Phenomenological comparison of Lattice calculations of the Collins-Soper kernel

Joint 20th International Workshop on Hadron Structure and Spectroscopy and 5th workshop on Correlations in Partonic and Hadronic Interactions (IWHSS-CPHI-2024)

30/09/2024-04/10/2024, Yerevan Armenia

UNIVERSITÀ DI TORINO

CHARLES UNIVERSITY
Faculty of mathematics and physics

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Conference participation supported by Charles university grant PRIMUS/22/SCI/017.

Phenomenology on TMD physics

Testing and comparing lattice calculations with experimental data

1) Extrapolation of lattice results

2) Fits on DY data at low scales

3) Postdiction on Z0 production at large scales

TMD factorization of D-Y

CSS formalism

Unpolarized TMD PDFs

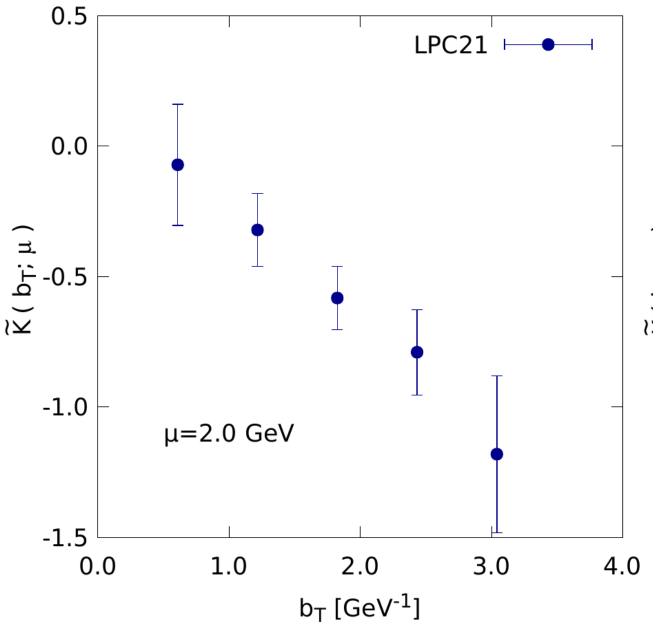
$$\frac{d\sigma}{dQ^2 dy dq_T} \sim \sum_{i,j} \underbrace{\mathcal{H}_{ij}(\mu/Q;\alpha_s(\mu))}_{\text{perturbative}} \left[\underbrace{F_{i/A}(x_A, k_{T,A};\zeta_A,\mu)}_{\text{non perturbative}} \otimes \underbrace{F_{j/B}(x_B, k_{T,B};\zeta_B,\mu)}_{\text{non perturbative}}\right]$$

CS kernel

$$\frac{\partial \ \log \tilde{F}(x, b_T;\mu,\zeta)}{\partial \ \log \sqrt{\zeta}} = \underbrace{\tilde{K}(b_T;\mu)}_{\text{non perturbative}}$$

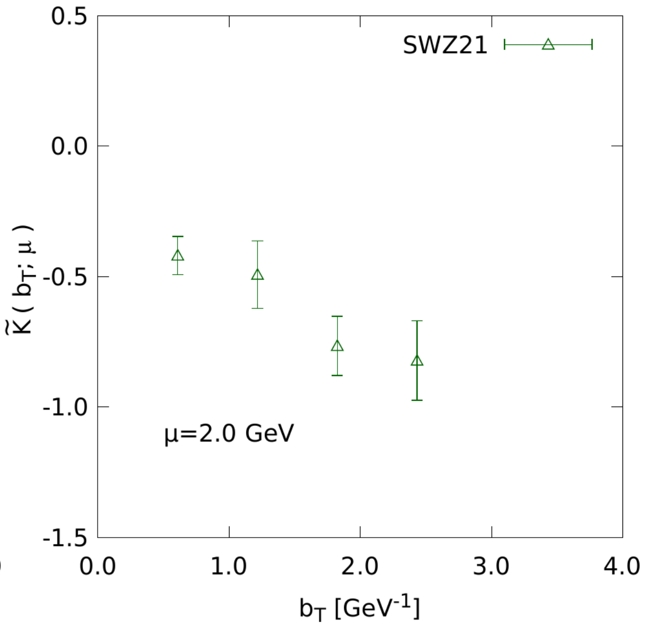
CS Kernel from the lattice

LPC21



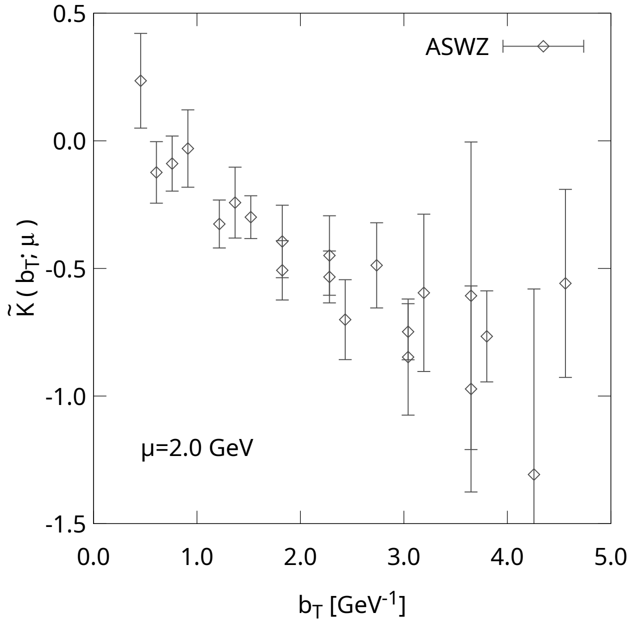
Chu et al. (2022)
Phys.Rev.D 106

SWZ21



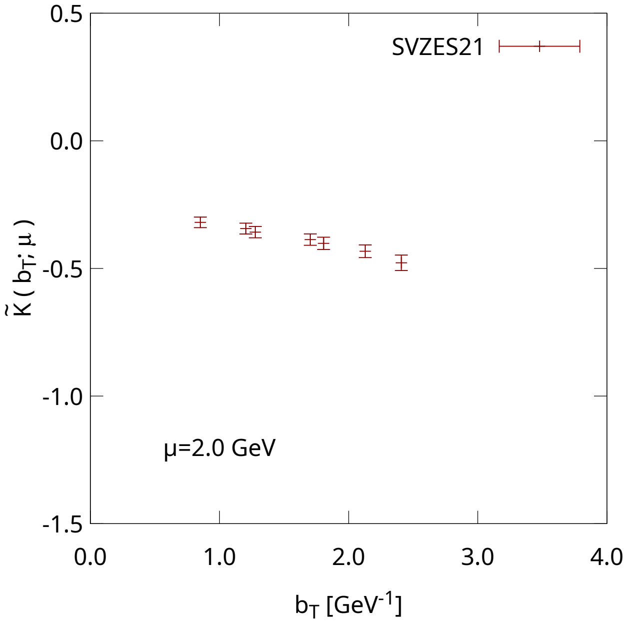
Shanahan et al. (2021)
Phys.Rev.D 104

ASWZ



Avkhadiev et al. (2024)
Phys.Rev.Lett. 132

SVZES21



Schlemmer et al. (2021)JHEP 08

Model formulation

Hadron structure oriented approach (HSO)

Order α_s

$$K_{input}(k_T;\mu_{Q_0}) = K_{fixed}(k_T;\mu_{Q_0}) + K_{core}(k_T)$$

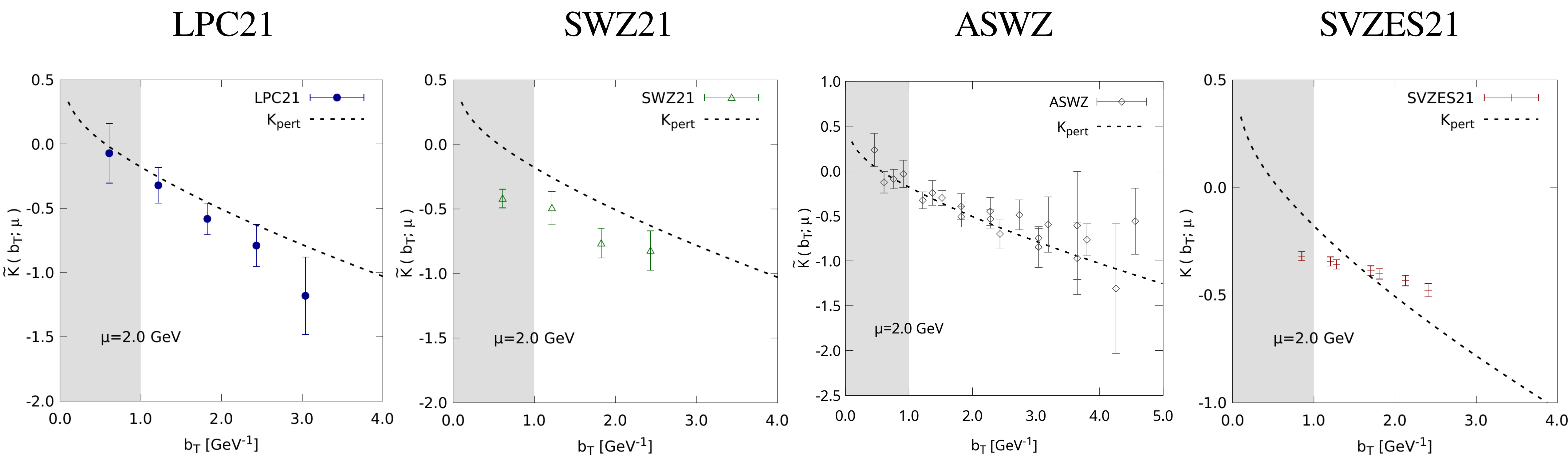
$$FT \Updownarrow$$

$$\tilde{K}_{input}(b_T;\mu_{Q_0}) = \tilde{K}_{fixed}(b_T;\mu_{Q_0}) + \tilde{K}_{core}(b_T)$$

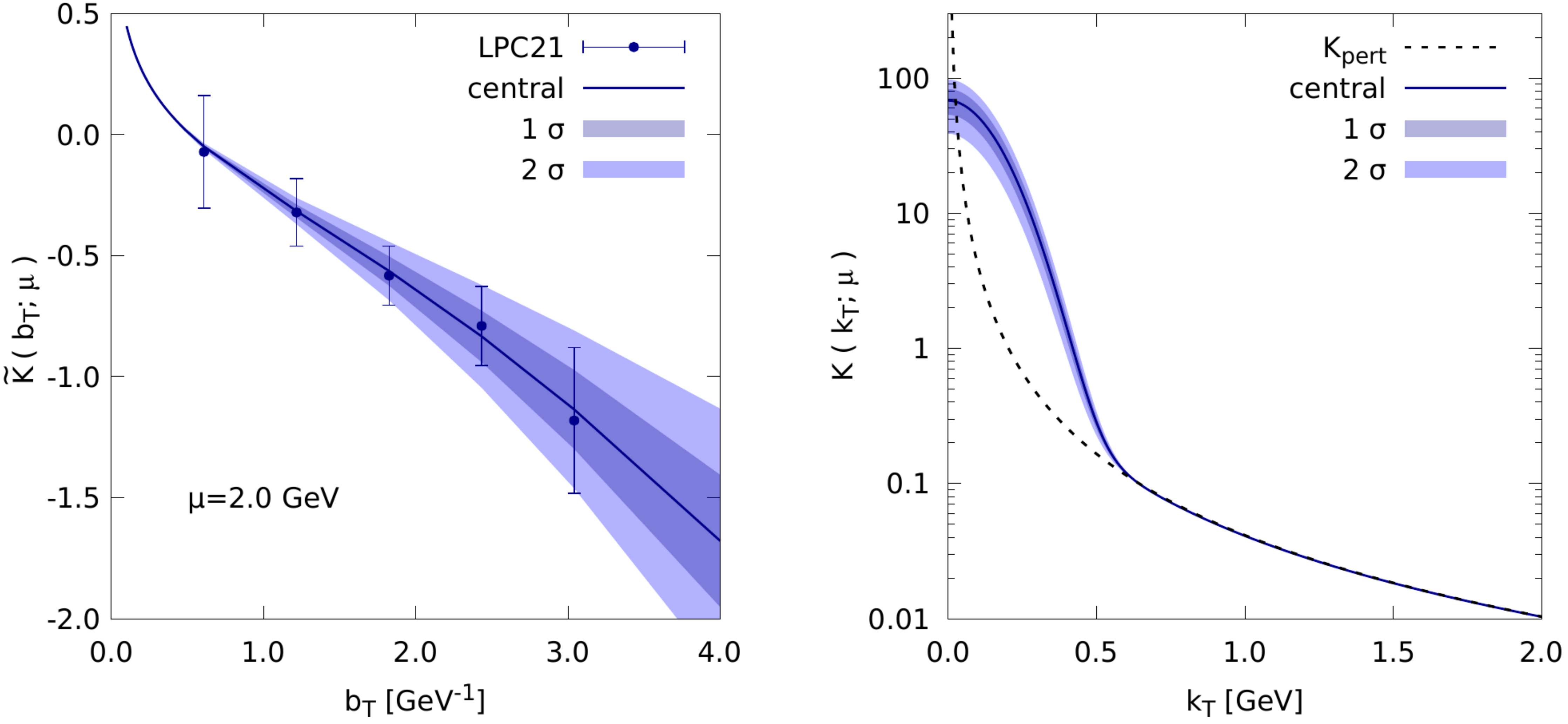
Gonzalez-Hernandez et al. (2022) Phys.Rev.D 106
Gonzalez-Hernandez et al. (2023) Phys.Rev.D 107

See talk by Tommaso Rainaldi

Lattice and pQCD

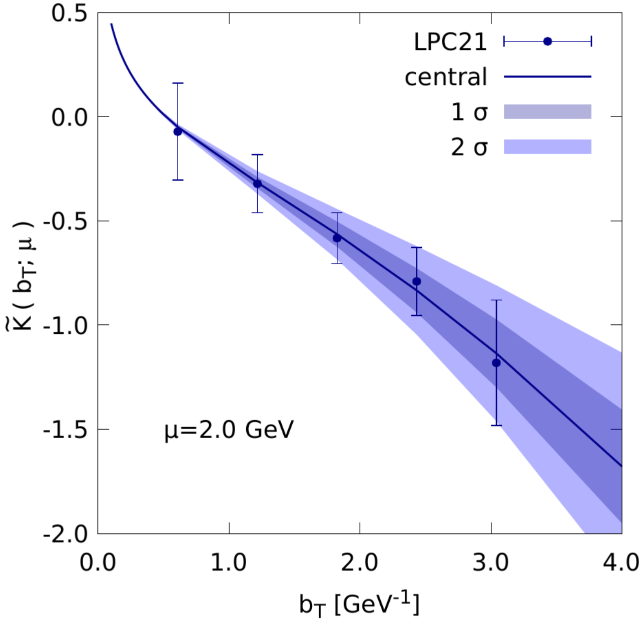


pQCD compatibility



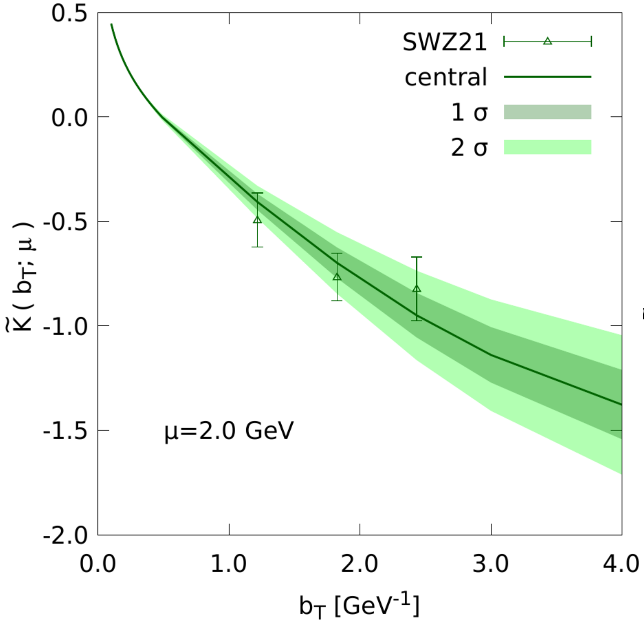
Final fits

LPC21



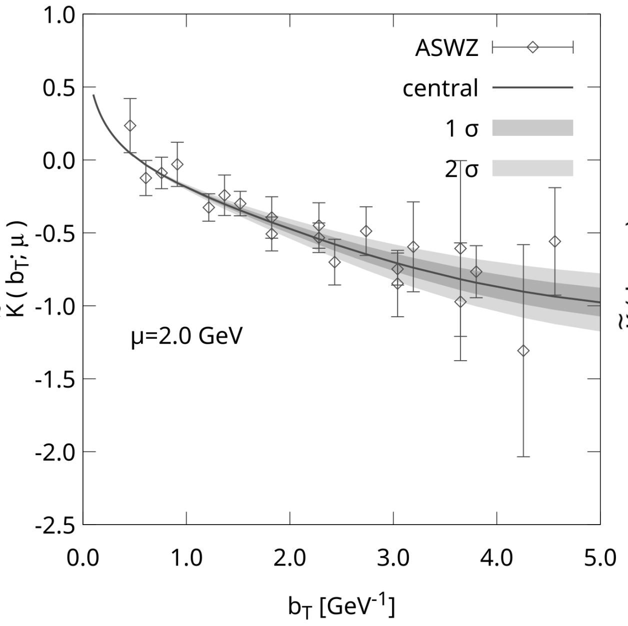
$$\chi^2_{d.o.f.} = 0.03$$

SWZ21



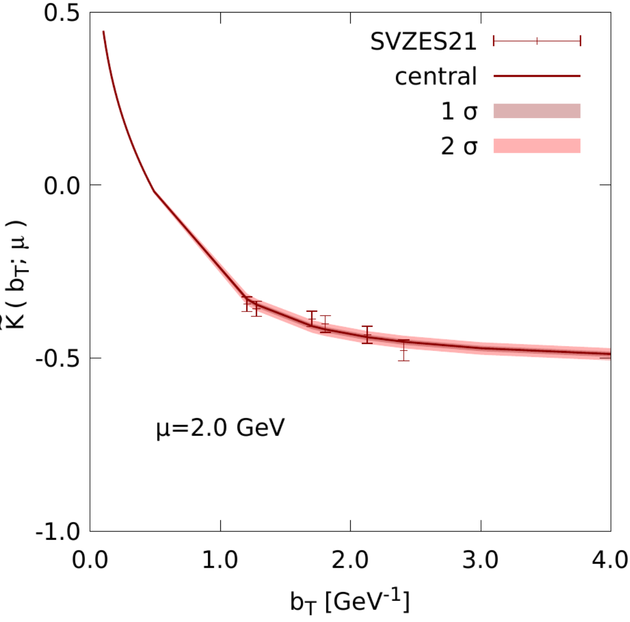
$$\chi^2_{d.o.f.} = 0.75$$

ASWZ



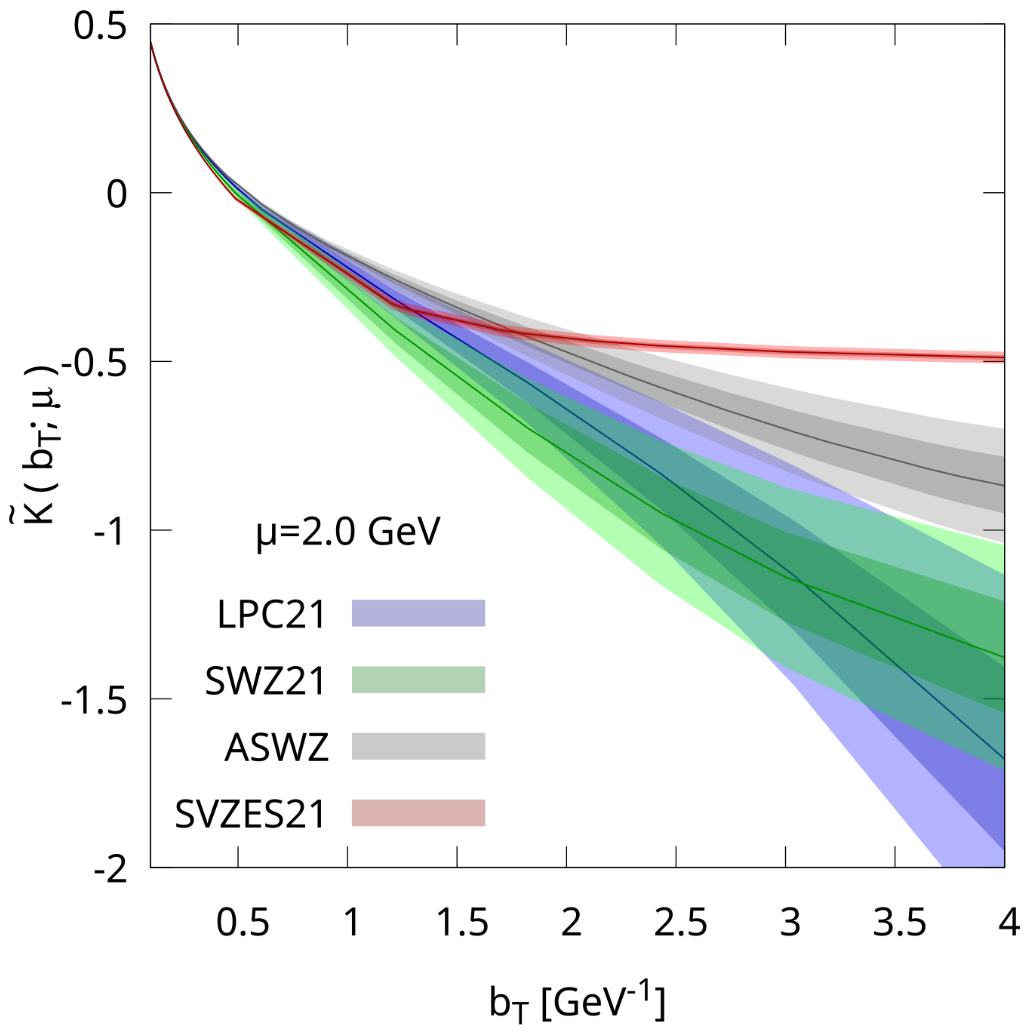
$$\chi^2_{d.o.f.} = 0.36$$

SVZES21

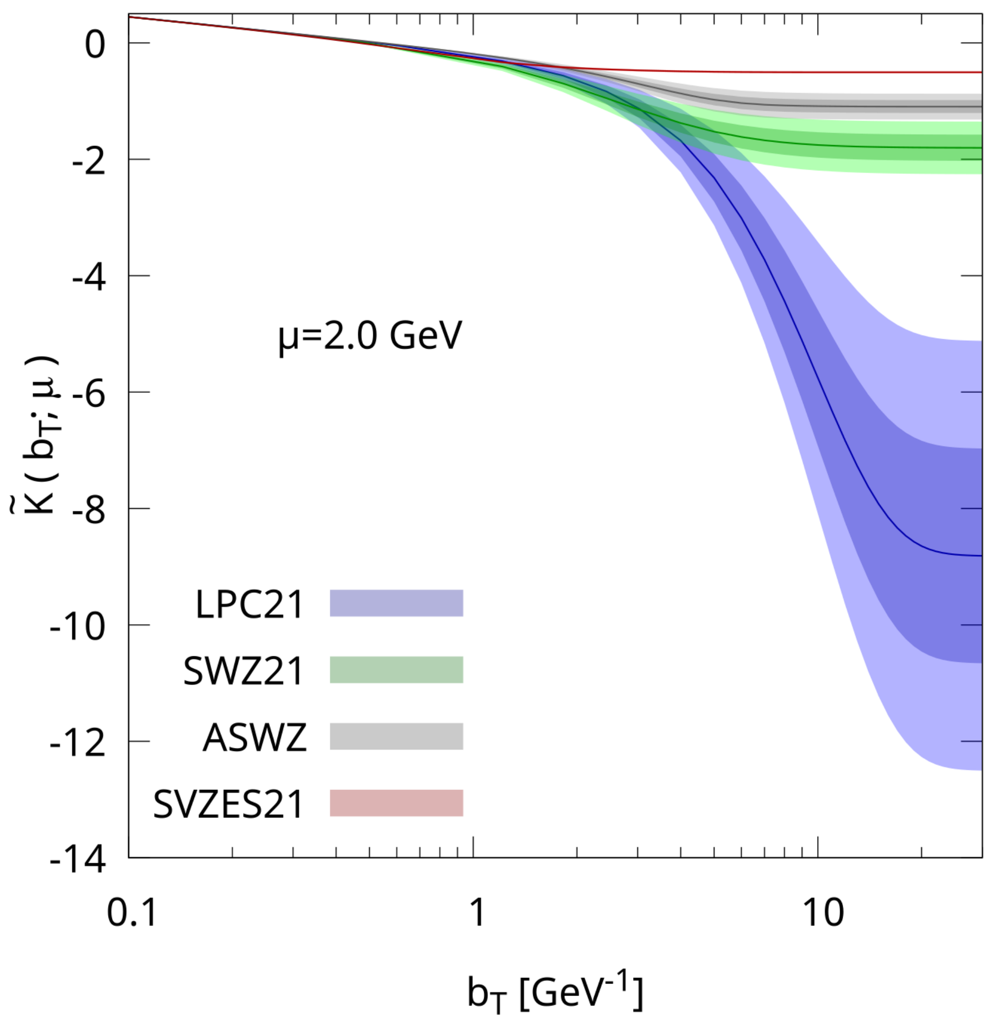


$$\chi^2_{d.o.f.} = 0.55$$

Final fits

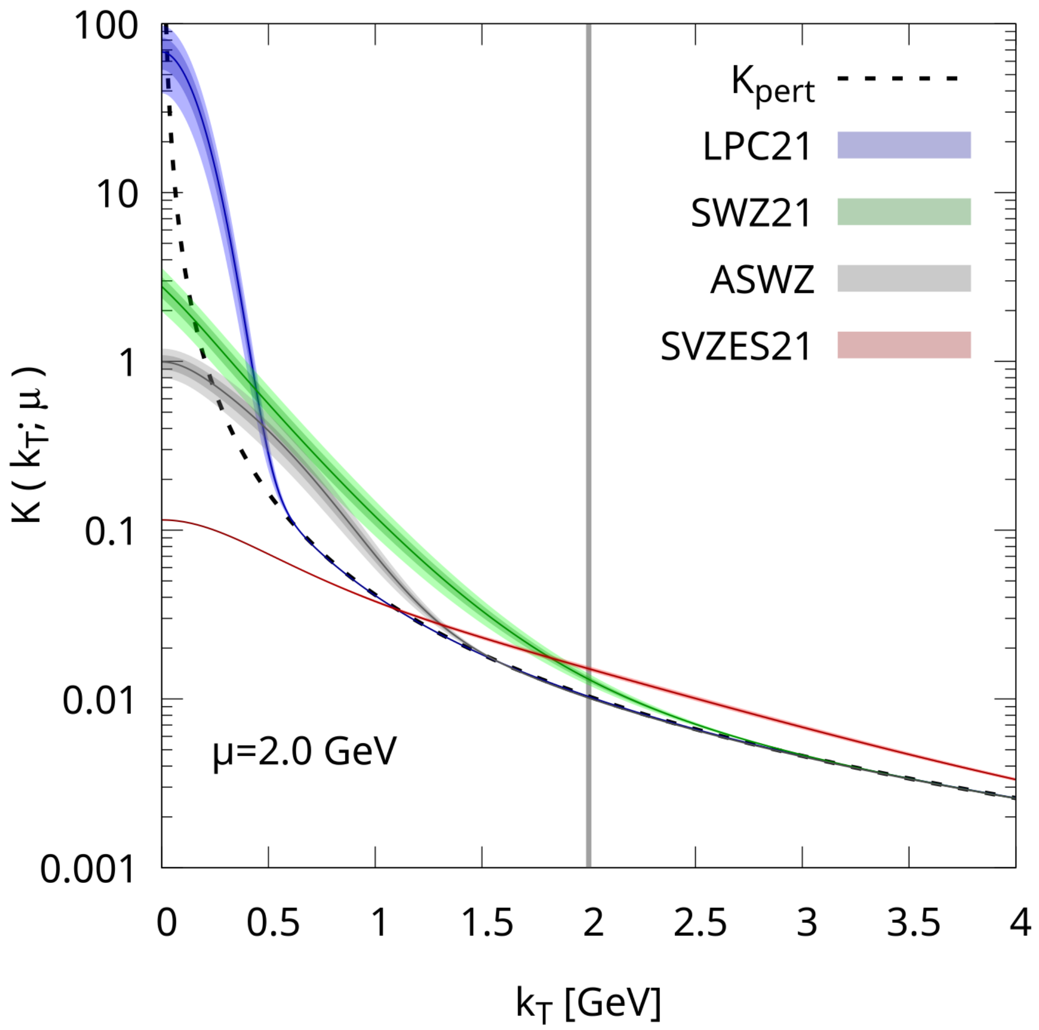


0.5
0
-0.5
-1
-1.5
-2
0.5
1
1.5
2
2.5
3
3.5
4
$\tilde{K}(b_T; \mu)$
b_T [GeV^{-1}]
μ=2.0 GeV
LPC21
SWZ21
ASWZ
SVZES21



0
-2
-4
-6
-8
-10
-12
-14
0.1
1
10
$\tilde{K}(b_T; \mu)$
b_T [GeV^{-1}]
μ=2.0 GeV
LPC21
SWZ21
ASWZ
SVZES21

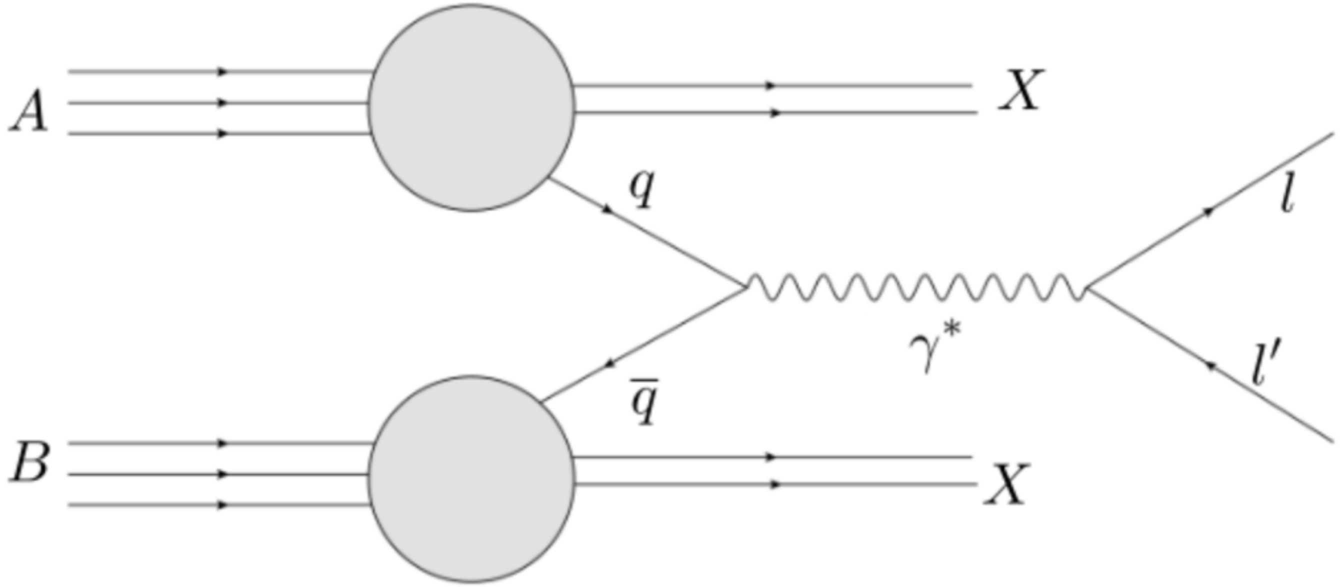
Final fits



100
10
1
0.1
0.01
0.001
K (k_T; μ)
0
0.5
1
1.5
2
2.5
3
3.5
4
k_T [GeV]
K_pert
LPC21
SWZ21
ASWZ
SVZES21
μ=2.0 GeV

E288 experiment

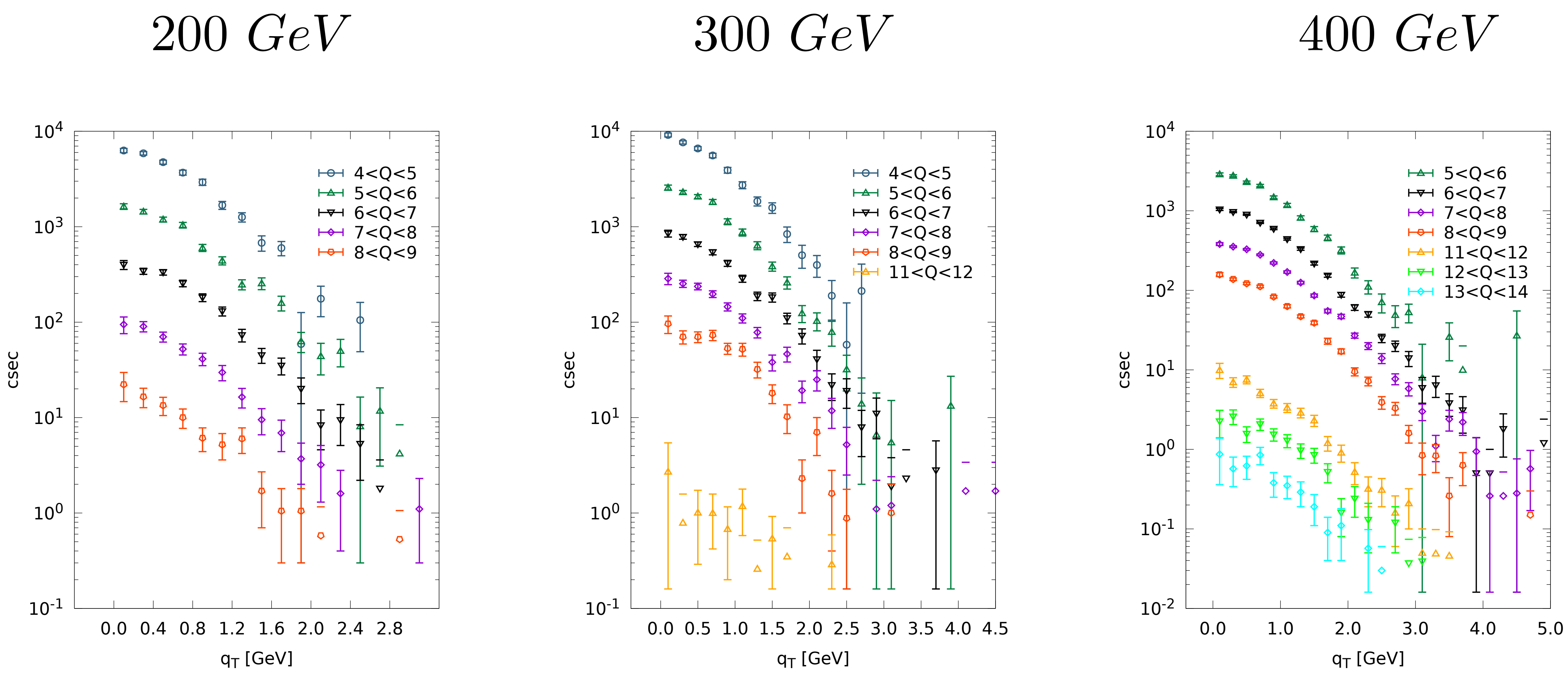
$$P + C_u \to \gamma \to \mu^+ + \mu^-$$



$$\mathcal{O} = \frac{1}{\pi} \int dQ^2 \frac{d^3\sigma}{q_T^2 dy_h dQ^2} \bigg|_{\substack{q_T = \langle q_T \rangle \\ y_h = \langle y_h \rangle}}$$

Ito et al. (1981) Phys.Rev.D 23

E288 data



$$q_T < 0.20\,Q$$

TMD PDF model

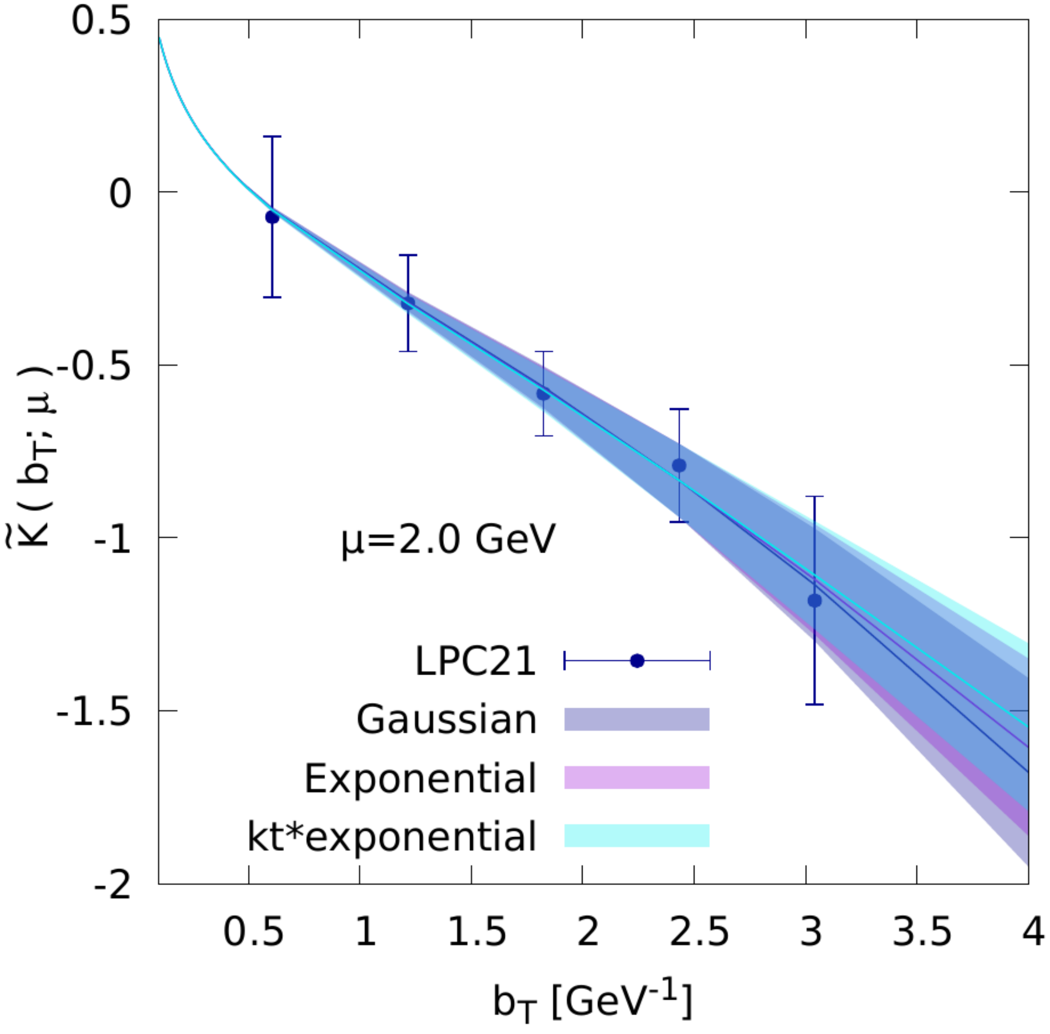
Built with HSO

$$F_{input,i/P}(x, k_T; \mu_{Q_0}, Q_0^2) = F_{fixed,i/P}(x, k_T; \mu_{Q_0}, Q_0^2) + C^f_{i/P} F_{core,i/P}(x, k_T; Q_0^2)$$

$$F^{Gauss}_{core,i/P}(x, k_T; Q_0^2) = \frac{e^{-\frac{k_T^2}{M_F^2(x)}}}{\pi M_F^2(x)}$$

$$M_F(x) = M_F + M_{F,log} \log \frac{1}{x}$$

Model dependence



Gaussian

$$K_{core}(k_T,\mu_{Q_0}) \propto e^{-k_T^2}$$

$$\chi^2_{d.o.f.} = 0.03$$

Exponential

$$K_{core}(k_T,\mu_{Q_0}) \propto e^{-k_T}$$

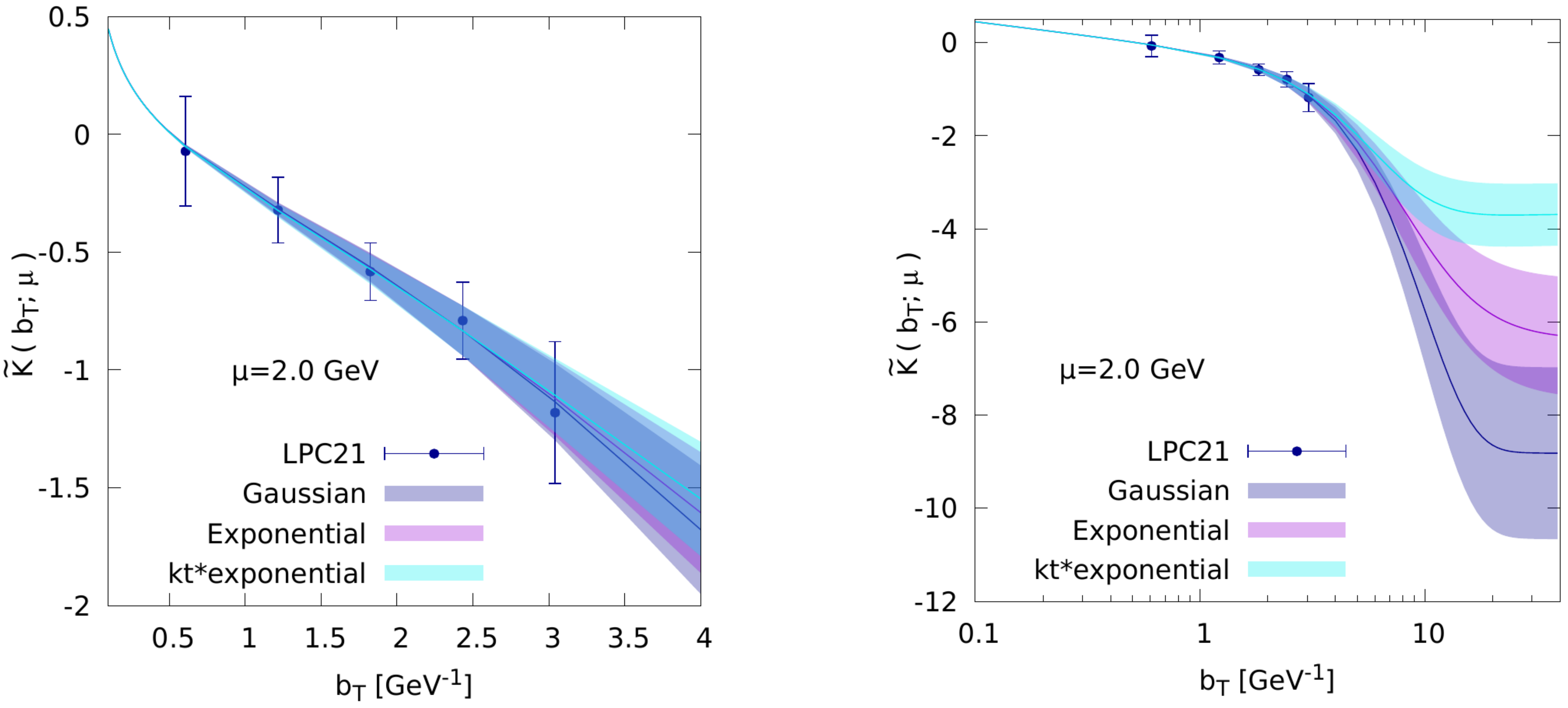
$$\chi^2_{d.o.f.} = 0.03$$

k_T* exponential

$$K_{core}(k_T,\mu_{Q_0}) \propto k_T e^{-k_T}$$

$$\chi^2_{d.o.f.} = 0.03$$

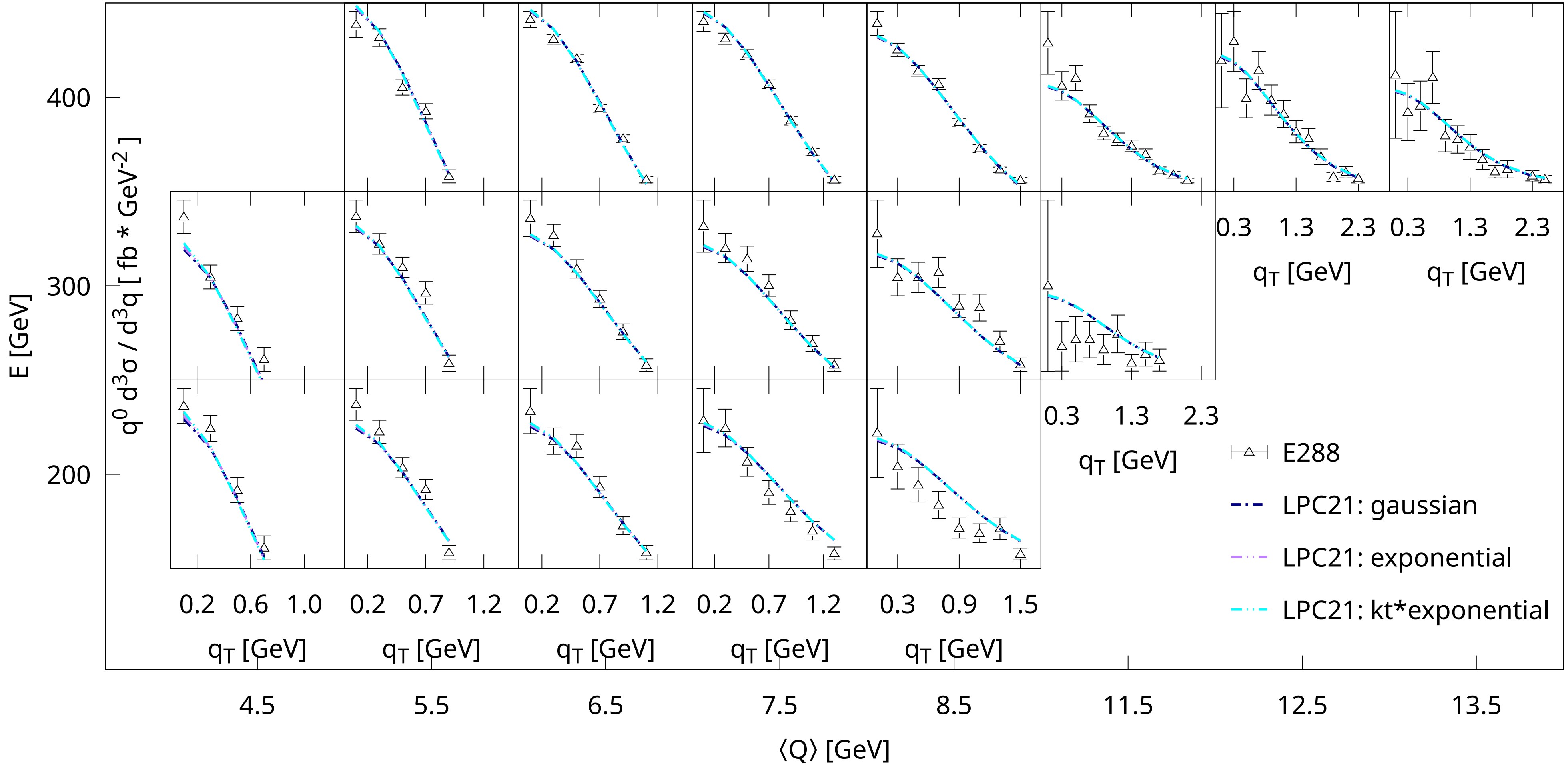
Model dependence



Model dependence

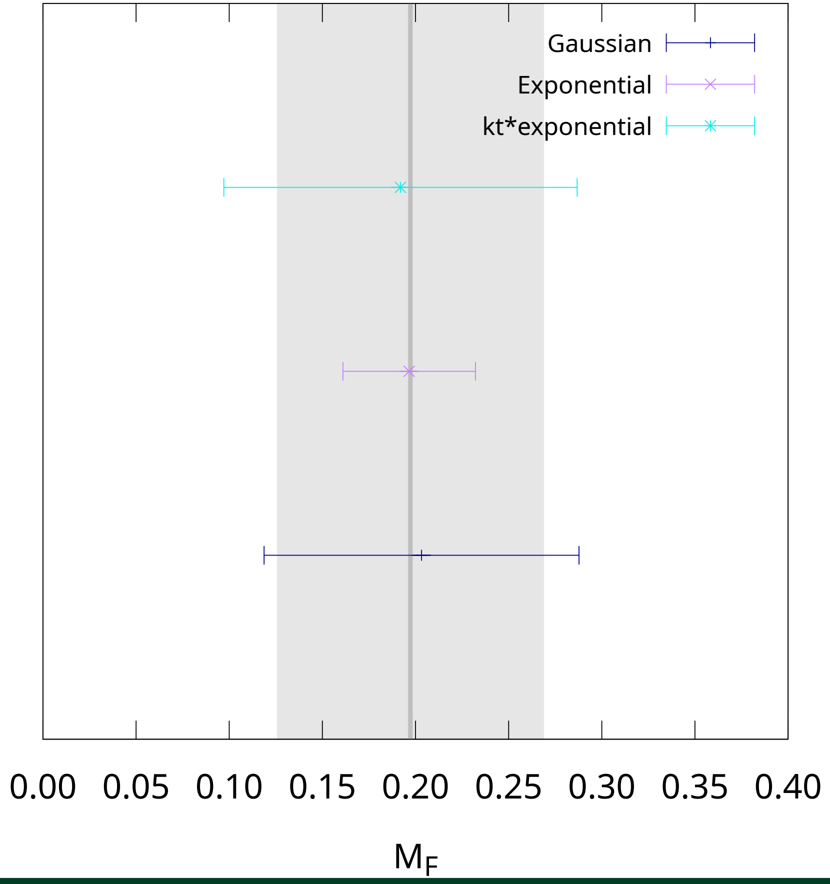
TMD model: Gaussian

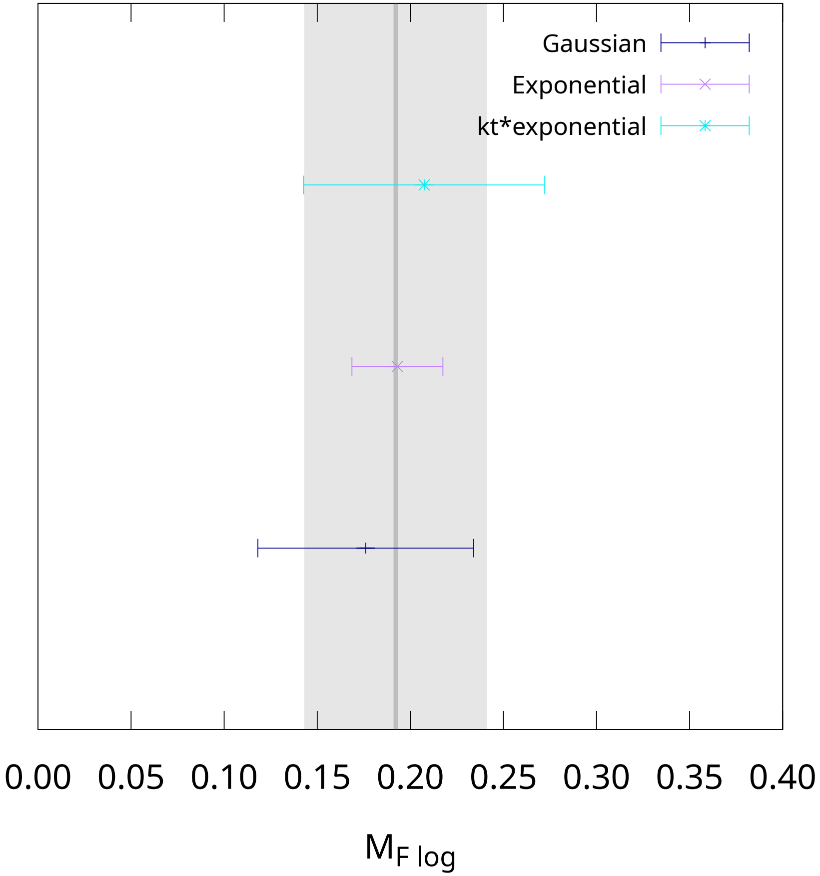
Set	LPC21	LPC21	LPC21
CS model	Gaussian	Exponential	k_T*exponential
$\chi^2_{\text{d.o.f.}}$	1.30	1.30	1.32



Model dependence

TMD model: Gaussian			
Set	LPC21	LPC21	LPC21
CS model	Gaussian	Exponential	k_T*exponential
$\chi^2_{\text{d.o.f.}}$	1.30	1.30	1.32
M_F	0.20 ± 0.09	0.20 ± 0.04	0.19 ± 0.10
$M_{F\ log}$	0.18 ± 0.06	0.19 ± 0.02	0.21 ± 0.07

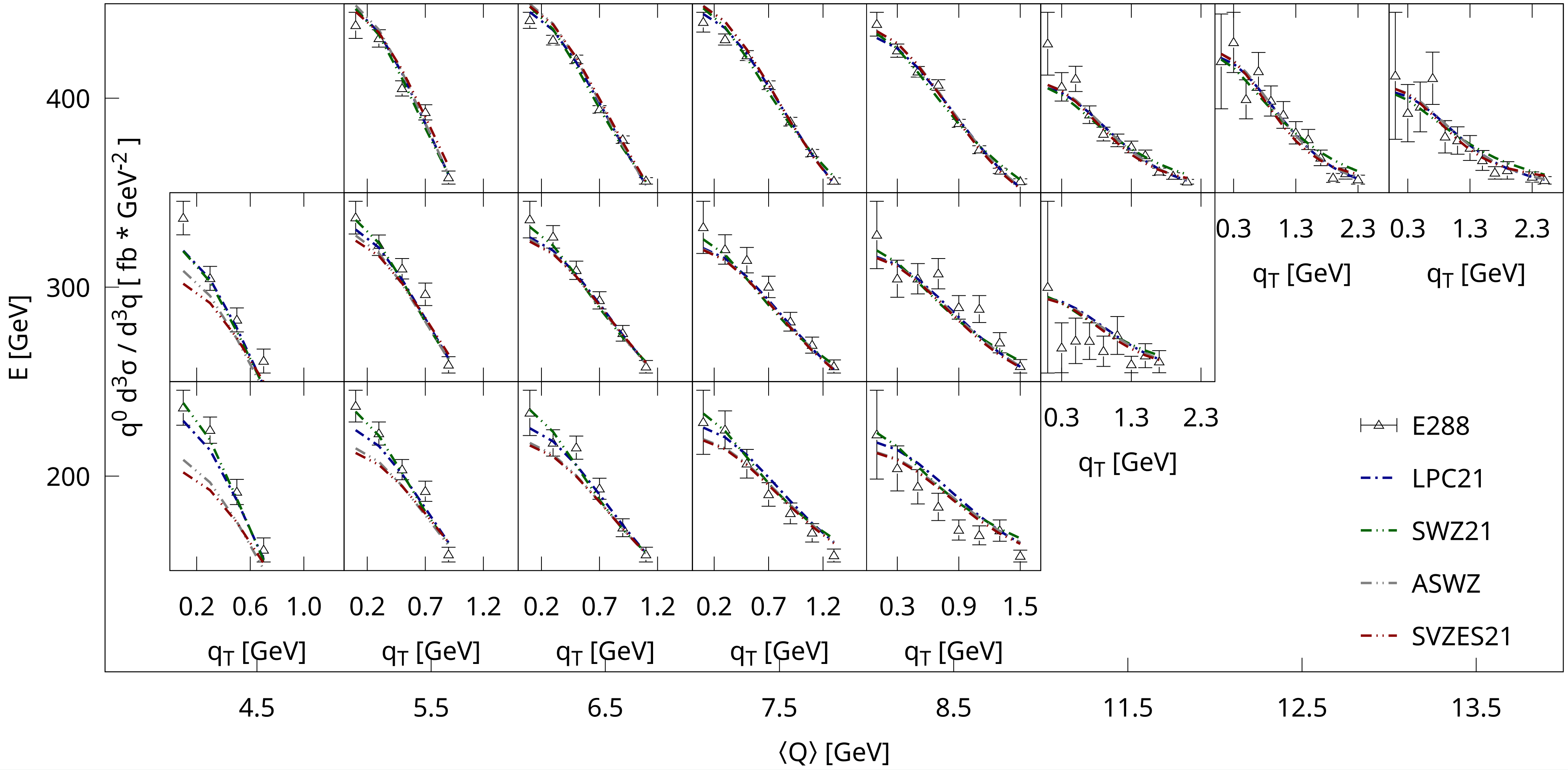




Lattice sets comparison

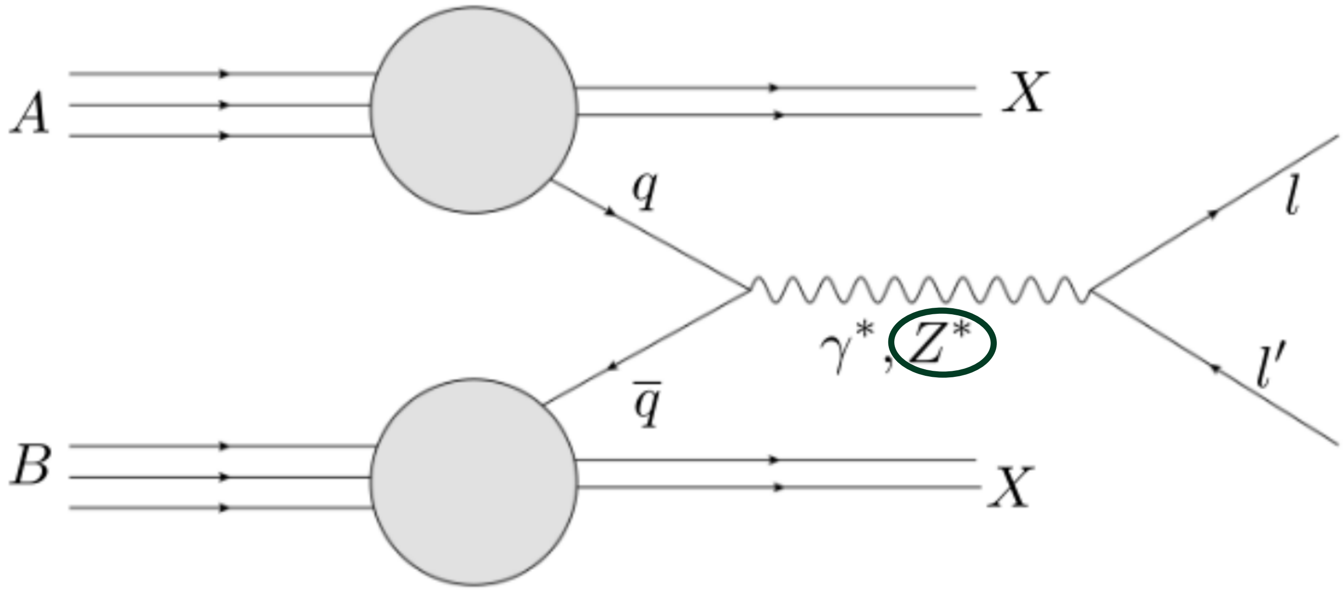
TMD model: Gaussian

Set	LPC21	SWZ21	ASWZ	SVZES21
CS model	Gaussian	Exponential	Gaussian	k_T^2∗exponential
$\chi^2_{\text{d.o.f.}}$	1.30	1.70	1.96	2.12



D0-I experiment

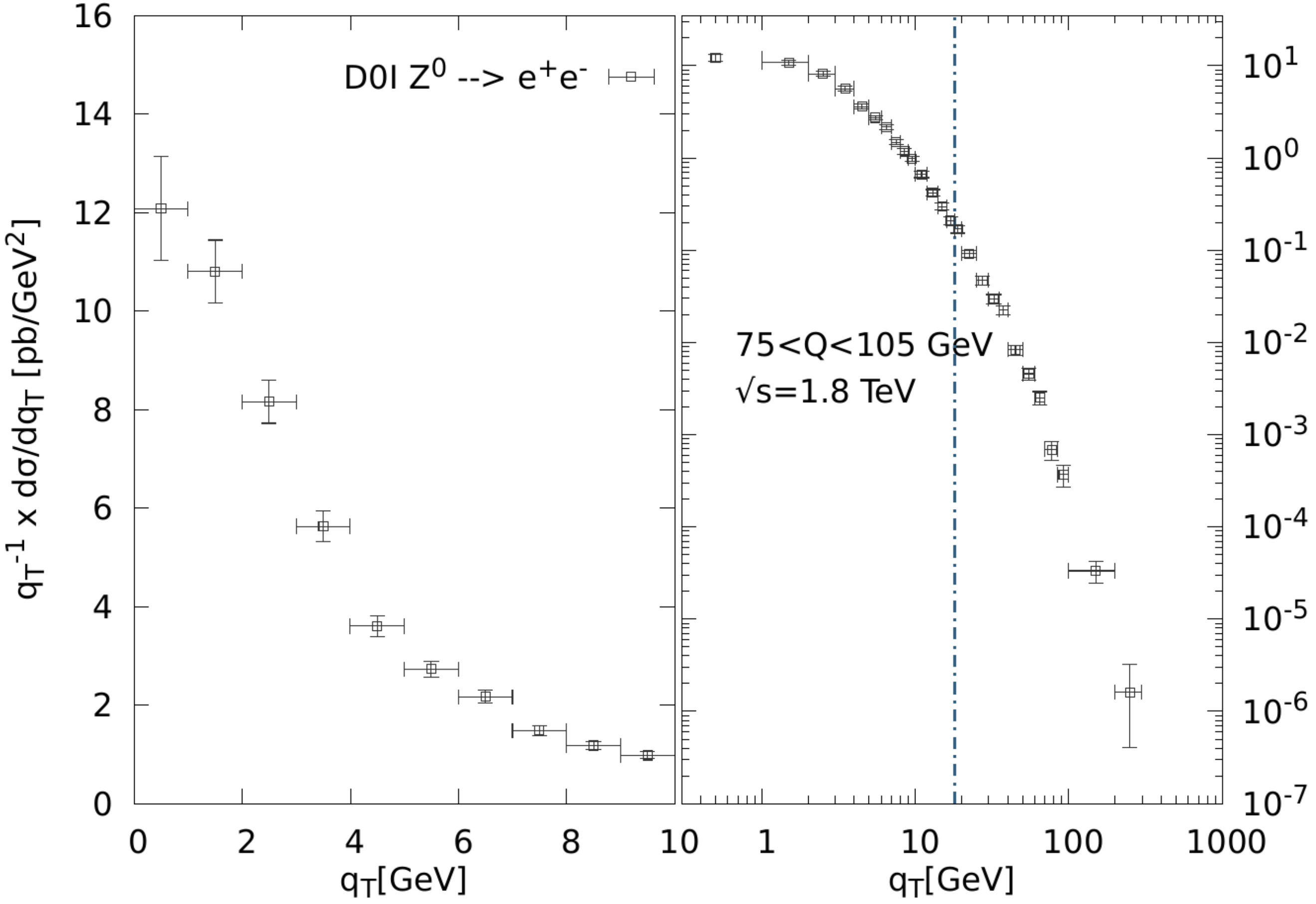
$$P \; + \; \overline{P} \to \; \gamma^*/Z \; \to \; e^+ + \; e^-$$



$$\mathcal{O} \propto \int dQ^2 \int dy_h \frac{d^3\sigma}{q_T^2 dy_h dQ^2}\bigg|_{q_T = \langle q_T \rangle}$$

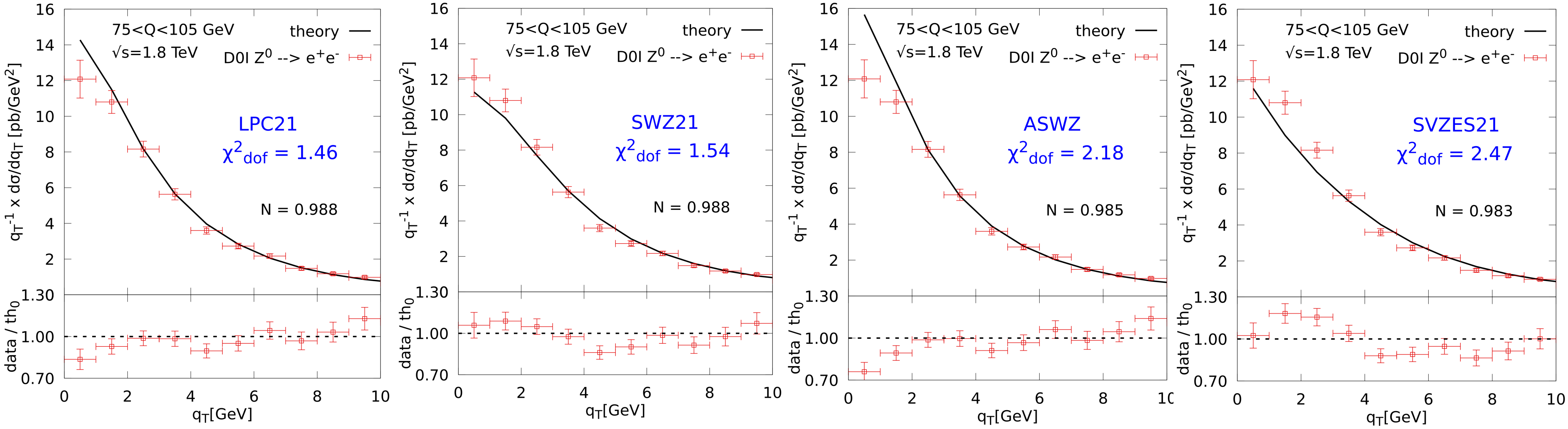
Abbott et al. (2000) Phys.Rev.D 61

D0-I data



$$q_T < 0.20\,Q$$

Lattice sets comparison



Appendix

Fourier transform of K and TMD

$$\tilde{K}_{input}(b_T;\mu) \equiv \int d^2k_T e^{ik_T b_T} K_{input}(k_T;\mu)$$

$$F_{i/A}(x,k_T;\mu,\zeta) \equiv \frac{1}{(2\pi)^2} \int d^2b_T e^{ik_T \cdot b_T} \tilde{F}_{i/A}(x,b_T;\mu,\zeta)$$

First theoretical constraint

$$\mathrm{K}^{(1)}_{input}(k_T;\mu_{Q_0}) \equiv \begin{cases} K^{(1)}(k_T;\mu_{Q_0}) = \frac{\alpha_s(\mu_{Q_0})C_F}{\pi^2}\frac{1}{k_T^2} + \mathcal{O}(\alpha_s(\mu_{Q_0})^2) & \text{if } k_T \gtrsim \mu_{Q_0}, \\ \text{nonperturbative parametrization} & \text{if } k_T < \mu_{Q_0} \end{cases}$$

$$\Downarrow$$

$$K_{input}(k_T;\mu_{Q_0}) = \frac{\alpha_s(\mu_{Q_0})C_F}{\pi^2}\frac{1}{k_T^2+m_K^2} + K_{core}(k_T)$$

Second theoretical constraint

$$\frac{d\tilde{K}^{(1)}_{input}(b_T;\mu_{Q_0})}{d\ \log\mu} = -\gamma_K^{(1)}(\alpha_s(\mu_{Q_0})) + \mathcal{O}(\alpha_s(\mu_{Q_0})^2) = 2\frac{\alpha_s(\mu_{Q_0})C_F}{\pi} + \mathcal{O}(\alpha_s(\mu_{Q_0})^2)$$

$$\Downarrow$$

$$K_{input}(k_T;\mu_{Q_0}) = \frac{\alpha_s(\mu_{Q_0})C_F}{\pi^2}\frac{1}{k_T^2+m_K^2} + C_K(\mu_{Q_0})\delta^{(2)}(k_T) + K_{core}(k_T)$$

$$C_K(\mu_{Q_0}) = \frac{2\alpha_s(\mu_{Q_0})C_F}{\pi}\log\left(\frac{m_K}{\mu_{Q_0}}\right) - b_K$$

Other theoretical constraint

$$\pi \int_0^{k_{max}^2} dk_T^2 K_{input}^{(n)}(k_T; \mu_{Q_0}) = \chi^{(n)}(k_{max}/\mu_{Q_0}, \alpha_s(\mu_{Q_0})) + \mathcal{O}\left(\frac{m}{\mu_{Q_0}}, \frac{m}{k_{max}}\right)$$

$$\frac{\partial}{\partial \log\ Q^2} W(Q, q_T) = \int d^2 k_T K(k_T; \mu) W(Q, q_T - k_T) + Const. \times W(Q, q_T)$$

Model formulation

$$K_{input}(k_T;\mu_{Q_0}) = \frac{\alpha_s(\mu_{Q_0})C_F}{\pi^2}\frac{1}{k_T^2+m_K^2} + C_K(\mu_{Q_0})\delta^{(2)}(k_T) + K_{core}(k_T)$$

$$FT \Updownarrow$$

$$\tilde{K}_{input}(b_T;\mu_{Q_0}) = \frac{2\alpha_s(\mu_{Q_0})C_F}{\pi}K_0(m_K b_T) + C_K(\mu_{Q_0}) + \tilde{K}_{core}(b_T)$$

Perturbative kernel

$$\tilde{K}^{(1)}(\mu, b_T) = -\frac{\alpha_s C_F}{\pi}[\log(\mu^2 b_T^2) - \log\ 4 + 2\gamma_E] + \mathcal{O}(\alpha_s^2)$$

Collins (2011) Camb.Monogr.Part.Phys.Nucl.Phys.Cosmol. 32

$$RG \Downarrow$$

$$\tilde{K}^{(1)}_{pert}(b_T; \mu_{Q_0}) = \tilde{K}^{(1)}(b_T; 2e^{-\gamma_E}/b_T) - \int_{2e^{-\gamma_E}/b_T}^{\mu_{Q_0}} \frac{d\mu'}{\mu'} \gamma_K^{(1)}(\alpha_s(\mu'))$$

$$\tilde{K}^{(1)}_{pert}(\mu, k_T) = \frac{\alpha_s C_F}{\pi^2 k_T^2} + \mathcal{O}(\alpha_s^2)$$

Methodology

LPC21, SWZ21, SVZES21

$$\chi^2 = \sum_i \left(\frac{\tilde{K}_{\text{lattice},i} - \tilde{K}_{\text{input}}(b_{T\,i};\mu_{Q_0})}{\delta \tilde{K}_{\text{lattice},i}} \right)^2$$

ASWZ

$$\chi^2 = \sum_i \left(\tilde{K}_{\text{lattice},i} - \tilde{K}_{\text{input}} \right) M^{-1} \sum_j \left(\tilde{K}_{\text{lattice},j} - \tilde{K}_{\text{input}} \right)^T$$

$$\Downarrow$$

$$\chi^2_{\text{d.o.f.}} = \frac{\chi^2}{\text{d.o.f.}} = \frac{\chi^2}{n_{\text{pts}} - n_{\text{par}}}$$

$\chi^2_{\text{d.o.f.}} \sim 1 \quad \Rightarrow$ Good fit

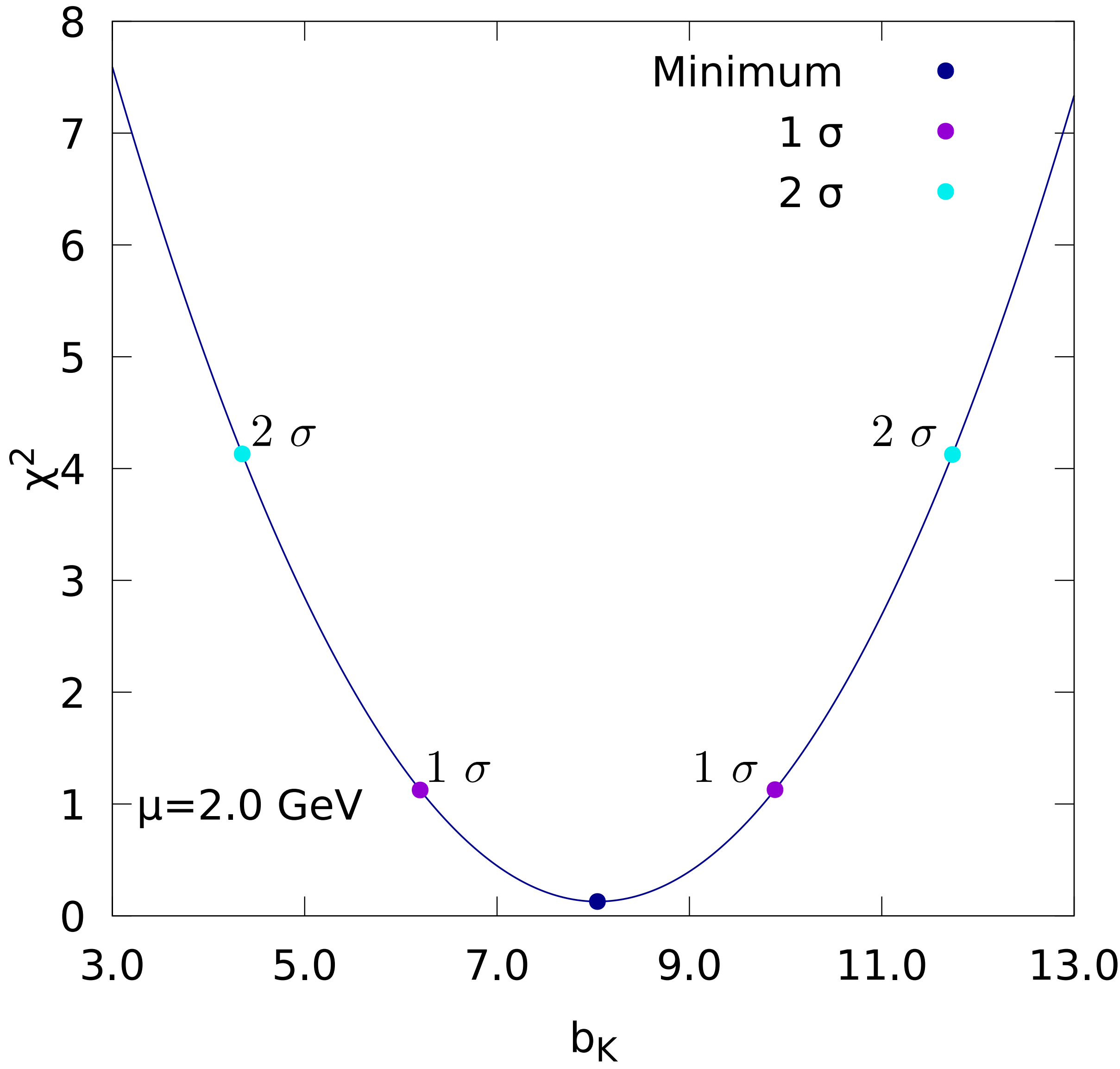
Fit uncertainty

$1\ \sigma:$

$$\chi^2 \leq \chi^2_{minimum} + 1$$

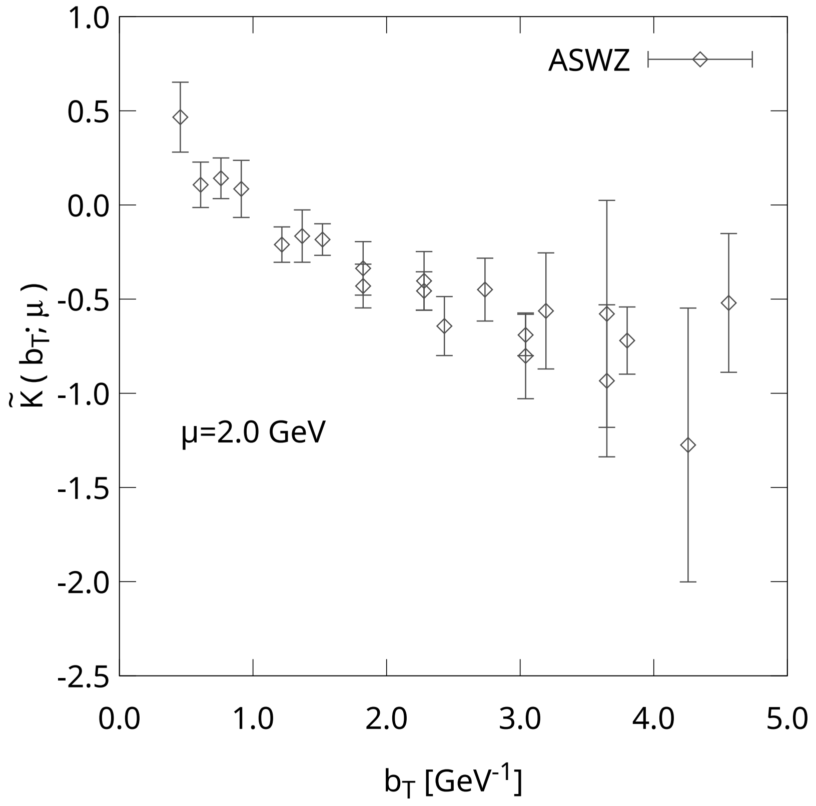
$2\ \sigma:$

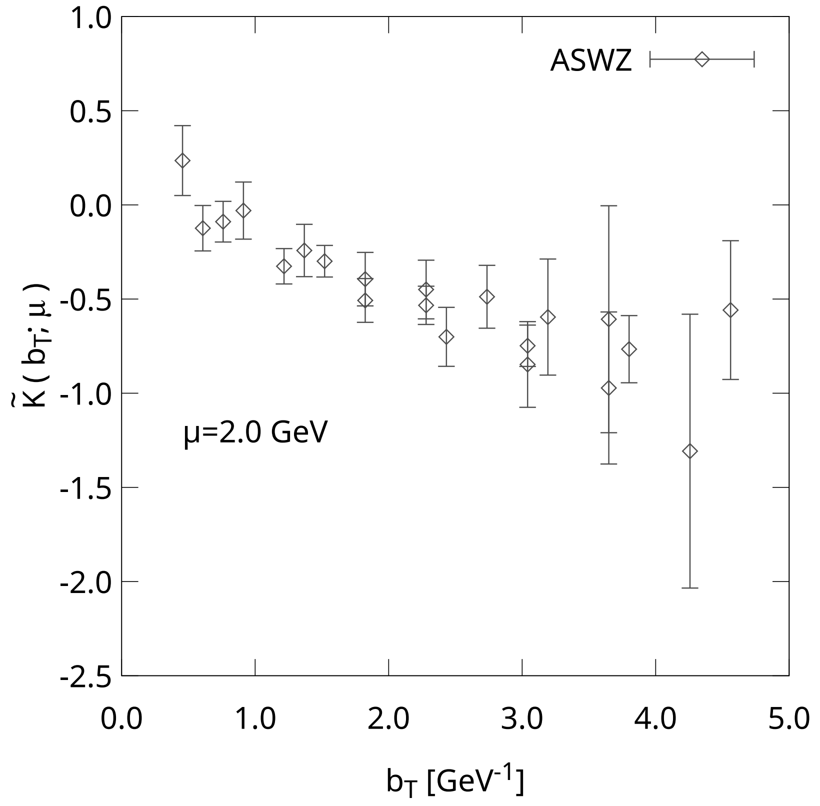
$$\chi^2 \leq \chi^2_{minimum} + 4$$



ASWZ rescaling

$$\tilde{K}_{input}(b_T;\mu_{Q_0}) = \tilde{K}_{fixed}(b_T;\mu_{Q_0}) + \ \tilde{K}_{core}(b_T) + k_1 \frac{a}{b_T}$$





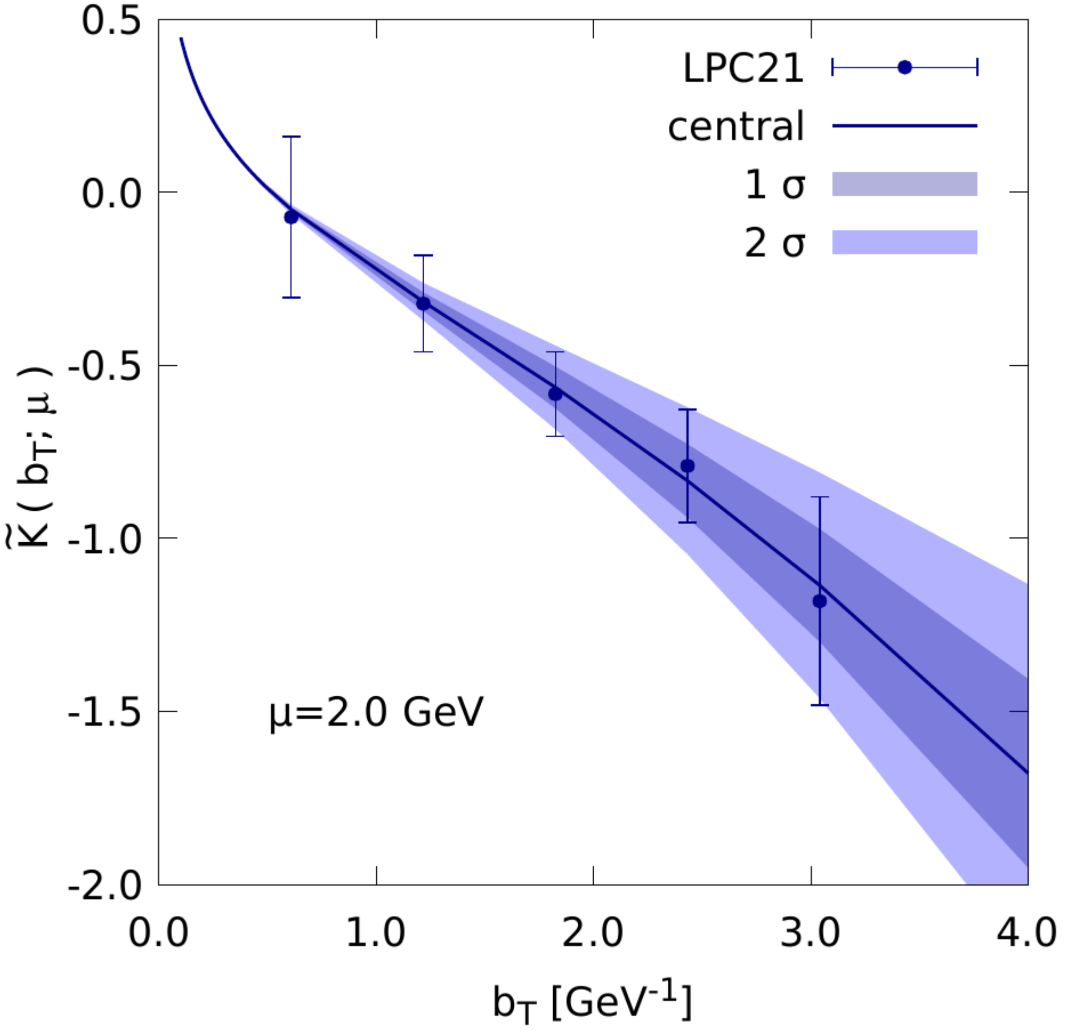
LPC21: gaussian

$$K_{core}(k_T, \mu_{Q_0}) = \frac{b_K}{4\pi m_K^2} e^{-\frac{k_T^2}{4m_K^2}}$$

$$FT \Updownarrow$$

$$\tilde{K}_{core}(b_T, \mu_{Q_0}) = b_K e^{-m_K^2 b_T^2}$$

$$\chi^2_{d.o.f.} = 0.03$$



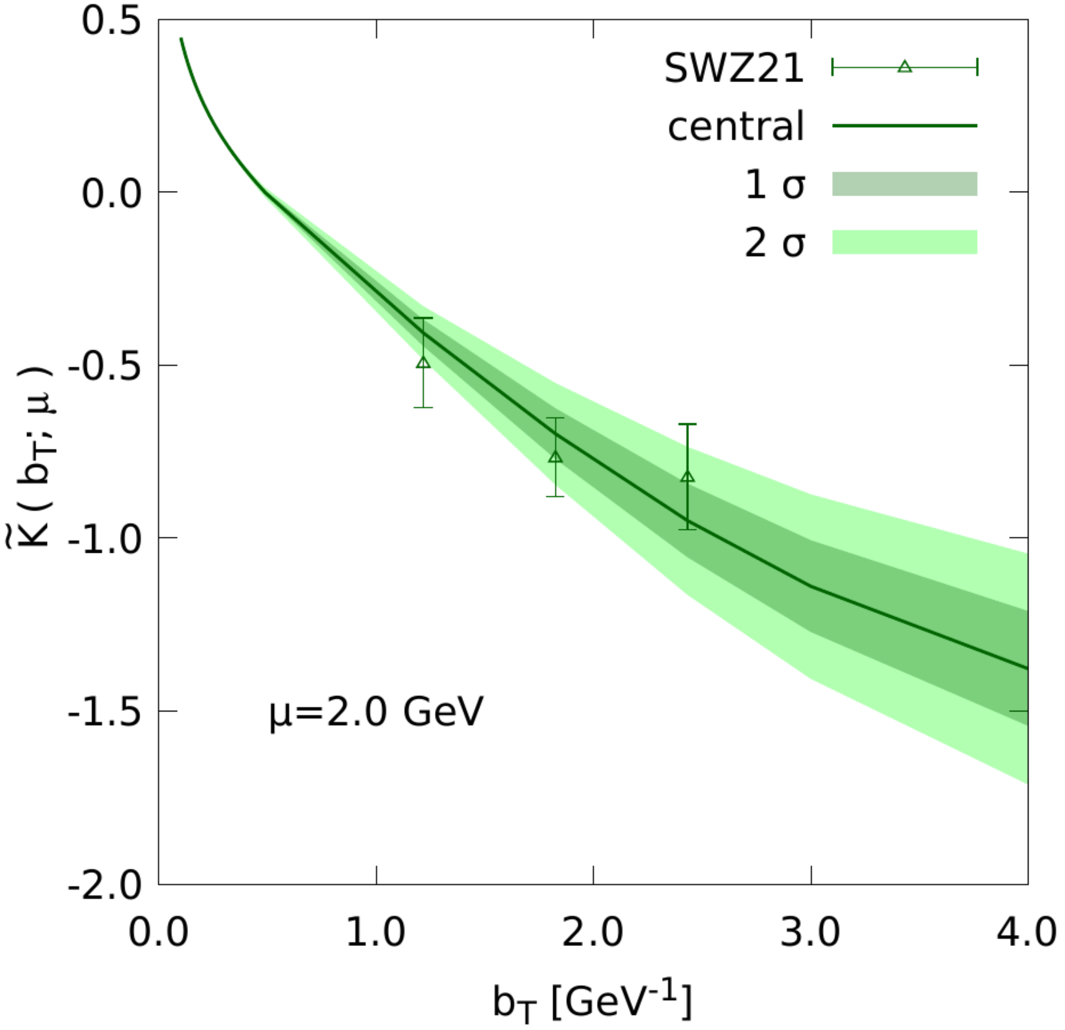
SWZ21: model comparison

$$K_{core}(k_T, \mu_{Q_0}) = \frac{b_K}{2\pi m_K^2} e^{-\frac{k_T}{m_K}}$$

$$FT \Updownarrow$$

$$\tilde{K}_{core}(b_T, \mu_{Q_0}) = \frac{b_K}{[1 + m_K^2 b_T^2]^{\frac{3}{2}}}$$

$$\chi^2_{d.o.f.} = 0.75$$



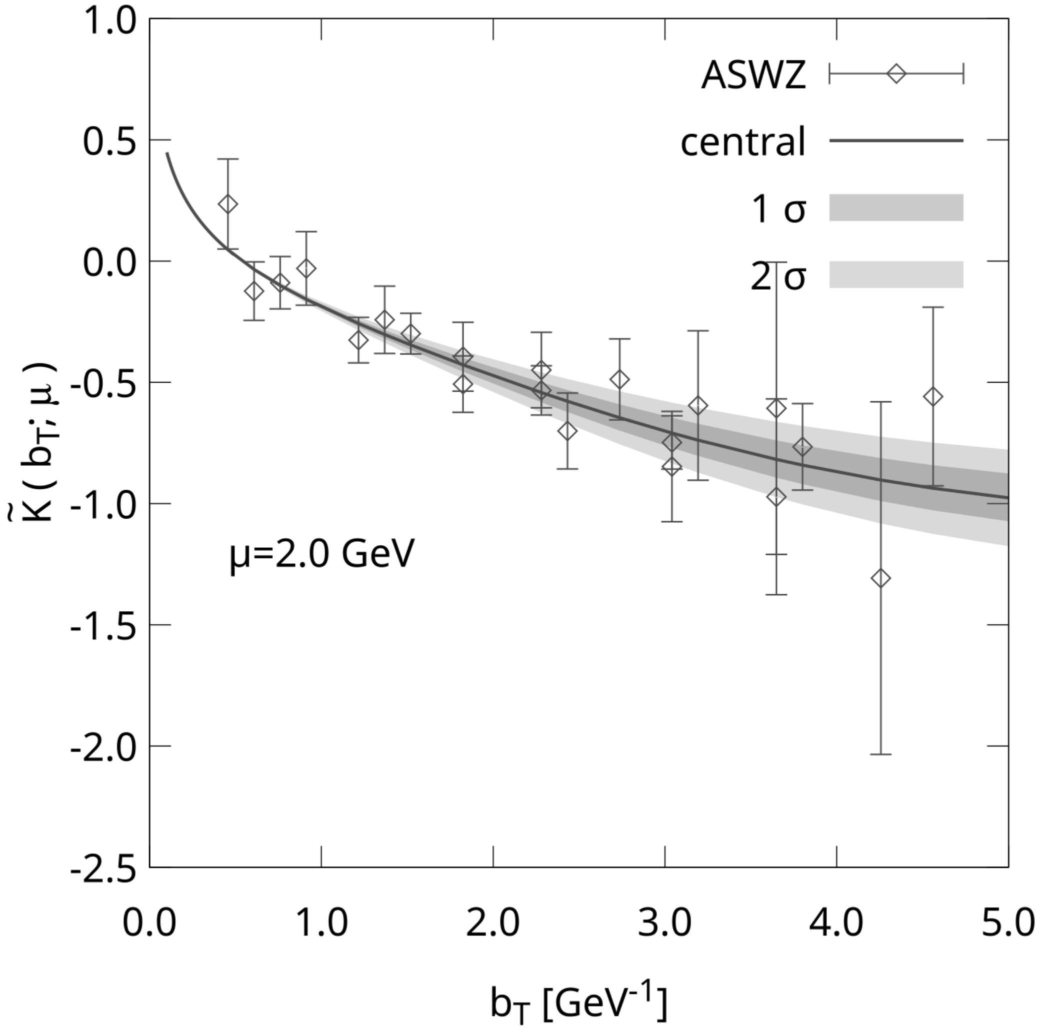
ASWZ: gaussian

$$K_{core}(k_T, \mu_{Q_0}) = \frac{b_K}{4\pi m_K^2} e^{-\frac{k_T^2}{4m_K^2}}$$

$$FT \Updownarrow$$

$$\tilde{K}_{core}(b_T, \mu_{Q_0}) = b_K e^{-m_K^2 b_T^2}$$

$$\chi^2_{d.o.f.} = 0.36$$



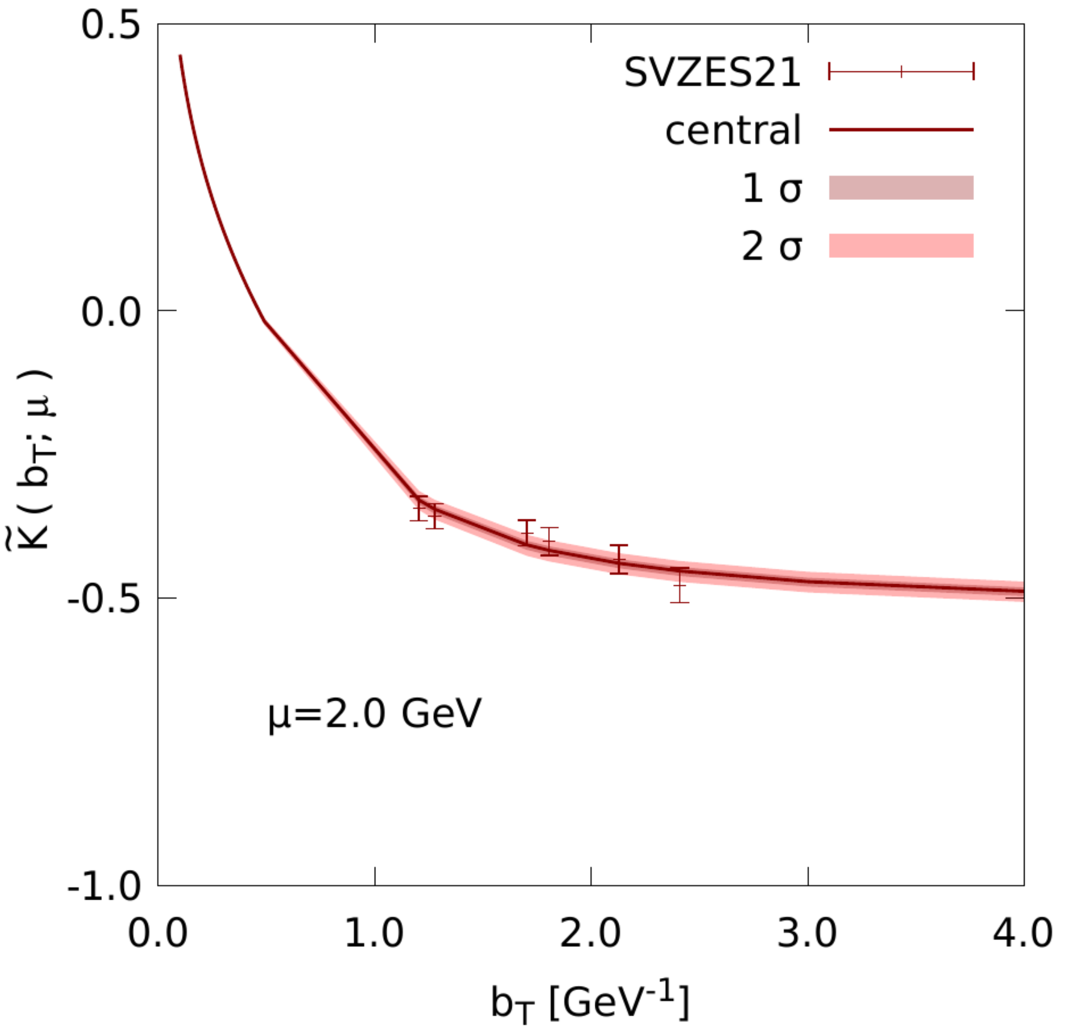
SVZES21: k_T^2 * exponential

$$K_{core}(k_T, \mu_{Q_0}) = \frac{b_K}{12\pi}\frac{k_T^2}{m_K^4}e^{-\frac{k_T}{m_K}}$$

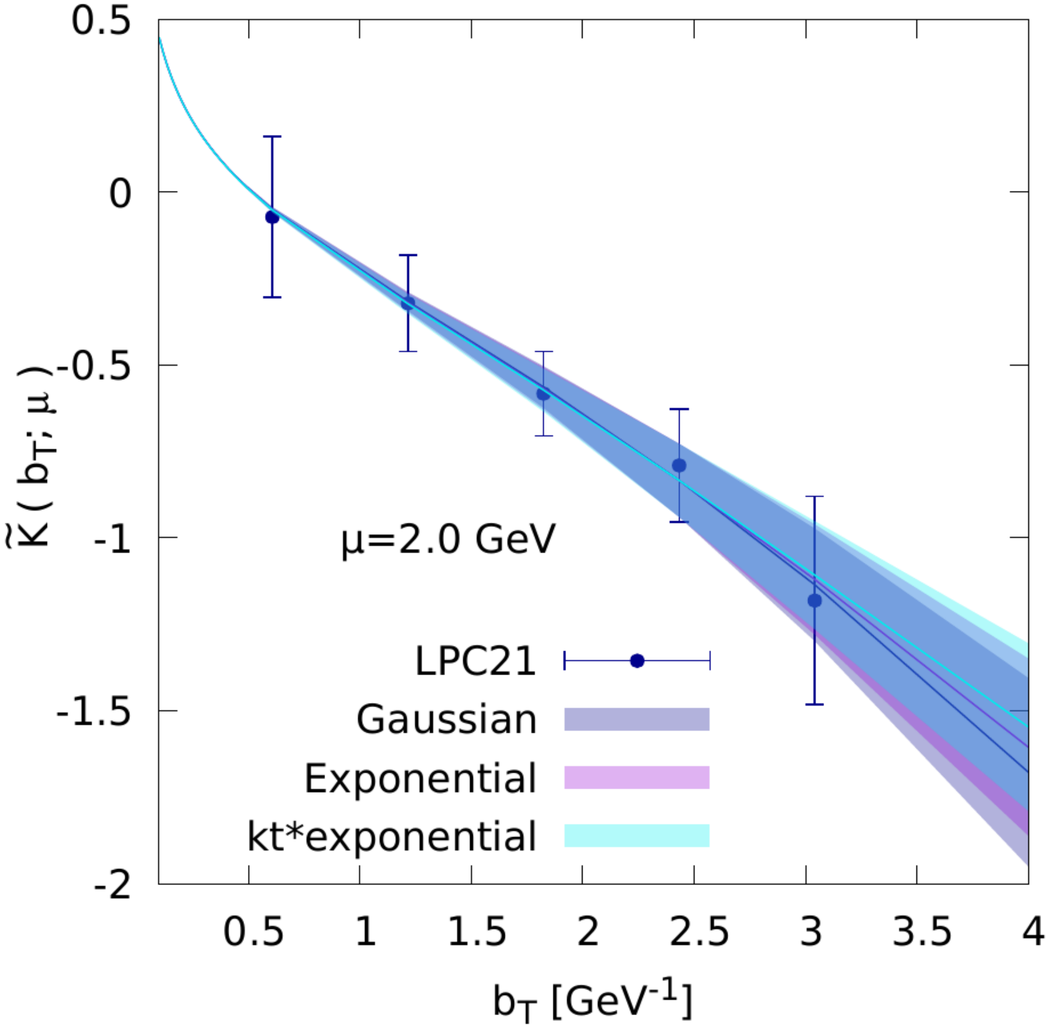
$$FT \Updownarrow$$

$$\tilde{K}_{core}(b_T, \mu_{Q_0}) = \frac{b_K}{2}\frac{2 - 3m_K^2 b_T^2}{[1 + m_K^2 b_T^2]^{\frac{7}{2}}}$$

$$\chi^2_{d.o.f.} = 0.55$$



Model dependence



Gaussian

$$K_{core}(k_T, \mu_{Q_0}) = \frac{b_K}{4\pi m_K^2} e^{-\frac{k_T^2}{4m_K^2}}$$

$$\chi^2_{d.o.f.} = 0.03$$

Exponential

$$K_{core}(k_T, \mu_{Q_0}) = \frac{b_K}{2\pi m_K^2} e^{-\frac{k_T}{m_K}}$$

$$\chi^2_{d.o.f.} = 0.03$$

k_T* exponential

$$K_{core}(k_T, \mu_{Q_0}) = \frac{b_K}{4\pi} \frac{k_T}{m_K^3} e^{-\frac{k_T}{m_K}}$$

$$\chi^2_{d.o.f.} = 0.03$$

Cross-section

$$\frac{d\sigma}{d^2q_T dQ^2 dy} = \mathcal{H}_{jj'}(\mu/Q;\alpha_s(\mu)) \int d^2b_T e^{iq_T\cdot b_T} \tilde{F}_{j/A}(x_A, b_T;\zeta,\mu)\tilde{F}_{j'/B}(x_A, b_T; Q^4/\zeta,\mu)$$
$$+ Y(Q, q_T) + \mathcal{O}(\frac{\Lambda}{Q}^a)$$

$$\Downarrow$$

$$q_T < 0.20\, Q$$

TMD PDF model

$$F_{input,i/P}(x, k_T; \mu_{Q_0}, Q_0^2) = \frac{1}{2\pi} \frac{1}{k_T^2 + m_{f_{i,P}}^2} \left[A_{i/P}^f(x; \mu_{Q_0}) + B_{i/P}^f(x; \mu_{Q_0}) \log \frac{Q_0^2}{k_T^2 + m_{f_{i,p}}^2} \right]$$
$$+ \frac{1}{2\pi} \frac{1}{k_T^2 + m_{f_{g,P}}^2} A_{i/P}^{f,g}(x; \mu_{Q_0}) + C_{i/P}^f F_{core,i/P}(x, k_T; Q_0^2)$$

Spectator model

$$F_{input,i/P}(x, k_T; \mu_{Q_0}, Q_0^2) = \frac{1}{2\pi} \frac{1}{k_T^2 + m_{f_{i,P}}^2} \left[A_{i/P}^{f}(x; \mu_{Q_0}) + B_{i/P}^{f}(x; \mu_{Q_0}) \log \frac{Q_0^2}{k_T^2 + m_{f_{i,p}}^2} \right]$$
$$+ \frac{1}{2\pi} \frac{1}{k_T^2 + m_{f_{g,P}}^2} A_{i/P}^{f,g}(x; \mu_{Q_0}) + C_{i/P}^{f} F_{core,i/P}(x, k_T; Q_0^2)$$

$$\mathrm{F}_{core,i/P}^{Spect}(x, k_T; Q_0^2) = \frac{1}{\pi} \frac{6L^6}{L^2+2(m_q+xM_p)^2} \frac{k_T^2+(m_q+xM_p)^2}{(k_T^2+L^2)^4}$$

$$L^2 = (1-x)\Lambda^2 + xM_X^2 - x(1-x)M_p^2$$

Methodology

$$\chi^2 = \frac{(1-N)^2}{\delta_N^2} + \sum_i \left(\frac{\mathcal{O}_i^{\mathrm{E288}} - \mathcal{O}_i^{\mathrm{theory}} N}{\delta \mathcal{O}_i^{\mathrm{E288}}} \right)^2 \qquad \Rightarrow \qquad \chi^2_{d.o.f.}$$

$$\delta_N = 25\%$$

$\chi^2_{\mathrm{d.o.f.}} \sim 1$ and $N \sim 1 \pm 0,25$ $\Rightarrow$ Good fit

Spectator model results

TMD model: Gaussian

Set	LPC21	LPC21	LPC21
CS model	Gaussian	Exponential	k_T∗exponential
$\chi^2_{\text{d.o.f.}}$	1.30	1.30	1.32

TMD model: Spectator

Set	LPC21	LPC21	LPC21
CS model	Gaussian	Exponential	k_T∗exponential
$\chi^2_{\text{d.o.f.}}$	1.38	1.38	1.39

TMD model: Gaussian

Set	LPC21	SWZ21	SVZES21
CS model	Gaussian	Exponential	k_T^2∗exponential
$\chi^2_{\text{d.o.f.}}$	1.30	1.70	2.12

TMD model: Spectator

Set	LPC21	SWZ21	SVZES21
CS model	Gaussian	Exponential	k_T^2∗exponential
$\chi^2_{\text{d.o.f.}}$	1.38	1.93	2.25