Gravitational Form Factors and Mechanical Properties of a Dressed Quark

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Gravitational Form Factors for the Nucleon

$$\langle P', S'|T_i^{\mu\nu}(0)|P,S\rangle = \bar{U}(P',S')\Big[-B_i(q^2)\frac{\bar{P}^\mu\bar{P}^\nu}{M} + (A_i(q^2)+B_i(q^2))\frac{1}{2}(\gamma^\mu\bar{P}^\nu + \gamma^\nu\bar{P}^\mu)$$
$$+ C_i(q^2)\frac{q^\mu q^\nu - q^2 g^{\mu\nu}}{M} + \bar{C}_i(q^2)M g^{\mu\nu}\Big]U(P,S),$$

$$\bar{P}^\mu = \tfrac{1}{2}(P'+P)^\mu,\ q^\mu = (P'-P)^\mu$$

$$P = (P^+, P_\perp, P^-) = \left(P^+, 0, \frac{M^2}{P^+}\right),$$
$$P' = (P'^+, P'_\perp, P'^-) = \left(P^+, q_\perp, \frac{q_\perp^2 + M^2}{P^+}\right)$$
$$q = P' - P = \left(0, q_\perp, \frac{q_\perp^2}{P^+}\right),$$

We choose Drell-Yan frame $Q^2 = -q^2 = \vec{q}_\perp^{\,2}$

GFFs give how matter couples to gravity

$A(Q^2)$ and $B(Q^2)$ are related to the mass and angular momentum of the proton

$$\int dx x[H_q(x,0,0) + E_q(x,0,0)] = A_q(0) + B_q(0) = 2J_q$$

X. Ji, PRD, 1997

H_q and E_q are generalized parton distributions (GPDs) that can be accessed in exclusive processes like DVCS or deeply virtual meson production

Gravitational Form Factors

Poincare invariance imposes the constraint :

$$\sum_{a=q,g} A_a(0) = 1, \quad \sum_{a=q,g} B_a(0) = 0, \quad \sum_{a=q,g} \bar{C}_a(t) = 0.$$

Total gravitomagnetic moment is zero follows from the equivalence principle of GTR

$\bar{C}(Q^2)$ arises due to non-conservation of EM tensor separately for quarks and gluons, and must vanish when summed over both

Lorce, Moutarde, Trawinski, EPJC (2019)

However, $C(Q^2)$, also called the D-term, is not related to any Poincare generator and is unconstrained

D term is related to the pressure and shear force distributions inside the nucleon

Polyakov and Schweitzer, IJMPA (2018)

Recent result from Jlab showed that the pressure distribution at the center of the nucleon is repulsive and confining towards the outer region

Burkert, Elouadrhiri, Girod, Nature(2018)

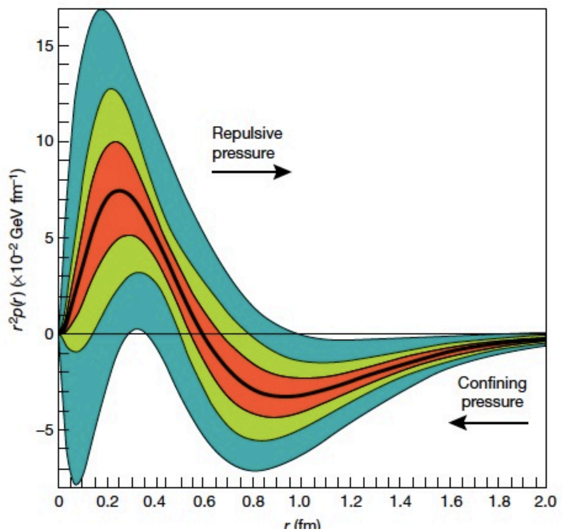
This also connects a set of collider observables (GPDs) to the investigation of the equation of state (EoS) of neutron stars

Rajan, Gorda, Liuti, Yagi (2018)

Quite a lot of theoretical calculations in recent days

Chakrabarti, Mondal, Mukherjee, Nair, Zhao; PRD(2020)

Pressure and shear distributions inside the nucleon

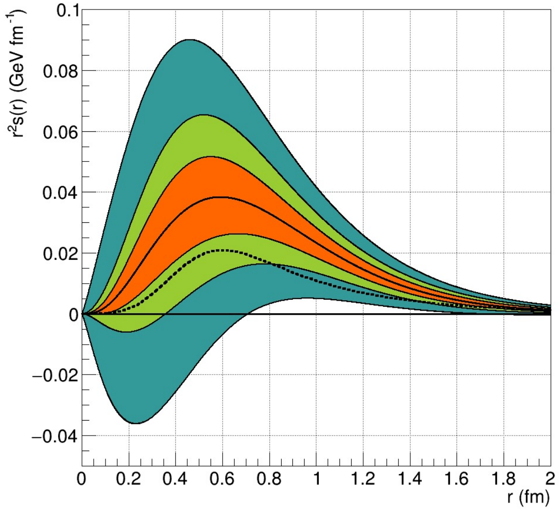


Pressure distribution obtained from fits to Jlab data to extract the GPDs, in particular the D-term

Pressure distribution is repulsive at the center of the nucleon and confining in the outer region

At the core it exceeds the pressure density of the most dense object that is neutron star , average peak pressure 10^{35} Pascals

Burkert, Elouadrhiri, Girod, Nature(2018)



Shear (tangential) force inside the nucleon from DVCS data at JLab

Maximum shear force at 0.6 fm from the center of the nucleon : confinement may be dominant

Shear forces change direction at r=0.45 fm from the center

Burkert, Elouadrhiri, Girod, 2104.02031[nucl-ex]

Quark state dressed with a gluon

$A(Q^2)$ and $B(Q^2)$ involves the 'good' components of the energy momentum tensor. But $C(Q^2)$ and $\bar{C}(Q^2)$

involve 'bad' components with the interaction terms

Instead of a proton state we take a simpler state with a gluon degree of freedom : a dressed quark

State is expanded in Fock space in terms of multiparton light-front wave functions (LFWFs) : two particle LFWF consists of a quark and a gluon

Fully relativistic spin -1/2 composite state that incorporates a gluonic degree of freedom

Two-particle LFWF can be calculated analytically using light front Hamiltonian perturbation theory

In order to calculate the interaction terms we use the two component formalism in light cone gauge

Fermionic field is decomposed as $\psi = \psi_+ + \psi_-,$ $\psi_\pm = \Lambda_\pm \psi$

In light cone gauge one uses a particular representation of gamma matrices so that

$$\psi_+ = \begin{bmatrix} \xi \\ 0 \end{bmatrix}, \qquad \psi_- = \begin{bmatrix} 0 \\ \eta \end{bmatrix},$$

ξ and **η** are two component fields

W. M . Zhang and A. Harindranath, PRD 48,4881 (1993)

$$\eta(y) = \left(\frac{1}{i\partial^+}\right)[\sigma^\perp \cdot (i\partial^\perp + gA^\perp(y)) + im]\xi(y),$$

Constrained field, can be eliminated using the equation of constraint

Quark state dressed with a gluon

Use two-component formalism in light-front Hamiltonian QCD : state expanded in Fock space in terms of multi-parton light-front wave functions (LEWFs)

$$|p,\sigma\rangle = \psi_1(p,\sigma) b^\dagger_\sigma(p)|0\rangle + \sum_{\lambda_1,\lambda_2}\int \frac{dk_1^+ d^2k_1^\perp dk_2^+ d^2k_2^\perp}{(16\pi^3)\sqrt{k_1^+ k_2^+}}$$
$$\times \sqrt{16\pi^3 p^+}\psi_2(p,\sigma|k_1,\lambda_1;k_2,\lambda_2)$$
$$\times \delta^{(3)}(p-k_1-k_2) b^\dagger_{\lambda_1}(k_1) a^\dagger_{\lambda_2}(k_2)|0\rangle.$$

Expansion can be truncated upto two-particle sector in a Boost invariant way, boost invariant LFWF :

$$\phi^{\sigma a}_{\lambda_1,\lambda_2}(x_i,\boldsymbol{\kappa}_i^\perp) = \sqrt{P^+}\psi_2(P,\sigma|k_1,\lambda_1;k_2,\lambda_2),$$

$$k_i^+ = x_i p^+, \qquad \boldsymbol{\kappa}_i^\perp = \boldsymbol{\kappa}_i^\perp + x_i \boldsymbol{p}^\perp,$$

$$\phi^{\sigma a}_{\lambda_1,\lambda_2}(x,\boldsymbol{\kappa}^\perp) = \frac{g}{\sqrt{2(2\pi)^3}}\left[\frac{x(1-x)}{\boldsymbol{\kappa}_\perp^2 + m^2(1-x)^2}\right]\frac{T^a}{\sqrt{1-x}}$$
$$\times \chi^\dagger_{\lambda_1}\left[-\frac{2(\boldsymbol{\kappa}^\perp\cdot\boldsymbol{\epsilon}^{\perp *}_{\lambda_2})}{1-x} - \frac{1}{x}(\tilde{\sigma}^\perp\cdot\boldsymbol{\kappa}^\perp)(\tilde{\sigma}^\perp\cdot\boldsymbol{\epsilon}^{\perp *}_{\lambda_2}) + im(\tilde{\sigma}^\perp\cdot\boldsymbol{\epsilon}^{\perp *}_{\lambda_2})\frac{1-x}{x}\right]\chi_\sigma\psi_1^\sigma,$$

Used two-component formalism of light-front QCD

Eliminated the constrained dof in light front gauge $A^+=0$

Two-particle LFWF can be obtained analytically using light-front eigenvalue equation

Zhang and Harindranath, PRD 48,4881 (1993)

GFFs for a dressed quark

$$\theta^{\mu\nu} = \frac{1}{2}\bar{\psi} i[\gamma^\mu D^\nu + \gamma^\nu D^\mu]\psi - F^{\mu\lambda a} F^{\nu}_{\lambda a} + \frac{1}{4} g^{\mu\nu} (F_{\lambda\sigma a})^2 - g^{\mu\nu}\bar{\psi}(i\gamma^\lambda D_\lambda - m)\psi.$$

GFFs are extracted by calculating the matrix elements of the energy momentum tensor

$$\mathcal{M}^{\mu\nu}_{SS'} = \frac{1}{2}[\langle P', S'|\theta^{\mu\nu}_{\mathrm{Q}}(0)|P, S\rangle]$$

Similarly for the gluon part

$$\mathcal{M}^{++}_{\uparrow\uparrow} + \mathcal{M}^{++}_{\downarrow\downarrow} = 2(P^+)^2 A_{\mathrm{Q}}(q^2),$$

$$\mathcal{M}^{++}_{\uparrow\downarrow} + \mathcal{M}^{++}_{\downarrow\uparrow} = \frac{iq^{(2)}}{M}(P^+)^2 B_{\mathrm{Q}}(q^2).$$

$$\mathcal{M}^{11}_{\uparrow\downarrow} + \mathcal{M}^{11}_{\downarrow\uparrow} + \mathcal{M}^{22}_{\uparrow\downarrow} + \mathcal{M}^{22}_{\downarrow\uparrow} = i\left[B_{\mathrm{Q}}(q^2)\frac{q^2}{4M} - C_{\mathrm{Q}}(q^2)\frac{3q^2}{M} + \bar{C}_{\mathrm{Q}}(q^2)2M\right]q^{(2)}.$$

$$\mathcal{M}^{11}_{\uparrow\downarrow} + \mathcal{M}^{11}_{\downarrow\uparrow} - \mathcal{M}^{22}_{\uparrow\downarrow} - \mathcal{M}^{22}_{\downarrow\uparrow} = i\left[\frac{B_{\mathrm{Q}}(q^2)}{4M} - \frac{C_Q(q^2)}{M}\right]((q^{(1)})^2 q^{(2)} - (q^{(2)})^3),$$

$$A_{\mathrm{Q}}(q^2) = 1 + \frac{g^2 C_F}{2\pi^2}\left[\frac{11}{10} - \frac{4}{5}\left(1 + \frac{2m^2}{q^2}\right)\frac{f_2}{f_1} - \frac{1}{3}\log\left(\frac{\Lambda^2}{m^2}\right)\right]$$

$$B_{\mathrm{Q}}(q^2) = \frac{g^2 C_F}{12\pi^2}\frac{m^2}{q^2}\frac{f_2}{f_1},$$

$$D_{\mathrm{Q}}(q^2) = \frac{5g^2 C_F}{6\pi^2}\frac{m^2}{q^2}(1 - f_1 f_2) = 4C_{\mathrm{Q}}(q^2),$$

$$\bar{C}_{\mathrm{Q}}(q^2) = \frac{g^2 C_F}{72\pi^2}\left(29 - 30 f_1 f_2 + 3\log\left(\frac{\Lambda^2}{m^2}\right)\right)$$

$$f_1 := \frac{1}{2}\sqrt{1 + \frac{4m^2}{q^2}},$$

$$f_2 := \log\left(1 + \frac{q^2(1 + 2f_1)}{2m^2}\right).$$

Contribution from the gluon part of the EMT

$$\theta^{\mu\nu} = \theta_Q^{\mu\nu} + \theta_G^{\mu\nu},$$

$$\theta_Q^{\mu\nu} = \frac{1}{2}\overline{\psi} i[\gamma^\mu D^\nu + \gamma^\nu D^\mu]\psi - g^{\mu\nu}\overline{\psi}(i\gamma^\lambda D_\lambda - m)\psi,$$

$$\theta_G^{\mu\nu} = -F^{\mu\lambda a}F^\nu_{\lambda a} + \frac{1}{4}g^{\mu\nu}(F_{\lambda\sigma a})^2.$$

$$\tilde{f}_1 := \sqrt{1 + \frac{4m^2x^2}{q^2(1-x)^2}},$$

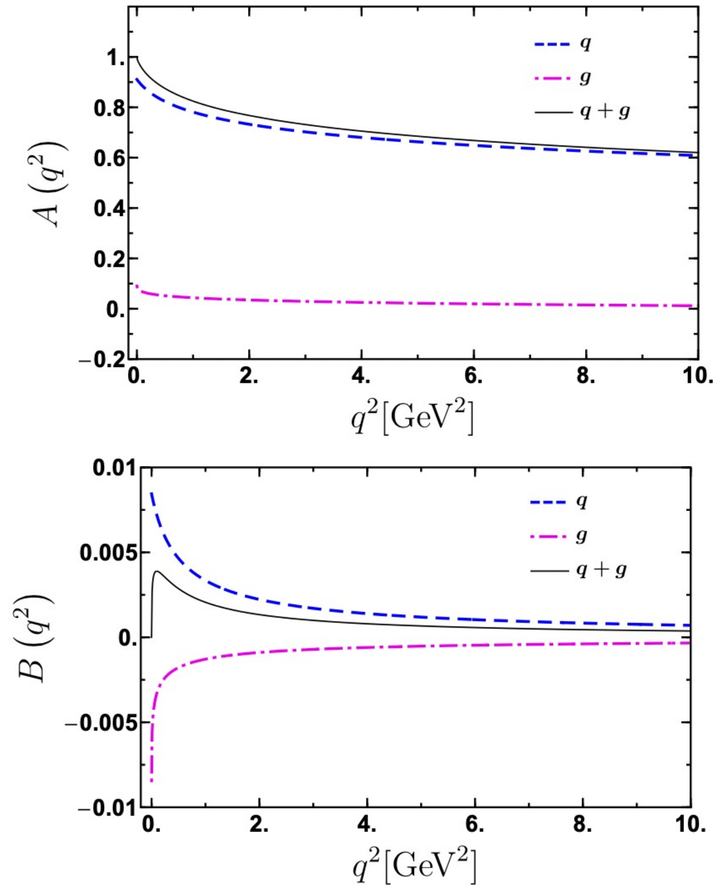
$$\tilde{f}_2 := \ln\left(\frac{1+\tilde{f}_1}{-1+\tilde{f}_1}\right).$$

$$A_G(q^2) = \frac{g^2 C_F}{8\pi^2}\left[\frac{29}{9} + \frac{4}{3}\ln\left(\frac{\Lambda^2}{m^2}\right) - \int dx\left(\left(1+(1-x)^2\right) + \frac{4m^2x^2}{q^2(1-x)}\right)\frac{\tilde{f}_2}{\tilde{f}_1}\right],$$

$$B_G(q^2) = -\frac{g^2 C_F}{2\pi^2}\int dx \frac{m^2x^2}{q^2}\frac{\tilde{f}_2}{\tilde{f}_1},$$

$$D_G(q^2) = \frac{g^2 C_F}{6\pi^2}\left[\frac{2m^2}{3q^2} + \int dx \frac{m^2}{q^4}(x((2-x)q^2 - 4m^2x))\right]\frac{\tilde{f}_2}{\tilde{f}_1},$$

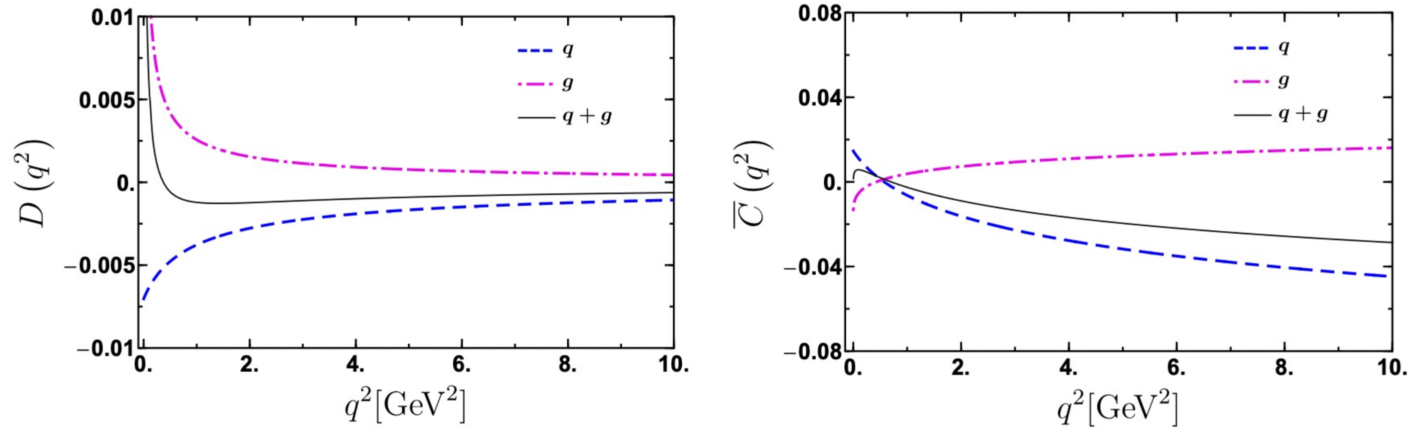
$$\overline{C}_G(q^2) = \frac{g^2 C_F}{72\pi^2}\left[10 + 9\int dx\left(x - \frac{4m^2x^2}{q^2(1-x)}\right)\frac{\tilde{f}_2}{\tilde{f}_1} - 3\ln\left(\frac{\Lambda^2}{m^2}\right)\right],$$



Cutoff dependence get canceled in q+g total GFF

Satisfy $\sum_{a=q,g} A_a(0) = 1, \quad \sum_{a=q,g} B_a(0) = 0,$

Numerical results of GFFs



Plots of the GFF $D(q^2)$ and $\overline{C}(q^2)$ as a function of q^2. The dashed blue curve and the dot-dashed magenta curve are for the quark (q) and gluon (g) form factors respectively. The solid black curve is for the sum of quark and gluon ($q+g$) contribution. Here $m = 0.3$ GeV and $\Lambda = 2$ GeV.

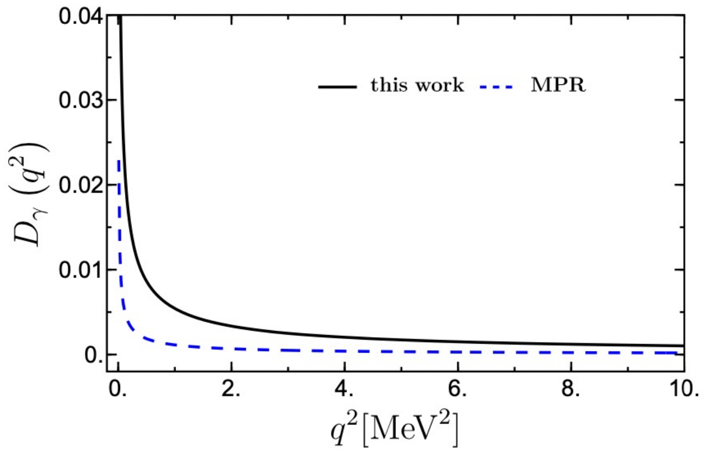
More, AM, Nair, Saha ; PRD 105, 056017 (2022); PRD 107, 116005 (2023)

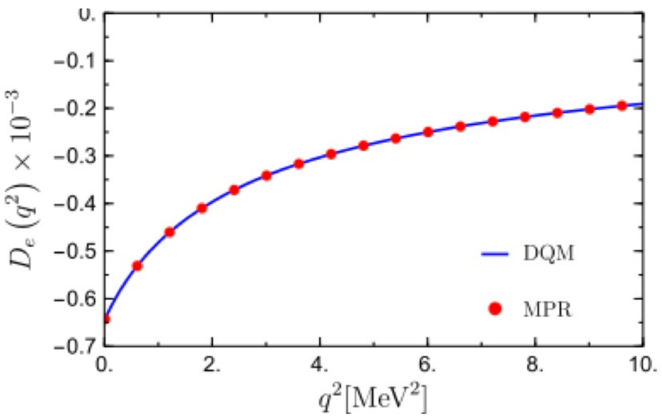
Total D-term negative except low Q^2 region, as expected in a bound system

Total $\bar{C}(Q^2)$ Not zero except at q^2 =0 : zero modes ?

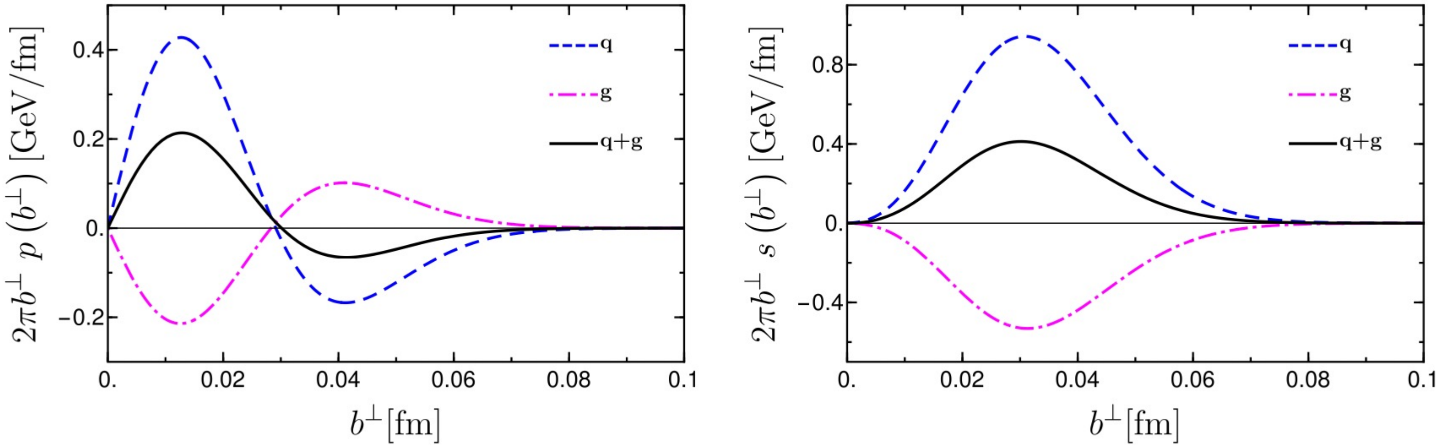
Comparison of D-term in QED with Metz, Pasquini, Rodini, PLB 820, 136501 (2021)

Photon D-term divergent due to long range Coulomb force





Pressure and shear distributions



Plots of the pressure distribution $2\pi b^{\perp} p(b^{\perp})$ and the shear force distribution $2\pi b^{\perp} s(b^{\perp})$ as a function of $b^{\perp}$. The dashed blue curve and the dot-dashed magenta curve are for the quark (q) and gluon (g) contributions respectively. The solid black curve is for the sum of quark and gluon ($q+g$) contribution. Here $\Delta = 0.2$.

$$p_i(b^{\perp}) = \frac{1}{8mb^{\perp}}\frac{d}{db^{\perp}}\left[b^{\perp}\frac{d}{db^{\perp}}D_i(b^{\perp})\right] - m\overline{C}_i(b^{\perp}),$$

$$s_i(b^{\perp}) = -\frac{b^{\perp}}{4m}\frac{d}{db^{\perp}}\left[\frac{1}{b^{\perp}}\frac{d}{db^{\perp}}D_i(b^{\perp})\right],$$

$$F(b^{\perp}) = \frac{1}{(2\pi)^2}\int d^2\boldsymbol{q}^{\perp} e^{-i\boldsymbol{q}^{\perp}\boldsymbol{b}^{\perp}}\mathcal{F}(q^2) = \frac{1}{2\pi}\int_0^{\infty} dq^{\perp}q^{\perp}J_0(q^{\perp}b^{\perp})\mathcal{F}(q^2),$$

We have taken a Gaussian wave packet

$$\frac{1}{16\pi^3}\int \frac{d^2\boldsymbol{p}^{\perp}dp^{+}}{p^{+}}\phi(p)|p^{+},\boldsymbol{p}^{\perp},\lambda\rangle,$$

$$\phi(p) = p^{+}\delta(p^{+}-p_0^{+})\phi(p^{\perp}) \quad \phi(p^{\perp}) = e^{-\frac{p^{\perp 2}}{2\Delta^2}},$$

Qualitative behaviour of q+g pressure distribution similar to that from Jlab data : repulsive core and attractive in the outer region : Von Laue stability condition is satisfied

$$\int d^2\boldsymbol{b}^{\perp} p(\boldsymbol{b}^{\perp}) = 0.$$

Total shear for is positive and resembles that of a stable hydrostatic system

More, AM, Nair, Saha ; PRD 105, 056017 (2022); PRD 107, 116005 (2023)

Angular momentum of quarks and gluons

Question : can one decompose the nucleon spin into quark and gluon parts, spin and orbital ?

For a long time, it was thought that the gluon angular momentum cannot be separated into orbital and spin part in a gauge invariant way

However, experiments, for example at RHIC, have measured spin asymmetries that are sensitive to the gluon intrinsic spin. Experimental observables are gauge invariant. How to relate the experimentally measured quantity to the spin sum rule of the nucleon ?

Gauge invariant decomposition can be obtained by adding another term called potential angular momentum that can be added either to the quark or gluon part of the orbital angular momentum (OAM) – different decompositions

What is the density of the angular momentum due to the quarks and gluons ? Is the density different for different decompositions ?

In the next few slides, we discuss the key points for different decompositions taking QED as example. The decompositions are similar for QCD.

Jaffe and Manohar, Nucl. Phys. B (1990)
X. Ji, PRL (1997)
Chen et al, PRL (2008), (2009);
Wakamatsu, PRD(2010), PRD(2011)
Leader and Lorce, Phys. Rep, 2014

Different decompositions of angular momentum

In Belinfante decomposition, operator has purely orbital appearance, symmetric. Can be separated into electron and photon AM.

Each part gauge invariant

$$J^{\mu\nu\rho}_{\text{Bel,q}}(x) = x^\nu T^{\mu\rho}_{\text{Bel,q}}(x) - x^\rho T^{\mu\nu}_{\text{Bel,q}}(x);$$

In Ji's decomposition, the Belinfante AM is rewritten in such a way that the electron AM can be separated into an orbital and spin part. Each part is gauge invariant and measurable, photon AM coincides with Benlinfante.

$$J^{\mu\nu\rho}_{\text{kin,q}}(x) = L^{\mu\nu\rho}_{\text{kin,q}}(x) + S^{\mu\nu\rho}_{\text{q}}(x),$$

No further decomposition of photon AM into orbital and spin

$$L^{\mu\nu\rho}_{\text{kin,q}}(x) + S^{\mu\nu\rho}_{\text{q}}(x) = J^{\mu\nu\rho}_{\text{Bel,q}}(x) - \frac{1}{2}\partial_\sigma[x^\nu S^{\sigma\mu\rho}_{\text{q}}(x) - x^\rho S^{\sigma\mu\nu}_{\text{q}}(x)].$$

X. Ji, PRL (1997)

In Jaffe-Manohar or canonical decomposition, AM is separated into electron and photon spin and OAM parts

Each of them are generators of rotation following Noether's theorem, but not all of them are gauge invariant

$$\begin{aligned} J^{\mu\nu\rho}_{\text{q}}(x) &= L^{\mu\nu\rho}_{\text{q}}(x) + S^{\mu\nu\rho}_{\text{q}}(x) \\ &= \frac{1}{2}\bar{\psi}(x)\gamma^\mu x^{[\nu} i \overleftrightarrow{\partial}^{\rho]}\psi(x) + \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\bar{\psi}(x)\gamma_\sigma\gamma_5\psi(x). \end{aligned}$$

Jaffe and Manohar, Nucl. Phys. B (1990)

Different decompositions, contd.

Chen et al : photon field is split into two parts, pure and physical

$$\boldsymbol{A} = \boldsymbol{A}_{\text{pure}} + \boldsymbol{A}_{\text{phys}}. \qquad \boldsymbol{\nabla} \times \boldsymbol{A}_{\text{pure}} = \boldsymbol{0}, \qquad \boldsymbol{\nabla} \cdot \boldsymbol{A}_{\text{phys}} = 0.$$

AM decomposed into electron and photon spin and OAM parts : each gauge invariant

Fields involved are non-local; decomposition same as Jaffe-Manohar decomposition in Coulomb gauge

Chen et al, PRL (2008), (2009)

Wakamatsu decomposition : subtracts the potential angular momentum from the electron part and compensates this in the photon part

AM is separated into spin an OAM of electron and photon, each gauge invariant

Makes Coulomb gauge special, physical photon field is non-local

Wakamatsu, PRD(2010), PRD(2011)

Decompositions of angular momentum: contd.

Angular momentum decompositions can be divided into two two families, kinetic and canonical.

Kinetic : potential AM attributed to photon (Belinfante, Ji, Wakamatsu)
Canonical : Potential AM attributed to electron (Jaffe-Manohar, Chen et al)

Covariant generalization of Chen et al decomposition : gauge invariant canonical (gic)

Covariant generalization of Wakamatsu's decomposition : gauge invariant kinetic (gik)

Leader and Lorce, Phys. Rep, 2014

Different decompositions at the density level also differ by superpotential terms, which are terms that become surface terms upon integration, and vanish when the fields vanish at the boundary.

Decomposition of angular momentum in QCD can be made along similar lines.

TABLE I. Properties of all the angular momentum densities derived from EMTs of the kinetic and canonical family.

Class	EMT	AM densities	Gauge invariant	Follow $SU(2)$ algebra
Kinetic	Belinfante	$\boldsymbol{J}_{\text{Bel,q}}$	✓	✗
		$\boldsymbol{J}_{\text{Bel,g}}$	✓	✗
	Ji	$\boldsymbol{L}_{\text{Ji,q}}$	✓	✗
		$\boldsymbol{S}_{\text{Ji,q}}$	✓	✓
		$\boldsymbol{J}_{\text{Ji,g}}$	✓	✗
	Wakamatsu (gik)	$\boldsymbol{L}_{\text{gic,q}}$	✓	✗
		$\boldsymbol{S}_{\text{gic,q}}$	✓	✓
		$\boldsymbol{L}_{\text{gic,g}}$	✓	✗
		$\boldsymbol{S}_{\text{gic,g}}$	✓	✗
Canonical	Jaffe-Manohar	$\boldsymbol{L}_{\text{JM,q}}$	✗	✓
		$\boldsymbol{S}_{\text{JM,q}}$	✓	✓
		$\boldsymbol{L}_{\text{JM,g}}$	✗	✗
		$\boldsymbol{S}_{\text{JM,g}}$	✗	✗
	Chen *et al.* (gic)	$\boldsymbol{L}_{\text{gic,q}}$	✓	✓
		$\boldsymbol{S}_{\text{gic,q}}$	✓	✓
		$\boldsymbol{L}_{\text{gic,g}}$	✓	✗
		$\boldsymbol{S}_{\text{gic,g}}$	✓	✗

In order to be identified as the generator of rotation, the components of angular momentum have to obey SU(2) algebra. But that is not the case for individual components.

Leader and Lorce, Phys. Rep, (2014), (2019)

Also, individual components are not always gauge invariant

Model calculations of the different components in different decompositions is interesting

Model incorporating gluon ?

Ravi Singh, Sudeep Saha, AM, Nilmani Mathur, PRD 109, 016022(2024)

OAM Distribution

Orbital angular momentum distribution in the front form is given by

$$\langle L^z\rangle(\boldsymbol{b}^\perp) = -i\epsilon^{3jk}\int \frac{d^2\boldsymbol{\Delta}^\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}^\perp\cdot\boldsymbol{b}^\perp}\left[\frac{\partial\langle T^{+k}\rangle_{\rm LF}}{\partial\Delta^j_\perp}\right],$$

$$\langle T^{+k}\rangle_{\rm LF} = \frac{\langle p',s|T^{+k}(0)|p,s\rangle}{2\sqrt{p'^+p^+}}$$

Instead of a proton, we use a dressed quark state

We use light front gauge and Drell-Yan frame, where the average transverse momentum of the system is zero

$$T^{+k}_{\rm kin,q}(0) = \sum_{\lambda,\lambda'}\int \frac{dk'^+d^2k'^\perp dk^+d^2k^\perp}{(16\pi^3)^2\sqrt{k'^+k^+}} b^\dagger_{\lambda'}(k')b_\lambda(k)\chi^\dagger_{\lambda'}[k'^k+k^k]\chi_\lambda$$
$$+2g\sum_{\lambda',\lambda,\lambda_3}\int \frac{dk'^+d^2k'^\perp dk^+d^2k^\perp dk_3^+d^2k_3^\perp}{(16\pi^3)^3k_3^+\sqrt{k'^+k^+}}[\epsilon^k_{\lambda_3}b^\dagger_{\lambda'}(k')b_\lambda(k)a_{\lambda_3}(k_3)+\epsilon^{k*}_{\lambda_3}b^\dagger_{\lambda'}(k')b_\lambda(k)a^\dagger_{\lambda_3}(k_3)].$$

Contribution to the matrix element comes from both diagonal and off-diagonal overlaps

$$p^\mu = \left(P^+, -\frac{\boldsymbol{\Delta}^\perp}{2}, \frac{1}{P^+}\left(m^2+\frac{\boldsymbol{\Delta}^{\perp 2}}{4}\right)\right),$$

$$p'^\mu = \left(P^+, \frac{\boldsymbol{\Delta}^\perp}{2}, \frac{1}{P^+}\left(m^2+\frac{\boldsymbol{\Delta}^{\perp 2}}{4}\right)\right),$$

$$\boldsymbol{\Delta}^\mu = (p'-p)^\mu = (0, \boldsymbol{\Delta}^\perp, 0),$$

Orbital angular momentum density

OAM density in impact parameter space

$$\langle L^z\rangle(\boldsymbol{b}^\perp) = -i\epsilon^{3jk}\int \frac{d^2\boldsymbol{\Delta}^\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}^\perp\cdot\boldsymbol{b}^\perp}\left[\frac{\partial\langle T^{+k}\rangle_{\rm LF}}{\partial\Delta^j_\perp}\right],$$

Contribution coming from the quark part of the EMT for a dressed quark state

For the kinetic OAM, we obtain

$$\langle L^z_{\rm kin,q}\rangle(\boldsymbol{b}^\perp) = \frac{g^2C_F}{72\pi^2}\int \frac{d^2\boldsymbol{\Delta}^\perp}{(2\pi)^2} e^{-i\boldsymbol{b}^\perp\cdot\boldsymbol{\Delta}^\perp}\left[-7+\frac{6}{\omega}\left(1+\frac{2m^2}{\Delta^2}\right)\log\left(\frac{1+\omega}{-1+\omega}\right) - 6\log\left(\frac{\Lambda^2}{m^2}\right)\right],$$

Λ : cutoff on transverse momentum

$$\omega = \sqrt{1+\tfrac{4m^2}{\Delta^2}}$$

Off-diagonal term corresponds to the potential angular momentum and it vanishes

Also observed in scalar diquark model and QED

D.A. Amor-Quiroz, M. Burkardt, W. Focillon, and C. Lorcé, Eur. Phys. J. C 81, 589 (2021). X. Ji, A. Schäfer, F. Yuan, J.-H. Zhang, and Y. Zhao, Phys. Rev. D 93, 054013 (2016).

OAM density : details

We have calculated the matrix element of the EMT in terms of the LFWF of a dressed quark

$$\frac{\langle 2,\uparrow|T^{+k}_{\text{kin,q}}(0)|2,\uparrow\rangle}{2p^+} = \frac{1}{2}\sum_{\lambda_1',\lambda_1,\lambda_2}\int dx d^2\boldsymbol{\kappa}^\perp \phi^{*\uparrow}_{\lambda_1',\lambda_2}(x,\boldsymbol{\kappa}'^\perp)\chi^\dagger_{\lambda_1'}(2\boldsymbol{\kappa}^k + (1-x)\boldsymbol{\Delta}^k)\chi_{\lambda_1}\phi^\uparrow_{\lambda_1,\lambda_2}(x,\boldsymbol{\kappa}^\perp),$$

$$\boldsymbol{\kappa}'^\perp = \boldsymbol{\kappa}^\perp + (1-x)\boldsymbol{\Delta}^\perp.$$

Diagonal overlap

$$= g^2 C_F \int \frac{dx d^2\boldsymbol{\kappa}^\perp}{16\pi^3}\frac{(2\kappa^k + (1-x)\Delta^k)}{(1-x)D_1 D_2}[m^2(1-x)^4 + (1+x^2)\boldsymbol{\kappa}^{\perp 2} + (1-x)(1+x^2)\boldsymbol{\kappa}^\perp\cdot\boldsymbol{\Delta}^\perp + i(1-x)(1-x^2)(\kappa^{(1)}\Delta^{(2)} - \kappa^{(2)}\Delta^{(1)})],$$

$$D_1 = \kappa^{\perp 2} + m^2(1-x)^2,$$

$$D_2 = (\boldsymbol{\kappa}^\perp + (1-x)\boldsymbol{\Delta}^\perp)^2 + m^2(1-x)^2.$$

$$\frac{1}{2p^+}[\langle 1,\uparrow|T^{+k}_{\text{kin,q}}(0)|2,\uparrow\rangle + \langle 2,\uparrow|T^{+k}_{\text{kin,q}}(0)|1,\uparrow\rangle]$$

Non-diagonal overlap : gives zero

$$= \frac{g}{\sqrt{16\pi^3}}\sum_{\lambda_1,\lambda_2}\int \frac{dx d^2\boldsymbol{\kappa}^\perp}{\sqrt{1-x}}[\psi_1^*(P,\sigma')\chi^\dagger_{\sigma'}\epsilon^k_{\lambda_2}\chi_{\lambda_1}\phi^\sigma_{\lambda_1,\lambda_2}(x,\boldsymbol{\kappa}^\perp) + \phi^{*\sigma'}_{\lambda_1,\lambda_2}(x,\boldsymbol{\kappa}^\perp)\chi^\dagger_{\lambda_1}\epsilon^{k*}_{\lambda_2}\chi_\sigma\psi_1(P,\sigma)],$$

Spin density

$$\langle S^z\rangle(\boldsymbol{b}^\perp) = \frac{1}{2}\epsilon^{3jk}\int\frac{d^2\boldsymbol{\Delta}^\perp}{(2\pi)^2}e^{-i\boldsymbol{\Delta}^\perp\cdot\boldsymbol{b}^\perp}\langle S^{+jk}\rangle_{\mathrm{LF}}.$$

Kinetic spin angular momentum is calculated using the overlap

$$\frac{\langle 2,\uparrow|S_q^{+jk}(0)|2,\uparrow\rangle}{2p^+}$$

$$= \frac{1}{4}\epsilon^{+jk-}\sum_{\lambda_1,\lambda_1',\lambda_2}\int dx d^2\boldsymbol{\kappa}^\perp \phi^{*\uparrow}_{\lambda_1',\lambda_2}(x,\boldsymbol{\kappa}'^\perp)(\chi^\dagger_{\lambda_1'}\sigma^{(3)}\chi_{\lambda_1})\phi^{\uparrow}_{\lambda_1,\lambda_2}(x,\boldsymbol{\kappa}^\perp)$$

$$= \frac{g^2C_F}{4}\epsilon^{+jk-}\int\frac{dxd^2\boldsymbol{\kappa}^\perp}{8\pi^3}\frac{1}{(1-x)D_1D_2}$$
$$\times[\boldsymbol{\kappa}^{\perp 2}(1+x^2)+\boldsymbol{\kappa}^\perp\cdot\boldsymbol{\Delta}^\perp(1-x)(1+x^2)+i(1-x)(1-x^2)(\kappa^{(1)}\Delta^{(2)}-\kappa^{(2)}\Delta^{(1)})-m^2(1-x)^4].$$

$$\langle S^z_{\mathrm{kin,q}}\rangle(\boldsymbol{b}^\perp) = -\frac{g^2C_F}{32\pi^2}\int\frac{d^2\boldsymbol{\Delta}^\perp}{(2\pi)^2}e^{-i\boldsymbol{\Delta}^\perp\cdot\boldsymbol{b}^\perp}\int\frac{dx}{1-x}$$
$$\times\left[\omega(1+x^2)\log\left(\frac{1+\omega}{-1+\omega}\right)+\left(\frac{1-\omega^2}{\omega}\right)x\log\left(\frac{1+\omega}{-1+\omega}\right)-(1+x^2)\log\left(\frac{\Lambda^2}{m^2(1-x)^2}\right)\right].$$

Belinfante AM

$$\langle J^z_{\text{Bel}}\rangle(\boldsymbol{b}^\perp) = -i\epsilon^{3jk}\int\frac{d^2\boldsymbol{\Delta}^\perp}{(2\pi)^2}e^{-i\boldsymbol{\Delta}^\perp\cdot\boldsymbol{b}^\perp}\left[\frac{\partial\langle T^{+k}_{\text{Bel}}\rangle_{\text{LF}}}{\partial\Delta^j_\perp}\right].$$

Connection between Belinfante and kinetic AM is through the superpotential term

$$\langle M^z\rangle(\boldsymbol{b}^\perp) = \frac{1}{2}\epsilon^{3jk}\int\frac{d^2\boldsymbol{\Delta}^\perp}{(2\pi)^2}e^{-i\boldsymbol{\Delta}^\perp\cdot\boldsymbol{b}^\perp}\Delta^l_\perp\frac{\partial\langle S^{l+k}\rangle_{\text{LF}}}{\partial\Delta^j_\perp}.$$

$$\begin{aligned}\langle J^z_{\text{Bel,q}}\rangle(\boldsymbol{b}^\perp) = {}& g^2C_F\int\frac{d^2\boldsymbol{\Delta}^\perp}{(2\pi)^2}e^{-i\boldsymbol{b}^\perp\cdot\boldsymbol{\Delta}^\perp}\int\frac{dx}{16\pi^2}\frac{1}{(1-x)\Delta^4\omega^3}\\ &\times\Bigg[(8m^4(1-2x)(1-x(1-x))+6m^2(1-(2-x)x(1+2x))\Delta^2+(1-(2-x)x(1+2x))\Delta^4)\log\left(\frac{1+\omega}{-1+\omega}\right)\\ &-\omega\Delta^2\left(4m^2(1-(1-x)x)+(1+x^2)\Delta^2+(1-(2-x)x(1+2x))(4m^2+\Delta^2)\log\left(\frac{\Lambda^2}{m^2(1-x)^2}\right)\right)\Bigg].\end{aligned}$$

$$\begin{aligned}\langle M^z_{\text{q}}\rangle(\boldsymbol{b}^\perp) = {}& \frac{g^2C_F}{32\pi^2}\int\frac{d^2\boldsymbol{\Delta}^\perp}{(2\pi)^2}e^{-i\boldsymbol{\Delta}^\perp\cdot\boldsymbol{b}^\perp}\int\frac{dx}{(1-x)}\frac{1}{\omega^3\Delta^4}\\ &\times[\omega\Delta^2((4m^2+\Delta^2)(1+x^2)-4m^2x)\\ &-2m^2((4m^2+\Delta^2)(1+x^2)-4m^2x-2x\Delta^2)].\end{aligned}$$

Canonical, gic and gik distributions

$$\langle L^z_{\text{kin,q}}\rangle(\boldsymbol{b}^\perp) = \langle L^z_{\text{q}}\rangle(\boldsymbol{b}^\perp), \quad \langle S^z_{\text{kin,q}}\rangle(\boldsymbol{b}^\perp) = \langle S^z_{\text{q}}\rangle(\boldsymbol{b}^\perp).$$

This is because the nondiagonal matrix element in the kinetic term, that corresponds to quark-gluon interaction gives zero contribution. So effectively we have the same operator structure contributing in both kinetic and canonical, in light-front gauge

$$T^{+k}_{\text{gic,q}} = \frac{1}{2}\bar{\psi}(x)\gamma^+ i\overleftrightarrow{D}^k_{\text{pure}}\psi(x),$$

$$\overleftrightarrow{D}^\mu_{\text{pure}} = \overleftrightarrow{\partial}^\mu - 2igA^\mu_{\text{pure}} \qquad A^\mu_{\text{pure}} = A^\mu - A^\mu_{\text{phys}}.$$

$$T^{+k}_{\text{gic,q}} = \frac{1}{2}\bar{\psi}(x)\gamma^+ i\overleftrightarrow{\partial}^k\psi(x) = T^{+k}_{\text{q}}.$$

In light front gauge, the gic decomposition gives the same distributions as canonical

$$\langle L^z_{\text{gic,q}}\rangle(\boldsymbol{b}^\perp) = \langle L^z_{\text{q}}\rangle(\boldsymbol{b}^\perp), \quad \langle S^z_{\text{gic,q}}\rangle(\boldsymbol{b}^\perp) = \langle S^z_{\text{q}}\rangle(\boldsymbol{b}^\perp).$$

Finally the gik decomposition gives the same distribution as kinetic for a dressed quark

$$T^{\mu\nu}_{\text{gik,q}} = T^{\mu\nu}_{\text{kin,q}}.$$

$$\langle L^z_{\text{gik,q}}\rangle(\boldsymbol{b}^\perp) = \langle L^z_{\text{kin,q}}\rangle(\boldsymbol{b}^\perp), \quad \langle S^z_{\text{gik,q}}\rangle(\boldsymbol{b}^\perp) = \langle S^z_{\text{kin,q}}\rangle(\boldsymbol{b}^\perp).$$

OAM distributions

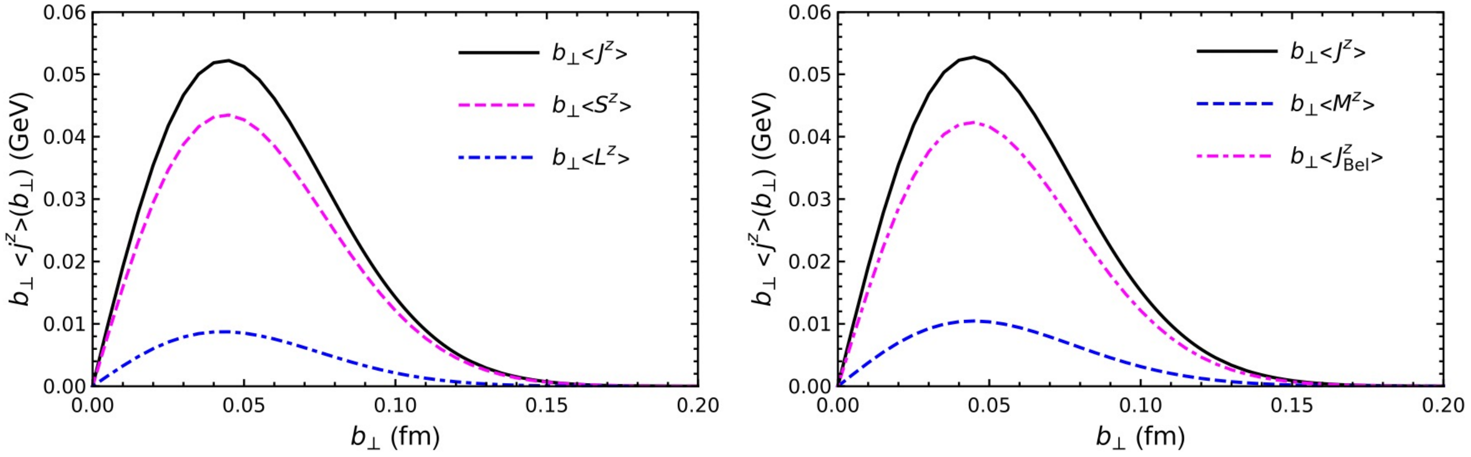


FIG. 1. Longitudinal angular momentum distribution of quarks as a function of impact parameter $b_\perp$. Left: Sum of the kinetic orbital AM $b_\perp \langle L^z \rangle$ (dot-dashed line) and spin AM $b_\perp \langle S^z \rangle$ (dashed line) given by kinetic total AM $b_\perp \langle J^z \rangle$ (solid line). Right: Kinetic total AM $b_\perp \langle J^z \rangle$ (solid line) is given by the sum of Belinfante total AM $b_\perp \langle J^z_{\mathrm{Bel}} \rangle$ (dot-dashed line) and the correction term corresponding to the total divergence $b_\perp \langle M^z \rangle$ (dashed line). Here, $m = 0.3$ GeV, $g = 1$, $C_f = 1$, and $\Lambda = 1.7$ GeV. We chose the Gaussian width $\sigma = 0.1$ GeV.

For the densities, we use a Gaussian wave packet state

$$\frac{1}{16\pi^3} \int \frac{d^2 \boldsymbol{p}^\perp dp^+}{p^+} \phi(p) |p^+, \boldsymbol{p}^\perp, \lambda\rangle$$

$$\phi(p) = p^+ \delta(p^+ - p_0^+) \phi(\boldsymbol{p}^\perp).$$

$$\phi(\boldsymbol{p}^\perp) = e^{-\frac{\boldsymbol{p}^{\perp 2}}{2\sigma^2}},$$

Spin distribution dominates over OAM distribution, similar to other calculations for proton

Superpotential term is positive throughout, in contrast to some other model calculations, where it has a positive core but negative near the periphery

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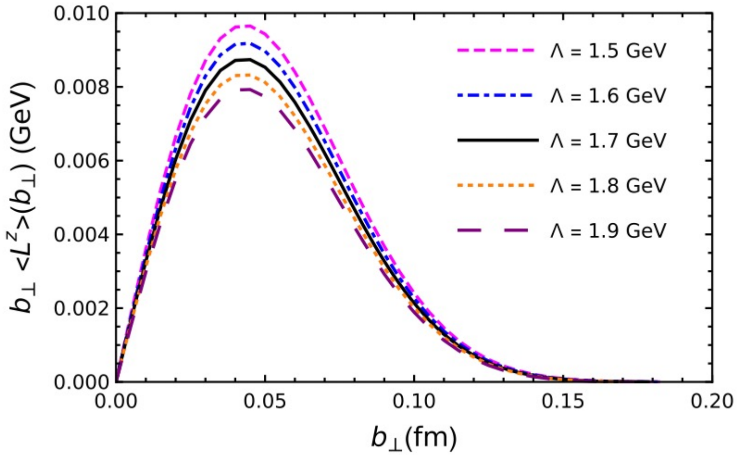
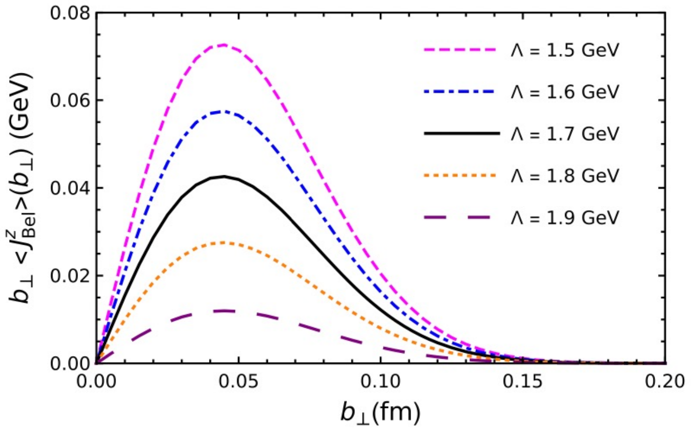
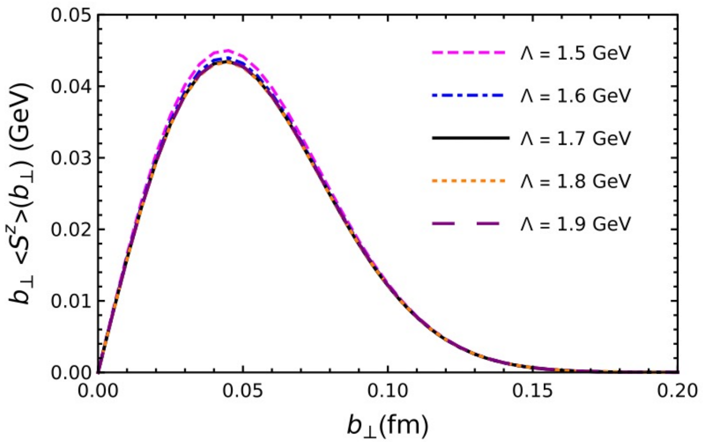
Scale dependence







Considered only quark part of EMT ; Contribution from gluon part needs to be calculated

Contribution from quark part depends on renormalization scale, in our approach cutoff on transverse momentum

This is because of the quark-gluon interaction present

Belinfante AM depends strongly on the cutoff

For other plots we have used Λ =1.7 GeV

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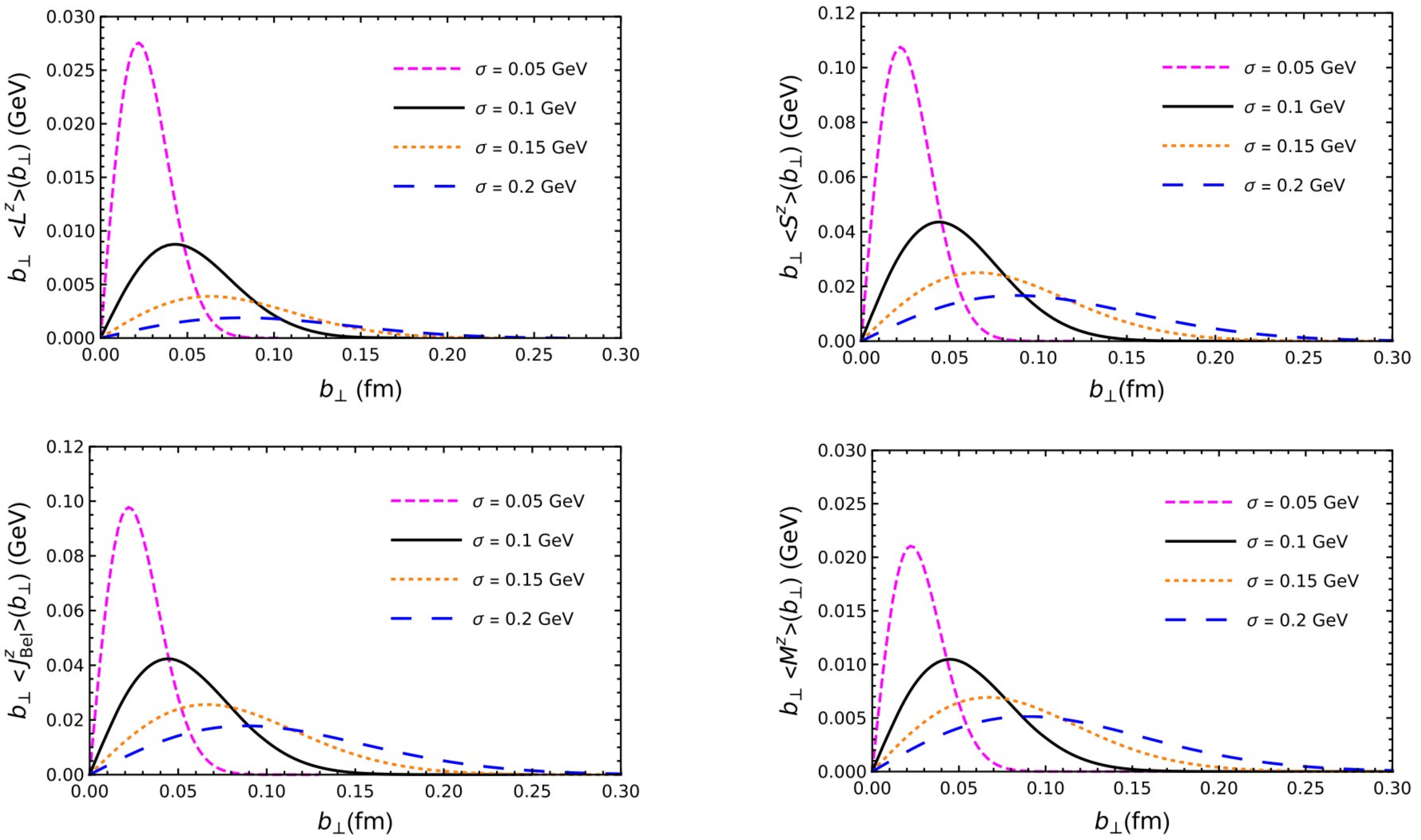
Dependence on Gaussian Width



Distributions spread and are broader for larger width of the Gaussian wave packet

Peak shifts towards larger impact parameter

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Axial vector form factor

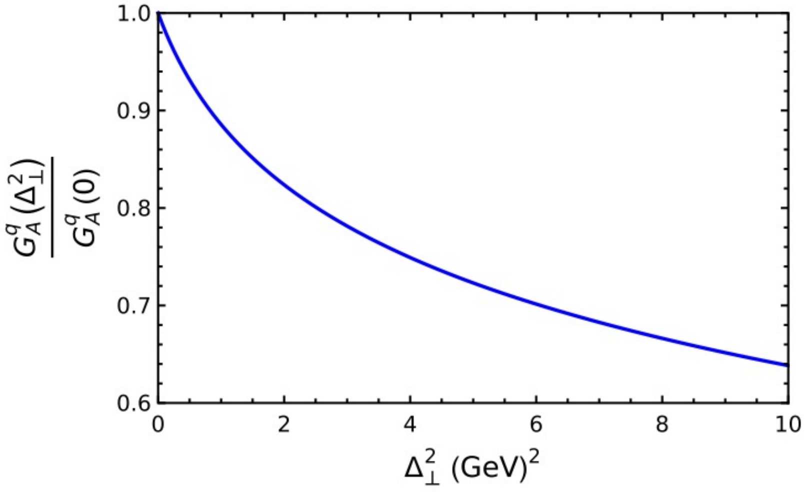

FIG. 4. Axial vector form factor $\frac{G_A^q(\Delta_\perp^2)}{G_A^q(0)}$ as a function of $\Delta_\perp^2$. Here $m = 0.3$ GeV, $g = 1$, $\Lambda = 1.7$ GeV, and $C_f = 1$.

Qualitative behavior similar to other models of the proton and lattice calculation

$$\langle p', s'|S^{\mu\alpha\beta}(0)|p, s\rangle = \frac{1}{2}\epsilon^{\mu\alpha\beta\lambda}\bar{u}(p', s')\left[\gamma_\lambda\gamma_5 G_A^q(t) + \frac{\Delta_\lambda\gamma_5}{2M}G_P^q(t)\right]u(p, s),$$

Matrix element of spin density operator is parametrized in terms of axial vector and pseudoscalar form factors

Antisymmetric part of EMT is related to the divergence of the spin operator

$$\bar{\psi}(x)[\gamma^\alpha i\overleftrightarrow{D}^\beta - \gamma^\beta i\overleftrightarrow{D}^\alpha]\psi(x) = -\epsilon^{\mu\alpha\beta\lambda}\partial_\mu[\bar{\psi}(x)\gamma_\lambda\gamma_5\psi(x)].$$

$$D_q(t) = -G_A^q(t). \qquad \frac{\langle p', s|S_q^{+jk}(0)|p, s\rangle}{2p^+} = \frac{1}{2}\epsilon^{+jk-}s^z G_A^q(t),$$

$$D_q(t) = -\frac{g^2 C_F}{16\pi^2}\int \frac{dx}{1-x}\left[\omega(1+x^2)\log\left(\frac{1+\omega}{-1+\omega}\right) + \left(\frac{1-\omega^2}{\omega}\right)x\log\left(\frac{1+\omega}{-1+\omega}\right) - (1+x^2)\log\left(\frac{\Lambda^2}{m^2(1-x)^2}\right)\right],$$

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Summary and conclusion

Presented a calculation of GFF and pressure distributions inside a quark state dressed with a gluon-simple spin-1/2 composite relativistic state

Used two-component formalism in light-front Hamiltonian QCD in light cone gauge, constrained fields are eliminated using equations of constraint

The D term and pressure distribution depend on quark-gluon interaction-compared with QED calculation

Pressure distribution satisfied stability condition

Presented a calculation of angular momentum in different decompositions

As we have calculated the contribution from the quark part of the EMT only, result depends on the renormalization scale, which is the cutoff on transverse momentum integral in this approach.

Potential term is zero, as already seen in QED

Contribution from the gluon part needs to be calculated to verify the spin sum rule.