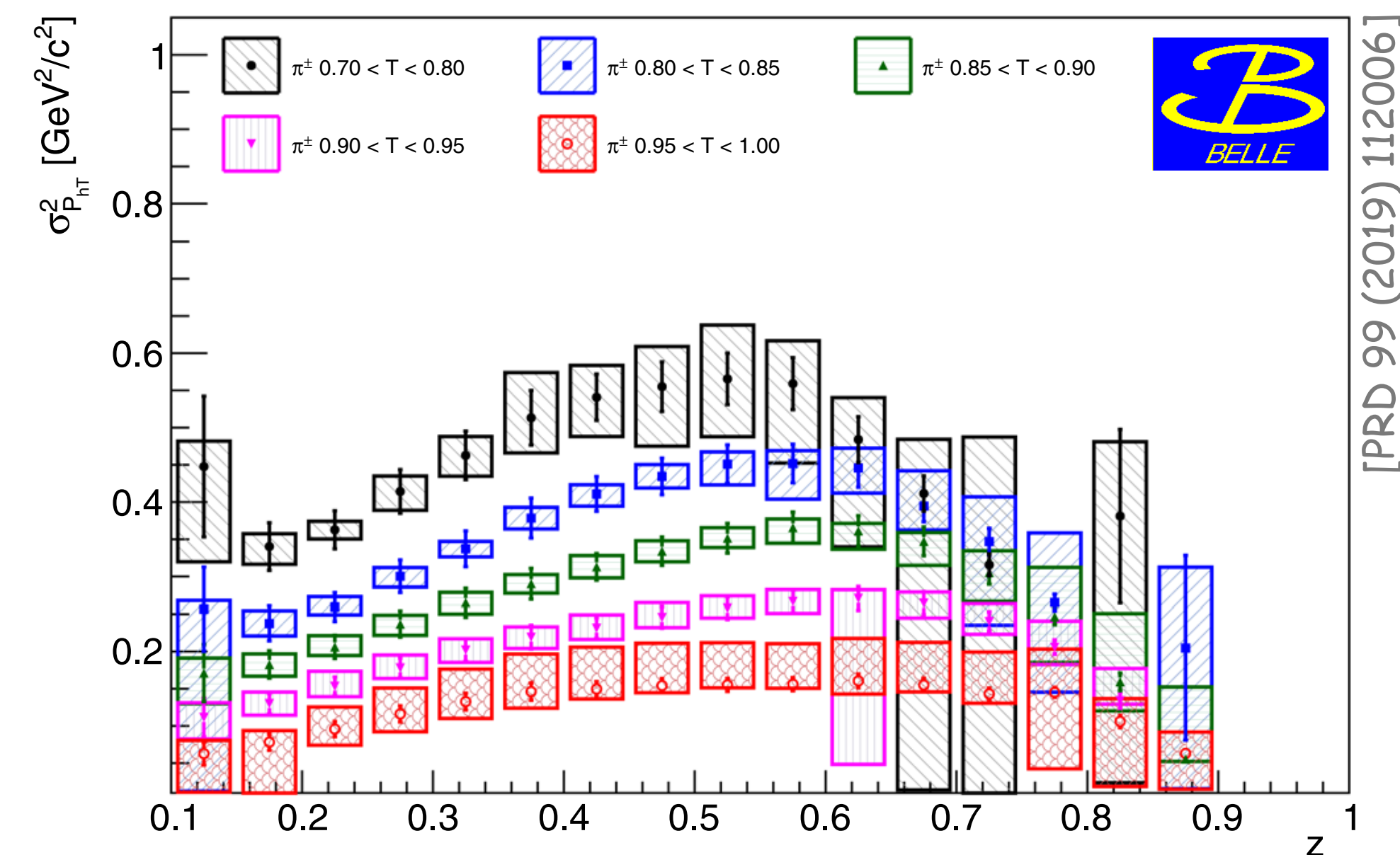
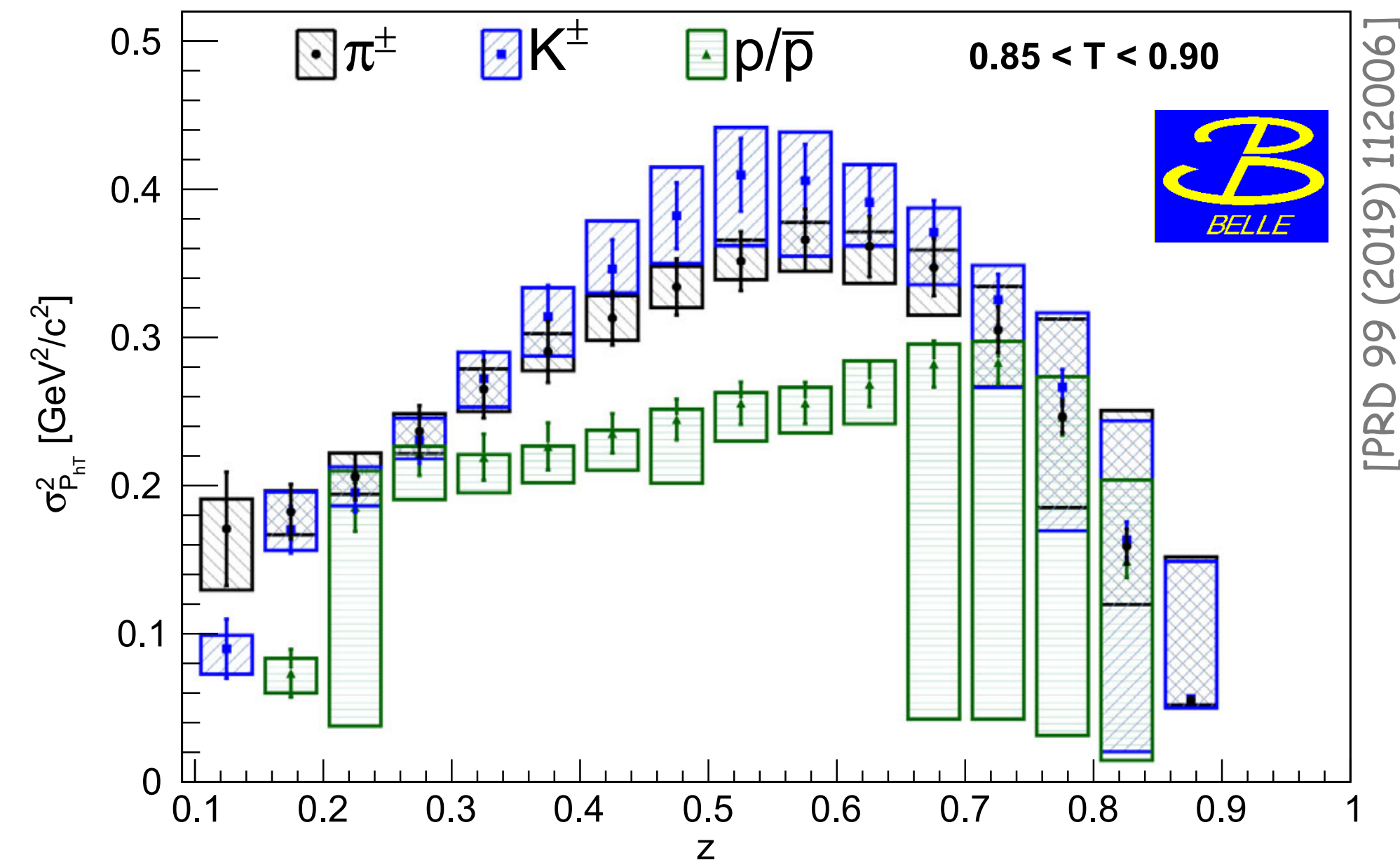
- $0.95 < T < 1.0$
- transverse momenta mostly Gaussian distributed
- widths very narrow as particles now very collimated



[PRD 99 (2019) 112006]

transverse-momentum: Gaussian widths

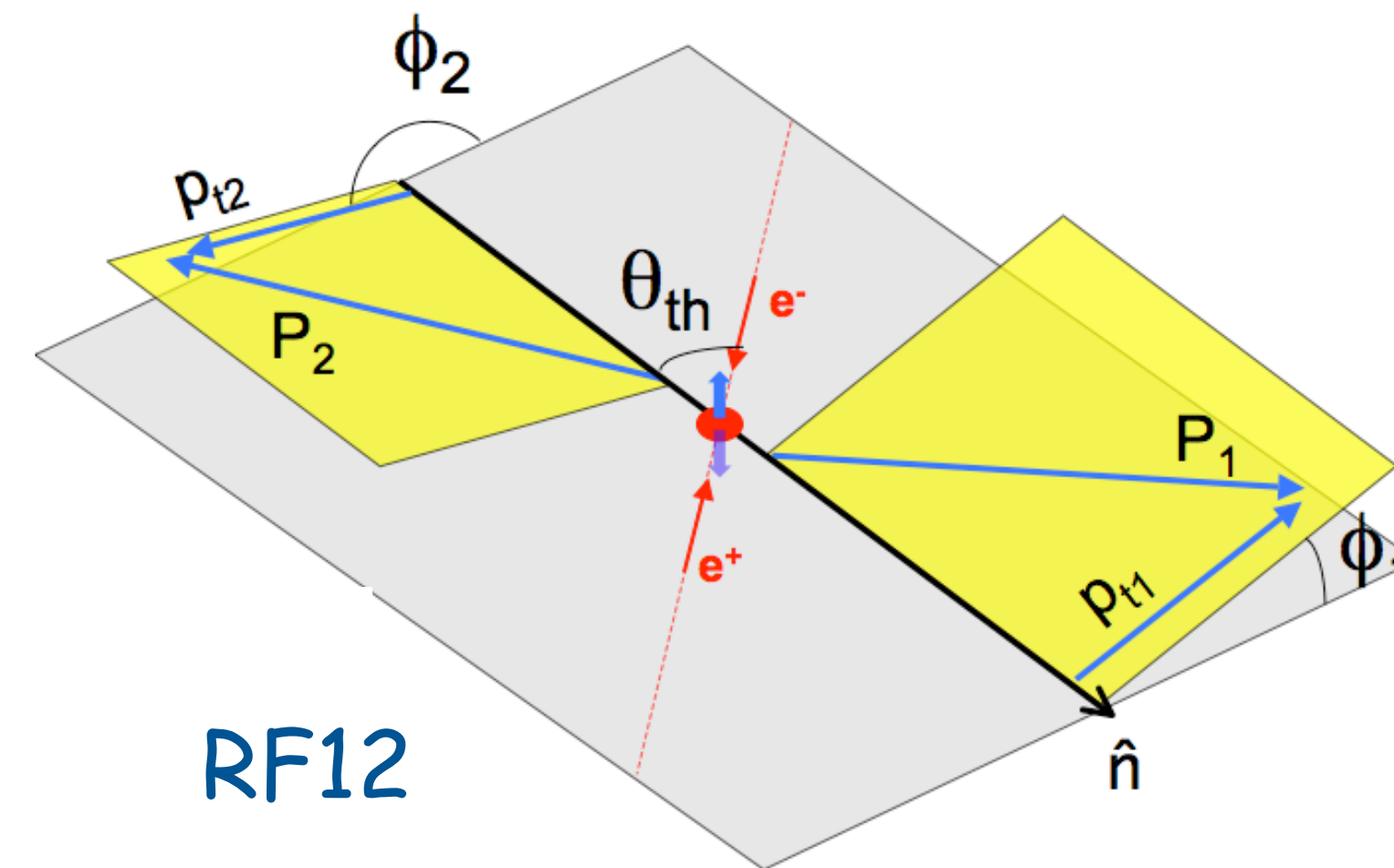
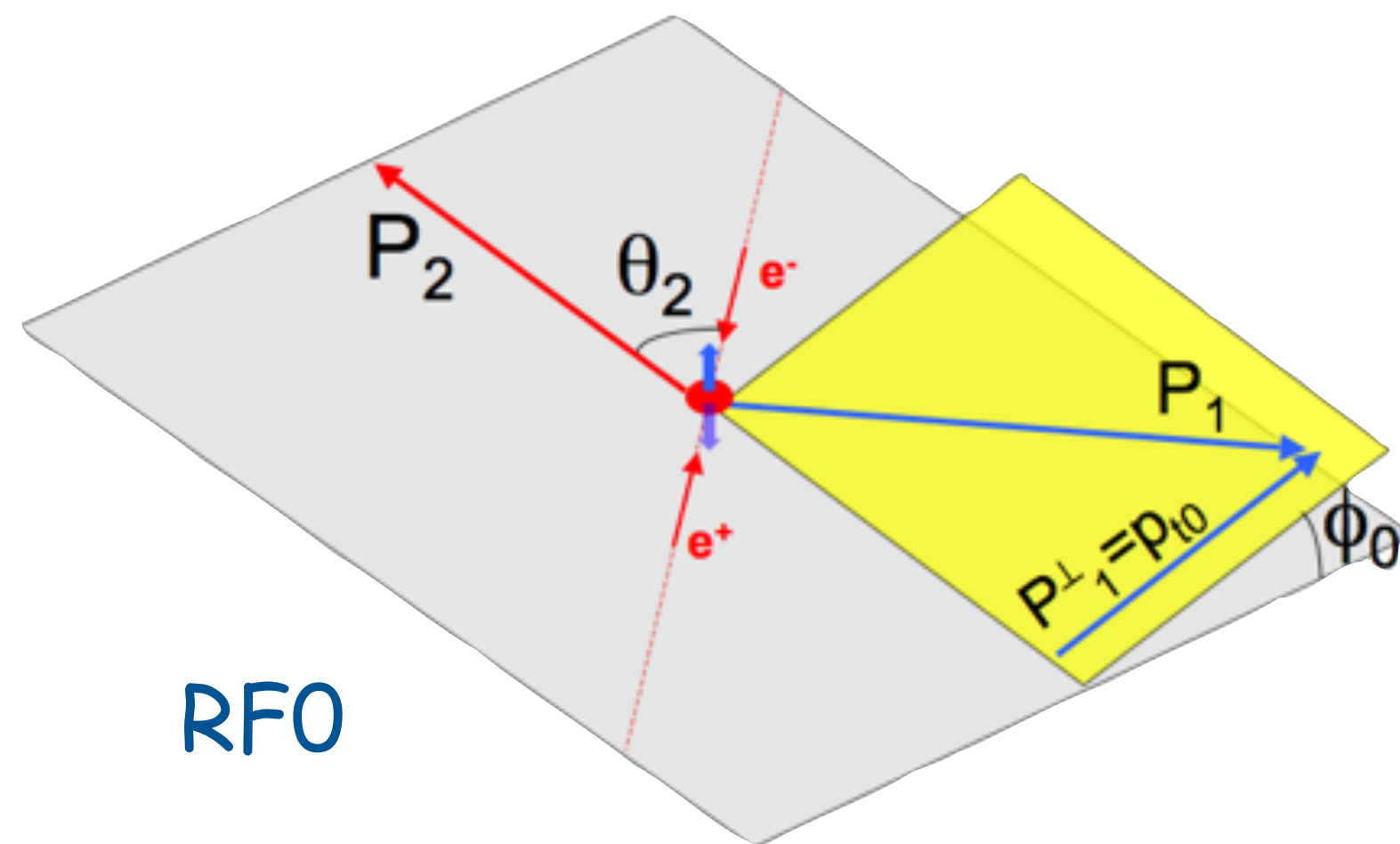
- fit Gaussian to low- P_{hT} data
- Gaussian widths depend on z and T
 - general increase with z with turnover at larger values of z
 - clear decrease of widths with increase of T
 - particles more and more collimated



polarization effects
despite unpolarized initial & states

hadron pairs: angular correlations

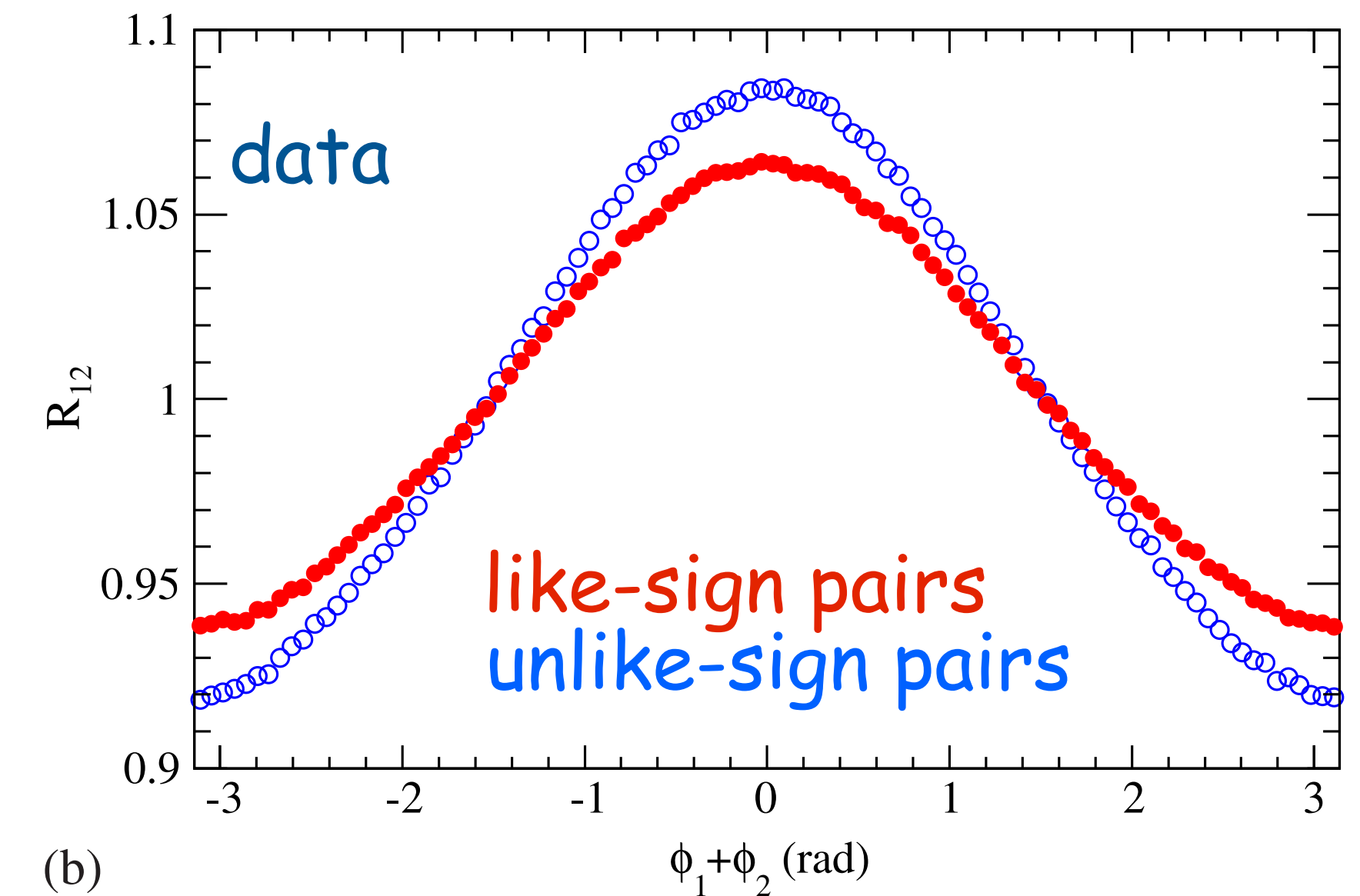
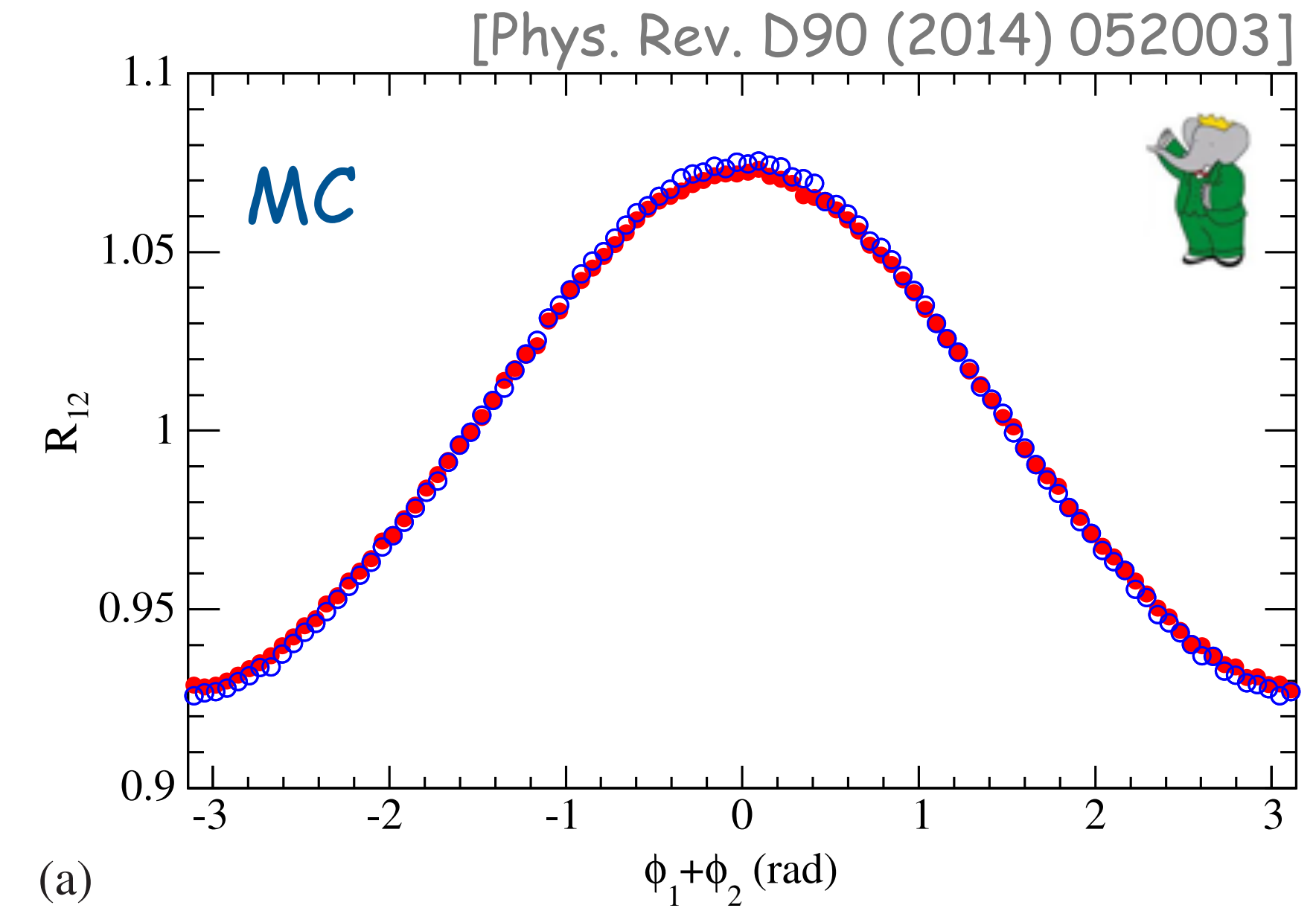
- angular correlations between nearly back-to-back hadrons used to tag transverse quark polarization -> Collins fragmentation functions
- RF0: one hadron as reference axis -> $\cos(2\phi_0)$ modulation
- RF12: thrust (or similar) axis -> $\cos(\phi_1+\phi_2)$ modulation



- RF0 and RF12: different convolutions over transverse momenta
- debatable: MC used to "correct" thrust axis to $q\bar{q}$ axis

hadron pairs: angular correlations

- challenge: large modulations even without Collins effect (e.g., in PYTHIA MC)

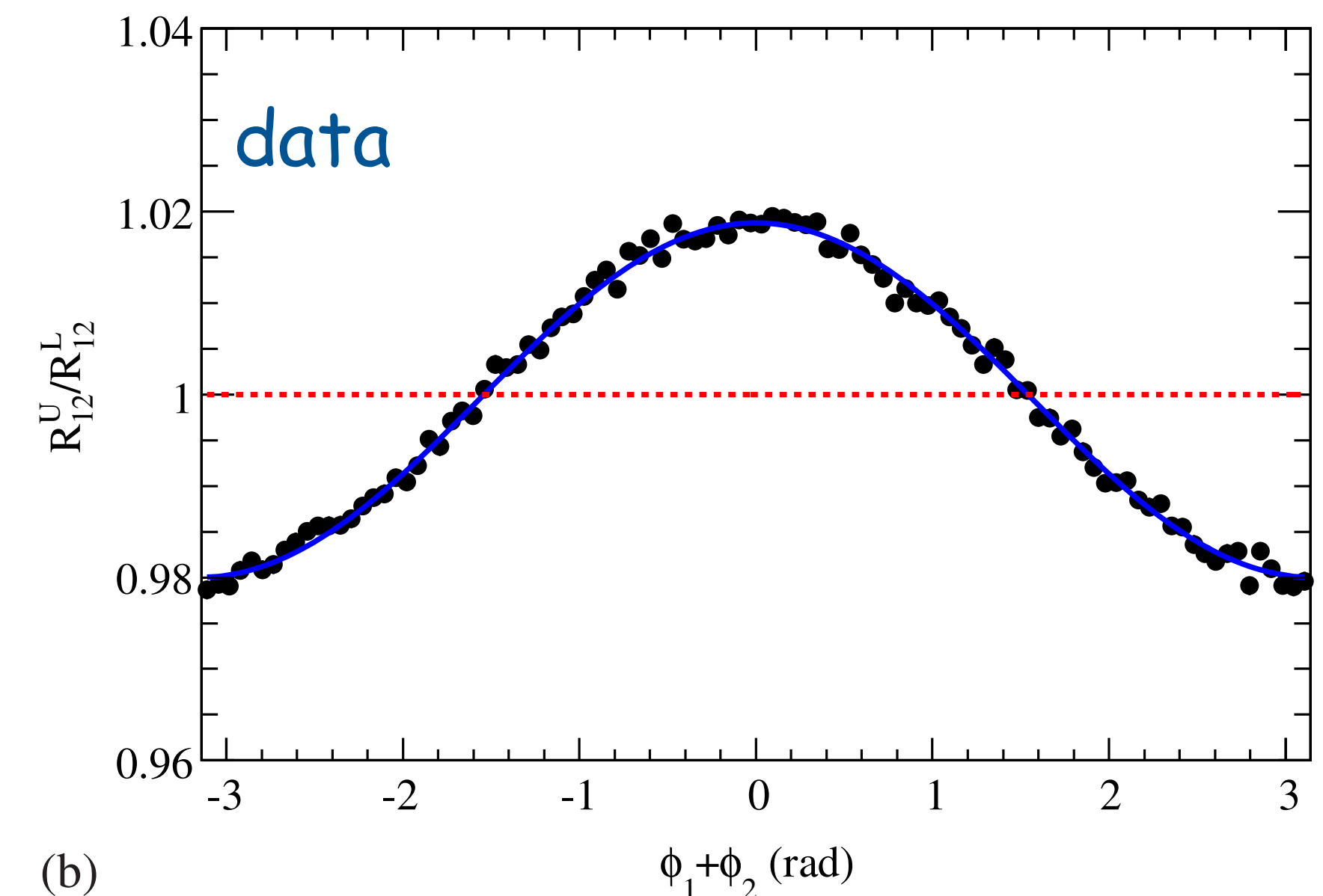
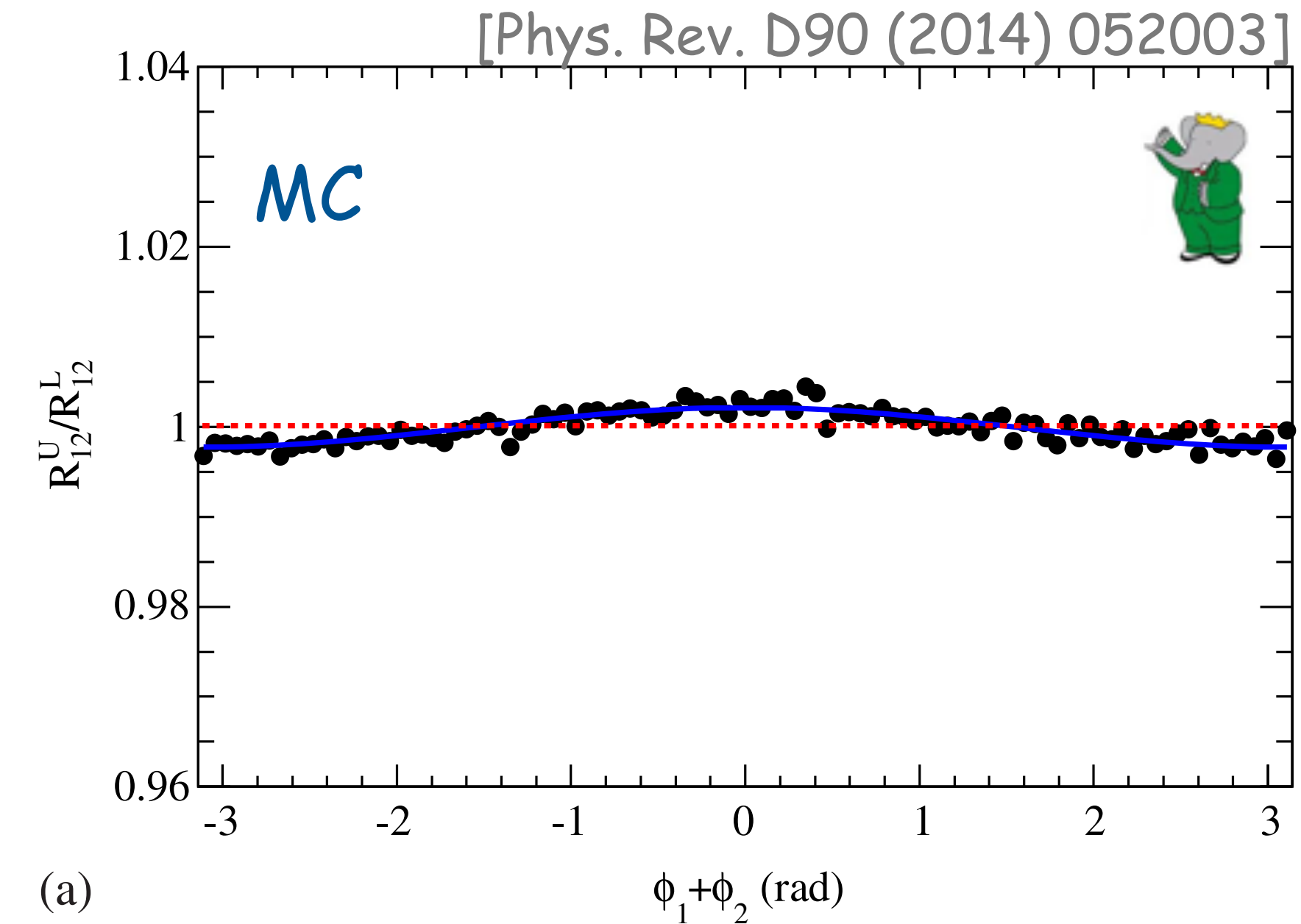


hadron pairs: angular correlations

- challenge: large modulations even without Collins effect (e.g., in PYTHIA MC)
- construct double ratio of normalized-yield distributions R_{12} , e.g. unlike-/like-sign:

$$\begin{aligned} \frac{R_{12}^U}{R_{12}^L} &\simeq \frac{1 + \left\langle \frac{\sin^2 \theta_{\text{th}}}{1 + \cos^2 \theta_{\text{th}}} \right\rangle G^U \cos(\phi_1 + \phi_2)}{1 + \left\langle \frac{\sin^2 \theta_{\text{th}}}{1 + \cos^2 \theta_{\text{th}}} \right\rangle G^L \cos(\phi_1 + \phi_2)} \\ &\simeq 1 + \left\langle \frac{\sin^2 \theta_{\text{th}}}{1 + \cos^2 \theta_{\text{th}}} \right\rangle \{G^U - G^L\} \cos(\phi_1 + \phi_2) \end{aligned}$$

- suppresses flavor-independent sources of modulations
- $G^{U/L}$: specific combinations of FFs
- remaining MC asymmetries \Rightarrow systematics

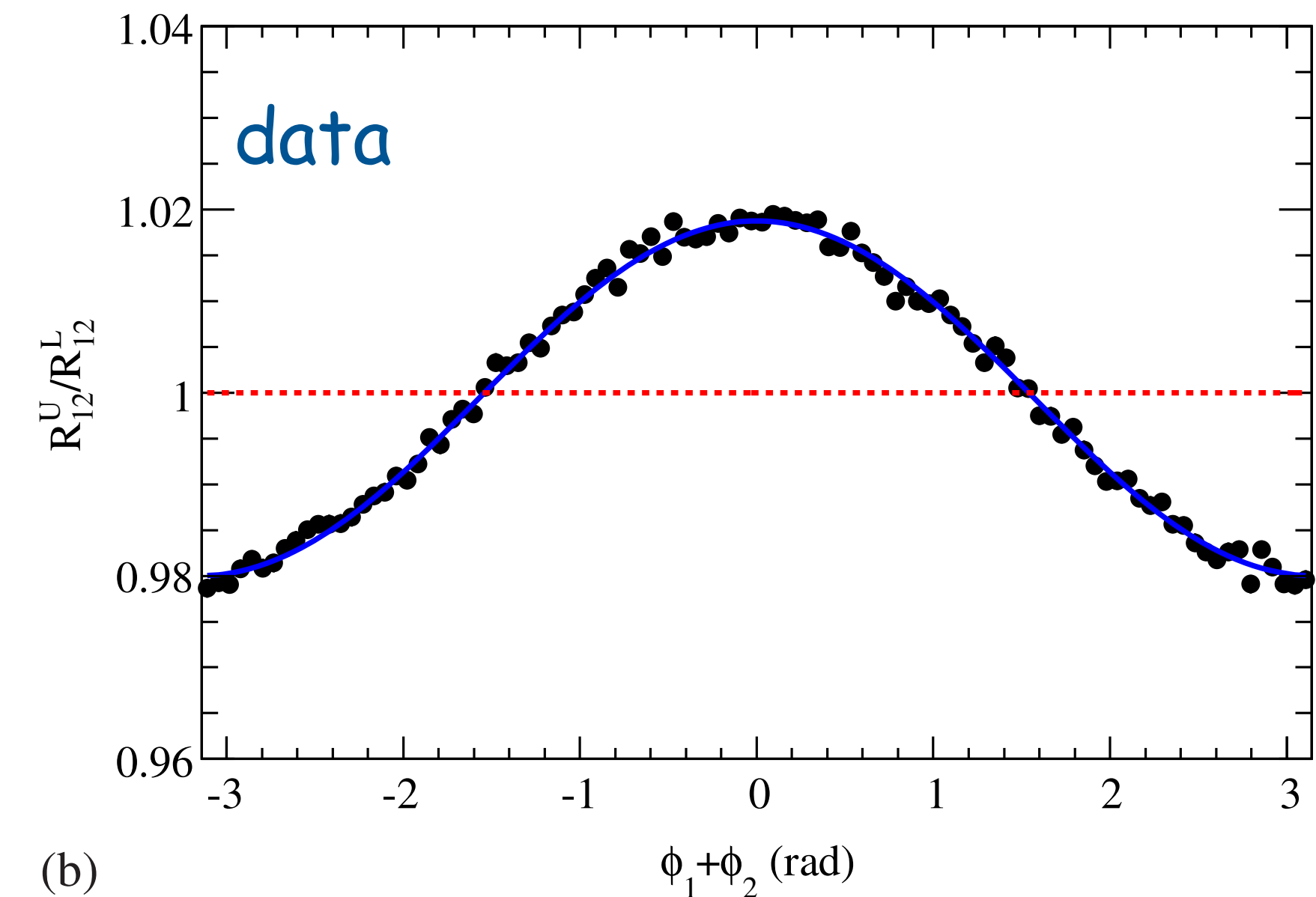
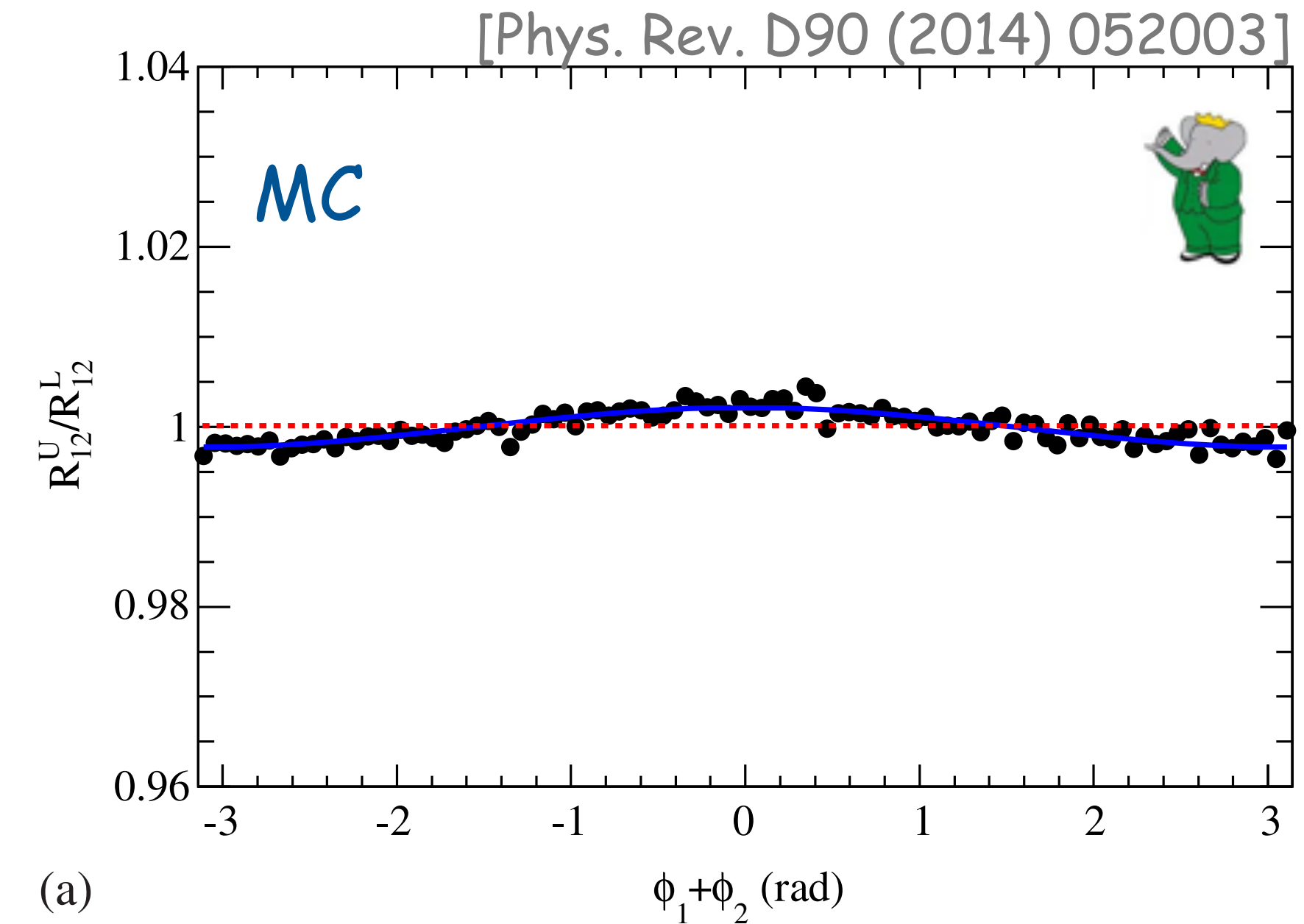


hadron pairs: angular correlations

- challenge: large modulations even without Collins effect (e.g., in PYTHIA MC)
- construct double ratio of normalized-yield distributions R_{12} , e.g. unlike-/like-sign:

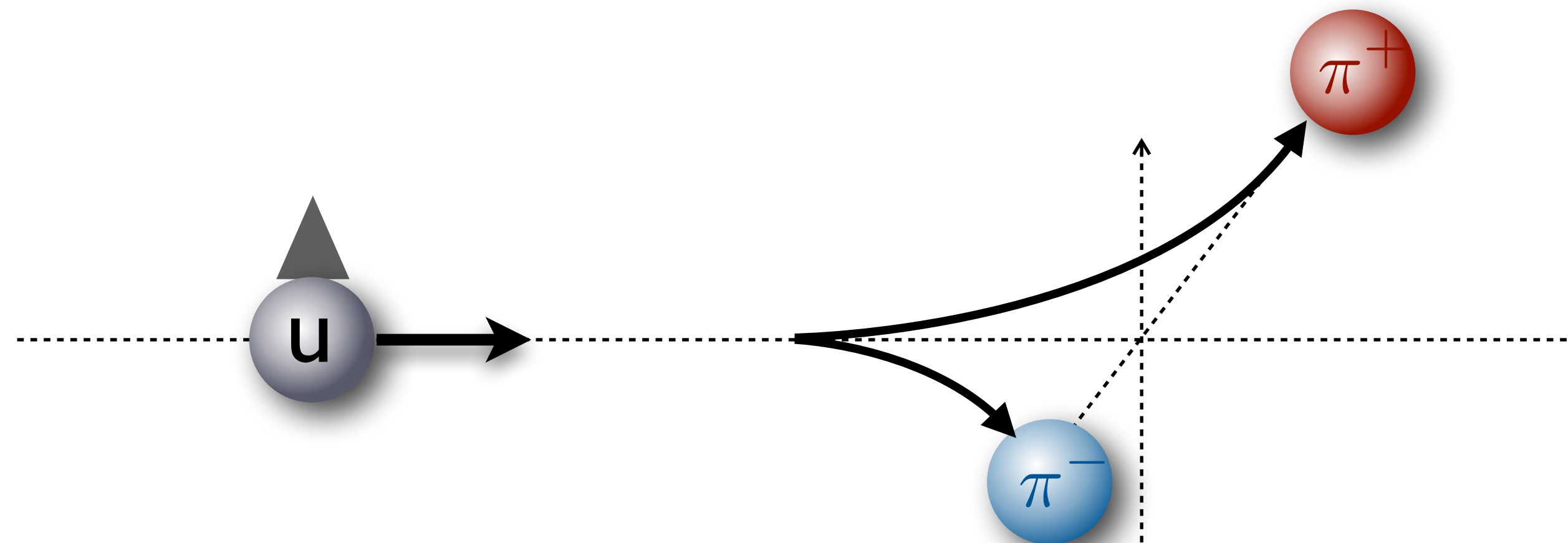
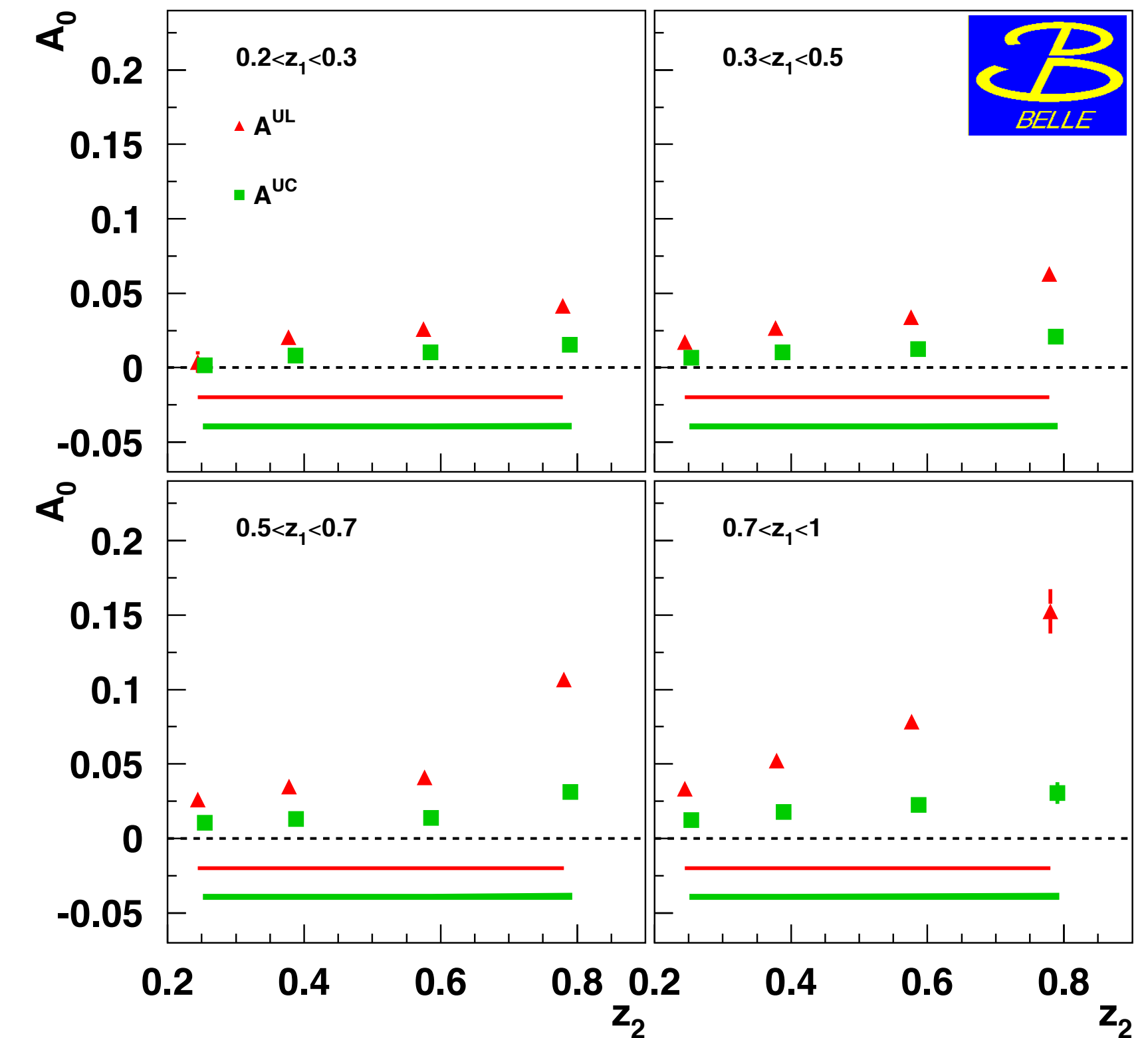
$$\begin{aligned} \frac{R_{12}^U}{R_{12}^L} &\simeq \frac{1 + \left\langle \frac{\sin^2 \theta_{\text{th}}}{1 + \cos^2 \theta_{\text{th}}} \right\rangle G^U \cos(\phi_1 + \phi_2)}{1 + \left\langle \frac{\sin^2 \theta_{\text{th}}}{1 + \cos^2 \theta_{\text{th}}} \right\rangle G^L \cos(\phi_1 + \phi_2)} \\ &\simeq 1 + \left\langle \frac{\sin^2 \theta_{\text{th}}}{1 + \cos^2 \theta_{\text{th}}} \right\rangle \{G^U - G^L\} \cos(\phi_1 + \phi_2) \end{aligned}$$

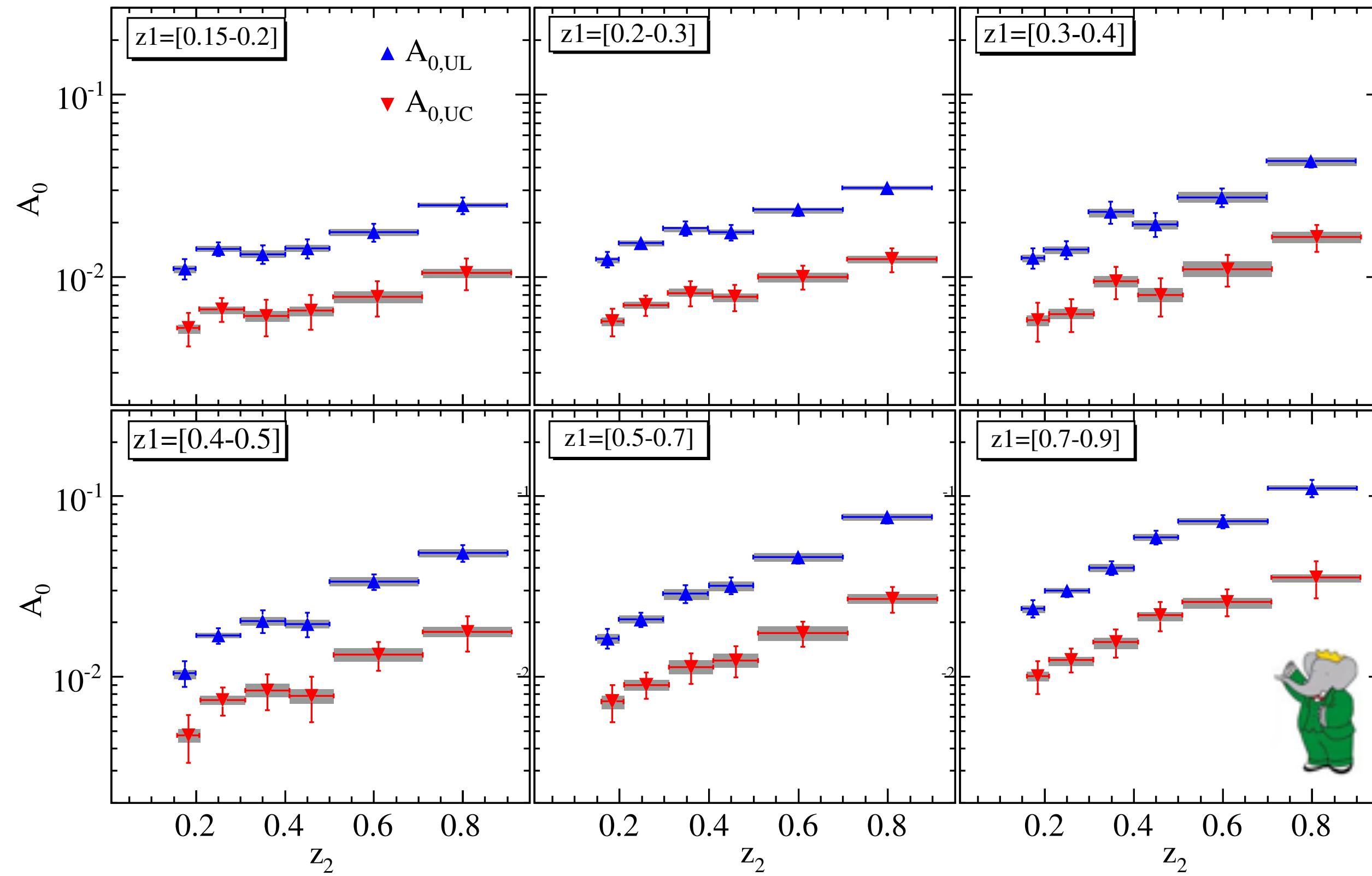
- suppresses flavor-independent sources of modulations
- $G^{U/L}$: specific combinations of FFs
- remaining MC asymmetries \Rightarrow systematics



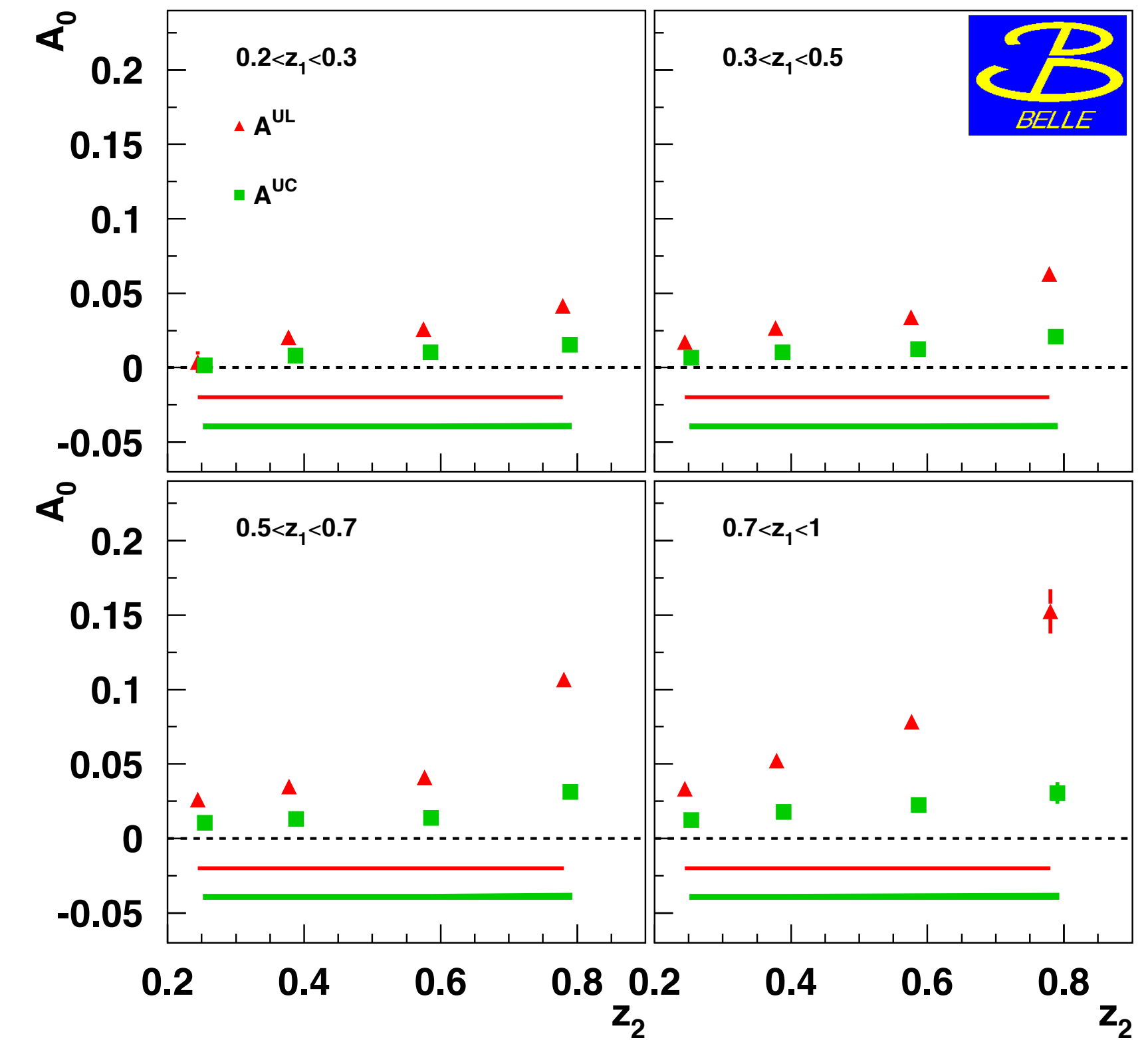
Collins asymmetries (RF0)

- first measurement of Collins asymmetries by Belle [PRL 96 (2006) 232002, PRD 78 (2008) 032011, PRD 86 (2012) 039905(E)]
- significant asymmetries rising with z
- used for first transversity and Collins FF extractions





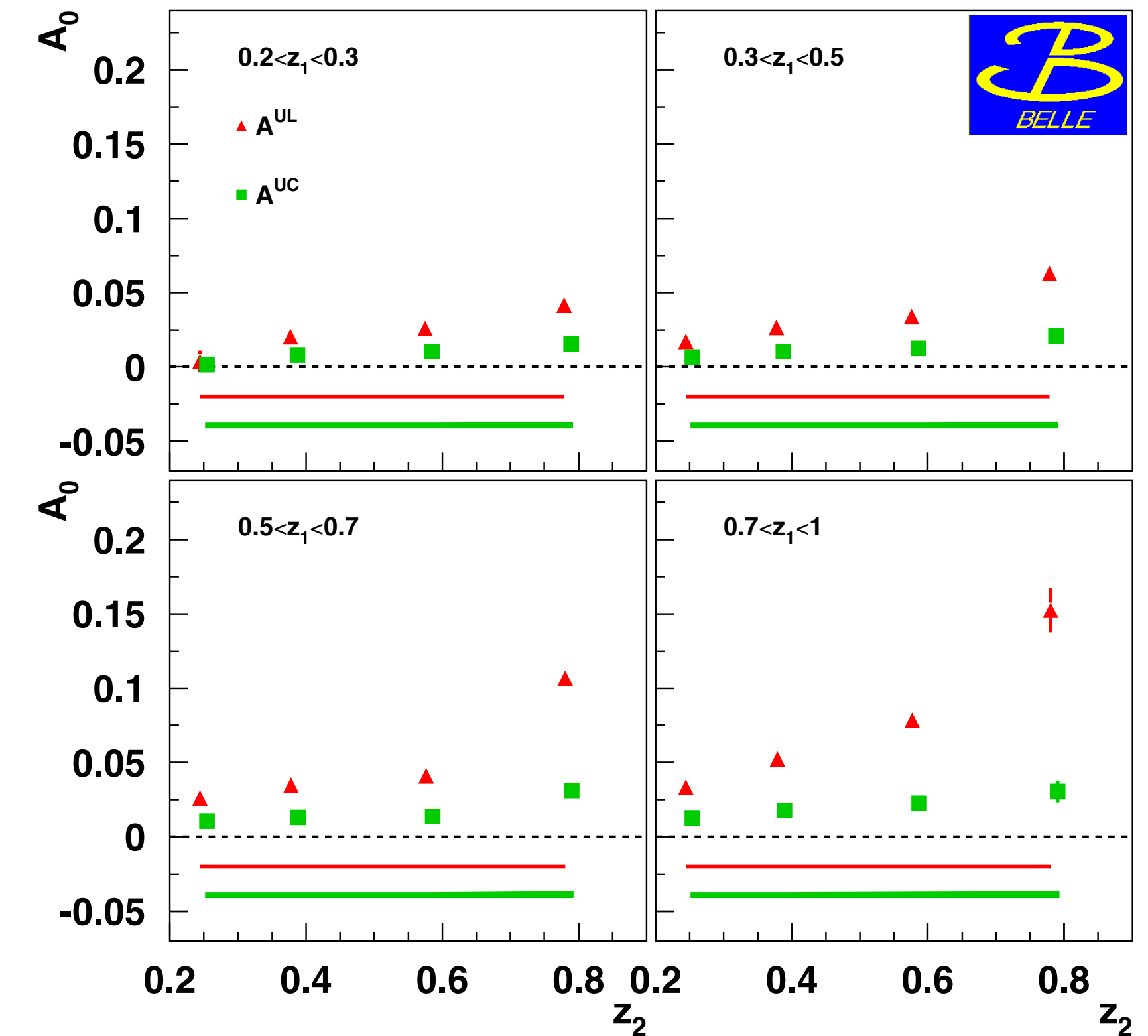
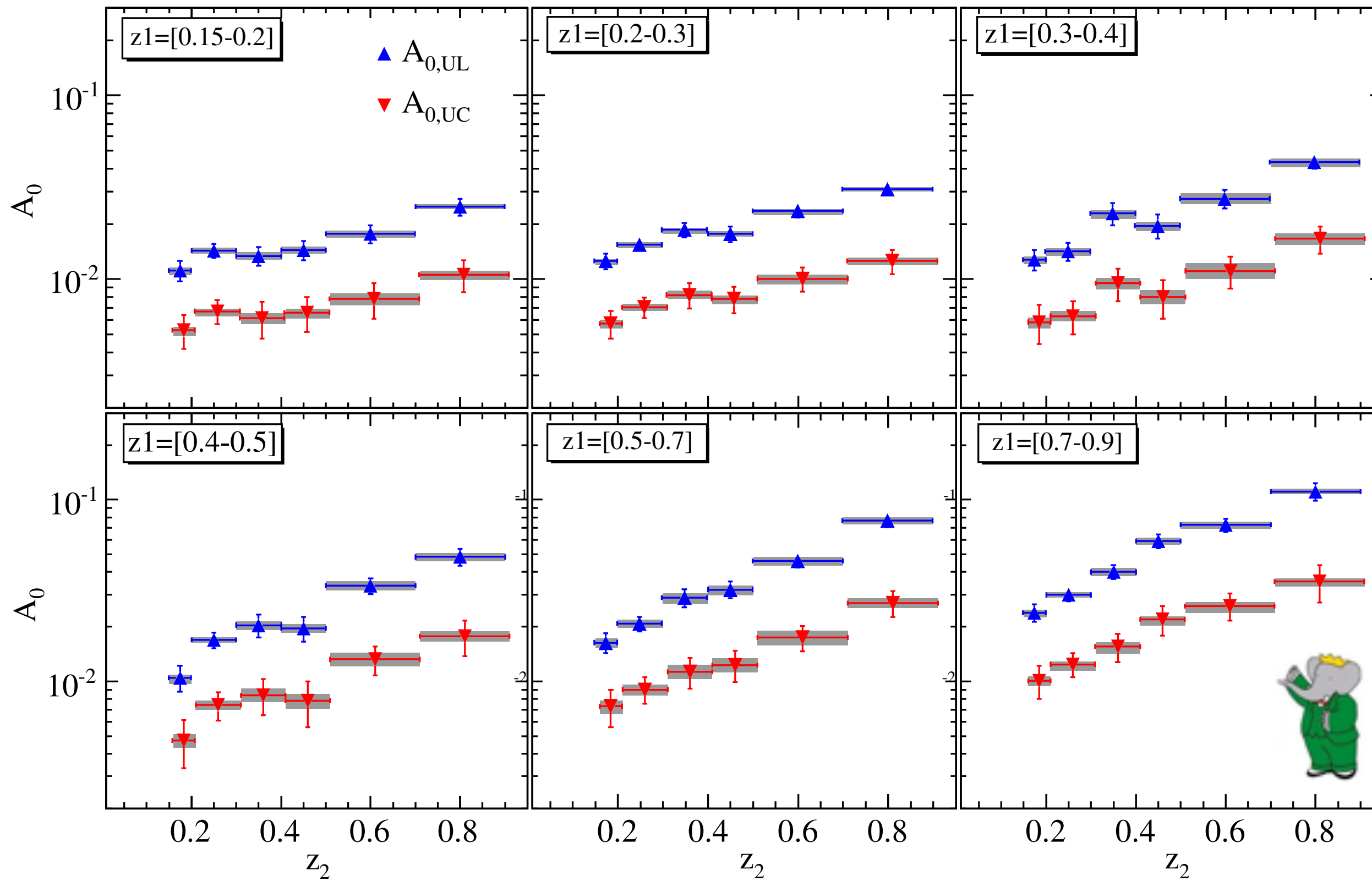
Collins asymmetries (RF0)



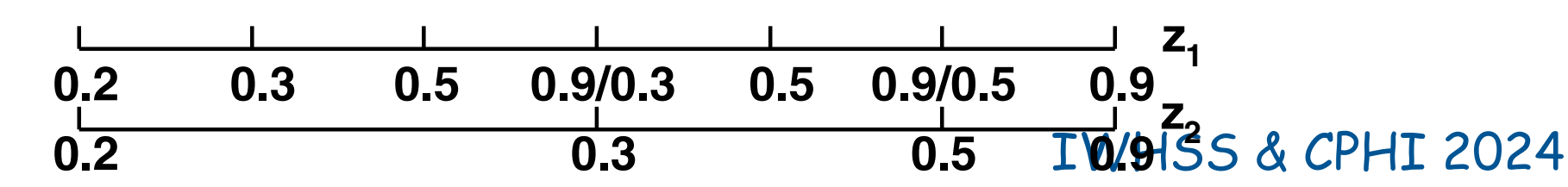
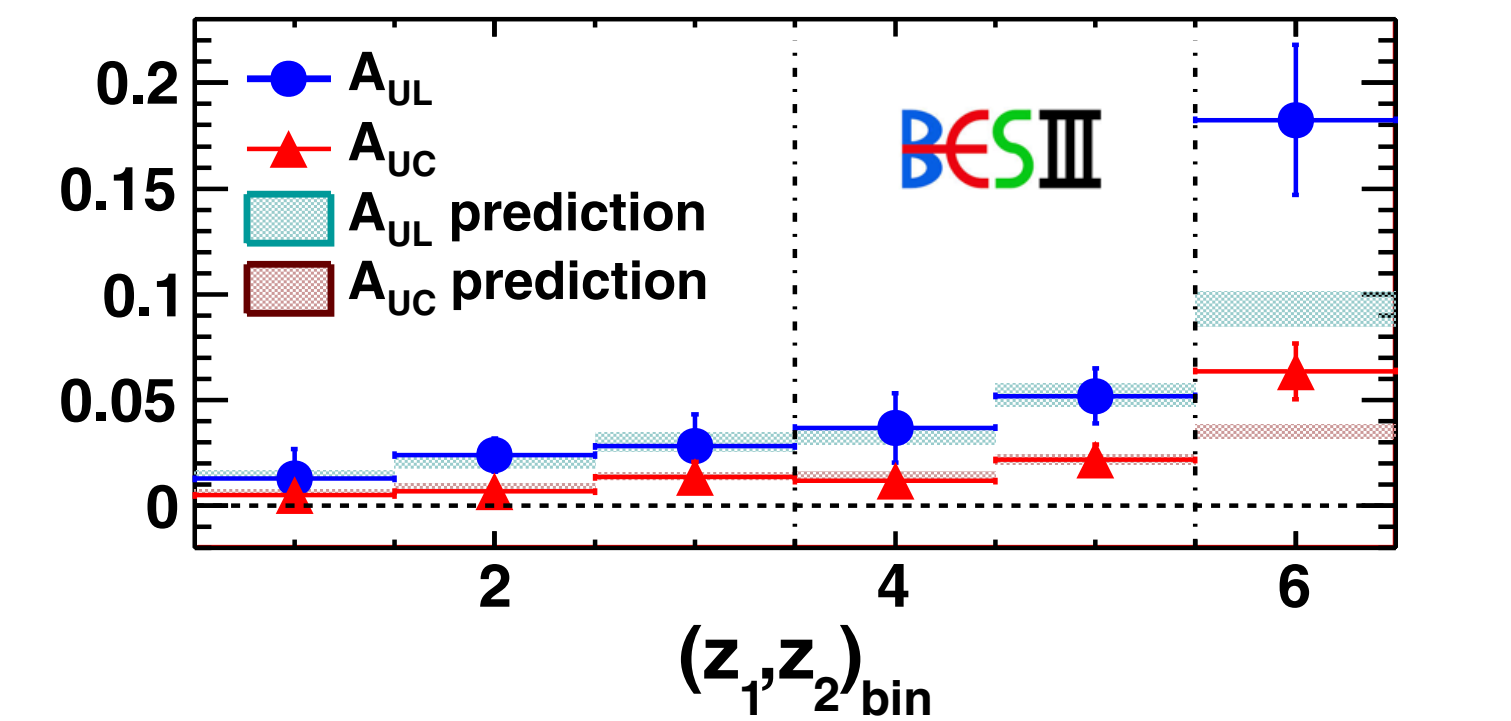
- BaBar results [PRD 90 (2014) 052003] consistent with Belle

Collins asymmetries (RFO)

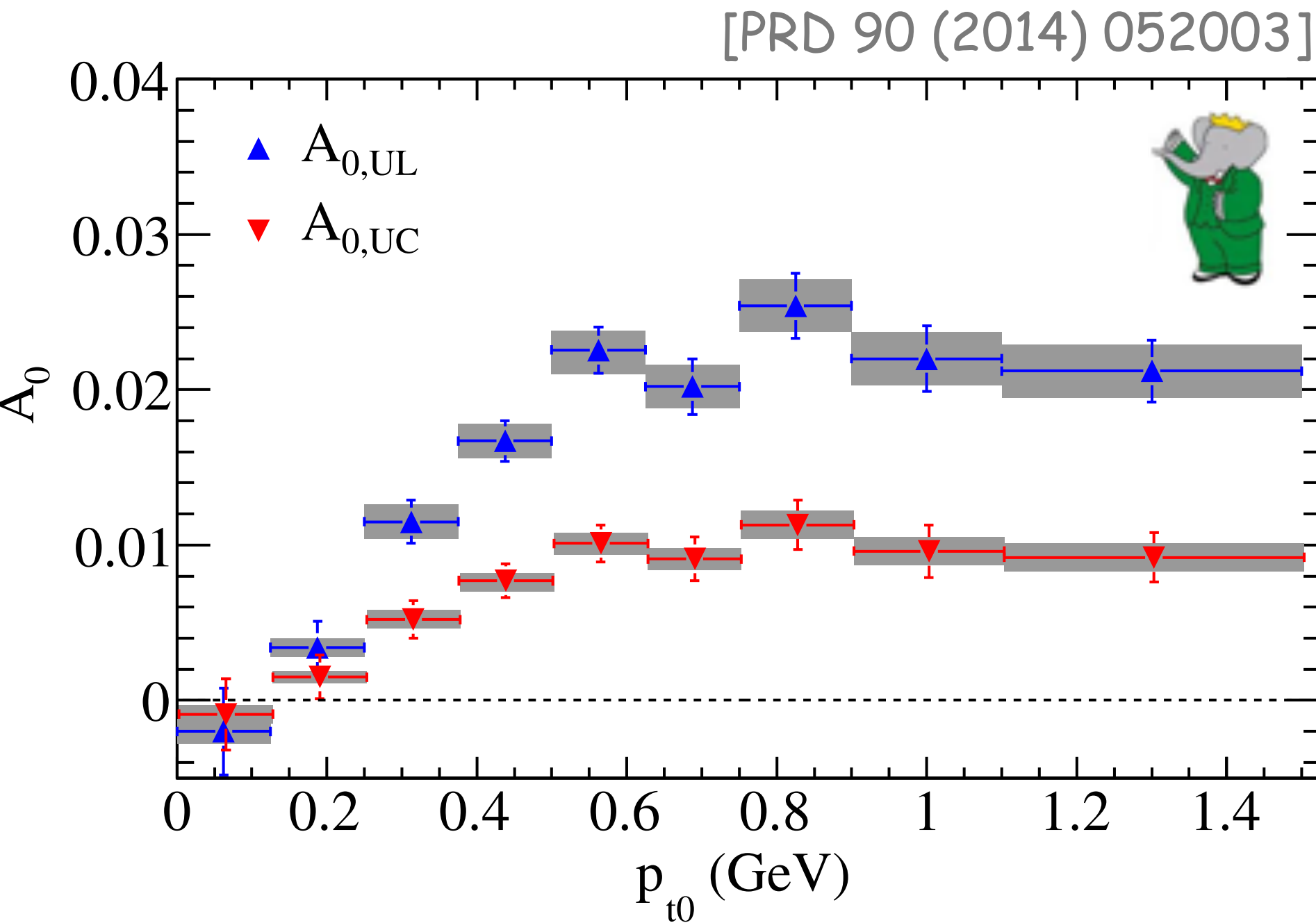
[Phys. Rev. D90 (2014) 052003]



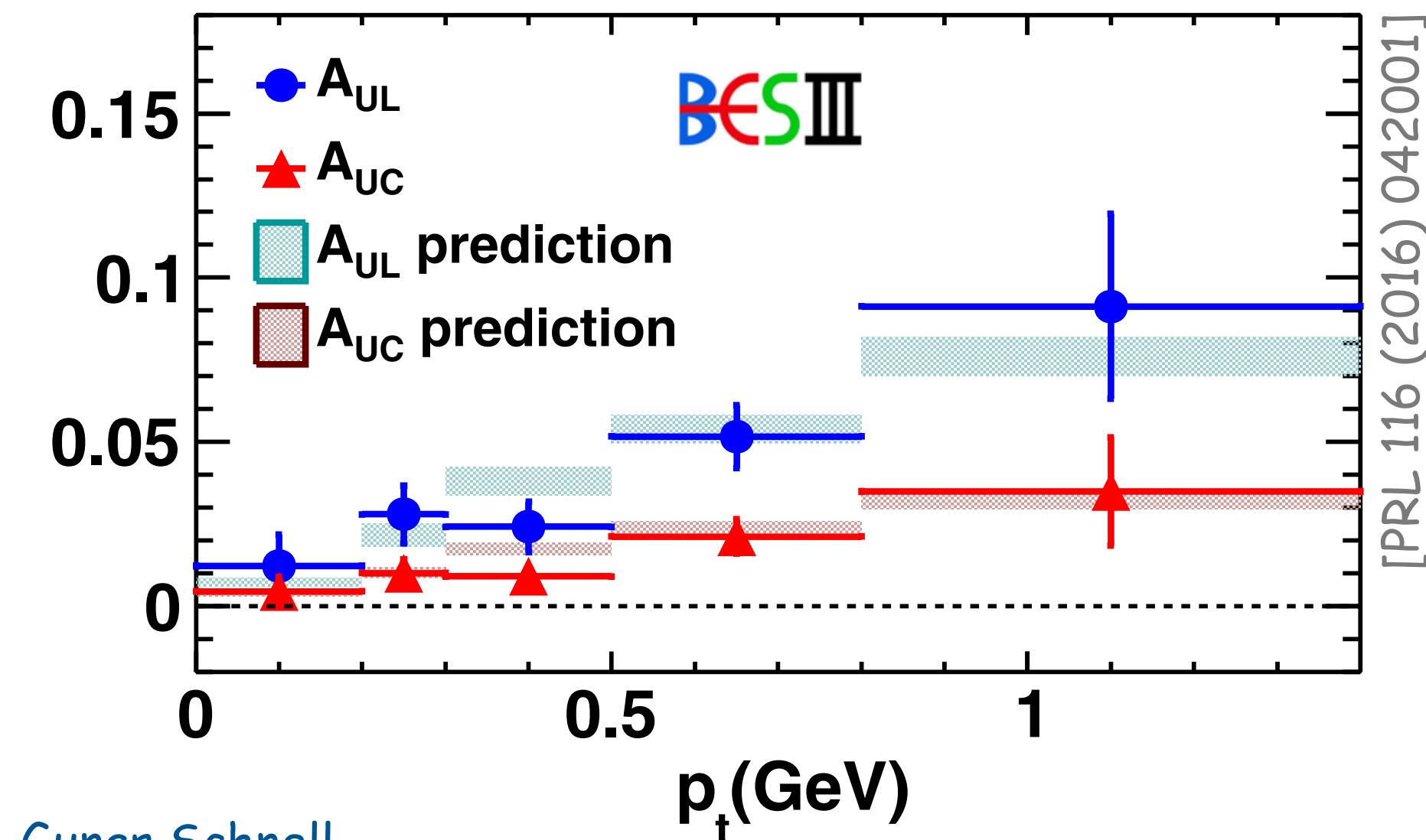
- BaBar results [PRD 90 (2014) 052003] consistent with Belle
- BESIII [PRL 116 (2016) 042001] (at smaller s) consistent with TMD evolution [Kang et al., PRD 93 (2016) 014009]



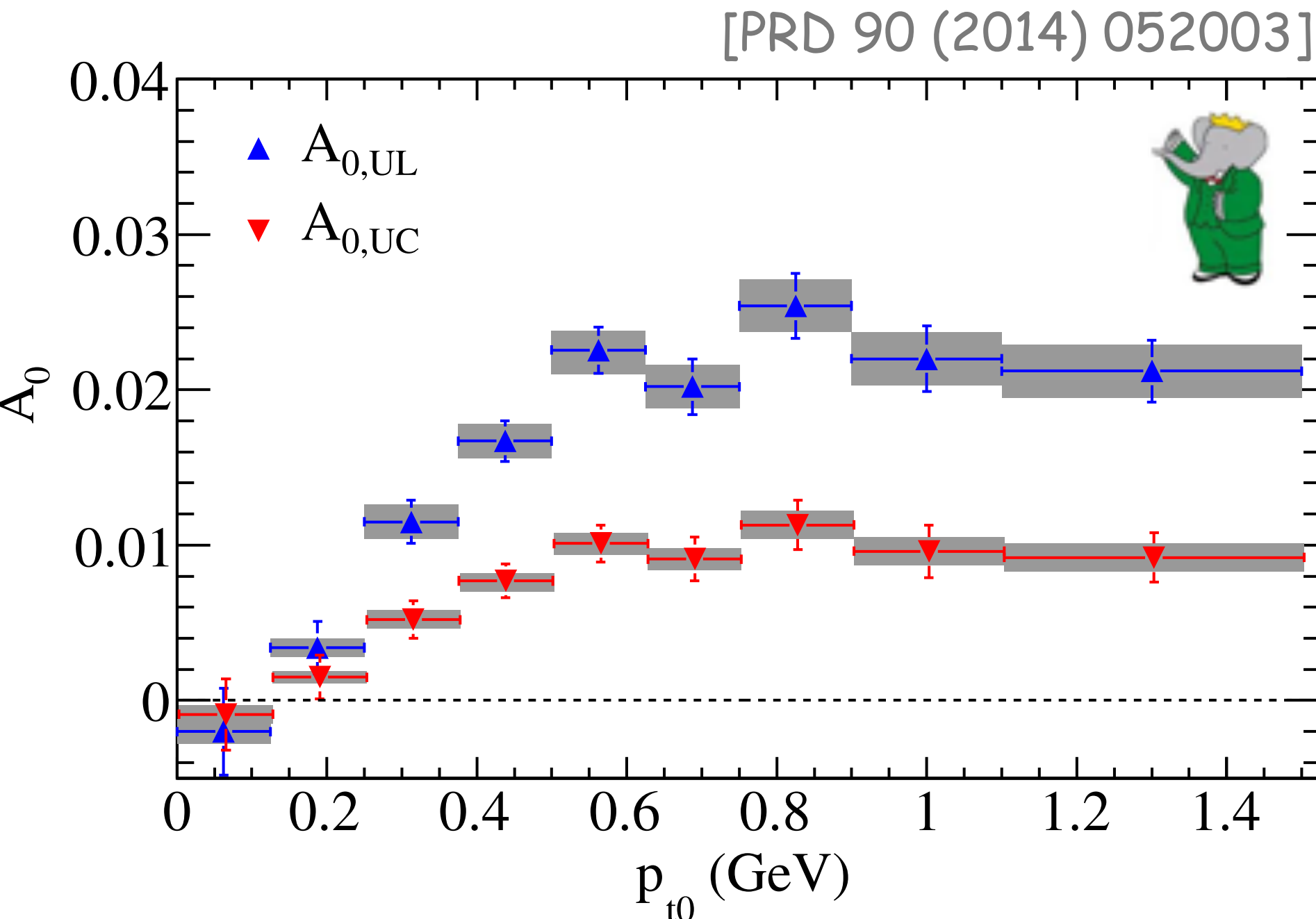
Collins asymmetries - going further



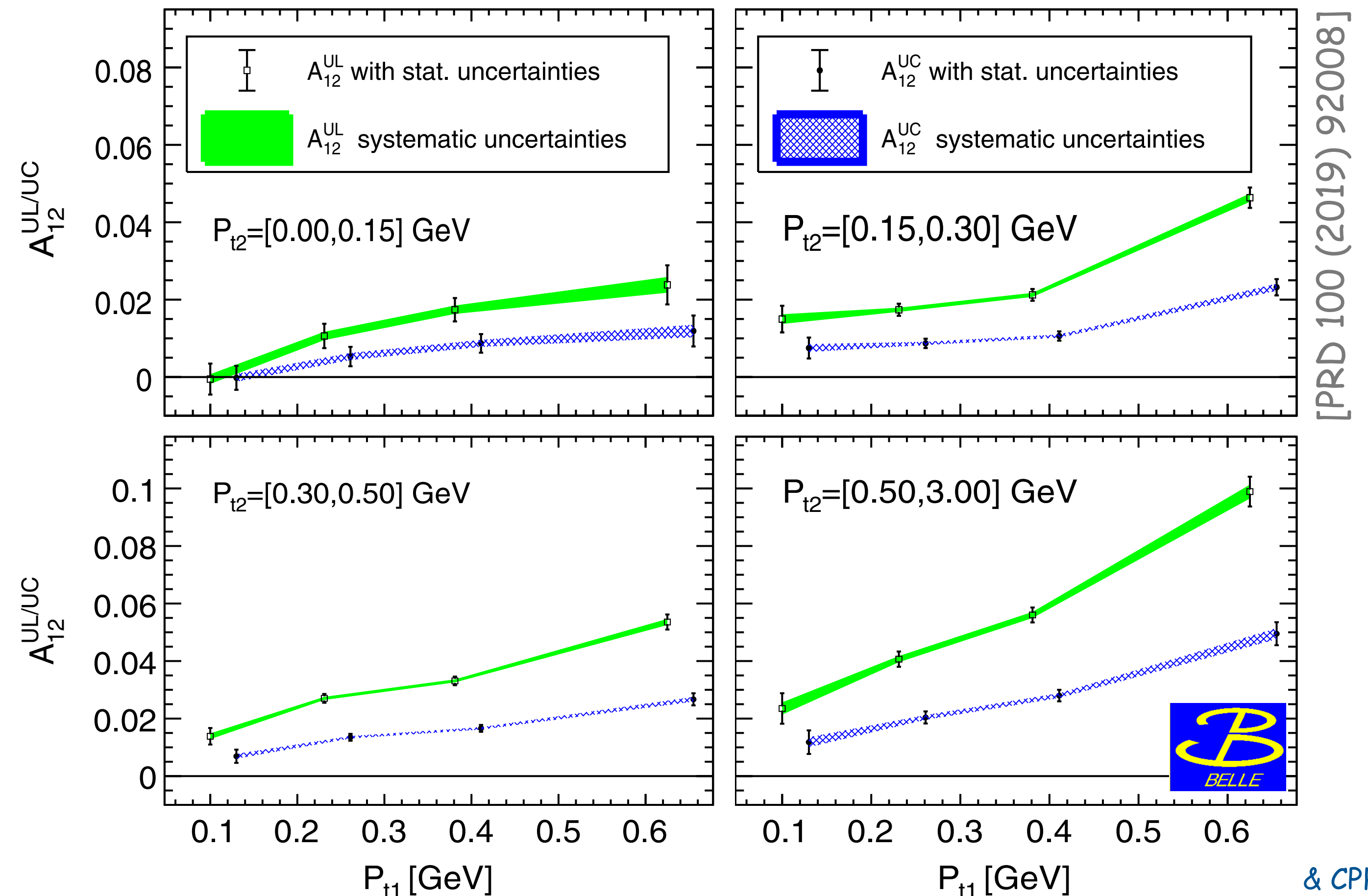
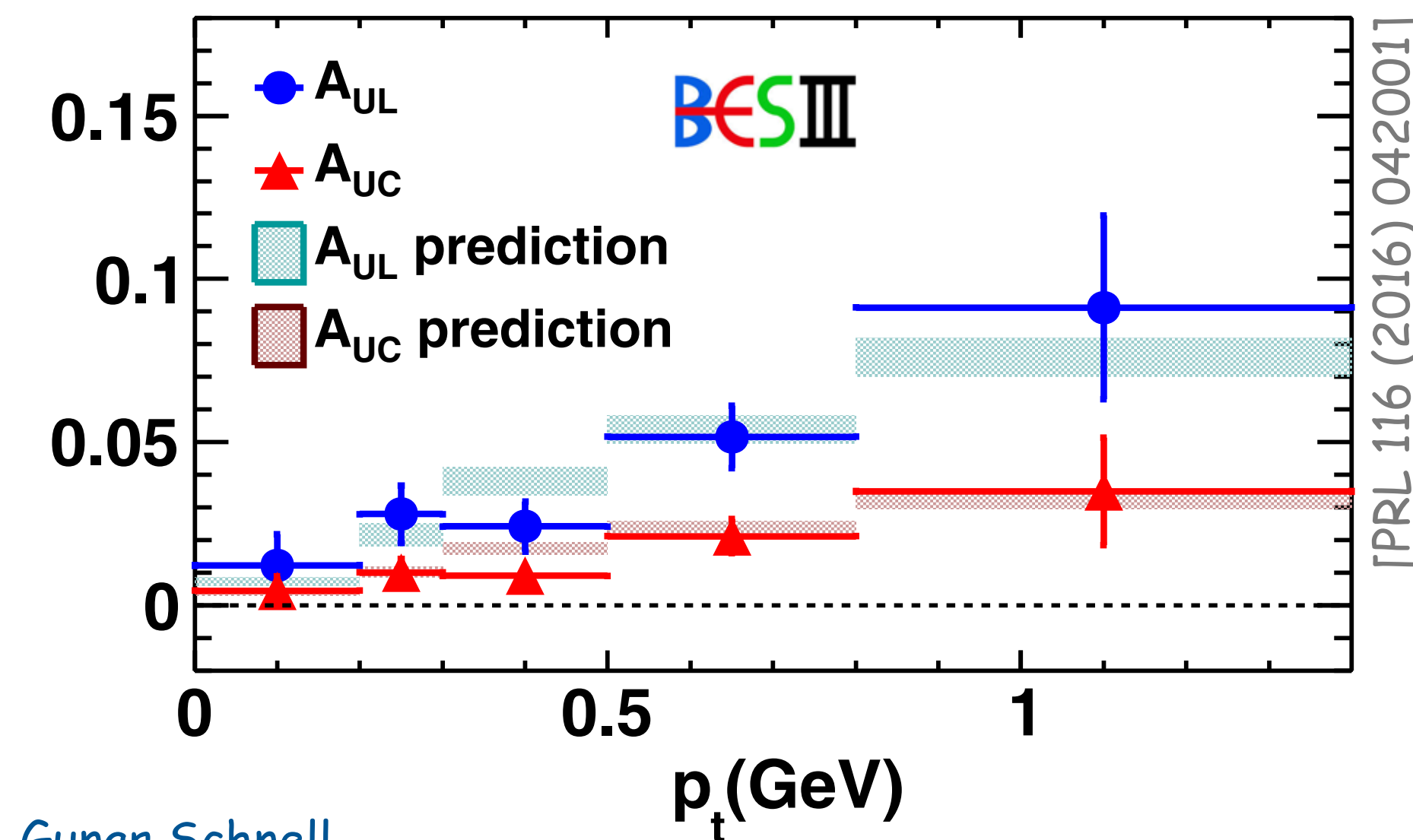
- p_T dependence for charged pions from BaBar & BESIII
- typical rise with p_T ; turnover around 0.8 GeV



Collins asymmetries - going further



- p_T dependence for charged pions from BaBar & BESIII
- typical rise with p_T ; turnover around 0.8 GeV
- ... now also from Belle in R12 frame:



Collins asymmetries - going further

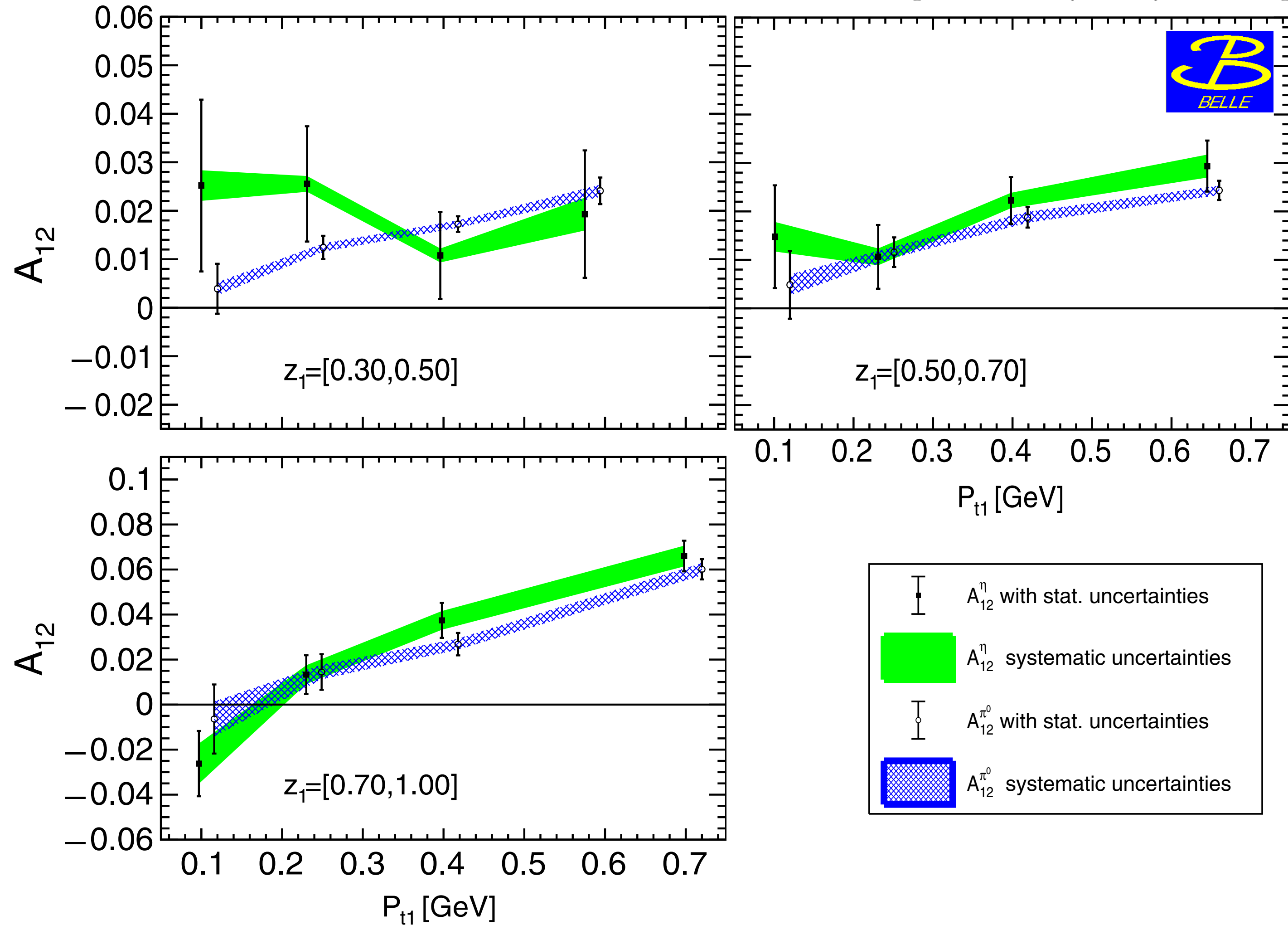
- ... as well as for neutral pion and eta

$$R_{12}^{\pi^0} = \frac{R_{12}^{0\pm}}{R_{12}^L} = \frac{\pi^0\pi^+ + \pi^0\pi^-}{\pi^+\pi^+ + \pi^-\pi^-}$$

$$R_{12}^{\eta} = \frac{R_{12}^{\eta\pm}}{R_{12}^L} = \frac{\eta\pi^+ + \eta\pi^-}{\pi^+\pi^+ + \pi^-\pi^-}$$

- no significant differences observed

[PRD 100 (2019) 92008]



Collins asymmetries - going further

$$R_{12}^{\pi^0} = \frac{R_{12}^{0\pm}}{R_{12}^L} \approx 1 + \cos(\phi_{12}) \frac{\sin^2(\theta)}{1 + \cos^2(\theta)}$$

$$\times \left\{ \frac{5(H_1^{\perp, fav} + H_1^{\perp, dis}) \otimes (H_1^{\perp, fav} + H_1^{\perp, dis}) + 4H_{1,s \rightarrow \pi}^{\perp, dis} \otimes H_{1,s \rightarrow \pi}^{\perp, dis}}{5(D_1^{fav} + D_1^{dis}) \otimes (D_1^{fav} + D_1^{dis}) + 4D_{1,s \rightarrow \pi}^{dis} \otimes D_{1,s \rightarrow \pi}^{dis}} \right.$$

$$\left. - \frac{5(H_1^{\perp, fav} \otimes H_1^{\perp, dis} + H_1^{\perp, dis} \otimes H_1^{\perp, fav}) + 2H_{1,s \rightarrow \pi}^{\perp, dis} H_{1,s \rightarrow \pi}^{\perp, dis}}{5(D_1^{fav} \otimes D_1^{dis} + D_1^{dis} \otimes D_1^{fav}) + 2D_{1,s \rightarrow \pi}^{dis} \otimes D_{1,s \rightarrow \pi}^{dis}} \right\}.$$

isospin
= $A_{12}^{UL} - A_{12}^{UC}$

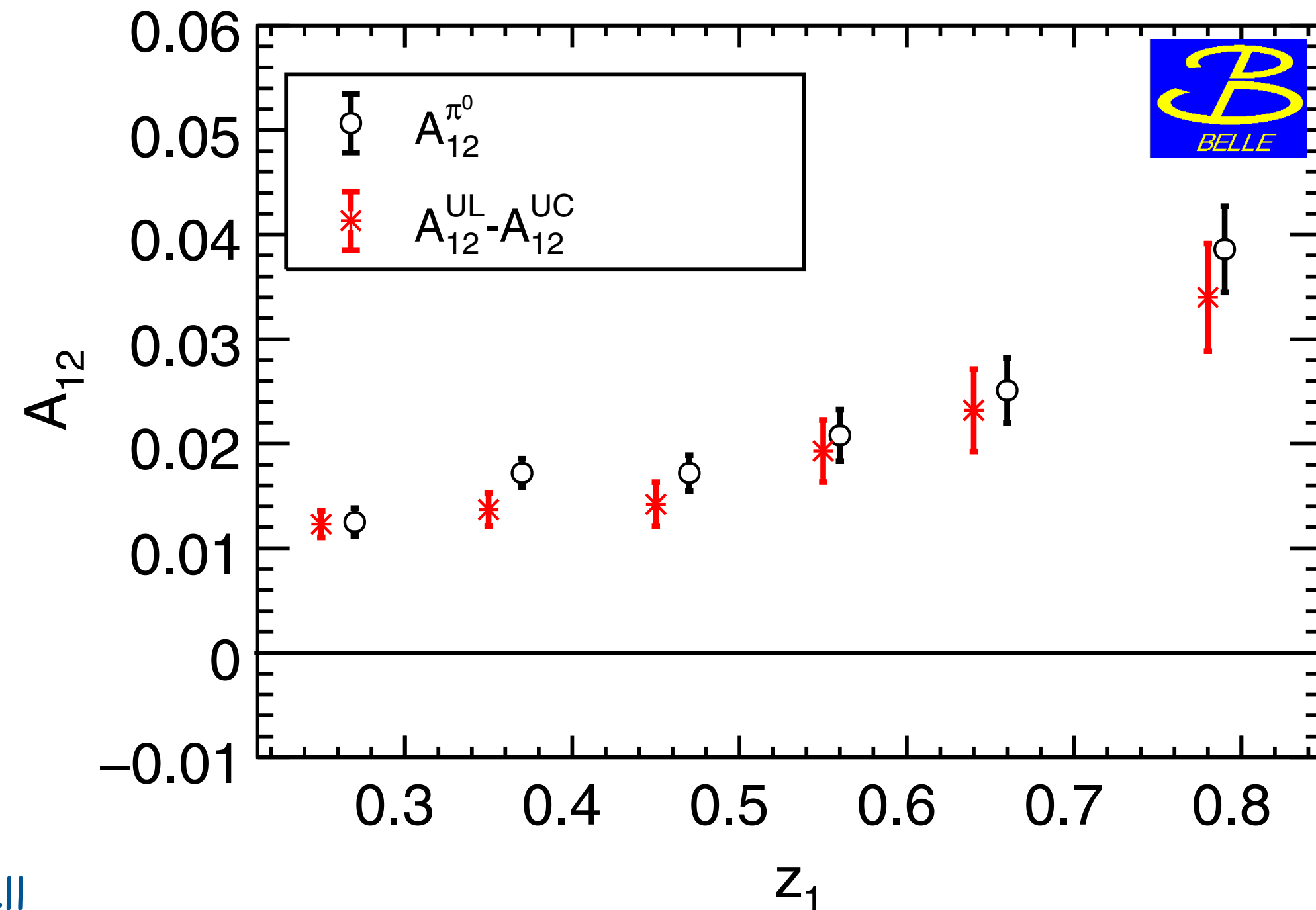
Collins asymmetries - going further

$$R_{12}^{\pi^0} = \frac{R_{12}^{0\pm}}{R_{12}^L} \approx 1 + \cos(\phi_{12}) \frac{\sin^2(\theta)}{1 + \cos^2(\theta)}$$

$$\times \left\{ \frac{5(H_1^{\perp, fav} + H_1^{\perp, dis}) \otimes (H_1^{\perp, fav} + H_1^{\perp, dis}) + 4H_{1,s \rightarrow \pi}^{\perp, dis} \otimes H_{1,s \rightarrow \pi}^{\perp, dis}}{5(D_1^{fav} + D_1^{dis}) \otimes (D_1^{fav} + D_1^{dis}) + 4D_{1,s \rightarrow \pi}^{dis} \otimes D_{1,s \rightarrow \pi}^{dis}} \right.$$

$$\left. - \frac{5(H_1^{\perp, fav} \otimes H_1^{\perp, dis} + H_1^{\perp, dis} \otimes H_1^{\perp, fav}) + 2H_{1,s \rightarrow \pi}^{\perp, dis} H_{1,s \rightarrow \pi}^{\perp, dis}}{5(D_1^{fav} \otimes D_1^{dis} + D_1^{dis} \otimes D_1^{fav}) + 2D_{1,s \rightarrow \pi}^{dis} \otimes D_{1,s \rightarrow \pi}^{dis}} \right\}$$

isospin
= $A_{12}^{UL} - A_{12}^{UC}$



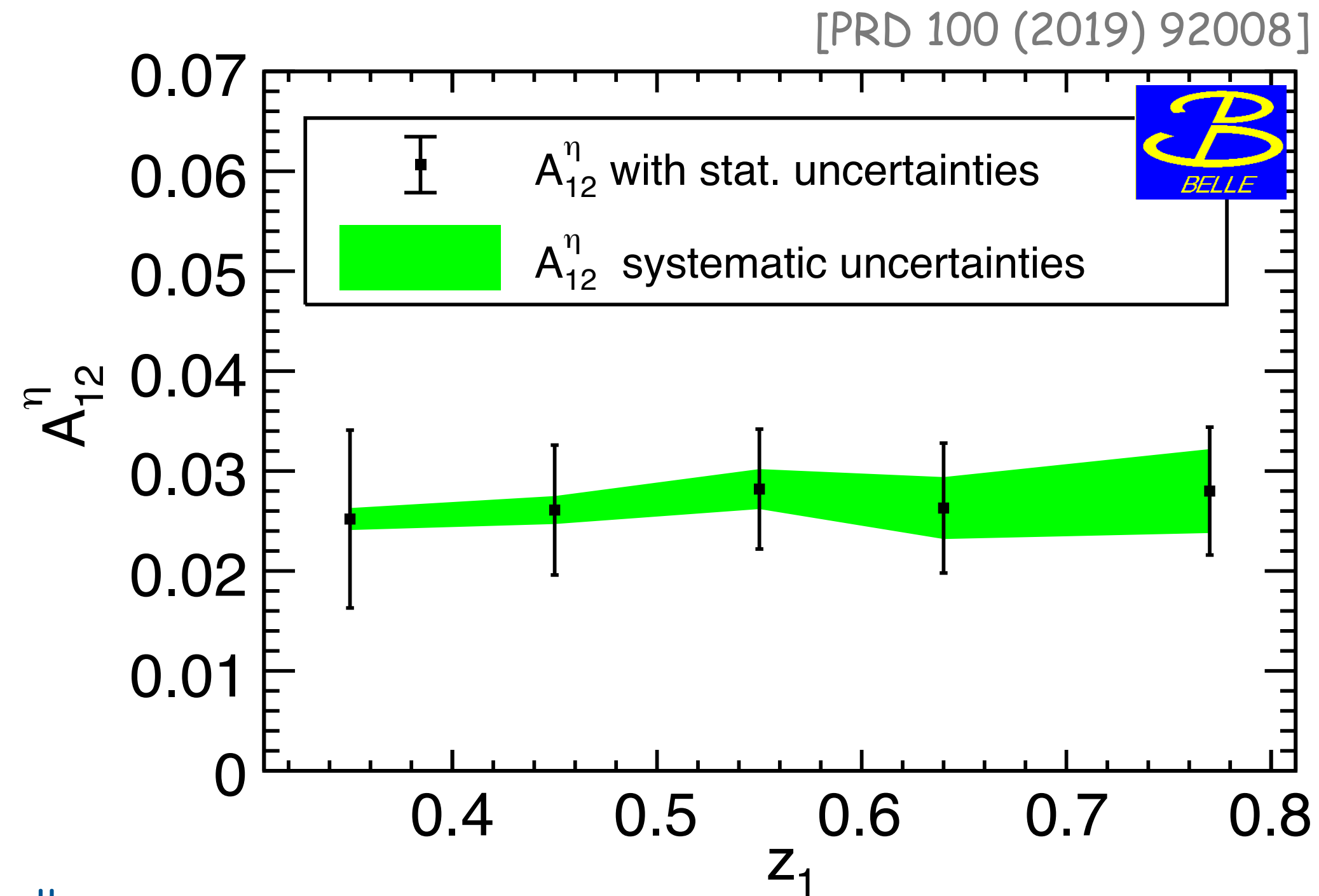
- consistency between neutral and charged pions
- typical rise with z also seen for neutral pions

Collins asymmetries - going further

$$R_{12}^{\pi^0} = \frac{R_{12}^{0\pm}}{R_{12}^L} \approx 1 + \cos(\phi_{12}) \frac{\sin^2(\theta)}{1 + \cos^2(\theta)}$$

$$\times \left\{ \frac{5(H_1^{\perp, fav} + H_1^{\perp, dis}) \otimes (H_1^{\perp, fav} + H_1^{\perp, dis}) + 4H_{1,s \rightarrow \pi}^{\perp, dis} \otimes H_{1,s \rightarrow \pi}^{\perp, dis}}{5(D_1^{fav} + D_1^{dis}) \otimes (D_1^{fav} + D_1^{dis}) + 4D_{1,s \rightarrow \pi}^{dis} \otimes D_{1,s \rightarrow \pi}^{dis}} \right. \\ \left. - \frac{5(H_1^{\perp, fav} \otimes H_1^{\perp, dis} + H_1^{\perp, dis} \otimes H_1^{\perp, fav}) + 2H_{1,s \rightarrow \pi}^{\perp, dis} H_{1,s \rightarrow \pi}^{\perp, dis}}{5(D_1^{fav} \otimes D_1^{dis} + D_1^{dis} \otimes D_1^{fav}) + 2D_{1,s \rightarrow \pi}^{dis} \otimes D_{1,s \rightarrow \pi}^{dis}} \right\}$$

} isospin $\underline{=}$ $A_{12}^{UL} - A_{12}^{UC}$



- consistency between neutral and charged pions
- typical rise with z also seen for neutral pions
- ... while basically flat for eta

Collins asymmetries - going further

$$R_{12}^{\pi^0} = \frac{R_{12}^{0\pm}}{R_{12}^L} \approx 1 + \cos(\phi_{12}) \frac{\sin^2(\theta)}{1 + \cos^2(\theta)}$$

$$\times \left\{ \frac{5(H_1^{\perp, fav} + H_1^{\perp, dis}) \otimes (H_1^{\perp, fav} + H_1^{\perp, dis}) + 4H_{1, s \rightarrow \pi}^{\perp, dis} \otimes H_{1, s \rightarrow \pi}^{\perp, dis}}{5(D_1^{fav} + D_1^{dis}) \otimes (D_1^{fav} + D_1^{dis}) + 4D_{1, s \rightarrow \pi}^{dis} \otimes D_{1, s \rightarrow \pi}^{dis}} \right.$$

$$\left. - \frac{5(H_1^{\perp, fav} \otimes H_1^{\perp, dis} + H_1^{\perp, dis} \otimes H_1^{\perp, fav}) + 2H_{1, s \rightarrow \pi}^{\perp, dis} H_{1, s \rightarrow \pi}^{\perp, dis}}{5(D_1^{fav} \otimes D_1^{dis} + D_1^{dis} \otimes D_1^{fav}) + 2D_{1, s \rightarrow \pi}^{dis} \otimes D_{1, s \rightarrow \pi}^{dis}} \right\}.$$

- non-zero π^0 or η results **not** direct sign of non-zero π^0 or η Collins FFs

Collins asymmetries - going further

$$R_{12}^{\pi^0} = \frac{R_{12}^{0\pm}}{R_{12}^L} \approx 1 + \cos(\phi_{12}) \frac{\sin^2(\theta)}{1 + \cos^2(\theta)}$$

$$\times \left\{ \frac{5(H_1^{\perp, fav} + H_1^{\perp, dis}) \otimes (H_1^{\perp, fav} + H_1^{\perp, dis}) + 4H_{1,s \rightarrow \pi}^{\perp, dis} \otimes H_{1,s \rightarrow \pi}^{\perp, dis}}{5(D_1^{fav} + D_1^{dis}) \otimes (D_1^{fav} + D_1^{dis}) + 4D_{1,s \rightarrow \pi}^{dis} \otimes D_{1,s \rightarrow \pi}^{dis}} \right.$$

$$\left. - \frac{5(H_1^{\perp, fav} \otimes H_1^{\perp, dis} + H_1^{\perp, dis} \otimes H_1^{\perp, fav}) + 2H_{1,s \rightarrow \pi}^{\perp, dis} H_{1,s \rightarrow \pi}^{\perp, dis}}{5(D_1^{fav} \otimes D_1^{dis} + D_1^{dis} \otimes D_1^{fav}) + 2D_{1,s \rightarrow \pi}^{dis} \otimes D_{1,s \rightarrow \pi}^{dis}} \right\} \quad \left. \vphantom{\frac{5(H_1^{\perp, fav} + H_1^{\perp, dis}) \otimes (H_1^{\perp, fav} + H_1^{\perp, dis}) + 4H_{1,s \rightarrow \pi}^{\perp, dis} \otimes H_{1,s \rightarrow \pi}^{\perp, dis}}{5(D_1^{fav} + D_1^{dis}) \otimes (D_1^{fav} + D_1^{dis}) + 4D_{1,s \rightarrow \pi}^{dis} \otimes D_{1,s \rightarrow \pi}^{dis}}} \right\} \text{contribution from charged pions}$$

- non-zero π^0 or η results **not** direct sign of non-zero π^0 or η Collins FFs
- double ratio dominated by terms involving charged-pion yields

Collins asymmetries - going further

$$R_{12}^{\pi^0} = \frac{R_{12}^{0\pm}}{R_{12}^L} \approx 1 + \cos(\phi_{12}) \frac{\sin^2(\theta)}{1 + \cos^2(\theta)}$$

$$\times \left\{ \frac{5(H_1^{\perp, fav} + H_1^{\perp, dis}) \otimes (H_1^{\perp, fav} + H_1^{\perp, dis}) + 4H_{1,s \rightarrow \pi}^{\perp, dis} \otimes H_{1,s \rightarrow \pi}^{\perp, dis}}{5(D_1^{fav} + D_1^{dis}) \otimes (D_1^{fav} + D_1^{dis}) + 4D_{1,s \rightarrow \pi}^{dis} \otimes D_{1,s \rightarrow \pi}^{dis}} \right.$$

$$\left. - \frac{5(H_1^{\perp, fav} \otimes H_1^{\perp, dis} + H_1^{\perp, dis} \otimes H_1^{\perp, fav}) + 2H_{1,s \rightarrow \pi}^{\perp, dis} H_{1,s \rightarrow \pi}^{\perp, dis}}{5(D_1^{fav} \otimes D_1^{dis} + D_1^{dis} \otimes D_1^{fav}) + 2D_{1,s \rightarrow \pi}^{dis} \otimes D_{1,s \rightarrow \pi}^{dis}} \right\}.$$

← contribution from π^0 or η

} contribution from charged pions



- non-zero π^0 or η results **not** direct sign of non-zero π^0 or η Collins FFs
- double ratio dominated by terms involving charged-pion yields
- only numerator of first term related to π^0 or η

Collins asymmetries - going further

$$R_{12}^{\pi^0} = \frac{R_{12}^{0\pm}}{R_{12}^L} \approx 1 + \cos(\phi_{12}) \frac{\sin^2(\theta)}{1 + \cos^2(\theta)}$$

$$\times \left\{ \frac{5(H_1^{\perp, fav} + H_1^{\perp, dis}) \otimes (H_1^{\perp, fav} + H_1^{\perp, dis}) + 4H_{1,s \rightarrow \pi}^{\perp, dis} \otimes H_{1,s \rightarrow \pi}^{\perp, dis}}{5(D_1^{fav} + D_1^{dis}) \otimes (D_1^{fav} + D_1^{dis}) + 4D_{1,s \rightarrow \pi}^{dis} \otimes D_{1,s \rightarrow \pi}^{dis}} \right.$$

$$\left. - \frac{5(H_1^{\perp, fav} \otimes H_1^{\perp, dis} + H_1^{\perp, dis} \otimes H_1^{\perp, fav}) + 2H_{1,s \rightarrow \pi}^{\perp, dis} H_{1,s \rightarrow \pi}^{\perp, dis}}{5(D_1^{fav} \otimes D_1^{dis} + D_1^{dis} \otimes D_1^{fav}) + 2D_{1,s \rightarrow \pi}^{dis} \otimes D_{1,s \rightarrow \pi}^{dis}} \right\}.$$

 contribution from π^0 or η
 contribution from charged pions

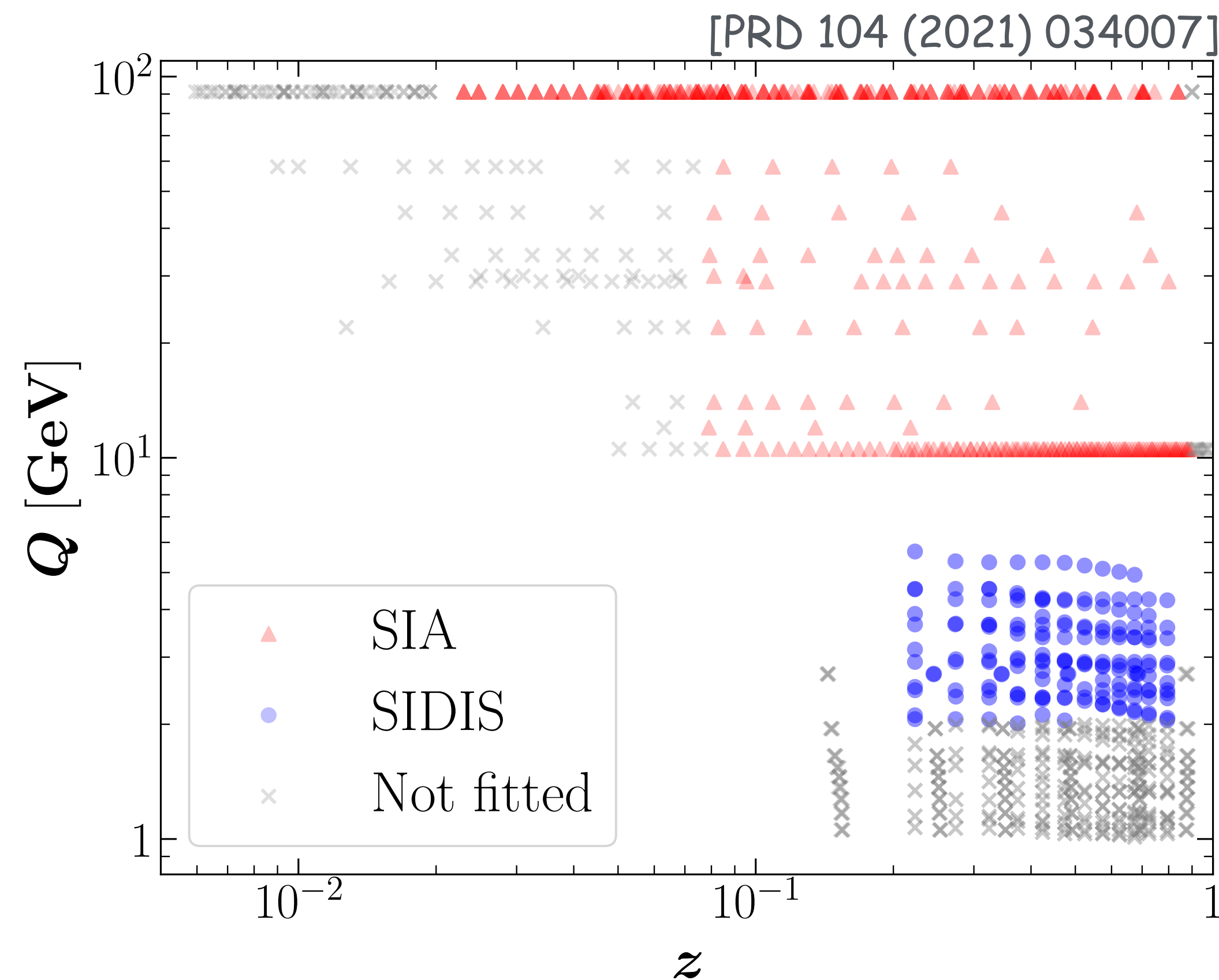
- non-zero π^0 or η results **not** direct sign of non-zero π^0 or η Collins FFs
- double ratio dominated by terms involving charged-pion yields
- only numerator of first term related to π^0 or η
- non-zero results could, in principle, arise entirely from charged-pion Collins FFs

a likely future

- several analyses still in the pipeline, e.g.,
 - k_T -dependent D_1 FFs (back-to-back hadrons)
(Belle, BESIII & possibly BaBar)
 - Collins asymmetries:
 - pion update w/ increased statistics (BESIII)
 - kaon & pion-kaon pairs; k_T dependence of Collins asymmetries (Belle, BESIII)
 - Collins asymmetries w/o double ratios (BaBar)
 - single-hadron production
 - **short-lived mesons and resonances** (Belle)
 - charged pions and kaon at lower s (BESIII)

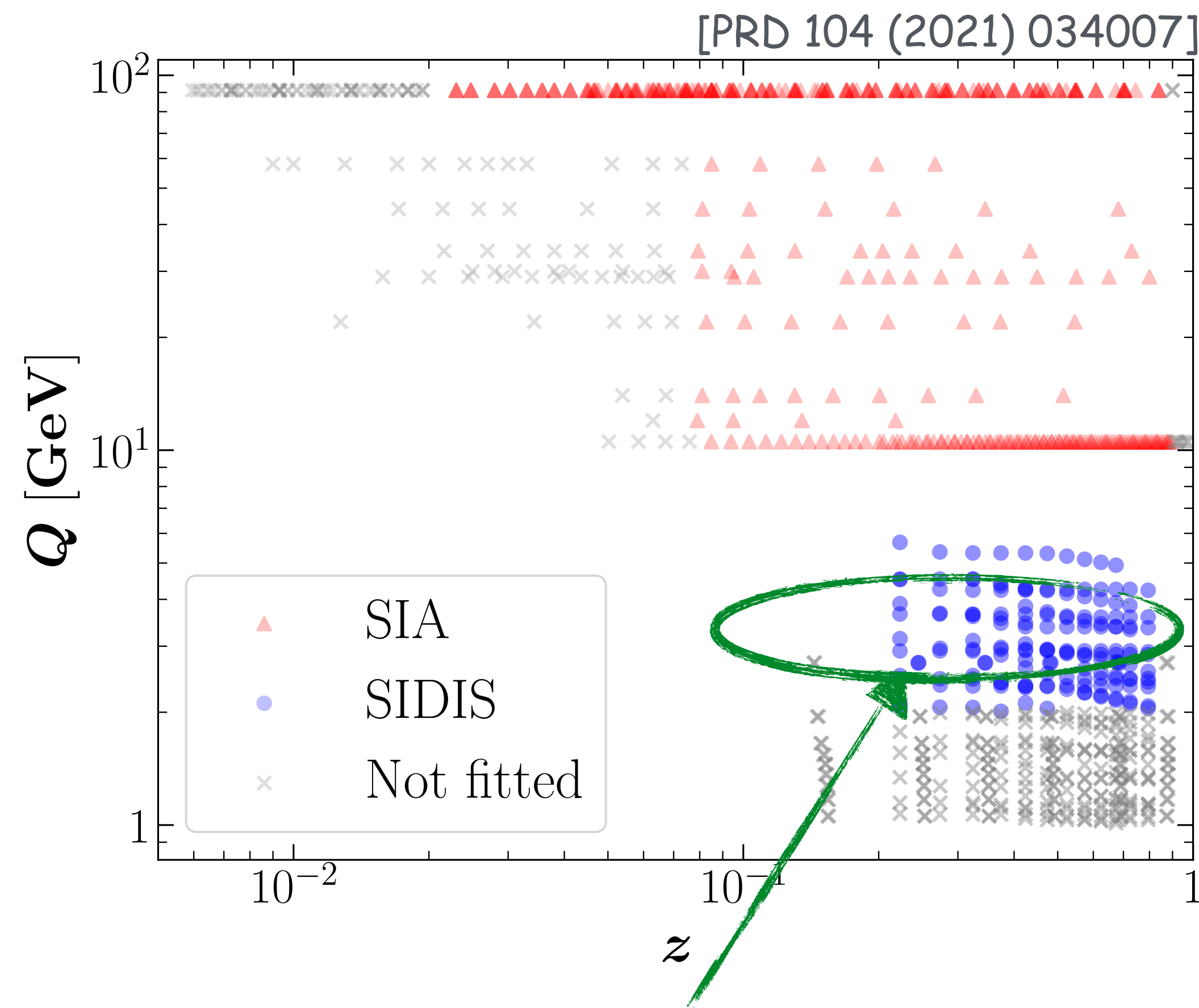
a likely future

- several analyses still in the pipeline, e.g.,
 - k_T -dependent D_1 FFs (back-to-back hadrons) (Belle, BESIII & possibly BaBar)
 - Collins asymmetries:
 - pion update w/ increased statistics (BESIII)
 - kaon & pion-kaon pairs; k_T dependence of Collins asymmetries (Belle, BESIII)
 - Collins asymmetries w/o double ratios (BaBar)
 - single-hadron production
 - short-lived mesons and resonances (Belle)
 - charged pions and kaon at lower s (BESIII)



a likely future

- several analyses still in the pipeline, e.g.,
 - k_T -dependent D_1 FFs (back-to-back hadrons) (Belle, BESIII & possibly BaBar)
 - Collins asymmetries:
 - pion update w/ increased statistics (BESIII)
 - kaon & pion-kaon pairs; k_T dependence of Collins asymmetries (Belle, BESIII)
 - Collins asymmetries w/o double ratios (BaBar)
 - single-hadron production
 - short-lived mesons and resonances (Belle)
 - charged pions and kaon at lower s (BESIII)

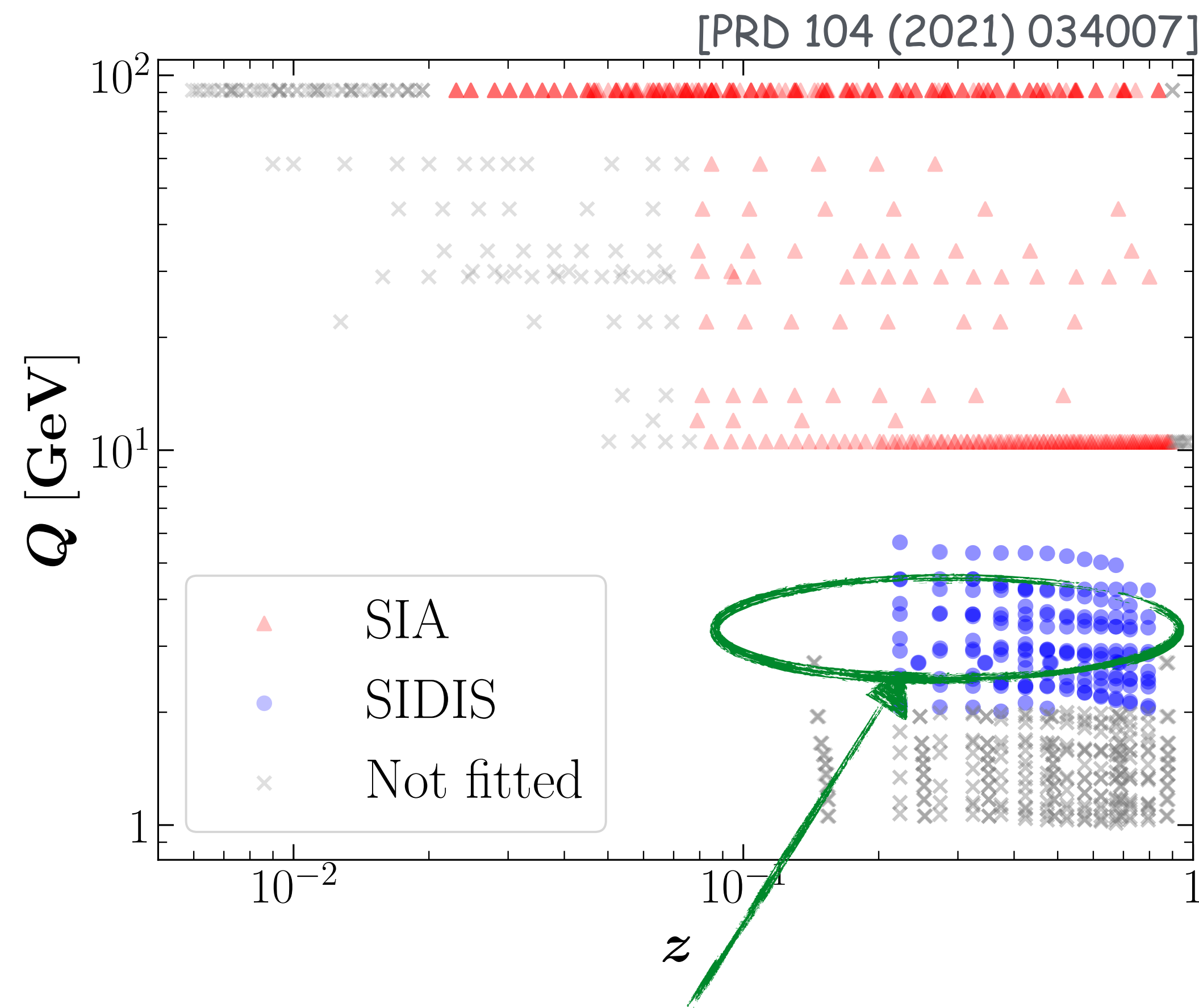


BESIII region

$\sim 62 \text{ pb}^{-1}$ @ 3.52 GeV used for Collins asym's
aim at 250 pb^{-1} data set

a likely future

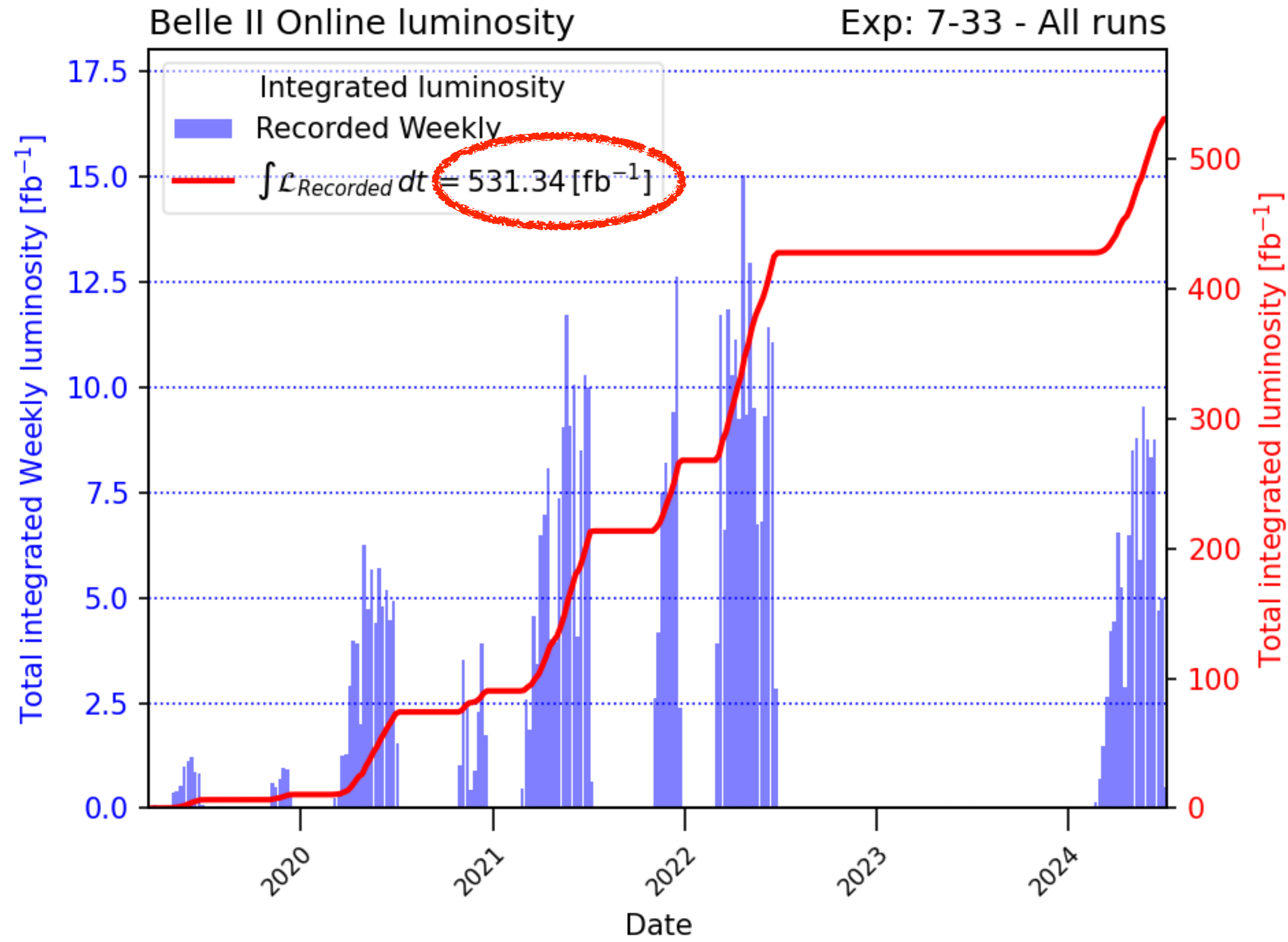
- several analyses still in the pipeline, e.g.,
 - k_T -dependent D_1 FFs (back-to-back hadrons) (Belle, BESIII & possibly BaBar)
 - Collins asymmetries:
 - pion update w/ increased statistics (BESIII)
 - kaon & pion-kaon pairs; k_T dependence of Collins asymmetries (Belle, BESIII)
 - Collins asymmetries w/o double ratios (BaBar)
 - single-hadron production
 - short-lived mesons and resonances (Belle)
 - charged pions and kaon at lower s (BESIII)
- new data from Belle II



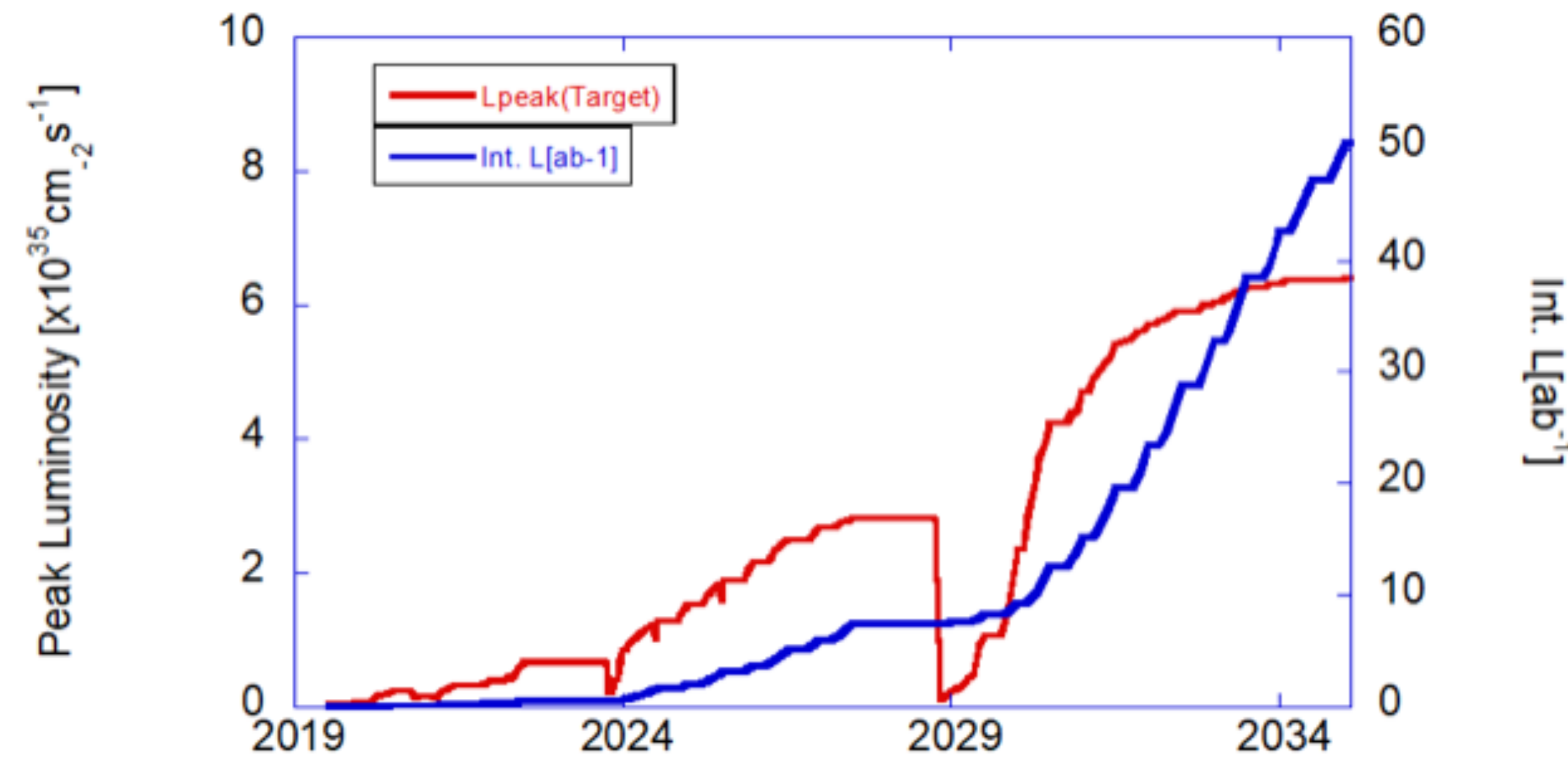
BESIII region

$\sim 62 \text{ pb}^{-1}$ @ 3.52 GeV used for Collins asym's
aim at 250 pb^{-1} data set

Belle II data taking



Updated on 2024/07/01 09:43 JST

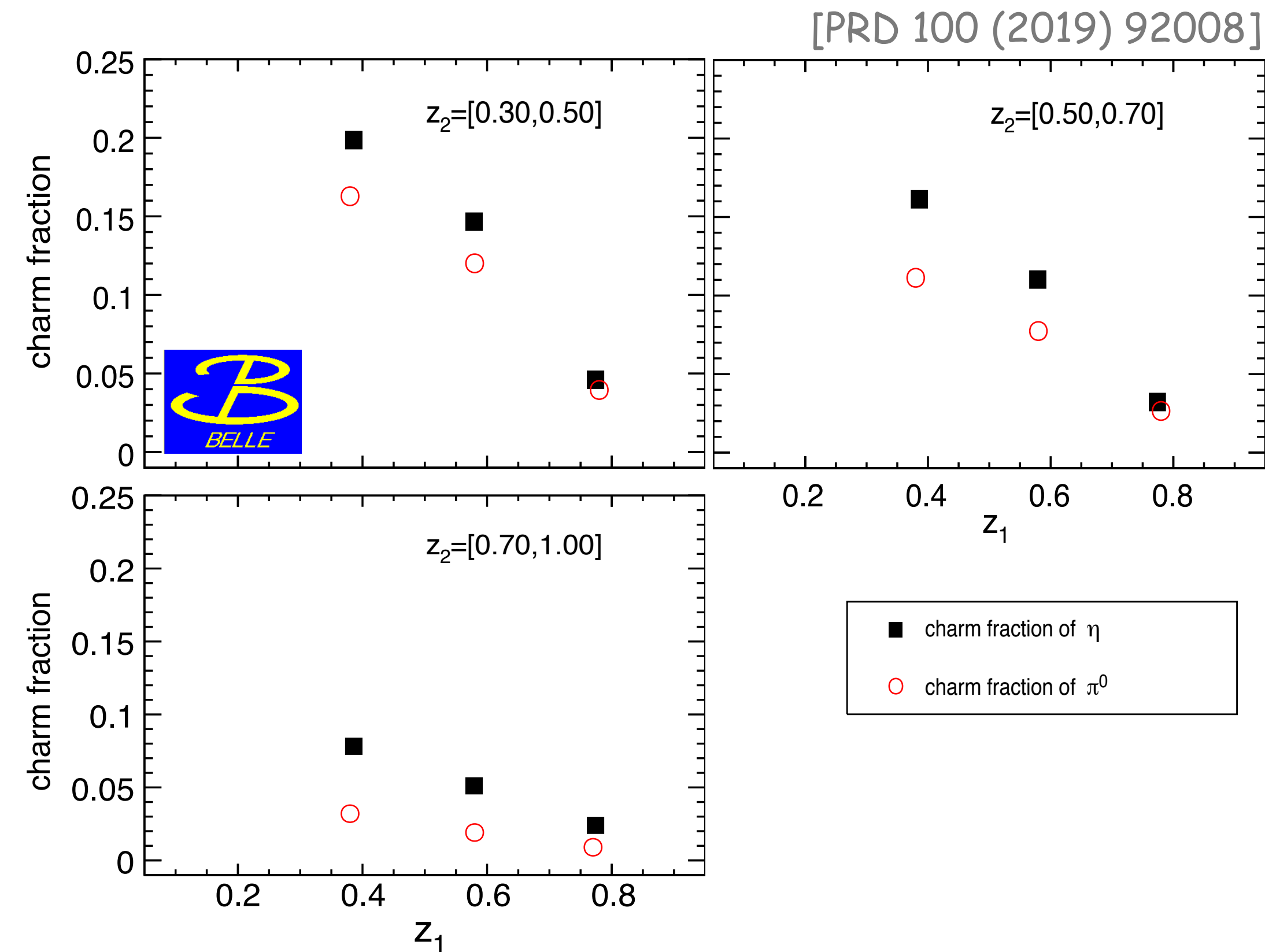


→ similar data sample as at 1st-generation B-factories "soonish"

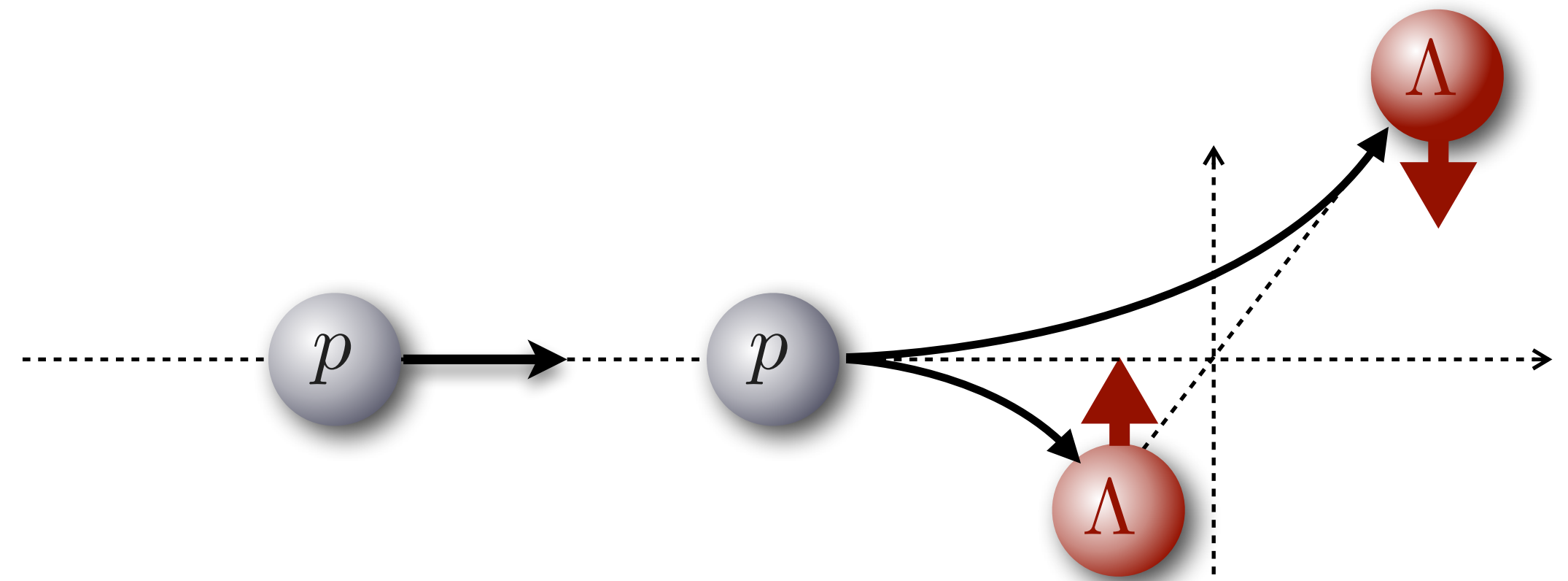
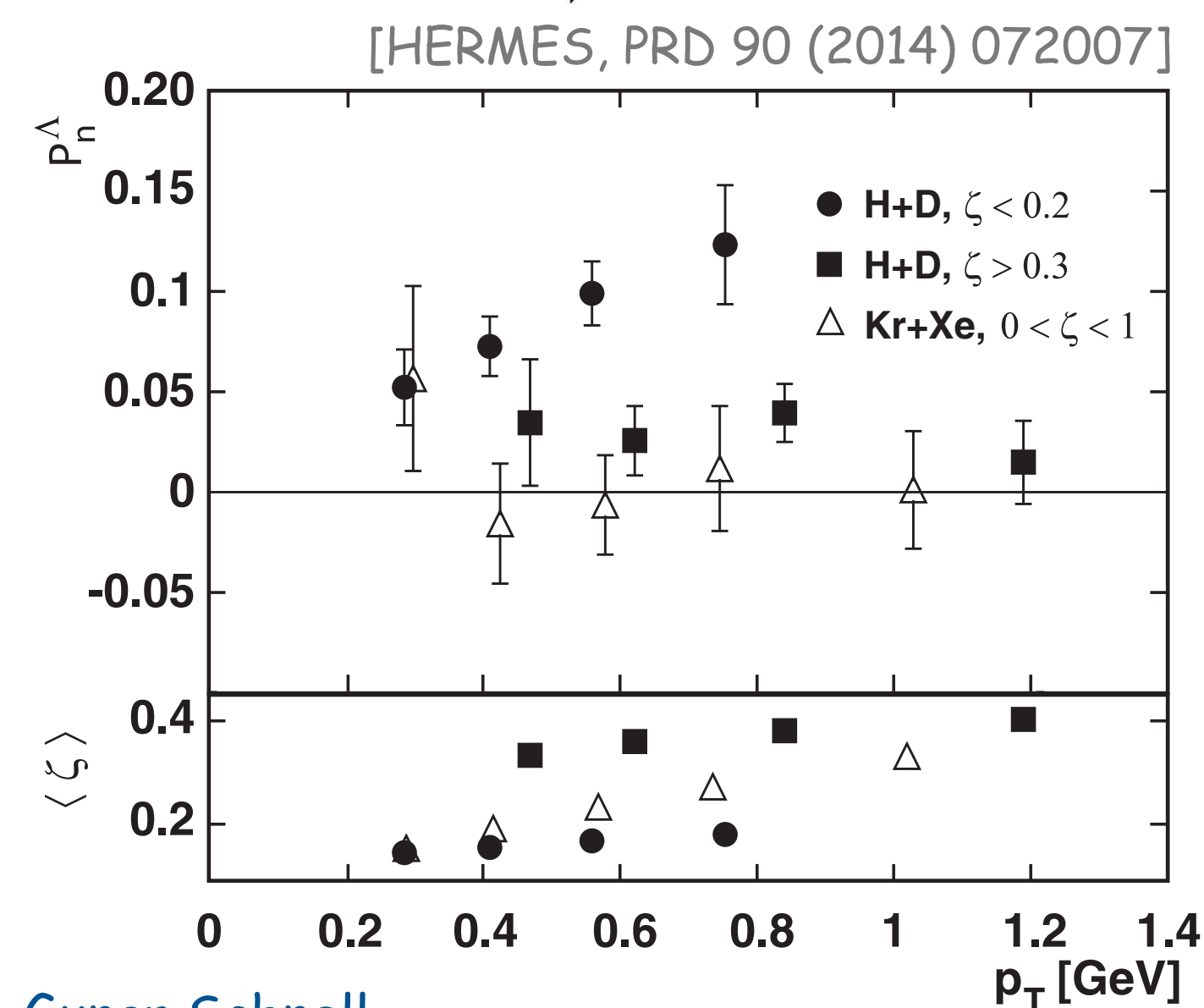
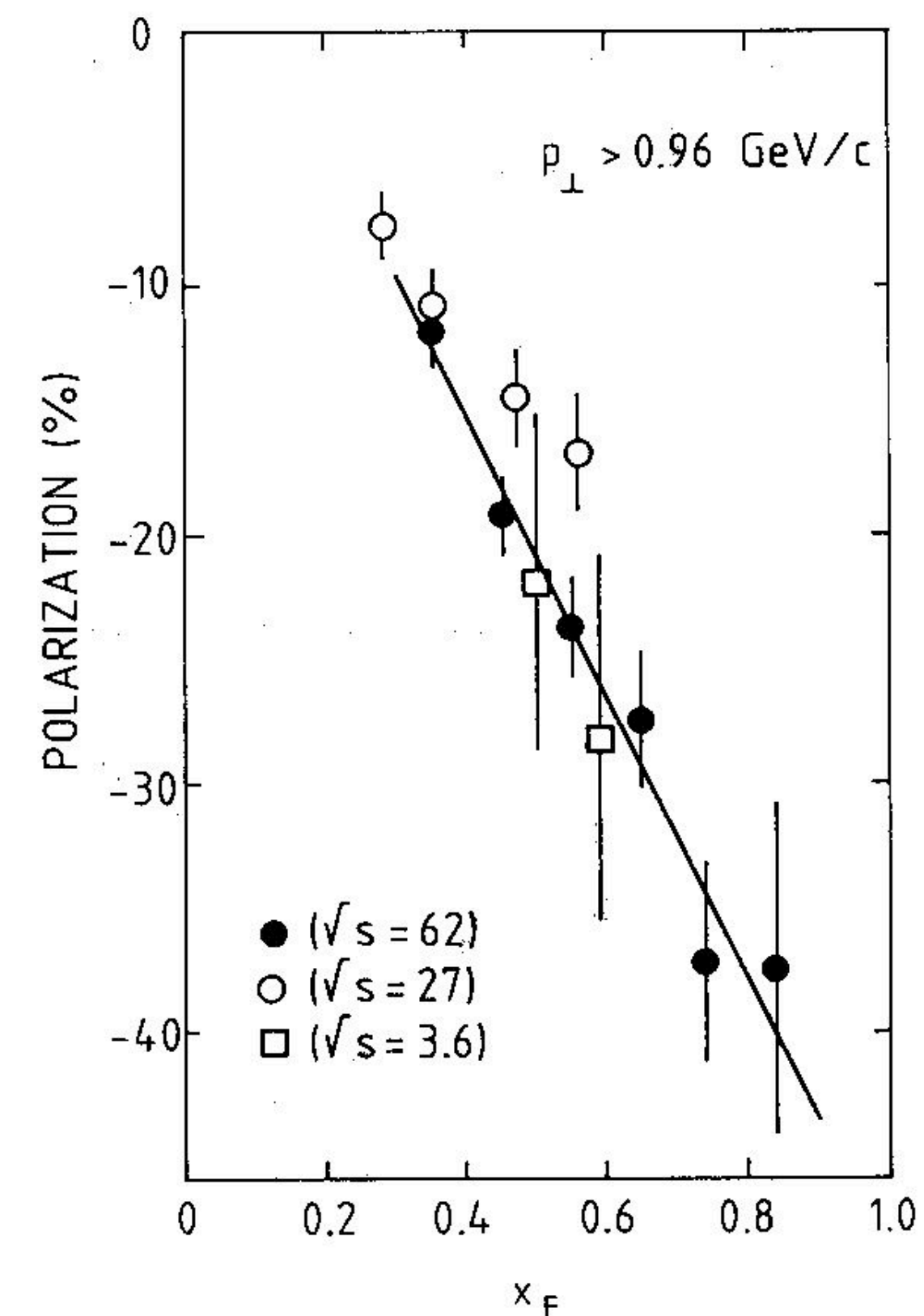
backup slides

Collins asymmetries - differences w.r.t. old analysis

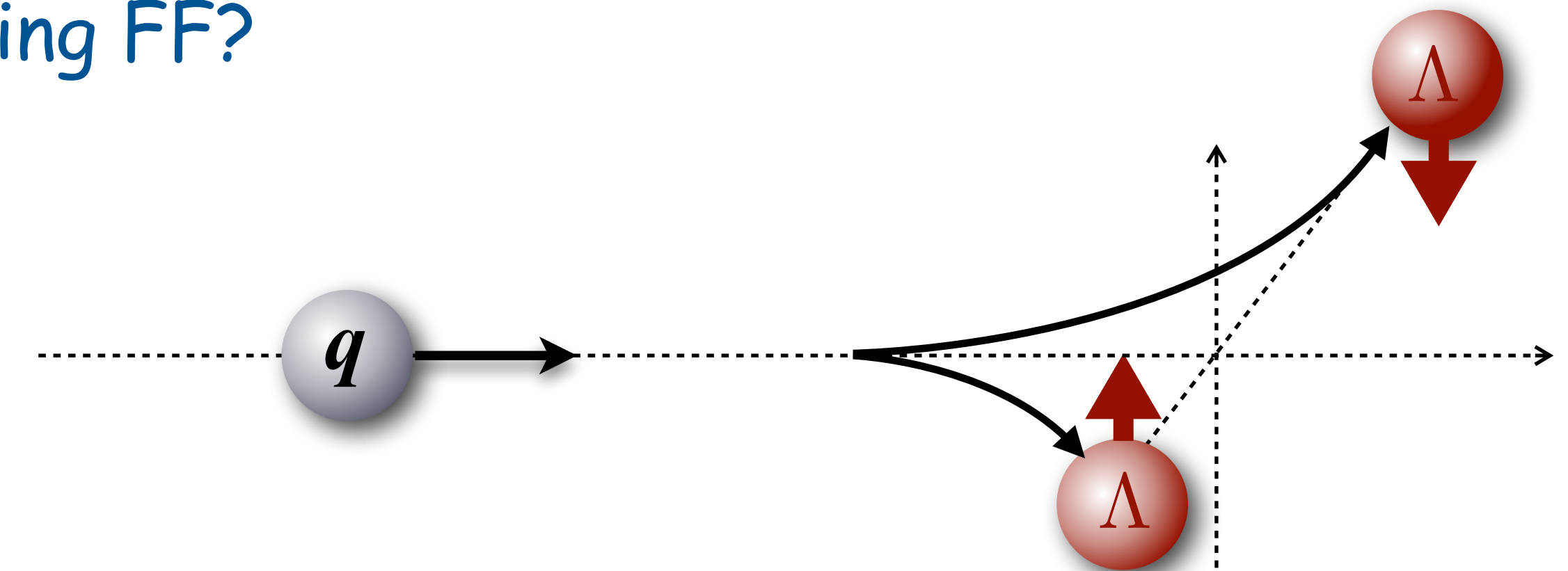
- qualitative changes in 2019 Belle analysis w.r.t. previous Belle analyses:
 - no correction to $q\bar{q}$ axis;
 - ⇒ rather to thrust axis, which is observable
 - upper limit on opening angle imposed
 - no correction for charm contribution;
 - ⇒ provide charm fraction



polarizing fragmentation

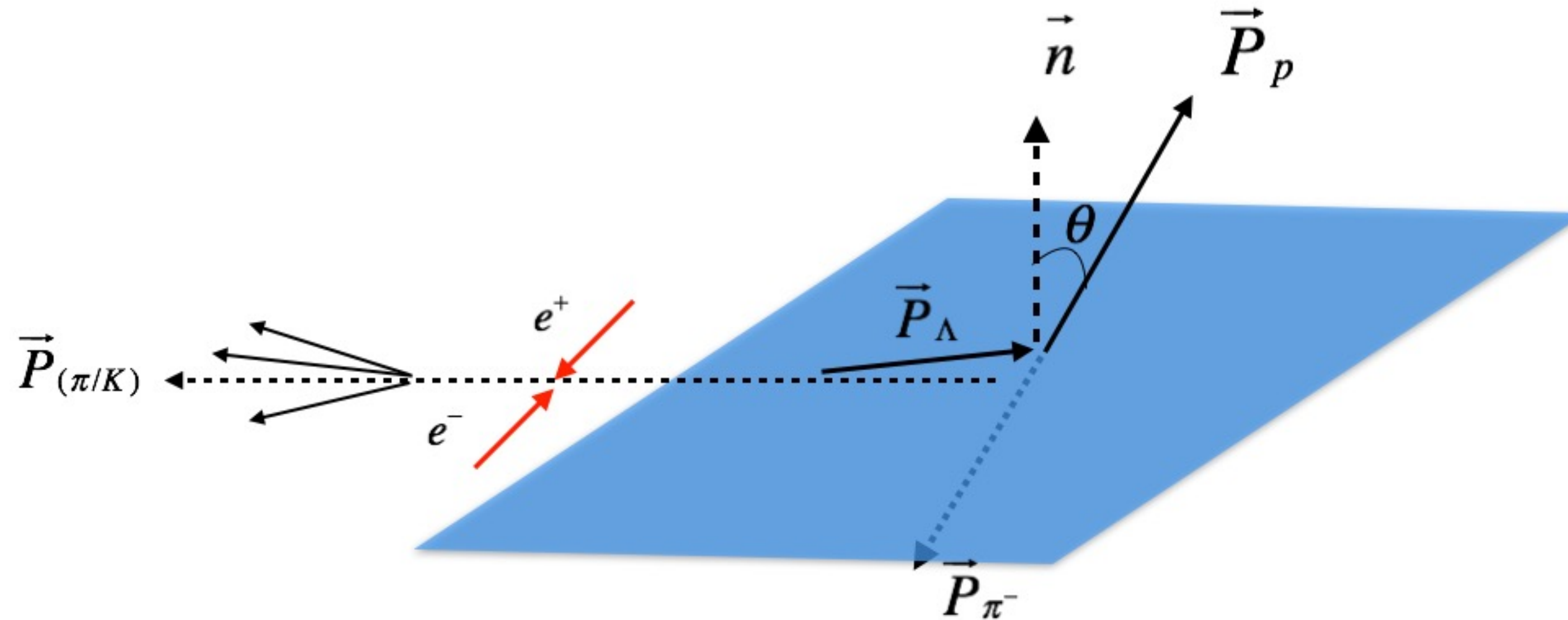


- large hyperon polarization in unpolarized hadron collision observed
- ... as well as in inclusive lepto-production
- caused by polarizing FF?



polarizing fragmentation function

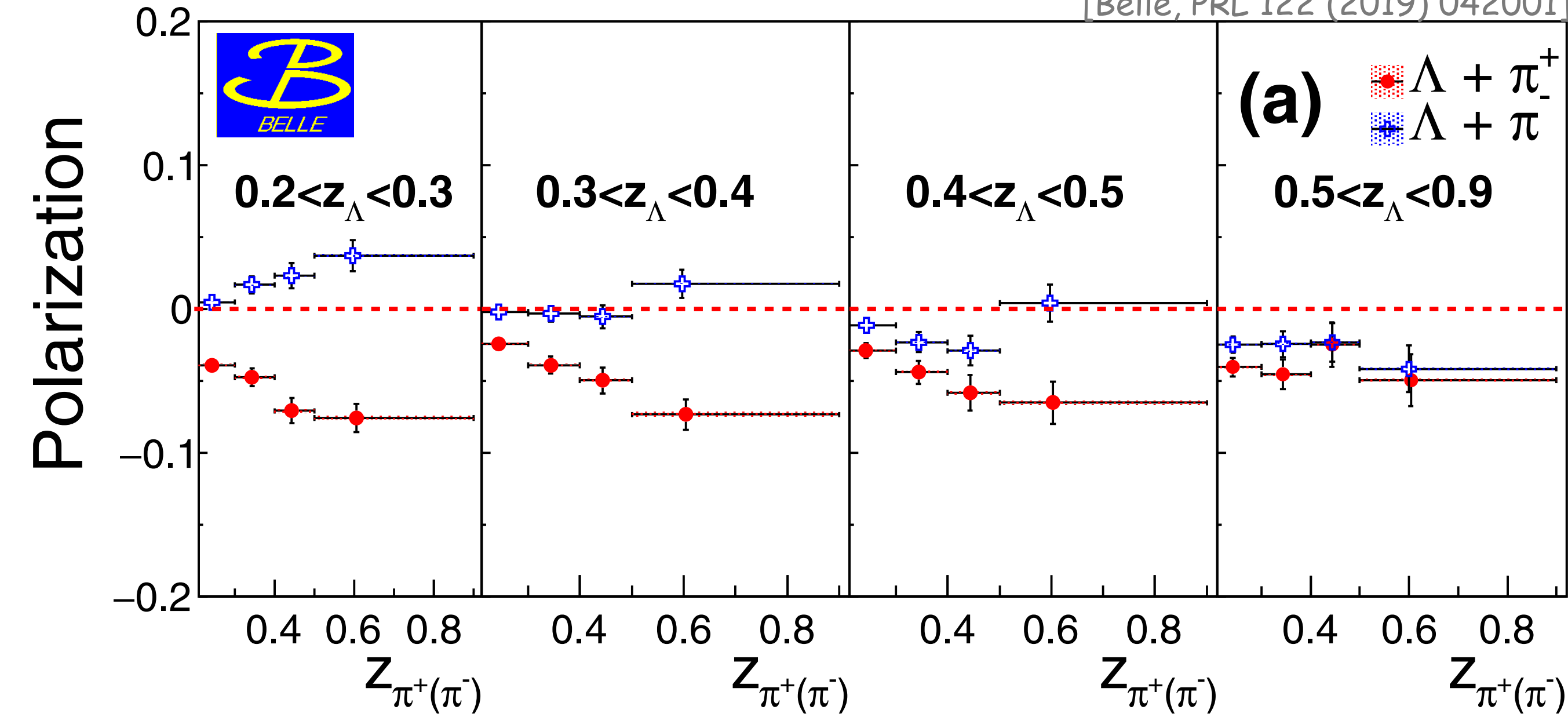
- polarization measured normal to production plane, i.e. $\propto (\vec{P}_q \times P_\Lambda)$



- reference axis to define transverse momentum:
 - "hadron frame" - use momentum direction of "back-to-back" hadron
 - "thrust frame" - use thrust axis
- exploit self-analyzing weak decay of Λ to determine polarization

polarizing fragmentation function

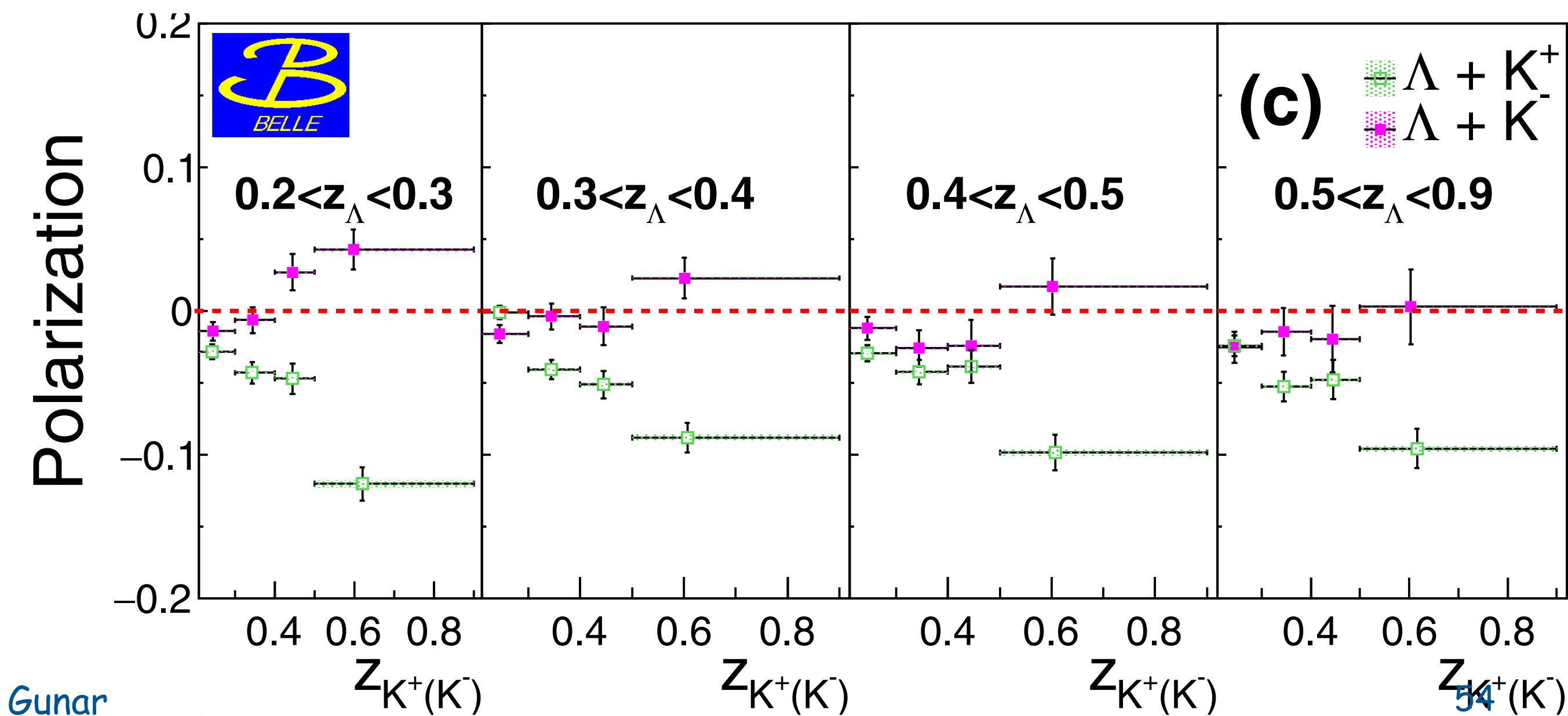
[Belle, PRL 122 (2019) 042001]



● flavor tagging through hadrons in opposite hemisphere:

● large- z_h hadrons tag quark flavor more efficiently

➔ enlarges differences between oppositely charged hadrons



$$z_h = \frac{E_h}{\sqrt{s}/2}$$