$\nu$ -nucleon Neutral Current scattering in the presence of Non-Standard Interaction

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Introduction to NC NSI

Effect of NC NSI on Deep Inelastic Scattering

Effect of NC NSI on the Quasi-Elastic scattering

Bounds on NC NSI as the results

The  ${\rm SU}(2)\times {\rm U}(1)$  gauge symmetry implies NC interaction which has been verified by the interaction between neutrino and matter fields at the Gargamelle experiment at CERN.

Gargamelle Neutrino Collaboration, Phys.Lett.B 46 (1973) 138-140



Wolfenstein's initial suggestion has been replaced by the neutrino mixing scheme, which is now the accepted explanation for neutrino oscillations and the solar neutrino anomaly.

L. Wolfenstein, Phys.Rev.D 17 (1978) 2369-2374

NSI has gained renewed interest as a subdominant effect to be discovered in current and upcoming precision neutrino experiments.

NSI could indeed be related to new gauge bosons and HNLs like sterile neutrinos within the seesaw mechanism and leptogenesis.

M. Malinsky et al., Phys.Rev.D 79 (2009) 011301

### Generalized $\nu$ NC interaction with matter in quark-level

$$\mathcal{L}_{\rm SM+NSI}^{\rm NC} = \frac{G_{\rm F}}{\sqrt{2}} \sum_{\substack{\alpha,\beta=e,\mu,\tau\\q=u,d,s}} \left[ \overline{\nu}_{\alpha}\gamma_{\mu} \left(1-\gamma^{5}\right)\nu_{\beta} \right] \underbrace{\left[ \overline{q}\gamma^{\mu} \left( f_{\alpha\beta}^{\rm Vq} + f_{\alpha\beta}^{\rm Aq}\gamma^{5} \right) q \right]}_{J^{\mu}(x)} \right]$$
$$f_{\alpha\beta}^{\rm Vq} = g^{\rm Vq}\delta_{\alpha\beta} + \epsilon_{\alpha\beta}^{\rm Vq}, \quad f_{\alpha\beta}^{\rm Aq} = g^{\rm Aq}\delta_{\alpha\beta} + \epsilon_{\alpha\beta}^{\rm Aq}$$
$$g^{\rm Vu} = \frac{1}{2} - \frac{4}{3}\sin^{2}\theta_{\rm W}, \quad g^{\rm Au} = \frac{-1}{2}, \quad g^{\rm Vd,s} = \frac{-1}{2} + \frac{2}{3}\sin^{2}\theta_{\rm W}, \quad g^{\rm Ad,s} = \frac{1}{2}$$

# Effect of NC NSI on Deep Inelastic Scattering

based on our JHEP 04 (2024) 038 in collaboration with Y. Farzan, S. Safari and S. Abbaslu

### NC DIS differential XS

$$\begin{aligned} \frac{d^2 \sigma_{\rm NC}^{\rm DIS} \left( \stackrel{-}{\nu_{\alpha}} + p \to \stackrel{-}{\nu_{\beta}} + X \right)}{dxdy} &= \frac{G_{\rm F}^2}{\pi} \left( M_{\rho} E_{\nu} \right) \left[ xy^2 F_1^{\rho}(x) + \left( 1 - y - \frac{xy M_{\rho}^2}{s} \right) F_2^{\rho}(x) \pm xy (1 - \frac{y}{2}) F_3^{\rho}(x) \right] \\ &\quad F_1^{\rho}(x) &= \frac{1}{2} \left\{ \left[ \left( f_{\alpha\beta}^{\rm Vu} \right)^2 + \left( f_{\alpha\beta}^{\rm Au} \right)^2 \right] \left( u(x) + \overline{u}(x) \right) \right. \\ &\quad + \left[ \left( f_{\alpha\beta}^{\rm Vd} \right)^2 + \left( f_{\alpha\beta}^{\rm Ad} \right)^2 \right] \left( d(x) + \overline{d}(x) \right) \right. \\ &\quad + \left[ \left( f_{\alpha\beta}^{\rm Vd} \right)^2 + \left( f_{\alpha\beta}^{\rm Ad} \right)^2 \right] \left( s(x) + \overline{s}(x) \right) \right\} \\ &\quad F_2^{\rho}(x) = 2x F_1^{Z\rho}(x) \\ &\quad F_3^{\rho}(x) = 2 \left\{ \Re[f_{\alpha\beta}^{\rm Vu}(f_{\alpha\beta}^{\rm Au})^*] \left( u(x) - \overline{u}(x) \right) \\ &\quad + \Re[f_{\alpha\beta}^{\rm Vd}(f_{\alpha\beta}^{\rm Ad})^*] \left( d(x) - \overline{d}(x) \right) \\ &\quad + \Re[f_{\alpha\beta}^{\rm Vs}(f_{\alpha\beta}^{\rm As})^*] \left( s(x) - \overline{s}(x) \right) \right\} \end{aligned}$$

One could find neutron XS using by charge symmetry  $\binom{(-)}{u}(x) \leftrightarrow \overset{(-)}{d}(x)$ 

# Is DIS enough for our purpose?



G.P. Zeller et al., Rev.Mod.Phys. 84 (2012) 1307-1341

For the CHIPS experiment with a simple water Cherenkov detector, it is shown that by invoking the neural network technique, the distinction DIS from Quasi-Elastic and resonance events.

# Effect of NC NSI on the Quasi-Elastic scattering

based on our upcoming work in collaboration with Y. Farzan, S. Safari and S. Abbaslu

# NC QE scattering differential XS

$$\frac{d\sigma_{\rm NC}^{\rm QE} \binom{(-)}{\nu_{\alpha}} + N \rightarrow \stackrel{(-)}{\nu_{\beta}} + N)}{dQ^2} = \frac{M_N^2 G_{\rm F}^2}{8\pi E_\nu^2} \left[ A \mp \frac{4M_N E_\nu - Q^2}{M_N^2} B + \frac{(4M_N E_\nu - Q^2)^2}{M^4} C \right]$$
$$A = \frac{Q^2}{M_N^2} \left[ (1+\tau)(\tilde{F}_A^N)^2 - (1-\tau)(\tilde{F}_1^N)^2 + \tau(1-\tau)(\tilde{F}_2^N)^2 + 4\tau \tilde{F}_1^N \tilde{F}_2^N \right]$$
$$B = \frac{Q^2}{M^2} \tilde{F}_A^N (\tilde{F}_1^N + \tilde{F}_2^N)$$
$$C = \frac{1}{4} \left[ (\tilde{F}_A^N)^2 + (\tilde{F}_1^N)^2 + \tau (\tilde{F}_2^N)^2 \right]$$
where  $\tau = Q^2/4M_N^2$ 

#### Modifed Form Factors

$$(\tilde{F}_{i}^{p})_{\alpha\beta} = (\frac{\delta_{\alpha\beta}}{2} - 2\delta_{\alpha\beta}\sin^{2}\theta_{W} + 2\epsilon_{\alpha\beta}^{Vu} + \epsilon_{\alpha\beta}^{Vd})F_{i}^{p} + (\frac{-\delta_{\alpha\beta}}{2} + \epsilon_{\alpha\beta}^{Vu} + 2\epsilon_{\alpha\beta}^{Vd})F_{i}^{n} + (\frac{-\delta_{\alpha\beta}}{2} + \epsilon_{\alpha\beta}^{Vu} + \epsilon_{\alpha\beta}^{Vd} + \epsilon_{\alpha\beta}^{Vs})F_{i}^{s}$$

$$(\tilde{F}_{i}^{n})_{\alpha\beta} = (\frac{\delta_{\alpha\beta}}{2} - 2\delta_{\alpha\beta}\sin^{2}\theta_{W} + 2\epsilon_{\alpha\beta}^{Vu} + \epsilon_{\alpha\beta}^{Vd})F_{i}^{n} + (\frac{-\delta_{\alpha\beta}}{2} + \epsilon_{\alpha\beta}^{Vu} + \epsilon_{\alpha\beta}^{Vd} + \epsilon_{\alpha\beta}^{Vs})F_{i}^{s}$$

$$\begin{split} (\tilde{F}^{p}_{A})_{\alpha\beta} &= (\frac{\delta_{\alpha\beta}}{2} + \frac{\epsilon^{Au}_{\alpha\beta} - \epsilon^{Ad}_{\alpha\beta}}{2})F_{A} + \frac{3}{2}(\epsilon^{Au}_{\alpha\beta} + \epsilon^{Ad}_{\alpha\beta})F^{(8)}_{A} + (\frac{\delta_{\alpha\beta}}{2} + \epsilon^{Au}_{\alpha\beta} + \epsilon^{Ad}_{\alpha\beta} + \epsilon^{As}_{\alpha\beta})F^{s}_{A} \\ (\tilde{F}^{n}_{A})_{\alpha\beta} &= (\frac{-\delta_{\alpha\beta}}{2} - \frac{\epsilon^{Au}_{\alpha\beta} - \epsilon^{Ad}_{\alpha\beta}}{2})F_{A} + \frac{3}{2}(\epsilon^{Au}_{\alpha\beta} + \epsilon^{Ad}_{\alpha\beta})F^{(8)}_{A} + (\frac{\delta_{\alpha\beta}}{2} + \epsilon^{Au}_{\alpha\beta} + \epsilon^{Ad}_{\alpha\beta} + \epsilon^{As}_{\alpha\beta})F^{s}_{A} \end{split}$$

$$F_1^N = \frac{\tau G_M^N + G_E^N}{1 + \tau}, \quad F_2^N = \frac{G_M^N - G_E^N}{1 + \tau}, \quad F_1^s = \frac{F_1^s(0)Q^2}{(1 + \tau)(1 + \frac{Q^2}{M_V^2})}, \quad F_2^s = \frac{F_2^s(0)}{(1 + \tau)(1 + \frac{Q^2}{M_V^2})}$$

$$F_A = rac{g_A}{(1+rac{Q^2}{M_A^2})^2}, \quad F_A^{(8)} = rac{g_A^{(8)}}{(1+rac{Q^2}{M_A^2})^2}, \quad F_A^s = rac{\Delta s}{(1+rac{Q^2}{M_A^2})^2}$$

# Bounds on NC NSI as the results

#### Bounds on vectorial part of NC NSI

From  $\nu$  oscillation and Coherent Elastic  $\nu$  Nucleus Scattering



P. Coloma et al., JHEP 12 (2020) 071

### Bounds on axial part of NC NSI

From DIS occures with high energy neutrino beam at

NuTeV

 $|\epsilon^{\mathcal{A}u}_{\mu\mu}| < 0.006, \quad |\epsilon^{\mathcal{A}d}_{\mu\mu}| < 0.018, \quad |\epsilon^{\mathcal{A}u}_{\mu au}|, |\epsilon^{\mathcal{A}d}_{\mu au}| < 0.01$ 

NuTeV Collaboration, Phys.Rev.Lett. 88 (2002) 091802



$$|\epsilon_{ee}^{Au}| < 1, \quad |\epsilon_{ee}^{Ad}| < 0.9, \quad |\epsilon_{e\tau}^{Au}|, |\epsilon_{e\tau}^{Ad}| < 0.5$$

CHARM Collaboration, Phys.Lett.B 180 (1986) 303-307

DUNE-like future experiments could improve bounds on all flavor elements

S. Abbaslu, M. Dehpour, S. Safari, Y. Farzan, JHEP 04 (2024) 038

From QE scattering?!

Thanks for your attention!

# Backup slides

#### Kinematic variables

The process under consideration is

$$u(p) + N(P) \rightarrow \nu(p') + X(P')$$

Energy, momentum and squared four-momentum transfer:

$$u = E_{\nu} - E'_{\nu} = E' - E, \quad \vec{q} = \vec{p} - \vec{p}' = \vec{P}' - \vec{P}, \quad q^2 = \nu^2 - \vec{q}^2 \equiv Q^2$$

squared total center-of-mass energy:

$$s = (p+P)^2 \equiv W^2$$

Bjorken scaling variable:

$$x = \frac{Q^2}{2P.q}$$

Fraction of lepton energy loss in lab. frame:

$$y = \frac{q.P}{p.P}$$

#### Lab. frame relations

Consider  $P = (M_N, 0)$  and reletivisic lepton and neglect their mass in comparison with their energy  $(E_{\nu} \simeq |\vec{p}|)$ 

$$Q^2\simeq 4EE'_
u\sin^2rac{ heta}{2}, \quad W^2\simeq 2M_NE_
u+M_N^2$$

$$x = \frac{Q^2}{2M_N\nu}, \quad y = \frac{\nu}{E_\nu}$$

#### 4-Fermi approximation

For scattering with low momentum transfer  $(|q^2| \ll M_Z)$  we can replace the propagator

$$rac{i}{q^2-M_Z^2}\left(-g^{\mu
u}+rac{q^\mu q^
u}{M_Z^2}
ight)
ightarrow rac{ig^{\mu
u}}{M_Z^2}$$

# DIS squared spin avraged matrix element

$$|\overline{\mathcal{M}}|^2 = \frac{G_{\mathrm{F}}}{2} L_{\mu\nu} W^{\mu\nu}$$

$$L_{\mu
u} = \sum_{\mathrm{initial\ spin\ spin$$

$$W^{\mu
u} = rac{1}{2}\sum_{\mathrm{initial}\ \mathrm{spin}\ \mathrm{states}} \sum_{X(P')|J^{\mu}(0)|N(P)>^{\dagger} < X(P')|J^{
u}(0)|N(P)>^{\dagger}$$

where u represent Dirac spinors

#### Leptonic and hadron tensor

$$L_{\mu
u} = p_{\mu}p'_{
u} + p'_{\mu}p_{
u} - g_{\mu
u}p.p' + i\epsilon_{\mu
u
ho\sigma}p^{
ho}p'^{\sigma}$$

$$\begin{aligned} \frac{1}{2M_N} W^{\mu\nu} &= -g^{\mu\nu} W_1 + \frac{P^{\mu} P^{\nu}}{M_N^2} W_2 + \frac{i\epsilon^{\mu\nu\rho\sigma} P_{\rho} P_{\sigma}}{2M_N^2} W_3 + \frac{q^{\mu} q^{\nu}}{M^2} W_4 \\ &+ \frac{P^{\mu} q^{\nu} + q^{\mu} P^{\nu}}{2M_N^2} W_5 + \frac{i(P^{\mu} q^{\nu} - q^{\mu} P^{\nu})}{2M_N^2} W_6 \end{aligned}$$

# DIS differential XS and scalling

$$\frac{d^2\sigma}{d\Omega dE'_{\nu}} = \frac{G_{\rm F}}{2\pi^2} \left(\frac{M_Z^2}{M_Z^2 + Q^2}\right)^2 (E'_{\nu})^2 \left[2W_1 \sin^2 \frac{1}{2}\theta + W_2 \cos^2 \frac{1}{2}\theta \mp W_3 \frac{E_{\nu} + E'_{\nu}}{M_N} \sin^2 \frac{1}{2}\theta\right]$$

$$F_{1}(x) \equiv \lim_{B_{j}} M_{N} W_{1}(x)$$
$$F_{2}(x) \equiv \lim_{B_{j}} \nu W_{2}(x)$$
$$F_{3}(x) \equiv \lim_{B_{j}} \nu W_{3}(x)$$

### Sum on final neutrino flavor

A DUNE-like detectors does not detect the final neutrino in the u-nucleus scattering so

$$\sigma_{\mathrm{NC}}^{(\stackrel{-}{\nu}_{\alpha})} = \sum_{\beta = e, \mu, \tau} \sigma_{\mathrm{NC}} (\stackrel{(-)}{\nu_{\alpha}} \mathsf{N} \rightarrow \stackrel{(-)}{\nu_{\beta}} + \mathsf{X})$$

# Deep Underground Neutrino Experiment



### $\nu$ oscillation corrections on XS in the DUNE-like FD

$$\mathcal{M}(\stackrel{(-)}{\nu}_{\mathrm{far}}+q \rightarrow \stackrel{(-)}{\nu}_{\beta}+q) = \sum_{lpha = e, \mu au} \stackrel{(-)}{\mathcal{A}}_{lpha} \mathcal{M}(\stackrel{(-)}{\nu}_{lpha}+q \rightarrow \stackrel{(-)}{\nu}_{eta}+q)$$

where  $\mathcal{A}_{lpha}$  is amplitude of neutrino oscilation which satisfied in

$$|\mathcal{A}_{lpha}|^2 = P(
u_{\mu} 
ightarrow 
u_{lpha})$$

### Number of events in the DUNE-like

$$\mathcal{N}_{(\nu)}^{\mathrm{ND}} = \int \phi_{(\nu)}^{\mathrm{ND}}(E) \left[ (\sigma_n)_{(\nu)} N_n^{\mathrm{ND}} + (\sigma_p)_{(\nu)} N_p^{\mathrm{ND}} \right] dE$$
$$\mathcal{N}_{(\nu)}^{\mathrm{FD}} = \int \phi_{(\nu)}^{\mathrm{FD}}(E) \left[ (\sigma_n)_{(\nu)} N_n^{\mathrm{FD}} + (\sigma_p)_{(\nu)} N_p^{\mathrm{FD}} \right] dE$$

$$N_p^{\rm ND/FD} = rac{18}{40} rac{M_{
m fid}^{
m ND/FD}}{M_p} \quad {
m and} \quad N_n^{
m ND/FD} = rac{22}{40} rac{M_{
m fid}^{
m ND/FD}}{M_p}$$

$${\cal B}^{
m ND/FD}_{
u/\overline{
u}} = \epsilon_{
m CC} ({\cal N}^{
m ND/FD}_{
m CC})_{
u/\overline{
u}} + \epsilon_{
m Res} ({\cal N}^{
m ND/FD}_{
m Res})_{
u/\overline{
u}}$$

# NSI bound forecast through DIS at DUNE Near Detector

Parameter	Flux	$\sigma_{\epsilon} = 0,  \sigma_{\omega} = 0$	$\sigma_{\epsilon}=10\%,\sigma_{\omega}=0$	$\sigma_{\epsilon}=10\%,\sigma_{\omega}=2\%$
_Au	CP	[-0.0098, 0.0098]	[-0.018, 0.018]	[-0.19, 0.19]
$\epsilon_{e\mu}$	$\tau$	[-0.0078, 0.0078]	[-0.015, 0.015]	[-0.19, 0.19]
_Au	CP	[-0.000099, 0.000099]	[-0.0007, 0.0007]	[-0.065, 0.065]
$\epsilon_{\mu\mu}$	$\tau$	$\left[-0.000062, 0.000062\right]$	[-0.0004, 0.0004]	[-0.065, 0.065]
Au	CP	[-0.0098, 0.0098]	[-0.018, 0.018]	[-0.19, 0.19]
Cµτ	$\tau$	[-0.0078, 0.0078]	[-0.014, 0.014]	[-0.19, 0.19]
Ad	CP	[-0.0096, 0.0096]	[-0.018, 0.018]	[-0.19, 0.19]
$\epsilon_{e\mu}$	$\tau$	[-0.0076, 0.0076]	[-0.014, 0.014]	[-0.19, 0.19]
Ad	CP	[-0.00009, 0.00009]	[-0.0034, 0.0034]	[-0.12, 0.12]
$\epsilon_{\mu\mu}$	$\tau$	[-0.00006, 0.00006]	[-0.0021, 0.0021]	[-0.12, 0.12]
Ad	CP	[-0.0095, 0.0095]	[-0.018, 0.018]	[-0.19, 0.19]
$\epsilon_{\mu\tau}$	$\tau$	[-0.0076, 0.0076]	[-0.014, 0.014]	[-0.19, 0.19]
-As	CP	[-0.027, 0.027]	[-0.051, 0.051]	[-0.55, 0.55]
$\epsilon_{e\mu}$	$\tau$	[-0.022, 0.022]	[-0.041, 0.041]	[-0.55, 0.55]
As	CP	[-0.00075, 0.00075]	[-0.0026, 0.0026]	[-1.21, 1.21]
$\epsilon_{\mu\mu}$	$\tau$	[-0.00048, 0.00048]	[-0.0016, 0.0016]	[-1.21, 1.21]
As	CP	[-0.027, 0.027]	[-0.051, 0.051]	[-0.55, 0.55]
$\epsilon_{\mu\tau}$	$\tau$	[-0.022, 0.022]	[-0.041, 0.041]	[-0.55, 0.55]
Au Ad	CP	[-0.0069, 0.0069]	[-0.013, 0.013]	[-0.14, 0.14]
$\epsilon_{e\mu}^{Au} = \epsilon_{e\mu}^{Au}$	$\tau$	[-0.0055, 0.0055]	[-0.010, 0.010]	[-0.14, 0.14]
_AuAd	CP	[-0.00074, 0.00074]	[-0.00085, 0.00085]	[-0.072, 0.072]
$\epsilon_{\mu\mu} = \epsilon_{\mu\mu}$	$\tau$	[-0.00045, 0.00045]	[-0.00054, 0.00054]	[-0.072, 0.072]
_AuAd	CP	[-0.0069, 0.0069]	[-0.013,0.013]	[-0.14,0.14]
$\epsilon_{\mu\tau} = \epsilon_{\mu\tau}$	$\tau$	[-0.0055, 0.0055]	[-0.010, 0.010]	[-0.14, 0.14]

NSI	bound	forecast	through	DIS	at	DUNE	Far	Detector
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Parameter	Flux	$\sigma_{\epsilon}=0,\sigma_{\omega}=0$	$\sigma_{\epsilon}=10\%,\sigma_{\omega}=0$	$\sigma_{\epsilon}=10\%,\sigma_{\omega}=2\%$
_Au	CP	[-1.28, 0.23]	[-2.7, 0.9]	[-2.8, 1.2]
$\epsilon_{ee}$	$\tau$	[-1.35, 0.33]	[-2.1, 2.5]	[-2.9, 2.7]
$\epsilon^{Au}$	CP	[-0.065, 0.038]	[-0.22, 0.23]	[-0.26, 0.29]
$c_{e\tau}$	$\tau$	[-0.078, 0.072]	[-0.22, 0.30]	[-0.32, 0.40]
$\epsilon^{Au}_{\tau\tau}$	CP	[-0.014, 0.014]	[-0.082, 0.072]	[-0.118, 0.100]
	$\tau$	[-0.021, 0.021]	$[-0.652, \! 0.503] \!+ \! [-0.117,  0.089]$	[-0.718, 0.156]
$\epsilon^{Ad}_{ee}$	CP	[-0.20, 0.35] + [0.93, 1.24]	[-0.7, 2.7]	[-0.96, 2.78]
	$\tau$	[-0.30, 1.29]	[-2.8, 1.2]	[-2.77, 2.37]
Ad	CP	[-0.051, 0.040]	[-0.18, 0.23]	[-0.23, 0.27]
$\epsilon_{e\tau}$	$\tau$	[-0.076, 0.052]	[-0.24, 0.23]	[-0.33, 0.32]
Ad	CP	[-0.014, 0.016]	[-0.11, 0.24]	[-0.14, 0.24]
$\epsilon_{\tau\tau}$	$\tau$	[-0.021, 0.021]	[-0.14, 0.37]	[-0.21, 0.44]
-As	CP	[-1.1, 2.1]	[-4.1, 5.0]	[-4.7, 5.7]
$\epsilon_{ee}$	$\tau$	[-1.5, 2.5]	[-6.1, 7.1]	[-7.6, 8.6]
-As	CP	[-0.54, 0.22]	[-0.72, 0.52]	[-0.85, 0.65]
$\epsilon_{e\tau}$	$\tau$	[-0.59, 0.27]	[-0.82, 0.57]	[-1.1, 0.84]
$\epsilon^{As}_{\tau\tau}$	CP	[-0.11, 0.15] + [0.85, 1.11]	[-0.29, 1.28]	[-0.39, 1.39]
	$\tau$	[-0.14, 0.21] + [0.79, 1.45]	[0.35, 1.35]	[-0.58,  1.58]
$\epsilon^{Au}_{ee} = \epsilon^{Ad}_{ee}$	CP	[-0.35, 0.41]	[-0.9, 1.3]	[-1.1, 1.5]
	$\tau$	[-0.45, 0.49]	[-1.9, 1.3]	[-2.1, 1.8]
$\epsilon^{Au}_{e\tau}{=}\epsilon^{Ad}_{e\tau}$	CP	[-0.089, 0.083]	[-0.13, 0.18]	[-0.16, 0.21]
	$\tau$	[-0.101, 0.095]	[-0.14, 0.20]	[-0.21, 0.26]
Au_Ad	CP	[-0.076, 0.084]	[-0.24, 0.09]	[-0.27, 0.13]
$\epsilon_{\tau\tau} = \epsilon_{\tau\tau}$	$\tau$	[-0.088, 0.099]	[-0.26, 0.11]	[-0.32, 0.18]

# Imaginary part of NSI parameter



# Strange axial coupling constant



KamLAND Collaboration, Phys.Rev.D 107 (2023) 7, 072006