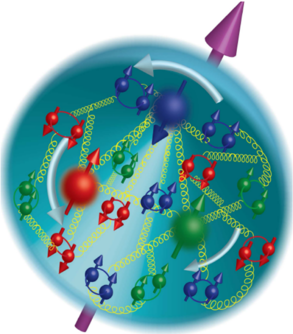
# $J/\psi$ production as a probe of gluon TMDs

Cristian Pisano
University and INFN Cagliari

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# Gluon TMDs

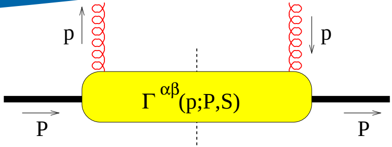
| GLUONS | *unpolarized* | *circular* | *linear* |
|---|---|---|---|
| U | $f_1^g$ | | $h_1^{\perp g}$ |
| L | | $g_{1L}^g$ | $h_{1L}^{\perp g}$ |
| T | $f_{1T}^{\perp g}$ | $g_{1T}^g$ | $h_{1T}^g$, $h_{1T}^{\perp g}$ |

Angeles-Martinez *et al.*, Acta Phys, Pol. B46 (2015)
Mulders, Rodrigues, PRD 63 (2001)
Meissner, Metz, Goeke, PRD 76 (2007)

- $h_1^{\perp\,g}$: $T$-even distribution of linearly polarized gluons inside an unp. hadron
- $h_{1T}^g$, $h_{1T}^{\perp\,g}$: helicity flip distributions like $h_{1T}^q$, $h_{1T}^{\perp\,q}$, but $T$-odd, chiral even!
- $h_1^g \equiv h_{1T}^g + \frac{\boldsymbol{p}_T^2}{2M_p^2}\, h_{1T}^{\perp\,g}$ does not survive under $p_T$ integration, unlike transversity

Gluon TMDs are almost unknown, but models have been proposed

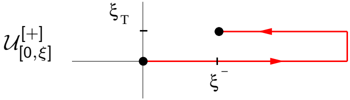
Bacchetta, Celiberto, Radici, Taels, EPJC 80 (2020)
Bacchetta, Celiberto, Radici, EPJC 84 (2024)
Chakrabarti, Choudhary, Gurjar, Kishore, Maji, Mondal, Mukherjee, PRD 108 (2023)

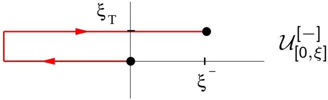


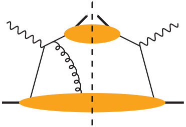

**Gauge invariant definition of $\Gamma^{\mu\nu}$**

$$\Gamma^{[\mathcal{U},\mathcal{U}']\alpha\beta} \propto \langle P,S|\,\mathrm{Tr_c}\left[\,F^{+\beta}(0)\,\mathcal{U}^{\mathcal{C}}_{[0,\xi]}\,F^{+\alpha}(\xi)\,\mathcal{U}^{\mathcal{C}'}_{[\xi,0]}\,\right]|P,S\rangle$$

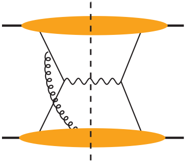
Mulders, Rodrigues, PRD 63 (2001)
Buffing, Mukherjee, Mulders, PRD 88 (2013)
Boer, Cotogno, Van Daal, Mulders, Signori, Zhou, JHEP 1610 (2016)



FSI in SIDIS



ISI in DY

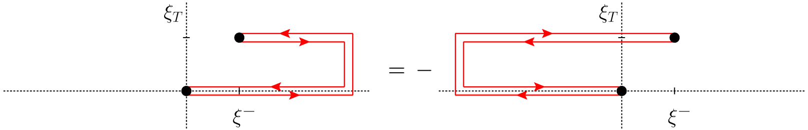
## Related Processes

$ep^\uparrow \to e' Q\overline{Q} X$, $ep^\uparrow \to e'\, \mathrm{jet\, jet}\, X$ probe GSF with $[++]$ gauge links (WW)

$p^\uparrow p \to \gamma\gamma X$ (and/or other CS final state) probe GSF with $[--]$ gauge links

**Analogue of the sign change of $f_{1T}^{\perp\, q}$ between SIDIS and DY (true also for $h_1^g$ and $h_{1T}^{\perp\, g}$)**

$$f_{1T}^{\perp\, g\, [e\, p^\uparrow \to e'\, Q\overline{Q}\, X]} = -f_{1T}^{\perp\, g\, [p^\uparrow\, p \to \gamma\, \gamma\, X]}$$

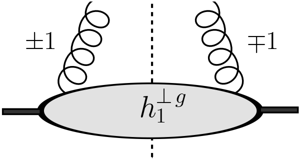



Boer, Mulders, CP, Zhou (2016)

Motivation to study gluon TMDs both at RHIC and the EIC

Gluons inside an unpolarized hadron can be linearly polarized

It requires nonzero transverse momentum




Interference between $\pm 1$ gluon helicity states

Like the unpolarized gluon TMD, it is $T$-even and exists in different versions:

- $[++] = [--]$ (WW) (SIDIS and DY-like process)

Gluons can be probed in heavy quark production in both *ep* and *pp* scattering

Mukherjee, Rajesh, EPJC 77 (2017)
Lansberg, CP, Scarpa, Schlegel, PLB 784 (2018)
Rajesh, Kishore, Mukherjee, PRD 98 (2018)
Bacchetta, Boer, CP, Taels, EPJC 80 (2020)

# $J/\psi$ production at the EIC

# $J/\psi$ production at the EIC

$e\,p \to e\,J/\psi\,X$ (with the inclusion of TMD shape functions)

Mukherjee, Rajesh, EPJ.C 77 (2017)
Kishore, Mukherjee, PRD 99 (2019)
Bacchetta, Boer, CP, Taels, EPJ.C 80 (2020)
Boer, Bor, Maxia, CP, Yuan, JHEP 08 (2023)

$e\,p \to e\,J/\psi\,\text{jet}\,X$

D'Alesio, Murgia, CP, Taels, PRD 100 (2019)
Kishore, Mukherjee, Pawar, Siddiqah, PRD 106 (2022)
Maxia, Yuan, 2403.02097
Kishore, Mukherjee, Pawar, Siddiqah, 2408.05698

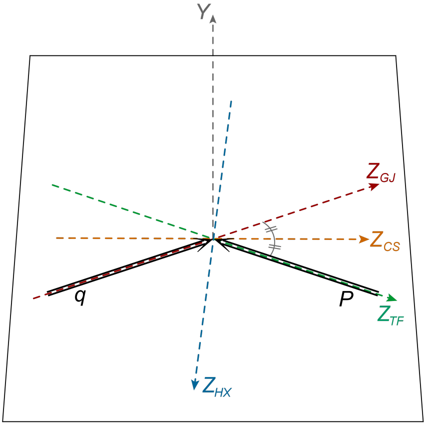
$e\,p \to e\,J/\psi\,\gamma\,X$

Chakrabarti, Kishore, Mukherjee, Rajesh, PRD 107 (2023)

$e\,p \to e\,J/\psi\,\pi\,X$

Banu, Mukherjee, Pawar, Rajesh, PRD 110 (2024)

We study $\gamma^*(q) + p(P) \to J/\psi(P_\psi) + X$ in the $J/\psi$ rest frame




HX: Helicity
TF: Target
CS: Collins-Soper
GJ: Gottfried-Jackson

The frames are related to each other by a rotation around the $Y$ axis

Model-independent arguments (gauge invariance, hermiticity, parity conservation) lead to eight independent helicity structure functions:

Lam, Tung, PRD 18 (1978)
Boer, Vogelsang, PRD 74 (2006)

$$\begin{aligned}
\mathcal{W}_T^{\mathcal{P}} &\equiv \mathcal{W}_{11}^{\mathcal{P}} = \mathcal{W}_{-1-1}^{\mathcal{P}} \\
\mathcal{W}_L^{\mathcal{P}} &\equiv \mathcal{W}_{00}^{\mathcal{P}} \\
\mathcal{W}_\Delta^{\mathcal{P}} &\equiv \sqrt{2}\,\mathrm{Re}\,\mathcal{W}_{10}^{\mathcal{P}} \\
\mathcal{W}_{\Delta\Delta}^{\mathcal{P}} &\equiv \mathcal{W}_{1-1}^{\mathcal{P}} = \mathcal{W}_{-11}^{\mathcal{P}}
\end{aligned}$$

- $\mathcal{P} = \perp, \parallel$: $\gamma^*$ polarization (w.r.t. $P, q$)
- $\Lambda = T, L, \Delta, \Delta\Delta$: $J/\psi$ helicity

However, by looking at the angular dependence of the decaying leptons only four linear combinations can be disentangled

$$\mathcal{W}_\Lambda \equiv \left[1 + (1-y)^2\right] \mathcal{W}_\Lambda^{\perp} + (1-y)\mathcal{W}_\Lambda^{\parallel} \qquad \text{with} \qquad \Lambda = T, L, \Delta, \Delta\Delta$$

Usual SIDIS variables:

$$Q^2 = -q^2\,, \qquad x_B = \frac{Q^2}{2P\cdot q}\,, \qquad y = \frac{P\cdot q}{P\cdot \ell}\,, \qquad z = \frac{P\cdot P_\psi}{P\cdot q}$$

Cross section differential in $\Omega = (\theta, \varphi)$, solid angle of the decaying lepton $\ell^+$

$$\mathrm{d}\sigma \equiv \frac{\mathrm{d}\sigma}{\mathrm{d}x_B\,\mathrm{d}y\,\mathrm{d}^4P_\psi\,\mathrm{d}\Omega}$$

$$\mathrm{d}\sigma \propto \frac{\alpha^2}{yQ^2}\left[\mathcal{W}_T(1+\cos^2\theta) + \mathcal{W}_L(1-\cos^2\theta) + \mathcal{W}_\Delta \sin 2\theta\cos\varphi + \mathcal{W}_{\Delta\Delta}\sin^2\theta\cos 2\varphi\right]$$

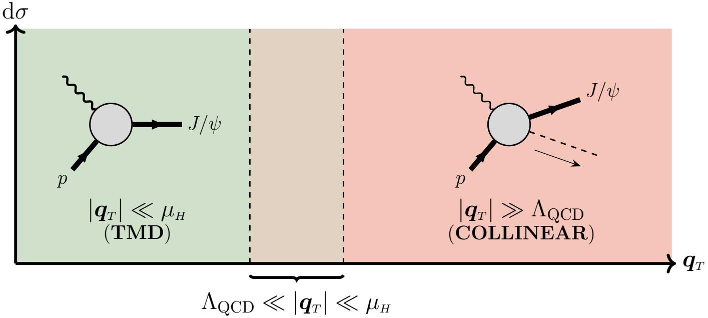
Alternatively, in terms of the polarization parameters $\lambda, \mu, \nu$ :

$$\mathrm{d}\sigma \propto \frac{\alpha^2}{yQ^2}\left(\mathcal{W}_T + \mathcal{W}_L\right)\left[1 + \lambda\cos^2\theta + \mu\sin 2\theta\cos\varphi + \frac{1}{2}\,\nu\,\sin^2\theta\cos 2\varphi\right]$$

$$\lambda = \frac{\mathcal{W}_T - \mathcal{W}_L}{\mathcal{W}_T + \mathcal{W}_L}\,, \qquad \mu = \frac{\mathcal{W}_\Delta}{\mathcal{W}_T + \mathcal{W}_L}\,, \qquad \nu = \frac{2\mathcal{W}_{\Delta\Delta}}{\mathcal{W}_T + \mathcal{W}_L}$$

Three physical scales, two theoretical tools

Bacchetta, Boer, Diehl, Mulders, JHEP 08 (2008)
Boer, D'Alesio, Murgia, CP, Taels, JHEP 09 (2020)
D'Alesio, Maxia, Murgia, CP, Rajesh, JHEP 037 (2022)
Boer, Bor, Maxia, CP, Yuan, JHEP 08 (2023)



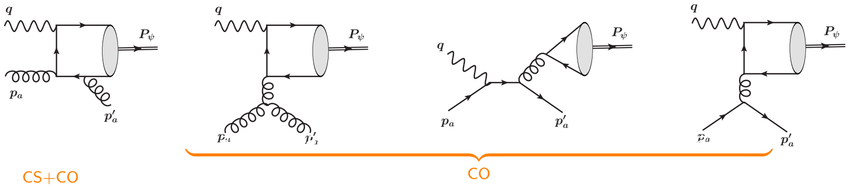

TMD factorization proven only for light hadron production in SIDIS

Matching in the intermediate region: a test of TMD factorization

The helicity structure functions can be calculated within the NRQCD framework

Contributing partonic subprocesses at the orders $\alpha_s^2$ and $v^4$

$$\gamma^*(q) + a(p_a) \to J/\psi(P_\psi) + a(p'_a) \qquad a = g, q, \bar{q}$$

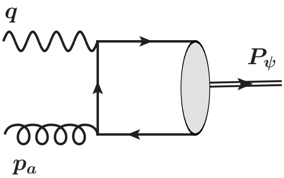



Fock states included in the calculation: ${}^3S_1^{[1]}$, ${}^1S_0^{[8]}$, ${}^3S_1^{[8]}$, ${}^3P_0^{[8]}$

($Q^2 = 0$) Beneke, Kramer, Vanttinen, PRD 57 (1998)
($W_T, W_L$) Yuan, Chao, PRD 63 (2001)
(unpolarized) Kniehl, Zwirner, NPB 621 (2002)

$q_T$: transverse momentum of the photon w.r.t. $P_\psi, P$

When $q_T^2 \ll Q^2$ at $\mathcal{O}(\alpha_s)$ only color-octet (CO) production channels dominate




Neglecting smearing effects in quarkonium formation:

$$\mathcal{W}_T^\perp = \widehat{w}_T^\perp\, f_1^g(x, \boldsymbol{q}_T^2) \qquad \mathcal{W}_T^\parallel = \widehat{w}_T^\parallel\, f_1^g(x, \boldsymbol{q}_T^2) \qquad \mathcal{W}_L^\parallel = \widehat{w}_L^\parallel\, f_1^g(x, \boldsymbol{q}_T^2)$$

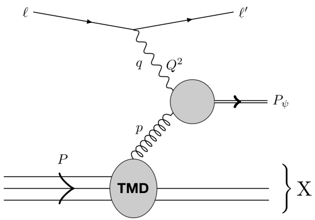
$$\mathcal{W}_{\Delta\Delta}^\perp = \widehat{w}_{\Delta\Delta}^\perp\, h_1^{\perp\, g}(x, \boldsymbol{q}_T^2)$$

$\mathcal{W}_{\Delta\Delta}^\perp$ gives access to $h_1^{\perp\, g}$ and to the poorly known ${}^3P_0$ LDME

Smearing effects encoded in $\Delta^{[n]}$ need to be included to match the result in the intermediate overlapping region $\Lambda_{\rm QCD}^2 \ll \boldsymbol{q}_T^2 \ll Q^2$

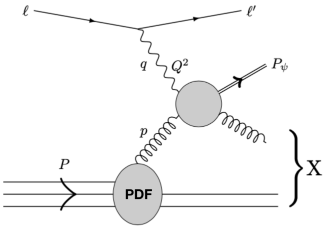
Imposing the matching of the TMD and collinear results in the overlapping region
$\Lambda^2_{\text{QCD}} \ll \boldsymbol{q}_T^2 \ll Q^2$: $f_1^g \longrightarrow \mathcal{C}[f_1^g\, \Delta^{[n]}]$

TMD




$\boldsymbol{q}_T^2 \ll Q^2$

Collinear




$\Lambda^2_{\text{QCD}} \ll \boldsymbol{q}_T^2$

Boer, D'Alesio, Murgia, CP, Taels, JHEP 09 (2020)
D'Alesio, Maxia, Murgia, CP, Rajesh, JHEP 037 (2022)
Boer, Bor, Maxia, CP, Yuan, JHEP 08 (2023)

Knowing the perturbative tail of the gluon TMD, we determine the one of $\Delta^{[n]}$

Factorization scale fixed to be: $\mu^2 = M_\psi^2 + Q^2$

$$\Delta^{[n]}(k_T^2, \mu^2) = -\frac{\alpha_S}{2\pi^2 k_T^2} C_A \left(1 + \log \frac{M_\psi^2}{M_\psi^2 + Q^2}\right) \langle \mathcal{O}^{[n]} \rangle \qquad \text{for} \quad k_T \gg \Lambda_{\mathrm{QCD}}$$

Boer, Bor, Maxia, CP, Yuan, JHEP 08 (2023)

Less divergent than fragmentation functions of light quarks $\propto \log Q^2/k_T^2$

Independent of $J/\psi$ polarization and CO quantum numbers

It should not depend on $Q^2$: hint of process dependence (photoproduction result is obtained by imposing $Q^2 = 0$)

Similar results obtained using the SCET formalism

Echevarria, Romera, Taels, JHEP 09 (2024)

# $J/\psi$ production at the LHC

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\mathrm{d}Y\mathrm{d}^2 q_T \mathrm{d}\Omega} \approx A\, f_1^g \otimes f_1^g + B\, f_1^g \otimes h_1^{\perp g} \cos(2\phi_{CS}) + C\, h_1^{\perp g} \otimes h_1^{\perp g} \cos(4\phi_{CS})$$

den Dunnen, Lansberg, CP, Schlegel, PRL 112 (2014)

- valid up to corrections $\mathcal{O}(q_T/Q)$
- $Y$: rapidity of the $J/\psi$-pair, along the beam in the hadronic c.m. frame
- $\mathrm{d}\Omega = \mathrm{d}\cos\theta_{CS}\,\mathrm{d}\phi_{CS}$: solid angle for $J/\psi$-pair in the Collins-Soper frame

Analysis similar to the one for $pp \to J/\psi\, \gamma^*\, X$ and $pp \to J/\psi\, J/\psi\, X$

Lansberg, CP, Schlegel, NPB 920 (2017)
Lansberg, CP, Scarpa, Schlegel, PLB 784 (2018)

The three contributions can be disentangled by defining the transverse moments

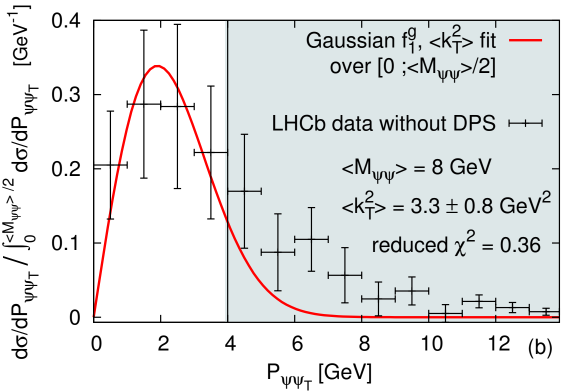
$$\langle \cos n\phi_{CS} \rangle \equiv \frac{\int_0^{2\pi} \mathrm{d}\phi_{CS}\, \cos(n\,\phi_{CS})\, \frac{\mathrm{d}\sigma}{\mathrm{d}Q\mathrm{d}Y\mathrm{d}^2 q_T \mathrm{d}\Omega}}{\int_0^{2\pi} \mathrm{d}\phi_{CS}\, \frac{\mathrm{d}\sigma}{\mathrm{d}Q\mathrm{d}Y\mathrm{d}^2 q_T \mathrm{d}\Omega}} \qquad (n = 2, 4)$$

$$\int \mathrm{d}\phi_{CS}\, \mathrm{d}\sigma \implies f_1^g \otimes f_1^g$$

$$\langle \cos 2\phi_{CS} \rangle \implies f_1^g \otimes h_1^{\perp g}$$

$$\langle \cos 4\phi_{CS} \rangle \implies h_1^{\perp g} \otimes h_1^{\perp g}$$

We consider $q_T = P_T^{\Psi\Psi} \leq M_{\Psi\Psi}/2$ in order to have two different scales



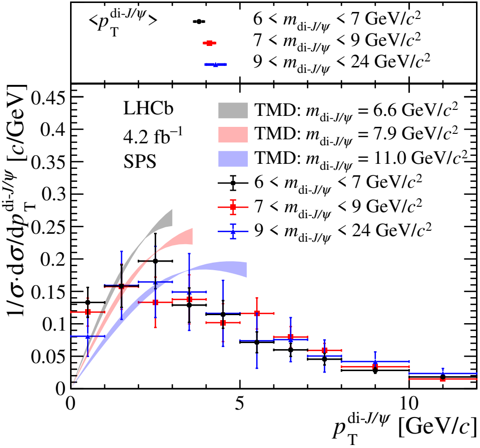

Lansberg, CP, Scarpa, Schlegel, PLB 784 (2018)
LHCb Coll., JHEP 06 (2017)

Gaussian model:
$$f_1^g(x, \boldsymbol{k}_T^2) = \frac{f_1^g(x)}{\pi \langle k_T^2 \rangle} \exp\left( -\frac{\boldsymbol{k}_T^2}{\langle k_T^2 \rangle} \right)$$

# $J/\psi$-pair production

## $p_T$-distribution at $\sqrt{s} = 13$ TeV

No obvious broadening can be seen due to the large uncertainties



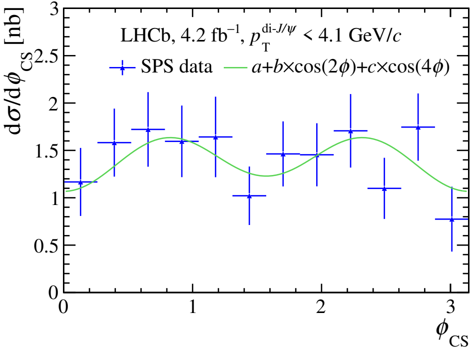

LHCb Coll., JHEP 03 (2024)
Scarpa, Boer, Echevarria, Lansberg, CP, Schlegel EPJC 80 (2020)

The average values of the $p_T$ distributions slightly increase with mass

$$\langle \cos 2\phi \rangle = -0.029 \pm 0.050\,(\text{stat}) \pm 0.009\,(\text{syst})$$
$$\langle \cos 4\phi \rangle = -0.087 \pm 0.052\,(\text{stat}) \pm 0.013\,(\text{syst})$$

Theoretical predictions consistent with measureaments

Scarpa, Boer, Echevarria, Lansberg, CP, Schlegel EPJC 80 (2020)

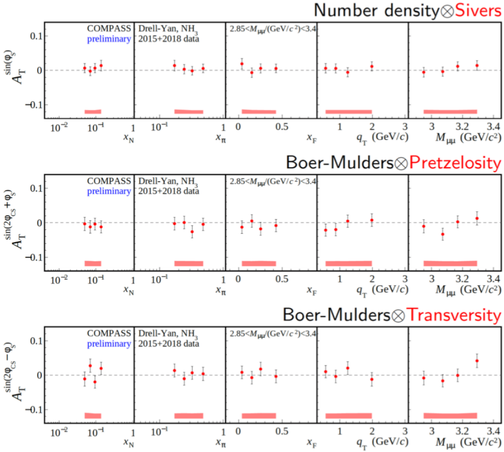



LHCb Coll., JHEP 03 (2024)

The results are consistent with zero, but the presence of an azimuthal asymmetry at a few percent level is allowed

# $\pi\, p^{\uparrow} \to J/\psi\, X$
## Preliminary COMPASS data

Single spin asymmetries compatible with zero



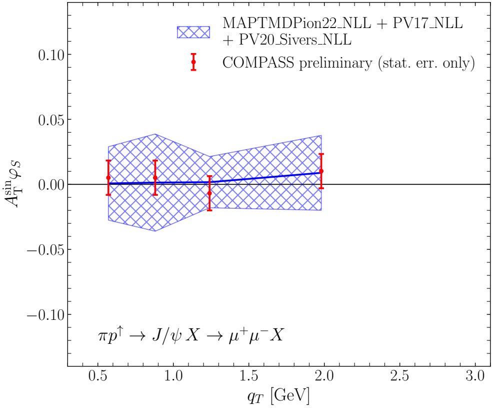

J. Matousek, talk at IWHSS (2022)

TMD factorization assumed but problematic: $q\bar{q} \to J/\psi$, $gg \to J/\psi$ via CO in NRQCD

No gluon Sivers function: cancellation due to the presence of both ISI/FSI

F. Yuan, PRD 78 (2008)

Very preliminary theoretical predictions, only quark contribution included




Courtesy of Carlo Flore
(Thanks also to F. Delcarro & L. Rossi)

$$A_{UT}^{\sin\phi_S} \approx \frac{f_1^{\bar{u}/\pi^-} \otimes f_{1T}^{u/p} + f_1^{d/\pi^-} \otimes f_{1T}^{\bar{d}/p}}{f_1^{q/\pi^-} \otimes f_1^{q/p}}$$

# Conclusions and outlook

- Heavy quarkonia and in particular $J/\psi$ are very good probes of gluon TMDs
- First extraction of unpolarized gluon TMD from LHC data on di-$J/\psi$ production
- Azimuthal asymmetries in $J/\psi$ production in SIDIS could give access to WW-type gluon TMDs (similar to SIDIS for quark TMDs)
- $pp \to \eta_c X$: similar to DY, in principle possible at LHCSpin

  L. Maxia, N. Kato, CP, PRD 110 (2024)
- $J/\psi$ production at AMBER/COMPASS: color entanglement and background from $q\bar{q}$ subprocess, but still very interesting!