

TMDPDFs extractions employing AI

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Outline

- Motivation
- A brief introduction to TMDs
- Sivers asymmetry from SIDIS
- DNN Approach for $SU(3)_{\text{flavor}}$
- DNN Method Testing
- DNN Fits & Results for Sivers function
- Summary and Outlook

Motivation

- TMDs: so far, model-dependent extractions → assumptions, limitations and biases
- Information from data: Has the full potential of the data been taken into account?

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- TMDs: so far, model-dependent extractions → assumptions, limitations and biases
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Introducing this ‘novel’ method with Deep Neural Network (DNN) **First time in TMD extractions**

- Capacity to handle complex patterns, relationships in data with multi-D dependence.
- Data-driven
- Minimally biased → un-biased
- Uncertainty propagation (from data) using bootstrap method by generating ‘replicas’
(Statistical & Systematic uncertainties from the experimental data are combined in quadrature)
- Recursive improvements to the DNN
- Systematic component can be quantified by a dedicated analysis

Intended to be
Exploratory & Instructional

TMD PDFs

$$\Phi(x, k_T; S) = \int \frac{d\xi^- d\xi_T}{(2\pi)^3} e^{ik \cdot \xi} \langle P, S | \bar{\psi}(0) \mathcal{U}_{[0, \xi]} \psi(\xi) | P, S \rangle |_{\xi^+ = 0}$$

At leading-twist, the Quark correlator can be decomposed into 8 components (6 T - even and 2 T -odd terms)

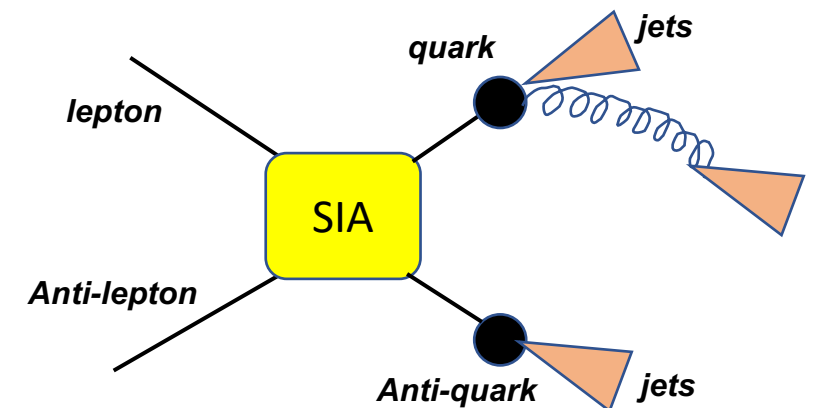
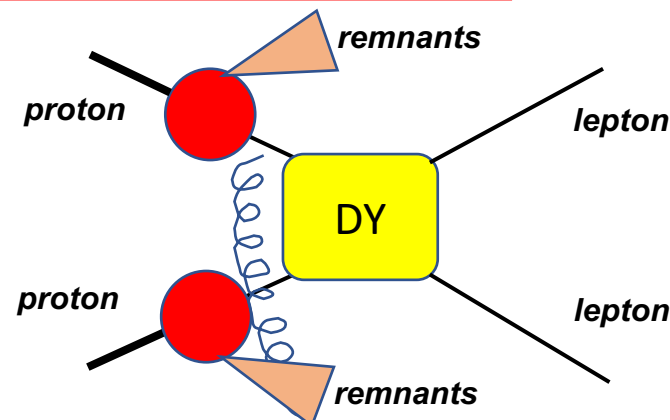
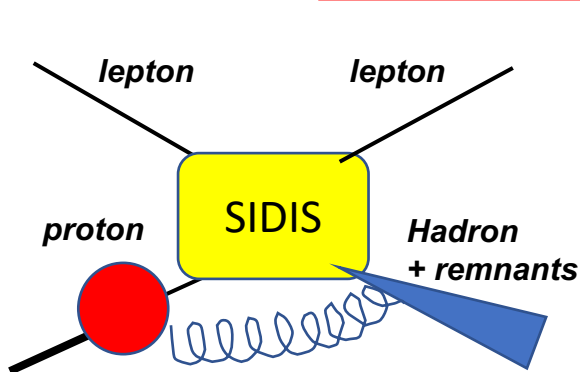
Leading Twist	Quark Polarization		
	Unpolarized [U]	Circular [L]	Linear [T]
Target Polarization	U f_1 Unpolarized		h_1^\perp Boer-Mulders
	L	g_1 Helicity	h_{1L}^\perp Worm-gear 1
	T f_{1T}^\perp Sivers	g_{1T} Worm-gear 2	h_1 Transversity h_{1T}^\perp Pretzelosity
TENSOR	$\theta_{LL}(x, k_T^2)$ $\theta_{TT}(x, k_T^2)$ $\theta_{LT}(x, k_T^2)$	$g_{1TT}(x, k_T^2)$ $g_{1LT}(x, k_T^2)$	$h_{1LL}^\perp(x, k_T^2)$ $h_{1TT}(x, k_T^2), h_{1TT}^\perp(x, k_T^2)$ $h_{1LT}(x, k_T^2), h_{1LT}^\perp(x, k_T^2)$

$$\begin{aligned} \Phi(x, k_T, P, S) = & f_1(x, k_T^2) \frac{\not{P}}{2} + \frac{h_{1T}(x, k_T^2)}{4} \gamma_5 [\not{S}_T, \not{P}] + \frac{S_L}{2} g_{1L}(x, k_T^2) \gamma_5 \not{P} + \frac{k_T \cdot S_T}{2M} g_{1T}(x, k_T^2) \gamma_5 \not{P} \\ & + S_L h_{1L}^\perp(x, k_T^2) \gamma_5 \frac{[k_T, \not{P}]}{4M} + \frac{k_T \cdot S_T}{2M} h_{1T}^\perp(x, k_T^2) \gamma_5 \frac{[k_T, \not{P}]}{4M} \end{aligned}$$

T-even

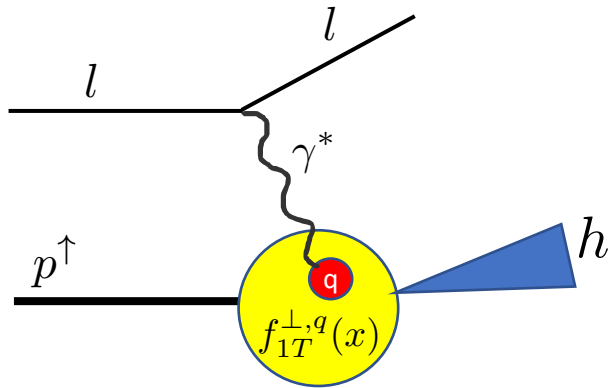
$$+ i h_1^\perp(x, k_T^2) \frac{[k_T, \not{P}]}{4M} - \frac{\epsilon_T^{k_T S_T}}{4M} f_{1T}^\perp(x, k_T^2) \not{P}$$

T-odd



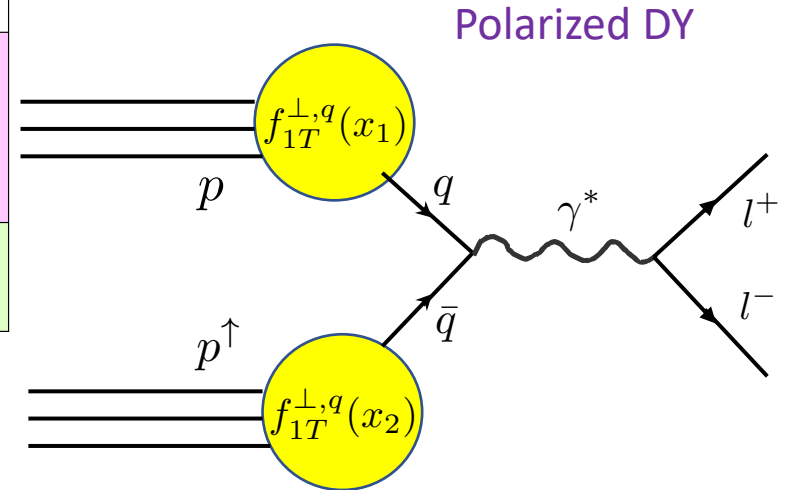
TMD PDFs

Polarized Semi Inclusive DIS



Leading Twist	Quark Polarization		
	Unpolarized [U]	Circular [L]	Linear [T]
Target Polarization	U f_1 Unpolarized		h_1^\perp Boer-Mulders
	L g_1 Helicity		h_{1L}^\perp Worm-gear 1
	T f_{1T}^\perp Sivers	g_{1T} Worm-gear 2	h_1 Transversity h_{1T}^\perp Pretzelosity
TENSOR	$\theta_{LL}(x, k_T^2)$ $\theta_{TT}(x, k_T^2)$ $\theta_{LT}(x, k_T^2)$	$g_{1TT}(x, k_T^2)$ $g_{1LT}(x, k_T^2)$	$h_{1LL}^\perp(x, k_T^2)$ $h_{1TT}(x, k_T^2), h_{1T}^\perp(x, k_T^2)$ $h_{1LT}(x, k_T^2), h_{1T}^\perp(x, k_T^2)$

* For these two processes
TMD factorization is proven



Polarized DY

$$\frac{d\sigma_{SIDIS}^{LO}}{dx dy dz dp_T^2 d\phi_h d\psi} = \left[\frac{\alpha}{xyQ^2} \frac{y^2}{2(1-\epsilon)} \left(1 + \frac{y^2}{2x} \right) \right]$$

$$\times (F_{UU,T} + \epsilon F_{UU,L}) \left\{ 1 + \cos 2\phi_h \left(\epsilon A_{UU}^{\cos 2\phi_h} \right) \right.$$

$$\left. + S_T \left[\sin(\phi_h - \phi_s) \left(A_{UT}^{\sin(\phi_h - \phi_s)} \right) + \sin(\phi_h + \phi_s) \left(\epsilon A_{UT}^{\sin(\phi_h + \phi_s)} \right) \right. \right.$$

$$\left. \left. + \sin(3\phi_h - \phi_s) \left(\epsilon A_{UT}^{\sin(3\phi_h - \phi_s)} \right) \right] \right\}$$

$$\frac{d\sigma^{LO}}{d\Omega} = \frac{\alpha_{em}^2}{F_q} F_v^1 \left\{ 1 + \cos^2 \theta + \sin^2 \theta \cos 2\phi_{CS} A_U^{\cos 2\phi_{CS}} \right.$$

$$\left. + S_T \left[(1 + \cos^2 \theta) \sin \phi_s A_T^{\sin \phi_s} + \sin^2 \theta \left(\sin(2\phi_{CS} + \phi_s) A_T^{\sin(2\phi_{CS} + \phi_s)} \right. \right. \right.$$

$$\left. \left. + \sin(2\phi_{CS} - \phi_s) A_T^{\sin(2\phi_{CS} - \phi_s)} \right) \right]$$

$$A_{UU}^{\cos 2\phi_h} \propto h_1^{\perp q} \otimes H_{1q}^{\perp h} \quad \text{BM} \otimes \text{CF}$$

$$A_{UT}^{\sin(\phi_h - \phi_s)} \propto f_{1T}^{\perp q} \otimes D_{1q}^h \quad \text{Sivers} \otimes \text{FF}$$

$$A_{UT}^{\sin(\phi_h + \phi_s)} \propto h_1^q \otimes H_{1q}^{\perp h} \quad \text{Transv} \otimes \text{CF}$$

$$A_{UT}^{\sin(3\phi_h - \phi_s)} \propto h_{1T}^{\perp q} \otimes H_{1q}^{\perp h} \quad \text{Pretz} \otimes \text{CF}$$

$$h_1^{\perp q} \Big|_{SIDIS} = -h_1^{\perp q} \Big|_{DY}$$

$$f_{1T}^{\perp q} \Big|_{SIDIS} = -f_{1T}^{\perp q} \Big|_{DY}$$

$$h_1^q \Big|_{SIDIS} = h_1^q \Big|_{DY}$$

$$h_{1T}^{\perp q} \Big|_{SIDIS} = h_{1T}^{\perp q} \Big|_{DY}$$

$$A_T^{\cos 2\phi_{CS}} \propto h_1^{\perp q} \otimes h_1^{\perp q} \quad \text{BM} \otimes \text{BM}$$

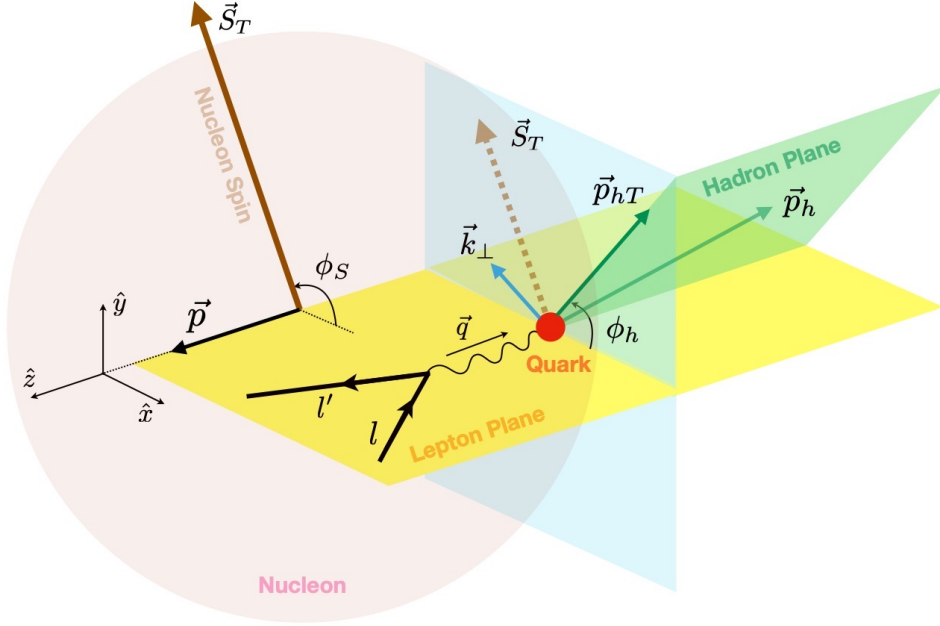
$$A_T^{\sin \phi_s} \propto f_1^q \otimes f_{1T}^{\perp q} \quad \text{PDF} \otimes \text{Sivers}$$

$$A_T^{\sin(2\phi_{CS} - \phi_s)} \propto h_1^{\perp q} \otimes h_1^q \quad \text{BM} \otimes \text{Transv}$$

$$A_T^{\sin(2\phi_{CS} + \phi_s)} \propto h_1^{\perp q} \otimes h_{1T}^{\perp q} \quad \text{BM} \otimes \text{Pretz}$$

Sivers Asymmetry from SIDIS

$$\frac{d^5\sigma^{lp\rightarrow lhX}}{dx dQ^2 dz d^2p_\perp} = \sum_q e_q^2 \int d^2\mathbf{k}_\perp \left(\frac{2\pi\alpha^2 \hat{s}^2 + \hat{u}^2}{x^2 s^2} \frac{1}{Q^4} \right) \times \hat{f}_{q/p^\uparrow}(x, \mathbf{k}_\perp) D_{h/q}(z, p_\perp) + \mathcal{O}(k_\perp/Q)$$



$$\hat{f}_{q/p^\uparrow}(x, \mathbf{k}_\perp) = f_{q/p}(x, \mathbf{k}_\perp) + \frac{1}{2} \Delta^N f_{q/p^\uparrow}(x, \mathbf{k}_\perp) \vec{S}_T \cdot (\hat{p} \times \hat{k}_\perp)$$

$$\Delta^N f_{q/p^\uparrow}(x, \mathbf{k}_\perp) = 2\mathcal{N}_q(x) h(k_\perp) f_{q/p}(x, \mathbf{k}_\perp)$$

Anselmino et al. (2017)

Single Spin Asymmetry (Sivers Asymmetry)

$$\langle p_\perp^2 \rangle = 0.12 \pm 0.01 \text{ GeV}^2$$

$$\langle k_\perp^2 \rangle = 0.57 \pm 0.08 \text{ GeV}^2$$

$$A_{UT}^{\sin(\phi_h - \phi_S)}(x, y, z, p_{hT}) = \frac{d\sigma^{l^\uparrow p \rightarrow hlX} - d\sigma^{l^\downarrow p \rightarrow hlX}}{d\sigma^{l^\uparrow p \rightarrow hlX} + d\sigma^{l^\downarrow p \rightarrow hlX}} \equiv \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$

$$\mathcal{A}_0(z, p_{hT}, m_1)$$

$$= \frac{\sqrt{2} e z p_{hT}}{m_1} \frac{[z^2 \langle k_\perp^2 \rangle + \langle p_\perp^2 \rangle] \langle k_S^2 \rangle^2}{[z^2 \langle k_S^2 \rangle + \langle p_\perp^2 \rangle]^2 \langle k_\perp^2 \rangle} \times \exp \left[- \frac{p_{hT}^2 z^2 (\langle k_S^2 \rangle - \langle k_\perp^2 \rangle)}{(z^2 \langle k_S^2 \rangle + \langle p_\perp^2 \rangle) (z^2 \langle k_\perp^2 \rangle + \langle p_\perp^2 \rangle)} \right]$$

$$\langle k_S^2 \rangle = \frac{m_1 \langle k_\perp^2 \rangle}{m_1^2 + \langle k_\perp^2 \rangle}$$

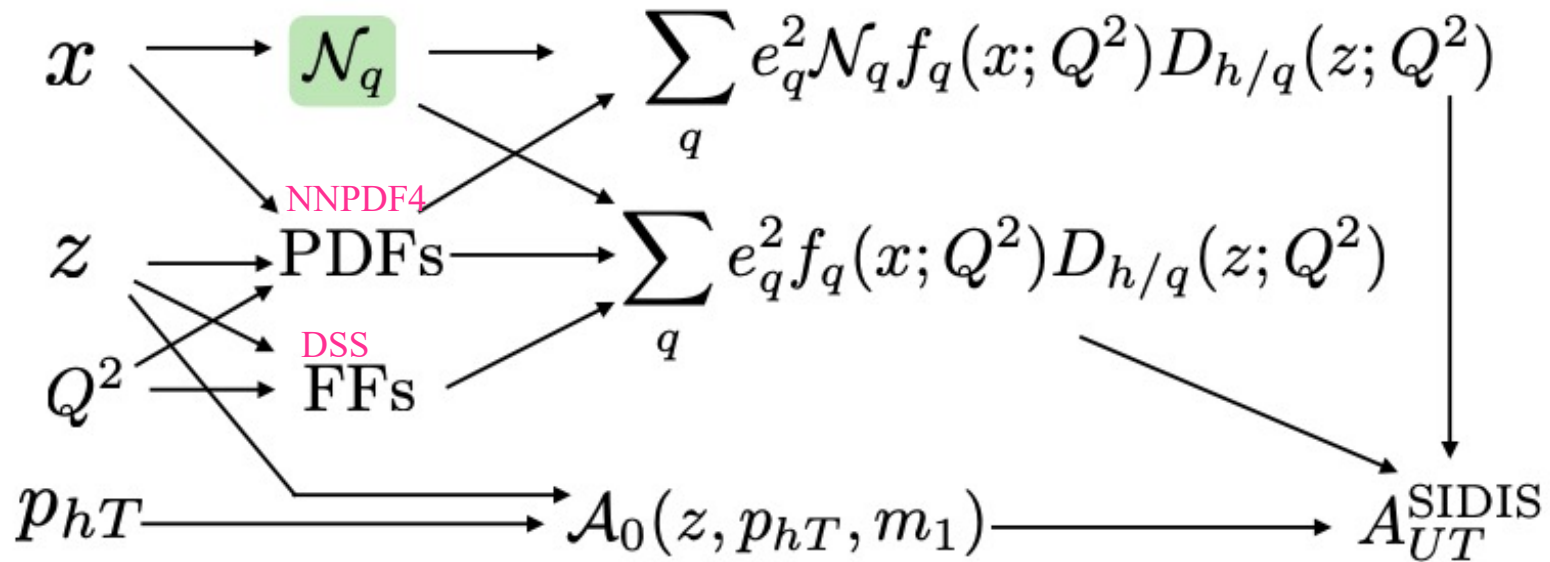
$$A_{UT}^{\sin(\phi_h - \phi_S)}(x, z, p_{hT}) = \mathcal{A}_0(z, p_{hT}, m_1) \left(\frac{\sum_q \mathcal{N}_q(x) e_q^2 f_q(x) D_{h/q}(z)}{\sum_q e_q^2 f_q(x) D_{h/q}(z)} \right)$$

$$\mathcal{N}_q(x) = N_q x^{\alpha_q} (1-x)^{\beta_q} \frac{(\alpha_q + \beta_q)^{(\alpha_q + \beta_q)}}{\alpha_q^{\alpha_q} \beta_q^{\beta_q}}$$

$$\mathcal{N}_{\bar{q}}(x) = N_{\bar{q}}$$

DNN Approach

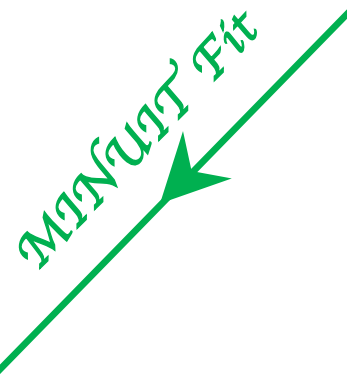
$$A_{UT}^{\sin(\phi_h - \phi_s)}(x, z, p_{hT}) = \mathcal{A}_0(z, p_{hT}, m_1) \left(\frac{\sum_q \mathcal{N}_q(x) e_q^2 f_q(x) D_{h/q}(z)}{\sum_q e_q^2 f_q(x) D_{h/q}(z)} \right)$$



The DNN Method for extracting TMDs

*I. P. Fernando and D. Keller
Phys. Rev. D.108.054007 (2023)*

Experimental-data



$$\mathcal{N}_q(x) = N_q x^{\alpha_q} (1-x)^{\beta_q} \frac{(\alpha_q + \beta_q)^{(\alpha_q + \beta_q)}}{\alpha_q^{\alpha_q} \beta_q^{\beta_q}}$$

generating function

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Experimental-data

MINUIT Fit

$$\mathcal{N}_q(x) = N_q x^{\alpha_q} (1-x)^{\beta_q} \frac{(\alpha_q + \beta_q)^{(\alpha_q + \beta_q)}}{\alpha_q^{\alpha_q} \beta_q^{\beta_q}}$$

generating function

Pseudo-data
(with exp. Uncert.)

DNN Fit

Tuned DNN model

- Use Multi-dimensional kinematic bins within the experimental kinematic phase-space
- Use replicas to propagate the experimental uncertainties
- Use accuracy and precision to ensure the DNN

$$\epsilon_q(x, k_{\perp}) = \left(1 - \frac{|\Delta^N f_{q/p\uparrow}^{(\text{true})} - \Delta^N f_{q/p\uparrow}^{(\text{mean})}|}{\Delta^N f_{q/p\uparrow}^{(\text{true})}} \right) \times 100\%$$

$$\sigma_q(x, k_{\perp}) = \sqrt{\frac{\sum_i \left(\Delta^N f_{q/p\uparrow}^{(i)} - \Delta^N f_{q/p\uparrow}^{(\text{mean})} \right)^2}{N}}$$

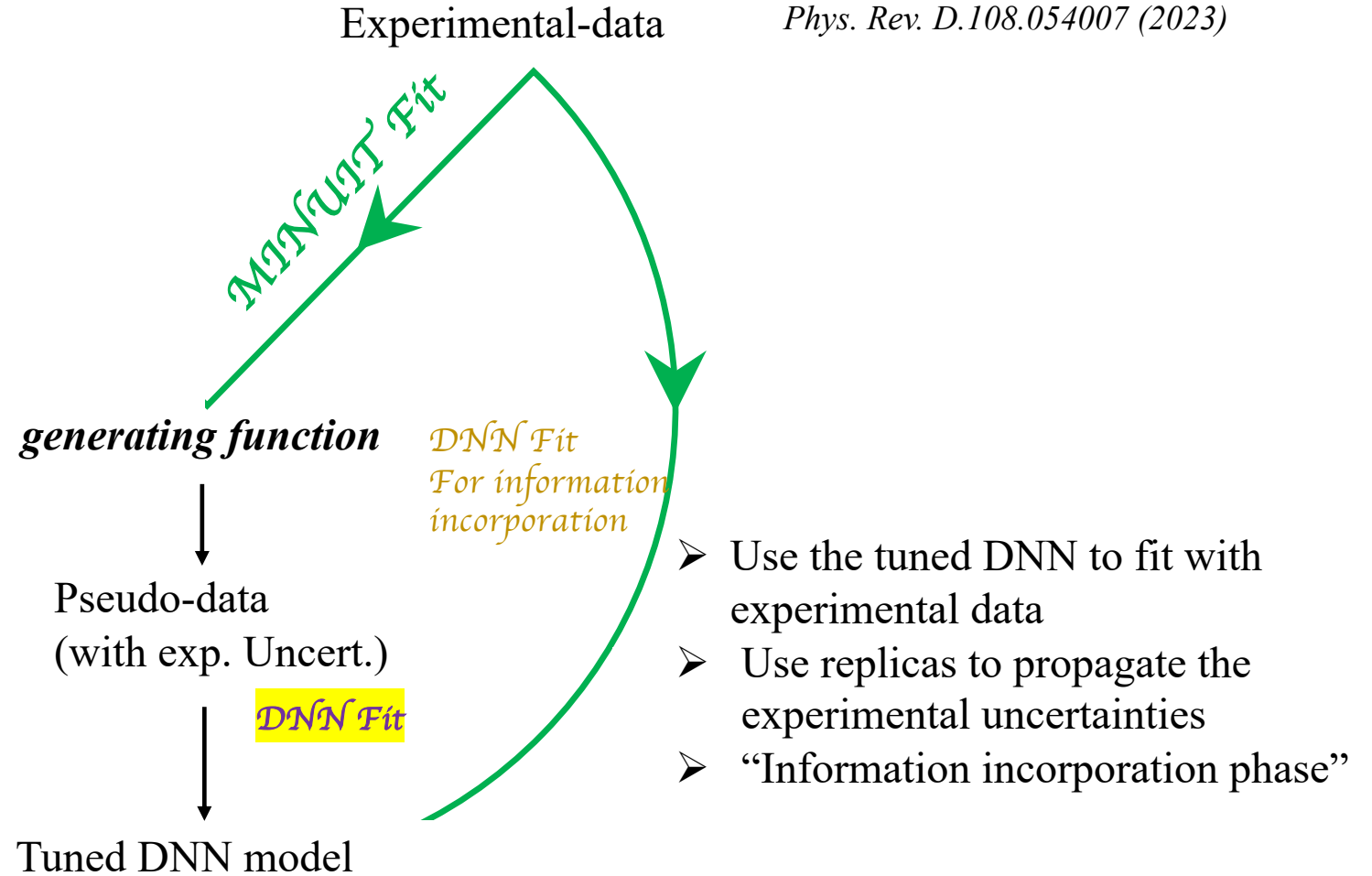
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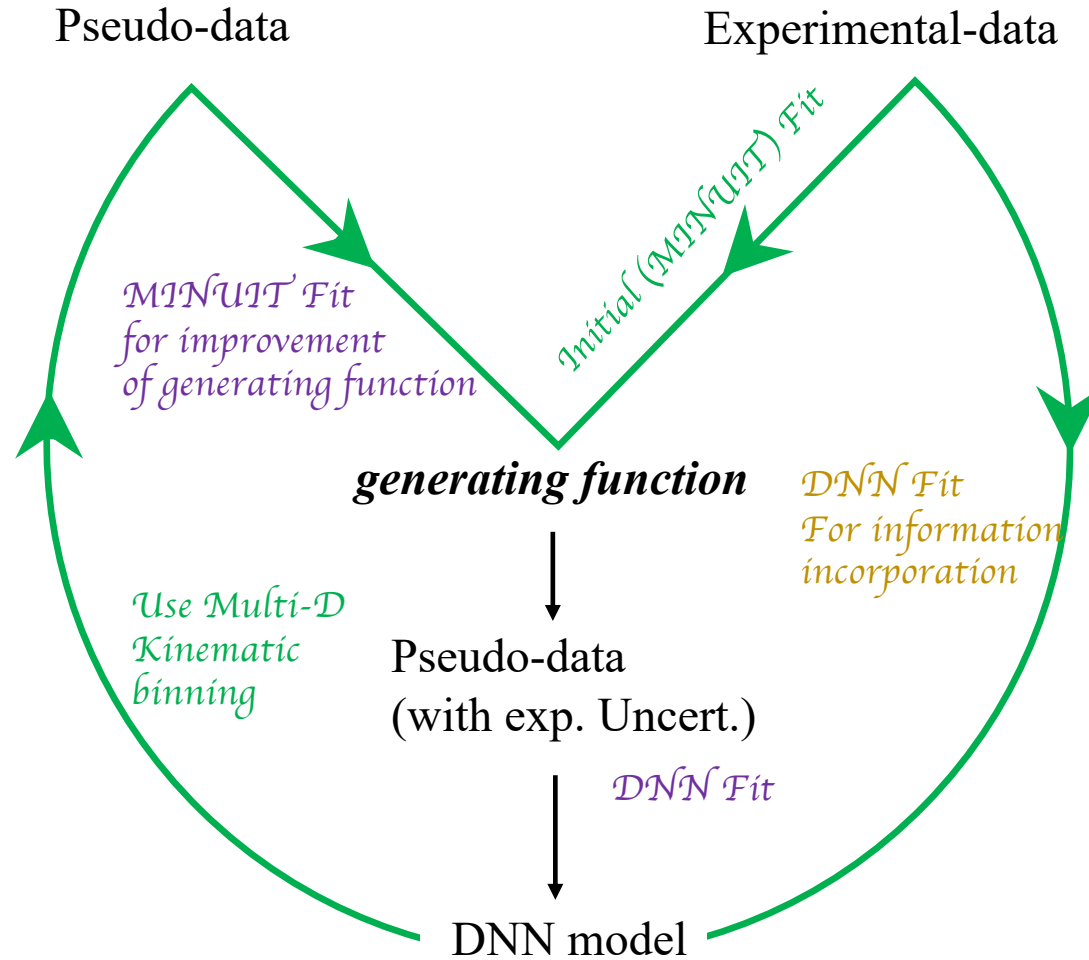
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The DNN Method for extracting TMDs

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Iterations!
 “Recursively
 Improving the DNN
 with multi-dimensional
 kinematic phase-space,
 and simultaneous
 information
 incorporation from
 experimental data
 (Nature)”

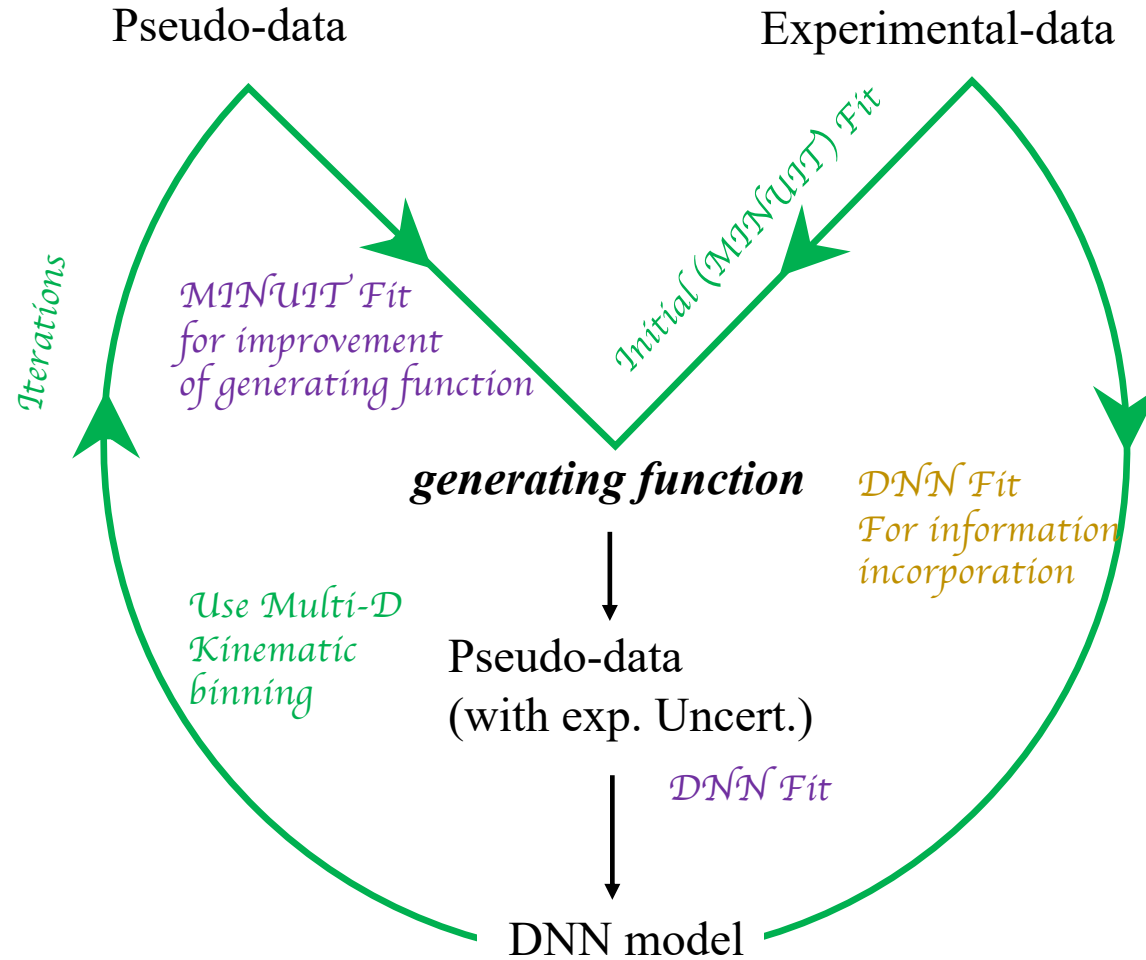
- We trained two separate models for “proton” and “neutron” (deuteron)
- To take full advantage of the information provided by the model testing in the previous slide, the steps from method testing with pseudo-data are performed again separately for proton and deuteron SIDIS data.

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The DNN Method for extracting TMDs

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- Systematic study for both DNN models were performed separately using various generating functions.

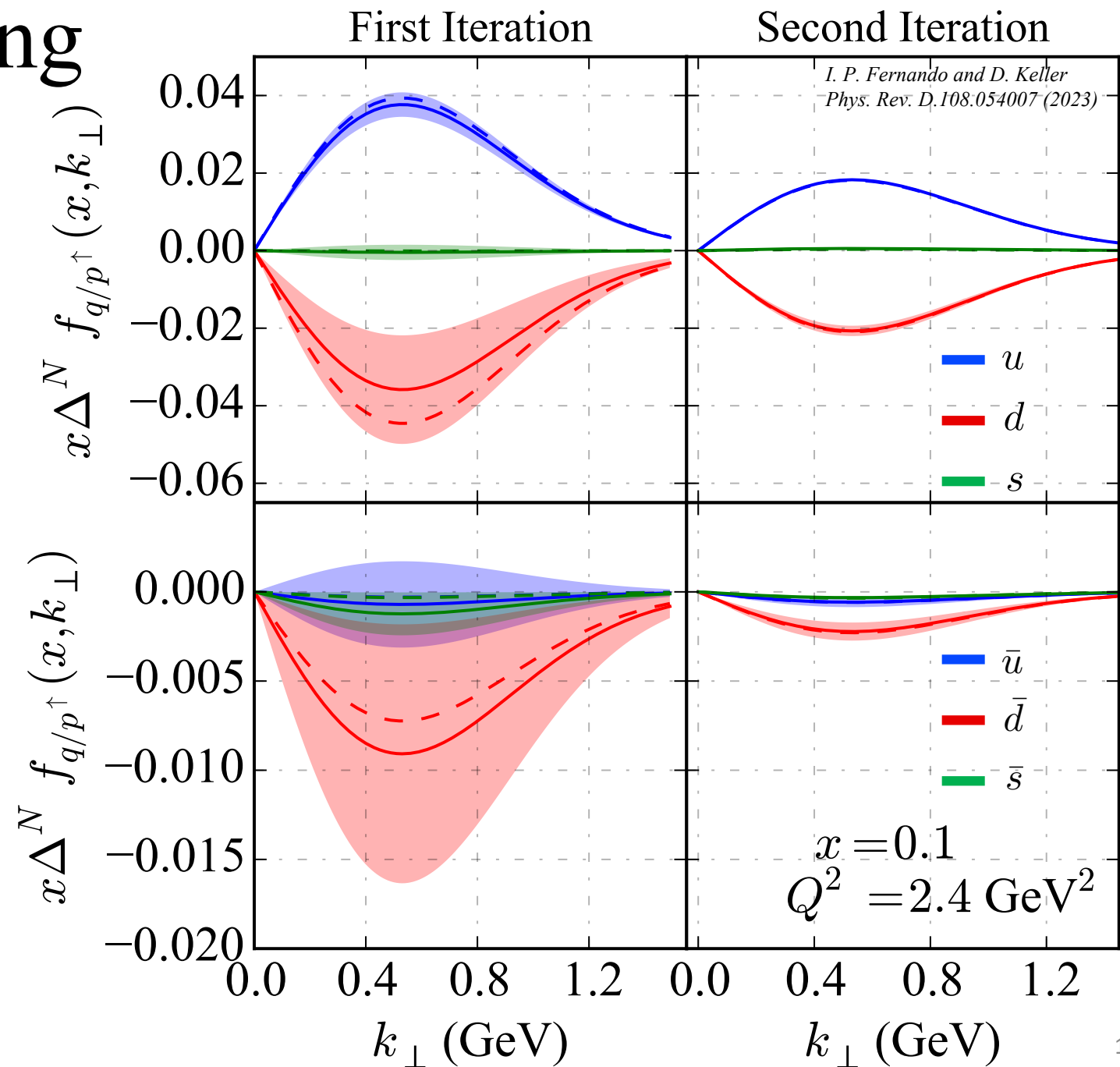
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$$\sigma_q(x, k_{\perp}) = \sqrt{\frac{\sum_i \left(\Delta^N f_{q/p\uparrow}^{(i)} - \Delta^N f_{q/p\uparrow}^{(\text{mean})} \right)^2}{N}}$$

DNN Method testing

- Dashed lines represent the **generating function** in each iteration.
- A comparison:
Improving the **generating function**
Fine-tuning the hyperparameters
- Solid-lines and the band represent the mean and 68% CL with 1000 replicas of the DNN model.



Data Selection

Dataset	Kinematic coverage	Reaction	Data points
HERMES2009 (SIDIS) [53]	$0.023 < x < 0.4$	$p^\uparrow + \gamma^* \rightarrow \pi^+$	21
	$0.2 < z < 0.7$	$p^\uparrow + \gamma^* \rightarrow \pi^-$	21
	$0.1 < p_{hT} < 0.9$	$p^\uparrow + \gamma^* \rightarrow \pi^0$	21
	$Q^2 > 1 \text{ GeV}^2$	$p^\uparrow + \gamma^* \rightarrow K^+$	21
		$p^\uparrow + \gamma^* \rightarrow K^-$	21
HERMES2020 (SIDIS) [55]	$0.023 < x < 0.6$	$p^\uparrow + \gamma^* \rightarrow \pi^+$	27, 64
	$0.2 < z < 0.7$	$p^\uparrow + \gamma^* \rightarrow \pi^-$	27, 64
	$0.1 < p_{hT} < 0.9$	$p^\uparrow + \gamma^* \rightarrow \pi^0$	27
	$Q^2 > 1 \text{ GeV}^2$	$p^\uparrow + \gamma^* \rightarrow K^+$	27, 64
		$p^\uparrow + \gamma^* \rightarrow K^-$	27, 64
COMPASS2015 (SIDIS) [54]	$0.006 < x < 0.28$	$p^\uparrow + \gamma^* \rightarrow \pi^+$	26
	$0.2 < z < 0.8$	$p^\uparrow + \gamma^* \rightarrow \pi^-$	26
	$0.15 < p_{hT} < 1.5$	$p^\uparrow + \gamma^* \rightarrow K^+$	26
	$Q^2 > 1 \text{ GeV}^2$	$p^\uparrow + \gamma^* \rightarrow K^-$	26
COMPASS2009 (SIDIS) [49]	$0.006 < x < 0.28$	$d^\uparrow + \gamma^* \rightarrow \pi^+$	26
	$0.2 < z < 0.8$	$d^\uparrow + \gamma^* \rightarrow \pi^-$	26
	$0.15 < p_{hT} < 1.5$	$d^\uparrow + \gamma^* \rightarrow K^+$	26
	$Q^2 > 1 \text{ GeV}^2$	$d^\uparrow + \gamma^* \rightarrow K^-$	26
JLAB2011 (SIDIS) [52]	$0.156 < x < 0.396$	${}^3\text{He}^\uparrow + \gamma^* \rightarrow \pi^+$	4
	$0.50 < z < 0.58$	${}^3\text{He}^\uparrow + \gamma^* \rightarrow \pi^-$	4
	$0.24 < p_{hT} < 0.43$ $1.3 < Q^2 < 2.7$		
COMPASS2017 (DY) [50]	$0.1 < x_N < 0.25$	$p^\uparrow + \pi^- \rightarrow l^+ l^- X$	15
	$0.3 < x_\pi < 0.7$		
	$4.3 < Q_M < 8.5$		
	$0.6 < q_T < 1.9$		

Proton DNN model

Deuteron DNN model

Projections from Deuteron DNN model

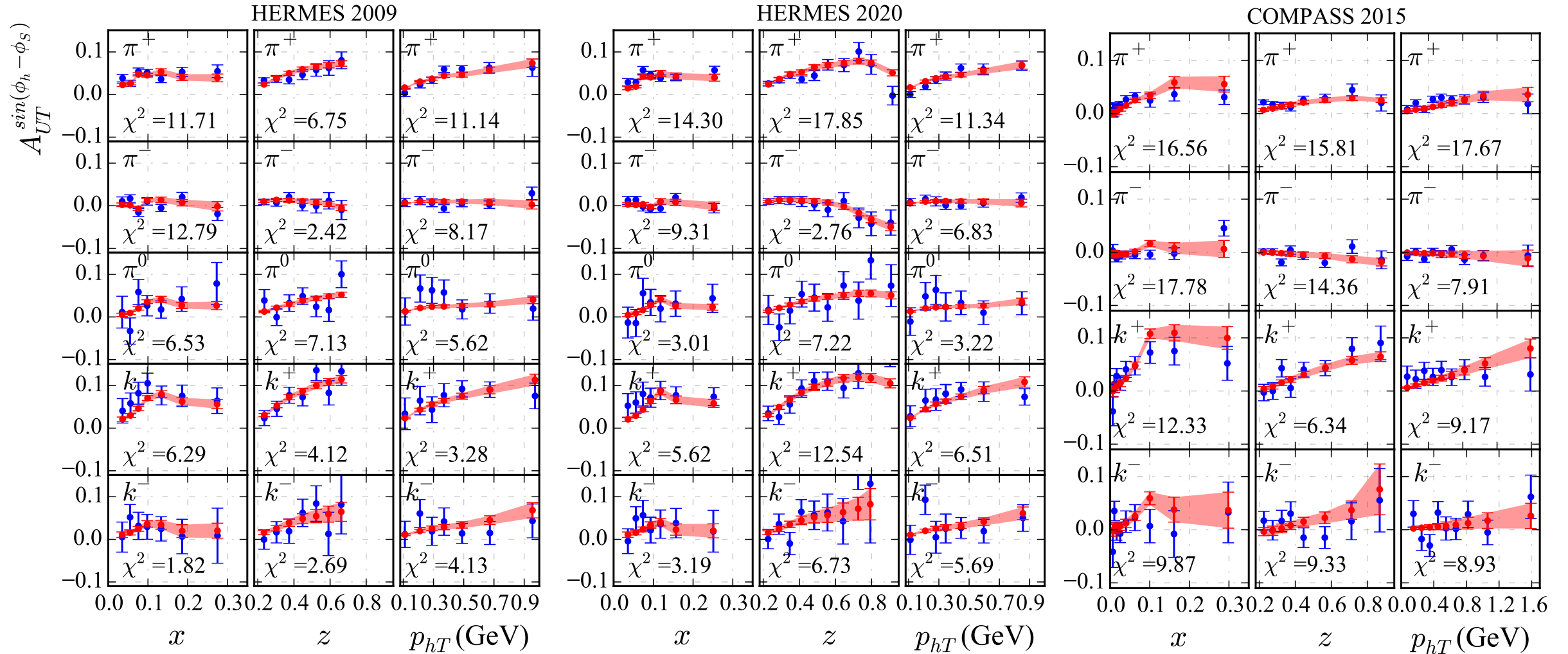
HERMES2020 3D binned data

Projections from Proton DNN model

$$\Delta^N f_{q/p^\uparrow}(x, k_\perp)|_{\text{SIDIS}} = - \Delta^N f_{q/p^\uparrow}(x, k_\perp)|_{\text{DY}}$$

Proton DNN Fit Results

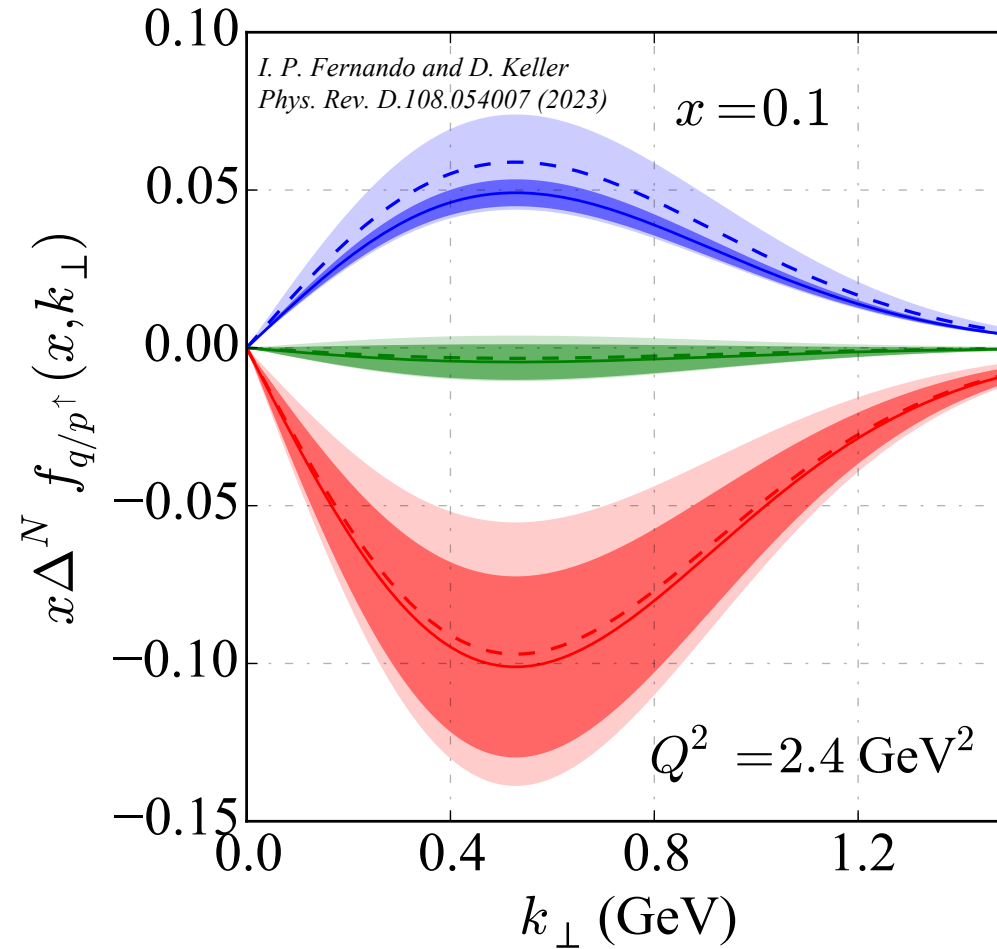
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- All data points are well-described by the proton-DNN model.
- No kinematic cuts were implemented.

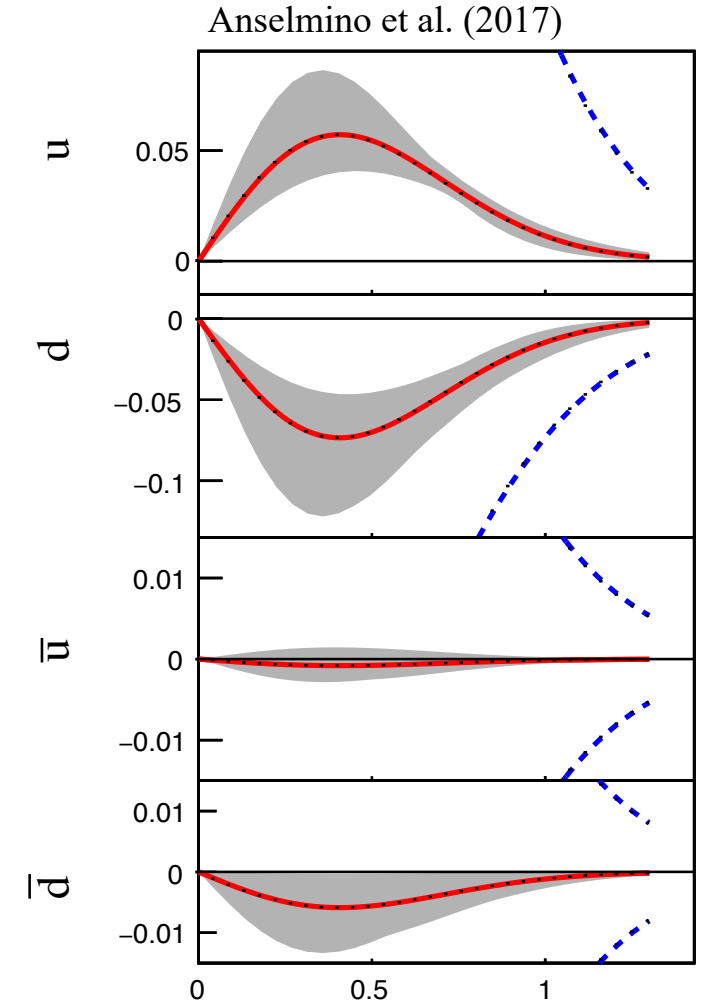
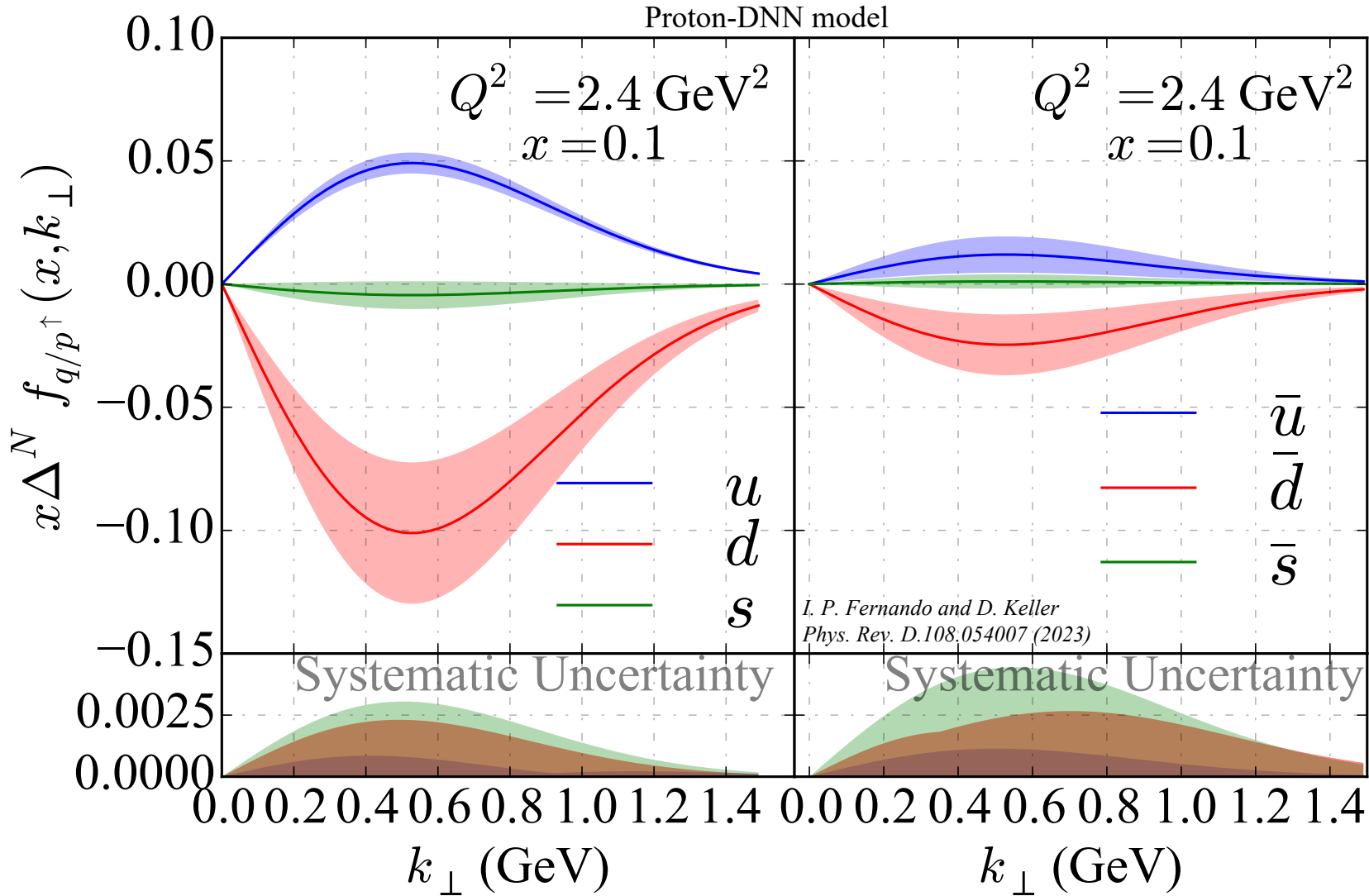
Calculated $\chi_{\text{total}}^2/N_{\text{pt}} = 1.04$

DNN Method: With Real data (Quality of the extraction)



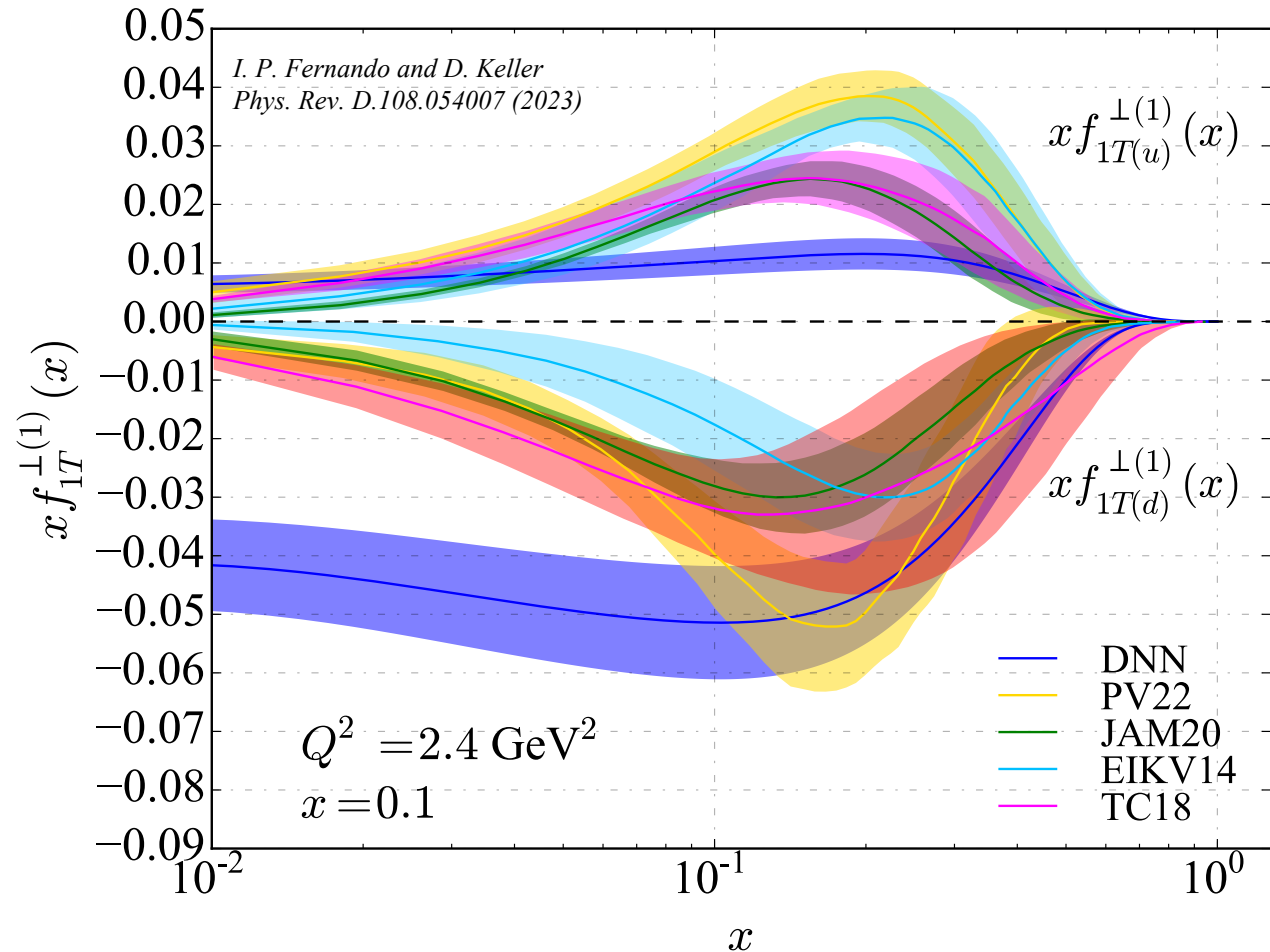
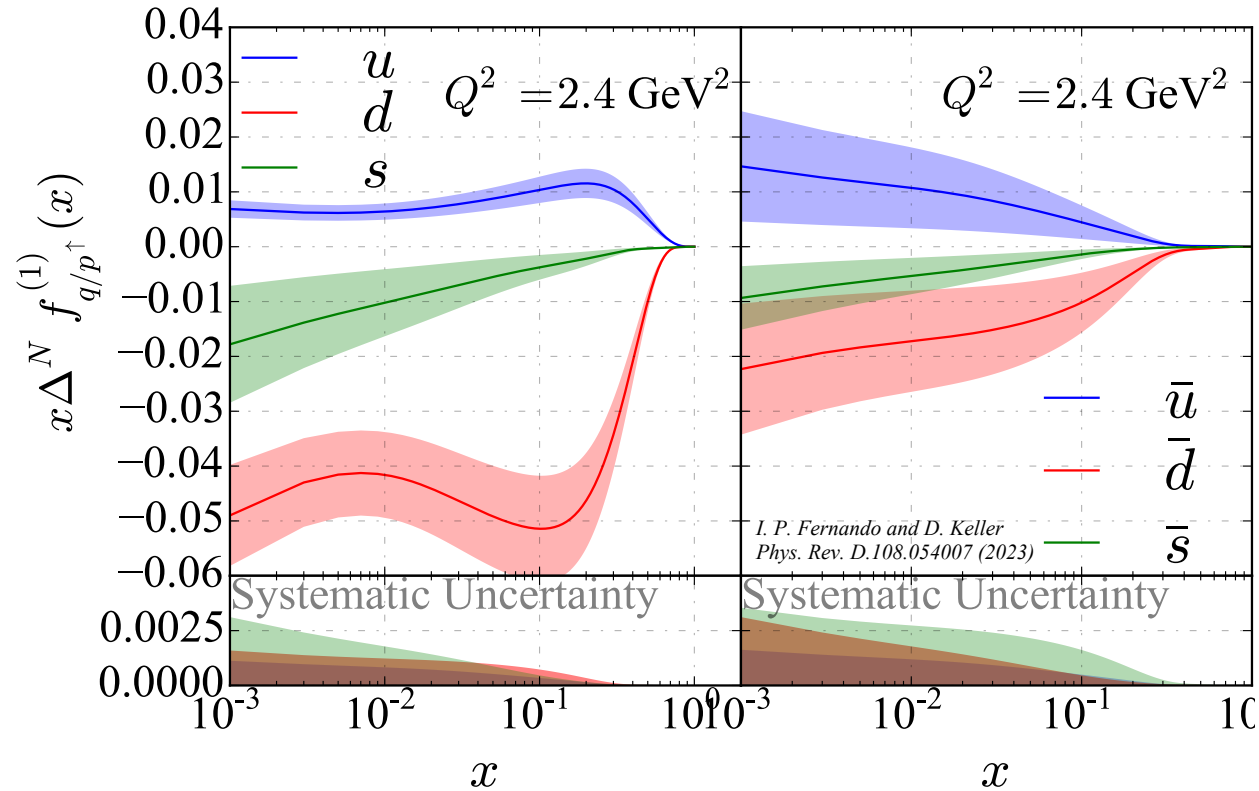
The qualitative improvement of the extracted Sivers functions for u (blue), d (red), and s (green) quarks at $x = 0.1$ and $Q^2 = 2.4 \text{ GeV}^2$ using the optimized proton-DNN model at the Second Iteration (solid-lines with dark-colored error bands with 68% CL), compared to the First Iteration (dashed-lines with light-colored error bands with 68% CL)

Sivers functions from the “Proton” DNN Model



Sivers 1st moments from the “Proton” Model

$$\Delta^N f_{q/p^\uparrow}^{(1)}(x) = \int d^2 k_\perp \frac{k_\perp}{4m_p} \Delta^N f_{q/p^\uparrow}(x, k_\perp) = -f_{1T}^{\perp(1)q}(x)$$

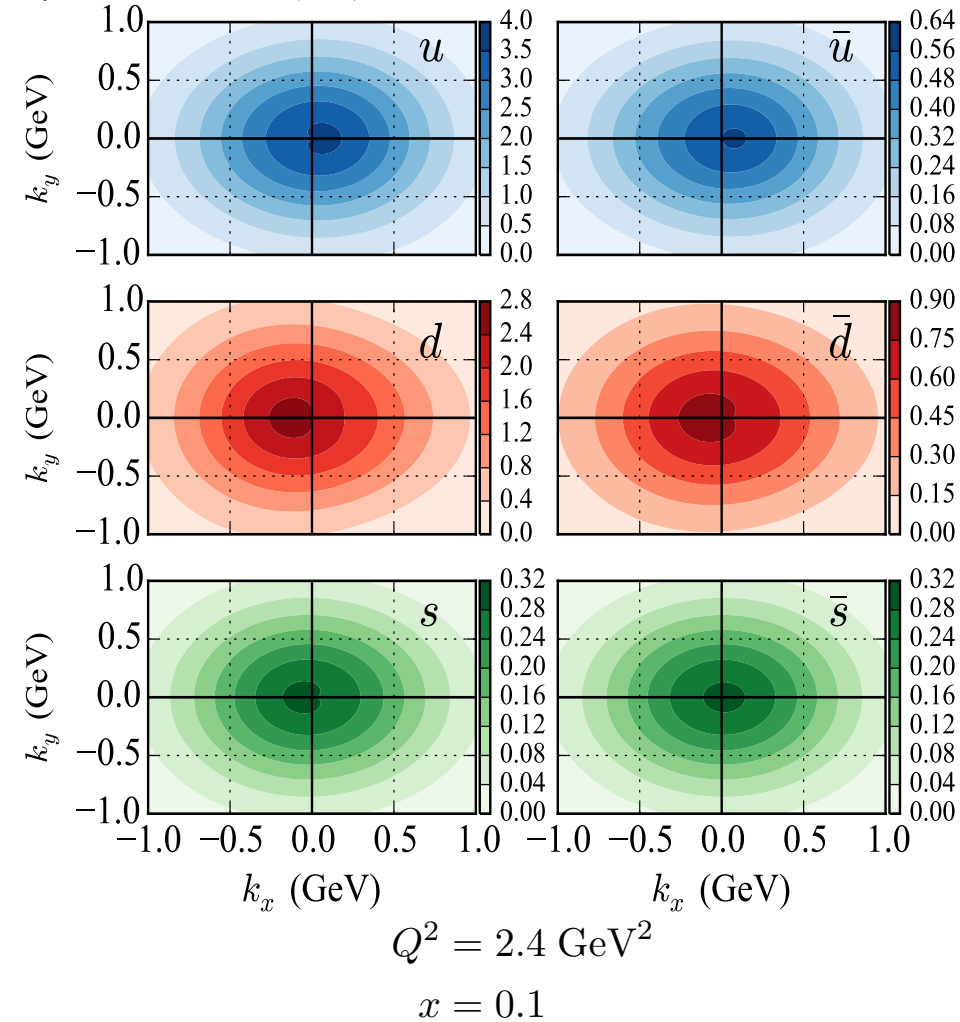


3D Tomography from the “Proton” DNN Model

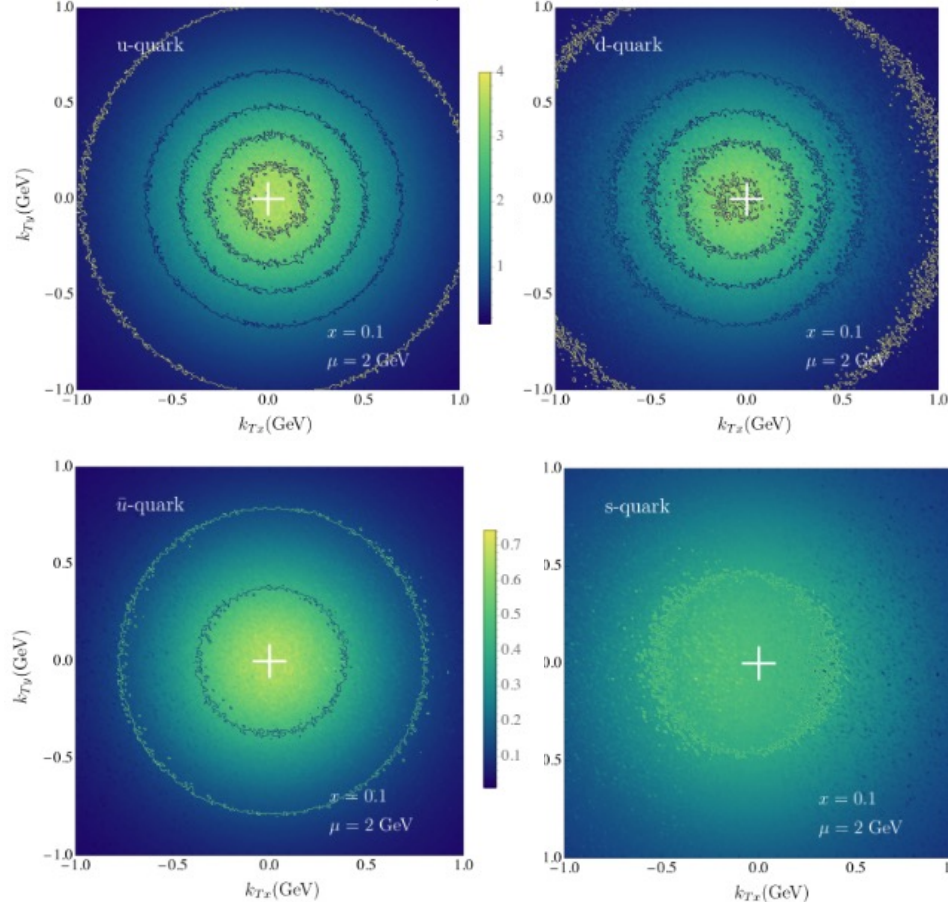
$$\rho_{p\uparrow}^a(x, k_x, k_y; Q^2) = f_1^a(x, k_\perp^2; Q^2) - \frac{k_x}{m_p} f_{1T}^{\perp a}(x, k_\perp^2; Q^2)$$

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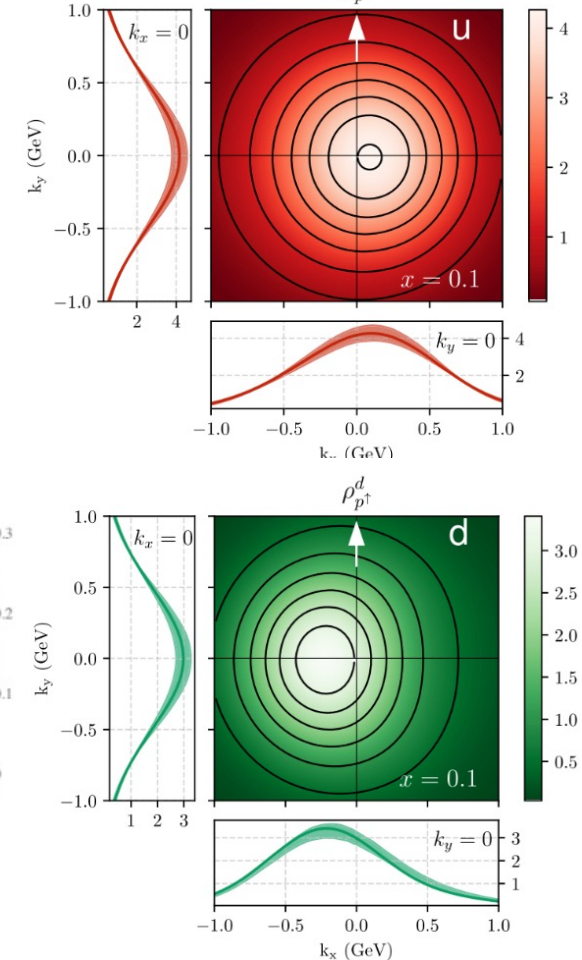
Proton-DNN model



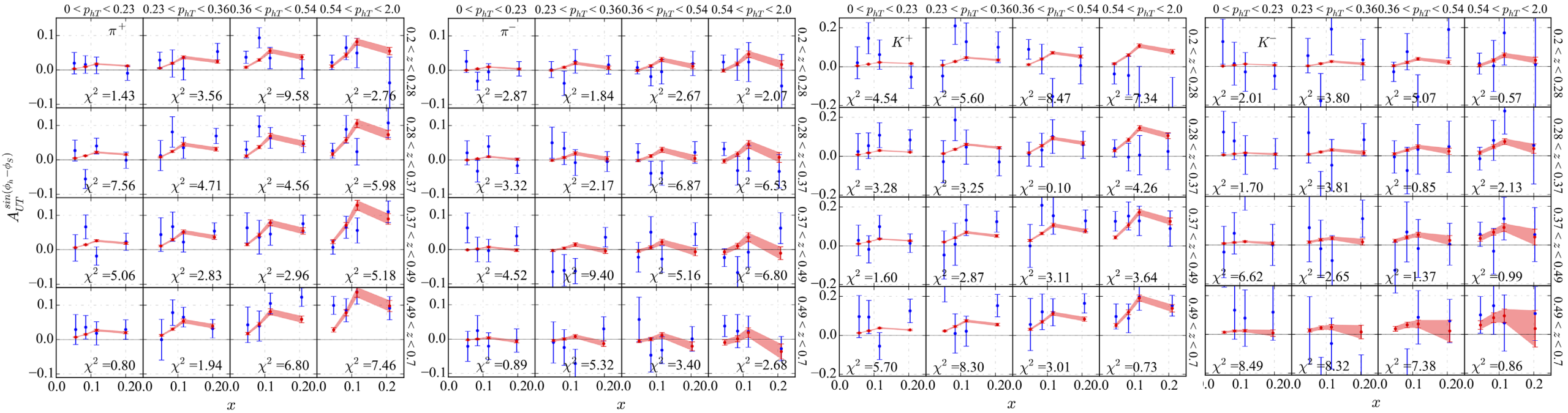
Bury et al (2021)



A. Bacchetta et al (2021) $\rho_{p\uparrow}^u$



Projections from the “Proton” DNN Model



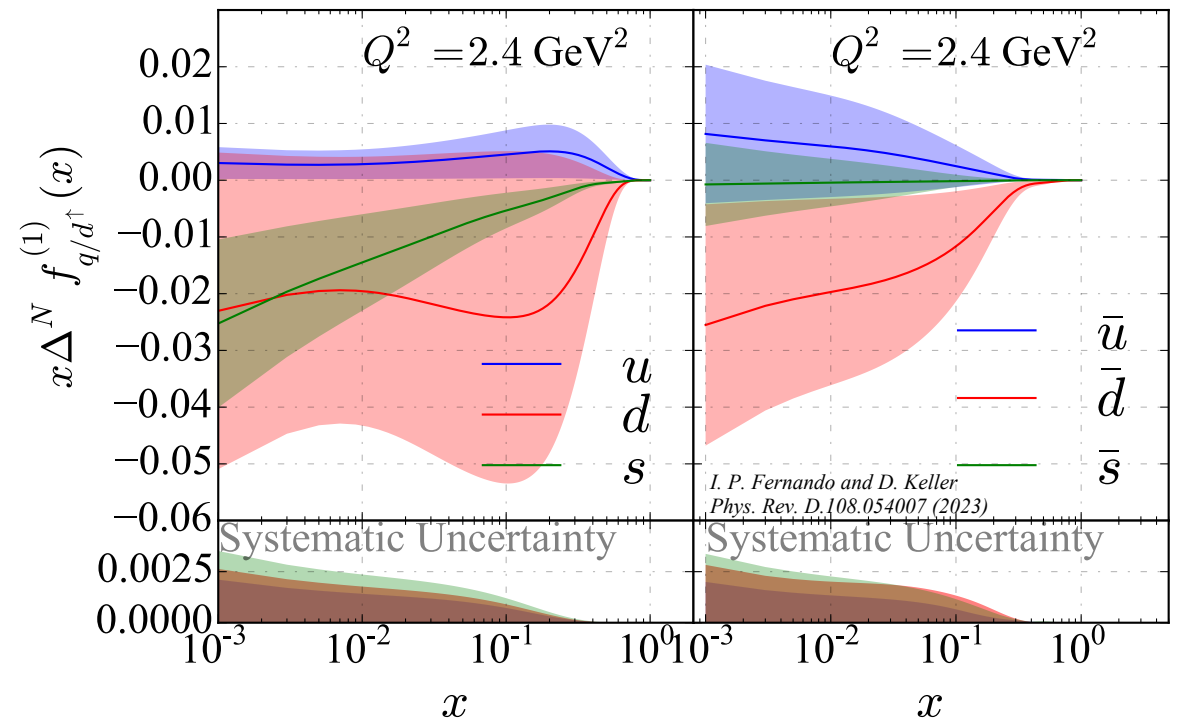
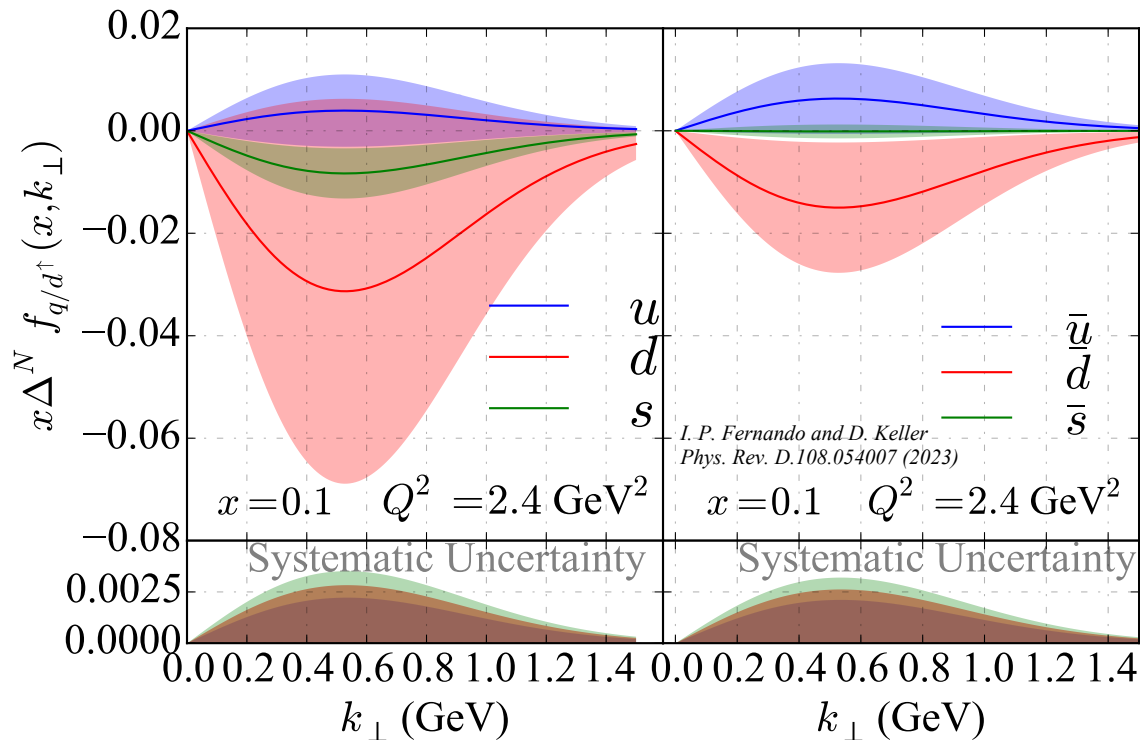
Projections of the of HERMES 2020 data for 3D kinematic bins, using the proton-DNN model including 68% C.L. error bands (in red) in comparison with the actual data points (in blue).

DNN Method: Results from the “Deuteron” Model

- Trained on COMPASS 2009 SIDIS data with Deuteron target.
- Did not imposed iso-spin symmetric conditions, or data cuts.

~~$$f_{1T,u\leftarrow d}^\perp = f_{1T,d\leftarrow d}^\perp = \frac{f_{1T,u\leftarrow p}^\perp + f_{1T,d\leftarrow p}^\perp}{2}$$~~

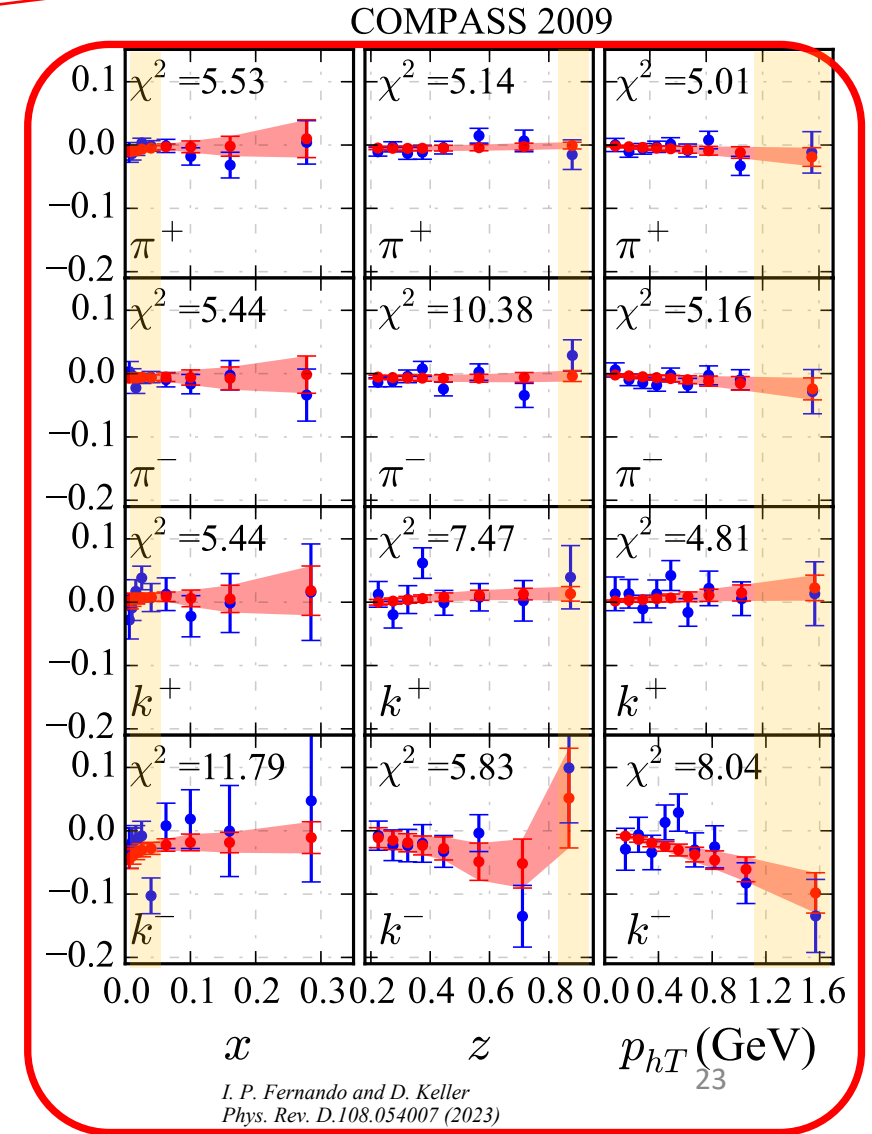
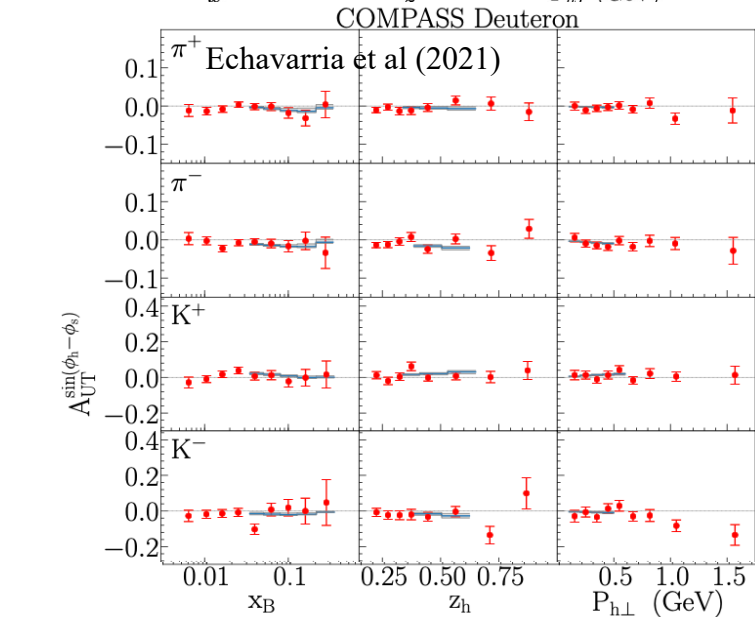
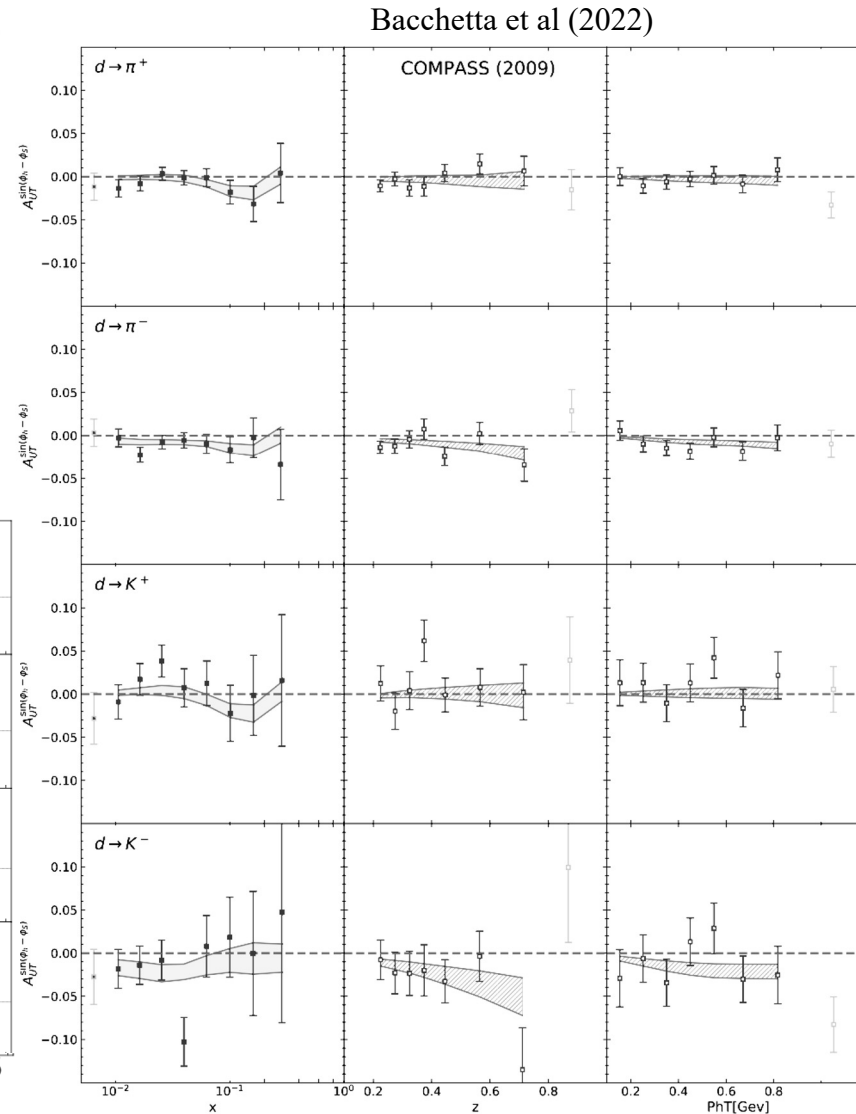
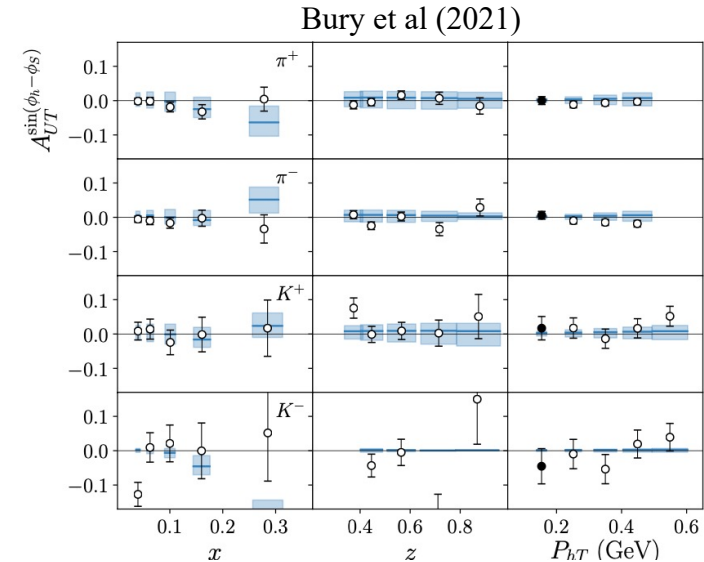
$$\Delta^N f_{q/p^\uparrow}^{(1)}(x) = \int d^2k_\perp \frac{k_\perp}{4m_p} \Delta^N f_{q/p^\uparrow}(x, k_\perp) = -f_{1T}^{\perp(1)q}(x)$$



Deuteron DNN Fit Results

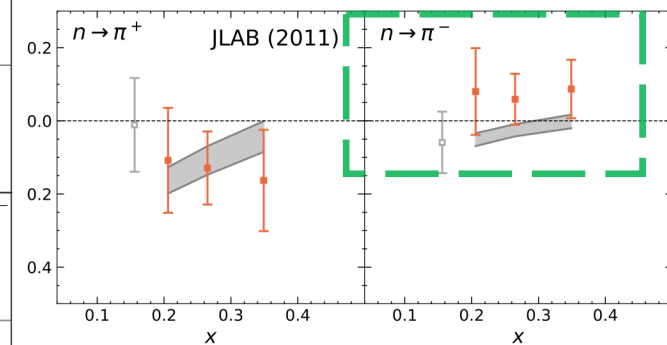
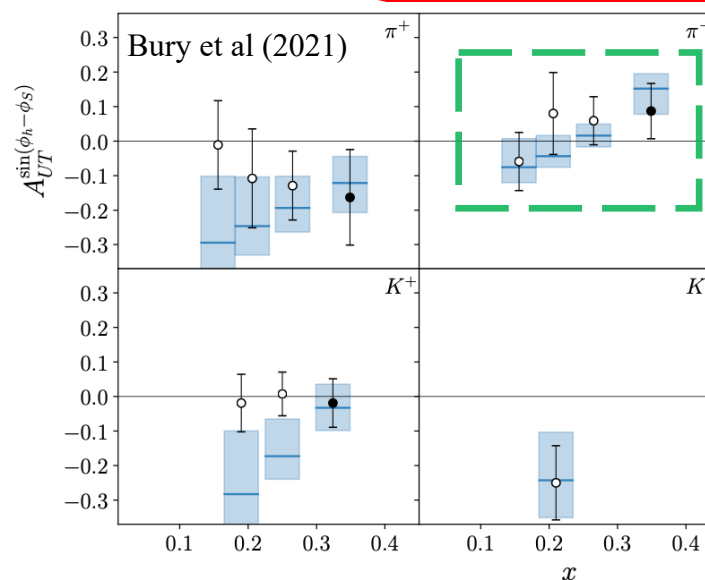
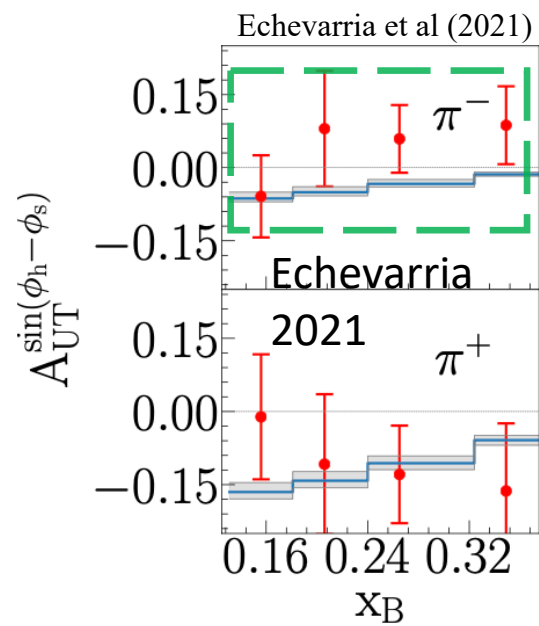
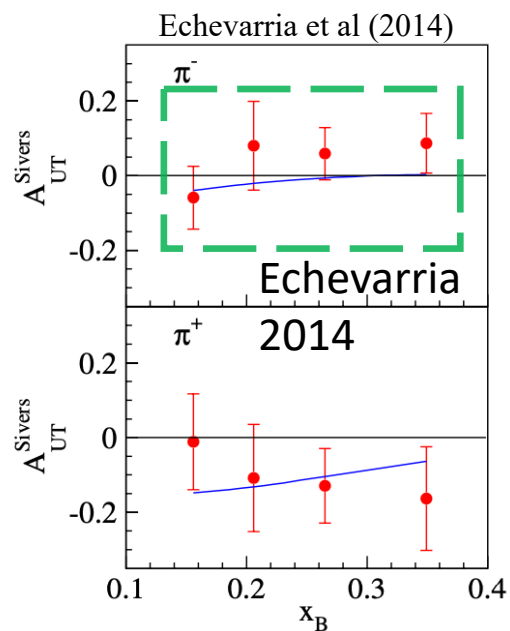
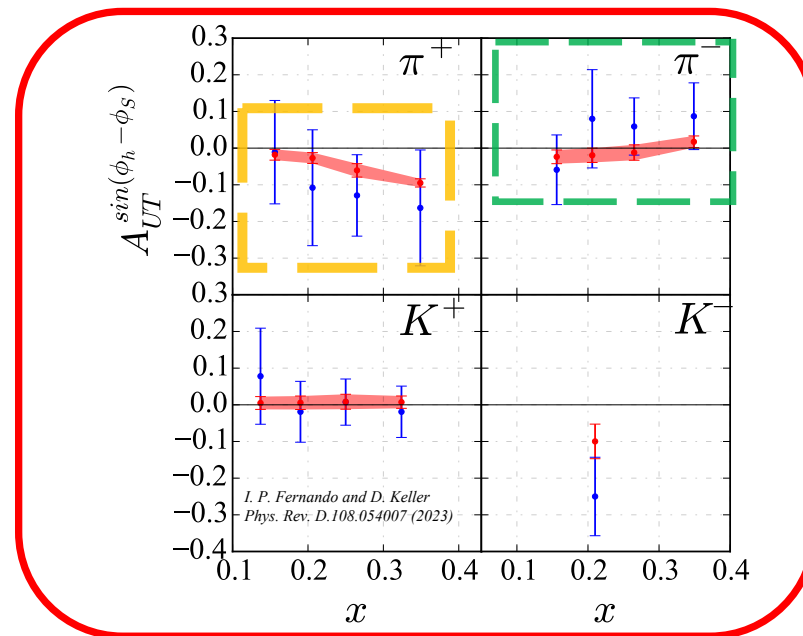
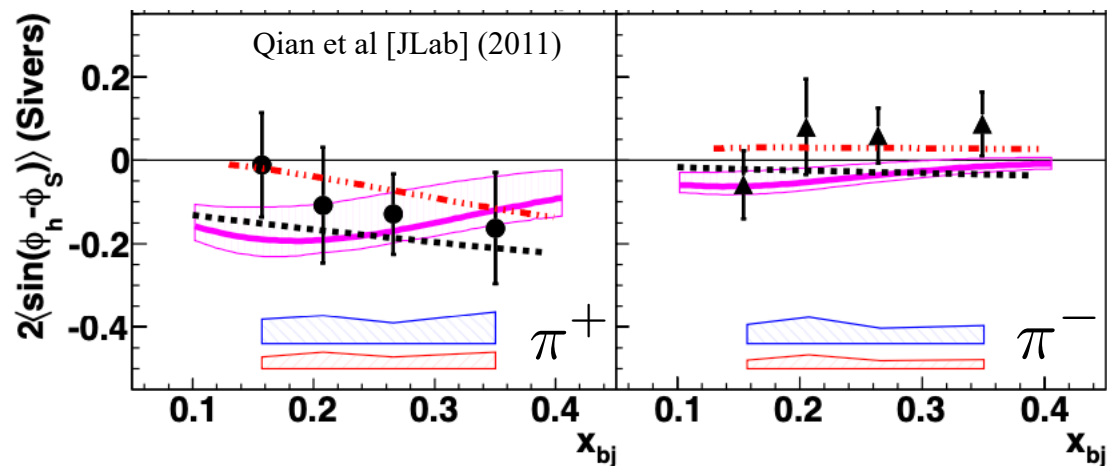
- No kinematic cuts are applied
- Deuteron-DNN model can describe data reasonably well
- No iso-spin symmetry conditions are applied

$$f_{1T,u\leftarrow d}^\perp = f_{1T,d\leftarrow d}^\perp = \frac{f_{1T,u\leftarrow p}^\perp + f_{1T,d\leftarrow p}^\perp}{2} \quad \chi^2/N_{\text{pt}} = 0.81$$



Deuteron DNN Projections for JLab Kinematics

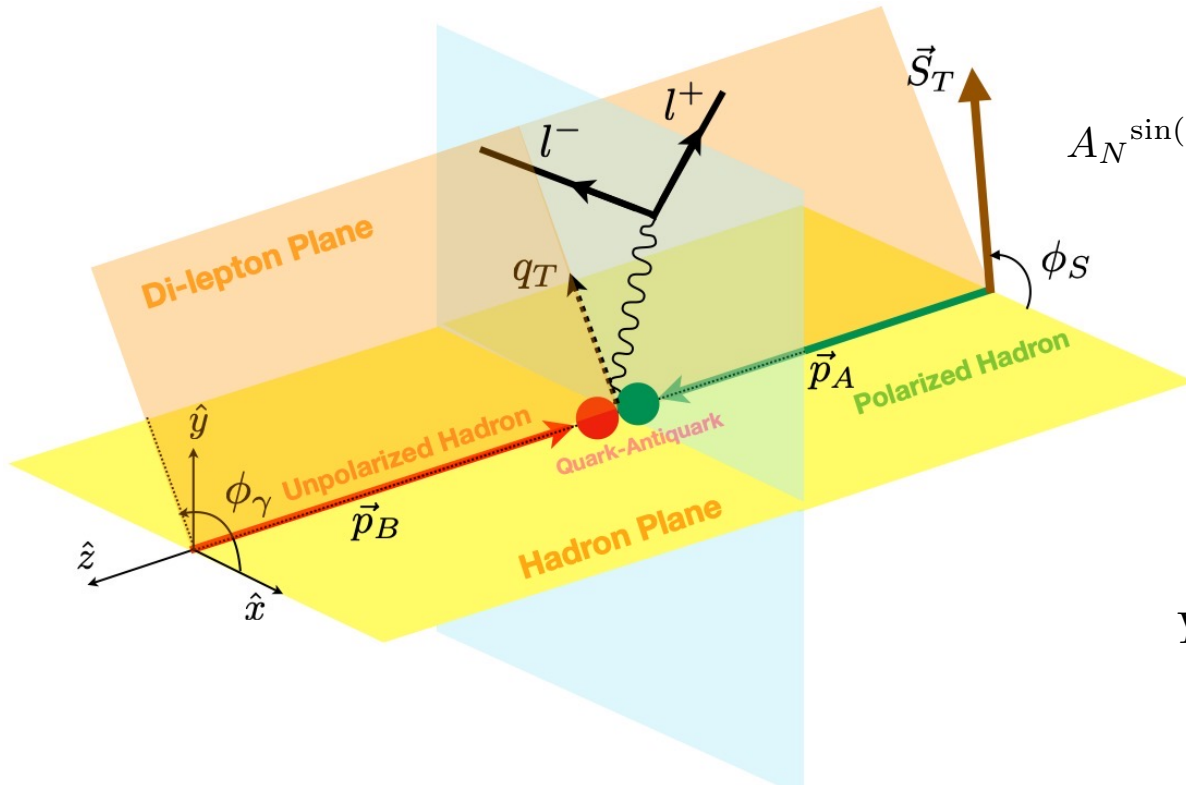
Deuteron-DNN



Bacchetta et al (2022)

DNN Model Projections: DY

Based on Anselmino et al. (2017)



$$A_N^{\sin(\phi_\gamma - \phi_S)}(x_F, M, q_T) = \mathcal{B}_0(q_T, m_1) \frac{\sum_q \frac{e_q^2}{x_1 + x_2} \mathcal{N}_q(x_1) f_{q/A}(x_1) f_{\bar{q}/B}(x_2)}{\sum_q \frac{e_q^2}{x_1 + x_2} f_{q/A}(x_1) f_{\bar{q}/B}(x_2)}$$

$$\mathcal{B}_0(q_T, m_1) = \frac{q_T \sqrt{2e}}{m_1} \frac{Y_1(q_T, k_S, k_{\perp 2})}{Y_2(q_T, k_{\perp 1}, k_{\perp 2})}$$

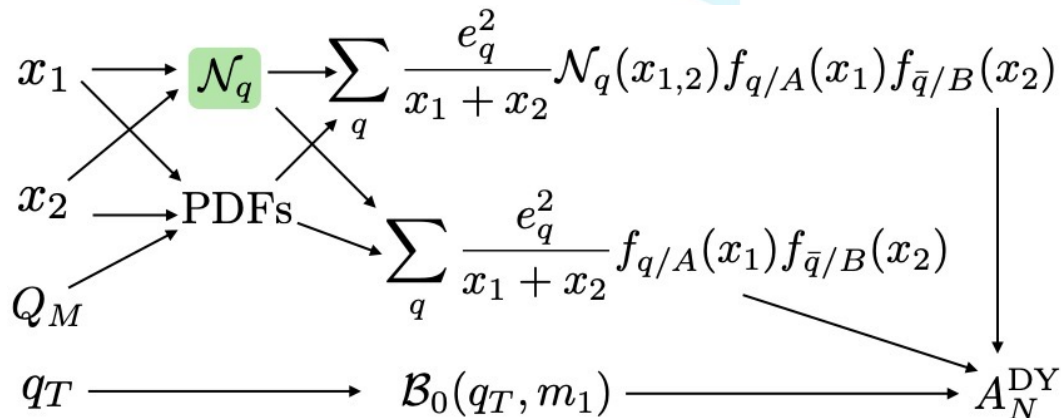
$$Y_1(q_T, k_S, k_{\perp 2}) = \left(\frac{\langle k_S^2 \rangle^2}{\langle k_{\perp 2}^2 \rangle (\langle k_S^2 \rangle + \langle k_{\perp 2}^2 \rangle)^2} \right) \times \exp \left(\frac{-q_T^2}{\langle k_S^2 \rangle + \langle k_{\perp 2}^2 \rangle} \right)$$

$$Y_2(q_T, k_{\perp 1}, k_{\perp 2}) = \left(\frac{1}{\langle k_{\perp 1}^2 \rangle + \langle k_{\perp 2}^2 \rangle} \right) \times \exp \left(\frac{-q_T^2}{\langle k_{\perp 1}^2 \rangle + \langle k_{\perp 2}^2 \rangle} \right)$$

$$\frac{1}{\langle k_S^2 \rangle} = \frac{1}{m_1^2} + \frac{1}{\langle k_{\perp 1}^2 \rangle}$$

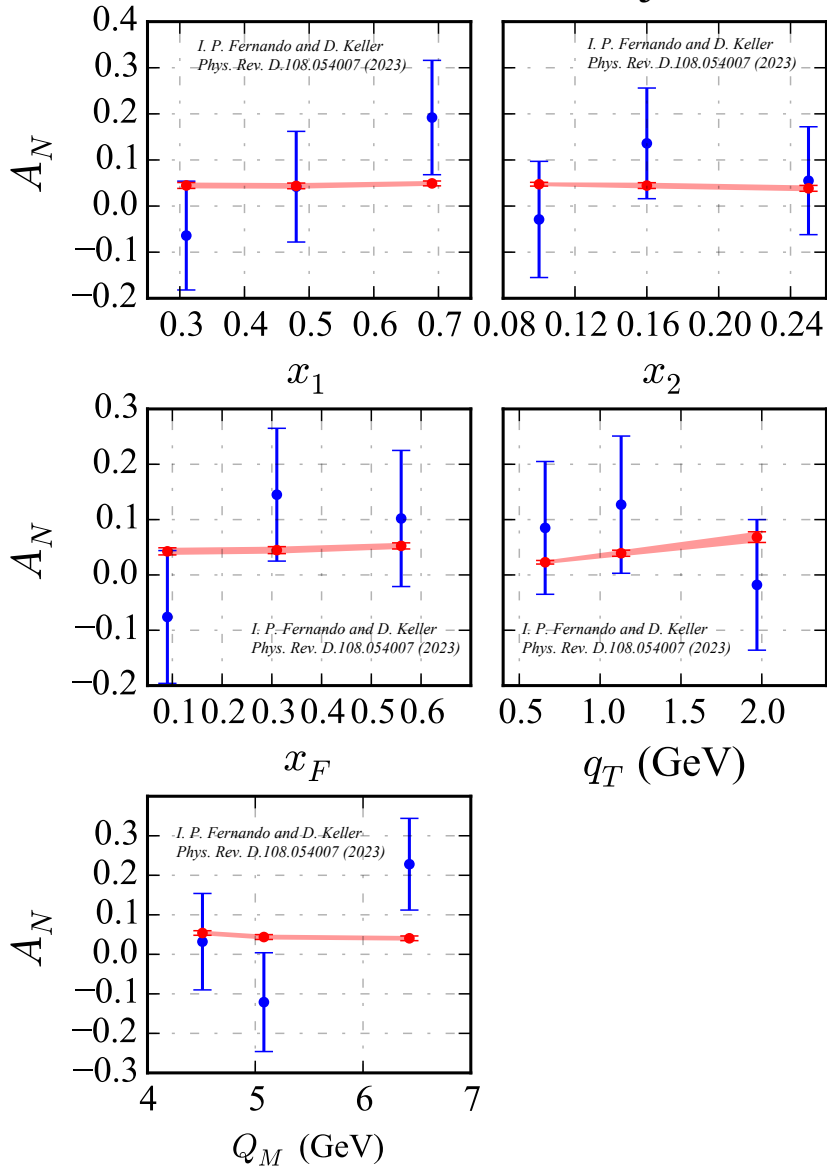
$$\langle k_{\perp 1}^2 \rangle = \langle k_{\perp 2}^2 \rangle = \langle k_{\perp}^2 \rangle = 0.25 \text{ GeV}^2$$

$$\Delta^N f_{q/p^\uparrow}(x, k_\perp)|_{\text{SIDIS}} = -\Delta^N f_{q/p^\uparrow}(x, k_\perp)|_{\text{DY}}$$



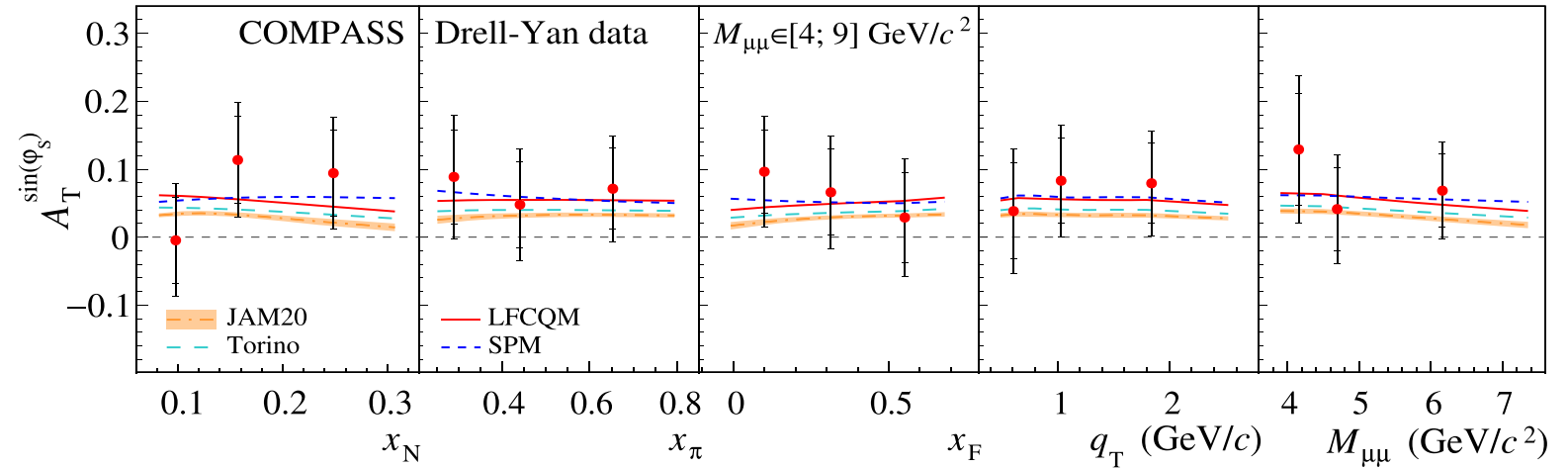
DNN (Proton) Model Projections: DY

COMPASS 2017 DY Projections



COMPASS 2024

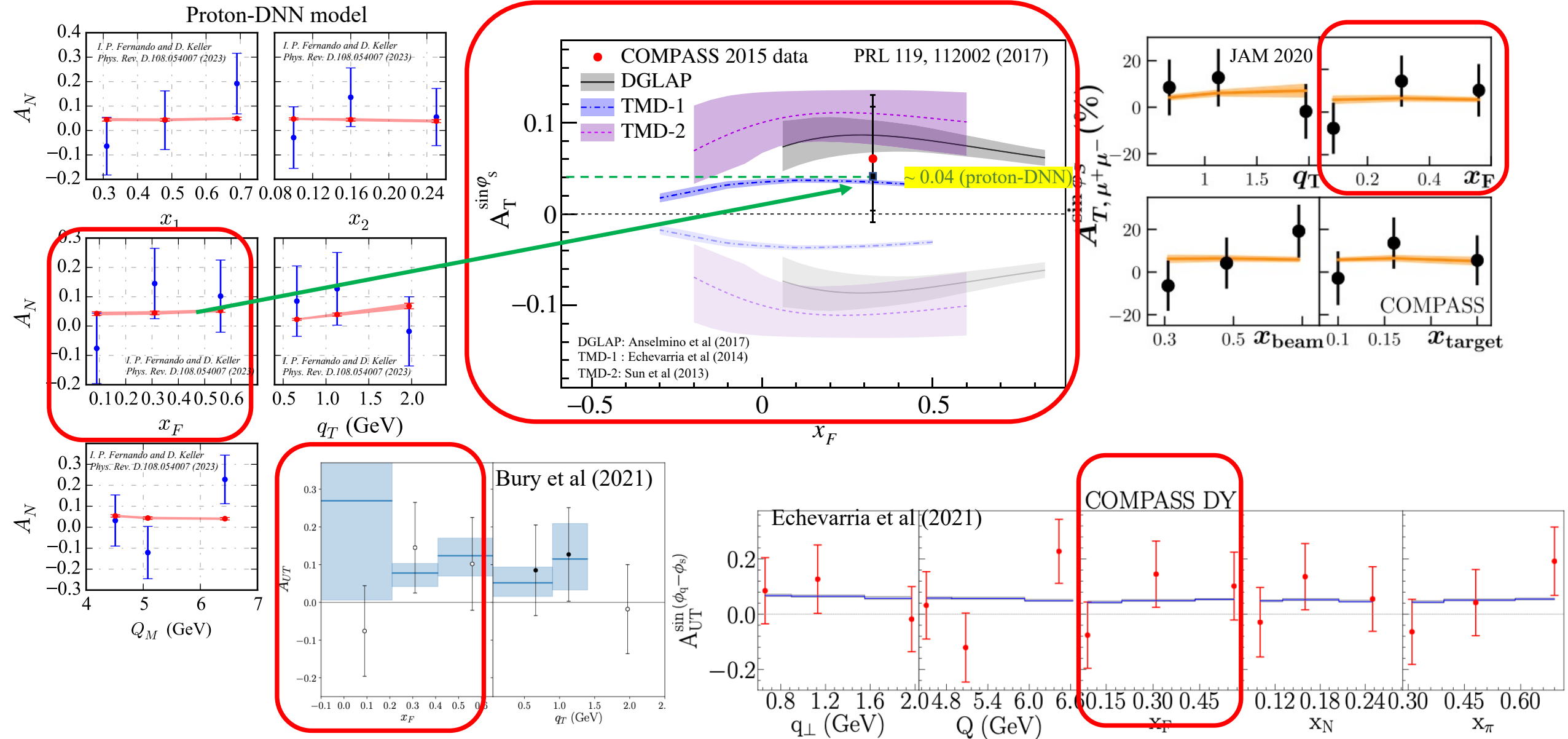
PHYSICAL REVIEW LETTERS **133**, 071902 (2024)



Note: These proton-DNN **projections** based on the assuming the sign-change of the Sivers functions

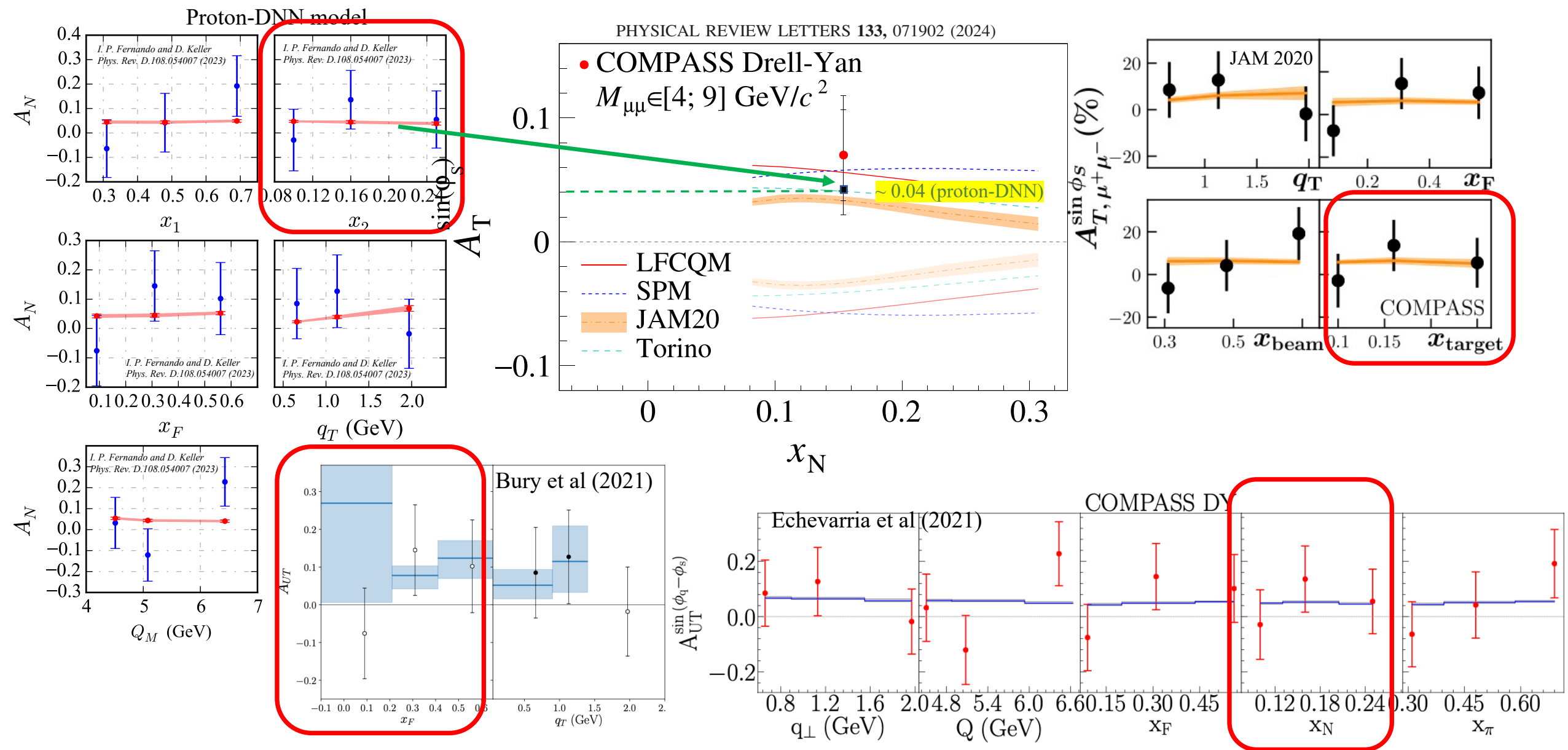
DNN Model Projections: DY

COMPASS 2017 DY Projections



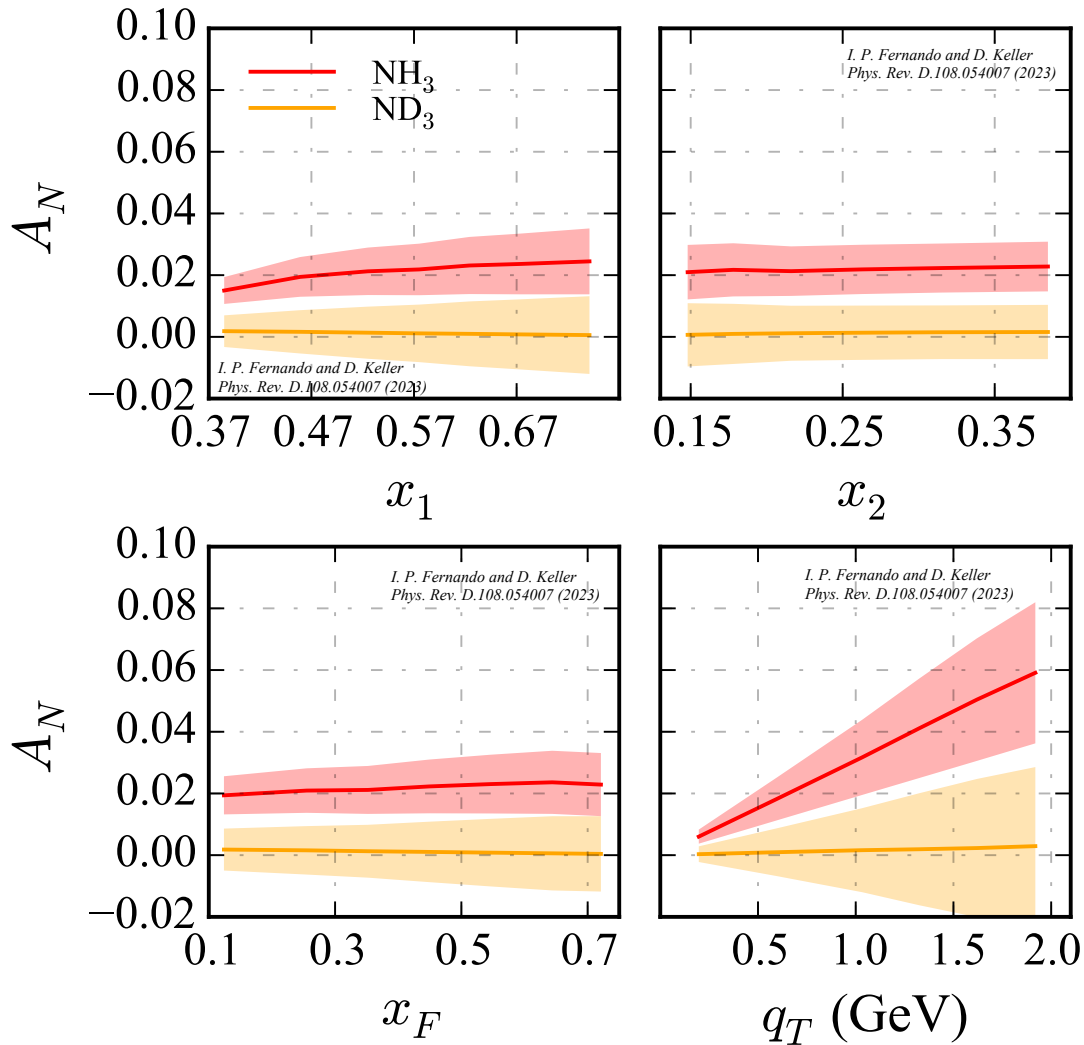
DNN Model Projections: DY

In Comparison with COMPASS 2024 Final



DNN Model Projections: DY @ SpinQuest

DNN Models



- SpinQuest (E1039) experiment at Fermilab is aiming to extract the Sivers function for the light-sea quarks.
- Unpolarized 120 GeV proton beam with polarized proton and deuteron targets (separately).
- Proton-DNN model predictions (Red)
Deuteron-DNN model predictions (Orange)

Systematic Studies: data cuts

$$\begin{aligned}
 W^{\mu\nu} = & \sum_f |\mathcal{H}_f(Q^2, \mu)|^{\mu\nu} \\
 & \times \int d^2k_\perp d^2p_\perp \delta^{(2)}(z_h k_\perp + p_\perp - p_{hT}) \\
 & \times F_{f/N^\uparrow}(x, z_h k_\perp, S; \mu, \zeta_F) D_{h/f}(z_h, p_\perp; \mu, \zeta_D) \\
 & + Y(p_{hT}, Q^2),
 \end{aligned}$$

Examples:

1. Bury et al JHEP 05 (2021) 151

$$Q > 2 \text{ GeV}$$

$$\delta = p_{hT}/zQ \leq 0.3$$

2. Echevarria et al JHEP 01 (2021) 126

$$q_T/Q < 0.75$$

3. JAM2020

$$Q^2 > 1.63 \text{ GeV}^2, \quad 0.2 < z < 0.6, \quad 0.2 < p_{hT} < 0.9 \text{ GeV}.$$

So far, the applicability of TMD factorization was ensured by applying cuts to SIDIS data based on various criteria in the literature.

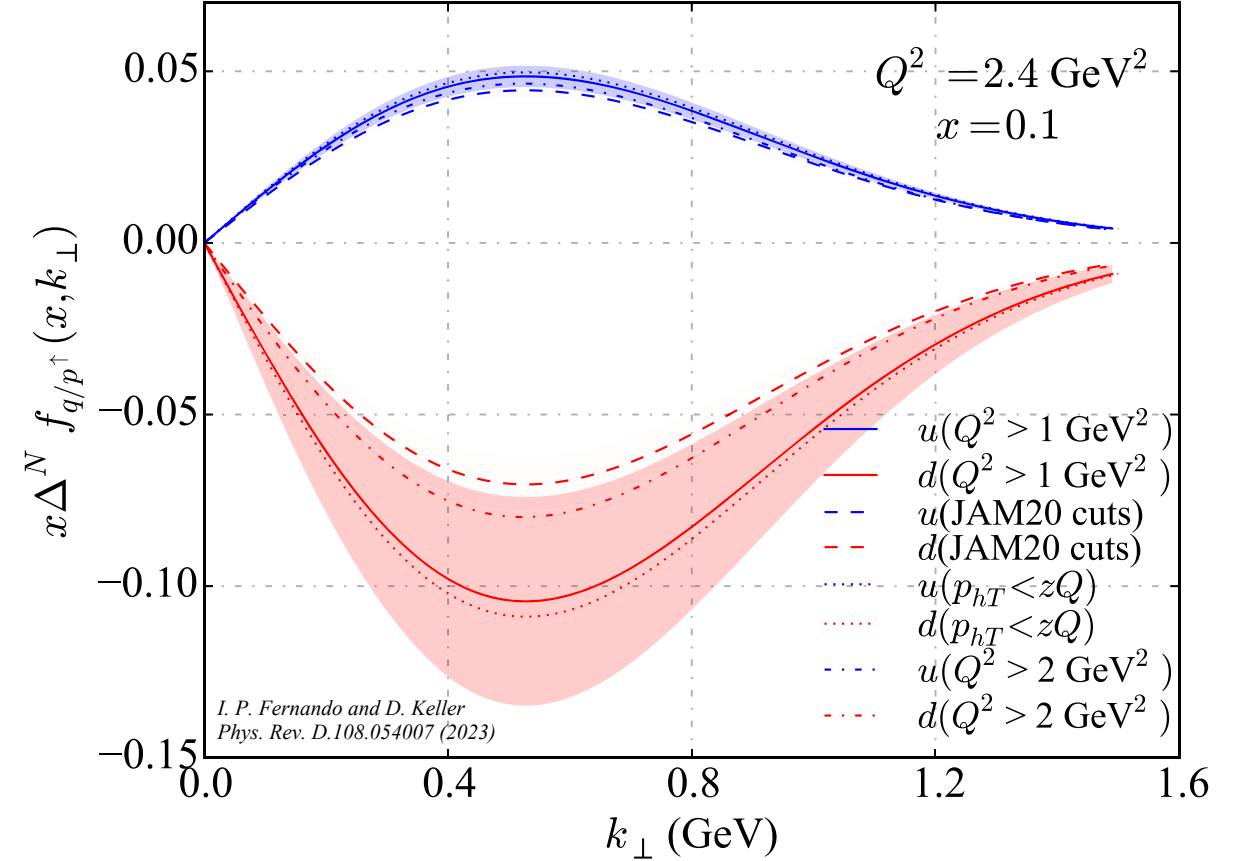


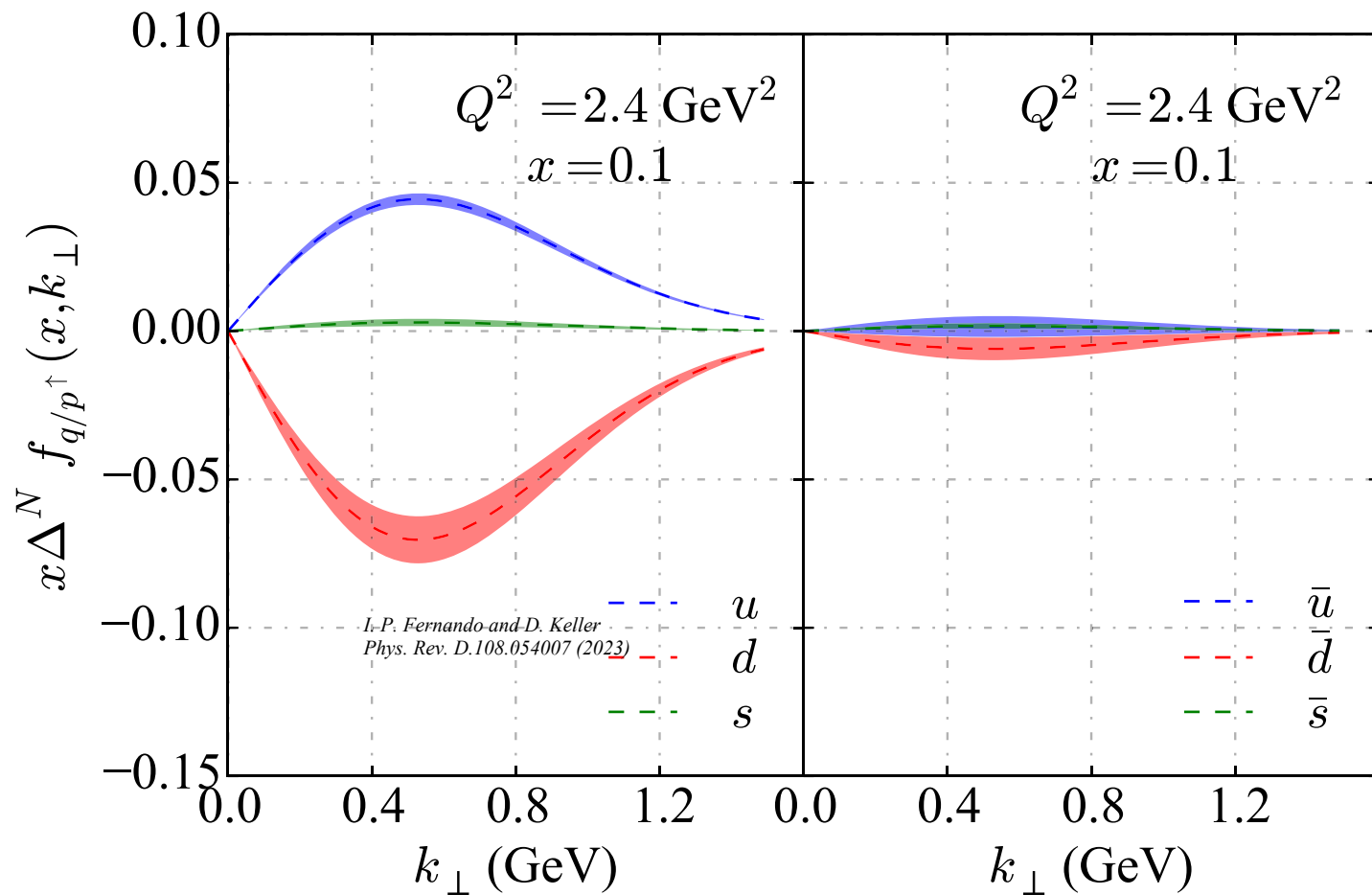
FIG. 17. Solid lines with light band represent the u (in blue), d (in red) Sivers functions using the cut $Q^2 > 1 \text{ GeV}^2$. These resulting DNN models made from the cuts from all tests are also shown.

Systematic Studies: data cuts

$$\begin{aligned}
 W^{\mu\nu} = & \sum_f |\mathcal{H}_f(Q^2, \mu)|^{\mu\nu} \\
 & \times \int d^2k_\perp d^2p_\perp \delta^{(2)}(z_h k_\perp + p_\perp - p_{hT}) \\
 & \times F_{f/N^\uparrow}(x, z_h k_\perp, S; \mu, \zeta_F) D_{h/f}(z_h, p_\perp; \mu, \zeta_D) \\
 & + Y(p_{hT}, Q^2),
 \end{aligned}$$

In addition to the basic data cut $Q^2 > 1 \text{ GeV}^2$ we performed $Q^2 > 2 \text{ GeV}^2$ and $p_{hT} < zQ$ cuts separately with the proton-DNN model to understand the impact on the extracted Sivers functions.

FIG. 18. Sivers functions from a retrained DNN model using the cuts [65] to the data demonstrating that being selective with the data can reduce the error bands of the fit but may also add an unintentional bias.



Systematic Studies: Choice of $h(k)$

$$\Delta^N f_{q/p^\uparrow}(x, k_\perp) = 2\mathcal{N}_q(x)h(k_\perp)f_{q/p}(x, k_\perp)$$

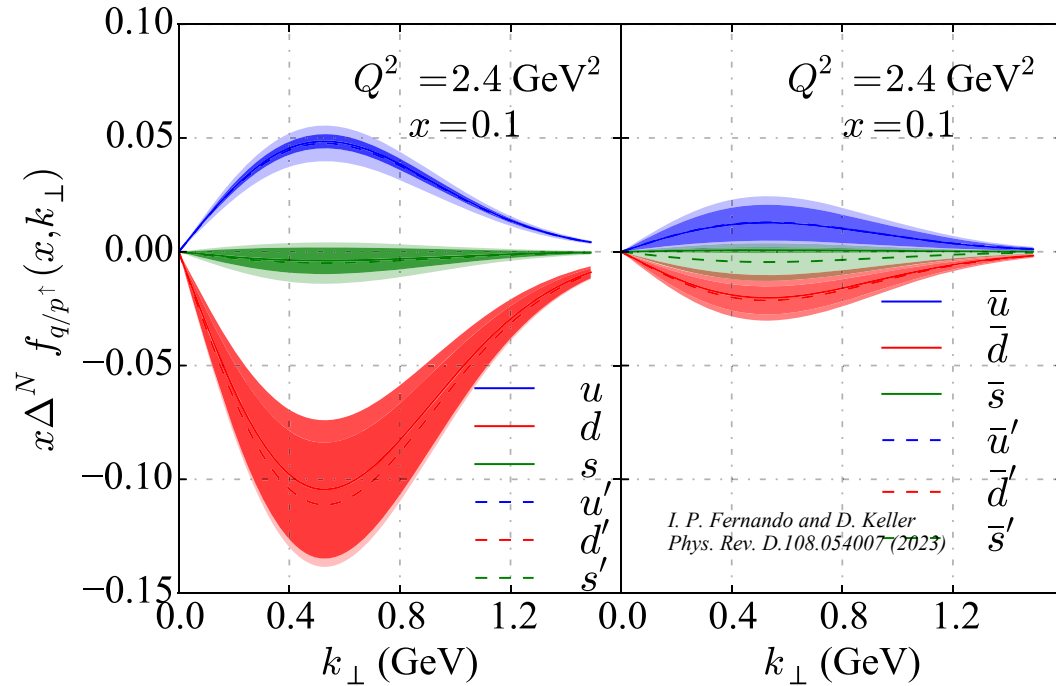


FIG. 19. Using two different $h(k_\perp)$. Solid line with dark band represents the Siversons functions with $h(k_\perp) = \sqrt{2}e \frac{k_\perp}{m_1} e^{-k_\perp^2/m_1^2}$, whereas the dashed line with light band represents the Siversons functions with $h(k_\perp) = \frac{2k_\perp m_1}{m_1^2 + k_\perp^2}$.

$$h(k_\perp) = \sqrt{2}e \frac{k_\perp}{m_1} e^{-k_\perp^2/m_1^2}$$

$$h(k_\perp) = \frac{2k_\perp m_1}{m_1^2 + k_\perp^2}$$

➤ It is clear that the DNN is capable of incorporating both types of $h(k)$ without affecting the Siversons functions in the final model as well as the asymmetries (with deviation less than 1%).

➤ This is because DNN demonstrates that it maps to the $h(k)$ such that the Siversons function is nearly unchanged.

Systematic Studies : TMD Evolution

The solution of the TMD evolution equations

$$\mu^2 \frac{dF(x, b; \mu, \zeta)}{d\mu^2} = \frac{\gamma_F(\mu, \zeta)}{2} F(x, b; \mu, \zeta)$$

$$\zeta \frac{dF(x, b; \mu, \zeta)}{d\zeta} = -\mathcal{D}(b, \mu) F(x, b; \mu, \zeta),$$

$$F(x, b; \mu, \zeta) = \left(\frac{\zeta}{\zeta_\mu(b)} \right)^{-\mathcal{D}(b, \mu)} F(x, b)$$

$$\mu \sim Q, \quad \zeta_F \zeta_D \sim Q^4, \quad \mu^2 = \zeta^2 = Q^2$$

$$\mathcal{N}_q(x) \longrightarrow \mathcal{N}_q(x, Q^2)$$

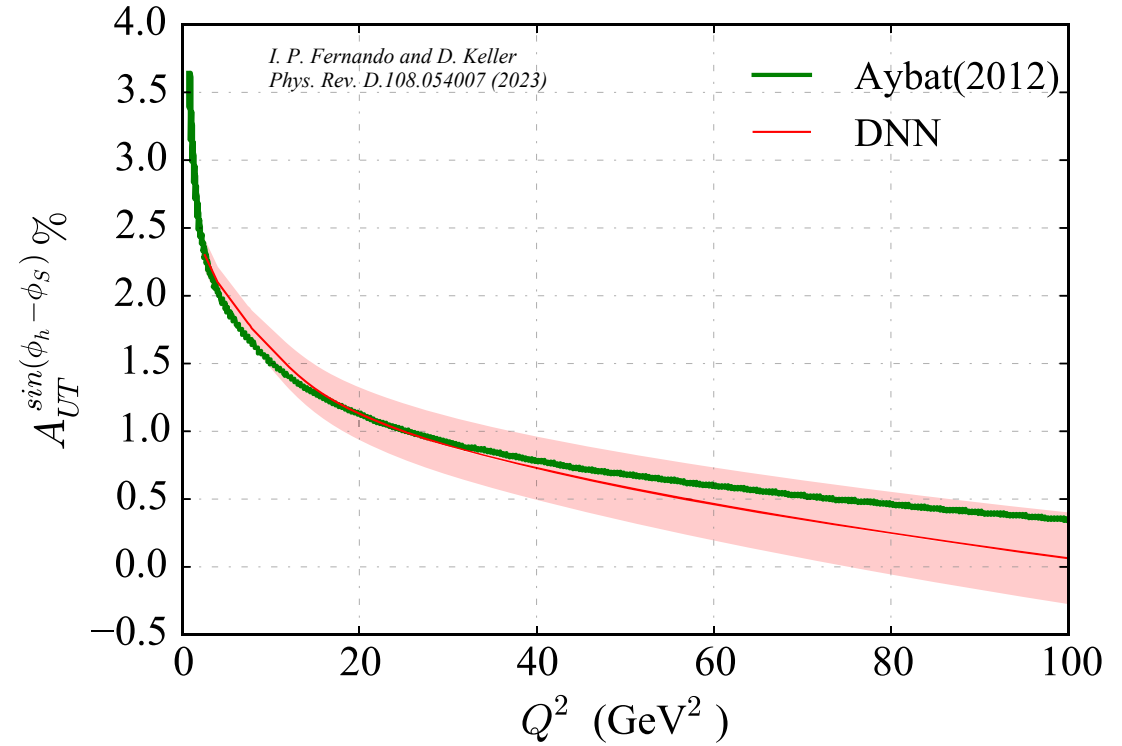


FIG. 21. The Sivers asymmetry evolution in Q^2 compared to the result from [6]. The red-colored solid line and the band represent the mean and standard deviation of the $A_{UT}^{\sin(\phi_h - \phi_s)}$ from 1000 replica models of the proton DNN at $x = 0.12$, $z = 0.32$, $p_{hT} = 0.14$ GeV.

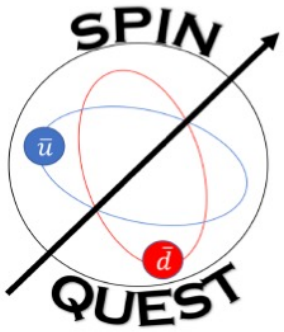
Summary & Outlook

- We proposed a method for performing global fits to extract TMDs employing DNNs (**first-ever application of DNNs in extracting TMDs**).
- Extracting Sivers function was performed as an example of this method based on utilizing DNNs and the use of generating function to ensure the accuracy and precision.
- We have successfully tested our method with pseudo-data, also a dedicated systematic study.
- We chose Deep Neural Net (DNN) method to incorporate all x-dependent features of $\mathcal{N}_q(x)$.
- We have already made a step-forward to consider incorporating TMD-evolution (needs more work)
- We performed global fit with experimental data: Separately on polarized SIDIS with Proton target and Deuteron target and obtained reasonably well description and extracted the Sivers functions for all light quark flavors in SU(3).
- We projected SIDIS and DY Sivers asymmetries: for already completed experiments (as a validation check: COMPASS) and upcoming experiments (such as SpinQuest).
- Currently working on Unpolarized TMDPDF extraction...

*“**Recursively Improving the DNN**
with **multi-dimensional kinematic phase-space**,
and simultaneous information incorporation
from experimental data (Nature)”*

Next:

- Applying the “DNN method” to extract other TMDs such as Transversity, Boer-Mulders function, as well as Spin-1 TMDs.

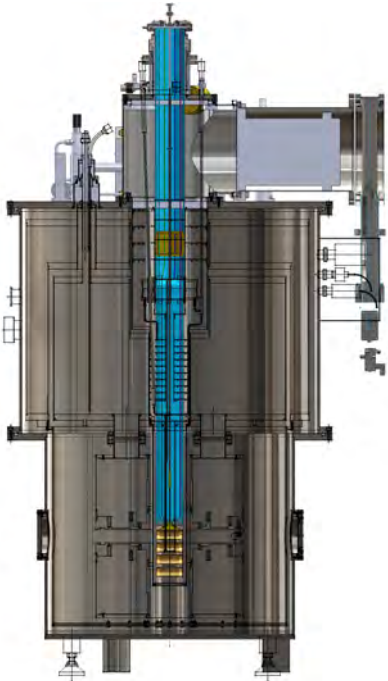


SpinQuest (E1039) Experiment at Fermilab

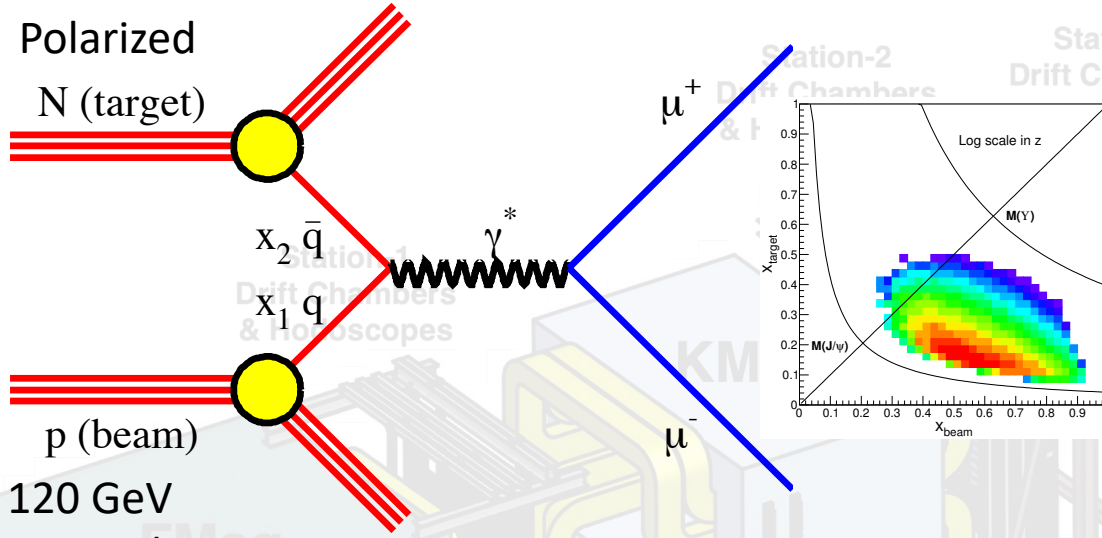


➤ Probing Sivers asymmetry from the (light) 'sea' quark contributions

$$pp^\uparrow(d^\uparrow) \rightarrow \mu^+\mu^-X, 4 < M_{\mu\mu} < 9 \text{ GeV}$$



Polarized
N (target)

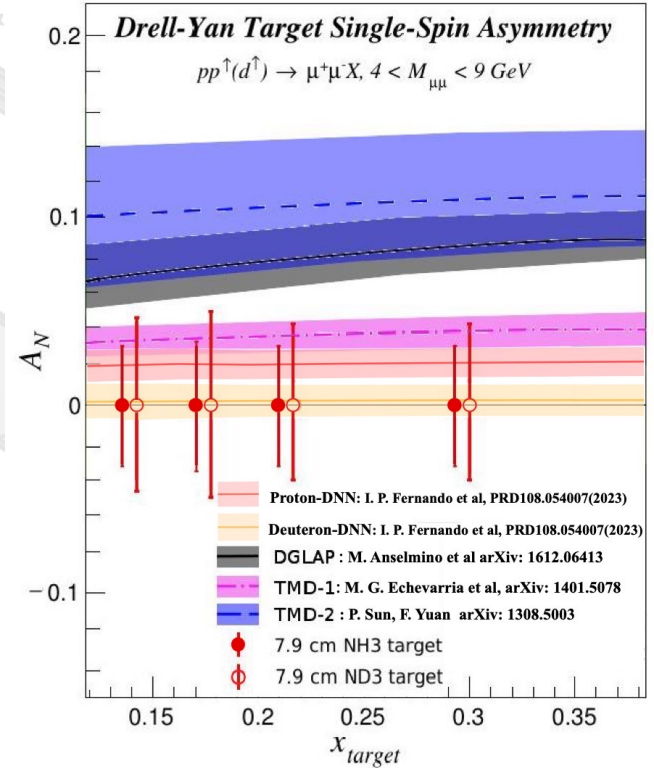


120 GeV
proton beam

$$\frac{d\sigma}{dx_1 dx_2} = \frac{4\pi\alpha^2}{9sx_1x_2} \sum_i e_i^2 (q_i^B(x_1, Q^2)\bar{q}_i^T(x_2, Q^2) + \bar{q}_i^B(x_1, Q^2)q_i^T(x_2, Q^2))$$

Station-4
Proportional tubes

Station-3
Drift Chambers
& Spectrometers



LANL-UVA

Polarized Target

<https://spinqest.fnal.gov/>

<http://twist.phys.virginia.edu/E1039/>

Please Join The Effort

Dustin Keller (dustin@virginia.edu)[Spokesperson]

Kun Liu (liuk@lanl.gov)[Spokesperson]

Highest beam intensity on a polarized target ever!

See Liliet's talk

Thank you

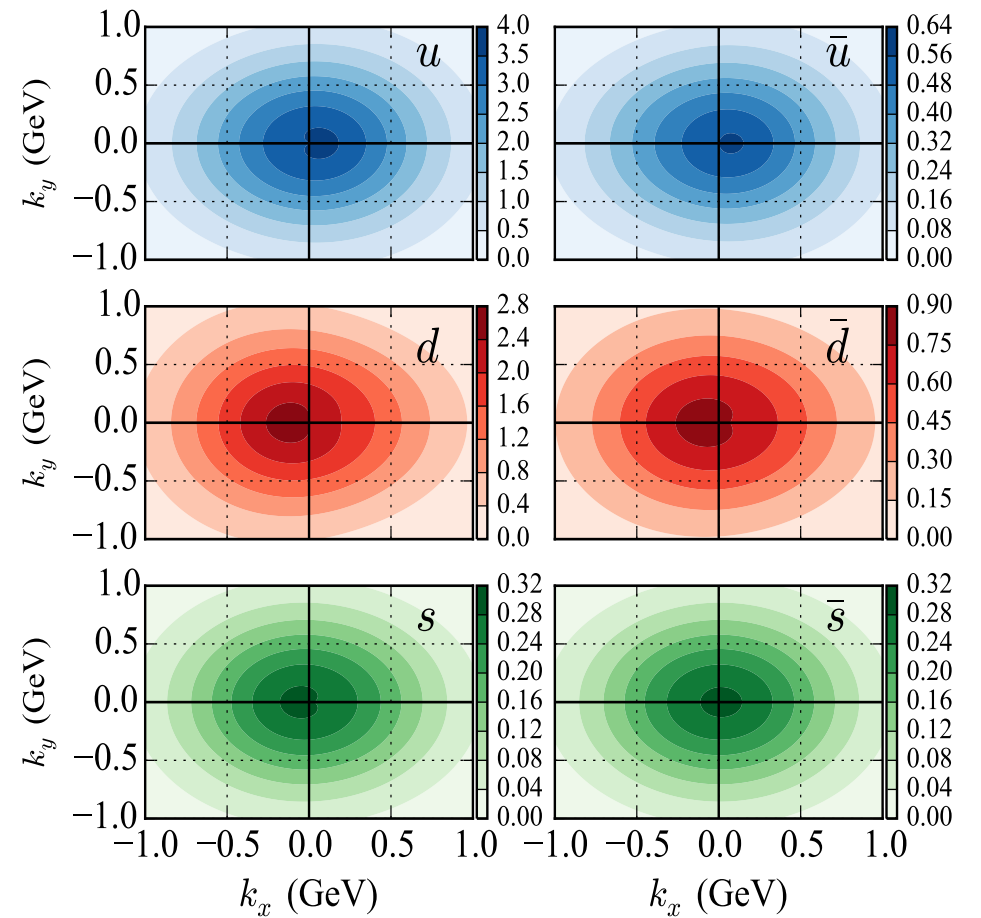


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ENERGY

Office of
Science



This work is supported by DOE contract DE-FG02-96ER40950

Backup Slides

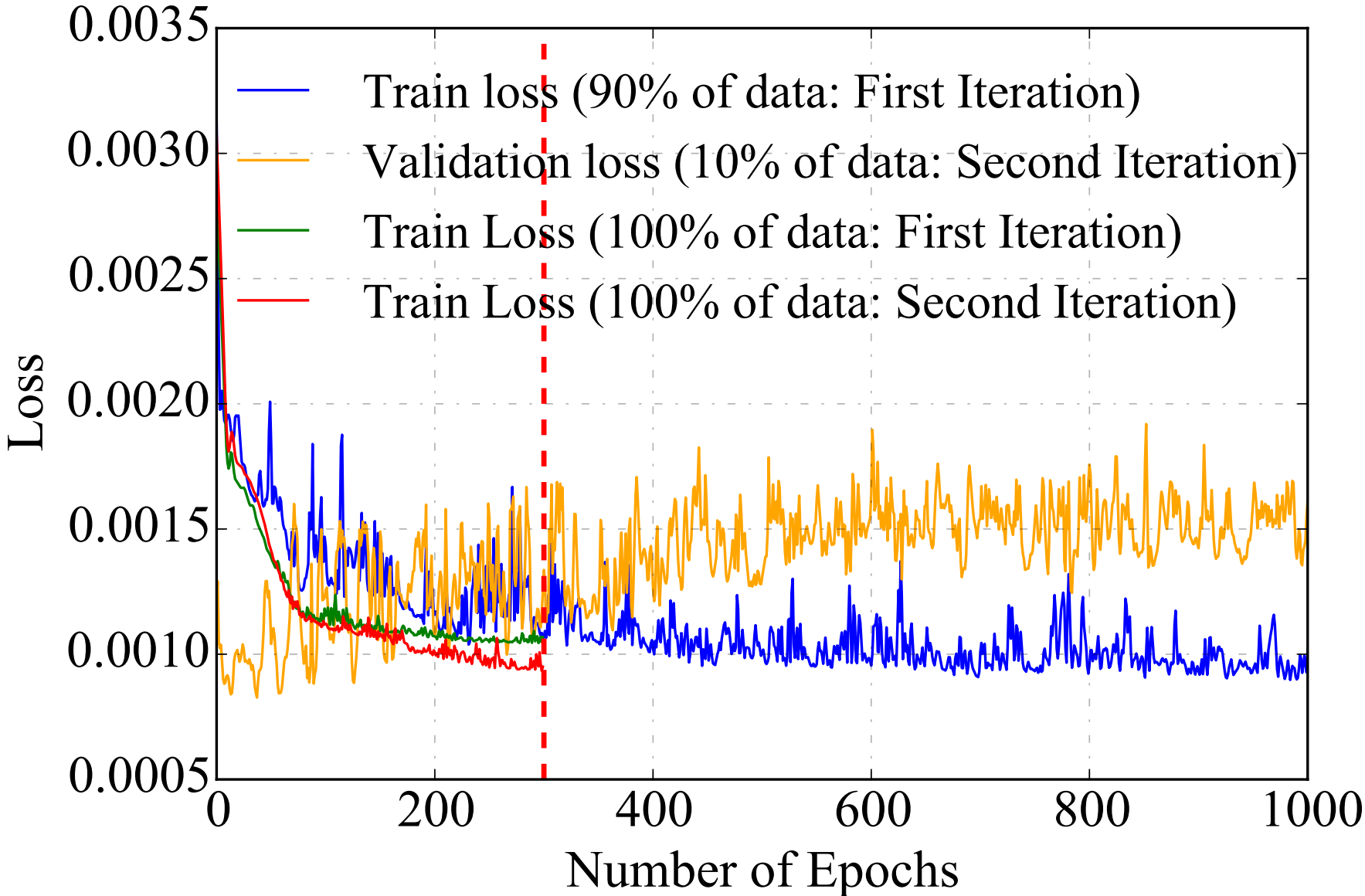


TABLE III. The summary of the optimized sets of hyperparameters: The indications in the table are \mathcal{C}_0^i and \mathcal{C}_0^f for results from the pseudodata from the generating function, \mathcal{C}_p^i , and \mathcal{C}_p^f for results from SIDIS data from experiments associated with the polarized-proton target, and \mathcal{C}_d^i and \mathcal{C}_d^f for results from SIDIS data from experiments associated with the polarized-deuterium target, where i and f indicate the *First Iteration* and *Second Iteration* respectively. The initial learning rate is also listed ($\times 10^{-4}$) as is the final training loss ($\times 10^{-3}$). The accuracy and precision in each case are the maxima over the phase space.

Hyperparameter	\mathcal{C}_0^i	\mathcal{C}_0^f	\mathcal{C}_p^i	\mathcal{C}_p^f	\mathcal{C}_d^i	\mathcal{C}_d^f
Hidden layers	5	7	5	7	5	8
Nodes/layer	256	256	550	550	256	256
Learning rate	1	0.125	5	1	10	1
Batch size	200	256	300	300	100	20
Number of epochs	1000	1000	300	300	200	200
Training loss	0.6	0.05	1.5	1	2	1
ϵ_u^{\max}	95.67	99.27	55.21	94.04	56.80	93.02
$\epsilon_{\bar{u}}^{\max}$	42.62	98.09	52.57	96.70	34.83	91.40
ϵ_d^{\max}	80.46	98.89	55.69	93.13	52.44	89.27
$\epsilon_{\bar{d}}^{\max}$	74.59	97.08	55.37	95.04	46.60	92.58
ϵ_s^{\max}	45.53	79.27	49.54	90.64	36.34	93.41
$\epsilon_{\bar{s}}^{\max}$	59.27	91.13	33.89	82.51	65.57	91.45
σ_u^{\max}	3	0.1	5	2	2	0.4
$\sigma_{\bar{u}}^{\max}$	2	0.2	6	2	8	2
σ_d^{\max}	10	1	20	6	2	1
$\sigma_{\bar{d}}^{\max}$	7	4	20	8	7	1
σ_s^{\max}	2	0.2	4	1	6	2
$\sigma_{\bar{s}}^{\max}$	1	0.1	4	2	6	3