Heavy- and light-quark mesons: A theory perspective

Alessandro Pilloni

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Università degli Studi di Messina

ExoHad Exotic Hadrons Topical Collaboration

INFN Istituto Nazionale di Fisica Nucleare

The richness of hadron spectrum

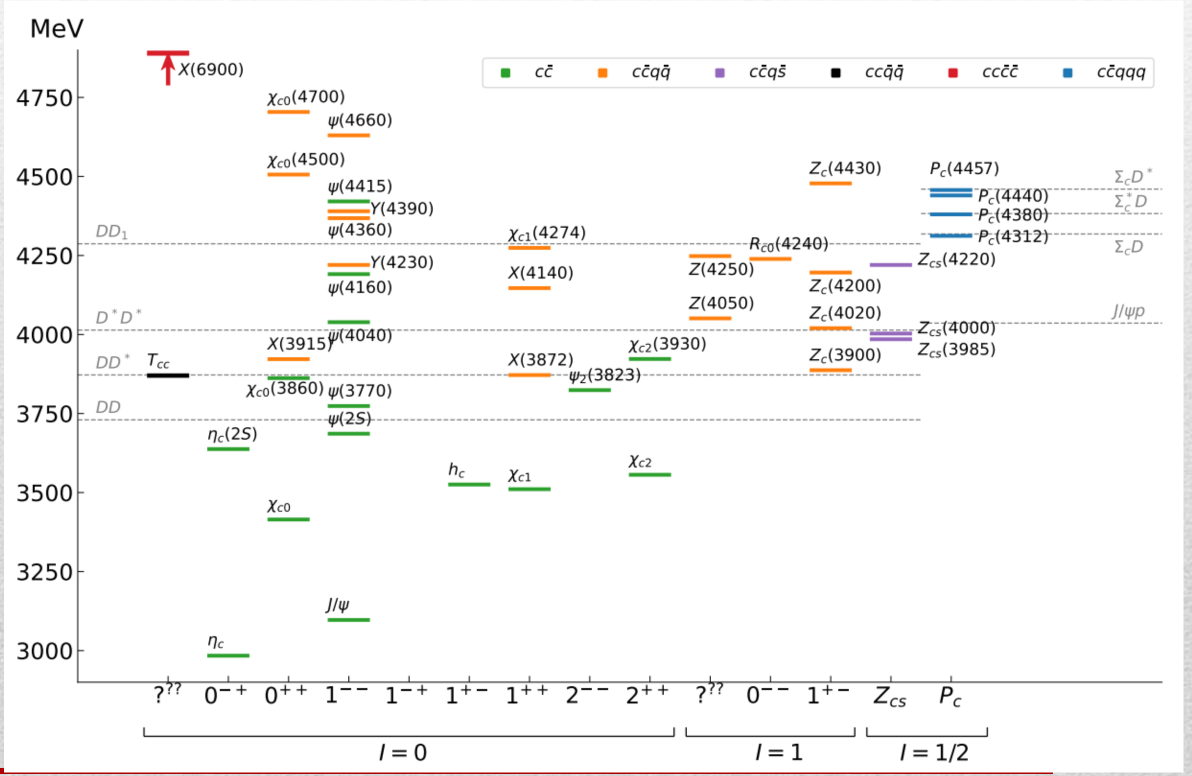
- The structures populating hadron reactions are extremely rich
- Ultimate goal is to understand them in term of QCD degrees of freedom
- To do so, the spectrum of resonances must be correctly reconstructed

JPAC, PPNP 127 (2022), 103981

The quark model is still the simplest tool to organize the spectrum

«Exotic hadrons» are Beyond the Standard (quark) Model

Organizing them teaches us some more about quark-gluon interactions



Potential problems

We can't afford being misguided by simplistic resonance extraction!
(life is already hard enough without red herrings)

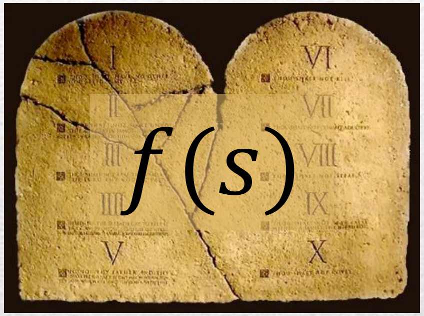
Amplitude analysis requires collaboration
of theorists and experimentalists

However, we don't have a universal recipe to do things right
We have to proceed:

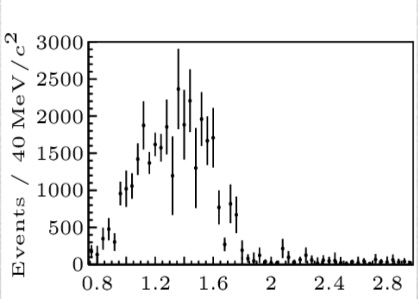
- Case-by-case
- Keep in mind the limitations of each approach
- Overstudy (the more approaches the better)
- No overstatement (do-no-harm)

Top-down approach

1) You are given a model/theory



2) You calculate the amplitude



3) You compare with data.
   Or you don't.

Predictive power ✓
Physical interpretation ✓
(within the model! ✘)
Biased by the input ✘

Options on the market

Quark-level calculations:

- Quark models, DSE, pNRQCD, Hybrids, ...
- Spectrum generally calculated as bound states of some potential
- Decay rates as «overlap integrals» between two static configurations

Comprehensive picture ✓ Little scattering dynamics ✘

Hadron level calculations:

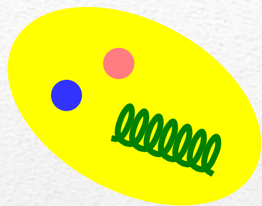
- Microscopic models inspired to EFT + Unitarization
- Compact states vs. molecules, triangles...
- States as poles of scattering amplitudes
- Couplings as residues at the poles

Scattering dynamics ✓ More case-by-case ✓✘

Models for every taste

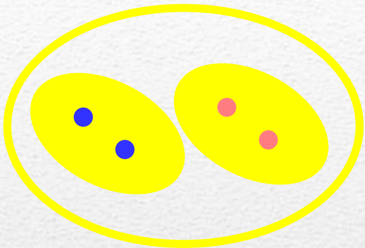
Compact

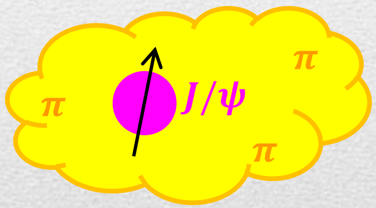
Extended



Hybrids
Containing gluonic degrees of freedom

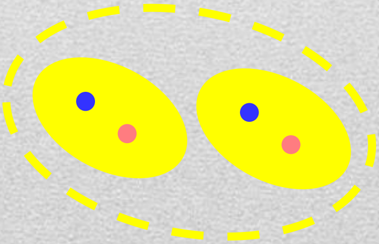
Multiquark
Several (cluster) of valence quarks





Hadroquarkonium
Heavy core interacting with a light cloud via Van der Waals forces

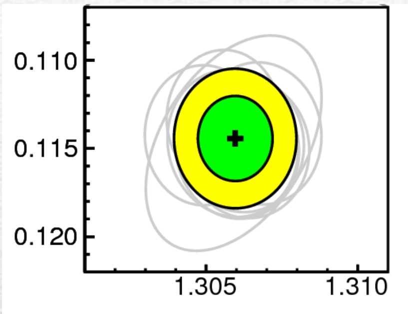
Rescattering effects
Structures generated by cross-channel rescattering, very process-dependent



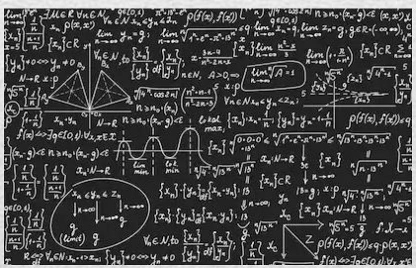
Molecule
Bound or virtual state generated by long-range exchange forces

Bottom-up approach

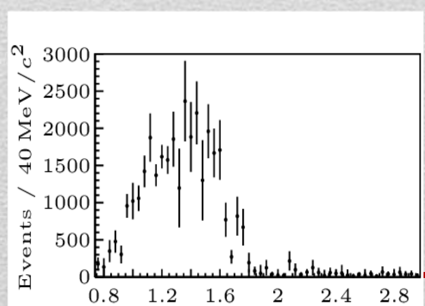
Less predictive power ✘
Some physical interpretation ✘
Minimally biased ✓



3) You extract physics

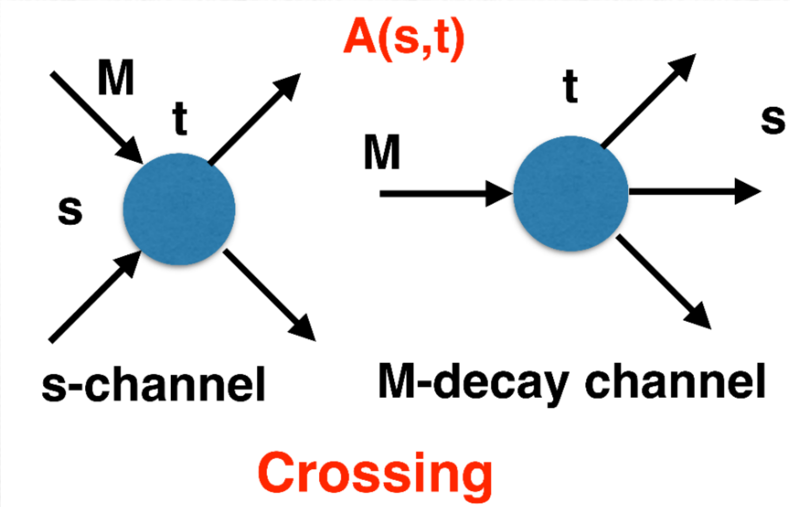


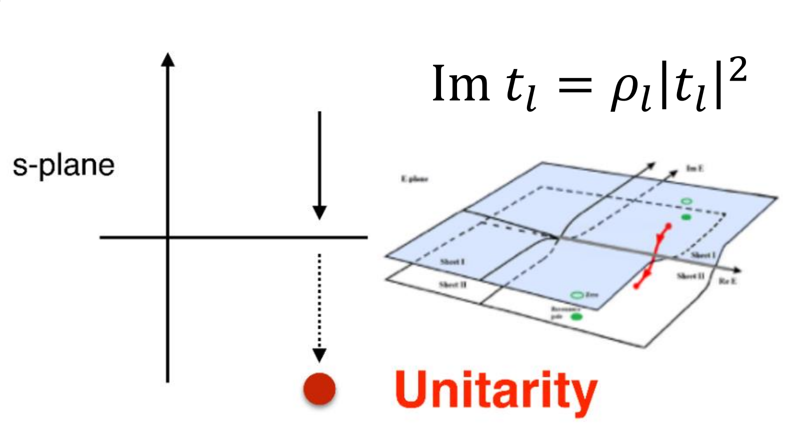
2) You choose a set of generic amplitudes

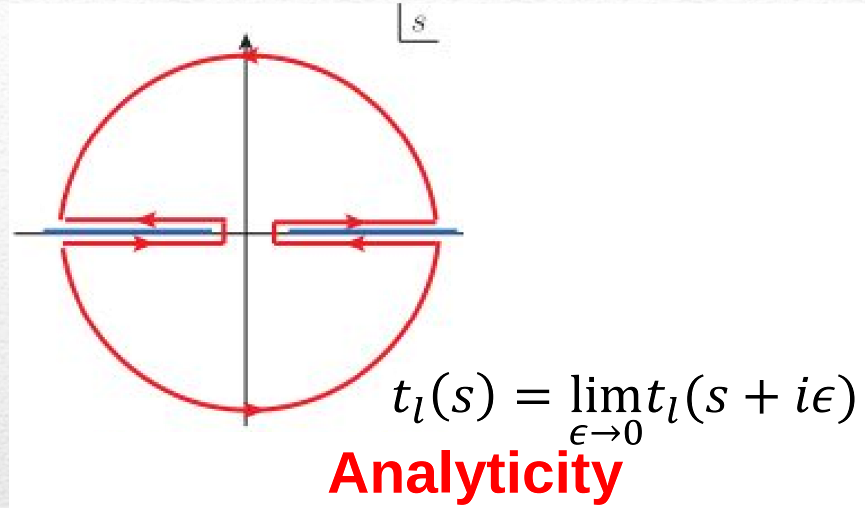


1) You start with data

S-Matrix principles







+ Lorentz, discrete & global symmetries

These are constraints the amplitudes have to satisfy, but do not fix the dynamics

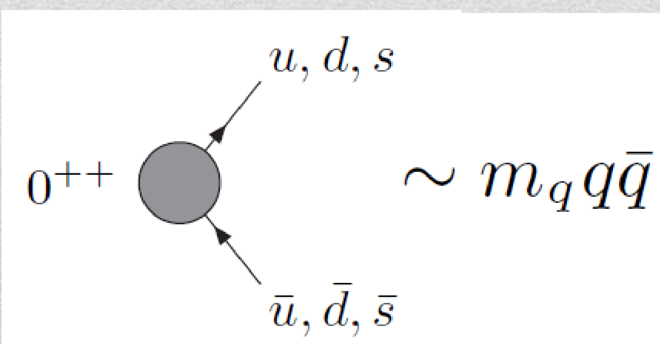
They can be imposed with an increasing amount of rigor, to extract robust physics information

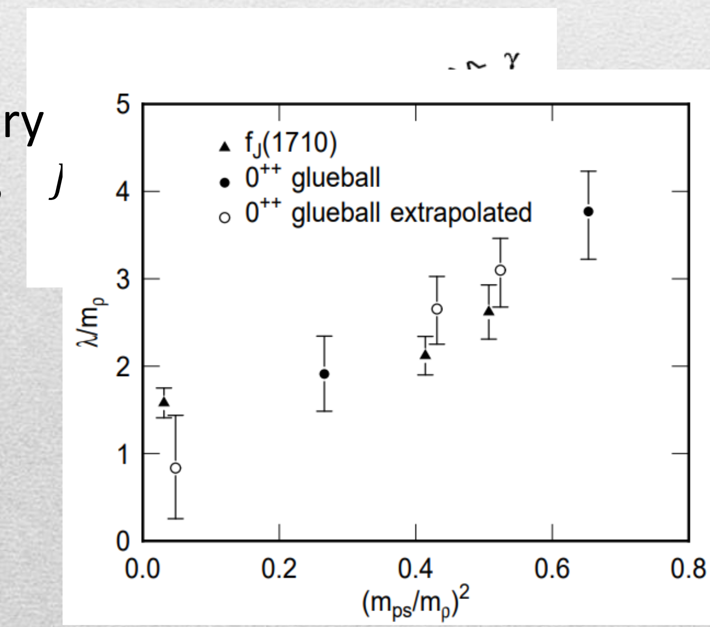
The «background» phenomena can be effectively parameterized in a controlled way

How to identify a glueball

You don’t. Since it mixes with light isoscalars, there is no model-independent way of saying which state is (mostly) the glueball. Only suggestions:

- There is one too many wrt QM. Indeed, $f_0(1370), f_0(1500), f_0(1710)$
- A glueball couples to photons only throughout mixing, so radiative widths should be small
- Their production is enhanced in gluon-rich processes, as J/ψ radiative decays
- It couples equally to mesons of all flavors (?)
  However, an argument based on chiral symmetry claims the coupling proportional to quark mass



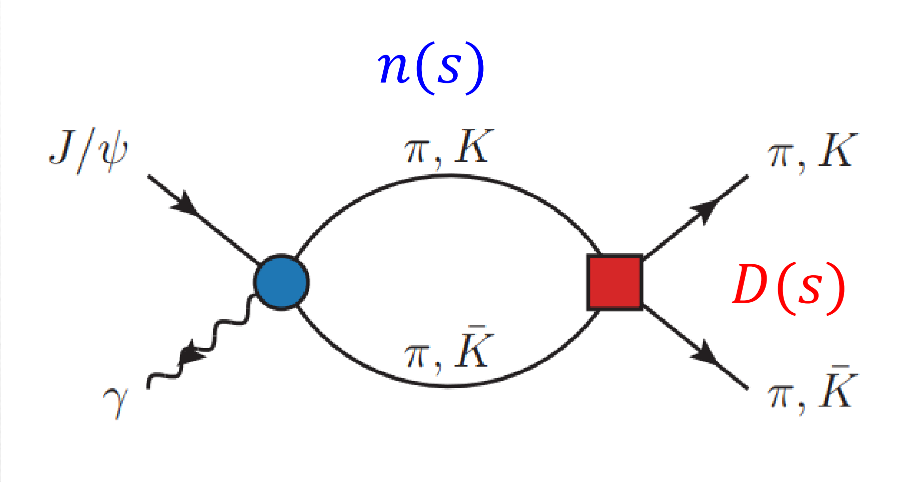


Amplitudes for $J/\psi \to \gamma PP$

Rodas *et al.*, EPJC 82, 1, 80

We build the partial wave amplitudes according to the N/D method

$$a(s) = \frac{n(s)}{D(s)}$$

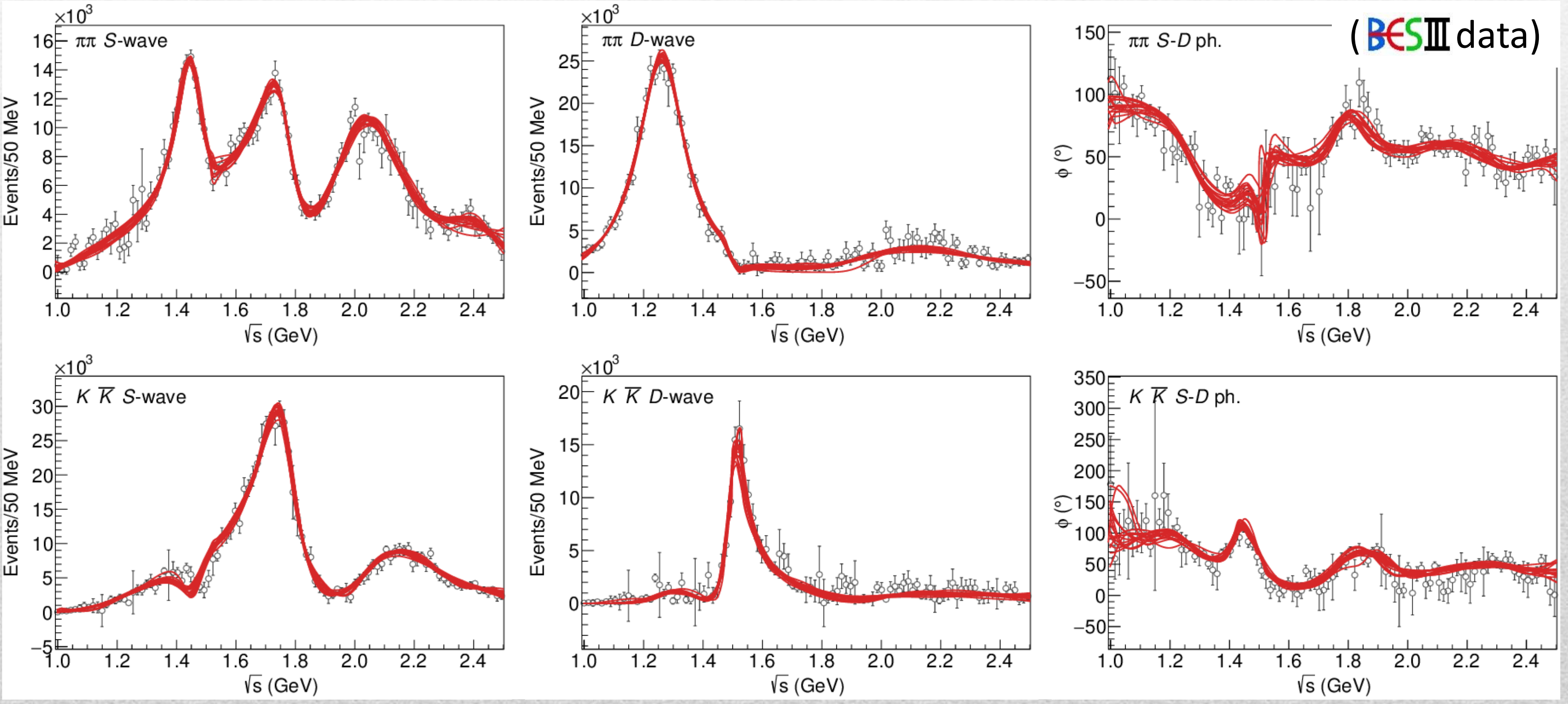


The $n(s)$ → background physics, process-dependent, smooth

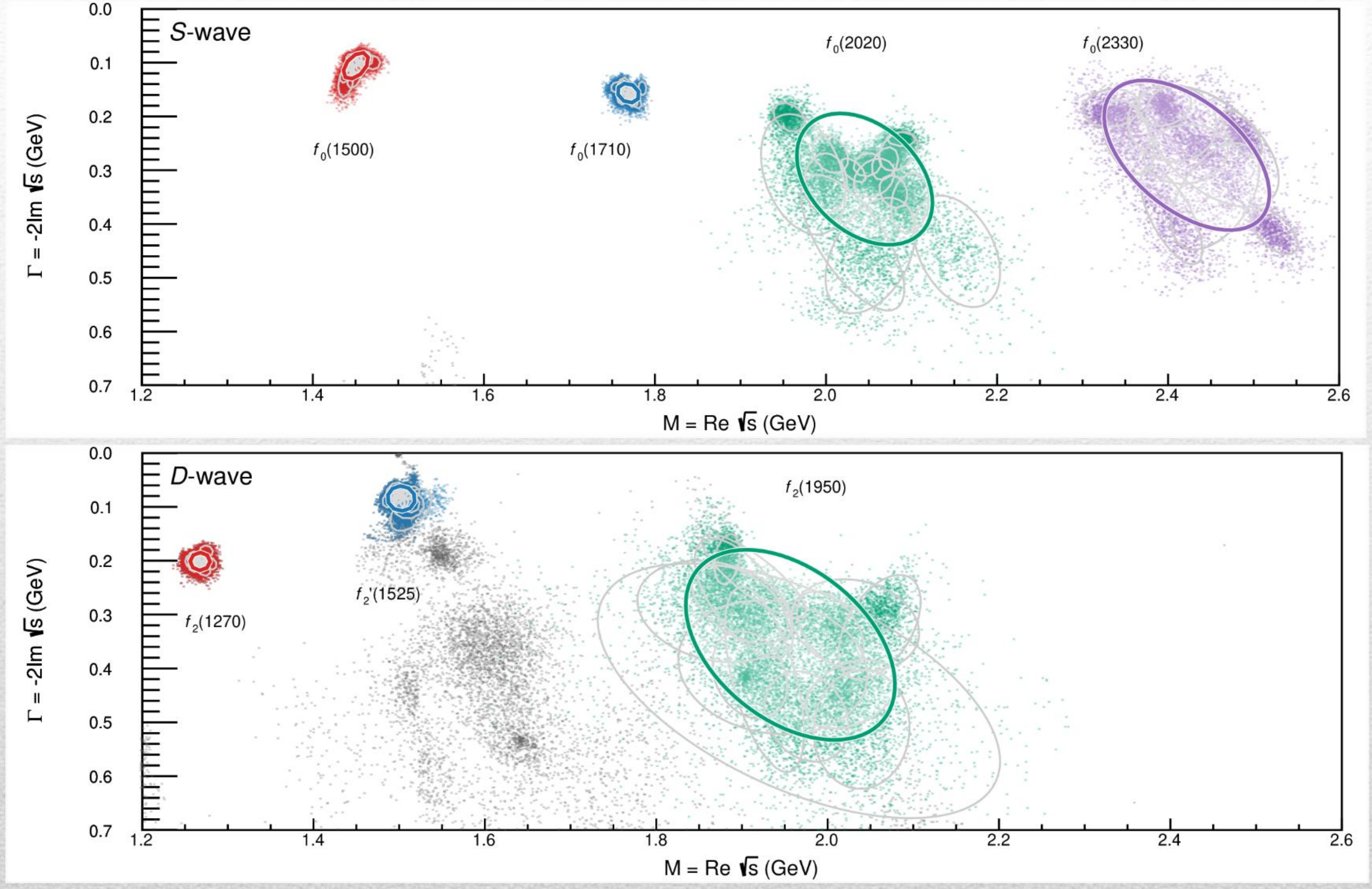
The $D(s)$ contains all the Final State Interactions constrained by unitarity → universal

$J/\psi \to \gamma\, \pi^0\pi^0$ and $\to \gamma\, K_S^0 K_S^0$

The fit is decent, but it's struggling in getting the f_2' right
Fit quality and local description improve when the third $\eta\eta$ channel is added
Being quasi-elastic, it's too constrained

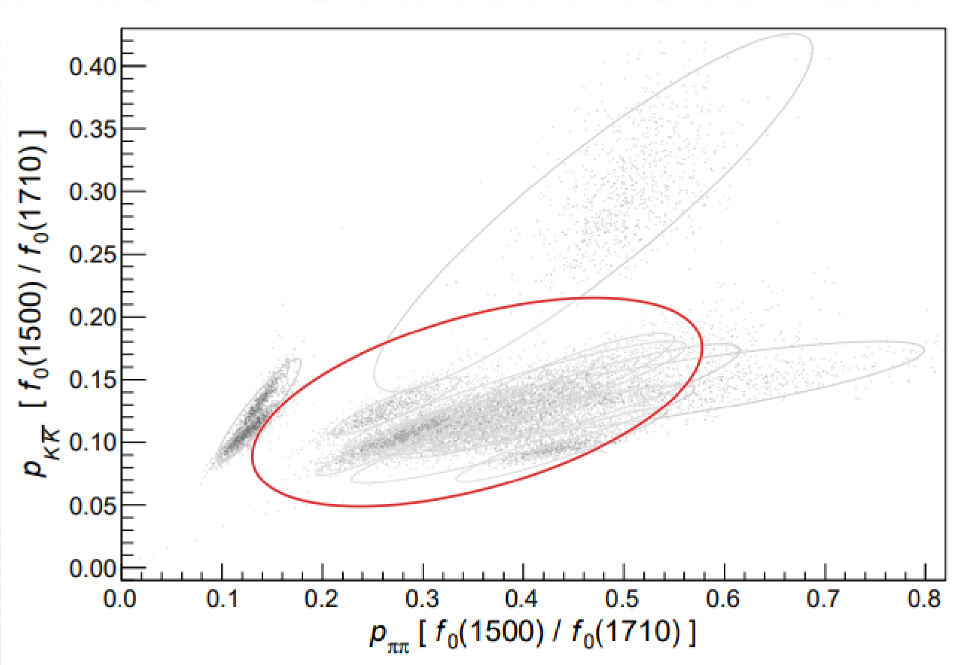


Pole extraction



For data-driven pole extractions, see also D. Binosi, A. Pilloni, R.-A. Tripolt PLB 839, 137809

Looking at the residues



Despite the large systematics, the $f_0(1710)$ couples to kaons more than the $f_0(1500)$

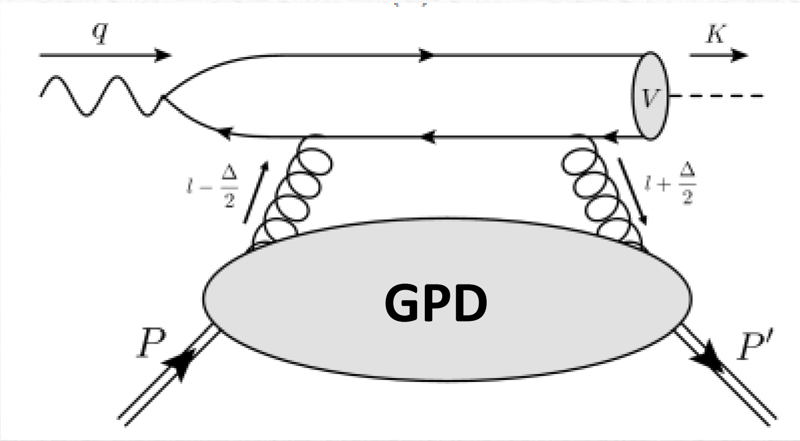
Also, the $f_0(1710)$ couples to the initial gluon-rich state more than the $f_0(1500)$

Both these fact suggest a sizeable glueball component for the $f_0(1710)$

Caveats:

- some of the relevant channels are neglected, hopefully ratios not affected
- Left-hand cuts of scattering processes contribute to scattering.
  Poorly known and neglected here.

J/ψ photoproduction at threshold



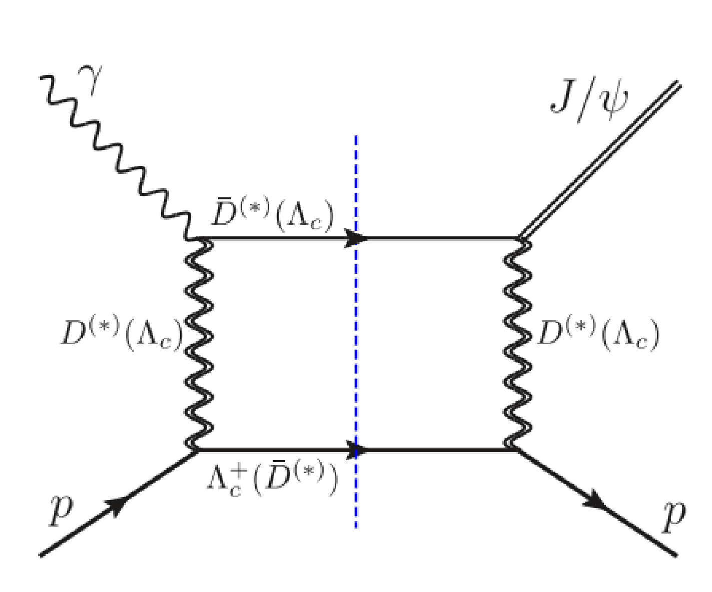
The common lore is that the study of vector quarkonium photoproduction at threshold is directly related to nucleon matrix elements

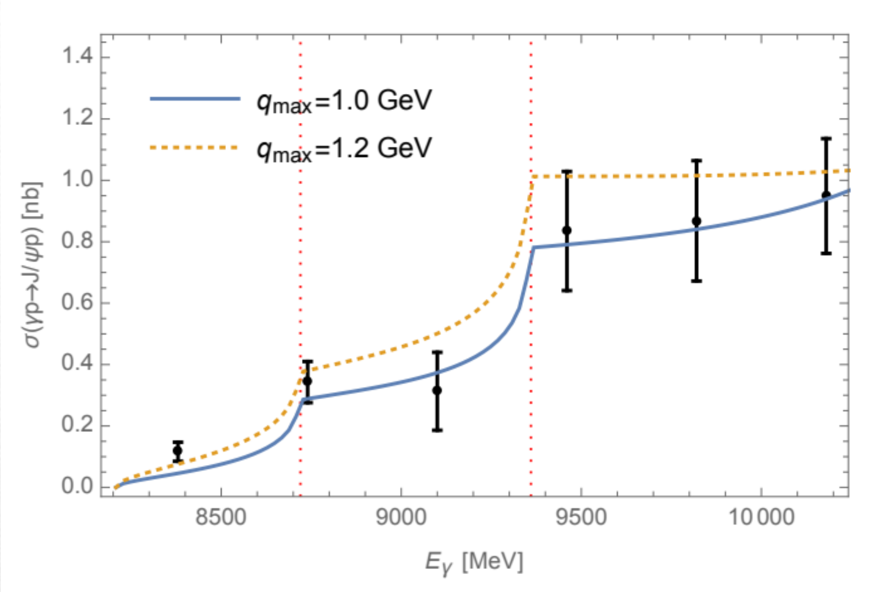
Assumptions:

1. Factorization proof for timelike DVCS can be extended at threshold as the heavy quark mass plays the role of a hard scale
2. The top vertex contains the trivial γ-ψ coupling $|\psi(0)|^2$
3. The exchange of anything but gluons is (OZI-, mass-) suppressed
4. The exchange is dynamically dominated by gluons carrying $J = 2$

Then one extracts matrix element of the energy momentum tensor $\langle p'|T_{\mu\nu}(0)|p\rangle$

Role of open charm



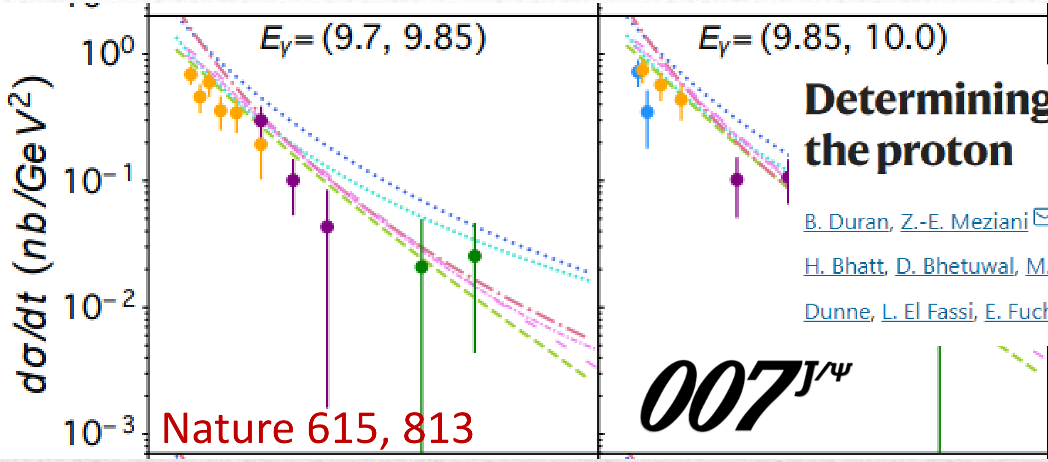


Du et al. Eur.Phys.J.C 80, 11, 1053

The role of intermediate open charm thresholds has been pointed out,
Maybe «all that glitters is not glue» ….

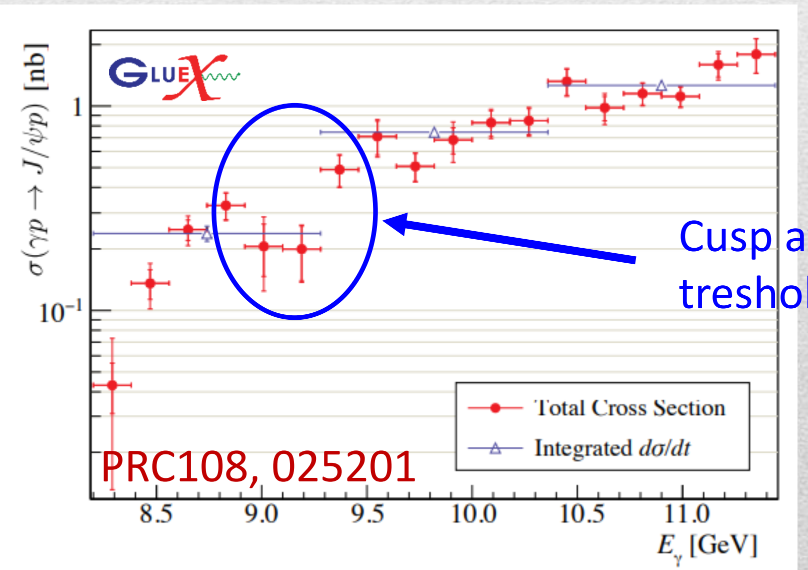
Calculation based on EFT and known couplings, S-wave saturates the cross section

J/ψ photoproduction at threshold

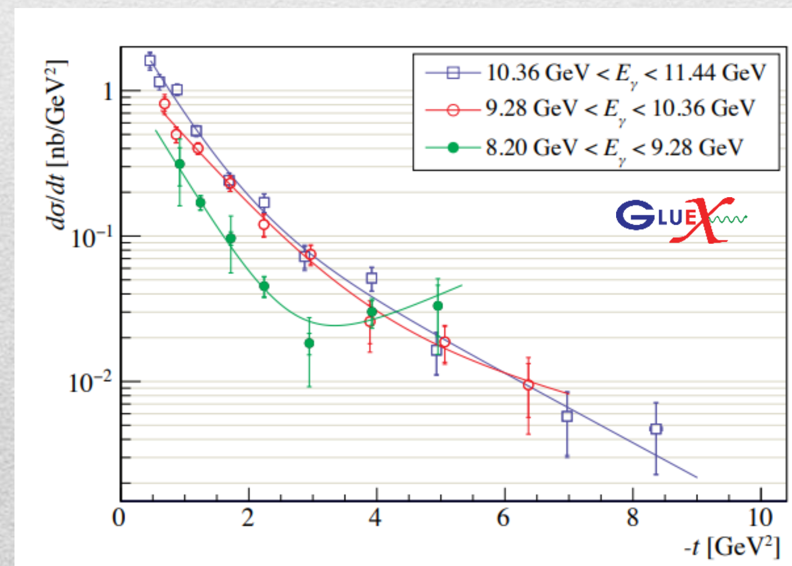


Determining the gluonic gravitational form factors of the proton

B. Duran, Z.-E. Meziani, S. Joosten, M. K. Jones, S. Prasad, C. Peng, W. Armstrong, H. Atac, E. Chudakov, H. Bhatt, D. Bhetuwal, M. Boer, A. Camsonne, J.-P. Chen, M. M. Dalton, N. Deokar, M. Diefenthaler, J. Dunne, L. El Fassi, E. Fuchey, H. Gao, D. Gaskell, O. Hansen, F. Hauenstein, ... Z. Zhao + Show authors



Cusp at the $\Lambda_c \overline{D}^*$ treshold



Unitary reanalysis

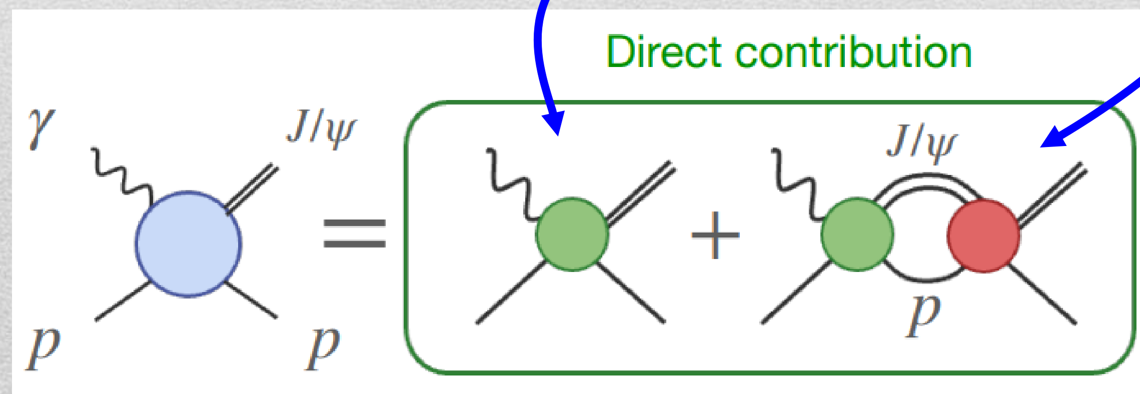
Winney *et al.* PRD 108, 054018

- Differential and total cross sections are fitted with a unitary model
- In lack of polarization observables and SDME, only orbital angular momentum is considered (spinless approx.)
- Truncated sum of PWs, $\ell \leq 3$

$$F(s,t) = \sum_{\ell} (2\ell+1)\, P_\ell(\cos\theta)\; F_\ell(s)$$

$$F_\ell(s) = f_\ell\,(1 + G\,T_\ell) = f_\ell\,(1 - G\,K_\ell)^{-1}$$
$$T_\ell(s) = K_\ell\,(1 - G\,K_\ell)^{-1}\,,$$

The dominant S-wave can include coupled channels,
for higher waves cusps are suppressed and there is no point



Contribution of open charm

$$\zeta_{\text{th}} = \frac{\left|F^{\psi p}_{\text{direct}}(s_{\text{th}})\right|}{\left|F^{\psi p}_{\text{direct}}(s_{\text{th}})\right| + \left|F^{\psi p}_{\text{indirect}}(s_{\text{th}})\right|} .$$

Naively Wilk's theorem says
the single channel is unfavored at 3.7σ
(no look elsewhere etc.),
indication but not the end of the story

Contribution of open charm $> 25\%$ at 90% CL

	1C	**2C**	**3C-NR**	**3C-R**
Parameters	9	13	15	15
χ^2	166	144	141	143
χ^2/dof	1.25	1.12	1.11	1.13
ζ_{th}	1	[0.56, 0.74]	[0.36, 0.63]	[0.03, 0.62]

Conclusions

Bottom-up approaches are important!

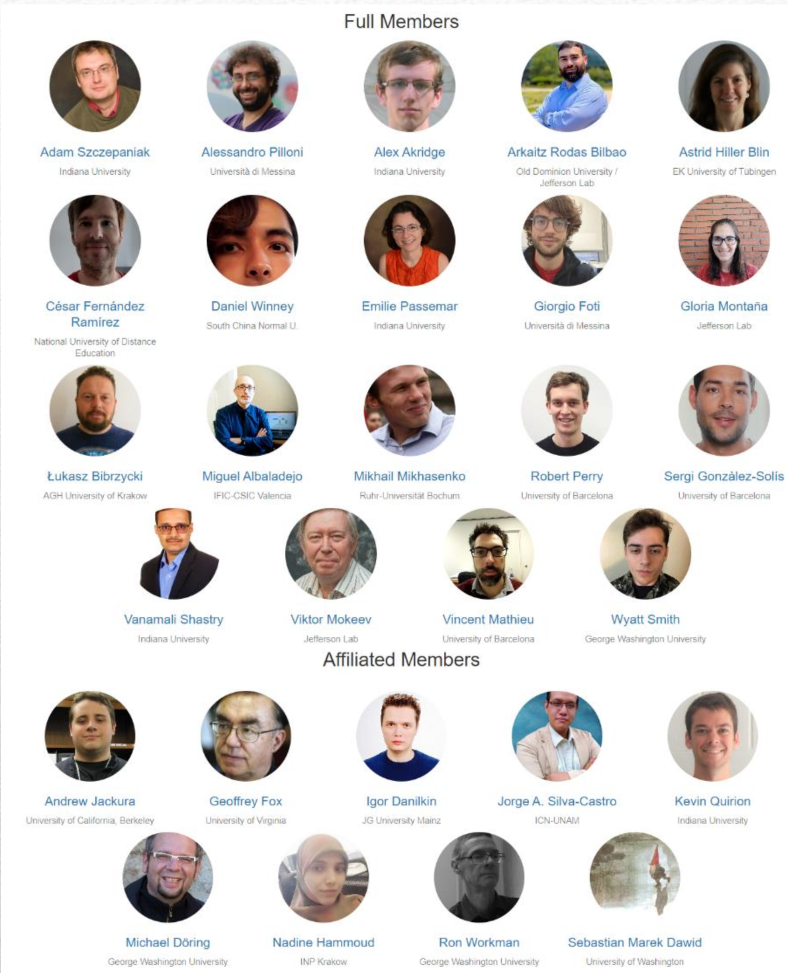
- They allow us to get the most out of high statistics data!
- JPAC and ExoHad will continue providing tools and results to understand the exciting spectrum of QCD

JPAC

EXOHAD
EXOTIC HADRONS TOPICAL COLLABORATION

Thank you!

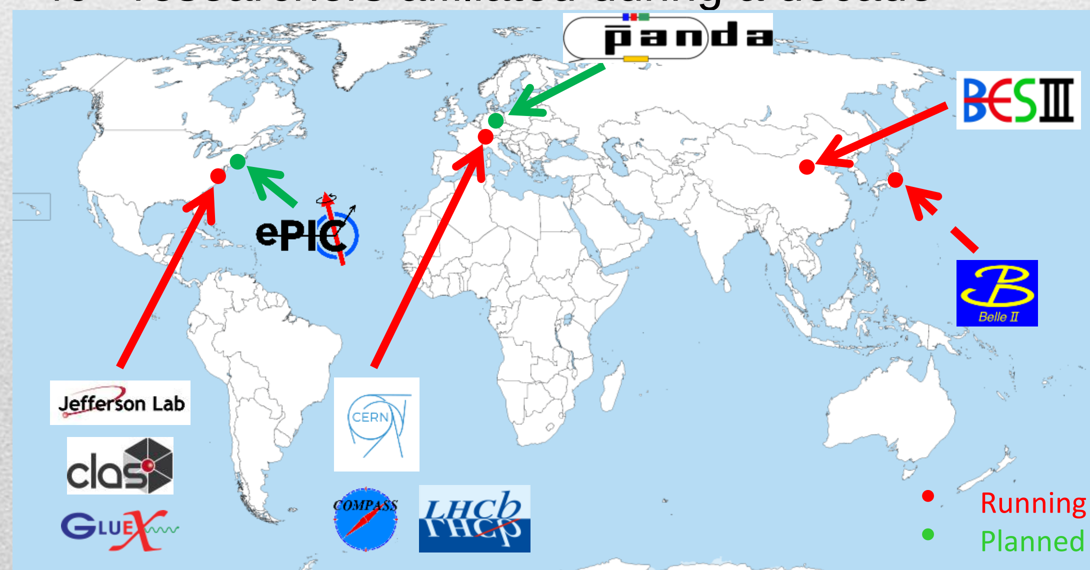
Joint Physics Analysis Center



Funded in 2013 as theory support originally for JLab exps, but grew up much larger

Creates bridges between theorists and experimentalists

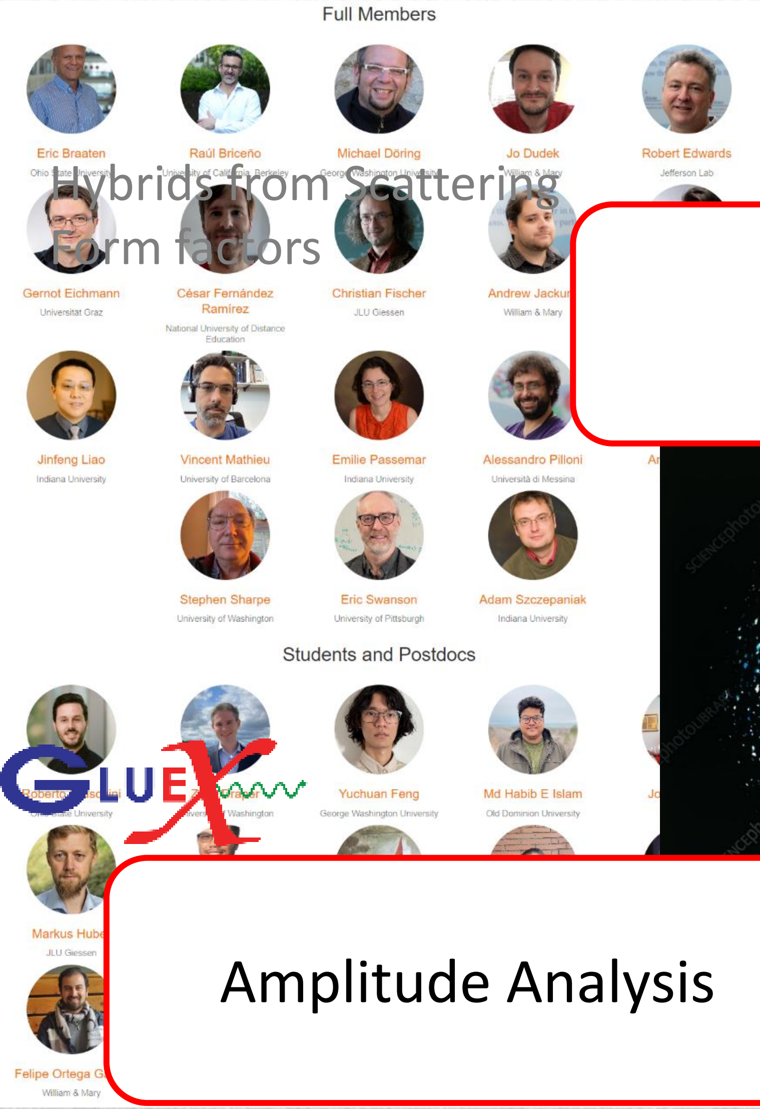
40+ researchers affiliated during a decade



BACKUP

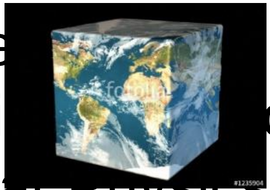
Exo(tic) Had(ron) TC

ExoHad
Exotic Hadrons Topical Collaboration



Hybrids from Scattering
Form factors

Funde[...] by DoE as
a Topi[...]oration in
Nucle[...]sics



[...]ring all
[...]otics
[...] GlueX)

Lattice QCD

Quark level calculations
Lessons from EFTs

Studying details of production mechanisms of exotics at GlueX

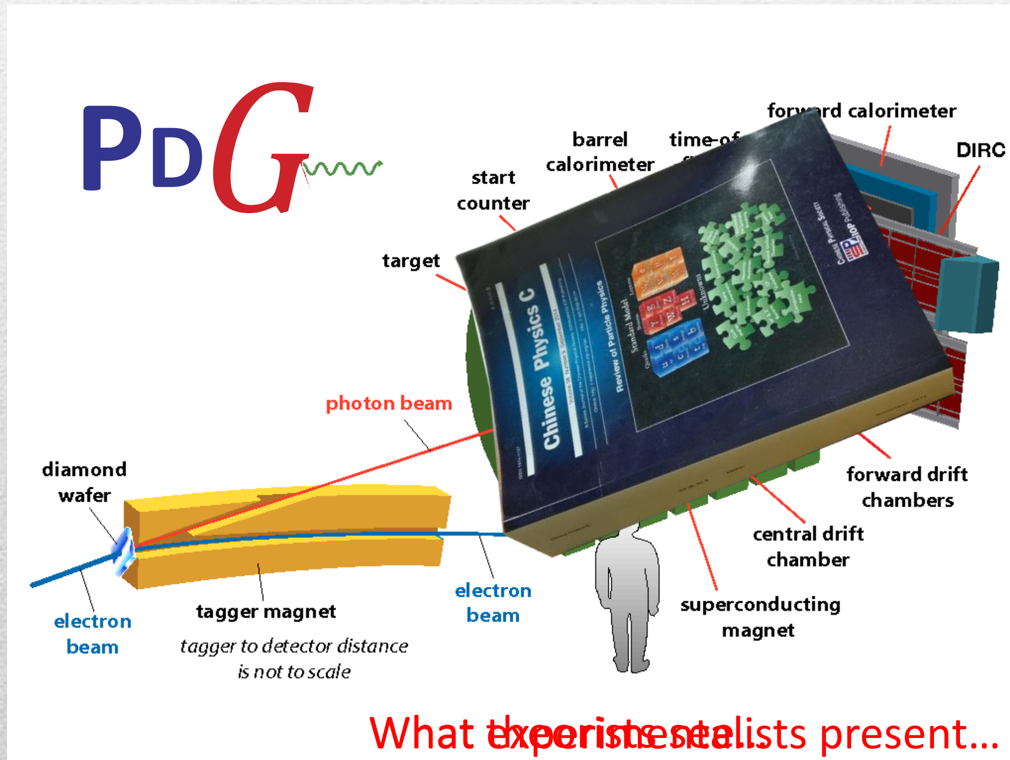
Finite energy sum rules

Amplitude Analysis

Phenomenology

How theorists think of experiments

All the information about hadrons is collected by the Particle Data Group



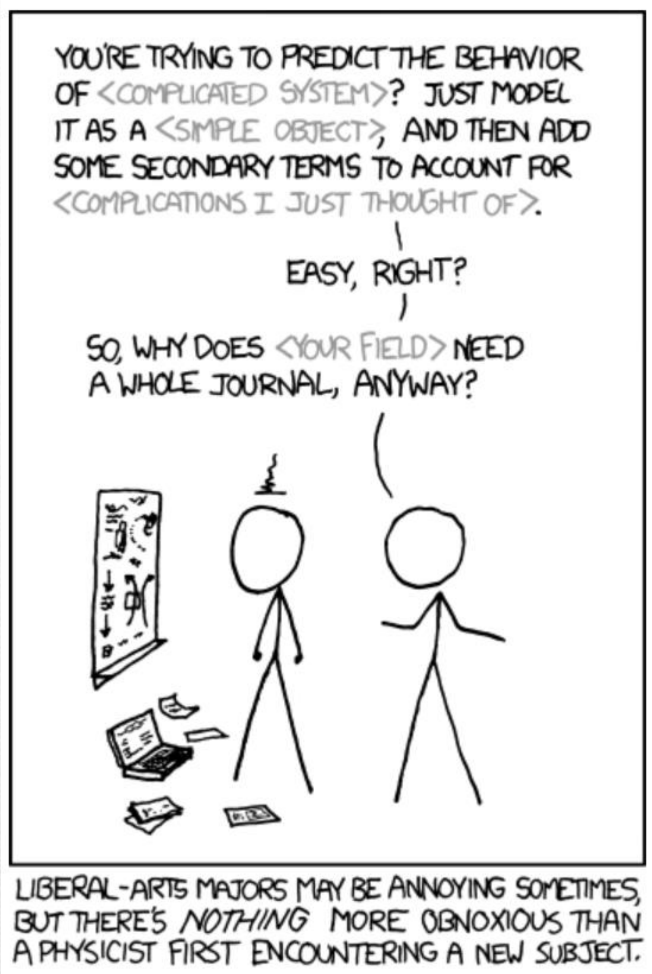
Changing beam and target, you can change the font

$a_1(1260)$ [i] $I^G(J^{PC}) = 1^-(1^{++})$

Mass $m = 1230 \pm 40$ MeV [j]
Full width $\Gamma = 250$ to 600 MeV

$a_1(1260)$ DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
3π	seen	577
$(\rho\pi)_{S-\text{wave}}$, $\rho \to \pi\pi$	seen	353
$(\rho\pi)_{D-\text{wave}}$, $\rho \to \pi\pi$	seen	353
$(\rho(1450)\pi)_{S-\text{wave}}$, $\rho \to \pi\pi$	seen	†
$(\rho(1450)\pi)_{D-\text{wave}}$, $\rho \to \pi\pi$	seen	†
$f_0(500)\pi$, $f_0 \to \pi\pi$	seen	–
$f_0(980)\pi$, $f_0 \to \pi\pi$	not seen	179
$f_0(1370)\pi$, $f_0 \to \pi\pi$	seen	†
$f_2(1270)\pi$, $f_2 \to \pi\pi$	seen	†
$\pi^+\pi^-\pi^0$	seen	576
$\pi^0\pi^0\pi^0$	not seen	577
$KK\pi$	seen	250
$K^*(892)K$	seen	†
$\pi\gamma$	seen	608

Real(istic) conversations between theorists and experimentalists



T: How could you not find the XXX quantum numbers?

E: Well, you know, 5D fit, O(100) parameters, local minima, systematics…

T: You are not that good, are you?

T: You shouldn't use Breit Wigners!

E: Gimme some better

T: I have an analytic unitary parametrization, but it works in a given mass window and only knows about energy dependence, you can't add background terms to it…

E: Breit Wigners.

T: Stop using complex couplings!

E: HAHAHAHHAHAHAHAHAHHHAHAHAHA

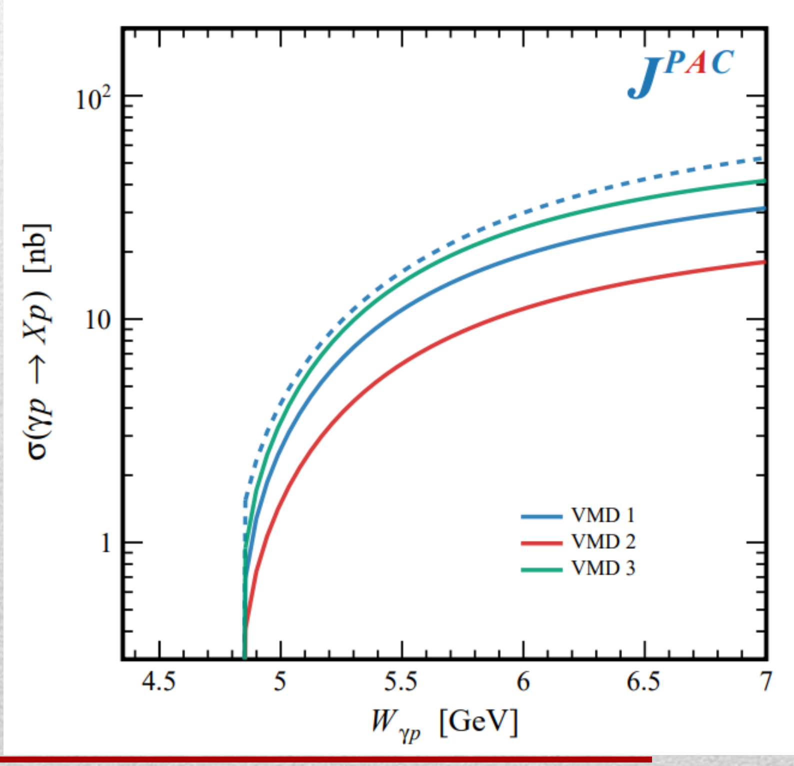
In defense of VMD

The weak point of the whole approach is the use of VMD
While in the light sector it works reasonably, for heavy states is not justified

The X(3872) observed in purely hadronic and photonic modes gives us unique clue to efficacy of VMD

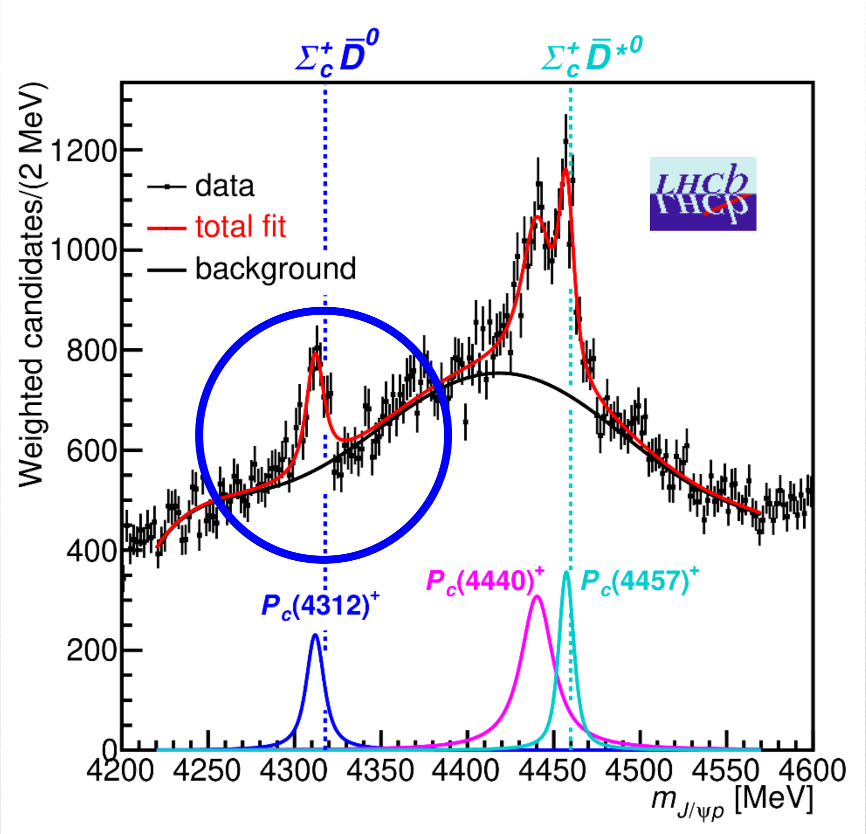
Belle extracted the coupling $X(3872) \to \gamma\gamma^*$,
that can be compared with the VMD predictions
from $X(3872) \to J/\psi\ \rho, \omega$

Q	V	$\mathcal{E}$	$g_{Q\gamma\gamma^*} \times 10^3$
$X(3872)$	γ	γ^*	3.2
	J/ψ	ρ	5.38
		ω	3.54



Other estimate lead to differences of a factor ~ 3

New pentaquarks discovered



The lowest $P_c(4312)$ appears
as an isolated peak
at the $\Sigma_c^+\bar{D}^0$ threshold

A detailed study of the lineshape provides insight on its nature

Bottom-up:
DON'T YOU DARE describing everything!!!
Focus on the peak region

Minimal(istic) model for $P_c(4312)$

$$\frac{dN}{d\sqrt{s}} = \rho(s)\left[|F(s)|^2 + b_0 + b_1 s\right]$$

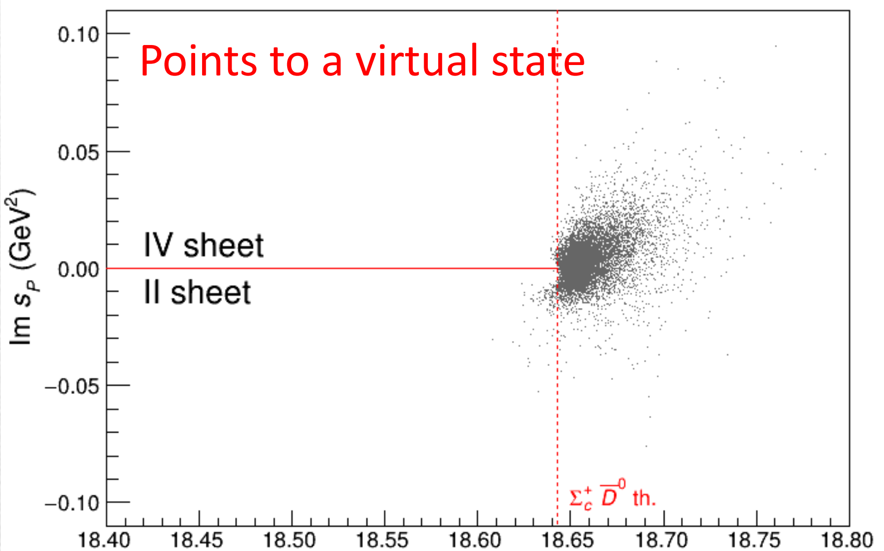
Fernandez-Ramirez *et al.* (JPAC), PRL 123, 092001

Effective range expansion

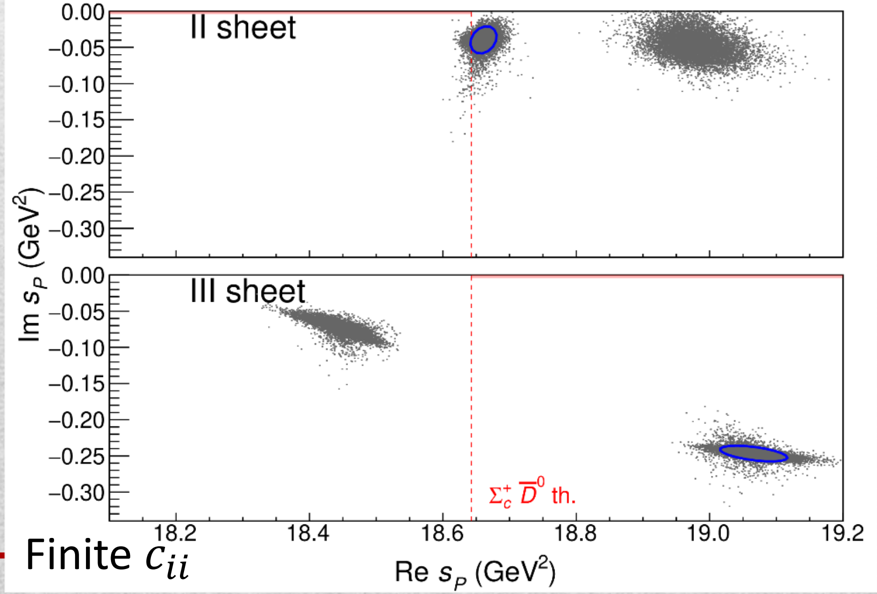
$$F(s) = (N_1 + N_2 s)\, T_{11}(s)$$

We can set $c_{ii} = 0$ to reduce to the scattering length approximation

$$T(s) = \begin{pmatrix} m_{11} - c_{11}s - i\rho_1(s) & m_{12} \\ m_{12} & m_{22} - c_{22}s - i\rho_2(s) \end{pmatrix}^{-1}$$



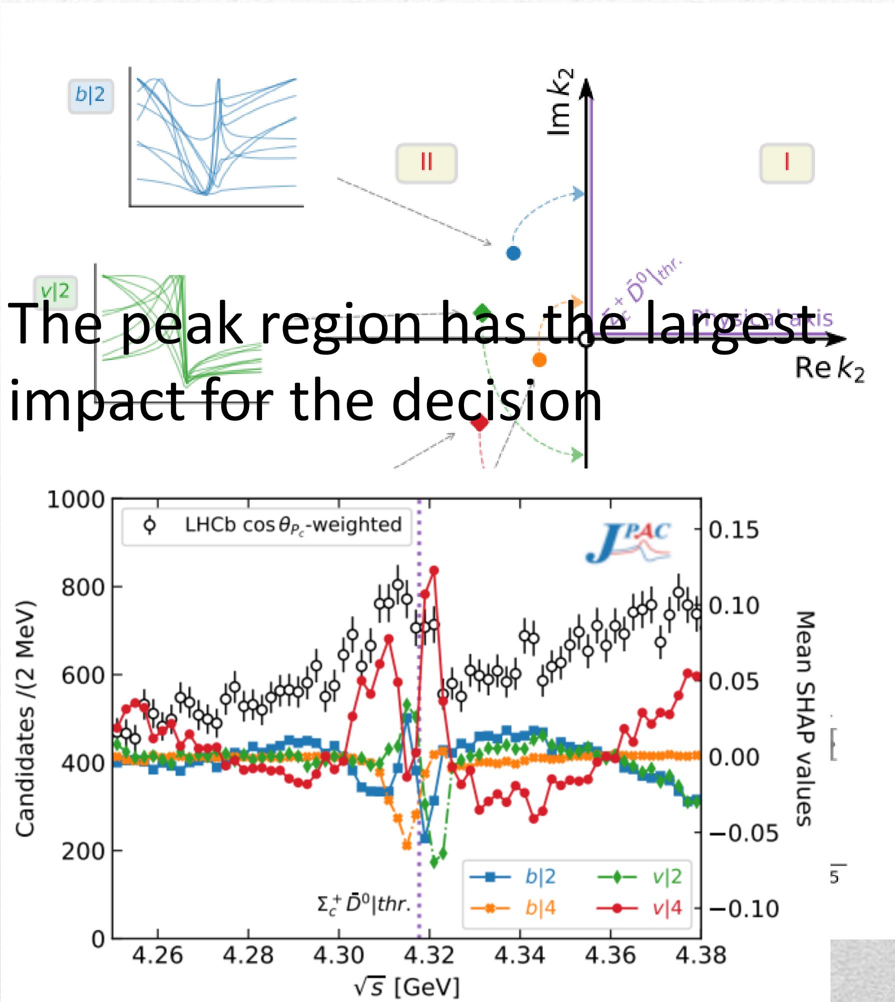
$c_{ii} = 0$



Finite c_{ii}

Minimal(istic) model with ANN

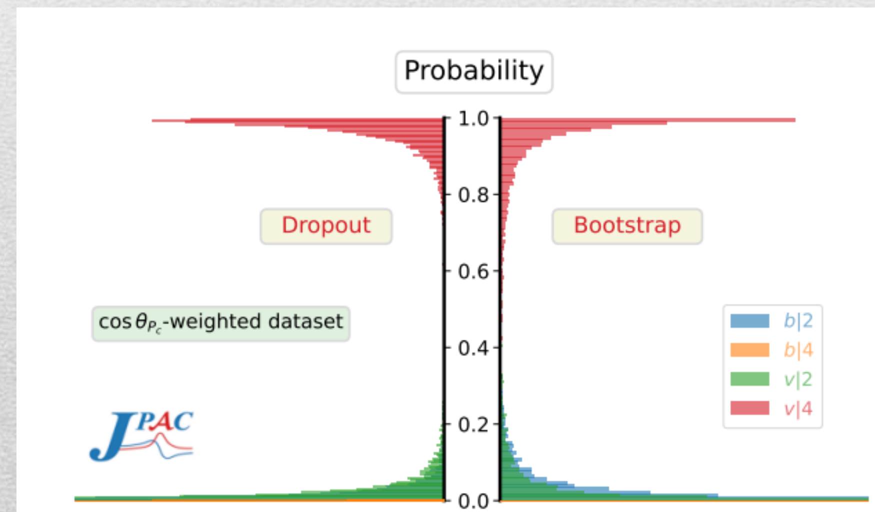
Ng, *et al.* (JPAC), PRD 105, L091501



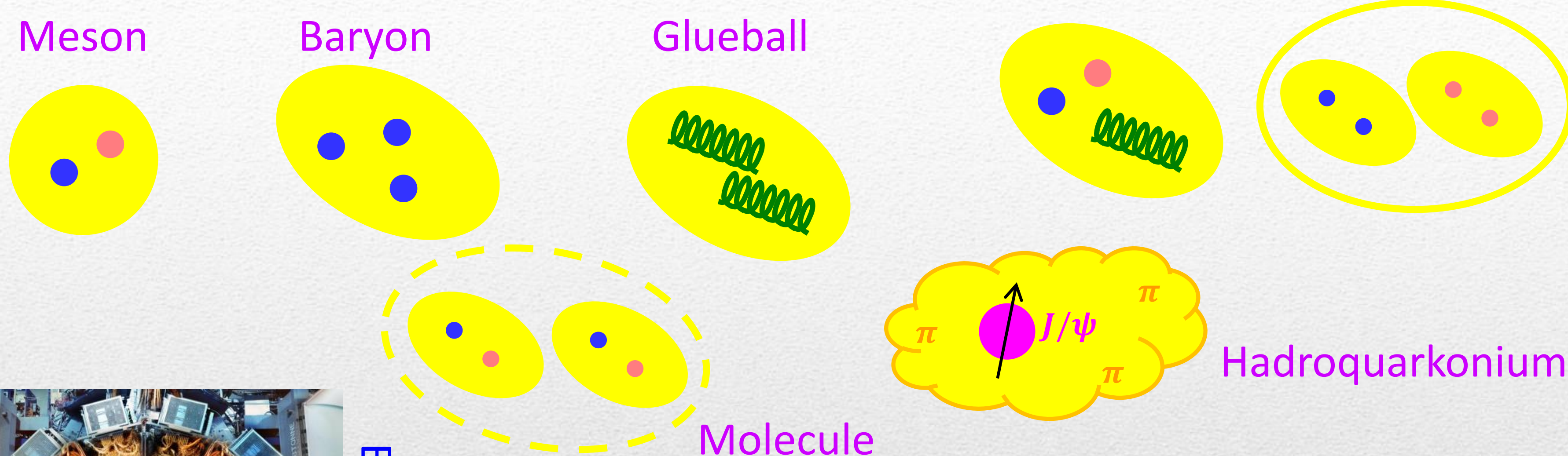
The peak region has the largest impact for the decision

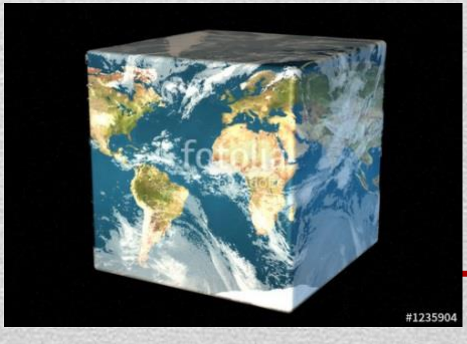
	$b\|2$	$b\|4$	$v\|2$	$v\|4$
$\cos\theta_{P_c}$-weighted	0.6%	$<$ 0.01%	1.1%	98.3%
$m_{Kp} > 1.9$ GeV	1.4%	$<$ 0.1%	1.6%	97.0%
m_{Kp} all	5.4%	$<$ 0.1%	21.0%	73.6%

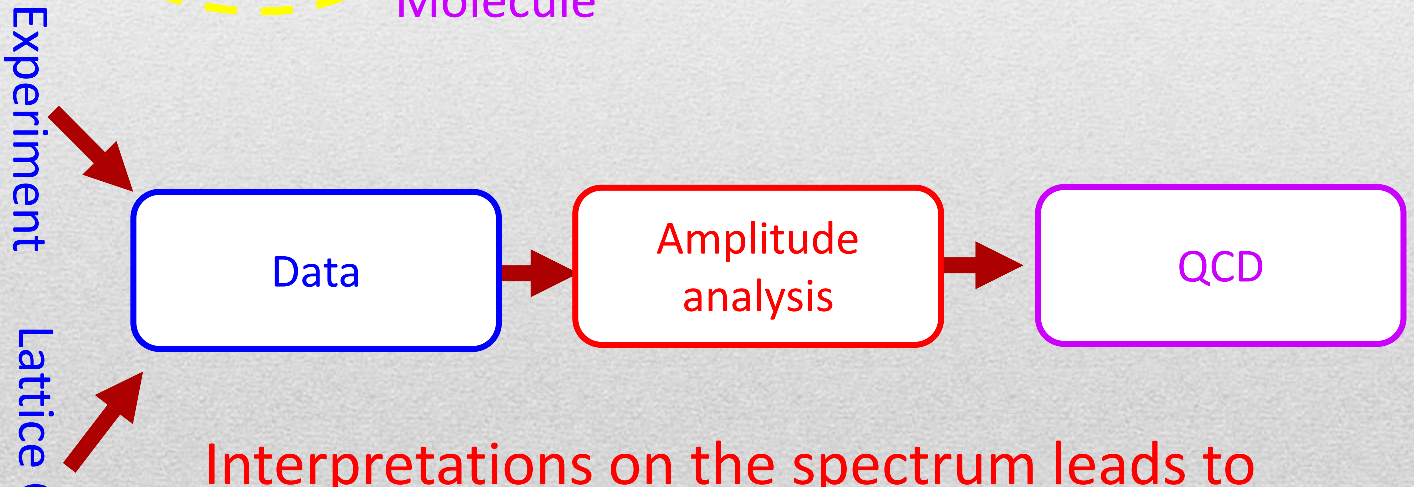
Highest probability for a virtual state in the IV sheet



Hadron Spectroscopy







Interpretations on the spectrum leads to understanding fundamental laws of nature

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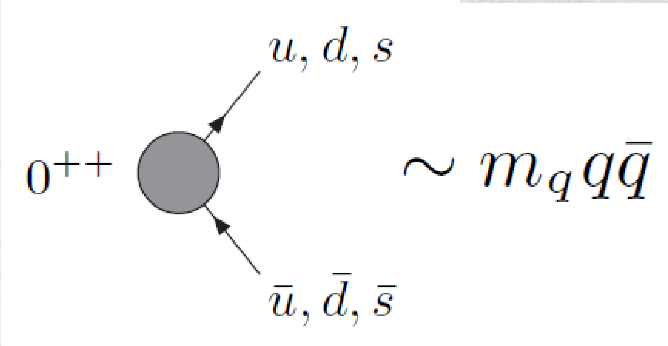
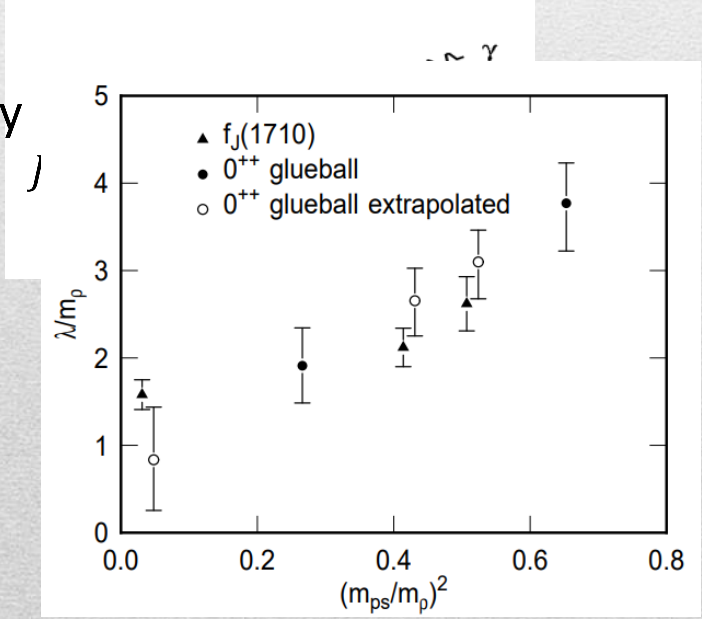
- It couples equally to mesons of all flavors (?)
  However, an argument based on chiral symmetry claims the coupling proportional to quark mass

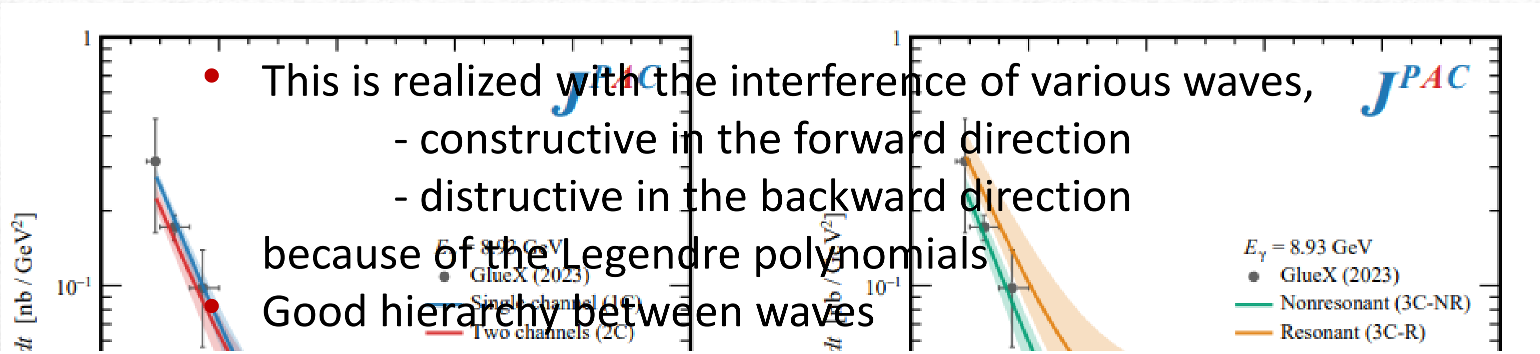


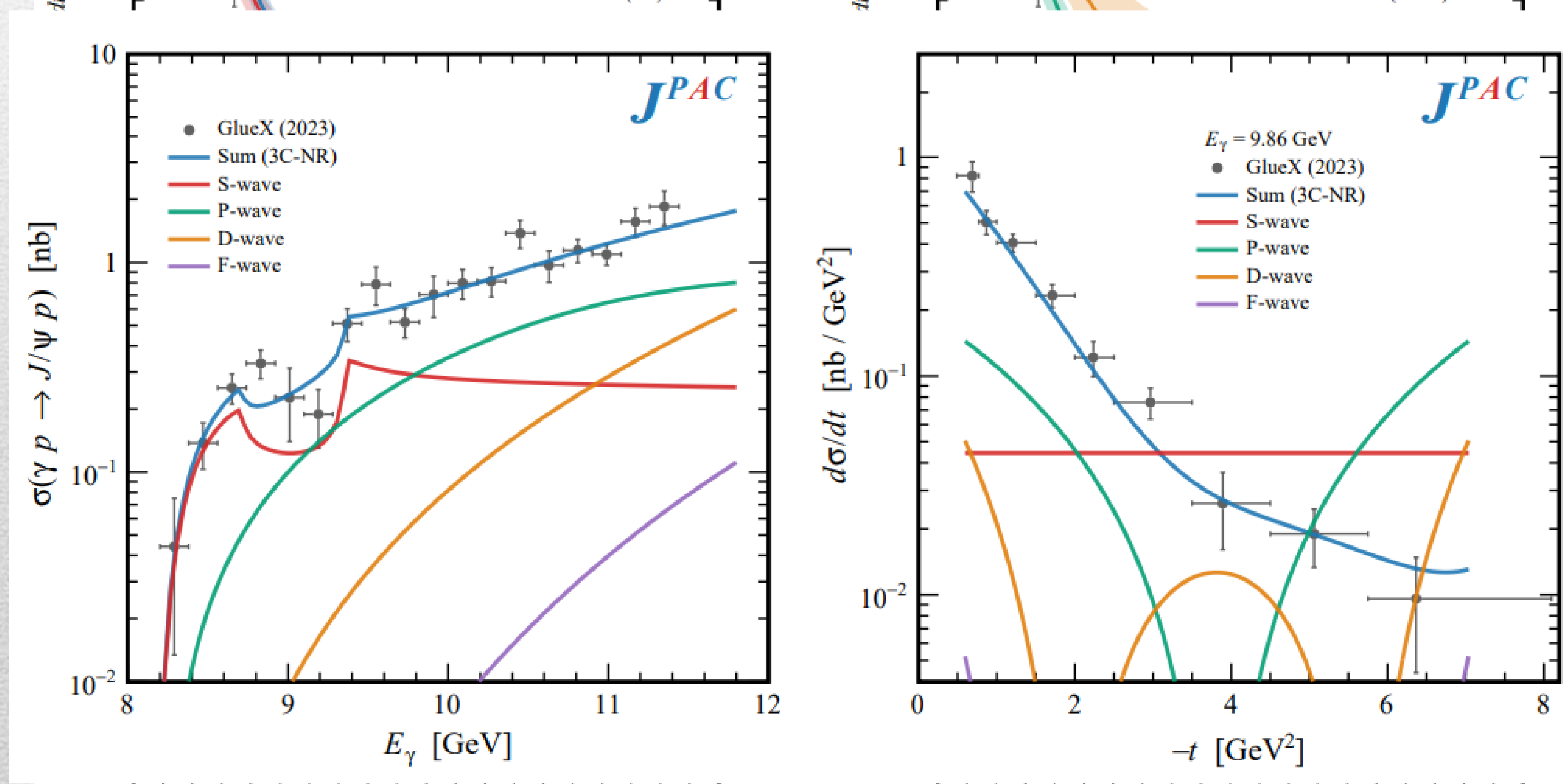


Differential cross section

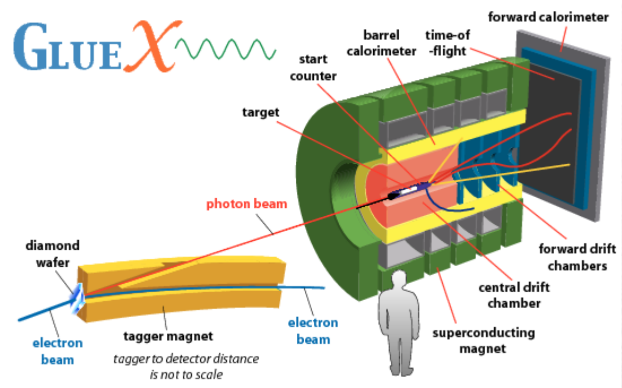
- This is realized with the interference of various waves,
  - constructive in the forward direction
  - distructive in the backward direction

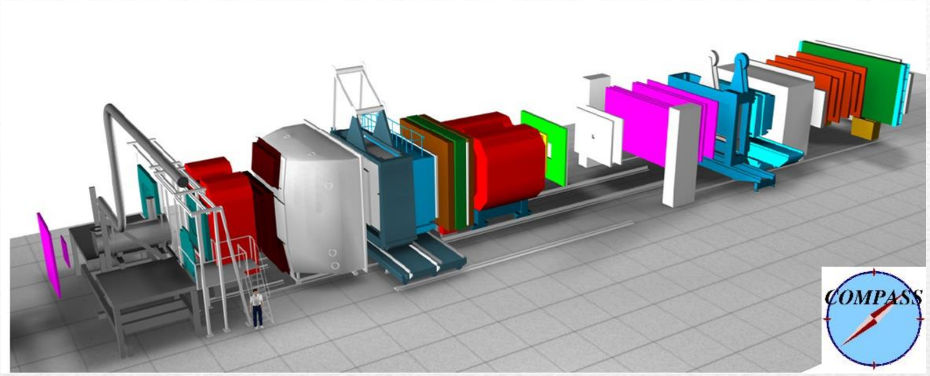
  because of the Legendre polynomials
- Good hierarchy between waves

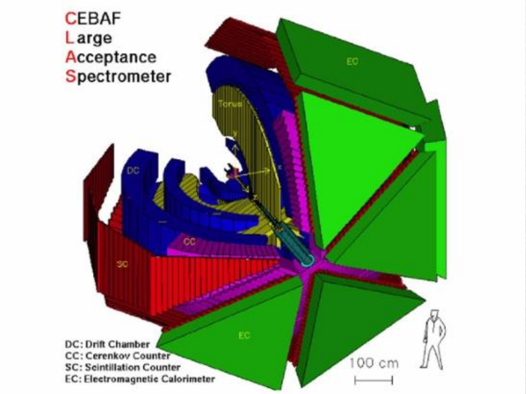




Spectroscopy from peripheral production

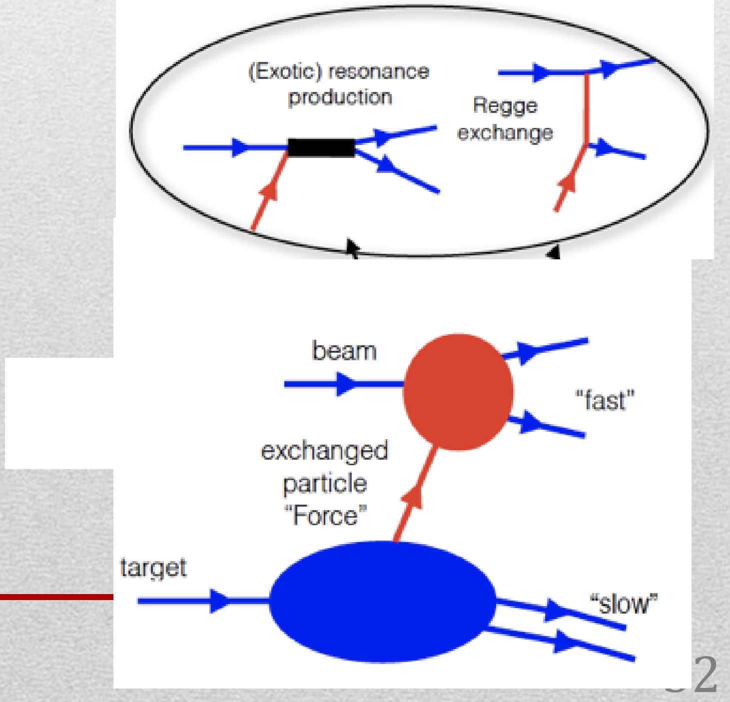




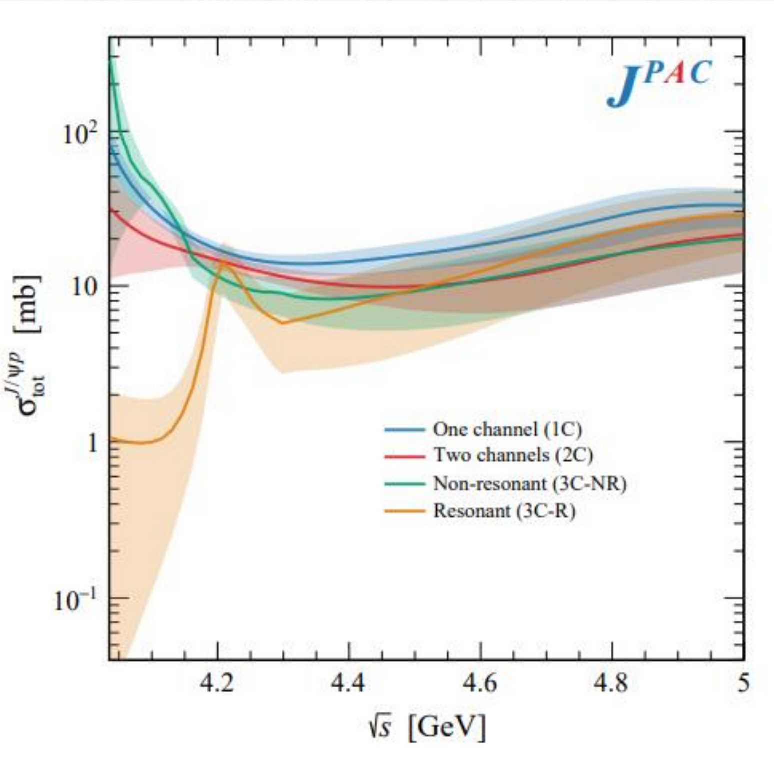


Need to establish Regge factorization

Single Regge pole dominate



Total cross section



With optical theorem one gets the total cross section $J/\psi\, p \to X$.

This has been estimated from the A-dependence of nuclear targets, 4mb at $\sqrt{s} = 6.2$ GeV.

VMD estimates provide values at least 1order of magnitude smaller

Options on the market

Quark-level calculations:

- Quark models, pNRQCD, Hybrids, ...
- Spectrum generally calculated as bound states of some potential
- Decay rates as «overlap integrals» between two static configurations

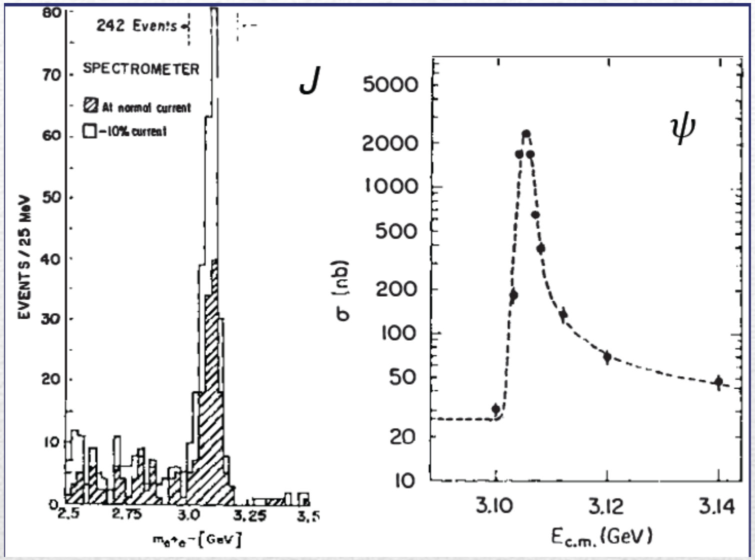
Comprehensive picture ✓ Little scattering dynamics ✘

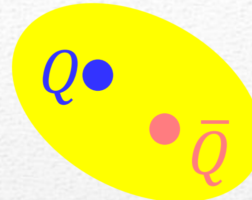
Hadron level calculations:

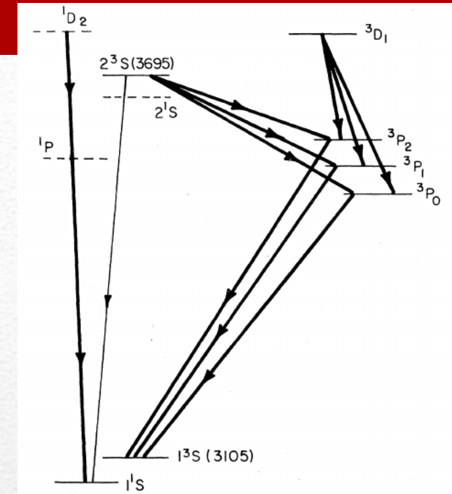
- Microscopic models inspired to EFT + Unitarization
- Compact states vs. molecules, triangles...
- States as poles of scattering amplitudes
- Couplings as residues at the poles

Scattering dynamics ✓ More case-by-case ✓✘

Quarkonium orthodoxy







$\alpha_s(M_Q) \sim 0.3$

(perturbative regime)

OZI-rule, QCD multipole

Heavy quark spin flip suppressed by quark mass, approximate heavy quark spin symmetry (HQSS)

Potential models

(meaningful when $M_Q \to \infty$)

$$V(r) = -\frac{C_F \alpha_s}{r} + \sigma r$$

(Cornell potential)

Solve NR Schrödinger eq. → spectrum

Effective theories

(HQET, NRQCD, pNRQCD...)

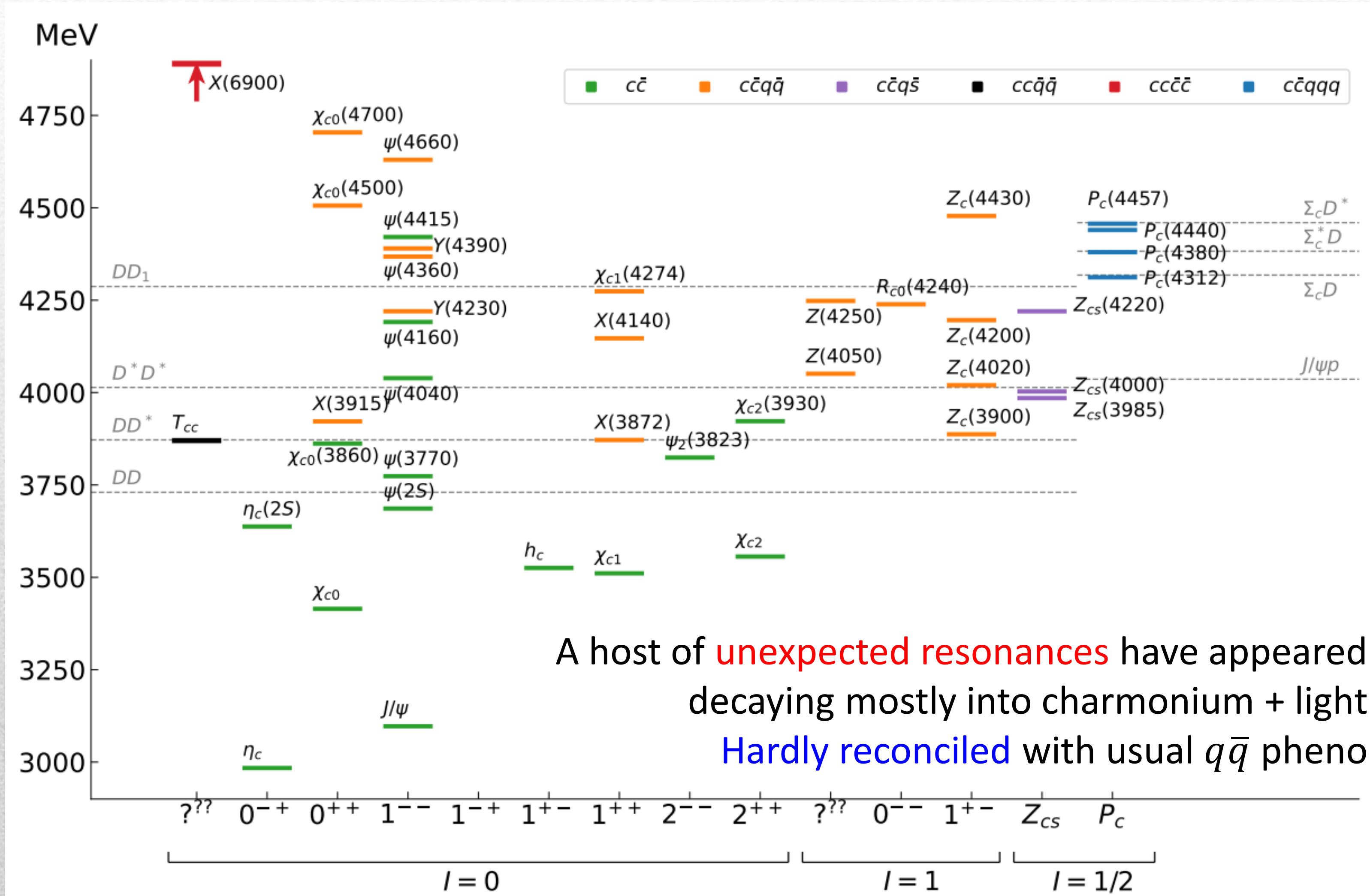
Integrate out heavy DOF

↓

(spectrum), decay & production rates

Exotic landscape in $c\bar{c}$

Esposito, AP, Polosa, Phys.Rept. 668
JPAC, PPNP



A host of unexpected resonances have appeared
decaying mostly into charmonium + light
Hardly reconciled with usual $q\bar{q}$ pheno

Exotic landscape

Broad mesons seen in b decay:
$X(4140)$, Z(4430), $Z_{cs}(4000)$…

Scarce consistency between various production mechanisms

Narrow structures seen in b decay:
$X(3872), P_c, (P_{cs})$

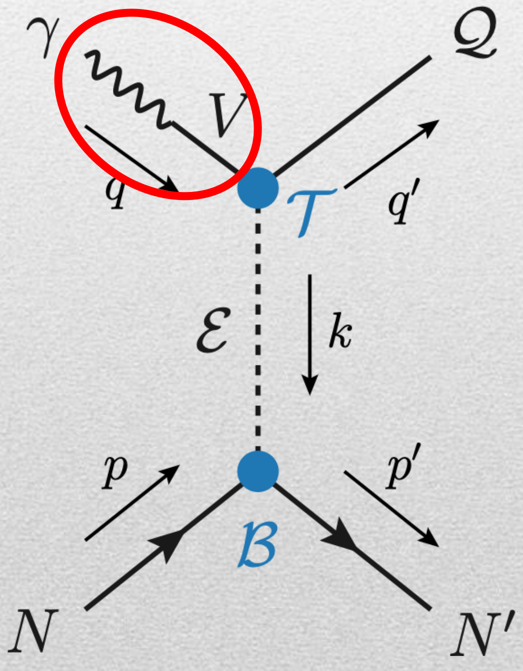
Narrow structures seen in e^+e^-:
$X(3872), Y(4260), Z_{c,b}^{(\prime)}$

Why photoproduction?

- It's new: no XYZ state has been uncontroversially seen so far
- Rescattering mechanisms that could mimic resonances in multibody decays can be controlled better (one can change the energy beam but not the B mass…)
- The framework is (relatively) clean from a theory point of view
- Radiative decays offer another way of discerning the nature of the states

Exclusive (quasi-real) photoproduction

- XYZ have so far not been seen in photoproduction: independent confirmation
- Not affected by 3-body dynamics: determination of resonant nature
- Experiments with high luminosity in the appropriate energy range are promising
- We study near-threshold (LE) and high energies (HE)
- Couplings extracted from data as much as possible, not relying on the nature of XYZ



M. Albaladejo et al. [JPAC], PRD

$$< \lambda_Q \lambda'_N | T | \lambda_\gamma \lambda_N >= \sum_{V,\mathcal{E}} \frac{e f_V}{m_V} \mathcal{T}^{\alpha_1 \cdots \alpha_j}_{\lambda_V = \lambda_\gamma, \lambda_Q} \mathcal{P}_{\alpha_1 \cdots \alpha_j; \beta_1 \cdots \beta_j} \mathcal{B}^{\beta_1 \cdots \beta_j}_{\lambda_N \lambda'_N}$$

VMD is used to couple the incoming photon
to a vector quarkonium V
….we'll come back to that!

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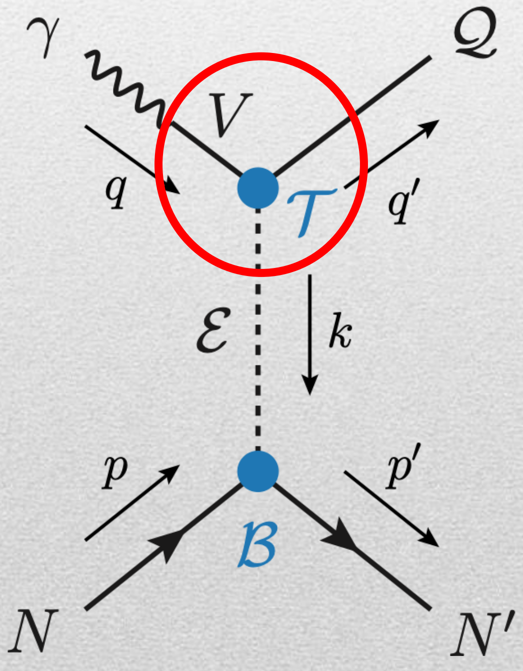

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Top vertex from measured $\mathcal{Q} \to V\mathcal{E}$ decay width

Exclusive (quasi-real) photoproduction

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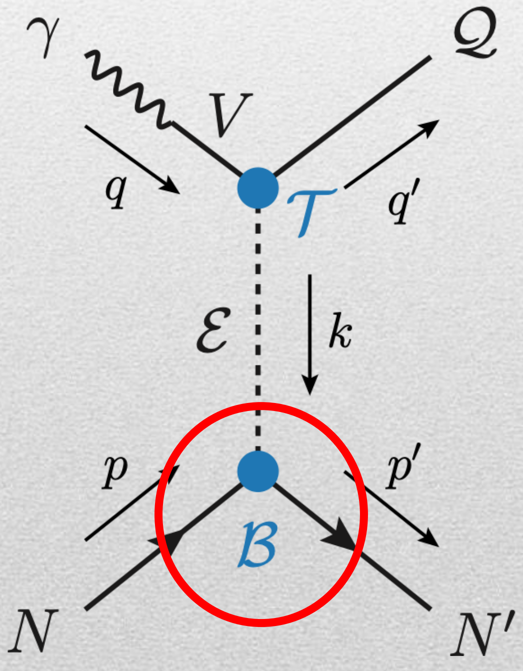

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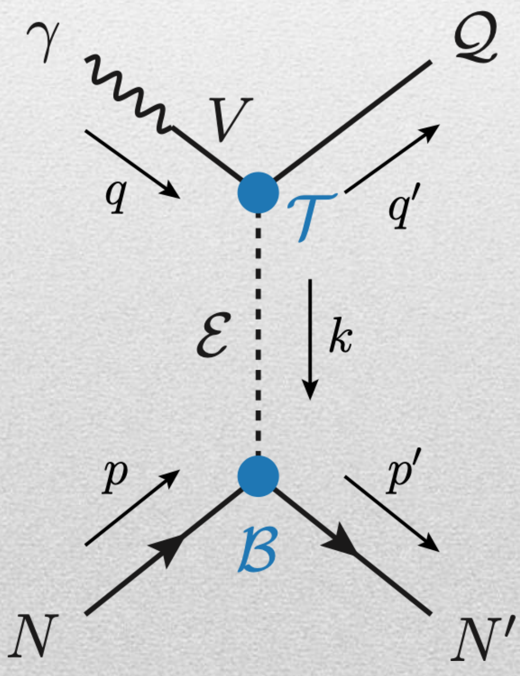
$$< \lambda_{\mathcal{Q}} \lambda'_N | T | \lambda_\gamma \lambda_N >= \sum_{V,\mathcal{E}} \frac{e f_V}{m_V} \mathcal{T}^{\alpha_1 \cdots \alpha_j}_{\lambda_V = \lambda_\gamma, \lambda_{\mathcal{Q}}} \mathcal{P}_{\alpha_1 \cdots \alpha_j; \beta_1 \cdots \beta_j} \mathcal{B}^{\beta_1 \cdots \beta_j}_{\lambda_N \lambda'_N}$$

Bottom vertex from standard photoproduction pheno, exponential form factors to further suppress large t

Threshold vs. high energy

- Fixed-spin exchanges expected to hold in the low energy region
- t channel grows as s^j, exceeding unitarity bound, Regge physics kicks in: Reggeized tower of particles with arbitrary spin at HE



$$s^j \longrightarrow s^{\alpha_0 + \alpha' t}$$

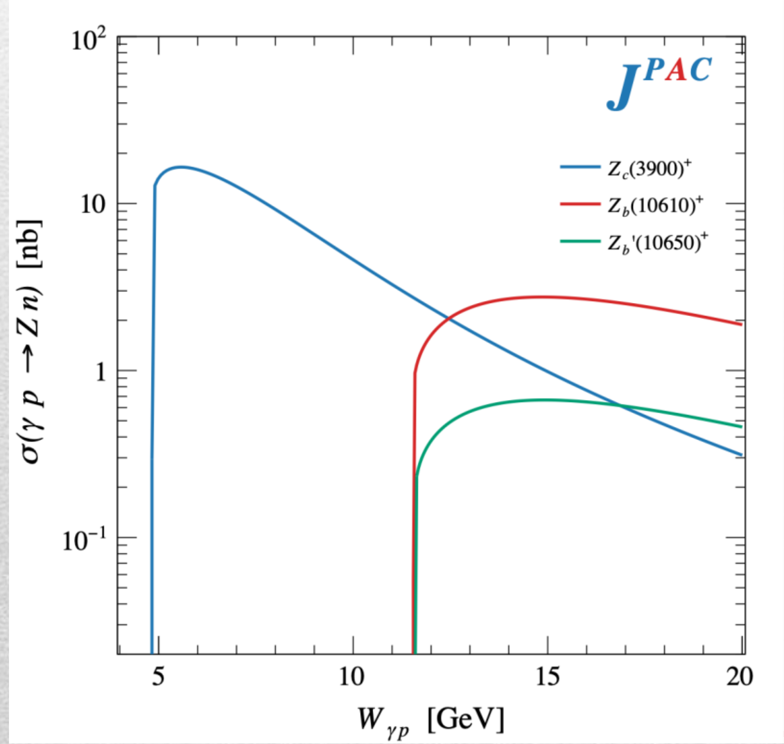
Holds at low energy, fixed spin

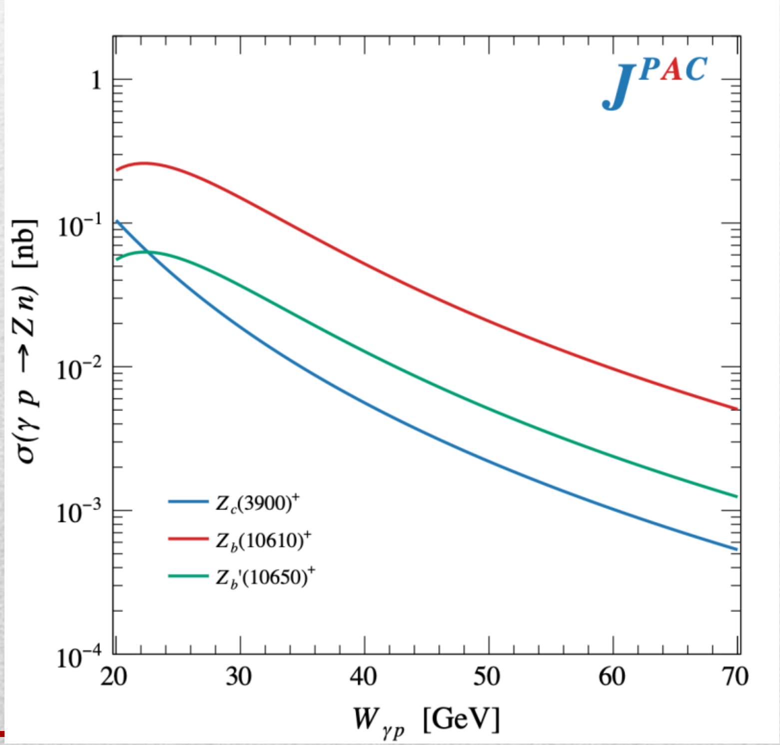
Holds at high energy, resummation of leading s power

- If $\varepsilon \neq \mathbb{P}$, $\alpha_0 < 1$, $d\sigma/dt$ decreases with energy
- Exchange of heavy particles further suppressed

Z photoproduction

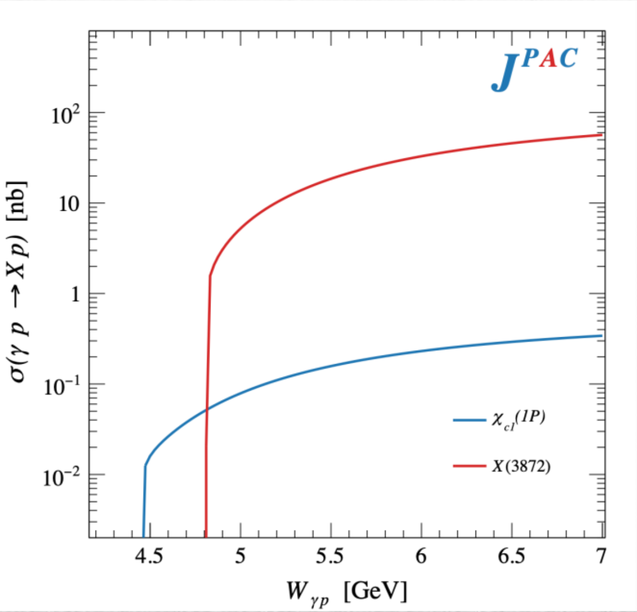
- The Zs are charged charmoniumlike 1^{+-} states close to open flavor thresholds
- Focus on $Z_c(3900)^+ \to J/\psi\, \pi^+, Z_b(10610)^+, Z_b'(10650)^+ \to \Upsilon(nS)\, \pi^+$
- The pion is exchanged in the t-channel



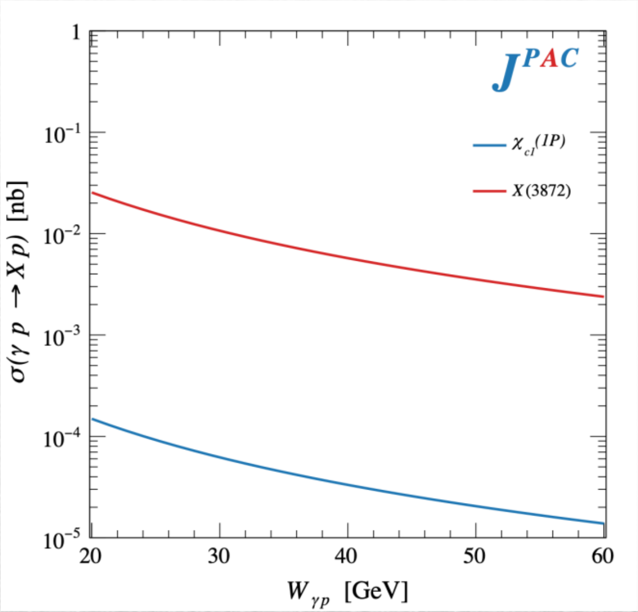


X photoproduction

- Focus on the famous 1^{++} $X(3872) \to J/\psi\, \rho, \omega$
- **ω** and **ρ** exchanges give main contributions:

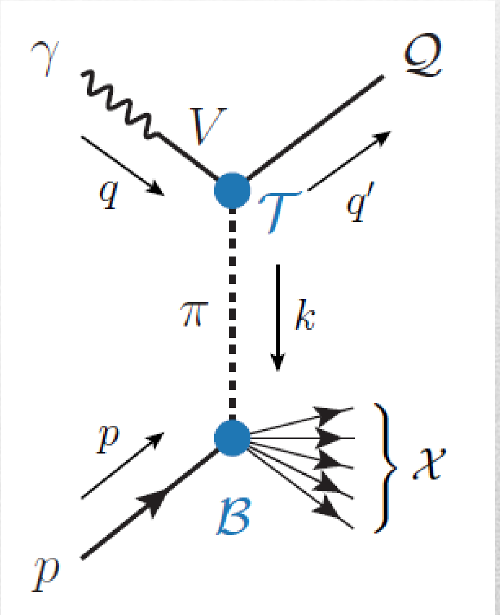


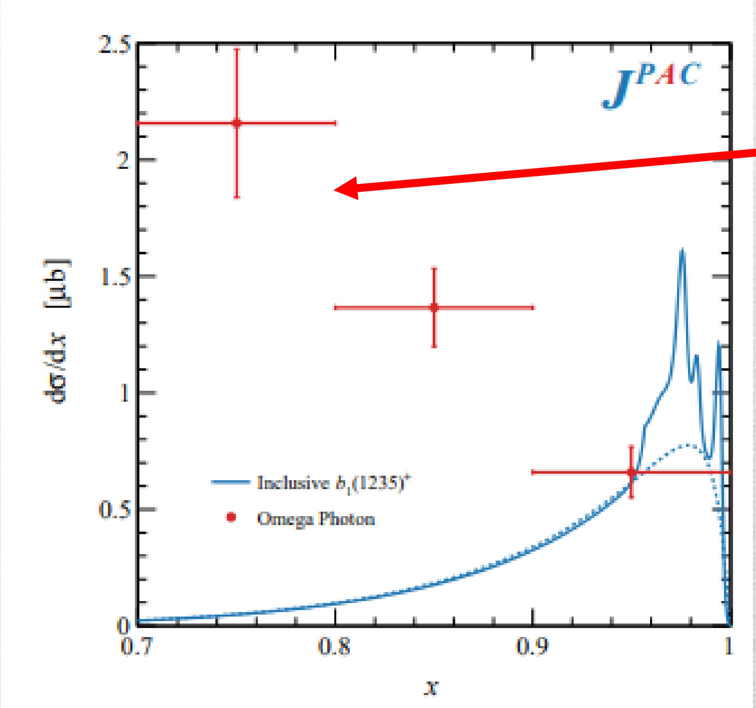
Large theory uncertainty
in the intermediate region



Semi-inclusive photoproduction

- Semi-inclusive cross sections are typically larger
- For small t and large x, one can assume the process to be dominated by pion exchange
- The bottom vertex depends on the (known) pion-proton total cross section
- The pion is exchanged in the t-channel
- Model benchmarked on b_1 production



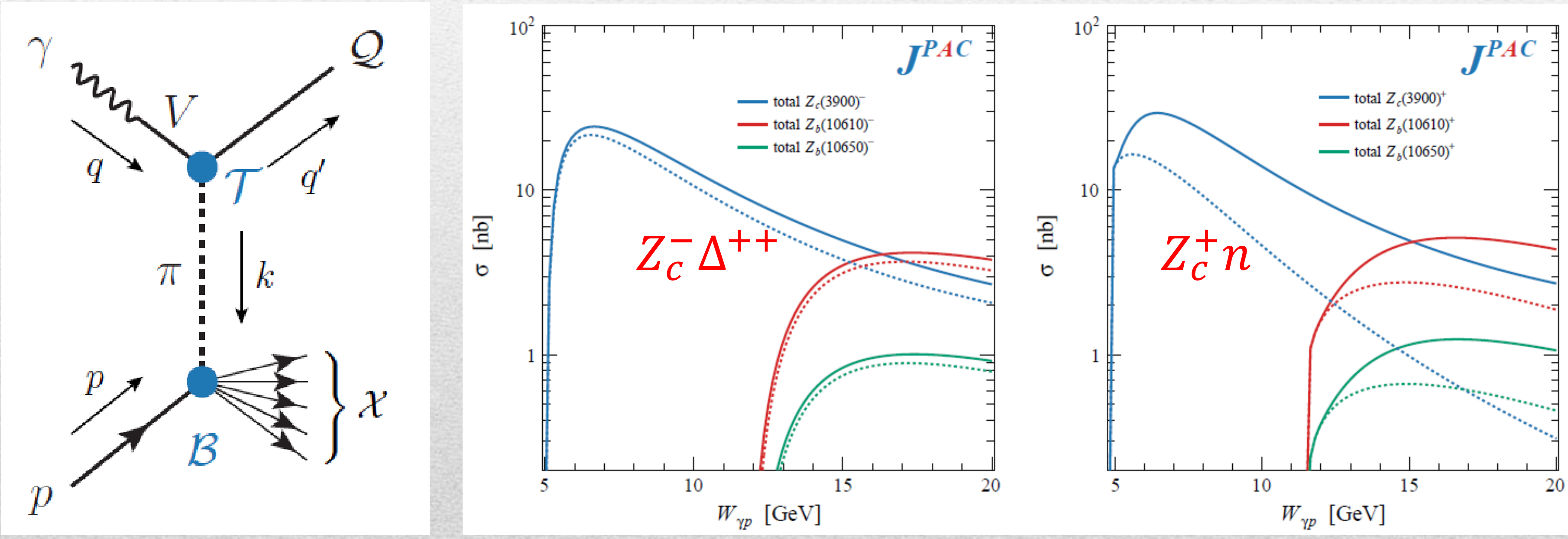


Model underestimates lower bins, conservative estimates

The model is expected to hold in the highest bin

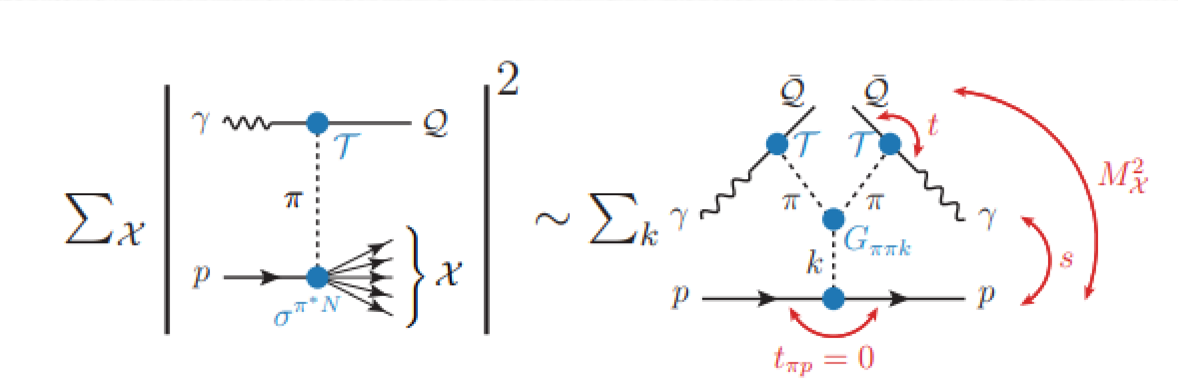
Semi-inclusive photoproduction

- For the Z_c^+, the inclusive cross section is sizably larger than the exclusive process



Semi-inclusive photoproduction

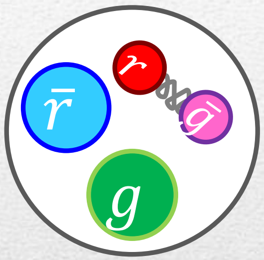
- At higher energies the triple Regge regime is reached, cross sections saturate



	$\sigma(\gamma p \to \mathcal{Q}^\pm\, \mathcal{X})$ [pb]			$\sigma(\gamma p \to \mathcal{Q}^+ n)$ [pb]		
$\mathcal{Q}$	30 GeV	60 GeV	90 GeV	30 GeV	60 GeV	90 GeV
$b_1(1235)$	$60 \cdot 10^3$	$60 \cdot 10^3$	$61 \cdot 10^3$	43	2.3	$< 10^{-8}$
$Z_c(3900)$	187	146	140	19	1.0	$< 10^{-8}$
$Z_b(10610)$	163	15	5	150	10	$< 10^{-8}$
$Z_b(10650)$	40	4	1	37	2.4	$< 10^{-8}$

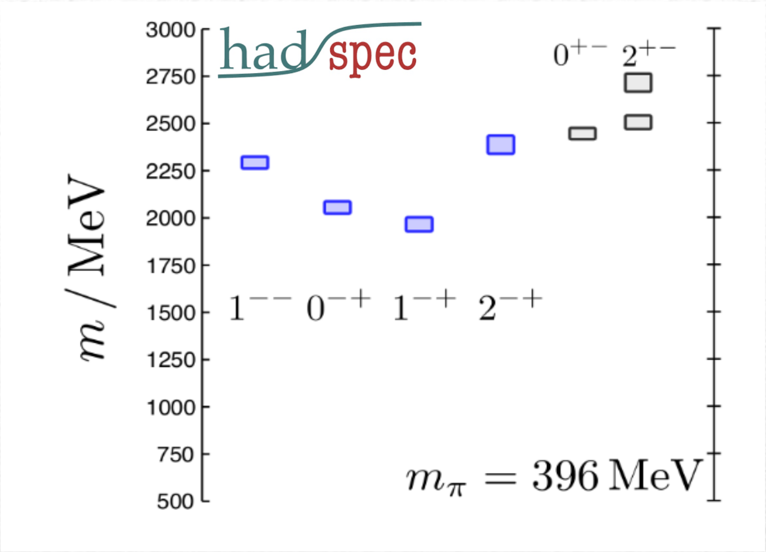
Hybrid hunting

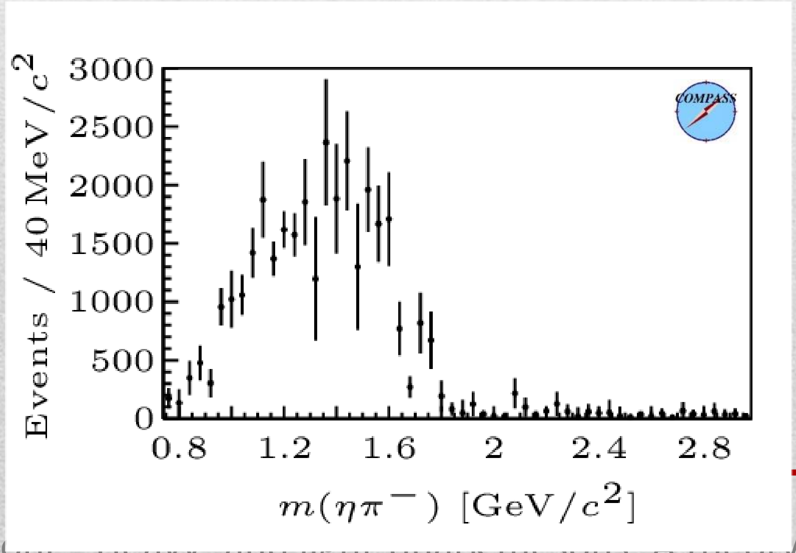
Constituent gluon (quasiparticle excitation),
$J^{PC} = 1^{+-}$, mass ~ 1.0–1.5 GeV

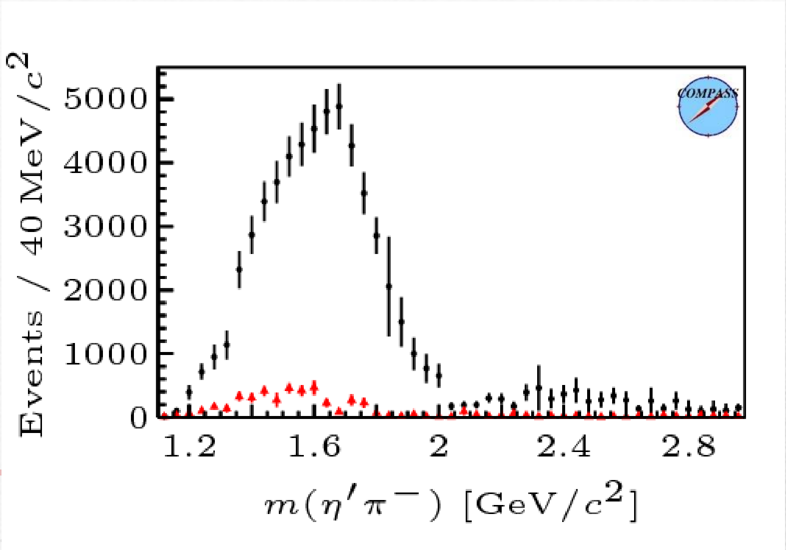


Look for a π_1 state with $\boldsymbol{J^{PC} = 1^{-+}}$

decaying into $\begin{cases} \eta\,\pi \text{ and } \eta'\pi \\ \rho\,\pi \to 3\pi \\ b_1\pi \to 5\pi \end{cases}$







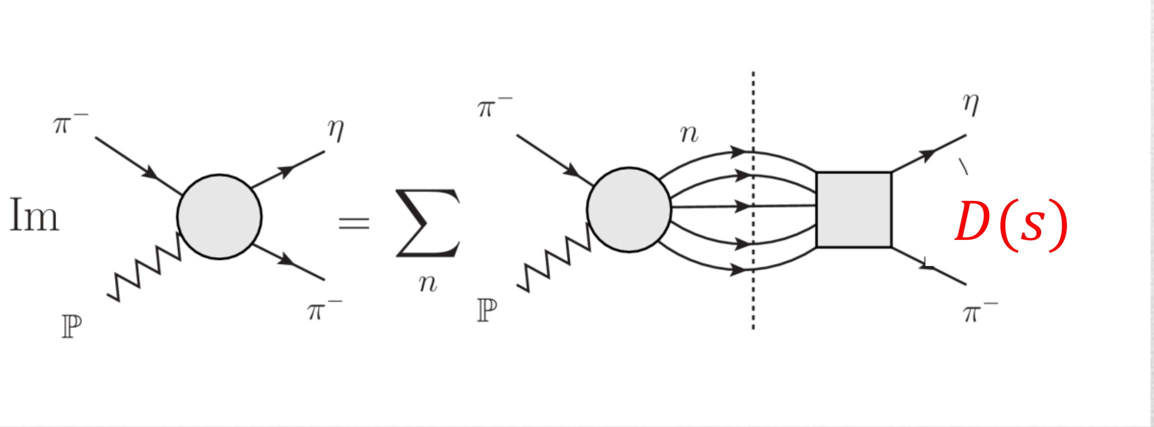
Amplitudes for $\eta^{(\prime)}\pi$

We build the partial wave amplitudes according to the N/D method

Jackura, Mikhasenko, AP *et al.* (JPAC & COMPASS), PLB

Rodas, AP *et al.* PRL

$$a(s) = \frac{n(s)}{D(s)}$$



The $D(s)$ contains all the Final State Interactions constrained by unitarity → universal

Amplitudes for $\eta^{(\prime)}\pi$

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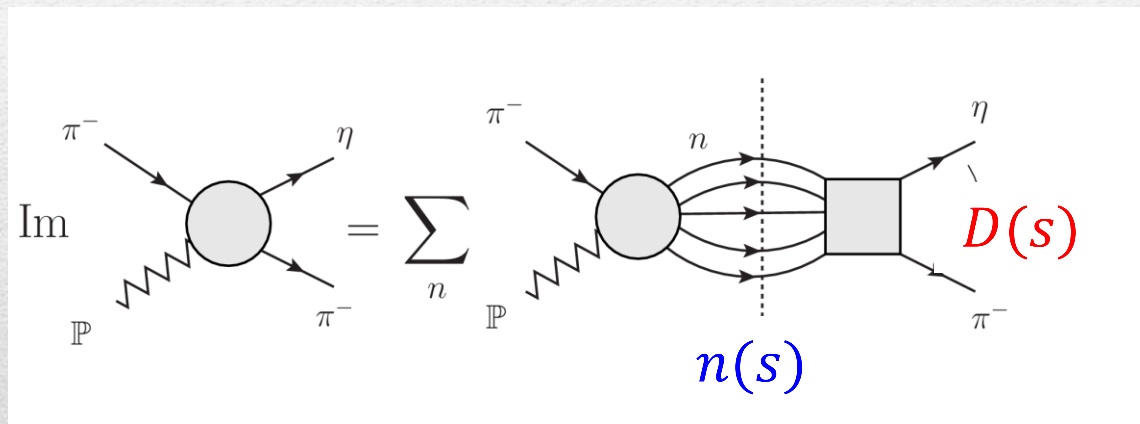
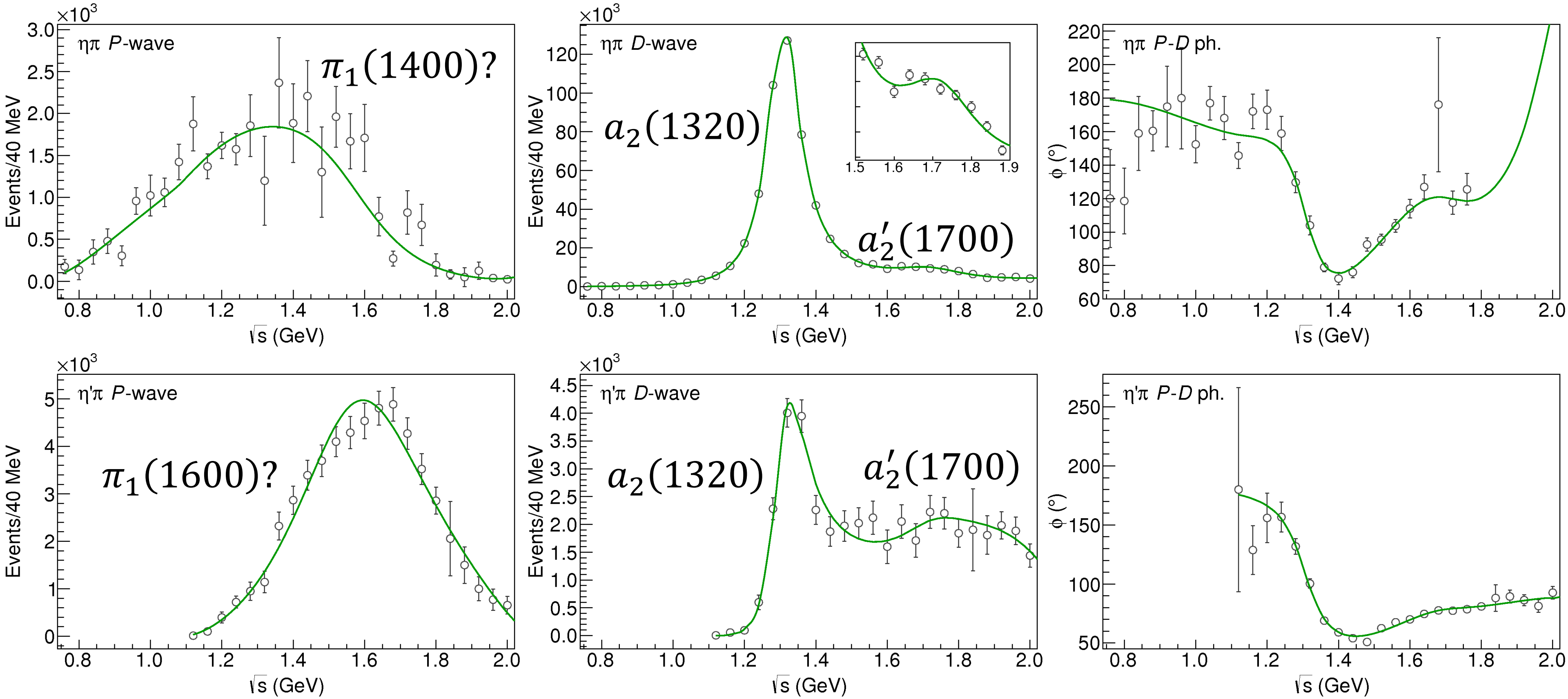


The $n(s)$ → background physics, process-dependent, smooth

Fit to $\eta^{(\prime)}\pi$

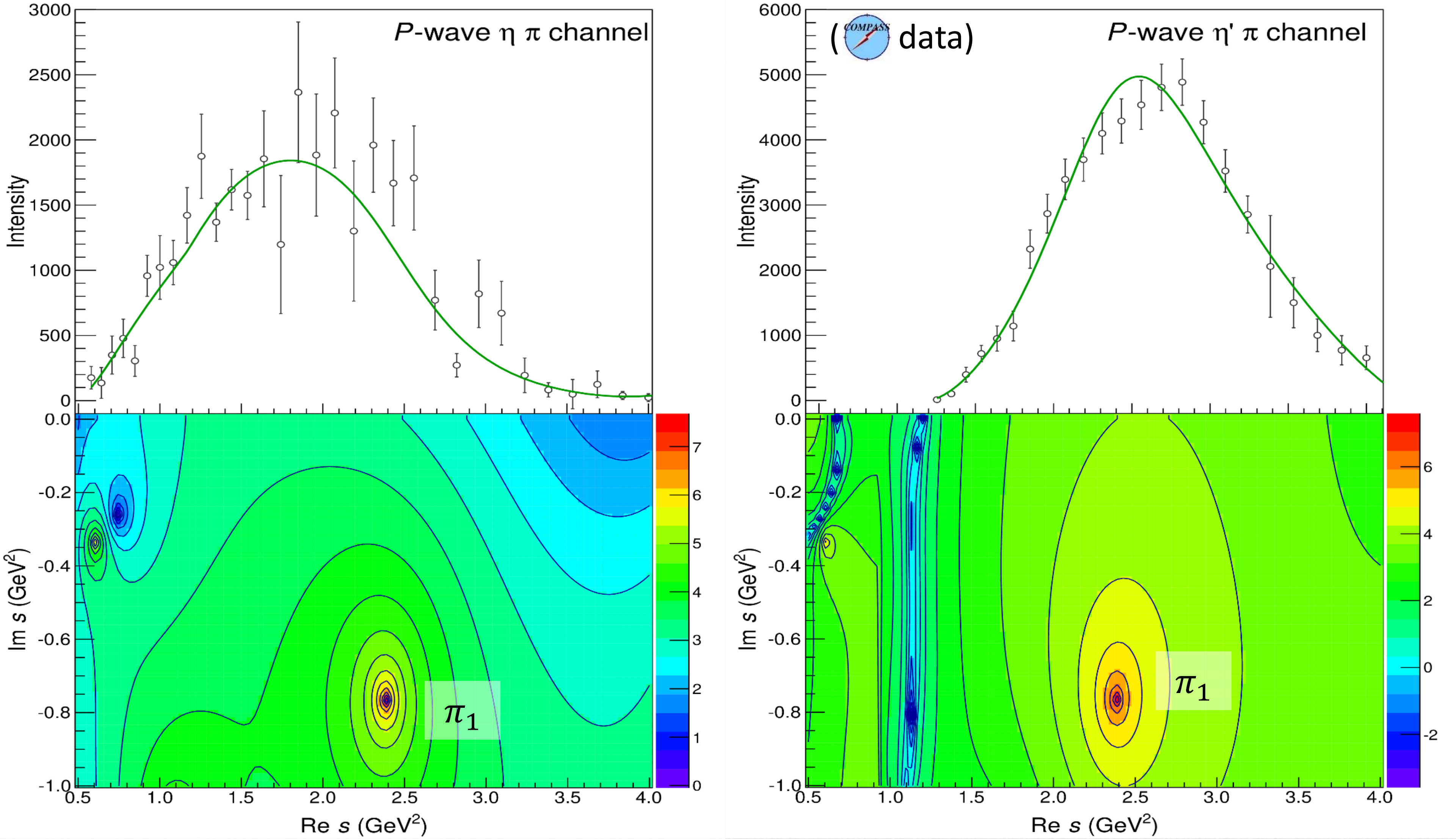
(data)



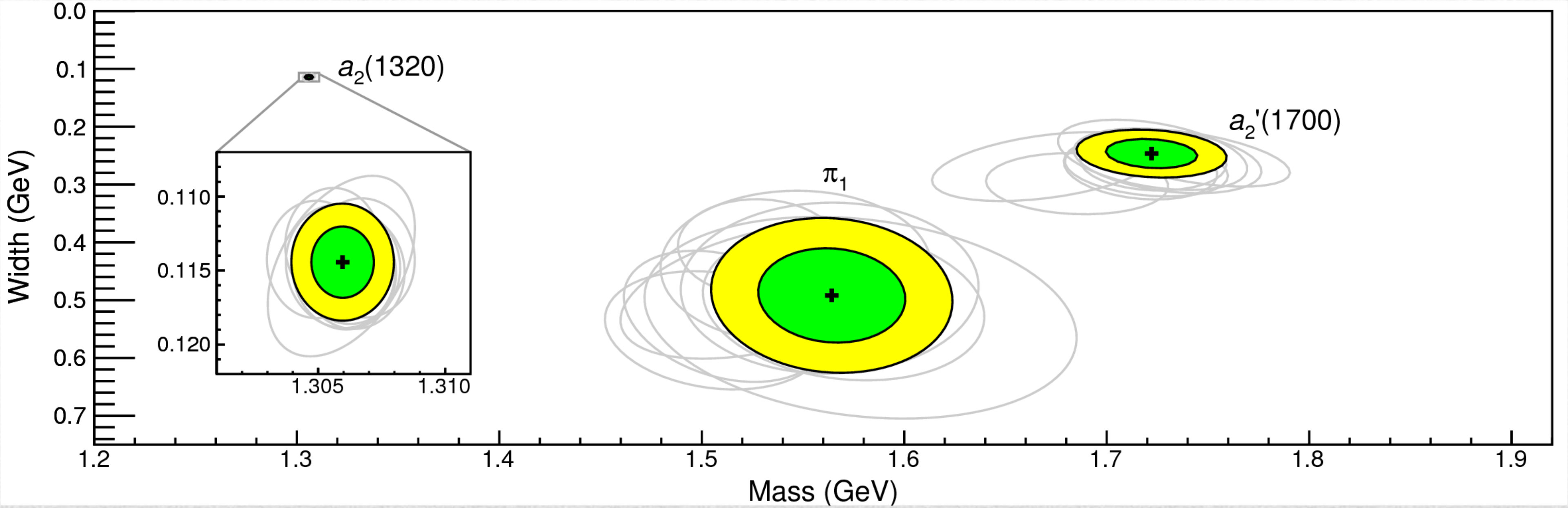
$J^{PC} = 1^{-+}$

$J^{PC} = 2^{++}$

Pole hunting



Final results



Poles	Mass (MeV)	Width (MeV)
$a_2(1320)$	$1306.0 \pm 0.8 \pm 1.3$	$114.4 \pm 1.6 \pm 0.0$
$a_2'(1700)$	$1722 \pm 15 \pm 67$	$247 \pm 17 \pm 63$
π_1	$1564 \pm 24 \pm 86$	$492 \pm 54 \pm 102$

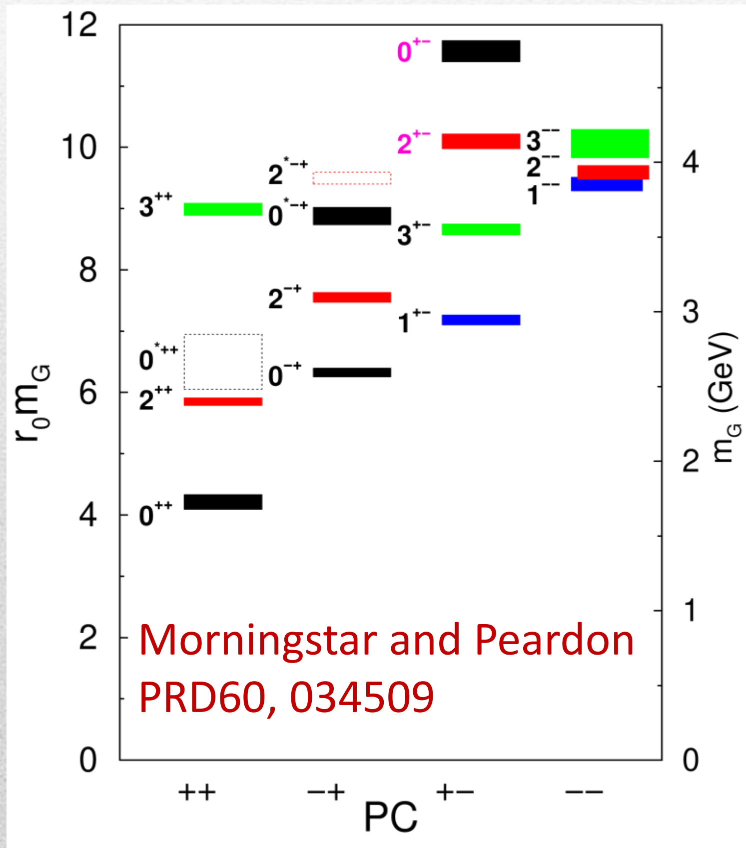
Agreement with Lattice is restored

That's the most rigorous extraction of an exotic meson available so far!

Glueballs

The clearest sign of confinement in pure Yang-Mills

The worst state to search in real life



Morningstar and Peardon PRD60, 034509

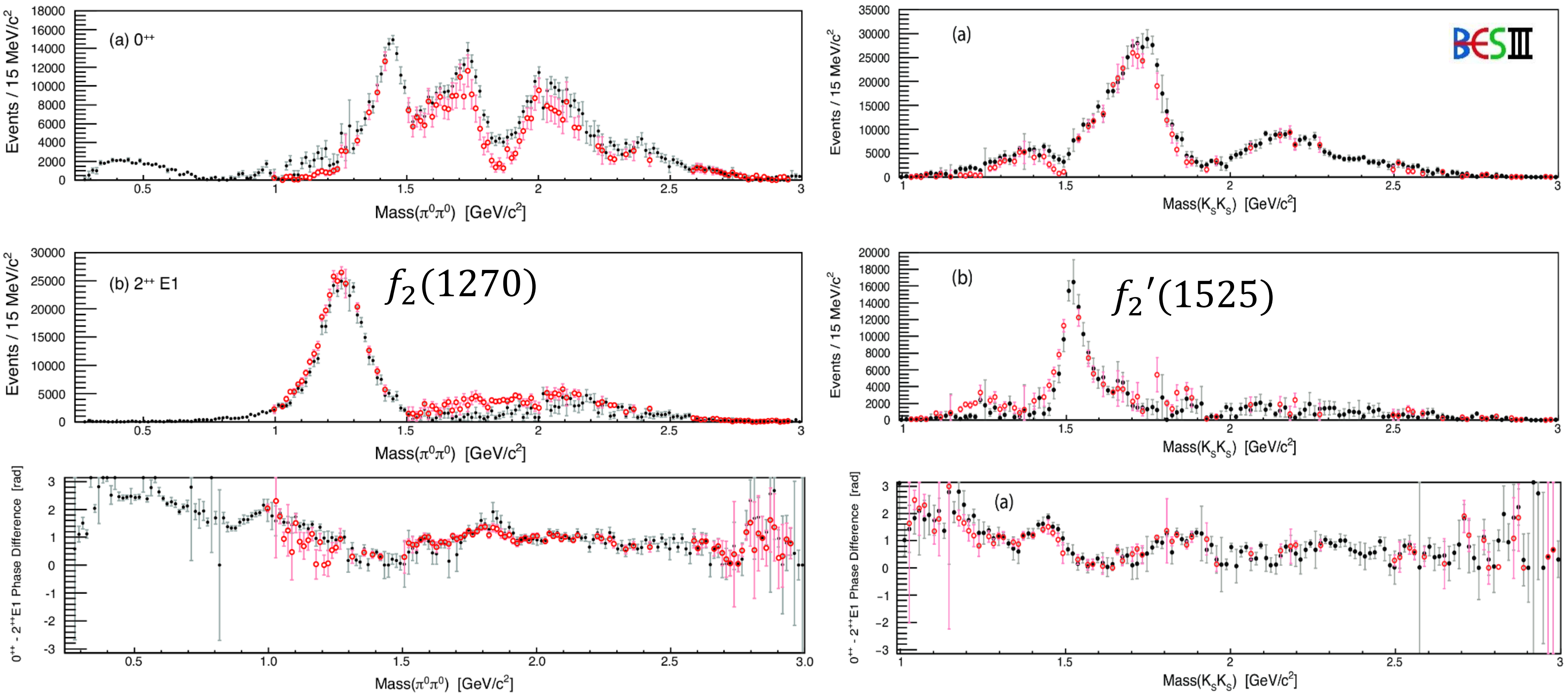
J^{PC}	Mass MeV			
	Unquenched	Quenched		
	This work	M&P	Ky	Meyer
0^{-+}		2590(40)(130)	2560(35)(120)	2250(60)(100)
2^{-+}	3460(320)	3100(30)(150)	3040(40)(150)	2780(50)(130)
0^{-+}	4490(590)	3640(60)(180)		3370(150)(150)
2^{-+}				3480(140)(160)
5^{-+}				3942(160)(180)
0^{--} (exotic)	5166(1000)			
1^{--}		3850(50)(190)	3830(40)(190)	3240(330)(150)
2^{--}	4590(740)	3930(40)(190)	4010(45)(200)	3660(130)(170)
2^{--}				3.740(200)(170)
3^{--}		4130(90)(200)	4200(45)(200)	4330(260)(200)
1^{+-}	3270(340)	2940(30)(140)	2980(30)(140)	2670(65)(120)
3^{+-}	3850(350)	3550(40)(170)	3600(40)(170)	3270(90)(150)
3^{+-}				3630(140)(160)
2^{+-} (exotic)		4140(50)(200)	4230(50)(200)	
0^{+-} (exotic)	5450(830)	4740(70)(230)	4780(60)(230)	
5^{+-}				4110(170)(190)
0^{++}	1795(60)	1730(50)(80)	1710(50)(80)	1475(30)(65)
2^{++}	2620(50)	2400(25)(120)	2390(30)(120)	2150(30)(100)
0^{++}	3760(240)	2670(180)(130)		2755(30)(120)
3^{++}		3690(40)(180)	3670(50)(180)	3385(90)(150)
0^{++}				3370(100)(150)
0^{++}				3990(210)(180)
2^{++}				2880(100)(130)
4^{++}				3640(90)(160)
6^{++}				4360(260)(200)

Gregory *et al.* JHEP1210, 170

$J/\psi \to \gamma\, \pi^0\pi^0$ and $\to \gamma\, K_S^0 K_S^0$

We consider the S and D wave by BESIII to use the information about their relative phase.

The D-wave is populated by two almost elastic resonances: the $f_2(1270)$ and $f_2'(1525)$



Same model as before

A. Rodas, AP *et al.* (JPAC) EPJC82, 1, 80 (2022)

Two/three channels, $i, k = \pi\pi, KK, \rho\rho$ Two waves, $J = S, D$ 40-56 parameters

$$D^J_{ki}(s) = \left[K^J(s)^{-1}\right]_{ki} - \frac{s}{\pi}\int_{s_k}^{\infty} ds' \frac{\rho N^J_{ki}(s')}{s'(s'-s-i\epsilon)}$$

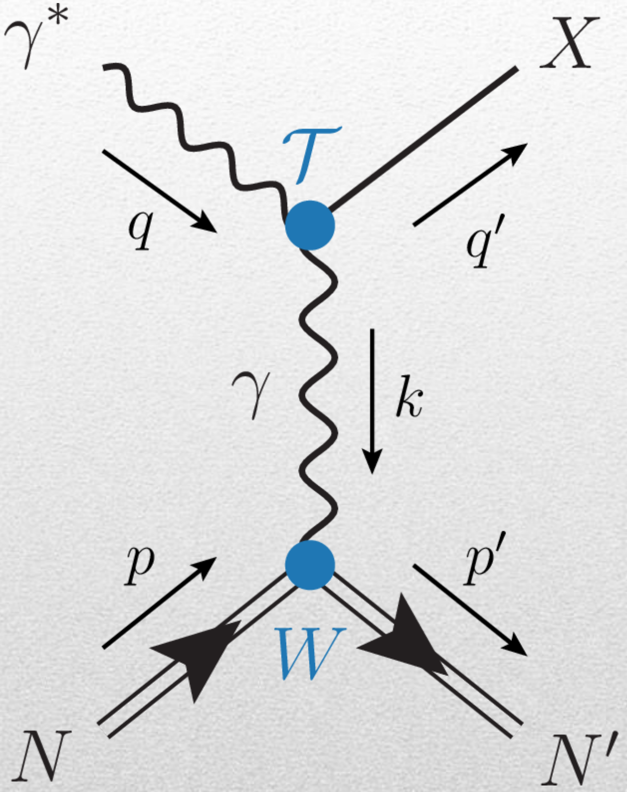
$$K^J_{ki}(s) = \sum_R \frac{g_k^{(R)} g_i^{(R)}}{m_R^2 - s} + c^J_{ki} + d^J_{ki}\, s$$

3 *K*-matrix pole for the S-wave
3 *K*-matrix poles for the D-wave

$$\rho N^J_{ki}(s') = \delta_{ki} \frac{\lambda^{J+1/2}\left(s', m^2_{\eta^{(\prime)}}, m^2_\pi\right)}{(s'+s_R)^{2J+1+\alpha}}$$

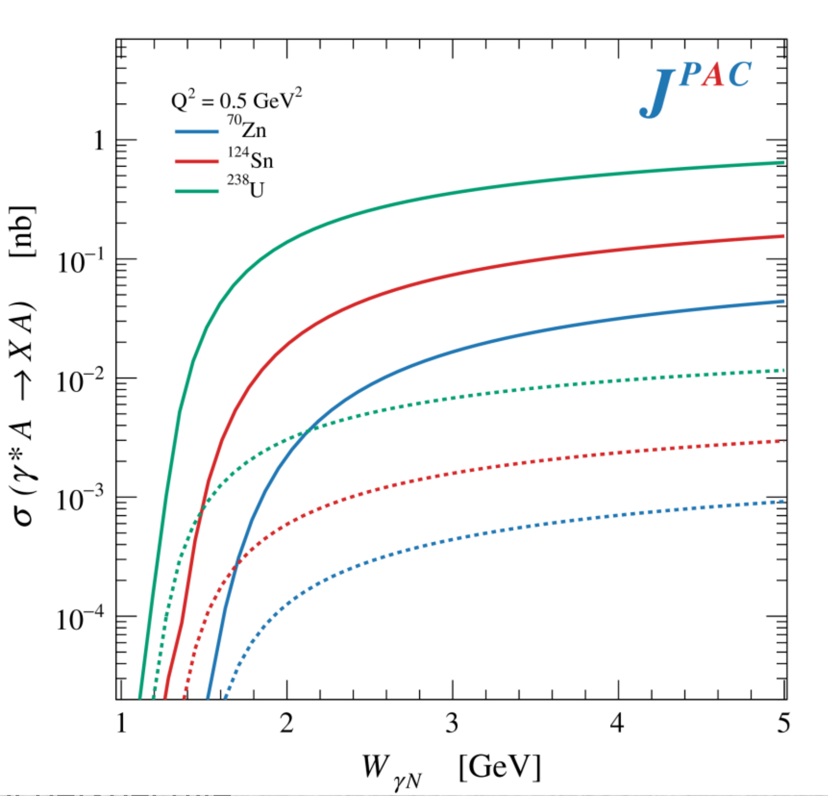
$$n^J_k(s) = \sum_{n=0}^{3} a_n^{J,k}\, T_n\left(\frac{s}{s+s_0}\right)$$

Primakoff X photoproduction



Using measurement of $\Gamma(X \to \gamma\gamma^*)$ from Belle, one can get predictions for Primakoff

Makes use of ion targets, enhancement of cross sections as Z^2



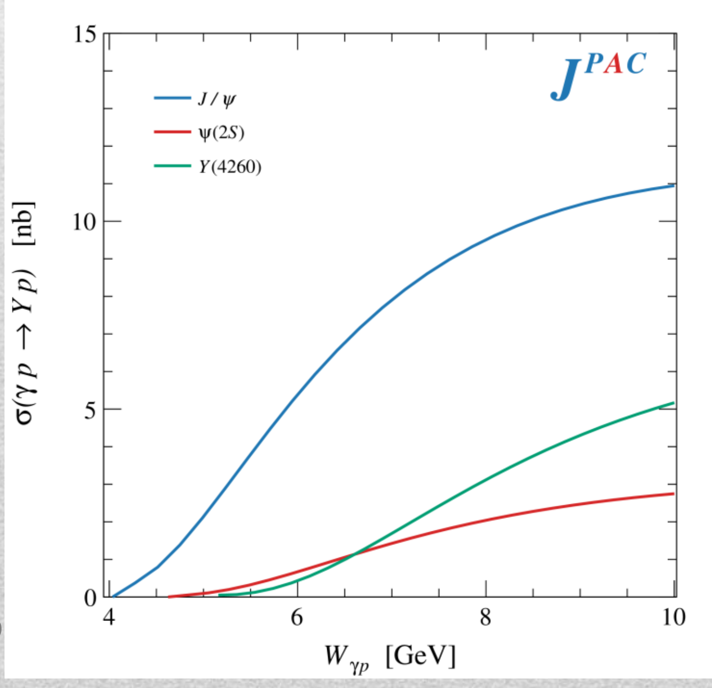
Y (vector) photoproduction

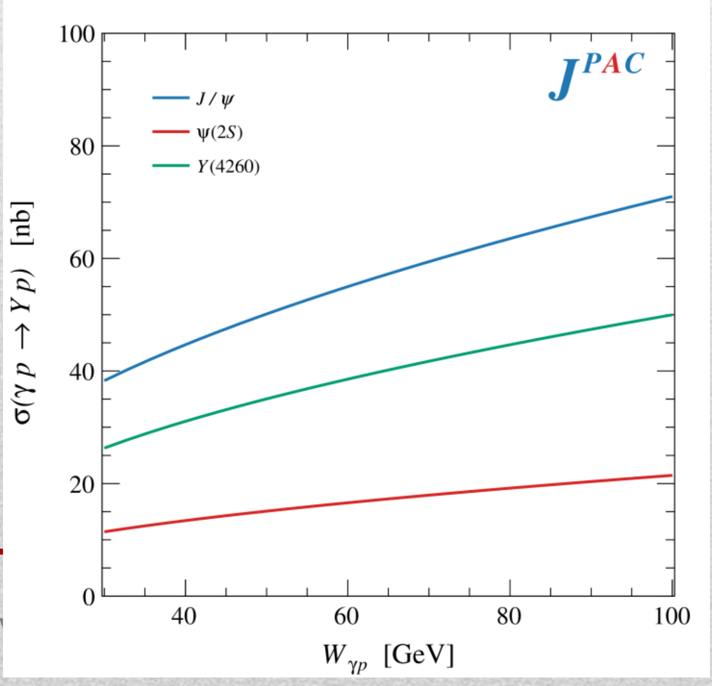
Diffractive production, dominated by Pomeron (2-gluon) exchange

$$R_Y = \frac{e f_\psi}{m_\psi} \sqrt{\frac{g^2(Y \to \psi\pi\pi)}{g^2(\psi \to \gamma g g)} \frac{g^2(\psi' \to \psi g g)}{g^2(\psi' \to \psi\pi\pi)}}$$

Existing data allow to put a 95% upper limit on the ratio of $\psi'/Y(4260)$ yields

Assuming previous formula, one gets:
$\Gamma_{ee}^{Y} = 930\ eV$
(cfr. hep-ex/0603024, 2002.05641)
$BR(Y \to J/\psi\pi\pi) = 0.96\%$
$R_Y = 0.84$





Y (vector) photoproduction

- Focus on the 1^{--} $Y(4260) \to J/\psi\, \pi^+\pi^-$, check with $\psi' \to J/\psi\, \pi^+\pi^-$
- Diffractive production, dominated by Pomeron (2-gluon) exchange
- Good candidates for EIC: diffractive production increases with energy!
- We have $\gamma\psi$-pomeron coupling from our analyses 1606.08912, 1907.09393

How to rescale from J/ψ to ψ' ?

$$R_{\psi'} = \sqrt{\frac{g^2(\psi' \to \gamma g g)}{g^2(\psi \to \gamma g g)}} \sim 0.55$$

$$g^2(\psi \to \gamma g g) = \frac{6 m_\psi \mathcal{B}(\psi \to \gamma g g)\Gamma_\psi}{\mathrm{PS}(\psi \to \gamma g g)}$$

Y (vector) photoproduction

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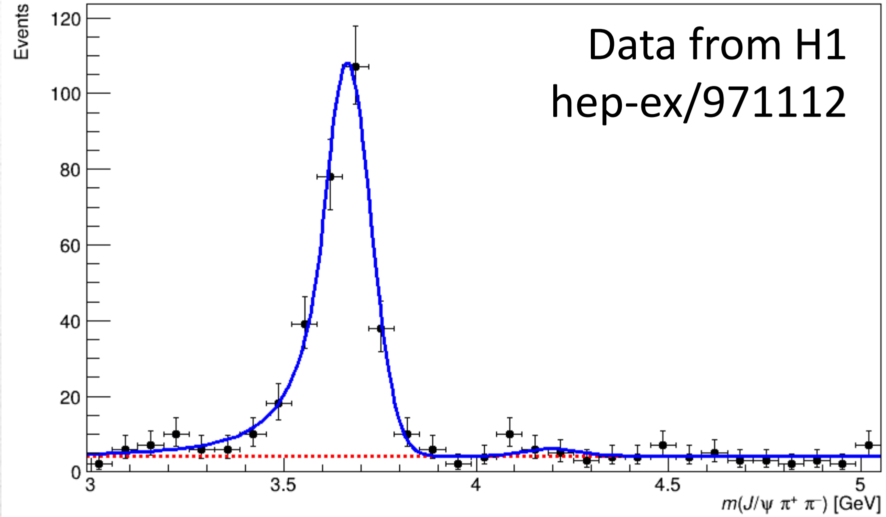
How to rescale from J/ψ to $Y(4260)$?

We assume VMD and $g^2(Y \to \psi\pi\pi) = g^2(Y \to \psi gg) \times g^2(gg \to \pi\pi)$ (Novikov & Shifman)

$$R_Y = \frac{e f_\psi}{m_\psi} \sqrt{\frac{g^2(Y \to \psi\pi\pi)}{g^2(\psi \to \gamma gg)} \frac{g^2(\psi' \to \psi gg)}{g^2(\psi' \to \psi\pi\pi)}}$$

Caveat : $BR(Y \to \psi\pi\pi)$ only known times the leptonic width Γ_{ee}^Y

Y (vector) photoproduction



Existing data allow to put a 95% upper limit on the ratio of $\psi'/Y(4260)$ yields

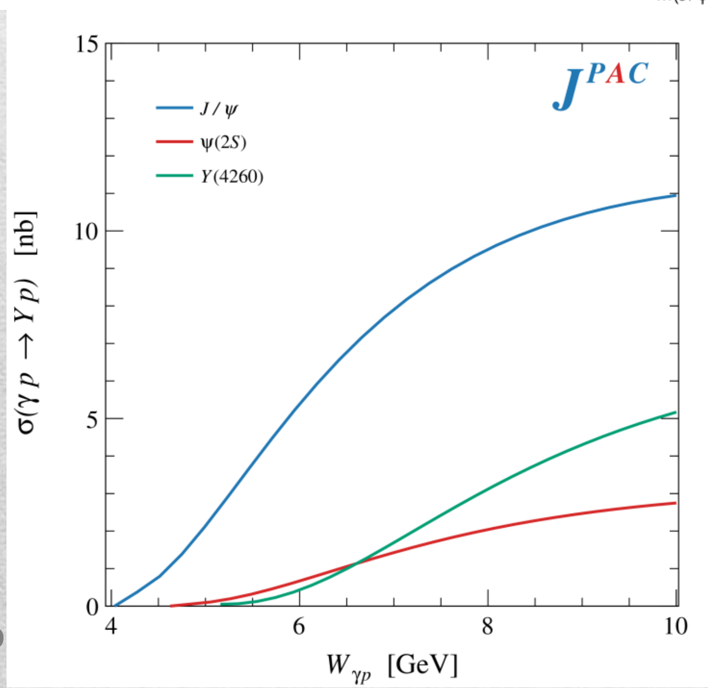
Assuming previous formula, one gets:

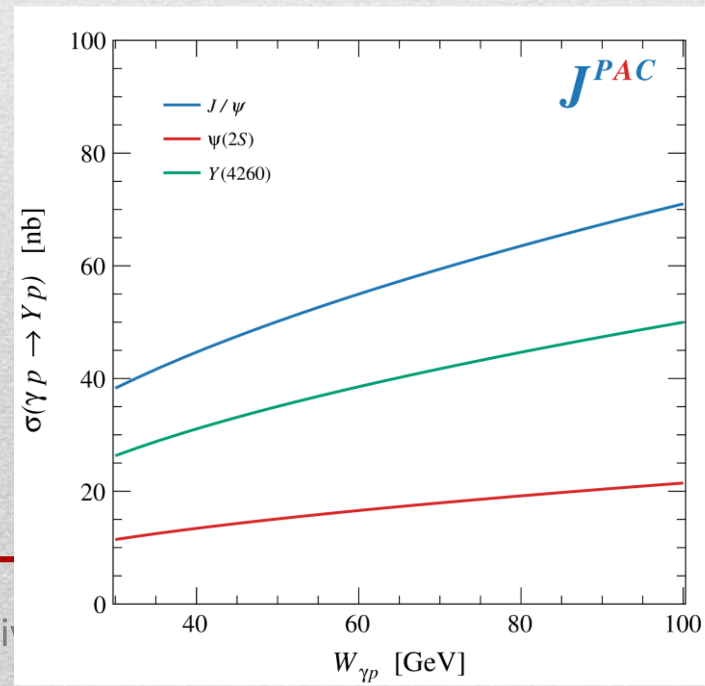
$\Gamma_{ee}^{Y} = 930\ eV$

(cfr. hep-ex/0603024, 2002.05641)

$BR(Y \to J/\psi\pi\pi) = 0.96\%$

$R_Y = 0.84$





Minimal(istic) model for $P_c(4312)$

(LHCb data)

Fernandez-Ramirez, AP *et al.* (JPAC), PRL 123, 092001

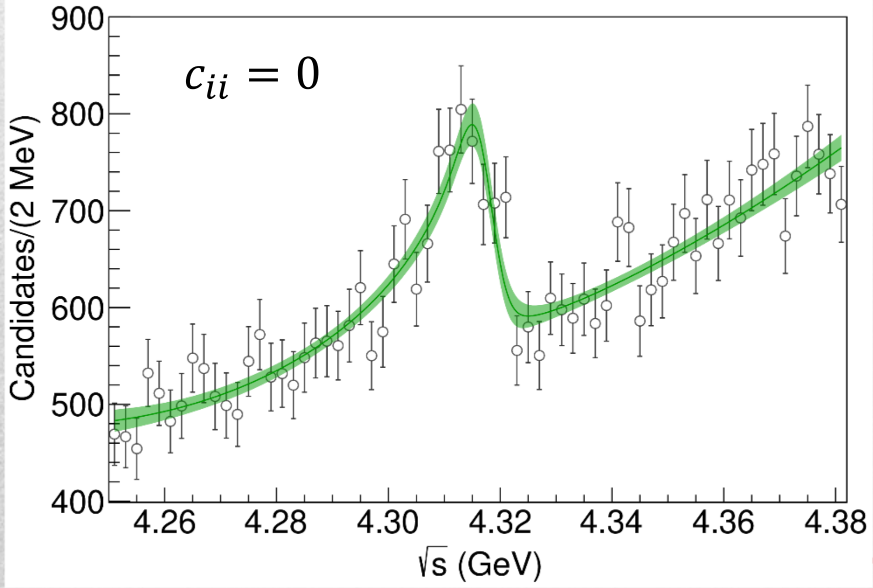
$$\frac{dN}{d\sqrt{s}} = \rho(s)\left[|F(s)|^2 + b_0 + b_1 s\right]$$

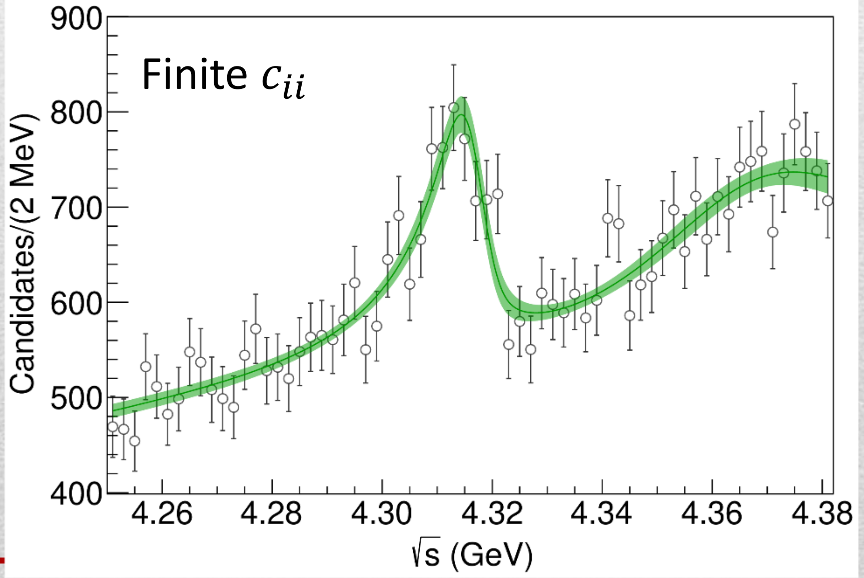
Effective range expansion

$$F(s) = (N_1 + N_2 s)\, T_{11}(s)$$

$$T(s) = \begin{pmatrix} m_{11} - c_{11}s - i\rho_1(s) & m_{12} \\ m_{12} & m_{22} - c_{22}s - i\rho_2(s) \end{pmatrix}^{-1}$$

We can set $c_{ii} = 0$ to reduce to the scattering length approximation





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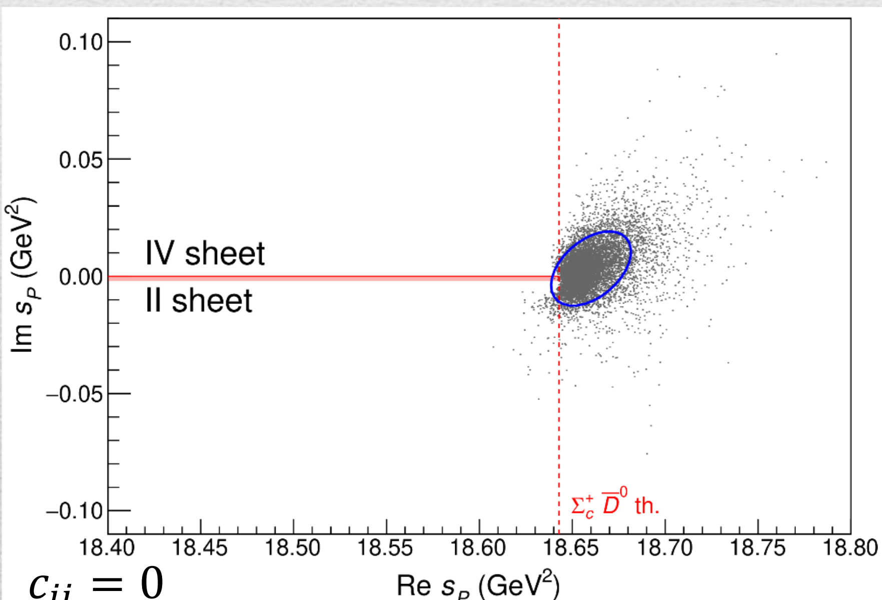

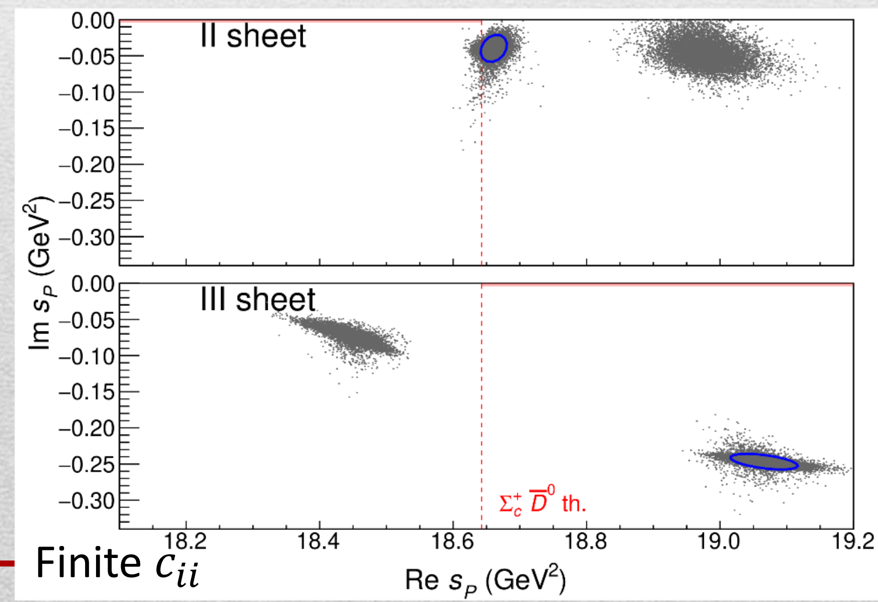

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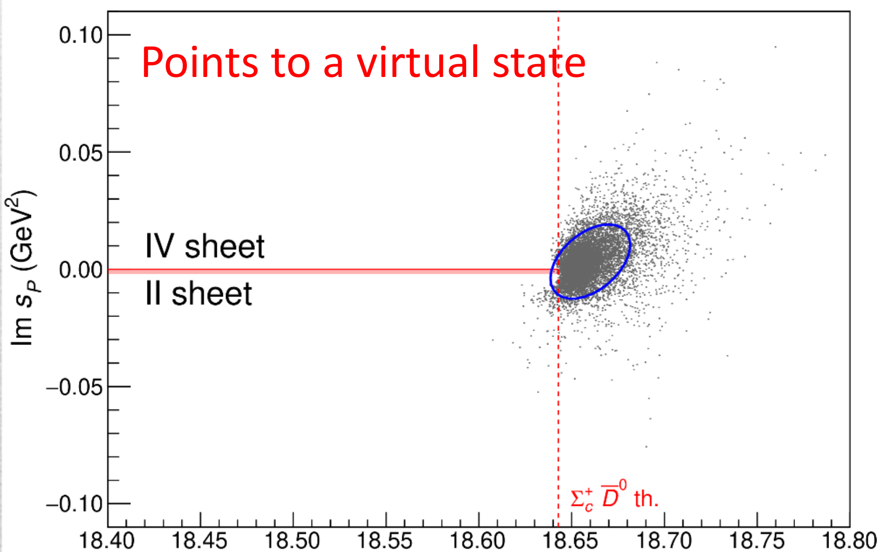
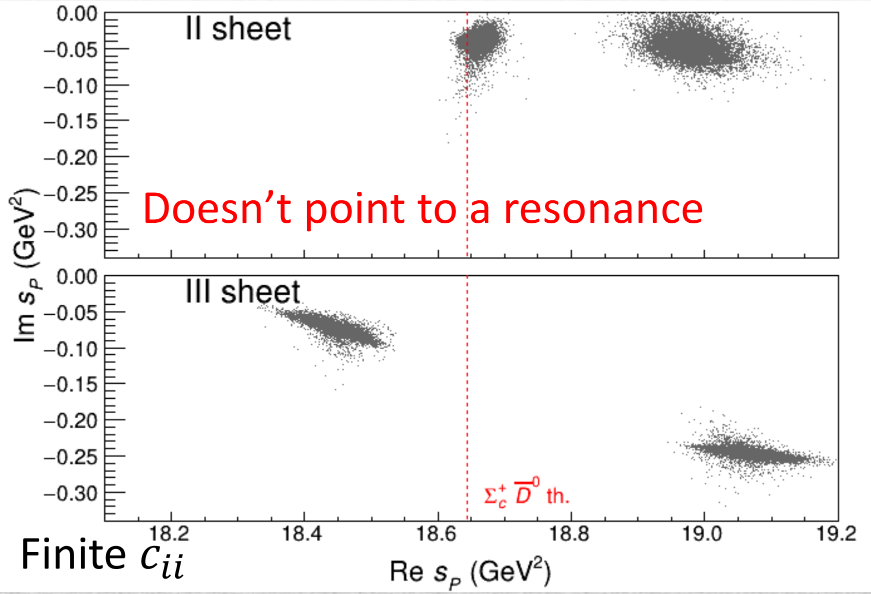


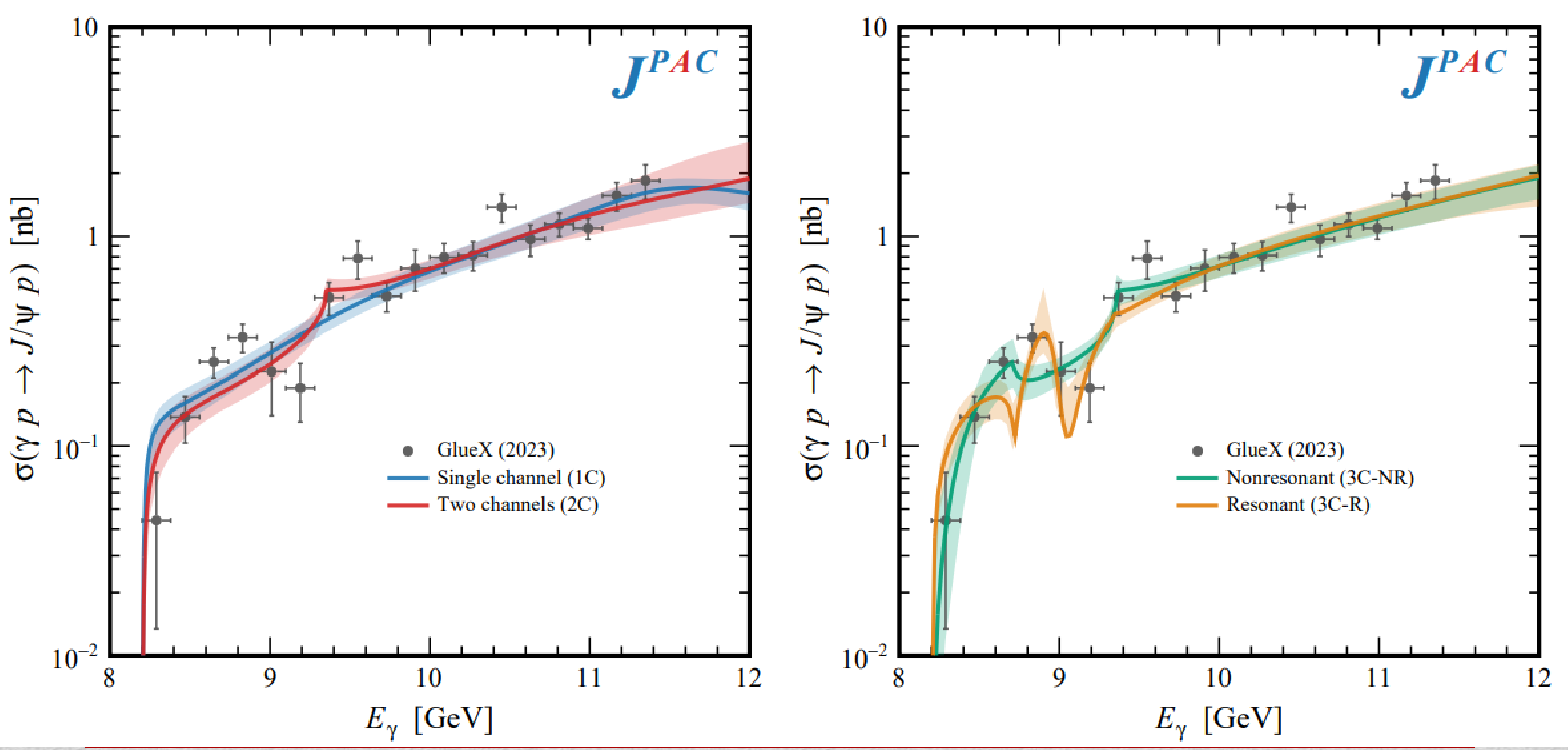
$c_{ii} = 0$



Finite c_{ii}

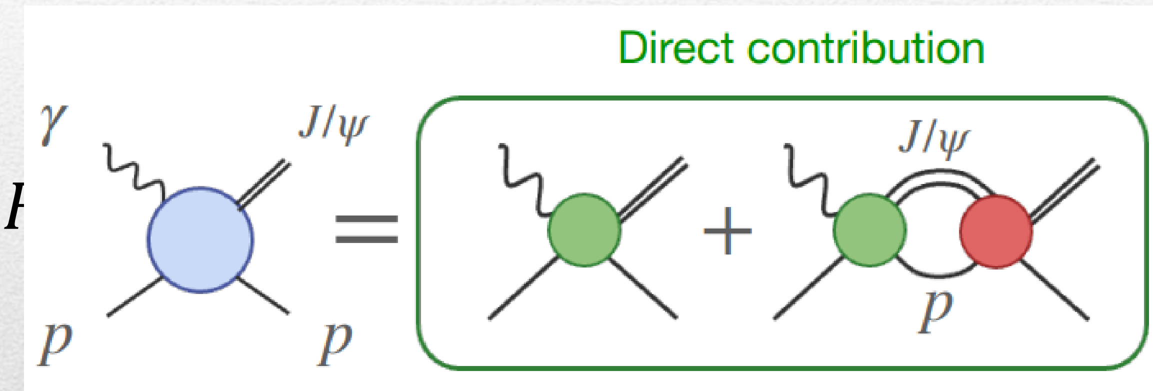
Total cross section

1. **Single channel (1C):** Only interactions involving the $J/\psi\, p$ are included;
2. **Two channels (2C):** We include contributions from an intermediate $\bar{D}^*\Lambda_c$ channel;[2]
3. **Three channels (3C):** We include both $\bar{D}^{(*)}\Lambda_c$ channels. In this case we find two classes of solutions which we discuss separately below.



Vector Meson Dominance

Since unitary model parametrize separately the production and scattering amplitude, one can compare with the predictions of VMD



ψ

p

$$F_\ell(s) = f_\ell\,(1 + G\,T_\ell)$$

But $G(s_{\text{th}}) = 0$

The value of the scattering amplitude at threshold is called scattering length
According to VMD, a small photoproduction cross sections implies a small scattering length O(1am)

With the unitary model, the value of the amplitude at threshold is an unrelated parameter, the scattering length enters with the energy dependence

$$\text{VMD: } F^{\psi p}(s_{\text{th}}, x) = -8\pi\sqrt{s_{\text{th}}}\, g_{\gamma\psi}\, a_{\psi p}$$

$$\text{1C: } F^{\psi p}(s \to s_{\text{th}}, \theta) = n_S^{\psi p}\,(1 - i\,q\,a_{\psi p})$$

Vector Meson Dominance

$$R_{\text{VMD}}(x) = \left| \frac{F^{\psi p}(s_{\text{th}}, x)/g_{\gamma\psi}}{T^{\psi p,\psi p}(s_{\text{th}}, x)} \right|$$

VMD badly excluded, except for the poorly constrained 3C-R model

Scattering lengths generally of O(1fm), but smaller ones are not excluded

Crucial to constrain better these fits by measuring open charm final states

	1C	**2C**	**3C-NR**	**3C-R**
Parameters	9	13	15	15
χ^2	166	144	141	143
χ^2/dof	1.25	1.12	1.11	1.13
ζ_{th}	1	$[0.56,\ 0.74]$	$[0.36,\ 0.63]$	$[0.03,\ 0.62]$
$R_{\text{VMD}}(\theta = 0)$	$[0.45,\ 0.73] \times 10^{-2}$	$[0.39,\ 1.62] \times 10^{-2}$	$[0.03,\ 1.74] \times 10^{-2}$	$[1.4 \times 10^{-2}, 0.58]$
$R_{\text{VMD}}(t = 0)$	$[1.3,\ 2.0] \times 10^{-2}$	$[1.3,\ 5.1] \times 10^{-2}$	$[0.08,\ 8.9] \times 10^{-2}$	$[5.4 \times 10^{-2}, 1.8]$
$a_{\psi p}$ [fm]	$[0.56,\ 1.00]$	$[0.11,\ 0.79]$	$[-2.77,\ 0.35]$	$[-0.04,\ 0.19]$