# Exploring patonic structures through the Target Fragmentation in SIDIS

Xuan-Bo Tong

University of Jyväskylä

IWHSS-CPHI 2024
Yerevan, Armenia

Oct 1st, 2024

Kaibao Chen, Jianping Ma, X.Tong

*JHEP* 08 (2024) 227, 2406.08559

*JHEP* 05 (2024) 298, 2402.15112

*PRD* 108 (2023) 9, 9, 2308.11251

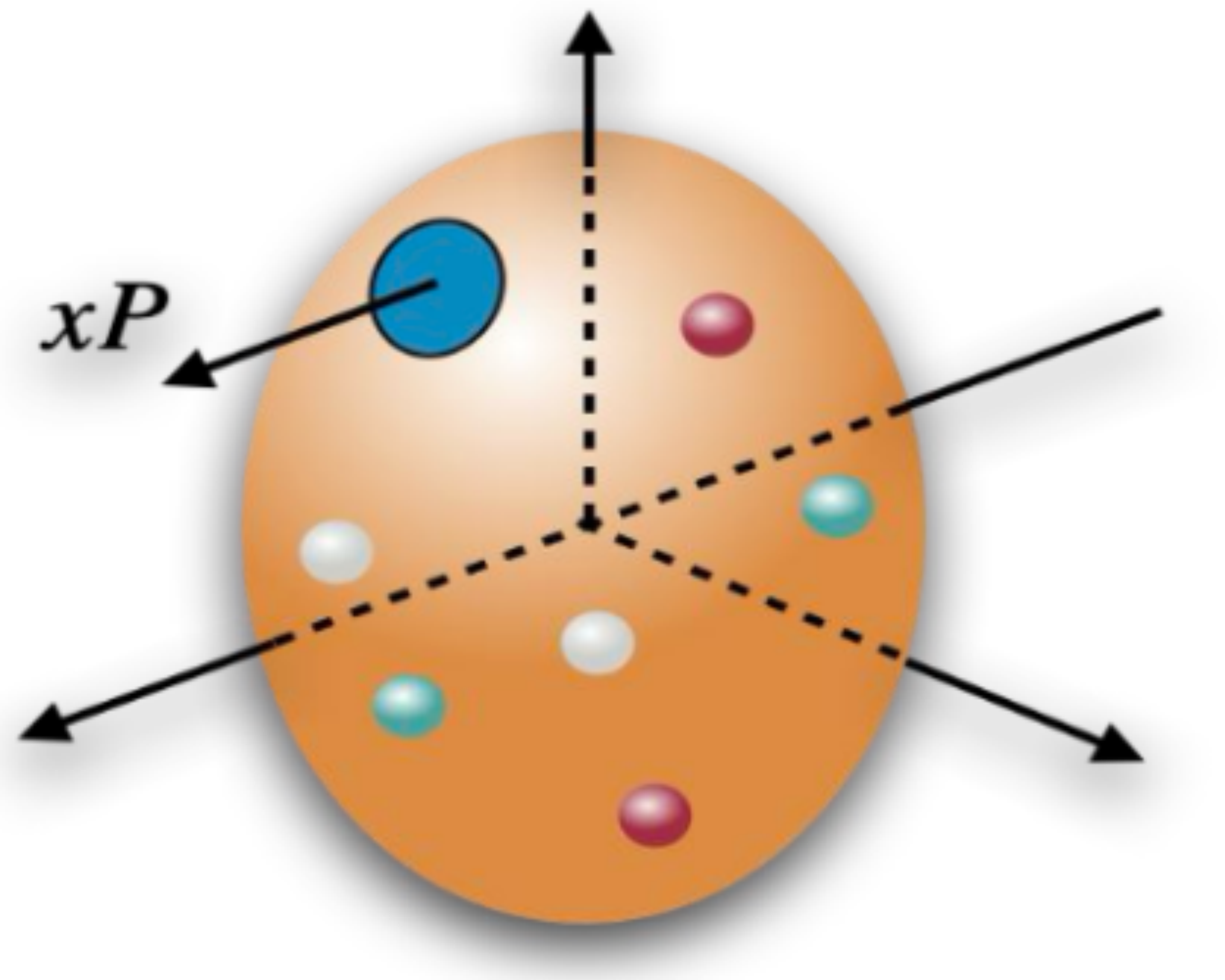
*JHEP* 11 (2021) 038, 2108.13582

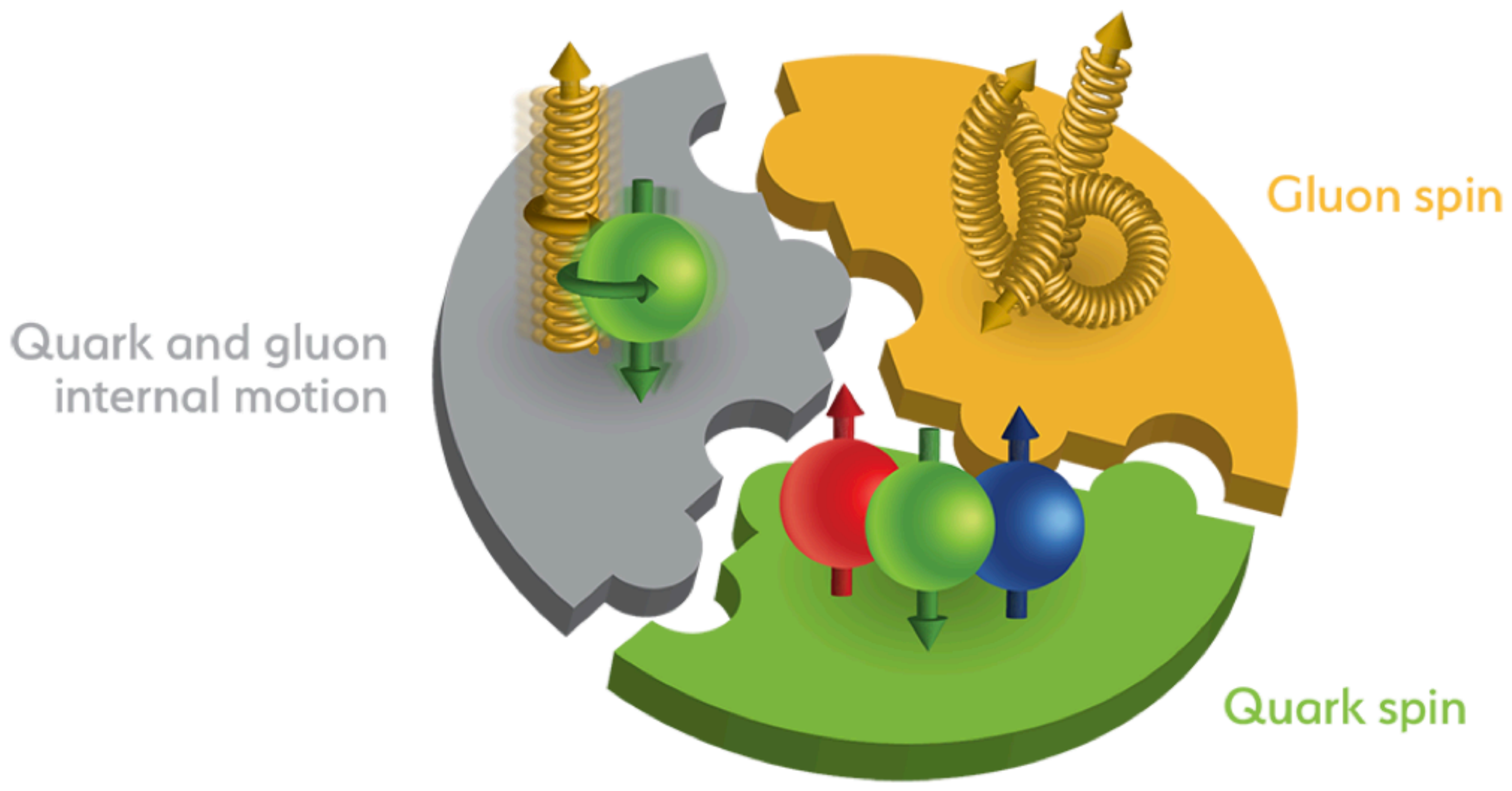
HELSINKI INSTITUTE OF PHYSICS

UNIVERSITY OF JYVÄSKYLÄ

Centre of Excellence in Quark Matter

# Nucleon tomography



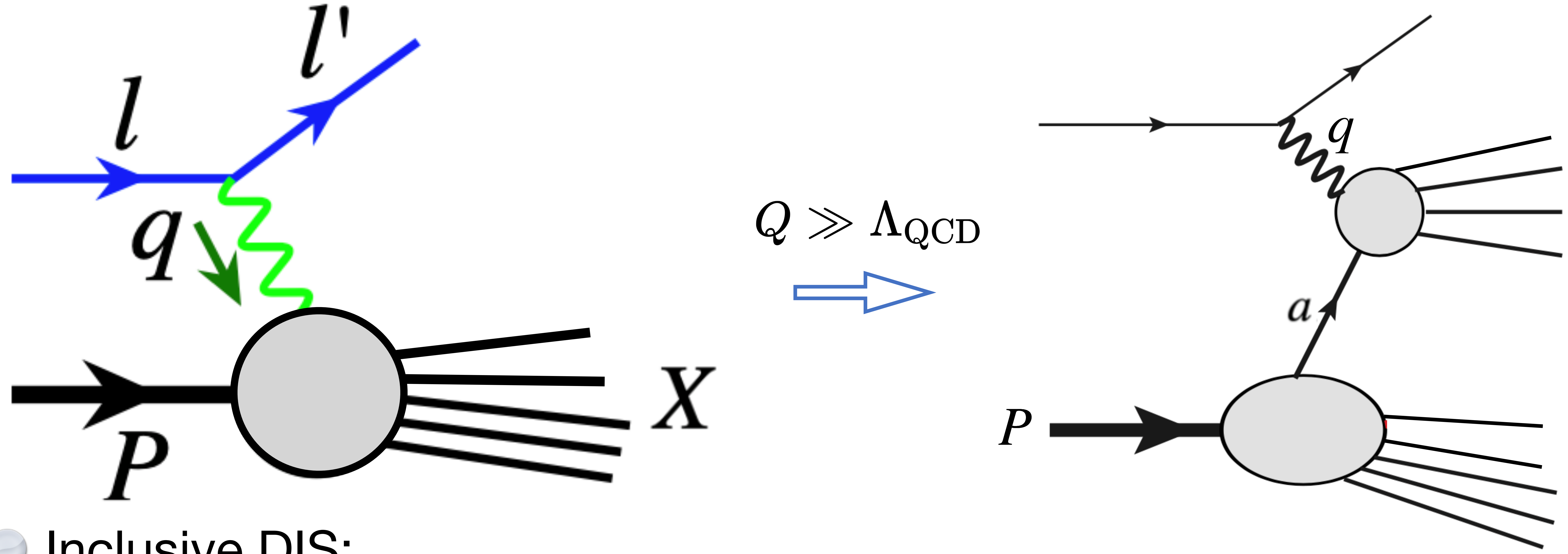




Questions:

- How is the momentum/spin of a nucleon distributed among quarks and gluons?
- Are there correlations between their momentum and spin orientations?

⇨ One of the main goals in HERA, JLab, Compass, EIC, EicC…

# Inclusive Deep-Inelastic Scattering




- Inclusive DIS:
  - Dominated by the hard scattering on a collinear parton
    - Soft gluons cancel
  - Collinear factorization: $\sigma \propto H(Q) \otimes f_{a/P}(x, \mu^2)$
  - However, too inclusive, lose information:
    - Fragmentation, final-state interactions
    - Transverse-momentum dependence

# Vanishing correlations in inclusive DIS

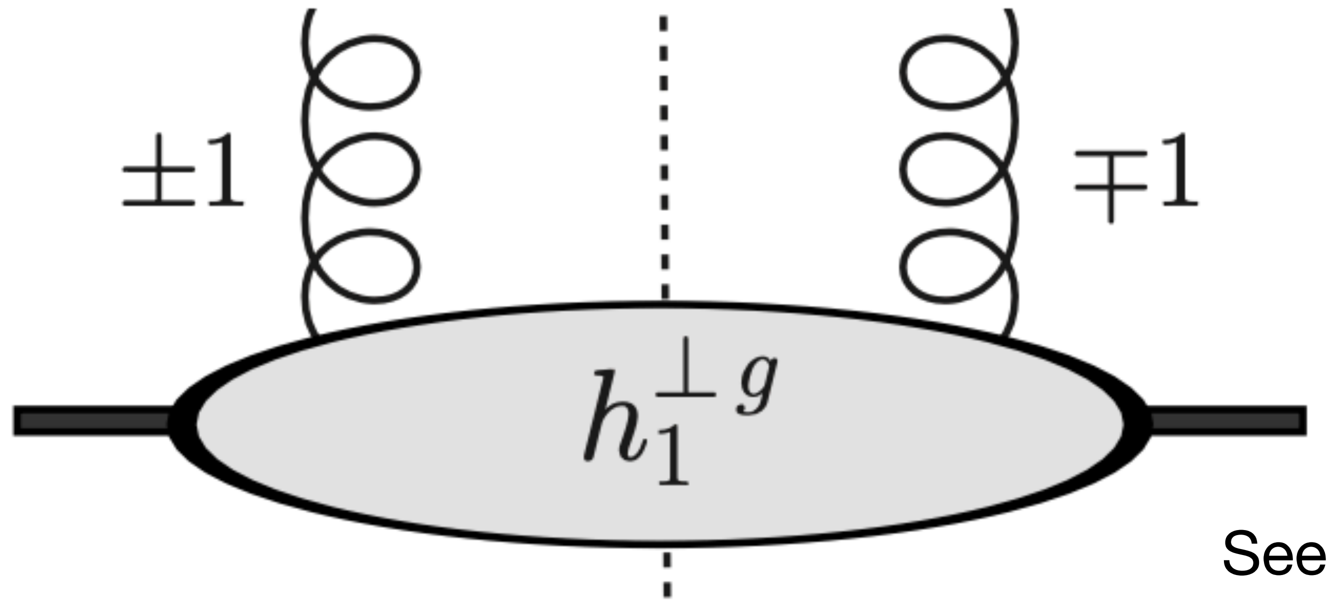
- Single transverse spin asymmetry

$$A_N \propto d\sigma(\vec{S}_\perp) - d\sigma(-\vec{S}_\perp)$$

  - T-odd effects
  - Require final/initial-state interactions

Otherwise, prohibited by time reversal invariance

- Linearly polarized gluons




See e.g. the talk by *Cristian Pisano*

  - Require a transverse reference direction

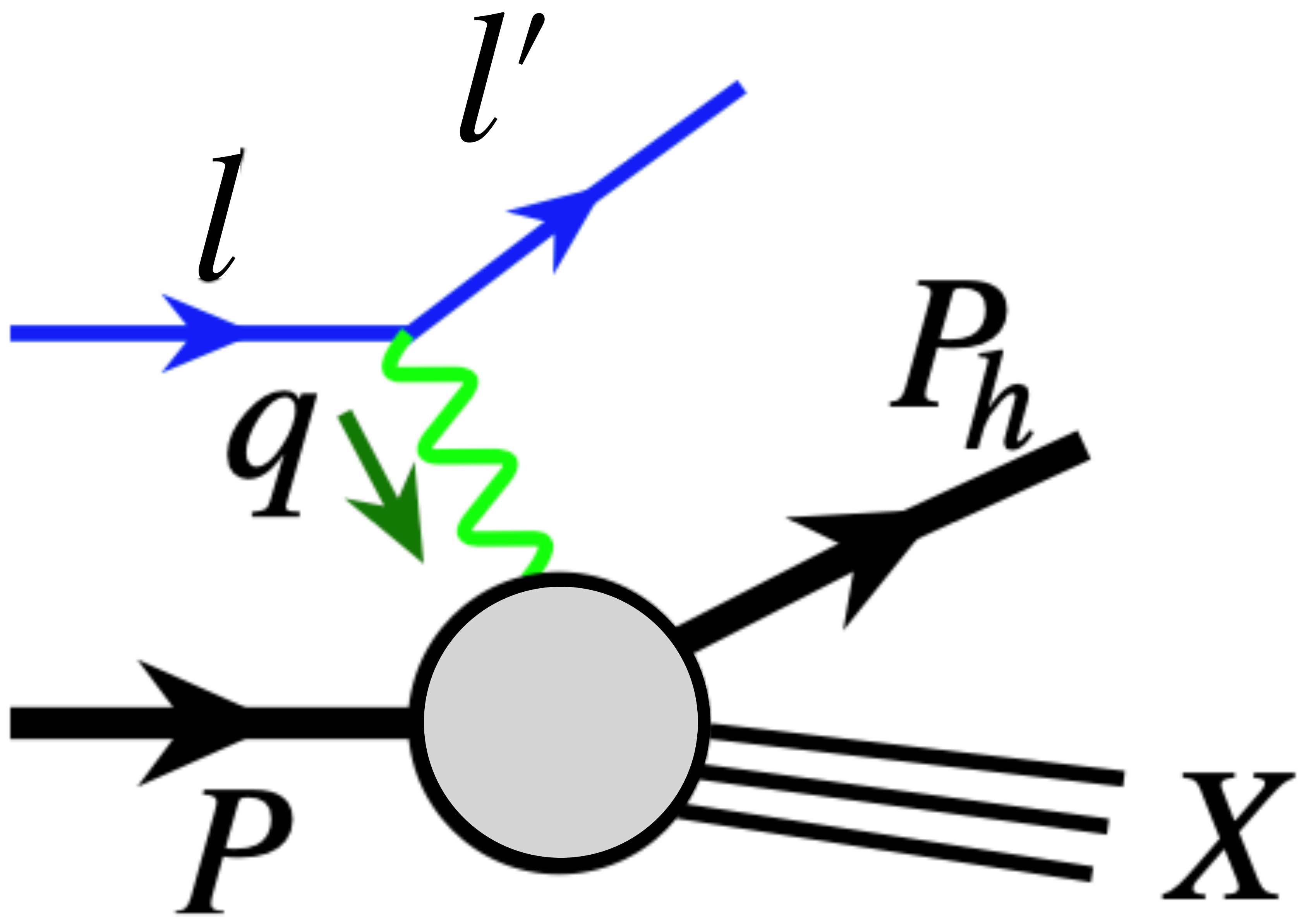
e.g. the initial parton $k_T$ [Mulders, Rodrigues, 2001]

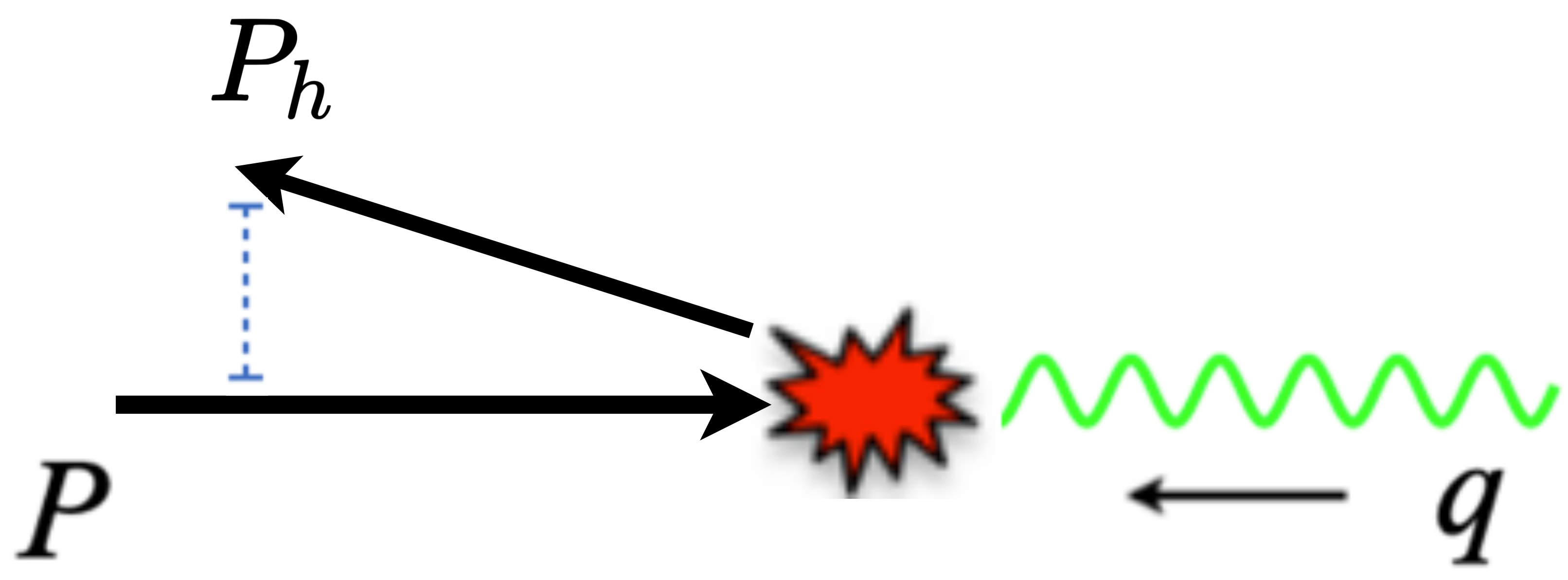
$$\langle P|F^{+\mu}F^{+\nu}|P\rangle \propto g_\perp^{\mu\nu} f_1^g - \frac{1}{M^2}\left(k_\perp^\mu k_\perp^\nu + g_\perp^{\mu\nu}\frac{\boldsymbol{k}_\perp^2}{2}\right)h_1^{\perp g}$$

Vanish after the $k_T$-integration

No associated collinear PDF

# Semi-inclusive Deep-Inelastic Scattering



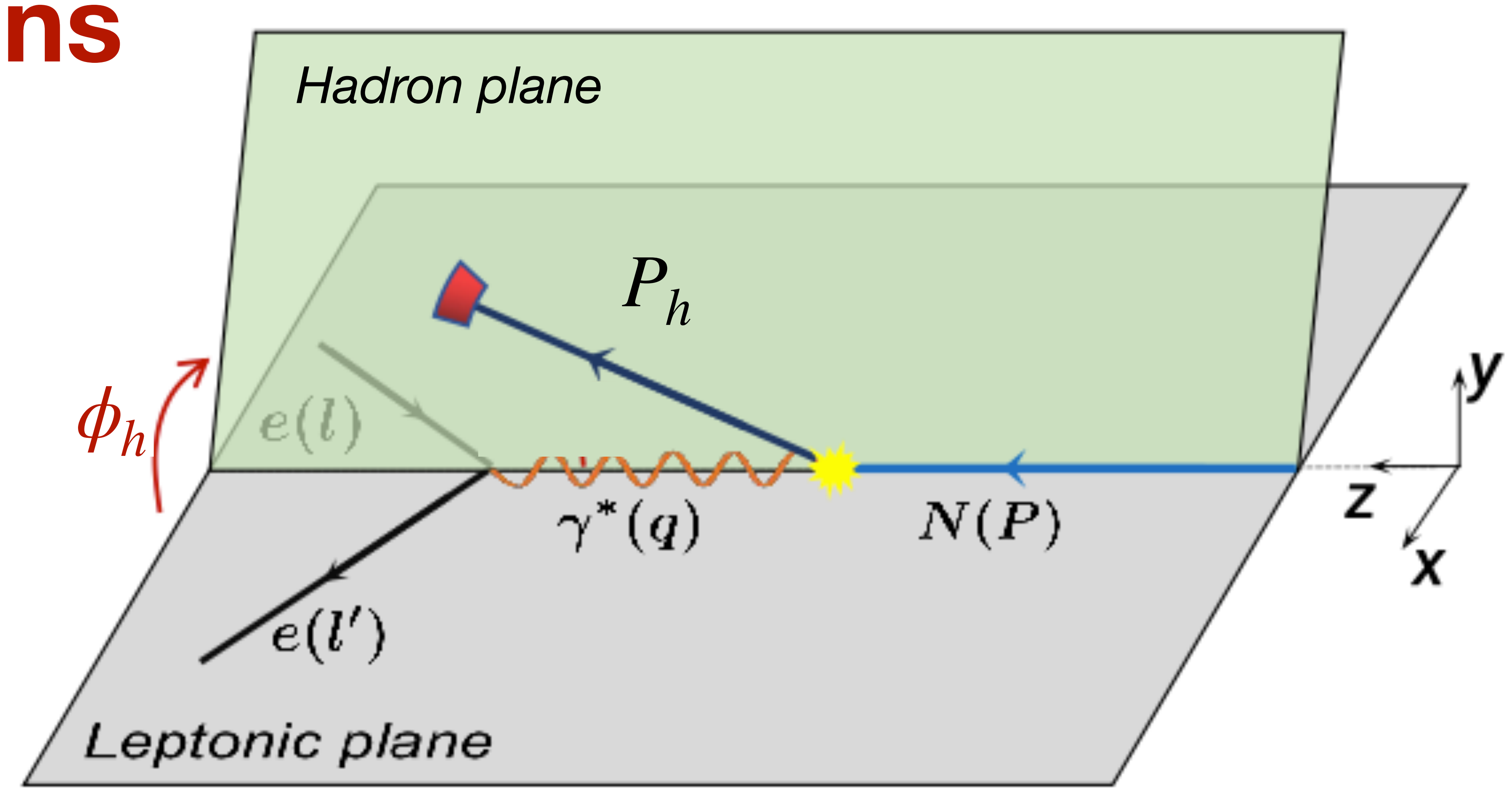




- SIDIS: a final-state hadron ($P_h$) is detected
  - Fragmentation of partons
  - A tunable transverse momentum, $\vec{P}_{h\perp}$
    - azimuthal correlation from the final/initial state

# SIDIS structure functions

SIDIS differential cross section    Bacchetta et al  JHEP 02 (2007) 093.

- 18 structure functions:

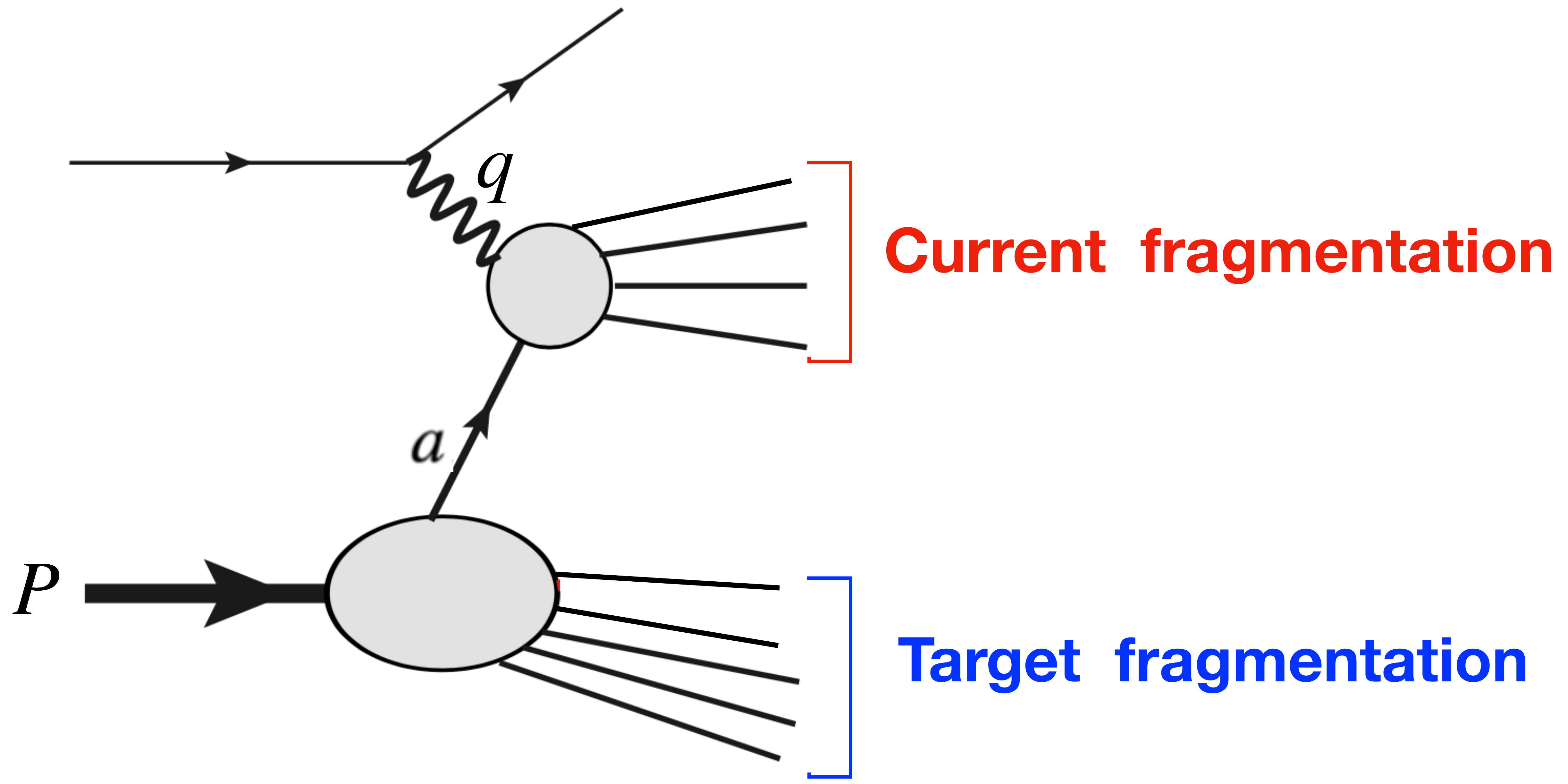



$$
\begin{aligned}
&\frac{d\sigma}{dx_B\,dy\,dz\,dP_{hT}^2\,d\phi_h\,d\phi_S} \\
&= \frac{\alpha^2}{x_B y Q^2}\frac{y^2}{2(1-\epsilon)}\left(1+\frac{\gamma^2}{2x_B}\right) \\
&\times\Big\{F_{UU,T}+\epsilon F_{UU,L}+\sqrt{2\epsilon(1+\epsilon)}F_{UU}^{\cos\phi_h}\cos\phi_h+\epsilon F_{UU}^{\cos 2\phi_h}\cos 2\phi_h+\lambda_e\sqrt{2\epsilon(1-\epsilon)}F_{LU}^{\sin\phi_h}\sin\phi_h \\
&+S_L\left[\sqrt{2\epsilon(1+\epsilon)}F_{UL}^{\sin\phi_h}\sin\phi_h+\epsilon F_{UL}^{\sin 2\phi_h}\sin 2\phi_h\right]+\lambda_e S_L\left[\sqrt{1-\epsilon^2}F_{LL}+\sqrt{2\epsilon(1-\epsilon)}F_{LL}^{\cos\phi_h}\cos\phi_h\right] \\
&+S_T\Big[\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\epsilon F_{UT,L}^{\sin(\phi_h-\phi_S)}\right)\sin(\phi_h-\phi_S)+\epsilon F_{UT}^{\sin(\phi_h+\phi_S)}\sin(\phi_h+\phi_S) \\
&+\epsilon F_{UT}^{\sin(3\phi_h-\phi_S)}\sin(3\phi_h-\phi_S)\quad+\sqrt{2\epsilon(1+\epsilon)}F_{UT}^{\sin\phi_S}\sin\phi_S+\sqrt{2\epsilon(1+\epsilon)}F_{UT}^{\sin(2\phi_h-\phi_S)}\sin(2\phi_h-\phi_S)\Big] \\
&+\lambda_e S_T\Big[\sqrt{1-\epsilon^2}F_{LT}^{\cos(\phi_h-\phi_S)}\cos(\phi_h-\phi_S) \\
&+\sqrt{2\epsilon(1-\epsilon)}F_{LT}^{\cos\phi_S}\cos\phi_S+\sqrt{2\epsilon(1-\epsilon)}F_{LT}^{\cos(2\phi_h-\phi_S)}\cos(2\phi_h-\phi_S)\Big]\Big\}
\end{aligned}
$$

Linearly polarized gluons, Boer-Mulder quark

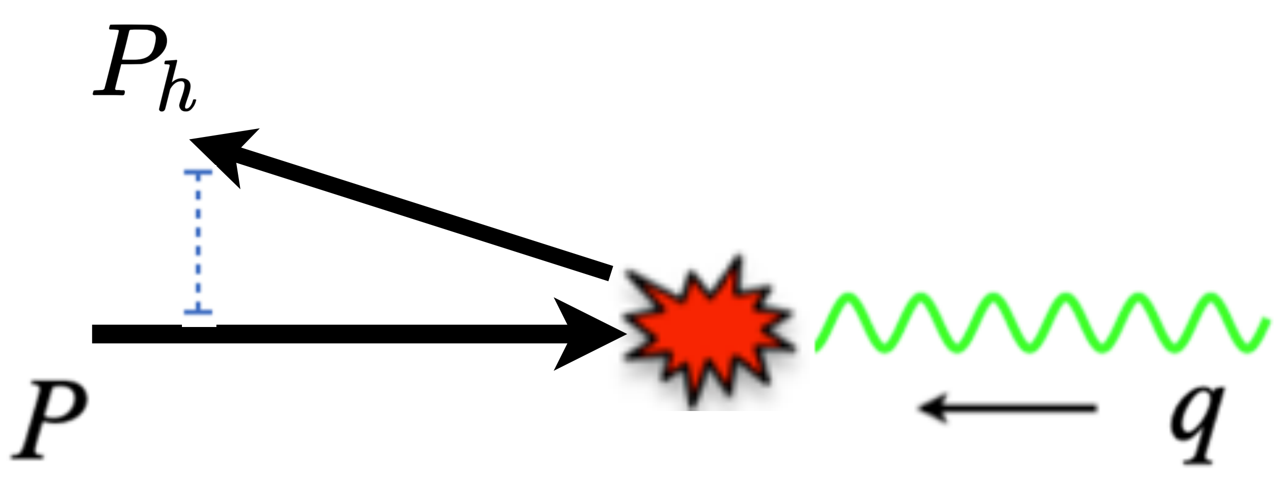
Single transverse spin, from e.g. Sivers effects,

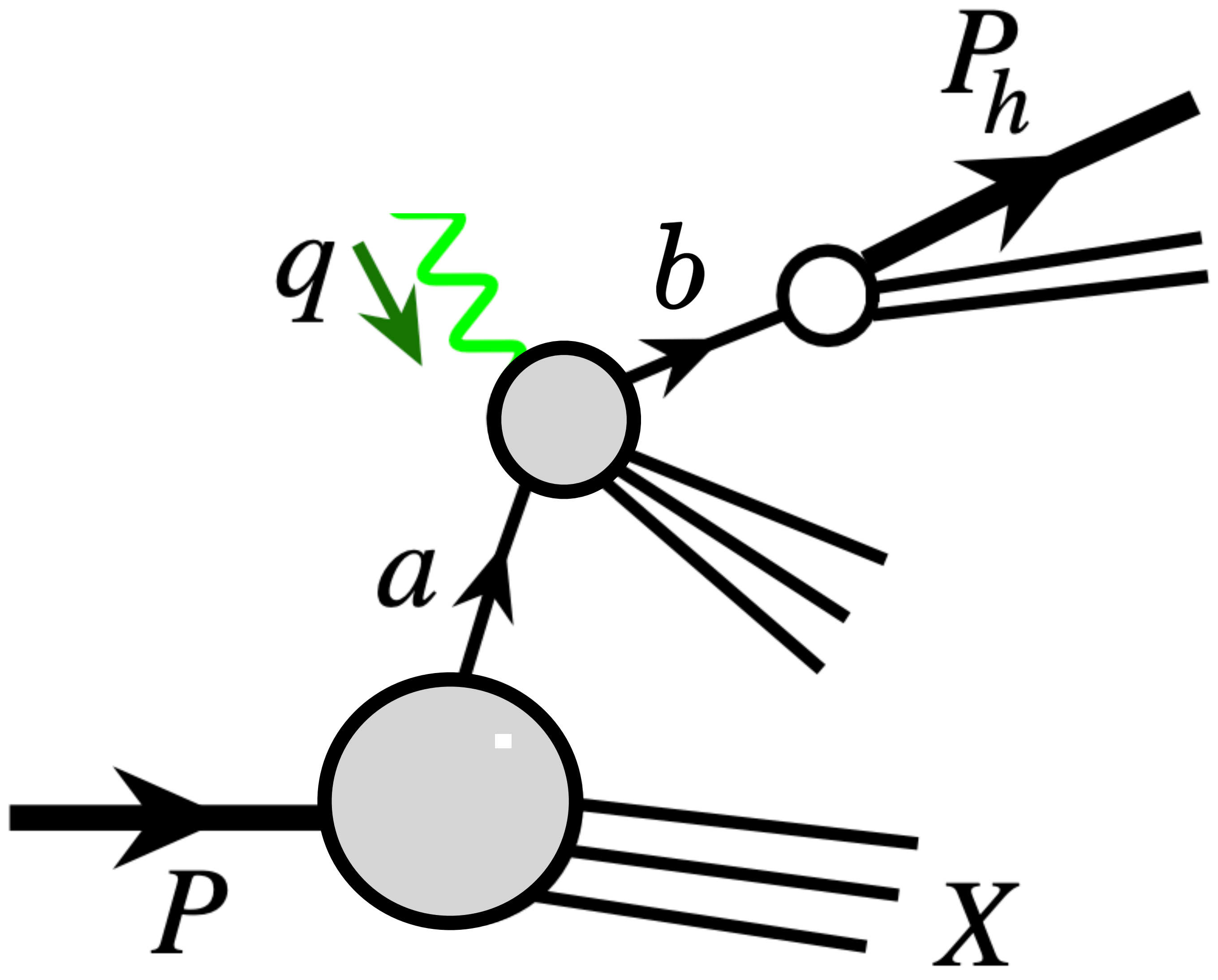
# Two regions in SIDIS




➡ Different interpretations for an azimuthal correlation

# Current Fragmentation Region (CFR)




✦ Collinear factorization: $P_{h_\perp} \gg \Lambda_{\mathrm{QCD}}$

$$\sigma \propto H(Q, P_{h\perp}) \otimes f_{a/P}(x, \mu^2) \otimes D_{h/b}(z, \mu^2)$$

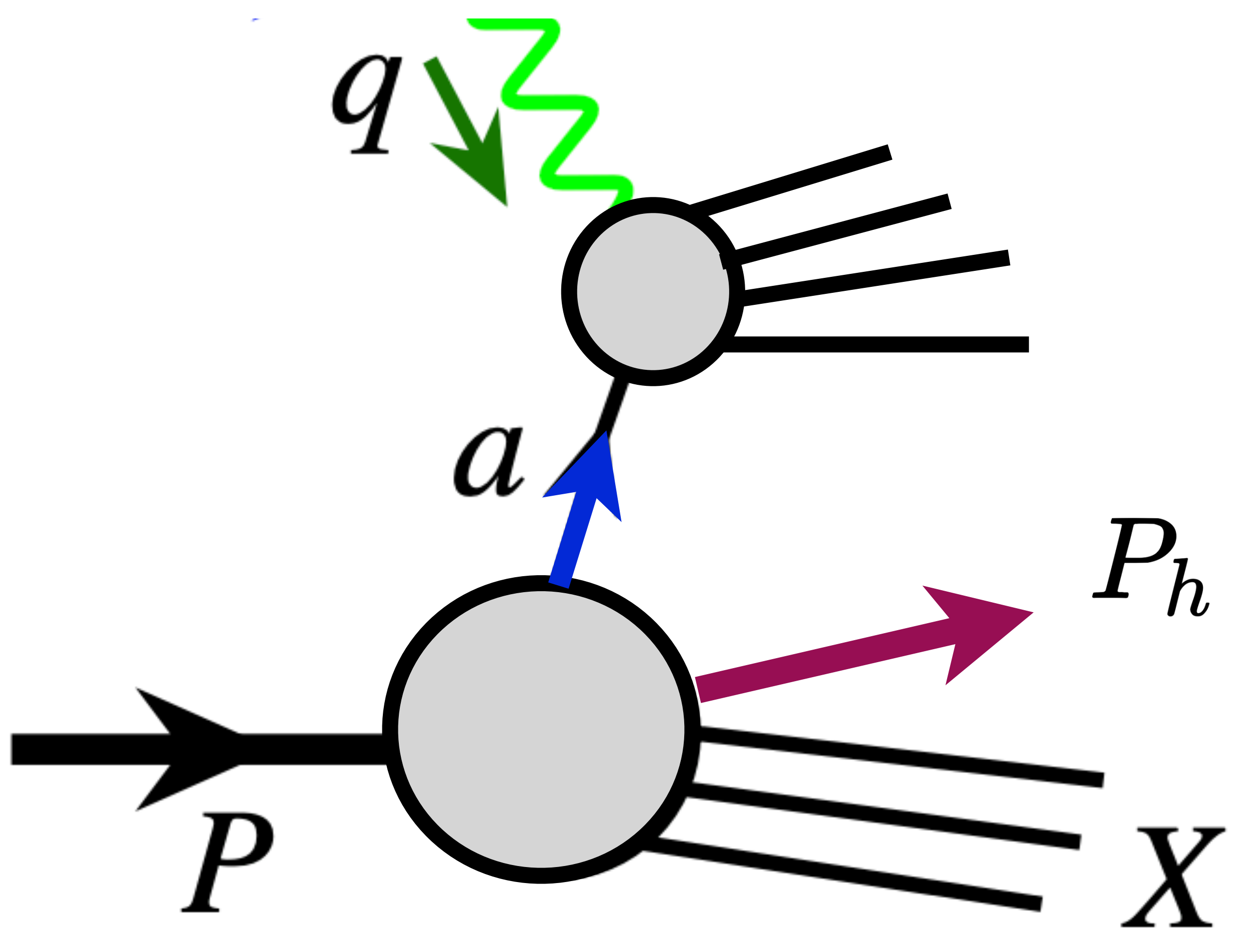
✦ TMD factorization: $P_{h\perp} \ll Q$

$$\sigma \propto H(Q) \otimes f_{a/P}(x, k_\perp, \mu^2) \otimes D_{h/b}(z, p_\perp, \mu^2)$$

- Because of $k_T$, there are more TMDs than collinear PDFs
  - Accommodate Sivers-effects, linearly polarized gluons, Boer-Mulder effects, etc.
- However, soft-gluon radiations play an important role
  - Sudakov effects
  - May generate asymmetries contaminate the interpretations

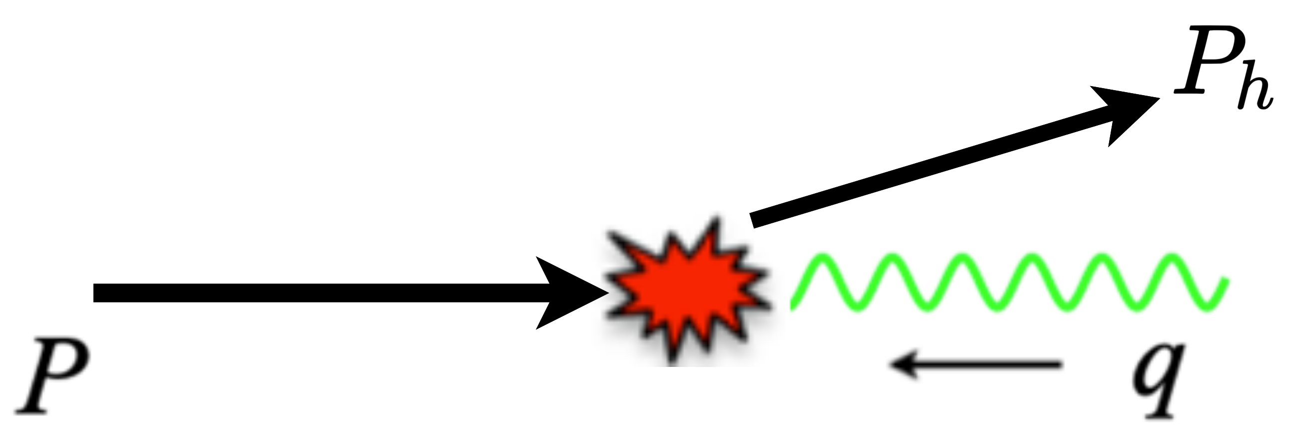
See e.g.. Hatta-Xiao-Yuan-Zhou, PRD 104, 054037 (2021).

# Target Fragmentation Region (TFR)

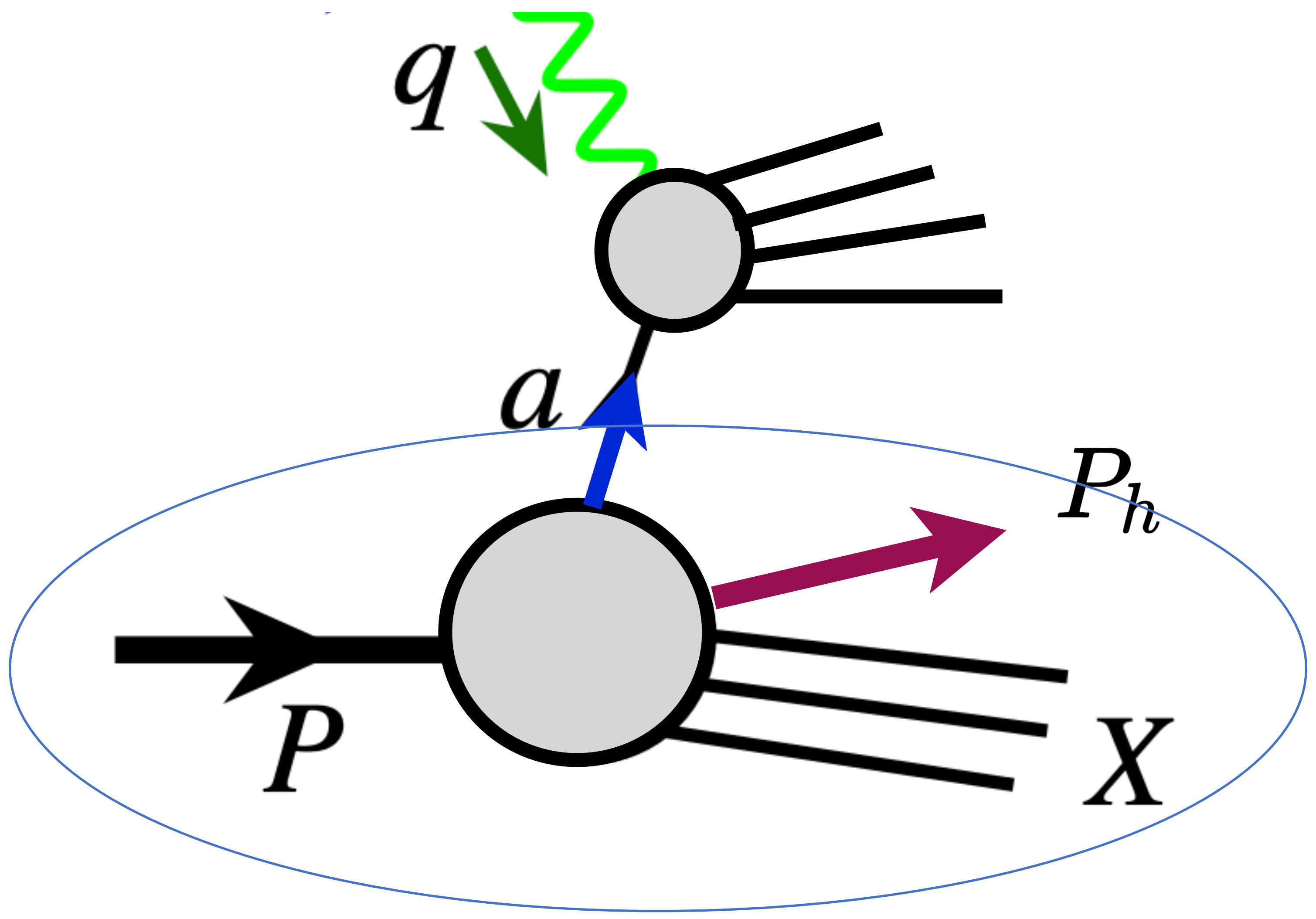



- Is there a probe:
  - ☑ Free of soft-gluon contributions, like inclusive DIS
  - ☑ Accommodate various correlation effects like TMDs

# Target Fragmentation Region (TFR)




- Fragmented from **the remnants of the target**, after a parton was struck out




$P_{h\perp} \ll Q$ ⇒

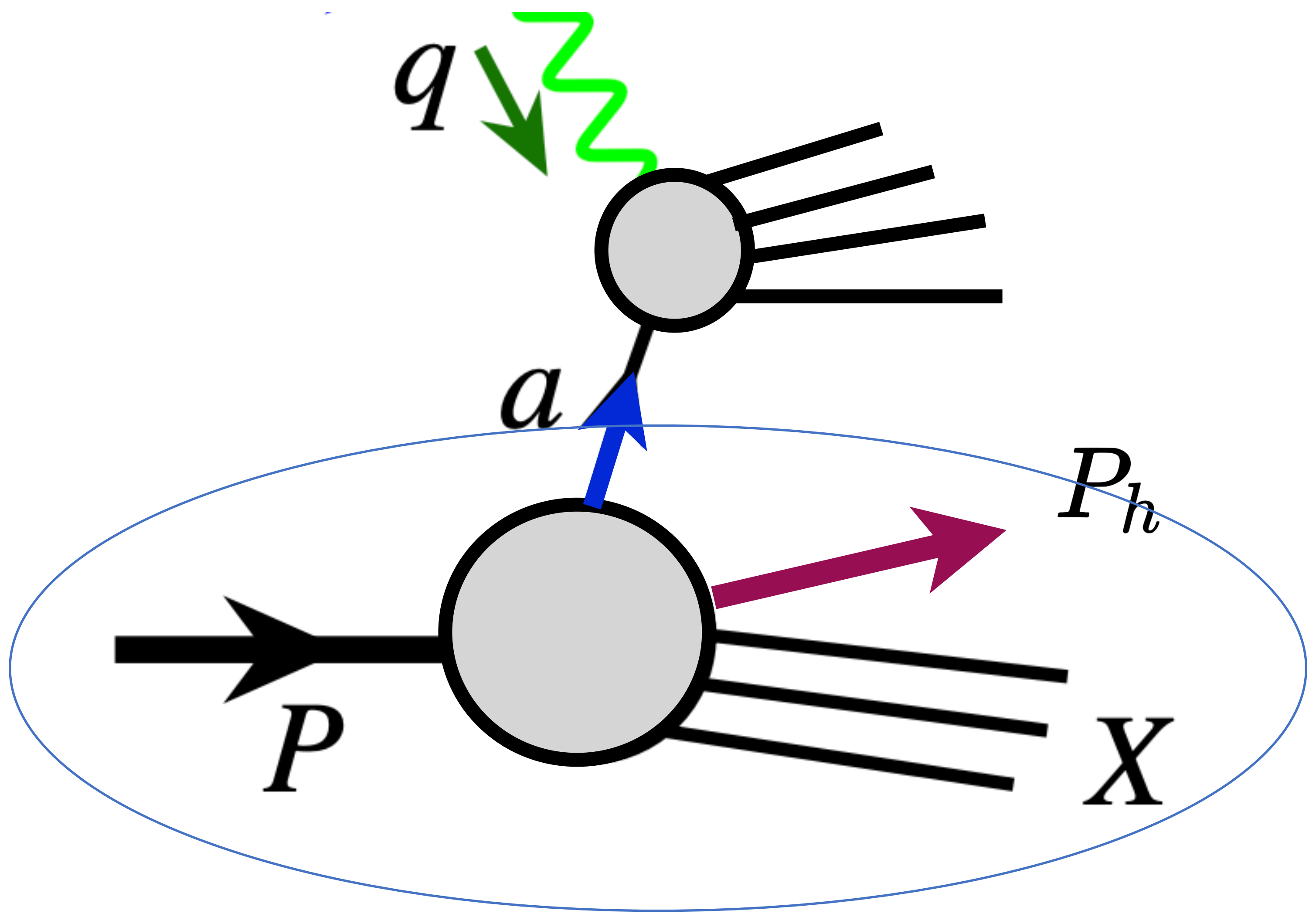
Strong correlations between
the initial patrons and the final hadron

# Fracture functions

Operator definition for collinear quark: [Berera-Soper *PRD 53 (1996) 6162*]

$$\mathcal{F}_{ij}(x,\xi_h,P_{h\perp}) \qquad \xi_h = \frac{P_h^+}{P^+}$$

$$= \frac{1}{2\xi_h(2\pi)^3}\int \frac{d\lambda}{2\pi} e^{-ixP^+\lambda} \sum_X \langle PS|[\bar{\psi}(\lambda n)\mathcal{L}_n^\dagger(\lambda n)]_j|XP_h\rangle\langle P_h X|[\mathcal{L}_n(0)\psi(0)]_i|PS\rangle$$

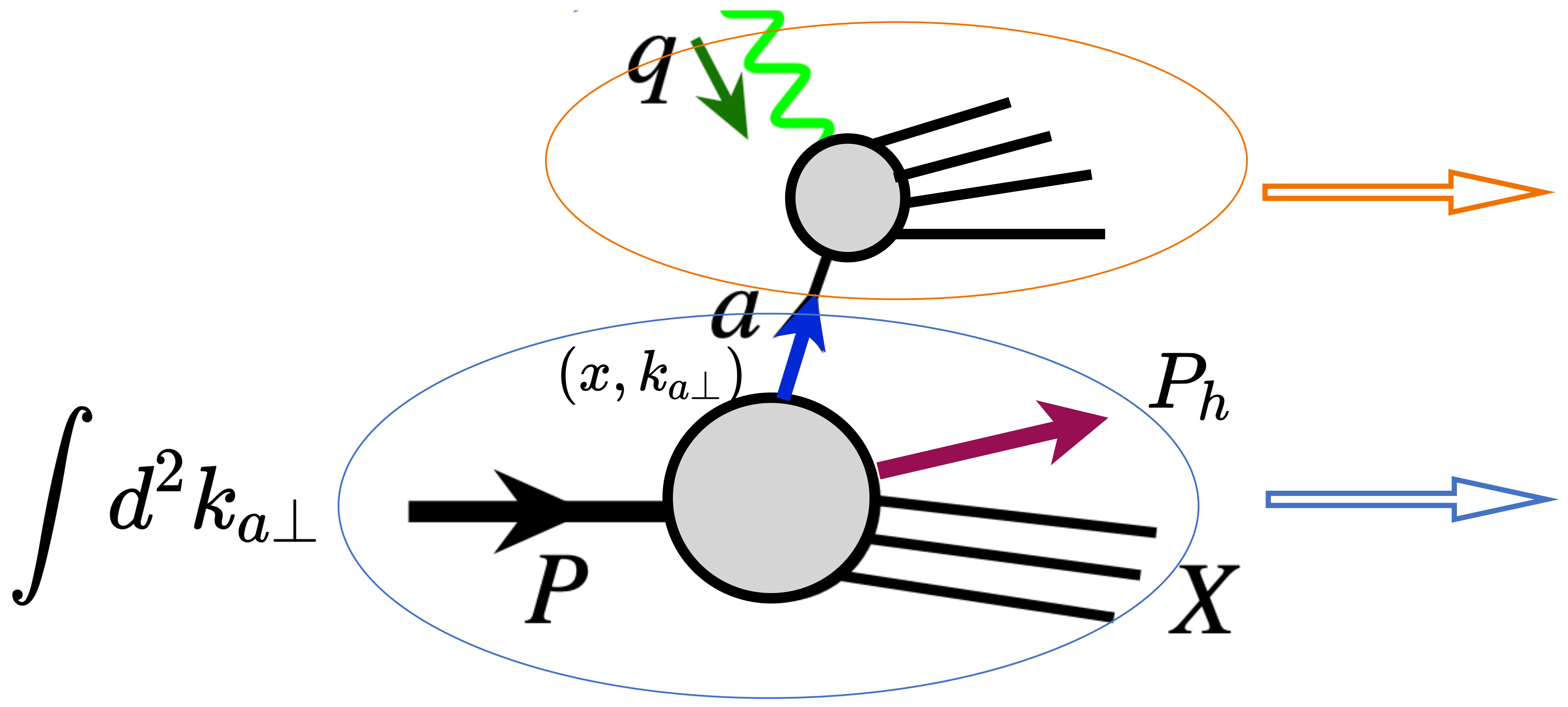



***Fracture*** **functions**

Trentadue-Veneziano
*PLB 323 (1994) 201*

- Describe the partonic ***structure*** of the target once it ***fragments*** into a given hadron $h$.
- Conditional probability; A combination between PDF and FFs
- h=P, also called diffractive PDFs

# SIDIS Factorization in the TFR




$k_{a\perp} \ll Q$ , thus neglected

$k_{a\perp}$-integrated fracture functions

- **Collinear factorization:** $\sigma \propto H(Q) \otimes \mathcal{F}_a(x, \xi, P_{h\perp})$ *Collins PRD 57 (1998) 3051*

  - Hard scattering, as simple as in inclusive DIS
    - Soft-gluon cancellation

✦ Collinear fracture functions: probe the nucleon structure by correlations between the initial state and final state

➡ **Azimuthal correlations** between $P_{h\perp}$ and **the parton/nucleon polarizations**

- parametrization analogous to TMDs

# Twist-2: Quark contributions

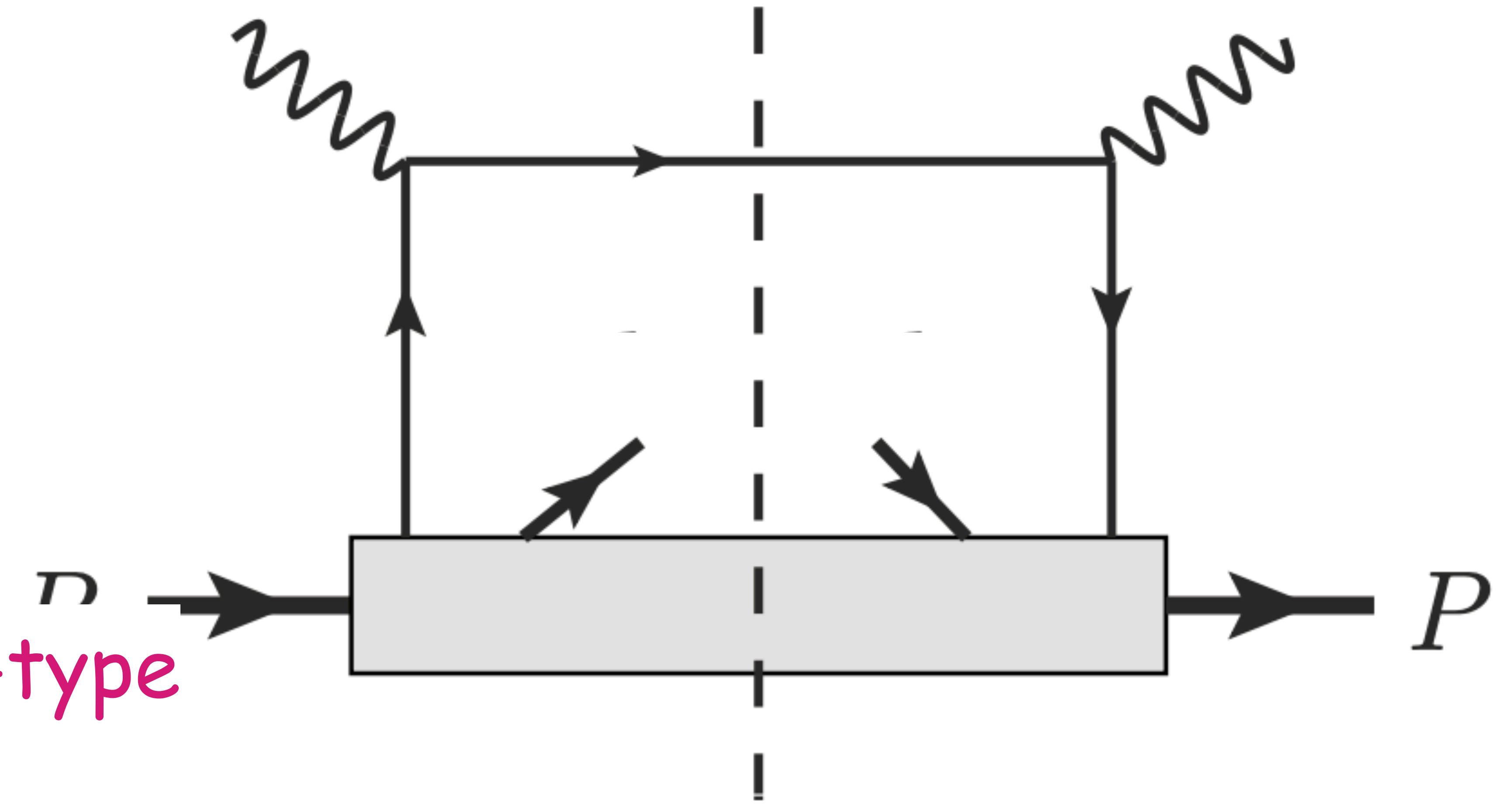
*Anselmino-Barone-Kotzinian,* PLB 699 (2011) 108

Quark polarization

| Nucleon polarization | U | L | T |
|---|---|---|---|
| U | $u_1$ | | $t_1^h$ |
| L | | $l_{1L}$ | $t_{1L}^h$ |
| T | $u_{1T}^h$ | $l_{1T}^h$ | $t_1, t_{1T}^h$ |

Boer-Mulder-type

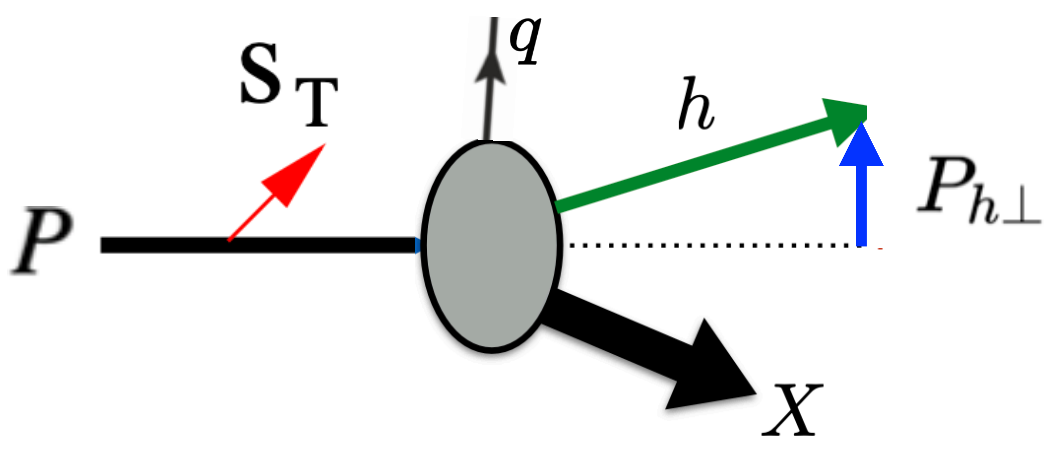
Sivers-type　Worm-gear




- Chiral even
- Four structure functions

$$F_{UU,T} = x_B u_1, \qquad F_{UT,T}^{\sin(\phi_h-\phi_S)} = \frac{|\vec{P}_{h\perp}|}{M} x_B u_{1T}^h,$$

$$F_{LL} = x_B l_{1L}, \qquad F_{LT}^{\cos(\phi_h-\phi_S)} = \frac{|\vec{P}_{h\perp}|}{M} x_B l_{1T}^h.$$



# Twist-2: Quark contributions

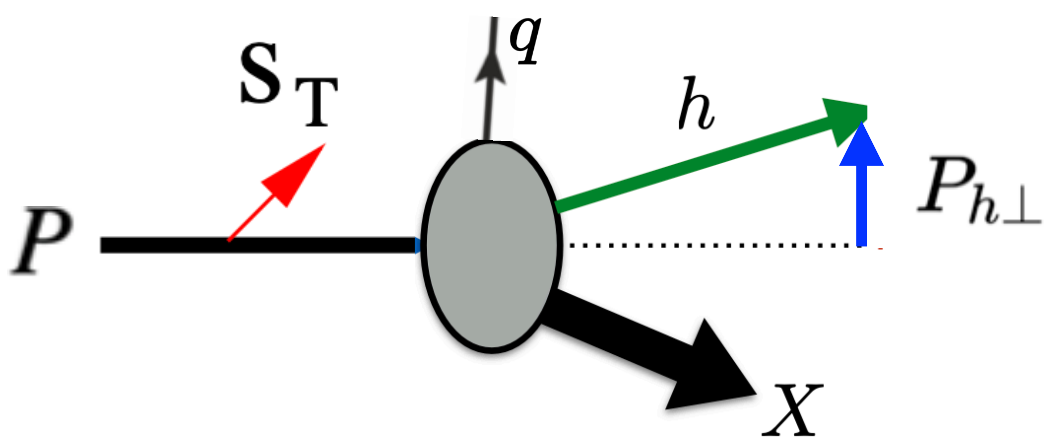
*Anselmino-Barone-Kotzinian,* PLB 699 (2011) 108

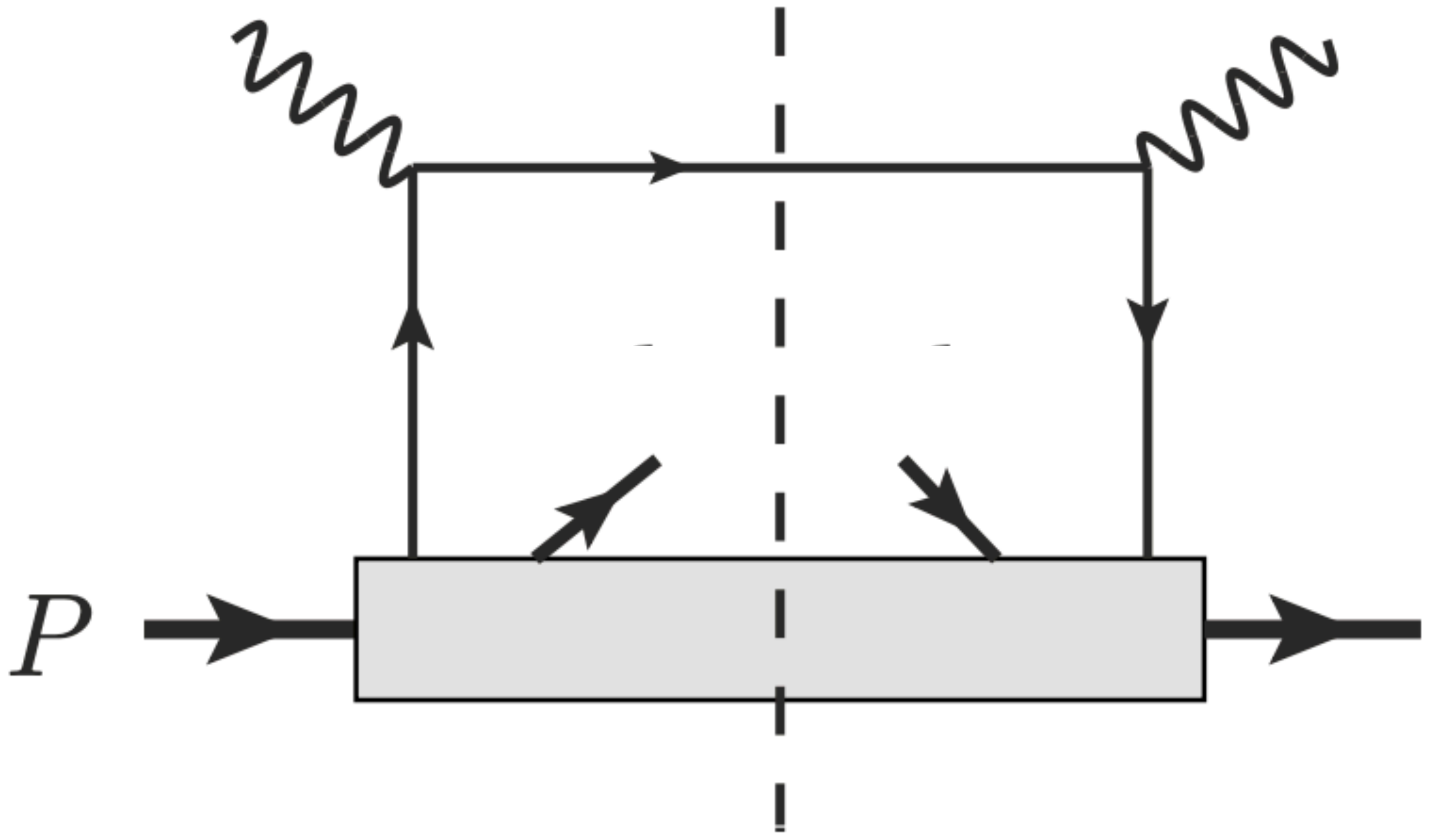
Quark polarization / Nucleon polarization

| | U | L | T |
|---|---|---|---|
| **U** | $u_1$ | | $t_1^h$ |
| **L** | | $l_{1L}$ | $t_{1L}^h$ |
| **T** | $u_{1T}^h$ | $l_{1T}^h$ | $t_1, t_{1T}^h$ |

Sivers-type  Worm-gear

Do not contribute!




- Chiral even
- Four structure functions

$$F_{UU,T} = x_B u_1, \qquad F_{UT,T}^{\sin(\phi_h-\phi_S)} = \frac{|\vec{P}_{h\perp}|}{M} x_B u_{1T}^h,$$

$$F_{LL} = x_B l_{1L}, \qquad F_{LT}^{\cos(\phi_h-\phi_S)} = \frac{|\vec{P}_{h\perp}|}{M} x_B l_{1T}^h.$$

The other 14 structure functions?

➡ Gluonic contributions and Twist-3 effects
(e.g., Linearly polarized gluons)

# Gluonic contributions

Chen-Ma-Tong, *JHEP* 05 (2024) 298

$$\mathcal{M}^{\alpha\beta} \propto \sum_X \langle PS| (G^{+\alpha}(\lambda n)\mathcal{L}_n^\dagger(\lambda n))^a |XP_h\rangle \langle P_h X| \left(\mathcal{L}_n(0) G^{+\beta}(0)\right)^a |PS\rangle$$

Gluon polarization

Nucleon polarization

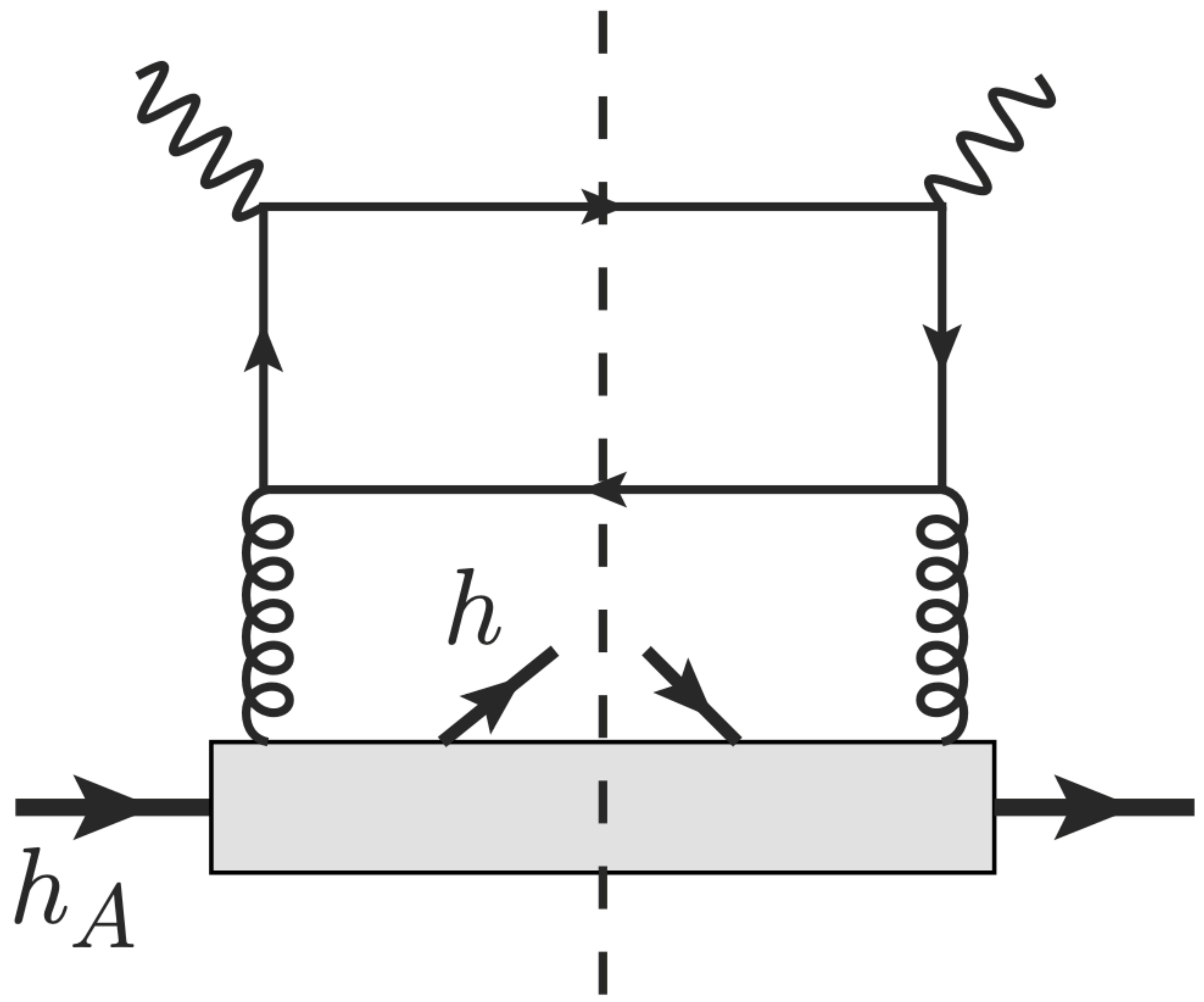
| | U | L | T |
|---|---|---|---|
| U | $u_{1g}$ | | $t_{1g}^h$ |
| L | | $l_{1gL}$ | $t_{1gL}^h$ |
| T | $u_{1gT}^h$ | $l_{1gT}^h$ | $t_{1gT}, t_{1gT}^h$ |

Unpolarized

Gluon helicity

Sivers-type

Worm-gear




- Start from one loop
- Eight gluonic fracture functions at twist 2

✦ Four of them mix with quark, only to yield the $\alpha_s$-corrections to the four structure functions at tree level

$$F_{UU,T}\ ,\ F_{LL}\ ,\ F_{UT,T}^{\sin(\phi_h-\phi_s)}\ ,\ F_{LT}^{\cos(\phi_h-\phi_s)}$$

# Gluonic contributions

Chen-Ma-Tong, *JHEP* 05 (2024) 298

$$\mathcal{M}^{\alpha\beta} \propto \sum_X \langle PS|\left(G^{+\alpha}(\lambda n)\mathcal{L}_n^\dagger(\lambda n)\right)^a |X P_h\rangle \langle P_h X| \left(\mathcal{L}_n(0) G^{+\beta}(0)\right)^a |PS\rangle$$

Gluon polarization

| Nucleon polarization | U | L | T |
|---|---|---|---|
| U | $u_{1g}$ | | $t_{1g}^h$ |
| L | | $l_{1gL}$ | $t_{1gL}^h$ |
| T | $u_{1gT}^h$ | $l_{1gT}^h$ | $t_{1gT}, t_{1gT}^h$ |

Linearly polarized gluon

Do not mix with quark!

$$\mathcal{M}_U^{\alpha\beta} = -\frac{1}{2} g_\perp^{\alpha\beta} u_{1g} + \frac{1}{2M^2}\left(P_{h\perp}^\alpha P_{h\perp}^\beta + \frac{1}{2} g_\perp^{\alpha\beta} P_{h\perp}^2\right) t_{1g}^h$$

$$\mathcal{M}_L^{\alpha\beta} = S_L\left(i\frac{\epsilon_\perp^{\alpha\beta}}{2} l_{1gL} + \frac{\tilde{P}_{h\perp}^{\{\alpha} P_{h\perp}^{\beta\}}}{4M^2} t_{1gL}^h\right)$$

$$\mathcal{M}_T^{\alpha\beta} = \frac{g_\perp^{\alpha\beta}}{2}\frac{P_{h\perp}\cdot \tilde{S}_\perp}{M} u_{1gT}^h + \frac{P_{h\perp}\cdot S_\perp}{M} i\frac{\epsilon_\perp^{\alpha\beta}}{2} l_{1gT}^h - \frac{P_{h\perp}\cdot S_\perp}{M}\frac{\tilde{P}_{h\perp}^{\{\alpha} P_{h\perp}^{\beta\}}}{4M^2} t_{1gT}^h + \frac{\tilde{P}_{h\perp}^{\{\alpha} S_\perp^{\beta\}} + \tilde{S}_\perp^{\{\alpha} P_{h\perp}^{\beta\}}}{8M} t_{1gT}$$

✦ These four generate four unique azimuthal modulations, without quark's interference !

$$F_{UU}^{\cos 2\phi_h}, F_{UL}^{\sin 2\phi_h}, F_{UT}^{\sin(3\phi_h-\phi_S)}, F_{UT}^{\sin(\phi_h+\phi_S)}$$

# Novel contributions from linearly polarized gluons

- Four azimuthal modulations uniquely determined by gluons

$$F_{UU}^{\cos 2\phi_h} = -\frac{\alpha_s T_F}{2\pi} x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B}^{1-\xi_h} \frac{dz}{z}\left(\frac{x_B}{z}\right)^2 \frac{P_{h\perp}^2}{2M^2} t_{1g}^h(z,\xi_h,P_{h\perp}),$$

T-even, linear polarized gluons in an unpolarized target

$$F_{UL}^{\sin 2\phi_h} = \frac{\alpha_s T_F}{2\pi} x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B}^{1-\xi_h} \frac{dz}{z}\left(\frac{x_B}{z}\right)^2 \frac{P_{h\perp}^2}{2M^2} t_{1gL}^h(z,\xi_h,P_{h\perp}),$$

$$F_{UT}^{\sin(3\phi_h-\phi_S)} = \frac{\alpha_s T_F}{2\pi} x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B}^{1-\xi_h} \frac{dz}{z}\left(\frac{x_B}{z}\right)^2 \frac{P_{h\perp}^3}{4M^3} t_{1gT}^h(z,\xi_h,P_{h\perp}),$$

T-odd, single spin asymmetry

$$F_{UT}^{\sin(\phi_h+\phi_S)} = \frac{\alpha_s T_F}{2\pi} x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B}^{1-\xi_h} \frac{dz}{z}\left(\frac{x_B}{z}\right)^2 \frac{P_{h\perp}}{2M}\left[t_{1gT}(z,\xi_h,P_{h\perp}) + \frac{P_{h\perp}^2}{2M^2} t_{1gT}^h(z,\xi_h,P_{h\perp})\right]$$

- Free of quark contributions to all loops;

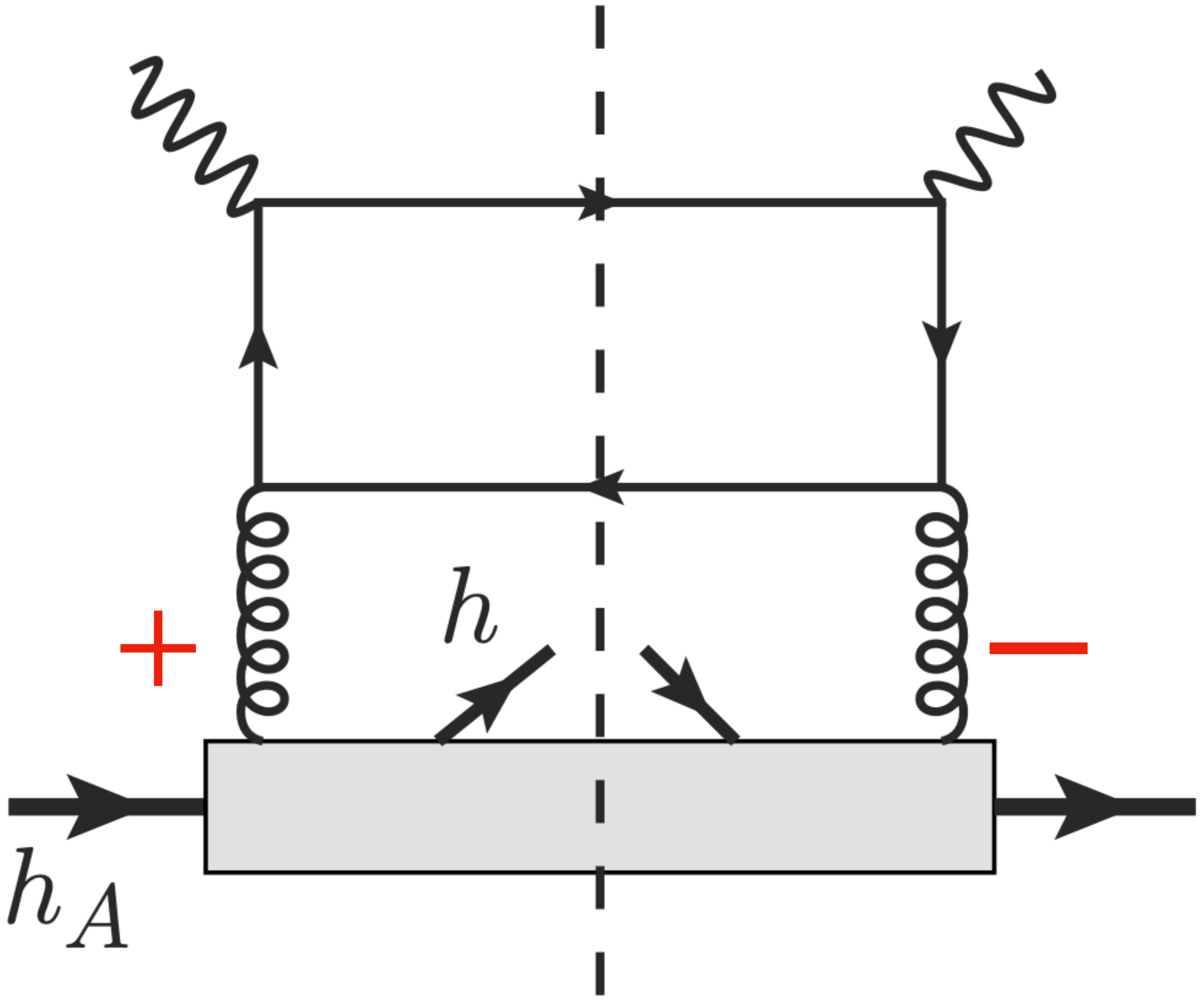
  ➡ Provide unique probes into gluon dynamics

- CFR: the Collins mechanism through the quark channels

Gluon polarization (columns) / Nucleon polarization (rows)

| | U | L | T |
|---|---|---|---|
| U | $u_{1g}$ | | $t_{1g}^h$ |
| L | | $l_{1gL}$ | $t_{1gL}^h$ |
| T | $u_{1gT}^h$ | $l_{1gT}^h$ | $t_{1gT}, t_{1gT}^h$ |

# Hadrons in TFR: a polarizing filter for gluons




- The TFR hadron dose not directly participate in the hard scattering
- However,  it can serves as a polarizing filter to select the initial gluons with linear polarizations

# Longitudinal photon

- At one loop,

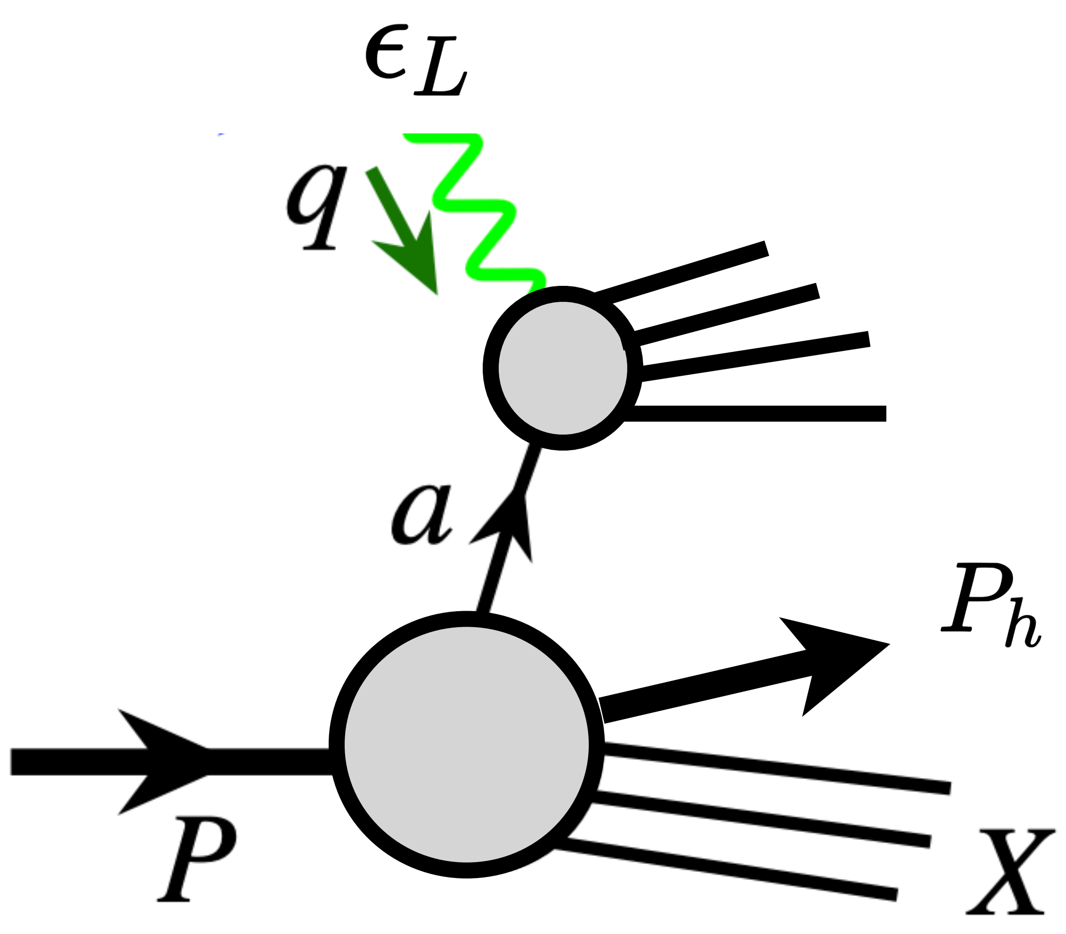
$$F_{UU,L} = \frac{\alpha_s}{2\pi} x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B}^{1-\xi_h} \frac{dz}{z} \left[4T_F z\bar{z} u_{1g}(z,\xi_h,P_{h\perp}) + 2C_F z u_1(z,\xi_h,P_{h\perp})\right]$$

Unpolarized gluon/quarks

$$F_{UT,L}^{\sin(\phi_h-\phi_S)} = \frac{\alpha_s}{2\pi} \frac{P_{h\perp}}{M} x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B}^{1-\xi_h} \frac{dz}{z} \left[4T_F z\bar{z} u_{1gT}^h(z,\xi_h,P_{h\perp}) + 2C_F z u_{1T}^h(z,\xi_h,P_{h\perp})\right]$$

Sivers-type gluon/quark

- Hard coefficients are the same as $F_L$ in the inclusive DIS
- TFR: $\sigma_L/\sigma_T \propto \alpha_s$ at twist-2
- CFR: the twist-4 TMDs at $\mathcal{O}(\alpha_s^0)$

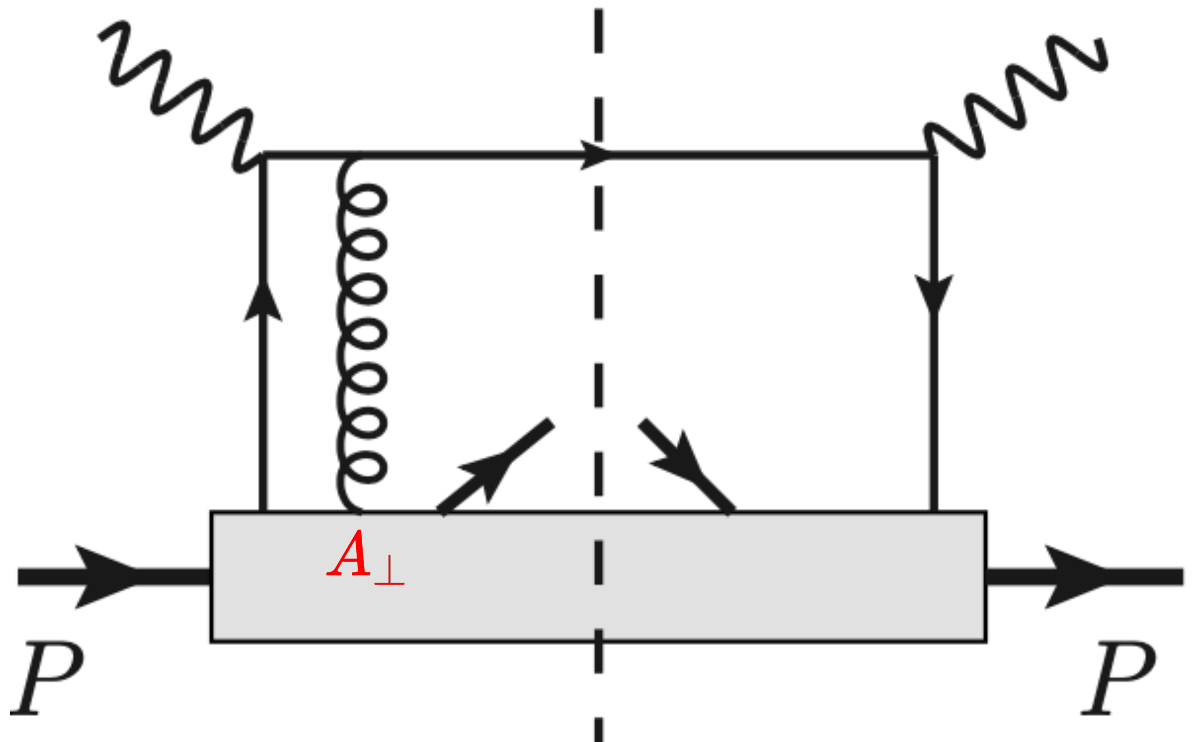


# Twist-3 contributions

*Chen-Ma-Tong, PRD* 108 (2023) 9, 9

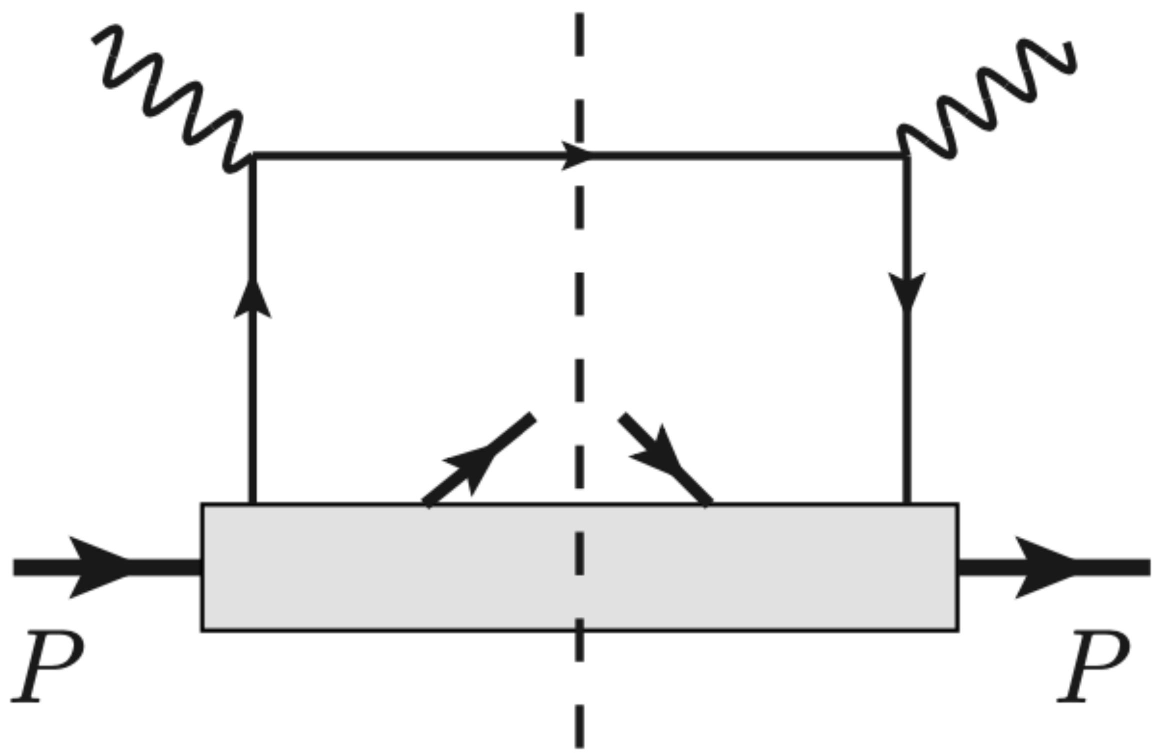
- Higher-twist effects are non-trivial even at tree level:

"Fast moving quark" + "slowly moving" gluon




$\langle PS|\bar\psi\gamma^+|P_hX\rangle\langle XP_h|D_\perp\psi|PS\rangle$

"slowly moving" quark




$\langle PS|\bar\psi\gamma_\perp|P_hX\rangle\langle XP_h|\psi|PS\rangle$

- Not independent, related by QCD equations of motion
- Expressed with the twist-3 quark distributions only:

$$\mathcal{M}^{[\gamma_\perp^\mu]} = \frac{1}{P^+}\left(P_{h\perp}^\mu u^h - S_L\tilde P_{h\perp}^\mu u_L^h - \tilde S_\perp^\mu M u_T - \frac{P_{h\perp}^\mu P_{h\perp}^\nu - \frac{1}{2}P_{h\perp}^2 g_\perp^{\mu\nu}}{M}\tilde S_{\perp\nu} u_T^h\right)$$

$$\mathcal{M}^{[\gamma_\perp^\mu\gamma_5]} = \frac{1}{P^+}\left(\tilde P_{h\perp}^\mu l^h + S_L P_\perp^\mu l_L^h + S_\perp^\mu M l_T - \frac{P_{h\perp}^\mu P_{h\perp}^\nu - \frac{1}{2}P_{h\perp}^2 g_\perp^{\mu\nu}}{M} S_{\perp\nu} l_T^h\right) \qquad \tilde a^\mu \equiv \epsilon_\perp^{\mu\nu} a_\nu$$

# Twist-3 contributions

*Chen-Ma-Tong*, *PRD* 108 (2023) 9, 9

- Eight structure functions contribute at twist-3, which are all missing at twist-2.

$$F_{UU}^{\cos\phi_h} = -\sum_q e_q^2 \frac{2|\vec{P}_{h\perp}|}{Q} x_B^2 u^h$$

$$F_{LU}^{\sin\phi_h} = \sum_q e_q^2 \frac{2|\vec{P}_{h\perp}|}{Q} x_B^2 l^h$$

$$F_{UL}^{\sin\phi_h} = -\sum_q e_q^2 \frac{2|\vec{P}_{h\perp}|}{Q} x_B^2 u_L^h$$

$$F_{LL}^{\cos\phi_h} = -\sum_q e_q^2 \frac{2|\vec{P}_{h\perp}|}{Q} x_B^2 l_L^h$$

$$F_{UT}^{\sin\phi_S} = -\sum_q e_q^2 \frac{2M}{Q} x_B^2 u_T$$

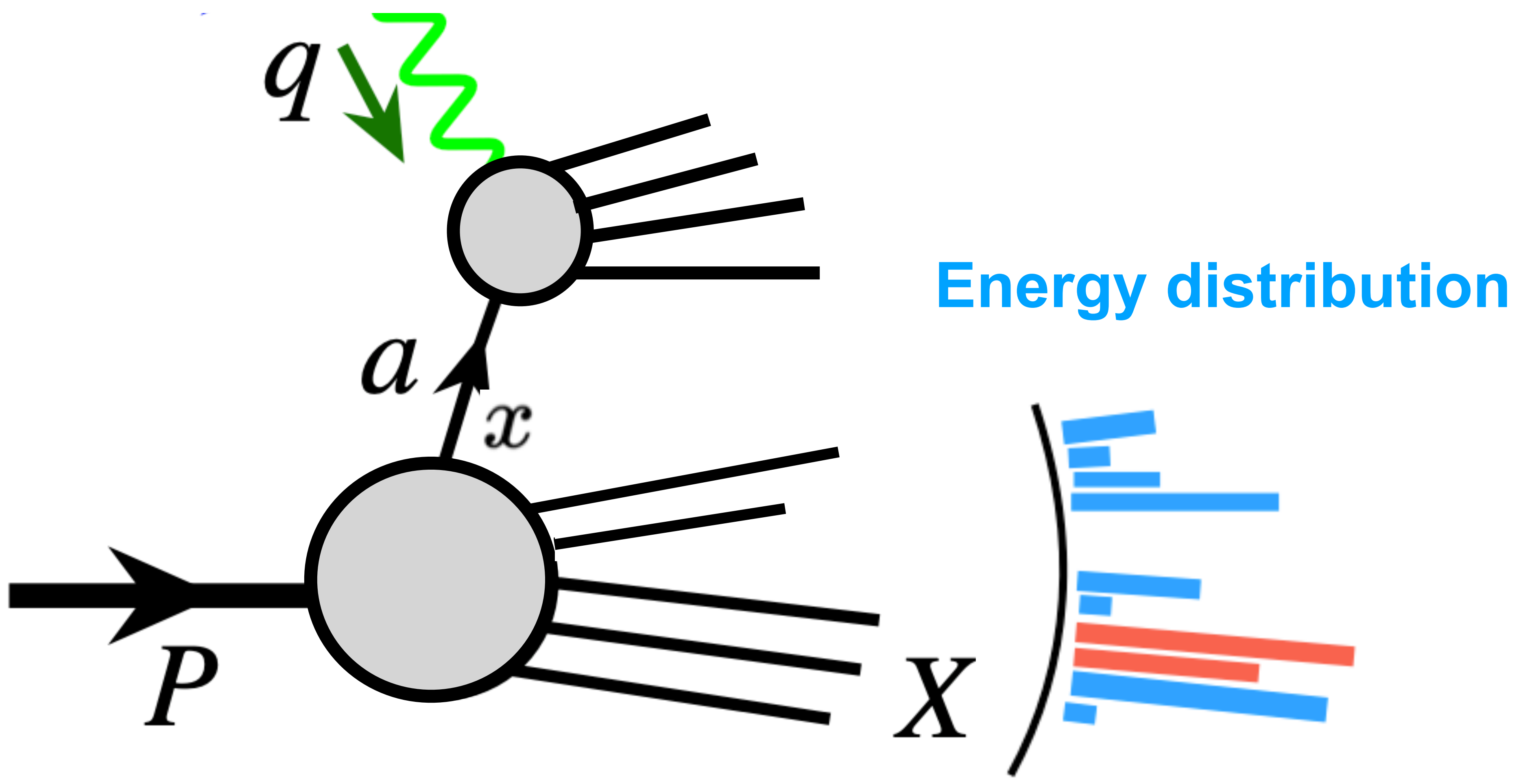
$$F_{UT}^{\sin(2\phi_h-\phi_S)} = -\sum_q e_q^2 \frac{\vec{P}_{h\perp}^2}{QM} x_B^2 u_T^h$$

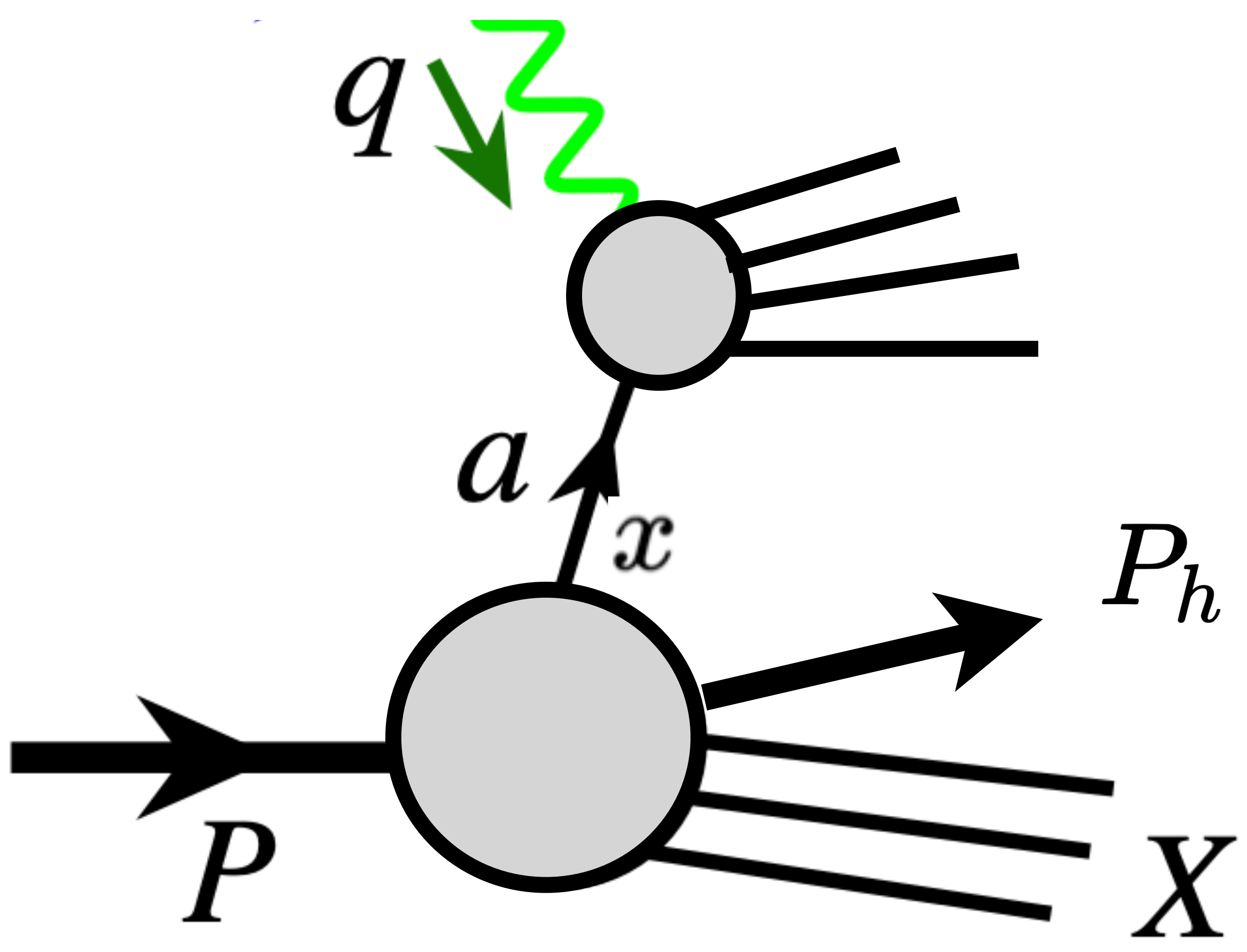
$$F_{LT}^{\cos\phi_S} = -\sum_a e_q^2 \frac{2M}{Q} x_B^2 l_T$$

$$F_{LT}^{\cos(2\phi_h-\phi_S)} = -\sum_q e_q^2 \frac{\vec{P}_{h\perp}^2}{QM} x_B^2 l_T^h$$

  - Connected to multi-parton correlations; Have no simple probability interpretation.

# DIS energy pattern in the TFR




$$\Sigma(\theta,\phi) = \sum_h \int d\sigma^{e+N\to e+h+X} \frac{E_h}{E_N} \delta\big(\theta^2 - \theta_h^2\big)\delta(\phi - \phi_h)$$

- Clearer probe, less sensitivity to the non-perturbative effects in final-state [Meng-Olness-Soper NPB 371 (1992) 79]
- Energy pattern in the TFR, factorized with **nucleon-energy correlator** [Liu-Zhu PRL 130, 2023]
- The factorization of SIDIS can be extended to the DIS energy pattern: [*Chen-Ma-Tong, JHEP* 08 (2024) 227]
  - ✓using the sum rule between fracture function and nucleon-energy correlator
  - ✓All 18 energy-pattern structure functions are derived

# Summary

- SIDIS in the TFR
  - Fracture function:
    - Probe the nucleon structure through the correlation between the initial state and the final state
  - All eighteen structure functions are derived in terms of fracture functions
    - Gluonic contributions and higher-twist effects
    - Hadrons in the TFR: a polarizing filter for gluons
  - Extended to the DIS energy pattern in the TFR using the connection between fracture function and nucleon-energy correlator

# Backup:

## Nucleon energy correlator Liu-Zhu PRL 130, 2023

$$\mathcal{M}^{q}_{ij,\mathrm{EEC}}(x,\theta,\phi) = \int \frac{d\eta^-}{2\pi} e^{-ixP^+\eta^-} \langle PS|\bar{\psi}_j(\eta^-)\mathcal{L}^\dagger_n(\eta^-)\mathcal{E}(\theta,\phi)\mathcal{L}_n(0)\psi_i(0)|PS\rangle$$

$$\mathcal{E}(\theta,\phi)|X\rangle = \sum_{a\in X} \delta(\theta^2-\theta_a^2)\delta(\phi-\phi_a)\frac{E_a}{E_N}|X\rangle$$

## Fracture functions are the parents functions of nucleon energy correlator

$$\mathcal{M}^{q}_{ij,\mathrm{EEC}}(x,\theta,\phi) = \sum_h \int_0^{1-x} \xi_h d\xi_h \int d^2\boldsymbol{P}_{h\perp}\delta\left(\theta^2-\theta_h^2\right)\delta(\phi-\phi_h)\mathcal{M}^{q}_{ij,\mathrm{FrF}}(x,\xi_h,\boldsymbol{P}_{h_\perp})$$

- One-to-one correspondence Chen-Ma-Tong, *JHEP* 08 (2024) 227, 2406.08559
- Parton-level proxy

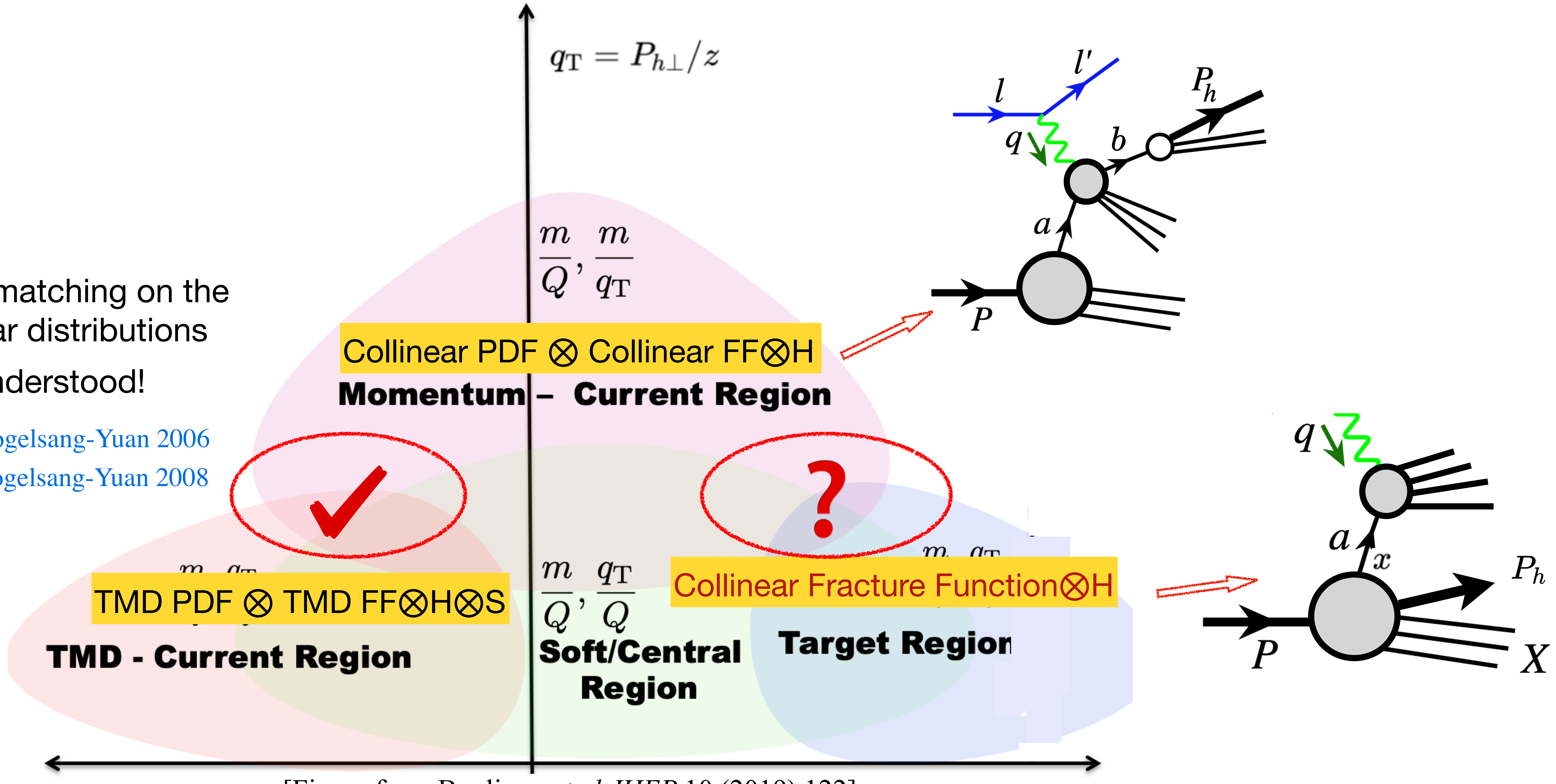
# Backup: Matching of Fracture Functions at large $P_{h\perp}$

TMDs matching on the collinear distributions

Well-understood!

Ji-Qiu-Vogelsang-Yuan 2006

Koike-Vogelsang-Yuan 2008




[Figure from Boglione *et al JHEP* 10 (2019) 122]

In the intermediate $P_{h\perp}$ region, how are the two approaches should be consistent

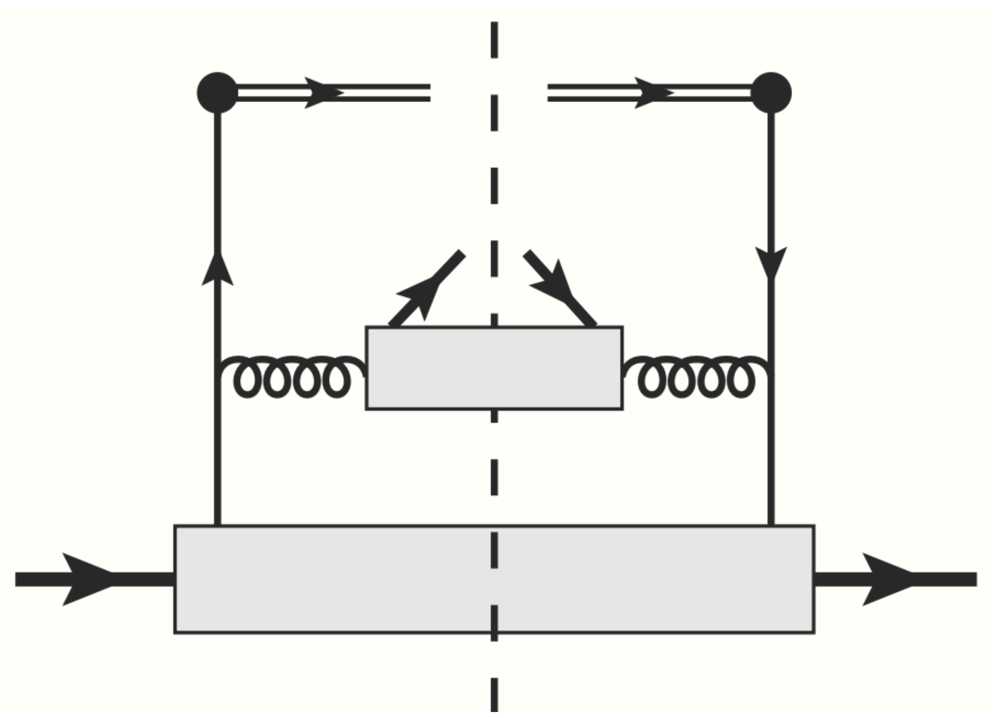
Chen-Ma-Tong JHEP 11 (2021) 038

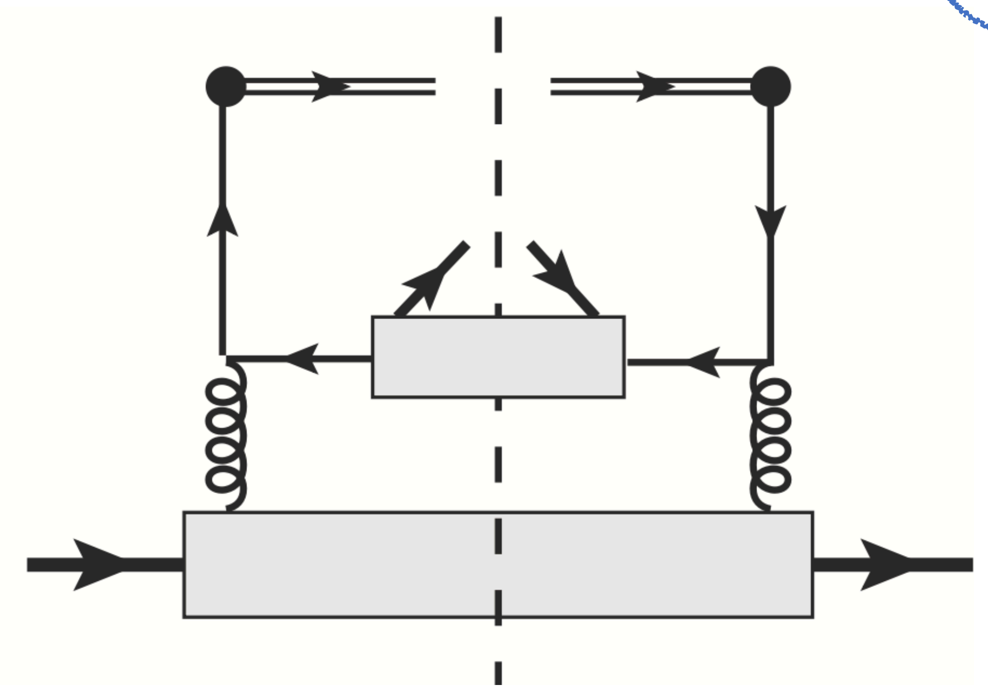
# Backup: Quark fracture functions at large $P_{h\perp}$

Chen-Ma-Tong JHEP 11 (2021) 038

## Unpolarized/helicity quark:

$$u_1, l_{1L} \propto \frac{1}{P_{h\perp}^2} \otimes \text{twist-2 FFs} \otimes \text{twist-2 PDFs}$$



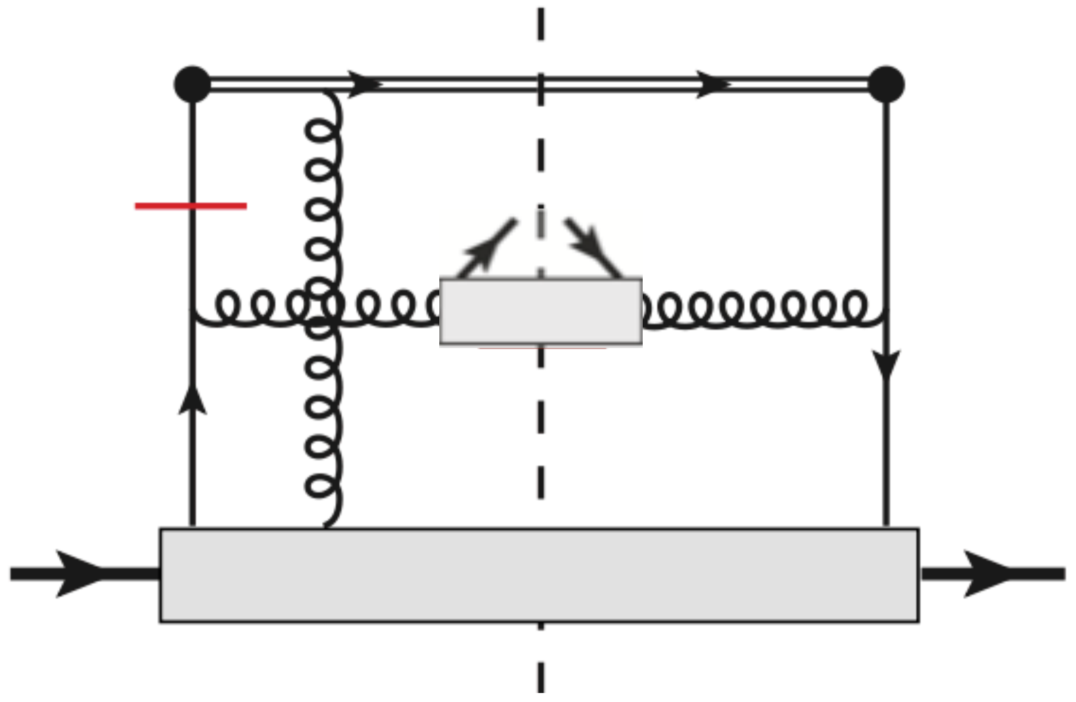


$$u_1^q\left(x,\xi_h,\boldsymbol{P}_{h\perp}^2\right) = \int_{\frac{\xi_h}{1-x}}^1 \frac{dz}{z^2} \int_x^1 dy\delta(x+\xi_h/z-y)\frac{\alpha_s z^2}{2\pi^2\xi_h \boldsymbol{P}_{h\perp}^2} \times\left[C_F\frac{x^2+y^2}{y^2}d_{h/g}(z)q(y)+T_F\left(1-\frac{x}{y}\right)\left[\frac{x^2}{y^2}+\left(1-\frac{x}{y}\right)^2\right]d_{h/\bar{q}}(z)g(y)\right]$$

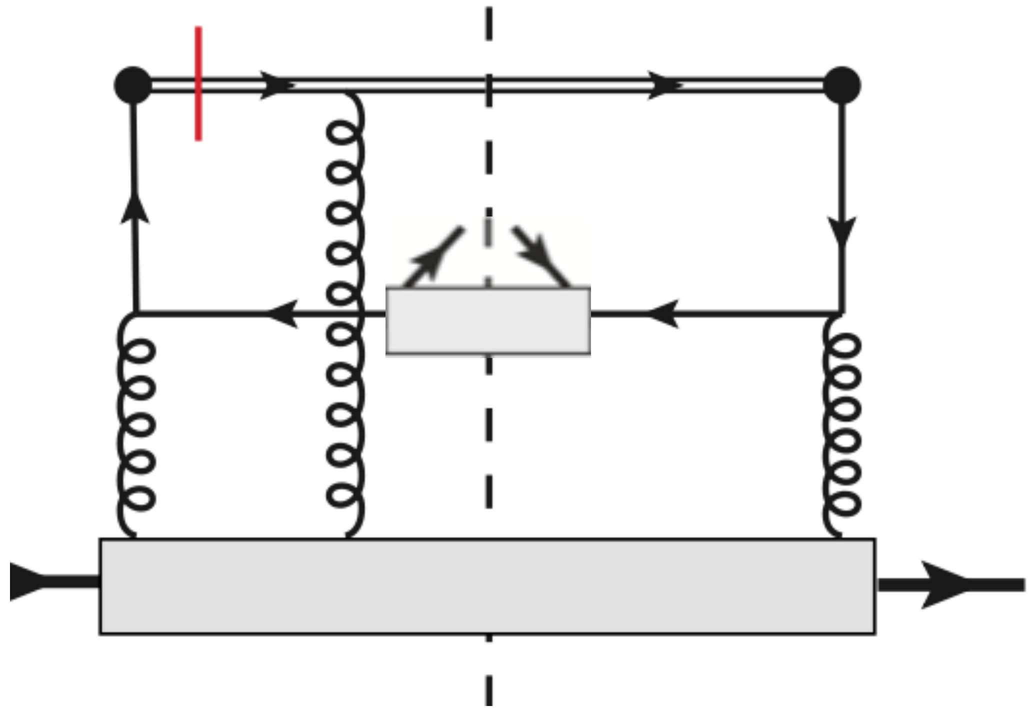
## Sivers & Worm-gear quark:

$$u_{1T}, l_{1T} \propto \frac{1}{P_{h\perp}^4} \otimes \text{twist-2 FFs} \otimes \text{twist-3 multi-parton distributions}$$

Follows the Efremov-Teryaev-Qiu-Sterman mechanism



Quark-gluon-quark



Three-gluon

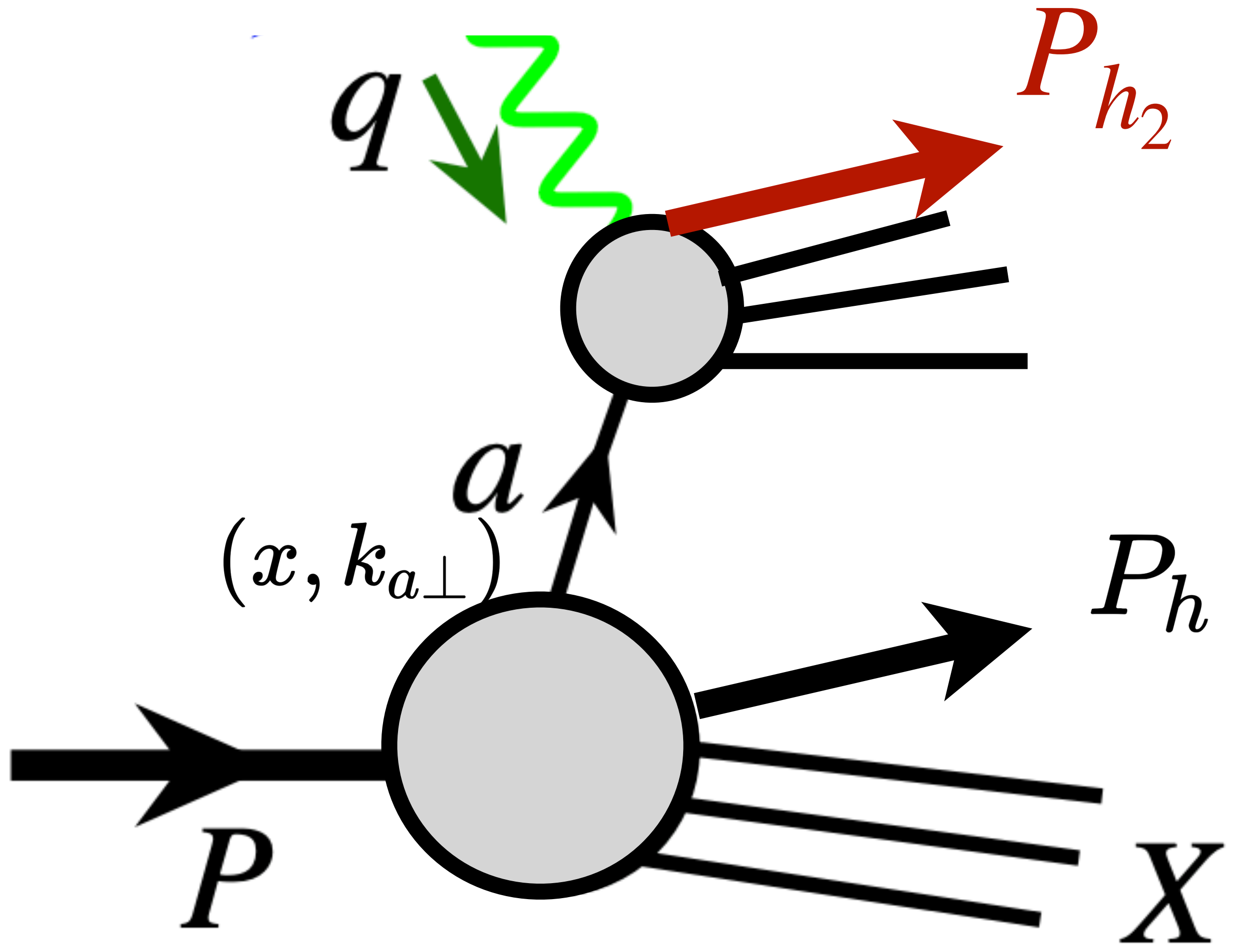
Sivers-type: T-odd, single spin asymmetry

Need a non-trivial phase to be generated in the perturbative region

Worm-gear-type: T-even, double spin asymmetry

# Backup: TMD fracture functions & Dihadron production

$$\int d^2 k_{a\perp}$$



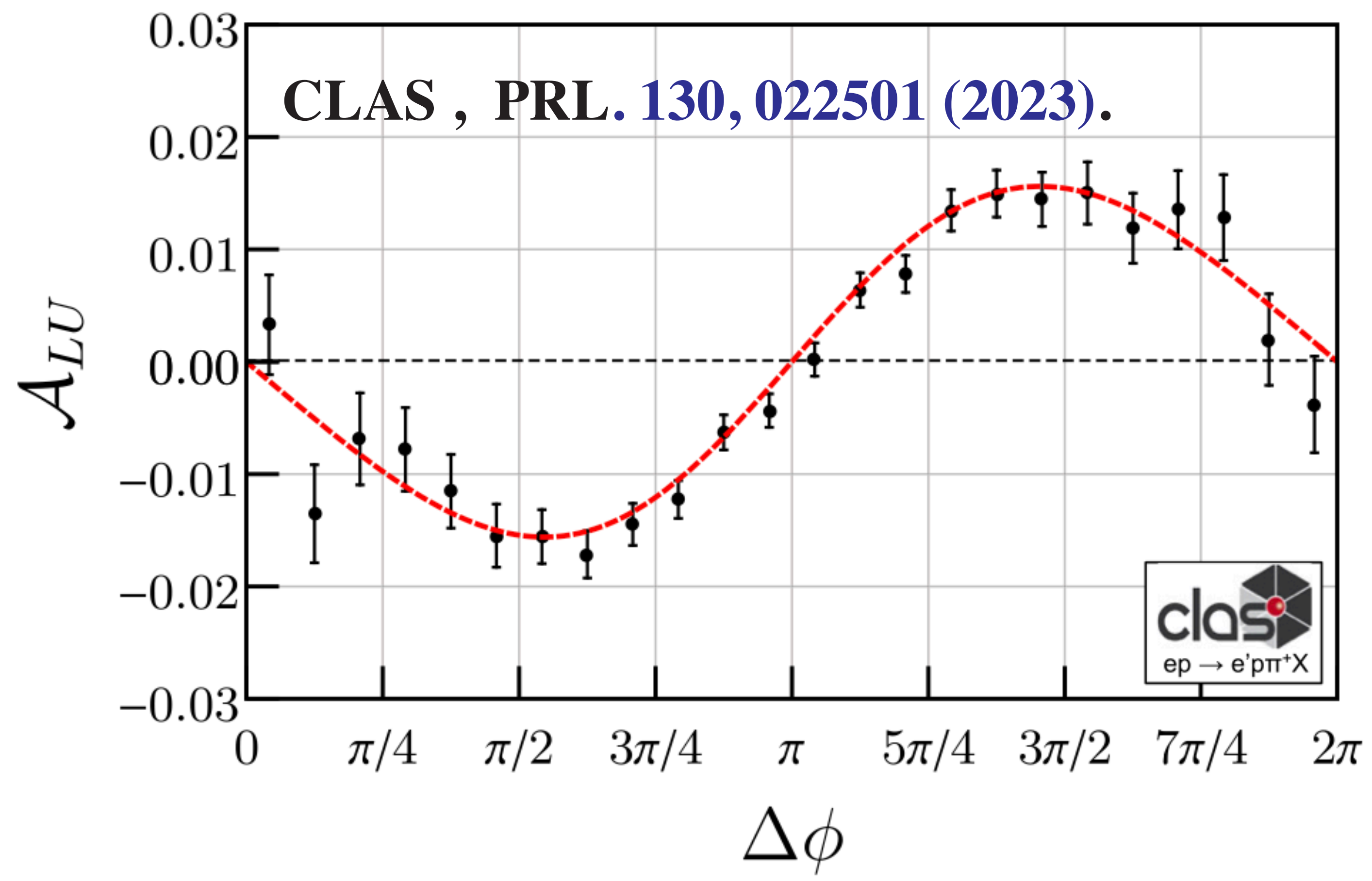

The hadron $P_{h_2\perp}$ in the CFR can resolve the initial $k_{a\perp}$

$k_{a\perp}$ un-integrated →TMD *fracture* functions

★**TMD factorization:** $\sigma \propto H(Q) \otimes \mathcal{F}_a(x, k_\perp, \xi, P_{h\perp})$

- Spin-dependence: Anselmino-Barone-Kotzinian, PLB 699 (2011) 108
  PLB 706, 46 (2011);
  PLB 713, 317 (2012).
- TMD Evolutions: Chen-Ma-Tong, JHEP 10 (2019) *285*
- Diffraction and small-x: Hatta-Xiao-Yuan PRD 106 (2022) 094015
  Hatta-Yuan PLB 854 (2024) 138738
  Iancu-Mueller-Triantafyllopoulos, PRL. 128 (2022) 202001

•Beam-spin asymmetry




-An explanation: *Guo-Yuan* 2312.01008