

ALESSANDRO BACCHETTA, PAVIA U. AND INFN

TMD OVERVIEW

slide from 2018 CPHI@Yerevan



slide from 2018 CPHI@Yerevan

It's the dawn of TMD global fits era



slide from 2018 CPHI@Yerevan

It's the dawn of TMD global fits era

...but there's still a lot of climbing to be done

In five to ten years

slide from 2018 CPHI@Yerevan

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- TMD multiplicities for pions and kaons, off protons and deuterons, from COMPASS and JLab

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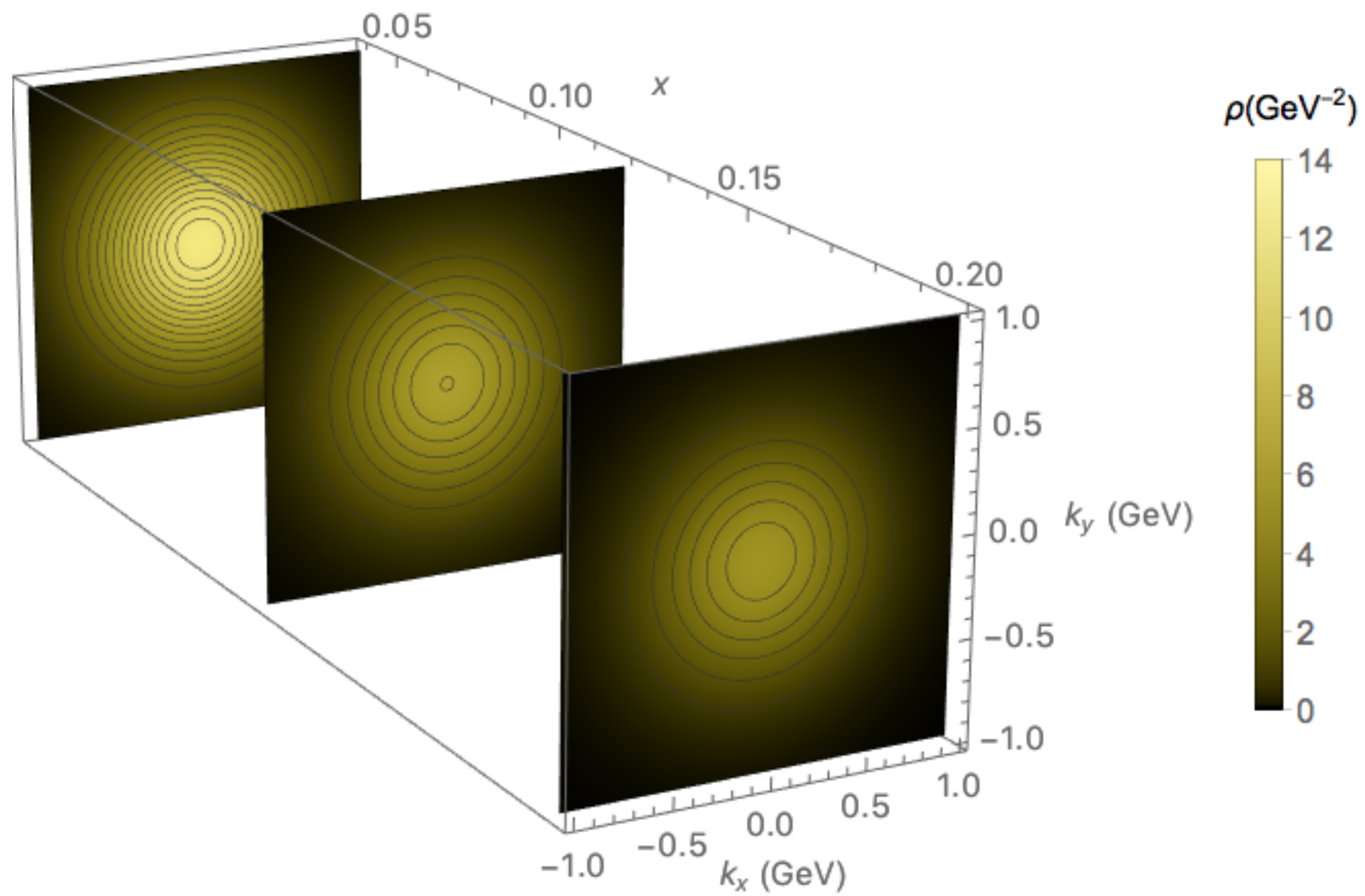
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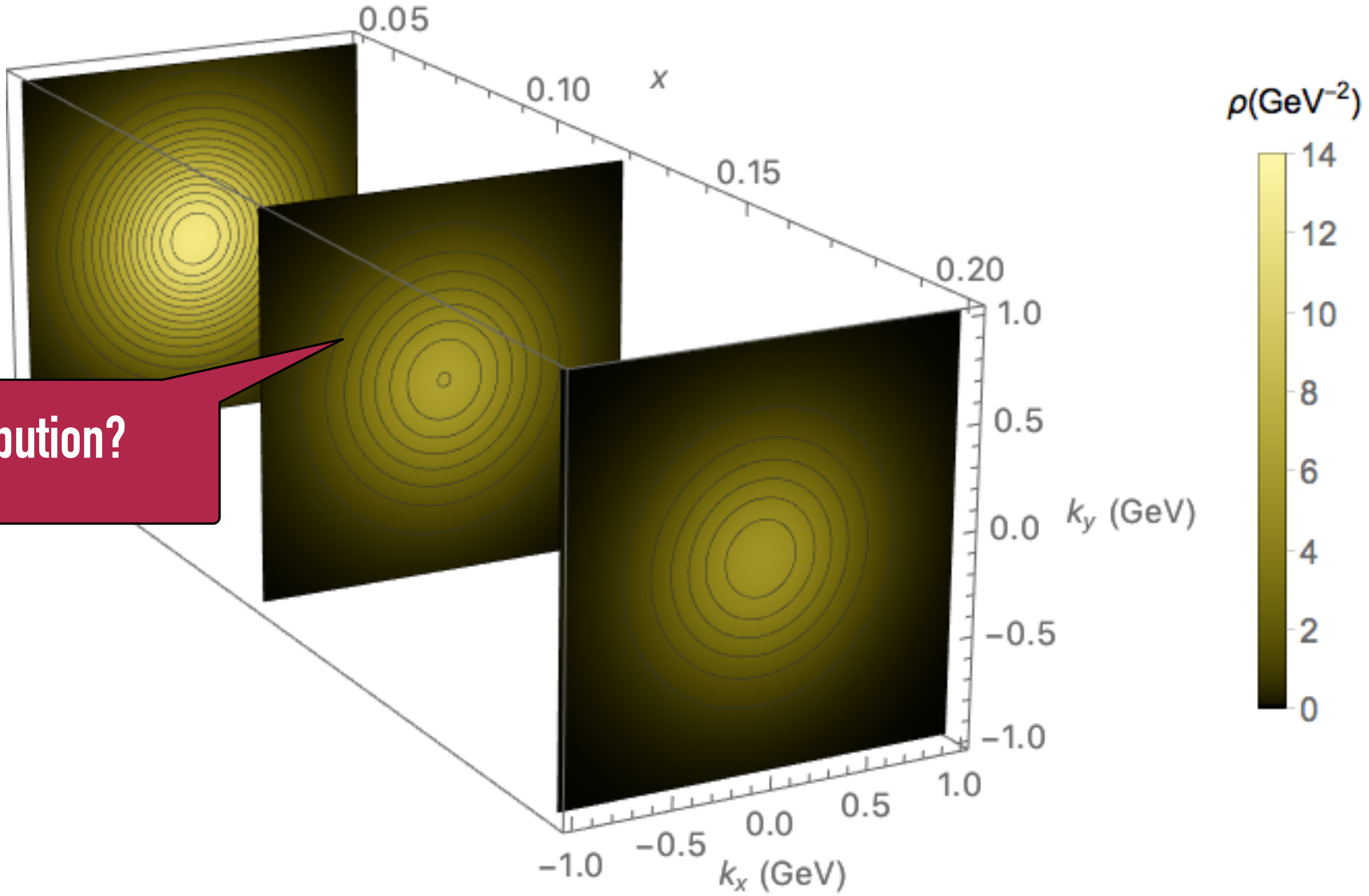
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- **READY TO USE EIC DATA**

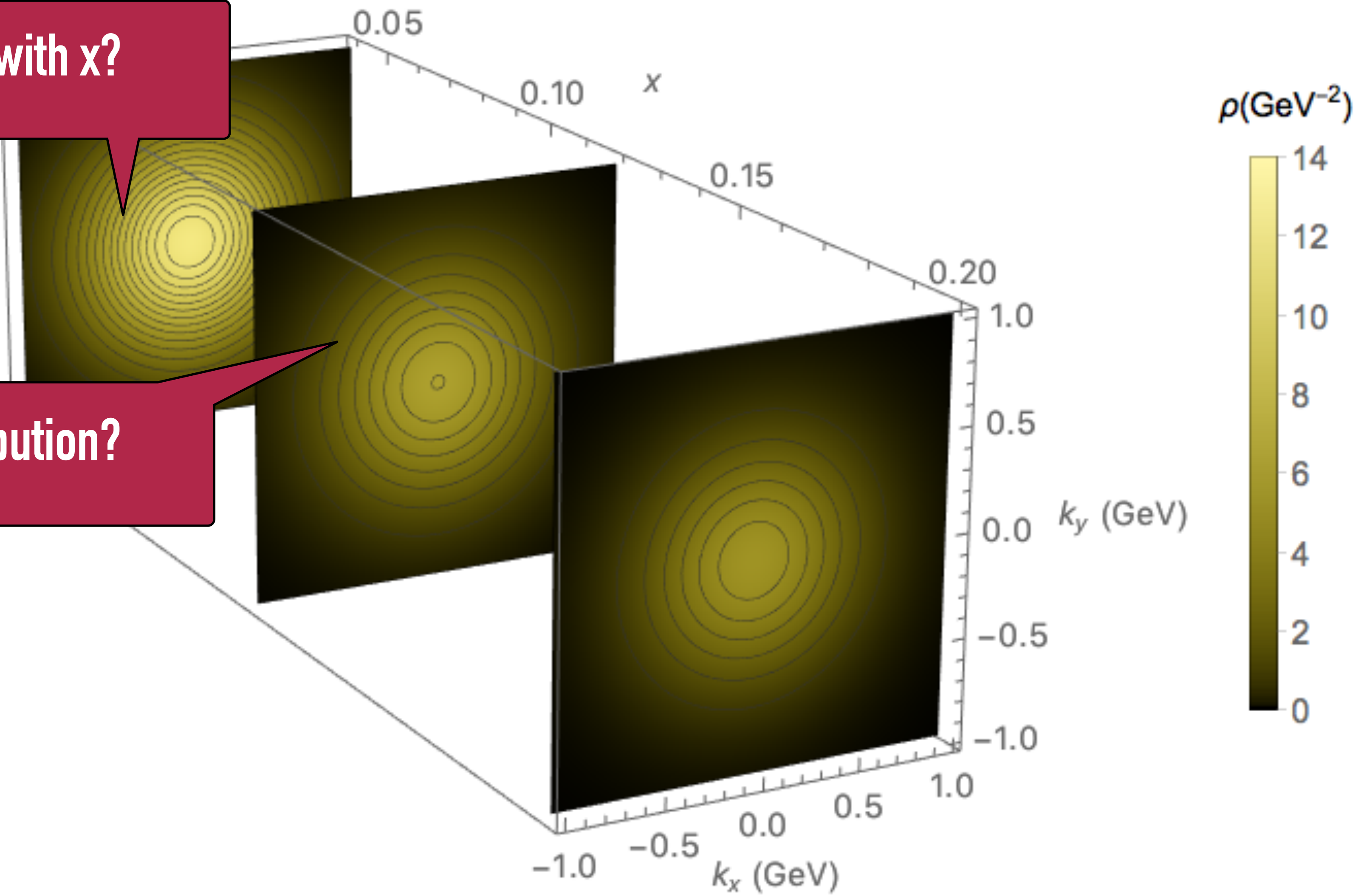


How "wide" is the distribution?



How does it change with x ?

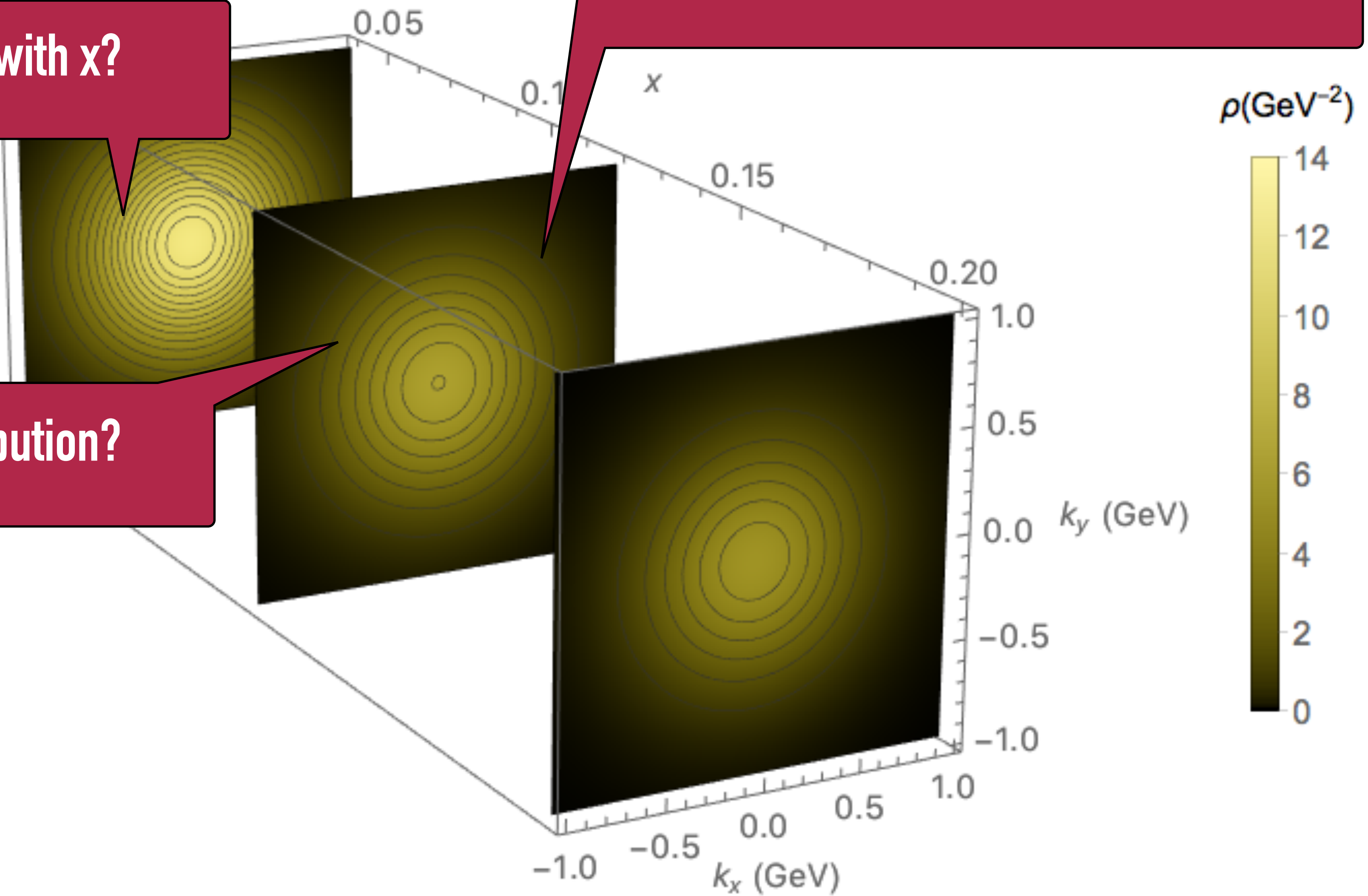
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Is there a difference between flavors?

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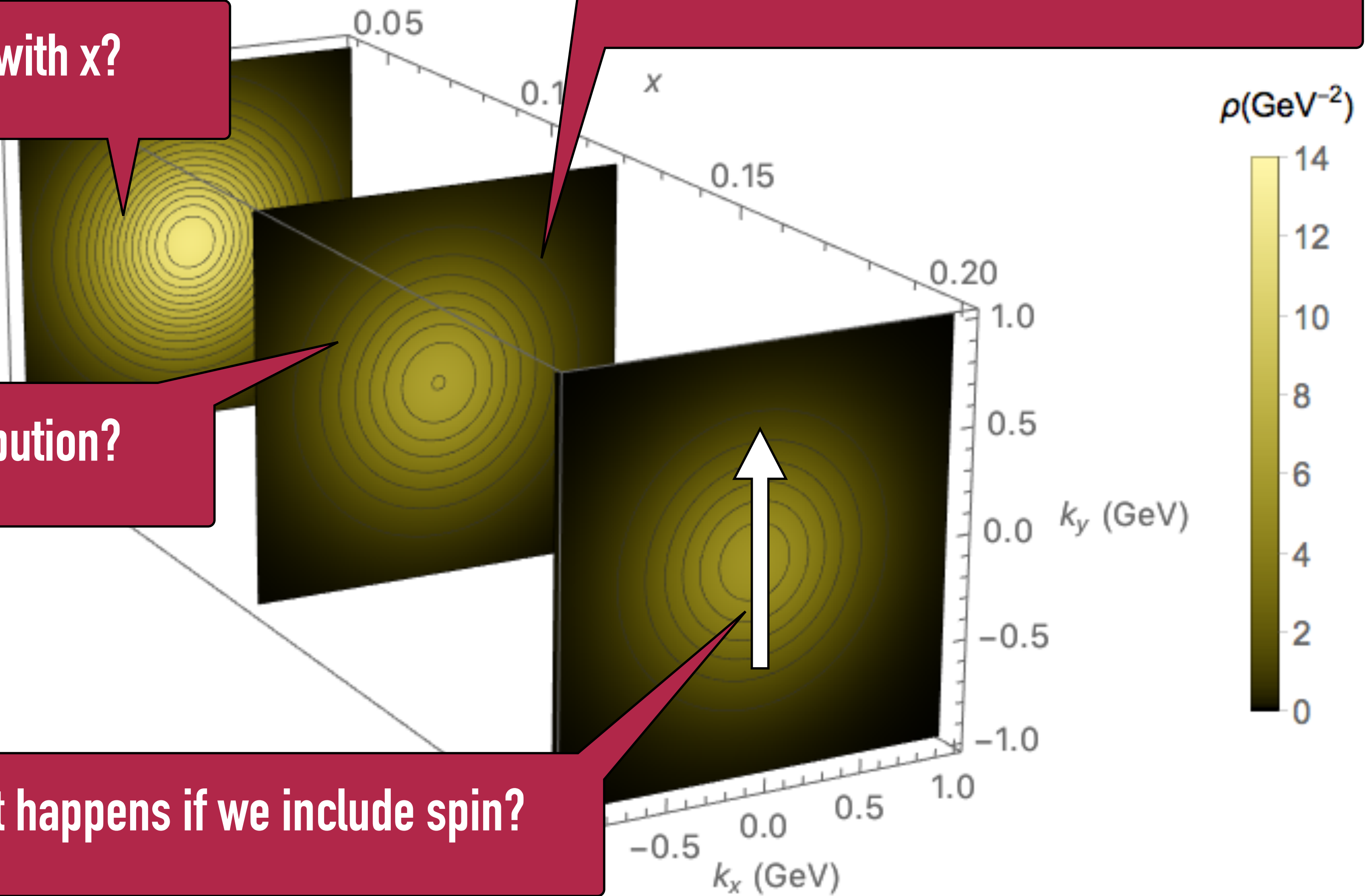


How does it change with x ?

Is there a difference between flavors?

How "wide" is the distribution?

What happens if we include spin?



[Mulders-Tangerman, NPB 461 \(96\)](#)

[Boer-Mulders, PRD 57 \(98\)](#)

quark pol.

	U	L	T
nucleon pol.	U		h_1^\perp
	L	g_{1L}	h_{1L}^\perp
	T	f_{1T}^\perp	h_1, h_{1T}^\perp

TMDs in **black** survive integration over transverse momentum

TMDs in **red** are time-reversal odd

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Transversity

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Transversity

- ▶ Very good knowledge of x dependence of f_1 and g_{1L}

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- ▶ Fair knowledge of Sivers and transversity (mainly x dependence)
- ▶ Some hints about all others

AVAILABLE EXTRACTIONS (NEWEST ONLY)

Unpol. TMD	MAP 22 arXiv:2206.07598 , ART23 2305.07473
Helicity	
Transversity	arXiv:1505.05589 , arXiv:1612.06413 , arXiv:2205.00999
Sivers	MAP20 arXiv:2004.14278 , arXiv:2009.10710 , arXiv:2103.03270 , arXiv:2205.00999 , arXiv:2304.14328
Boer-Mulders	arXiv:2004.02117 ,
Worm-gear g1T	arXiv:2110.10253 , arXiv:2210.07268
Worm-gear h1L	
Pretzelosity	arXiv:1411.0580

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Not mentioned: pion TMDs, TMD fragmentation functions, nuclear TMDs

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- ▶ Lots of progress from the theory side
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- ▶ All others unknown

Mulders, Rodrigues, PRD63, 2001

gluon pol.

nucleon pol.

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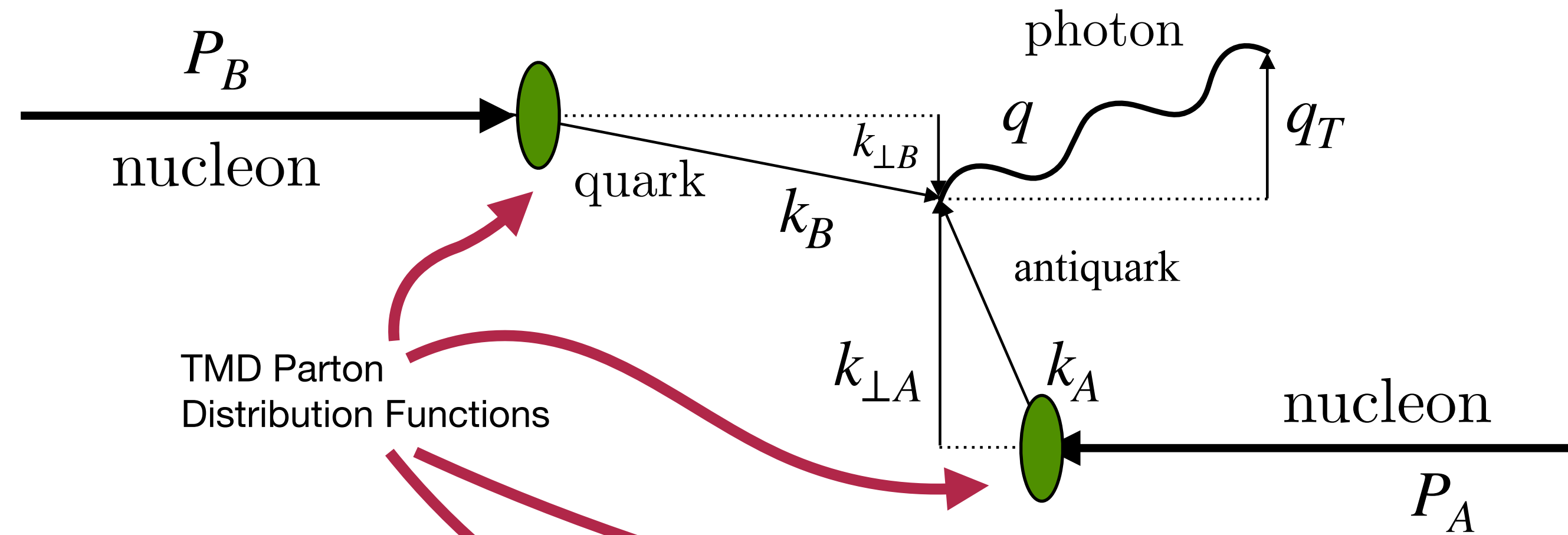
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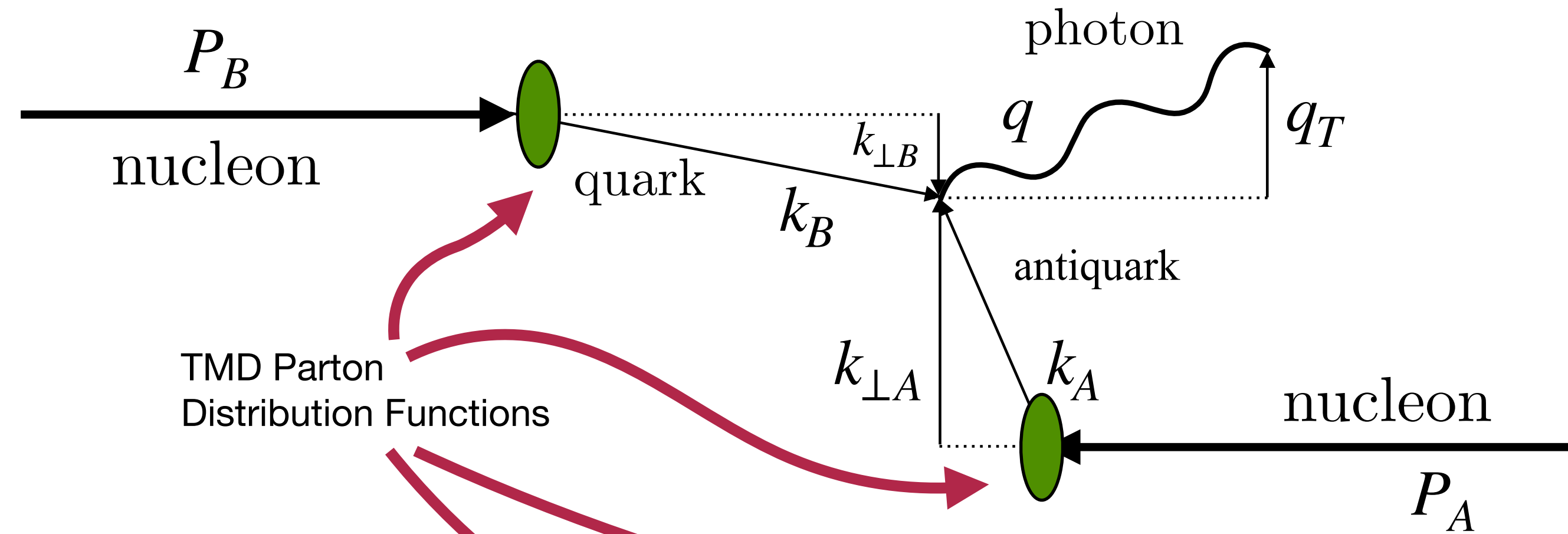
See talk by Cristian Pisano

UNPOLARIZED TMD



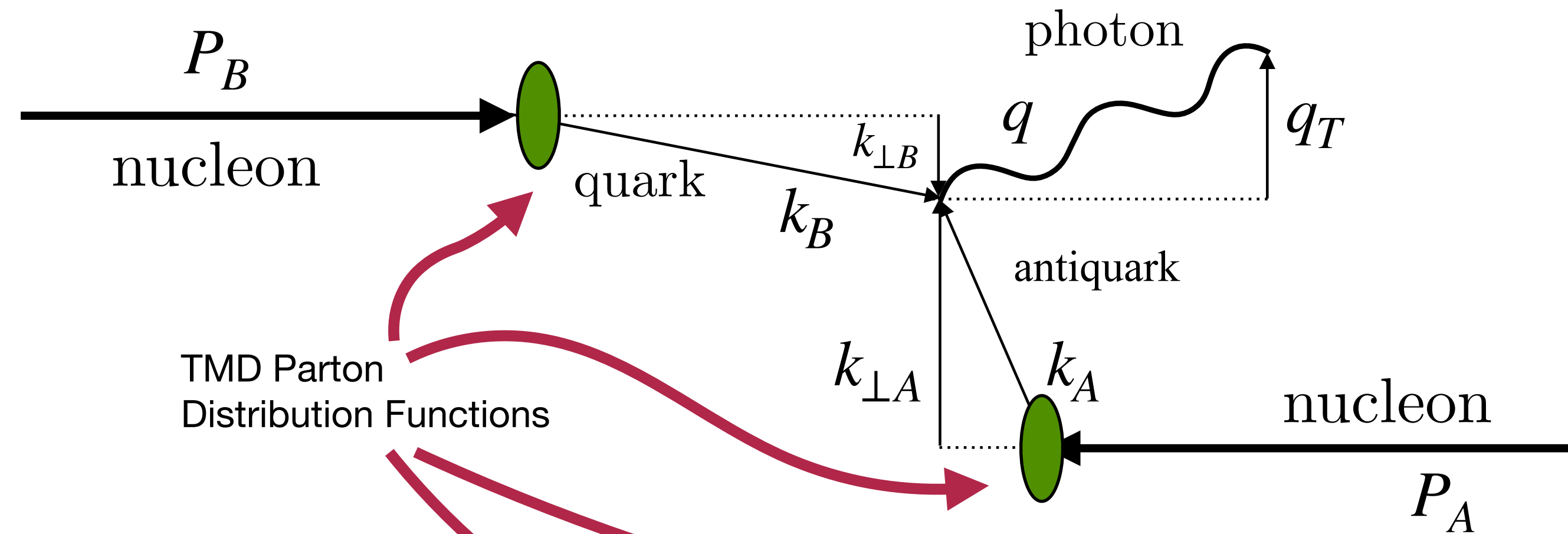
$$F_{UU}^1(x_A, x_B, \mathbf{q}_T^2, Q^2)$$

$$\approx \sum_q \mathcal{H}_{UU}^{1q}(Q^2, \mu^2) \int d^2\mathbf{k}_{\perp A} d^2\mathbf{k}_{\perp B} f_1^q(x_A, \mathbf{k}_{\perp A}^2; \mu^2) f_1^{\bar{q}}(x_B, \mathbf{k}_{\perp B}^2; \mu^2) \delta^{(2)}(\mathbf{k}_{\perp A} - \mathbf{q}_T + \mathbf{k}_{\perp B})$$



$$\begin{aligned}
 F_{UU}^1(x_A, x_B, \mathbf{q}_T^2, Q^2) & \\
 & \approx \sum_q \mathcal{H}_{UU}^{1q}(Q^2, \mu^2) \int d^2\mathbf{k}_{\perp A} d^2\mathbf{k}_{\perp B} f_1^q(x_A, \mathbf{k}_{\perp A}^2; \mu^2) f_1^{\bar{q}}(x_B, \mathbf{k}_{\perp B}^2; \mu^2) \delta^{(2)}(\mathbf{k}_{\perp A} - \mathbf{q}_T + \mathbf{k}_{\perp B}) \\
 & = \sum_q \mathcal{H}_{UU}^{1q}(Q^2, \mu^2) \int db_T b_T J_0(b_T |\mathbf{q}_T|) \hat{f}_1^q(x_A, b_T^2; \mu^2) \hat{f}_1^{\bar{q}}(x_B, b_T^2; \mu^2)
 \end{aligned}$$

The analysis is usually done in Fourier-transformed space

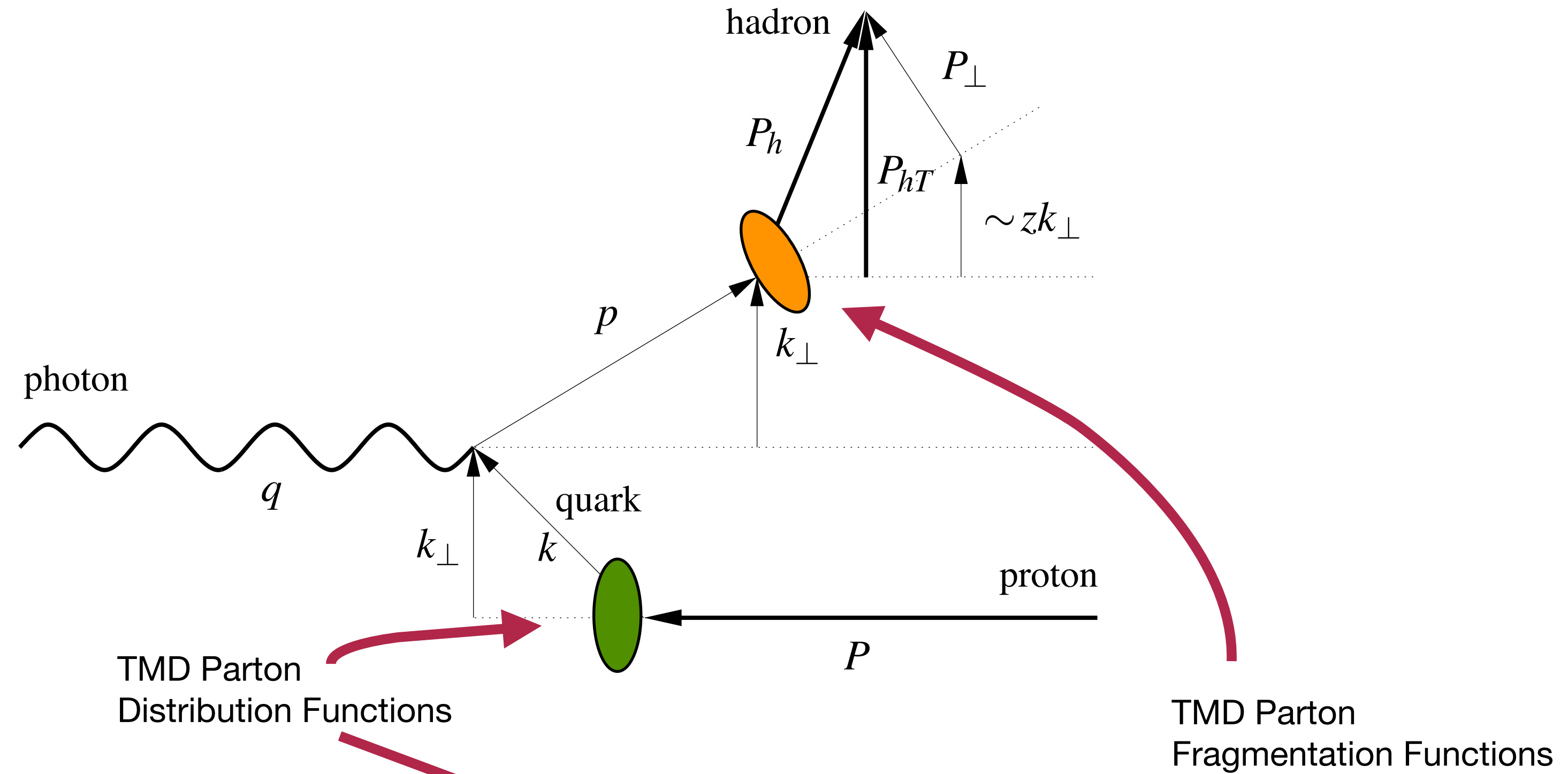


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$$= \sum_q \mathcal{H}_{UU}^{1q}(Q^2, \mu^2) \int db_T b_T J_0(b_T |\mathbf{q}_T|) \hat{f}_1^q(x_A, b_T^2; \mu^2) \hat{f}_1^{\bar{q}}(x_B, b_T^2; \mu^2)$$

The analysis is usually done in Fourier-transformed space
 TMDs formally depend on two scales, but we set them equal.



$$\begin{aligned}
 F_{UU,T}(x, z, \mathbf{P}_{hT}^2, Q^2) &= x \sum_q \mathcal{H}_{UU,T}^q(Q^2, \mu^2) \int d^2 \mathbf{k}_\perp d^2 \mathbf{P}_\perp f_1^a(x, \mathbf{k}_\perp^2; \mu^2) D_1^{a \rightarrow h}(z, \mathbf{P}_\perp^2; \mu^2) \delta(z \mathbf{k}_\perp - \mathbf{P}_{hT} + \mathbf{P}_\perp) \\
 &= x \sum_a \mathcal{H}_{UU,T}^a(Q^2, \mu^2) \int db_T b_T J_0(b_T |\mathbf{P}_{h\perp}|) \hat{f}_1^q(x, z^2 b_\perp^2; \mu^2) \hat{D}_1^{a \rightarrow h}(z, b_\perp^2; \mu^2)
 \end{aligned}$$

$$\hat{f}_1^a(x, |\mathbf{b}_T|; \mu, \zeta) = \int d^2\mathbf{k}_\perp e^{i\mathbf{b}_T \cdot \mathbf{k}_\perp} f_1^a(x, \mathbf{k}_\perp^2; \mu, \zeta)$$

see, e.g., Collins, "Foundations of Perturbative QCD" (11)
TMD collaboration, "TMD Handbook," arXiv:2304.03302

$$\hat{f}_1^a(x, |\mathbf{b}_T|; \mu, \zeta) = \int d^2\mathbf{k}_\perp e^{i\mathbf{b}_T \cdot \mathbf{k}_\perp} f_1^a(x, \mathbf{k}_\perp^2; \mu, \zeta)$$

$$\hat{f}_1^a(x, b_T^2; \mu_f, \zeta_f) = [C \otimes f_1](x, \mu_{b_*}) e^{\int_{\mu_{b_*}}^{\mu_f} \frac{d\mu}{\mu} (\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta_f}}{\mu})} \left(\frac{\sqrt{\zeta_f}}{\mu_{b_*}} \right)^{K_{\text{resum}} + g_K}$$

$$\mu_b = \frac{2e^{-\gamma_E}}{b_T}$$

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perturbative Sudakov form factor

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collinear PDF

Collins-Soper kernel (perturbative and nonperturbative)

$$\mu_b = \frac{2e^{-\gamma_E}}{b_T}$$

matching coefficients (perturbative)

[see, e.g., Collins, "Foundations of Perturbative QCD" \(11\)](#)
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perturbative Sudakov
form factor

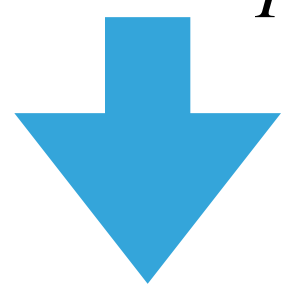
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collinear PDF

Collins-Soper kernel
(perturbative and
nonperturbative)

matching coefficients
(perturbative)

$$\mu_b = \frac{2e^{-\gamma_E}}{b_T}$$



$$\mu_{b_*} = \frac{2e^{-\gamma_E}}{\bar{b}_*}$$

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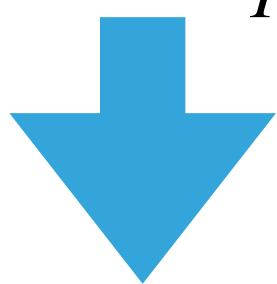
collinear PDF

Collins-Soper kernel (perturbative and nonperturbative)

nonperturbative part of TMD

matching coefficients (perturbative)

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[see, e.g., Collins, "Foundations of Perturbative QCD" \(11\)](#)
[TMD collaboration, "TMD Handbook," arXiv:2304.03302](#)

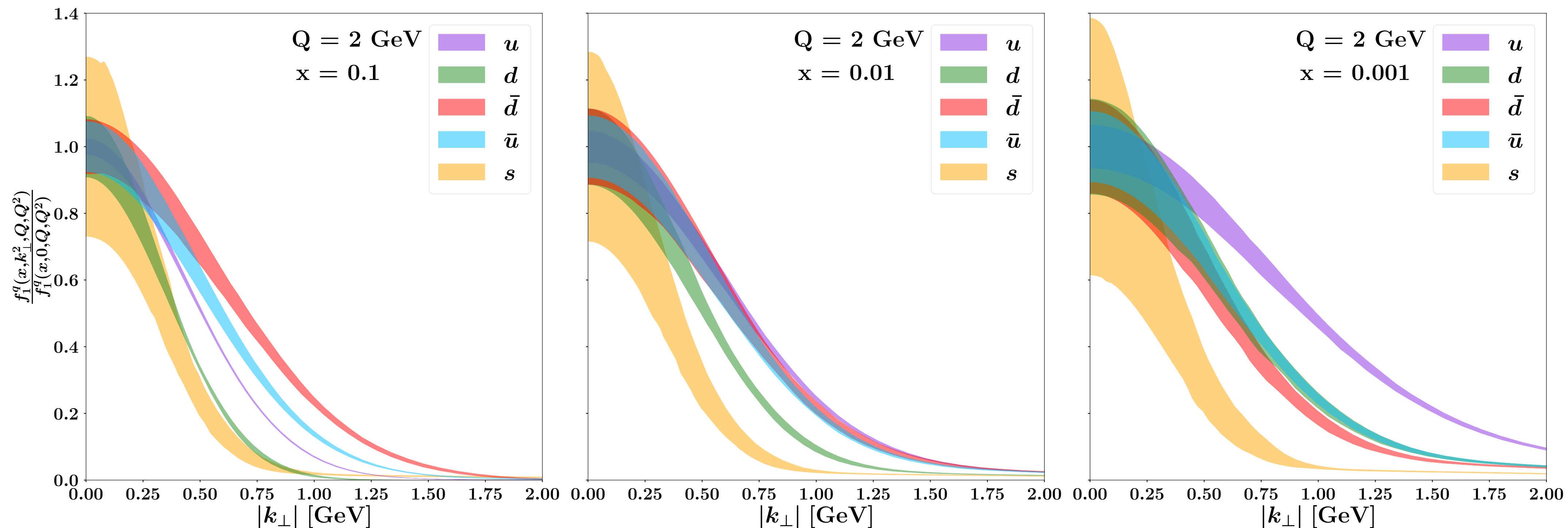
	Accuracy	SIDIS HERMES	SIDIS COMPASS	DY fixed target	DY collider	N of points	χ^2/N_{points}
Pavia 2017 arXiv:1703.10157	NLL	✓	✓	✓	✓	8059	1.55
SV 2019 arXiv:1912.06532	N ³ LL ⁻	✓	✓	✓	✓	1039	1.06
MAP22 arXiv:2206.07598	N ³ LL ⁻	✓	✓	✓	✓	2031	1.06
ART23 arXiv:2305.07473	N ⁴ LL	✗	✗	✓	✓	627	0.96
MAP24 arXiv:2405.13833	N ³ LL	✓	✓	✓	✓	2031	1.08

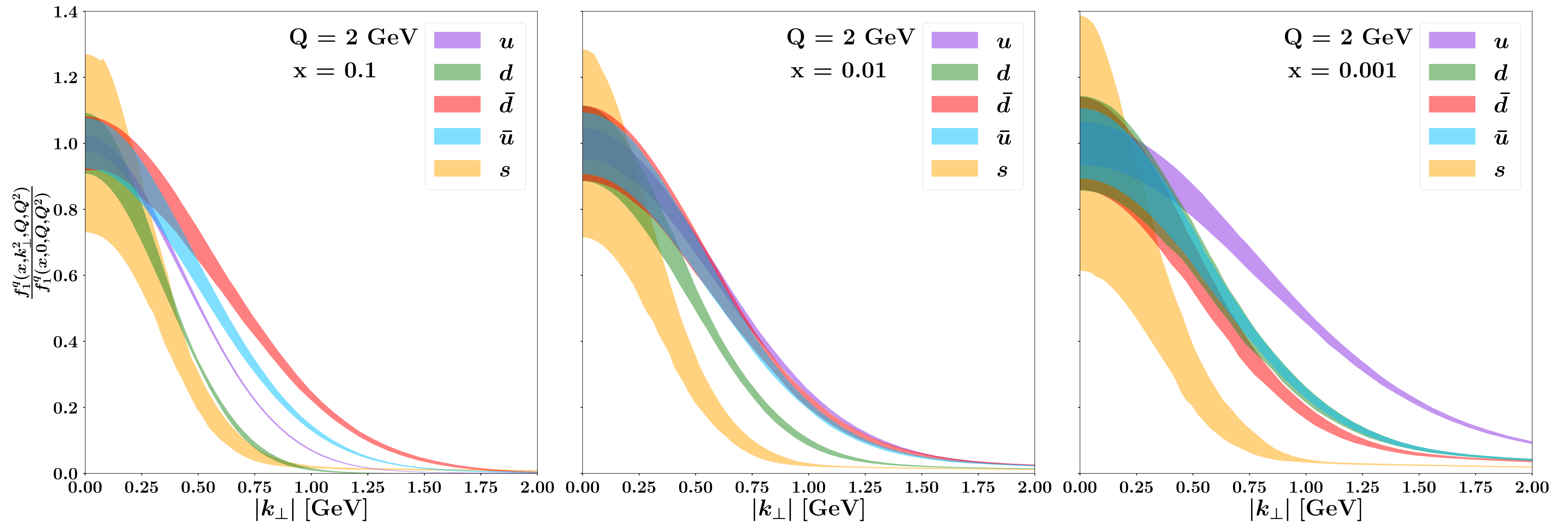
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See talks by Filippo Delcarro and Valentin Moos

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See talks by Filippo Delcarro and Valentin Moos
see also Parton Branching approach, talk by Louis Moureaux

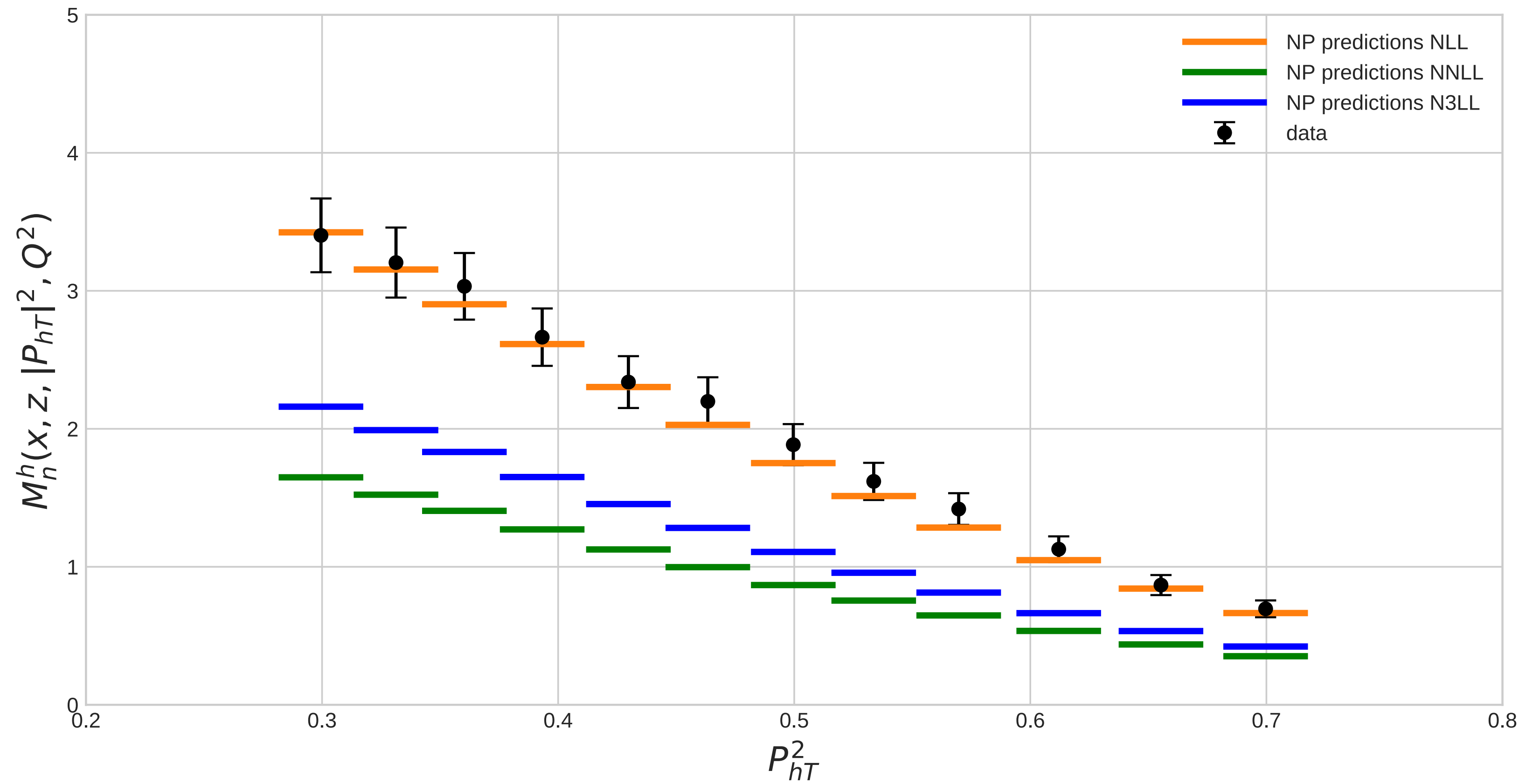




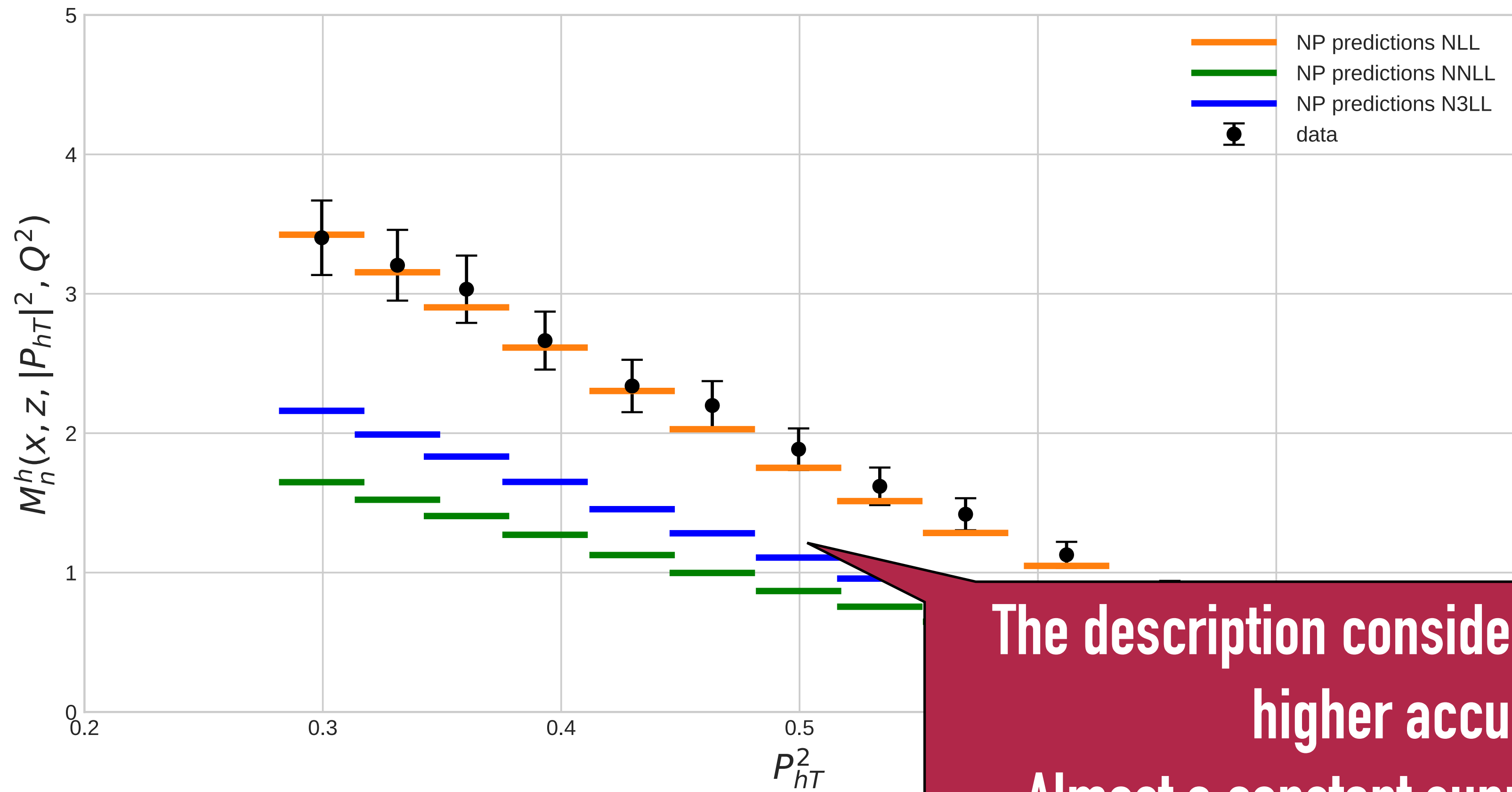
See talk by Filippo Delcarro

SOME OPEN ISSUES . . .

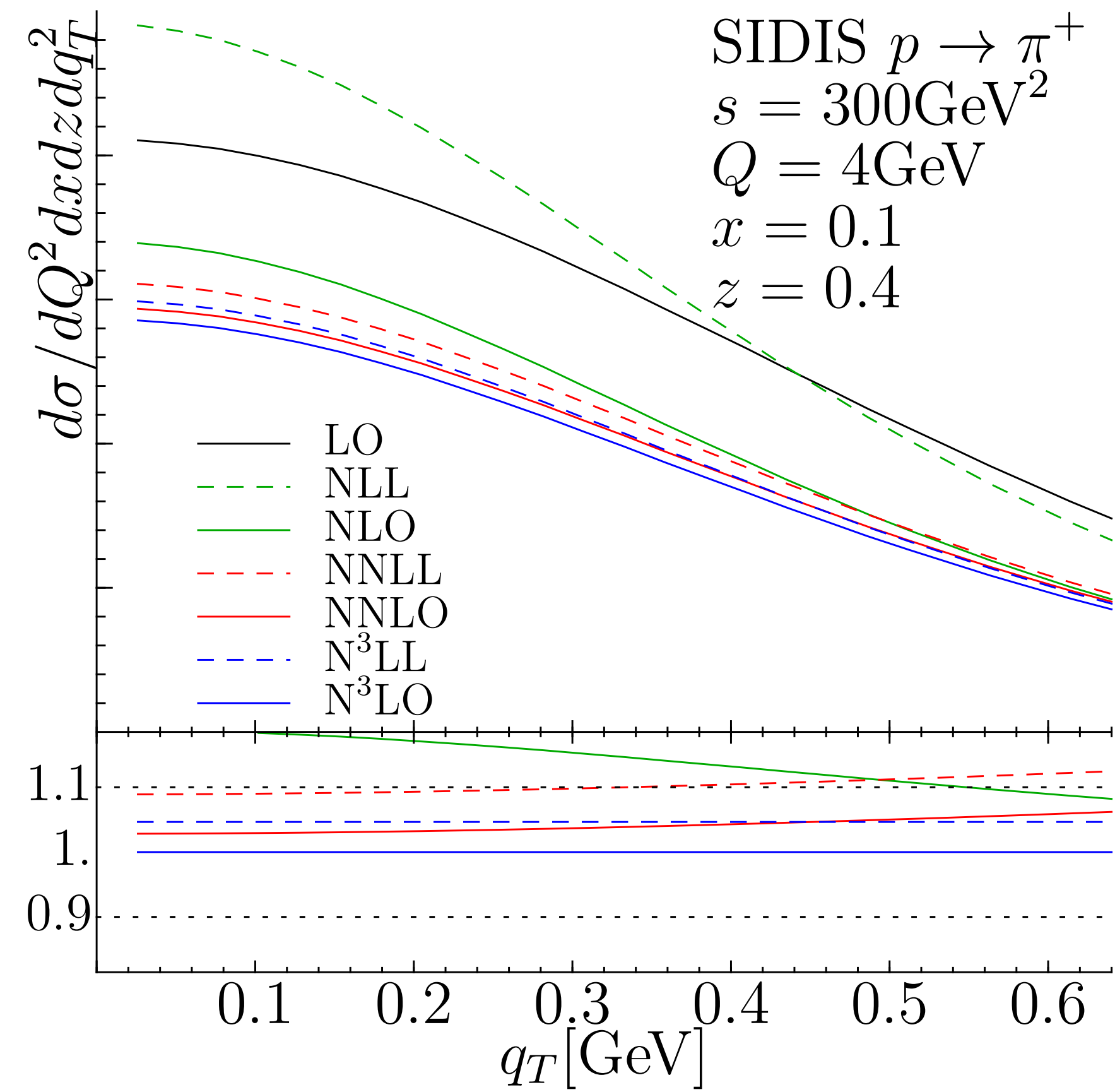
COMPASS multiplicities (one of many bins)

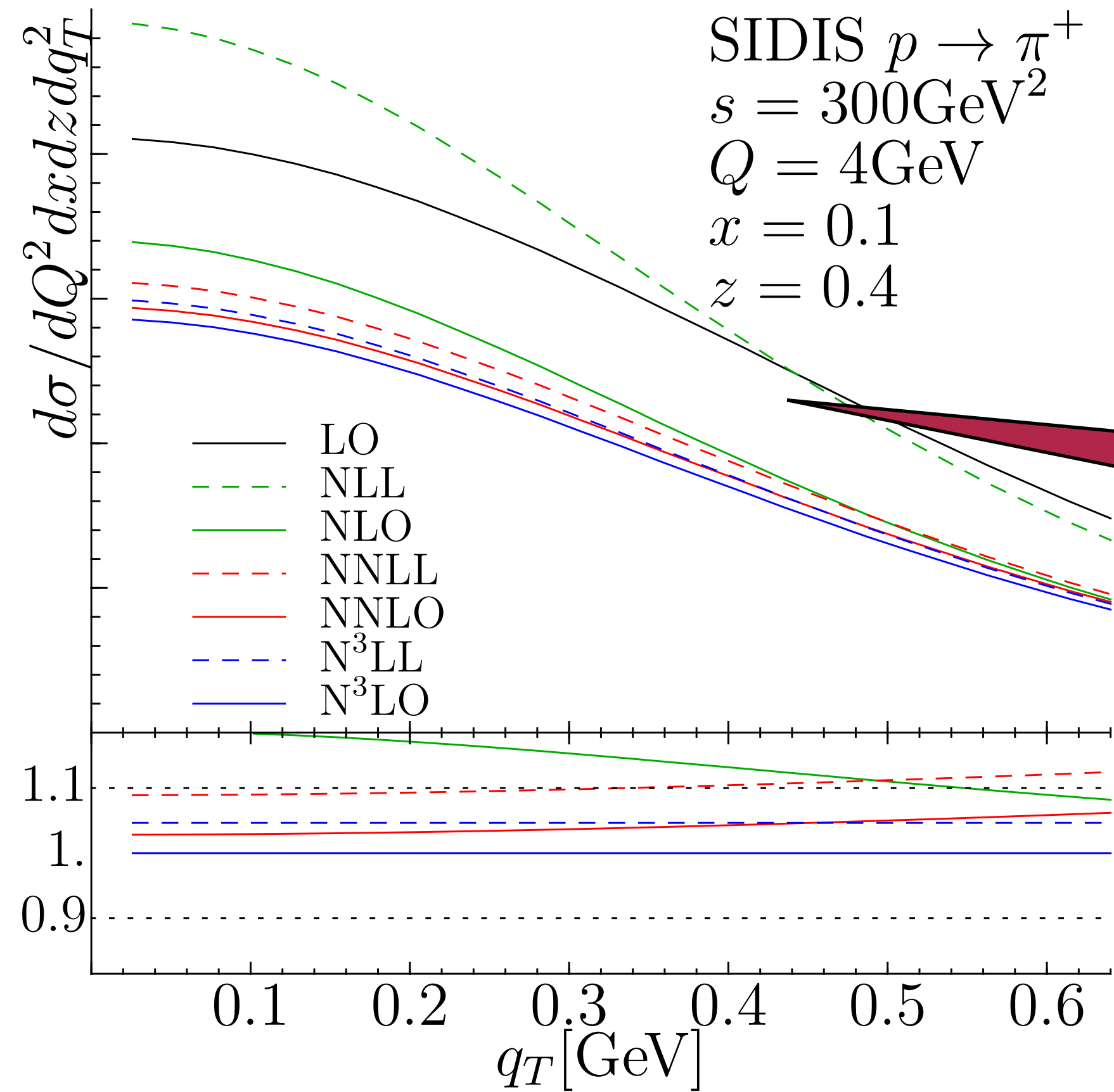


COMPASS multiplicities (one of many bins)



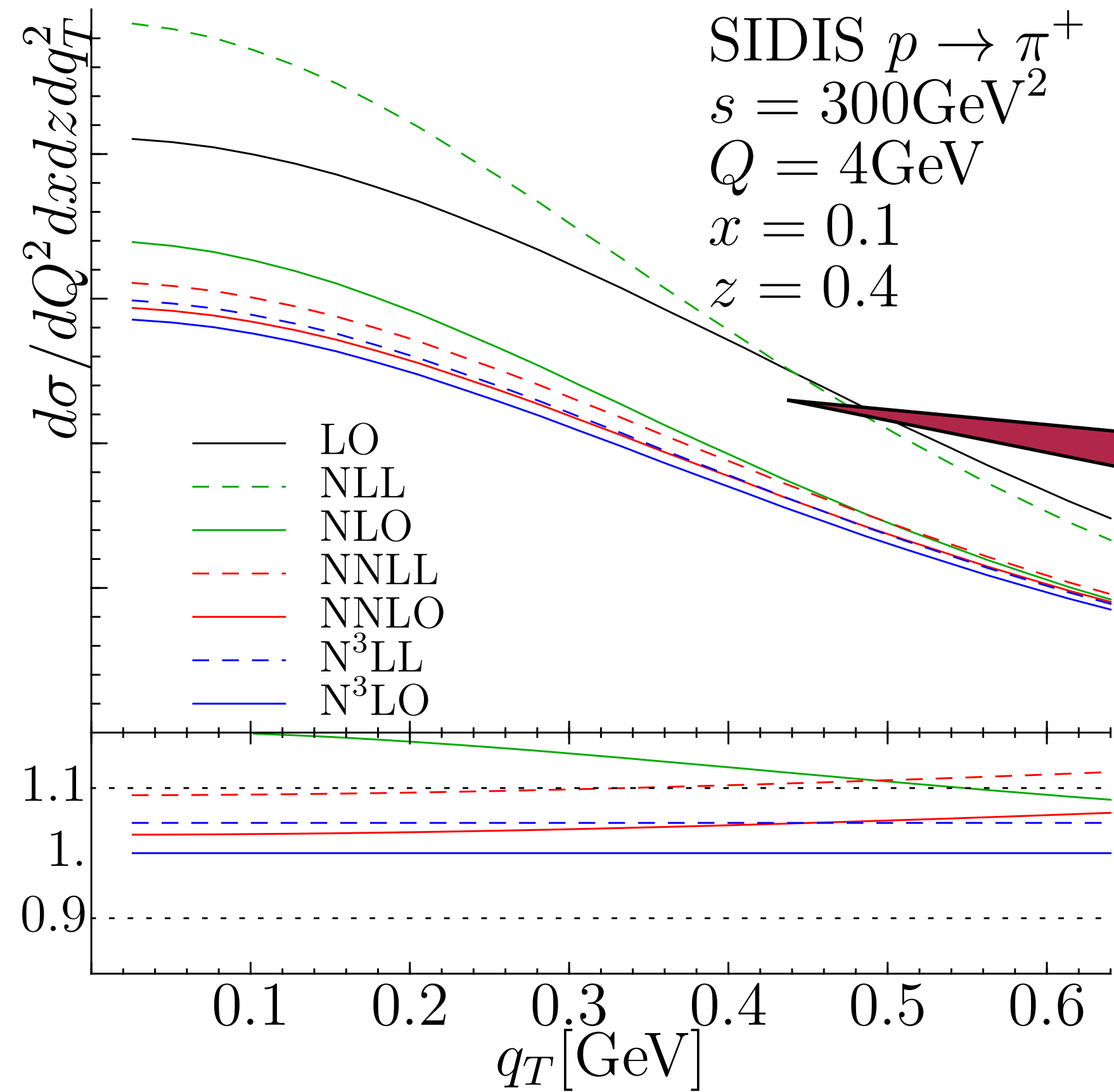
The description considerably worsens at higher accuracy.
Almost a constant suppression factor.





Also in the SV19 study, the overall decrease is evident

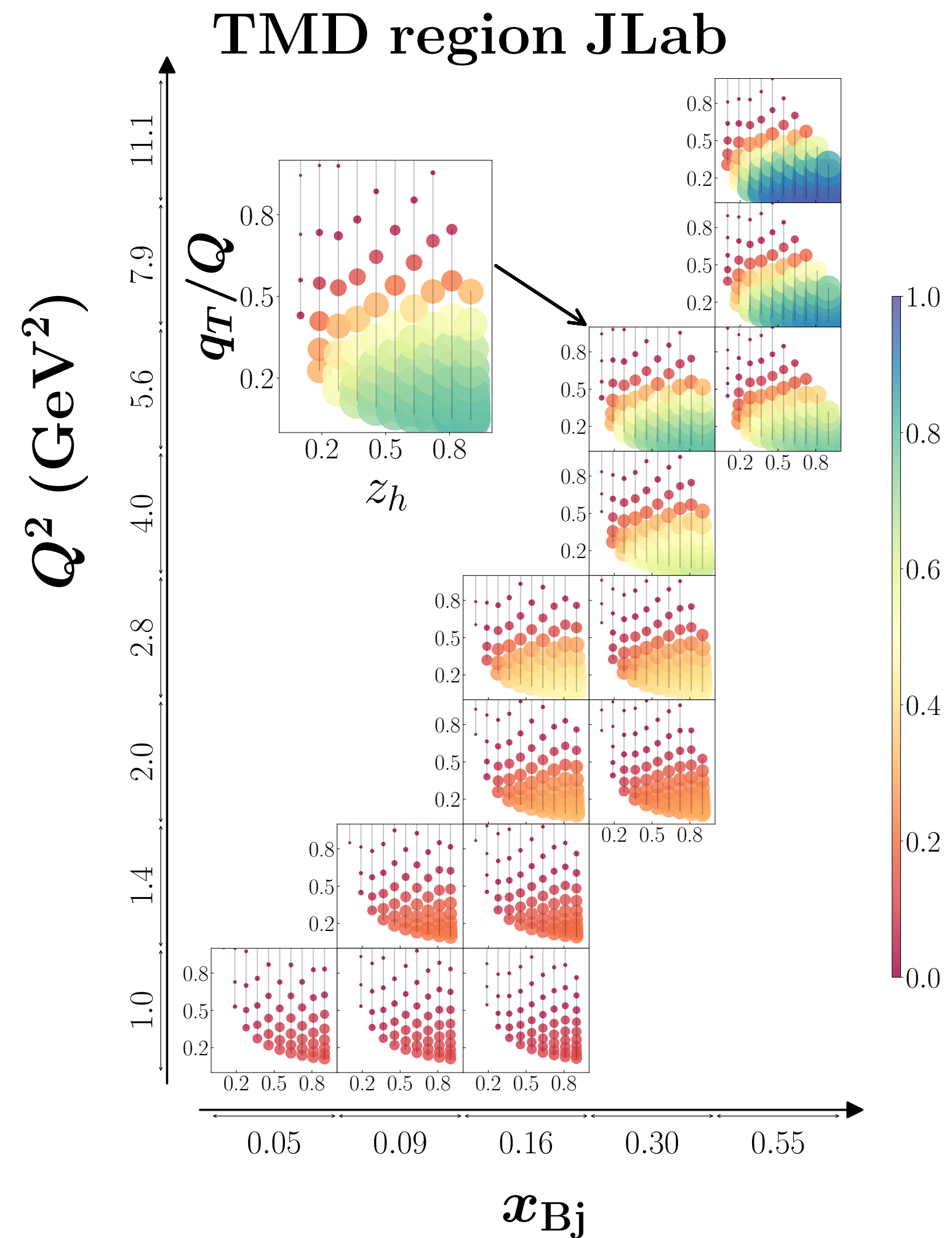
[Scimemi, Vladimirov, arXiv:1912.06532](#)



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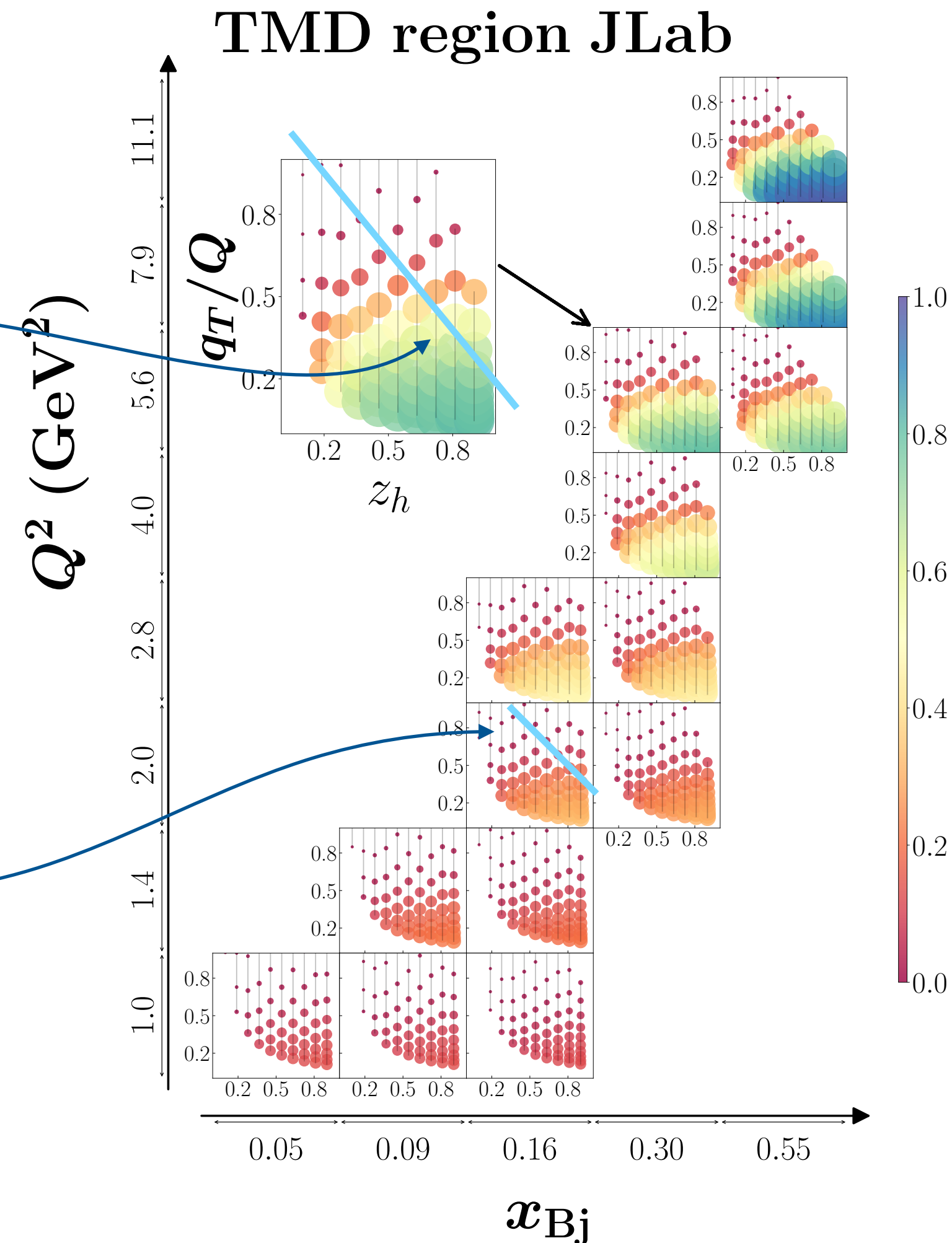
See talk by Valentin Moos

$$|q_T| = |P_{hT}|/z \ll Q$$



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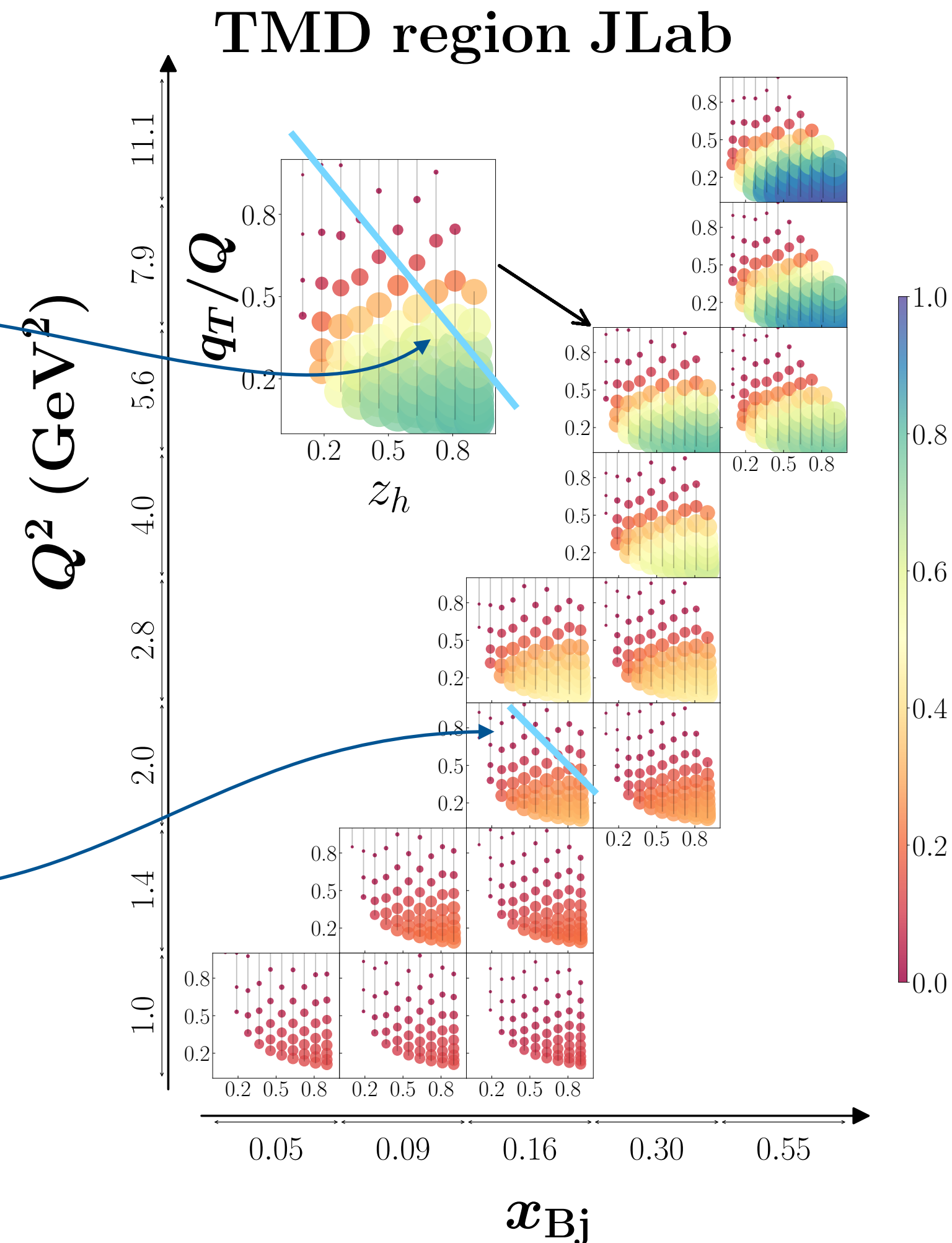
Approximate region
corresponding to
MAP22 cuts



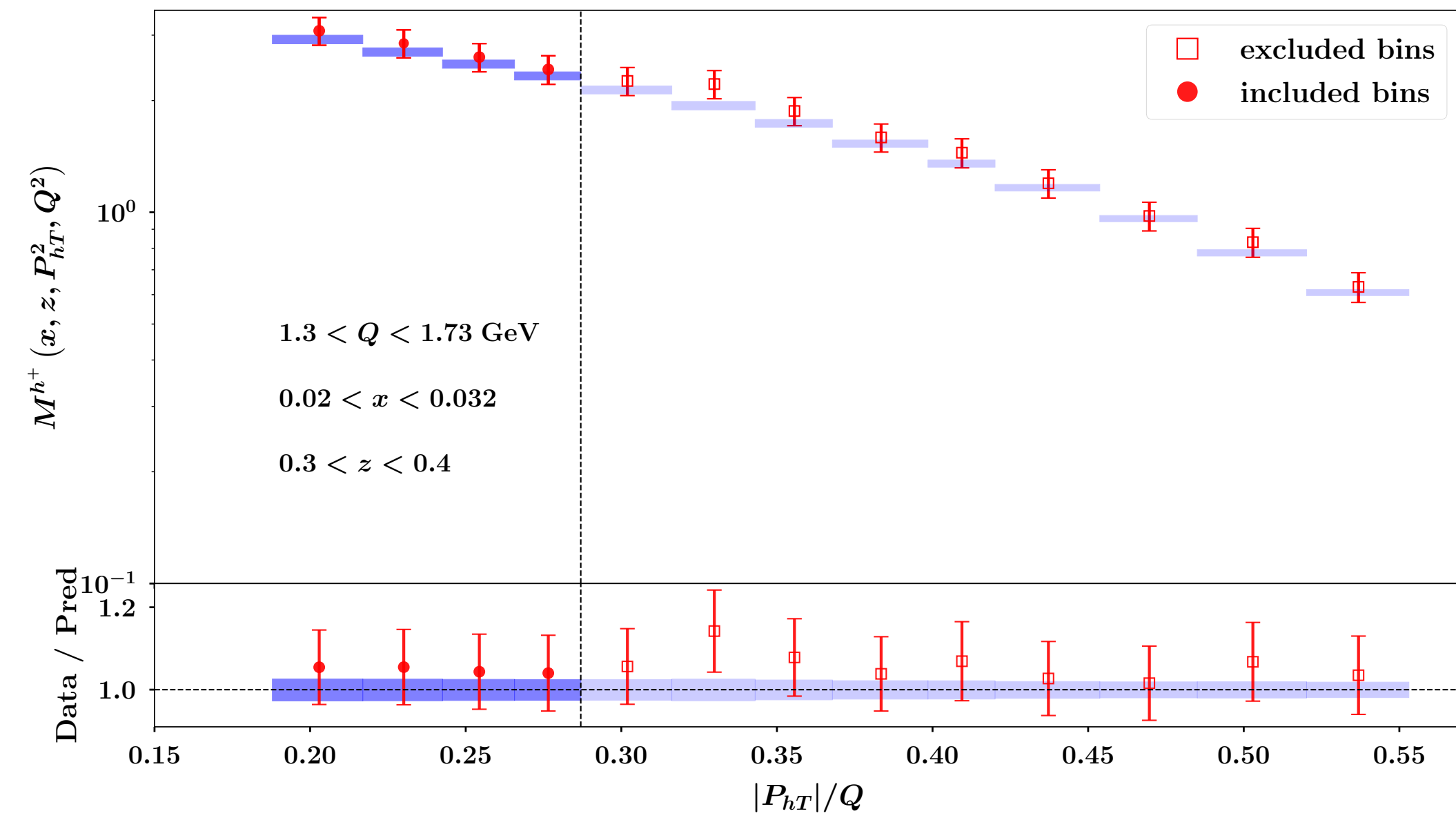
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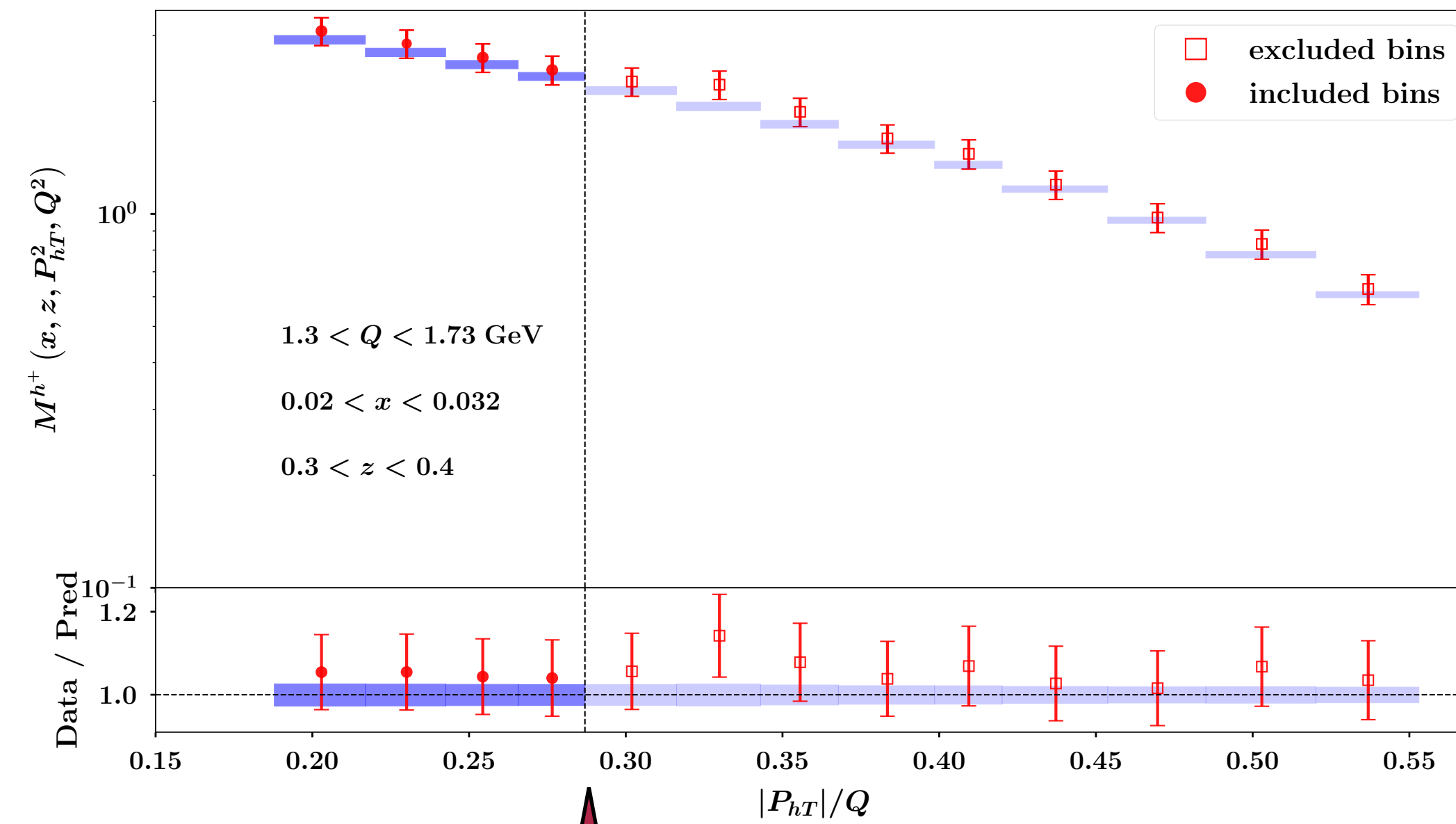
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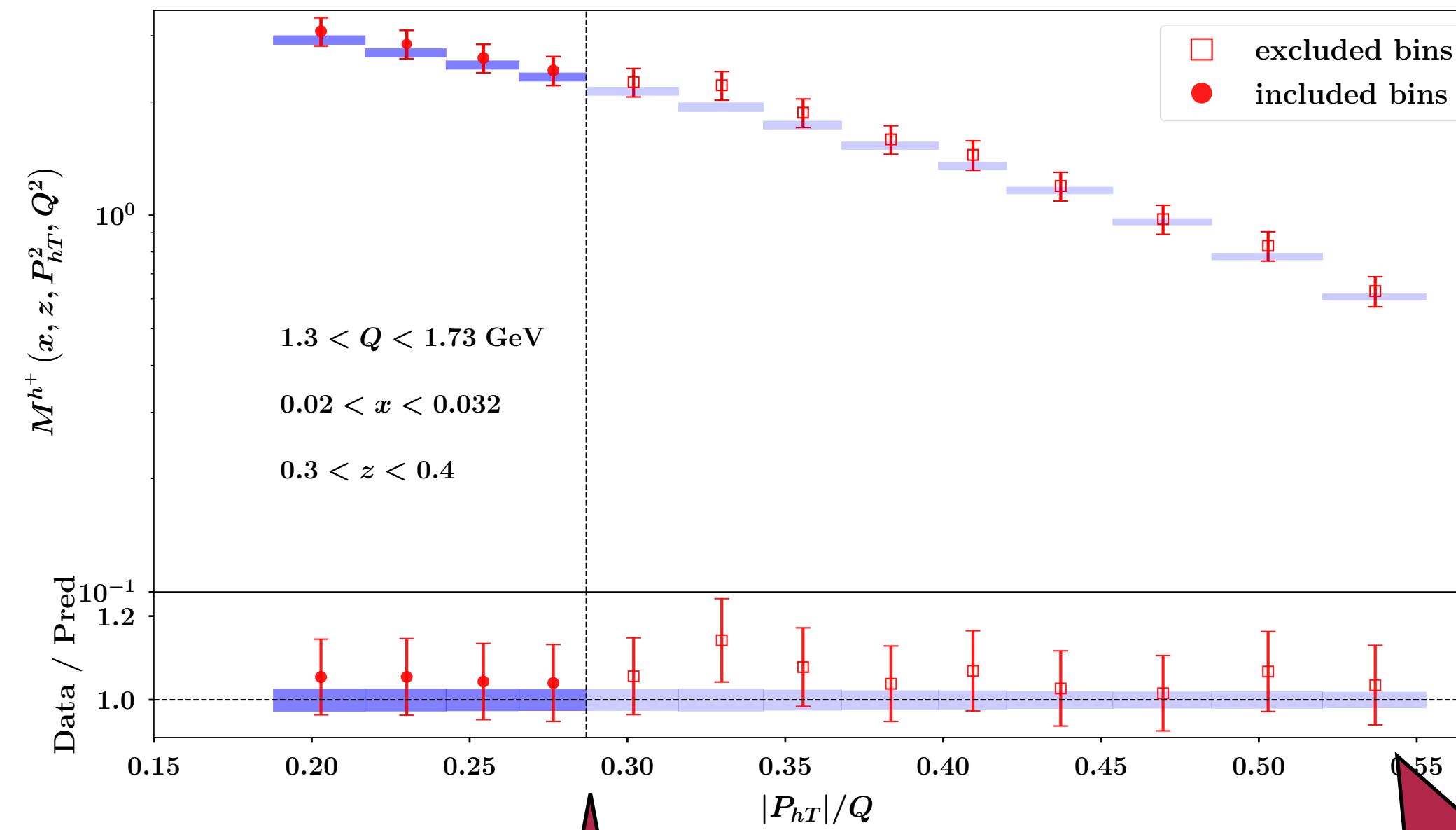


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MAP22 cut

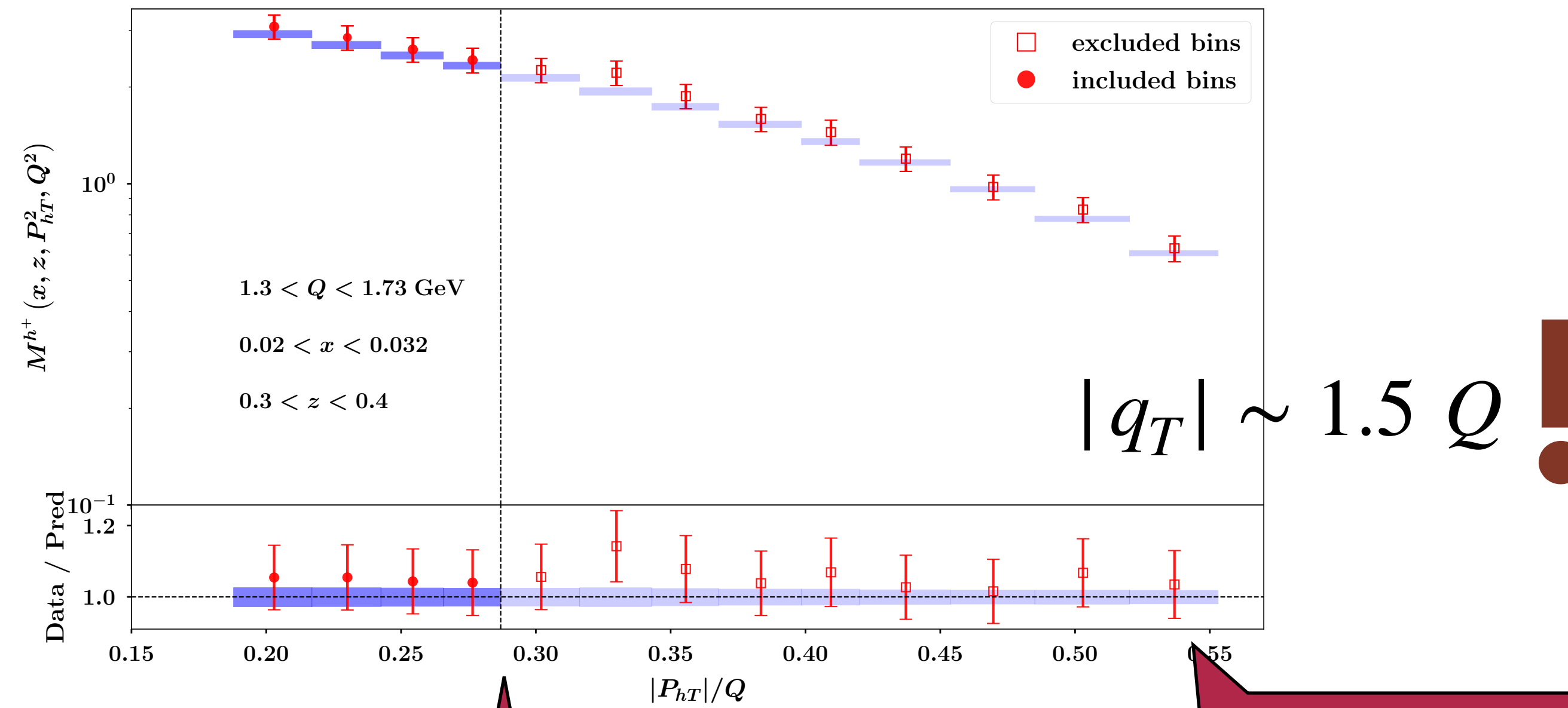
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MAP22 cut

MAP22
extrapolation

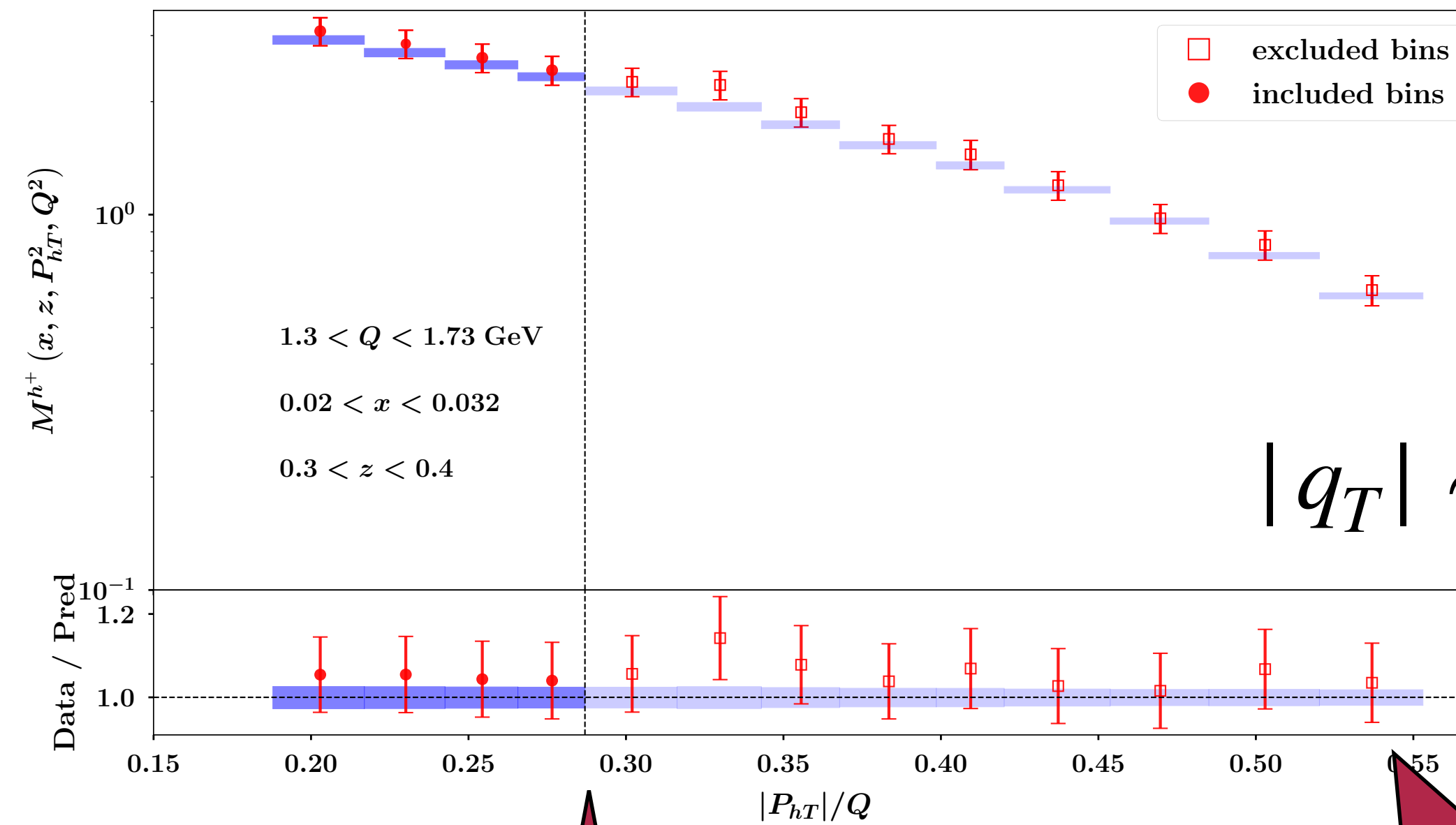
$$|q_T| = |P_{hT}|/z \ll Q$$



MAP22 cut

MAP22 extrapolation

$$|q_T| = |P_{hT}|/z \ll Q$$

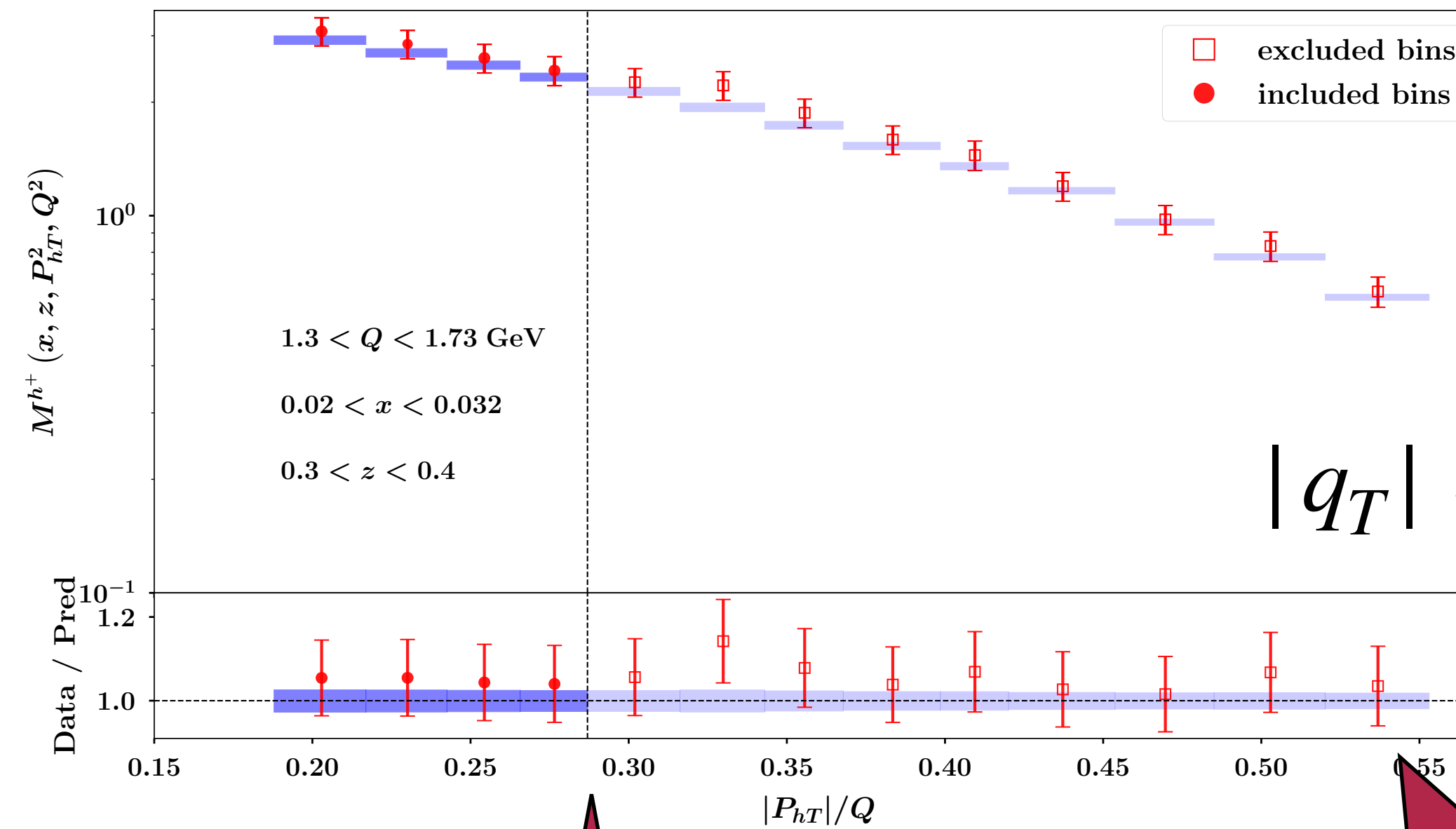


MAP22 cut

MAP22 extrapolation

The MAP22 cut is already considered to be “generous”, but the physics seems to be the same for a much wider transverse momentum

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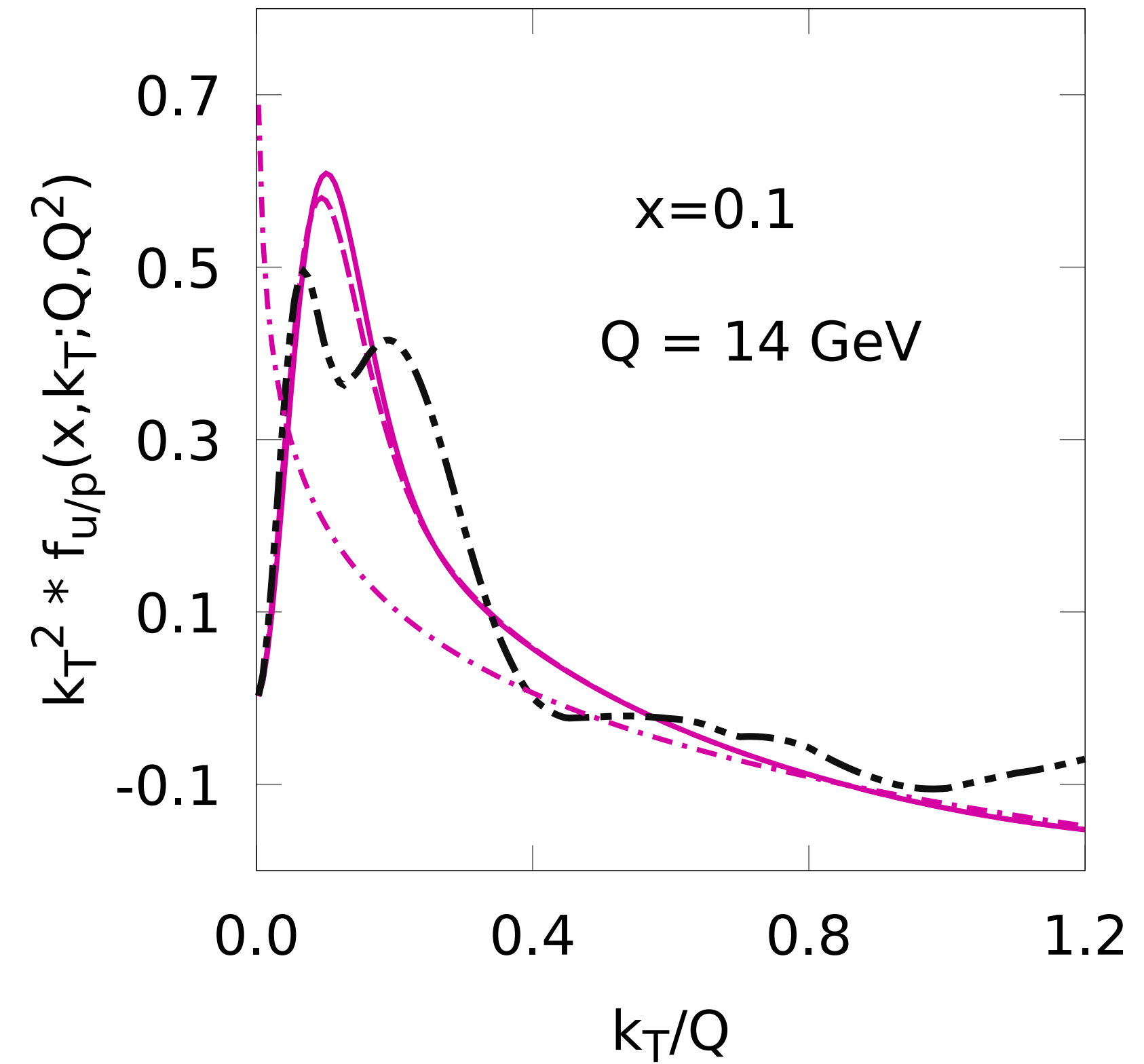
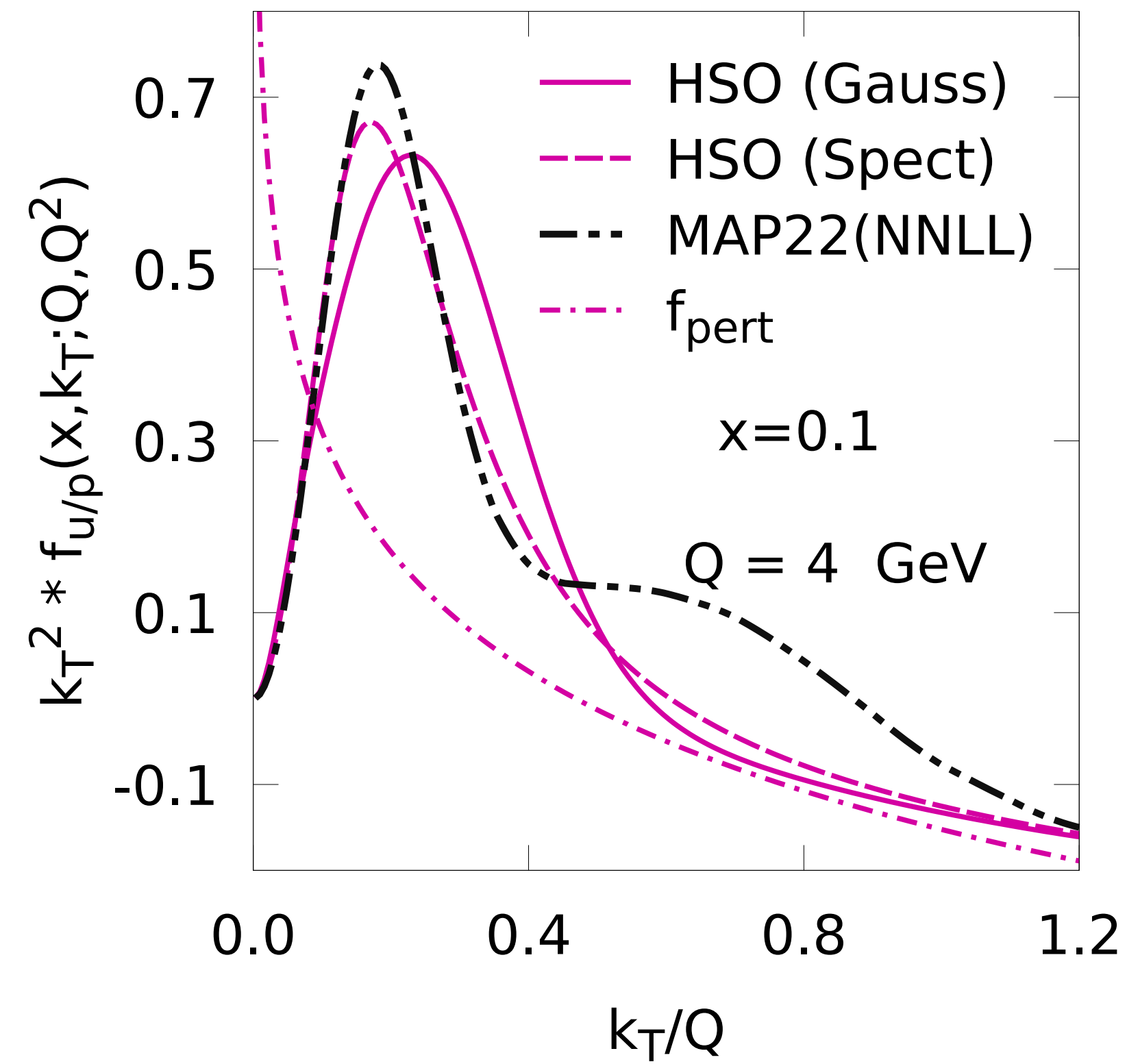
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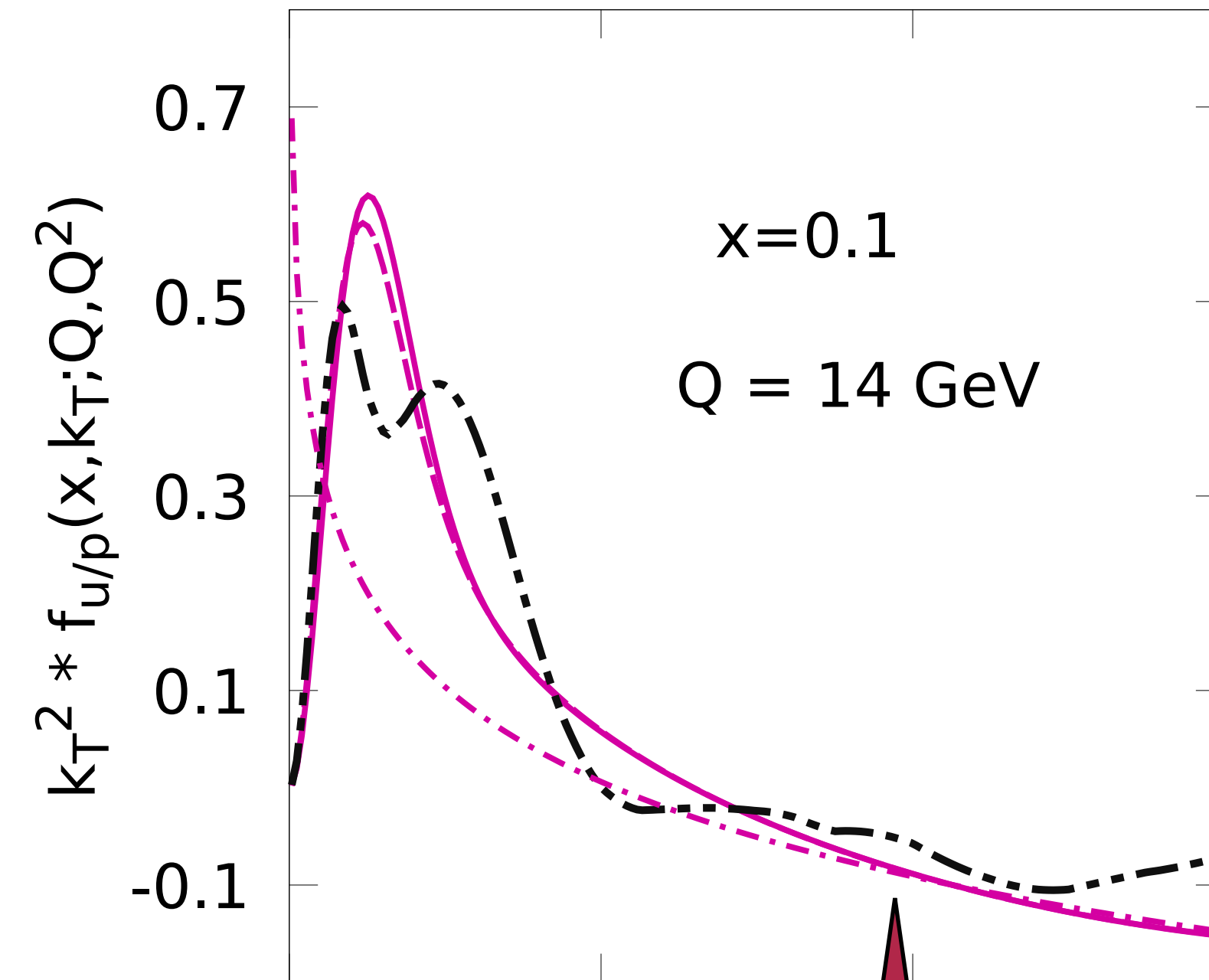
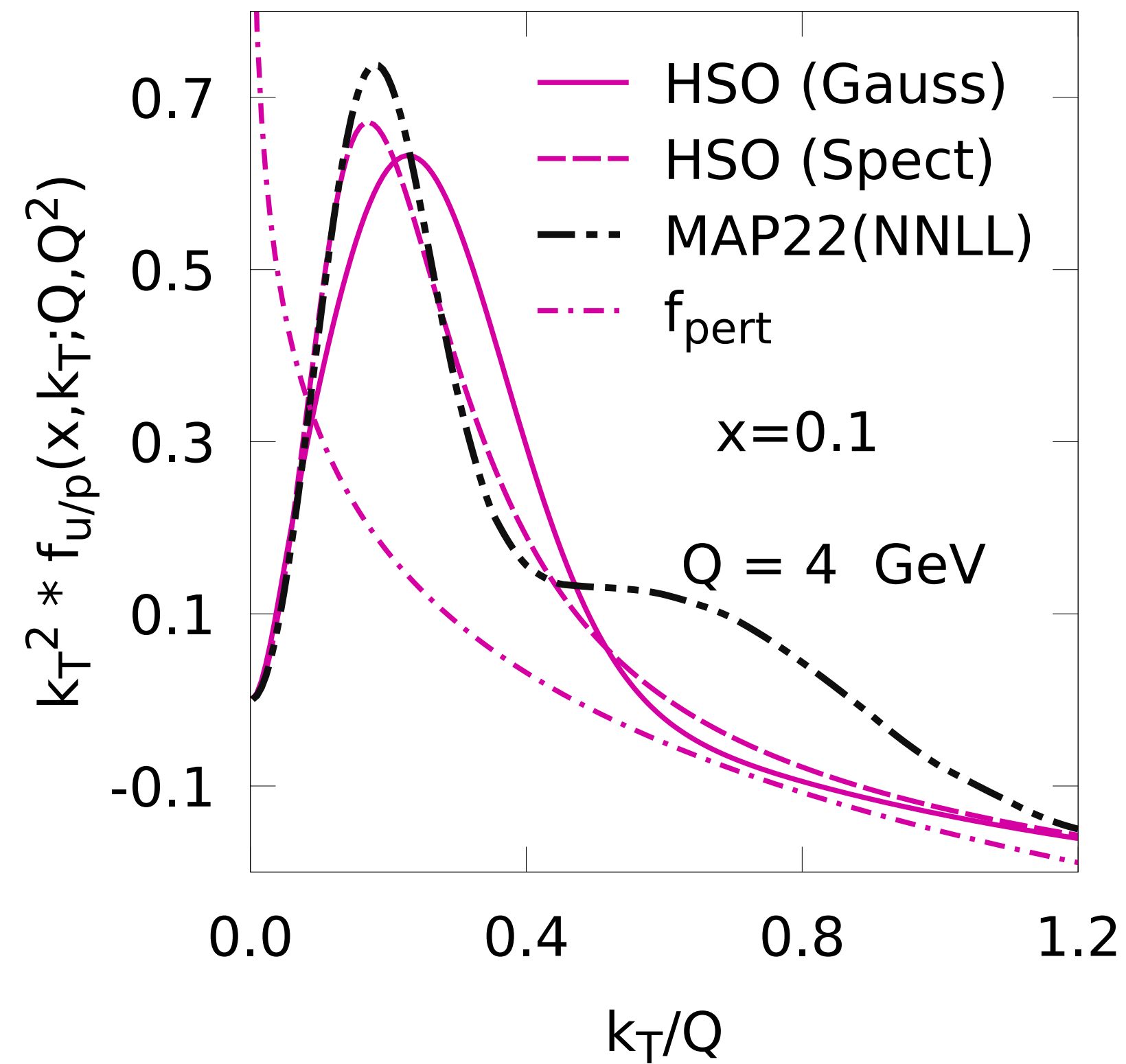
RECENT DISCUSSION ABOUT “HSO” APPROACH

Aslan, Boglione, Gonzalez-Hernandez, Rainaldi, Rogers, Simonelli, 2401.14266



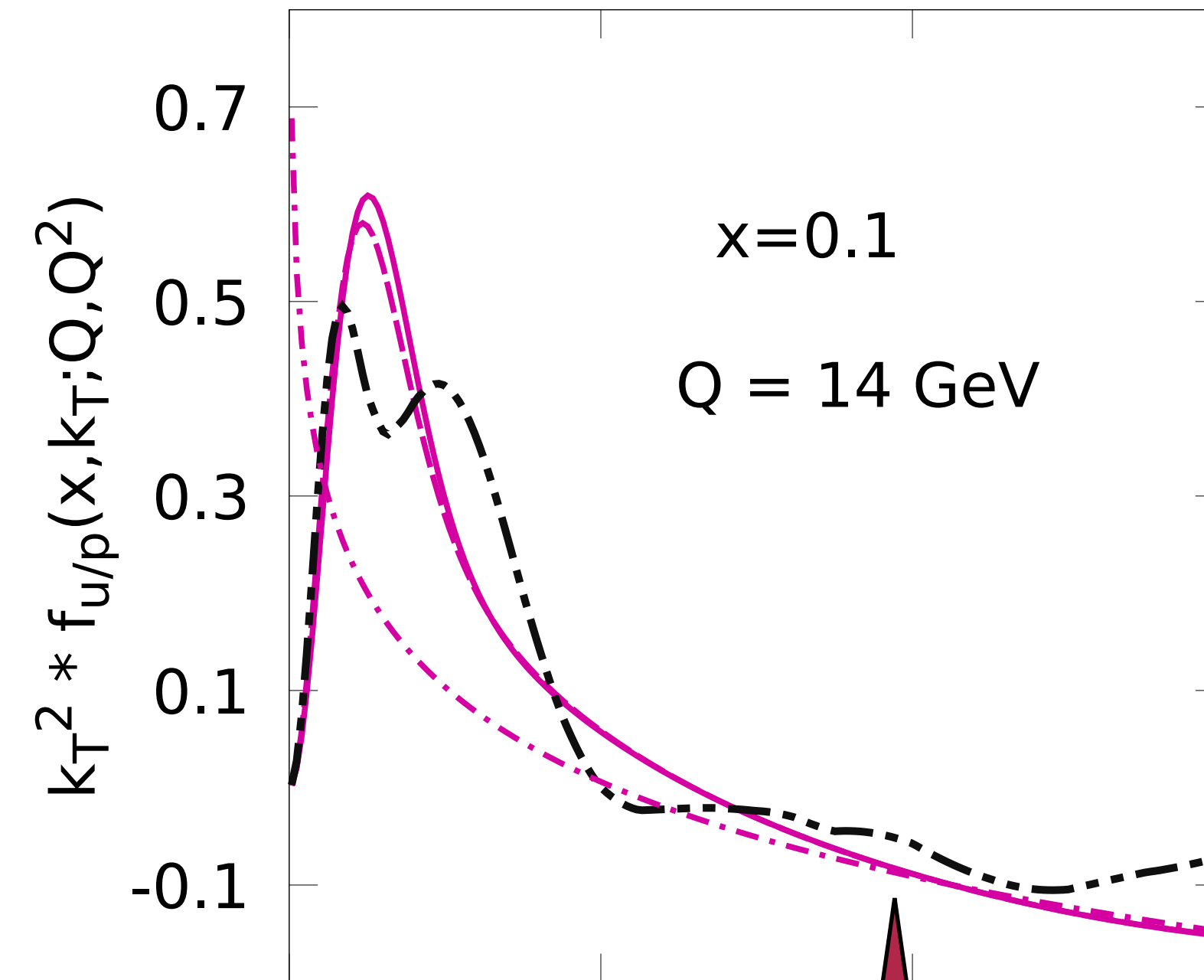
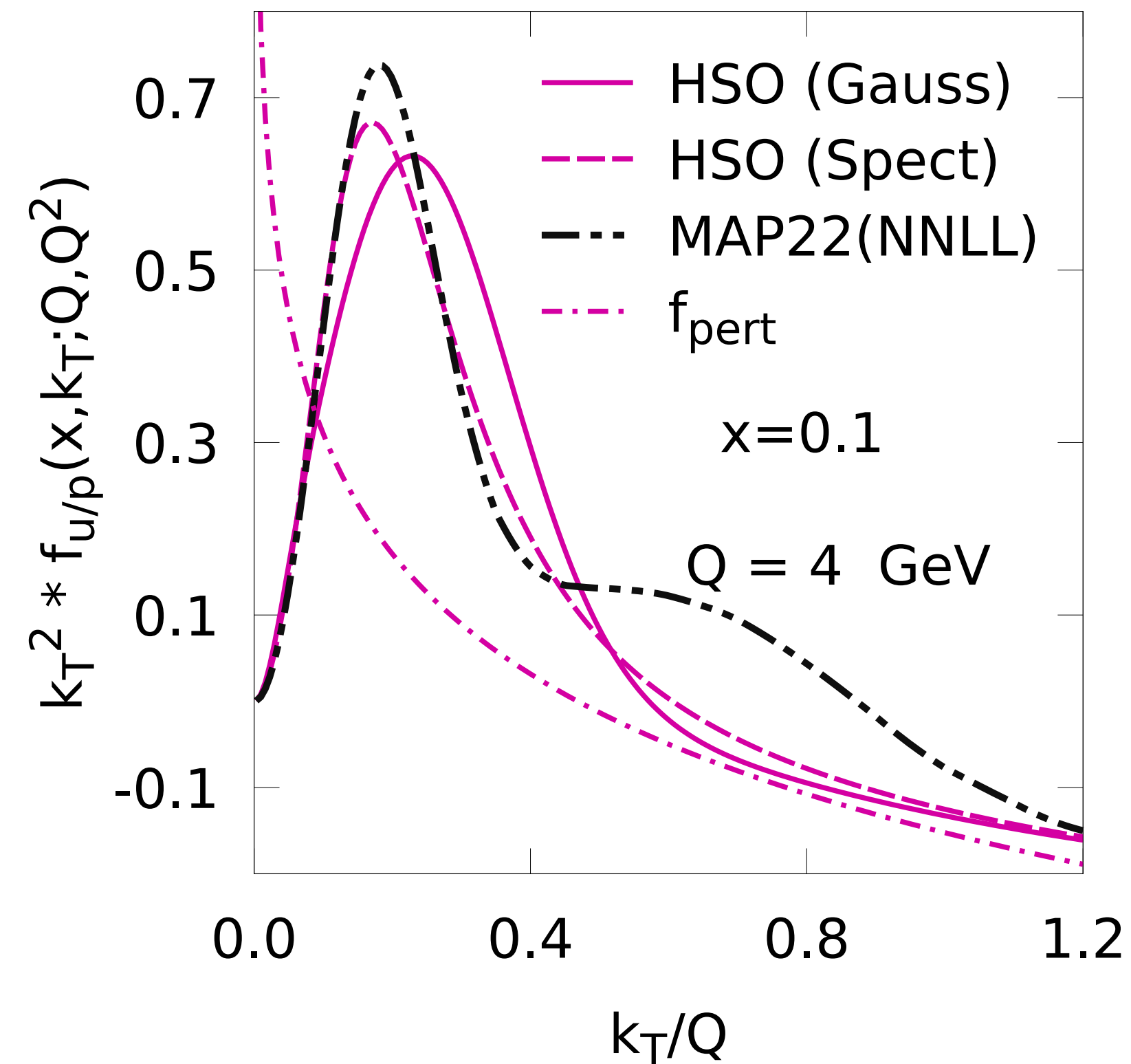
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The k_T^2 weighing exposes the tails

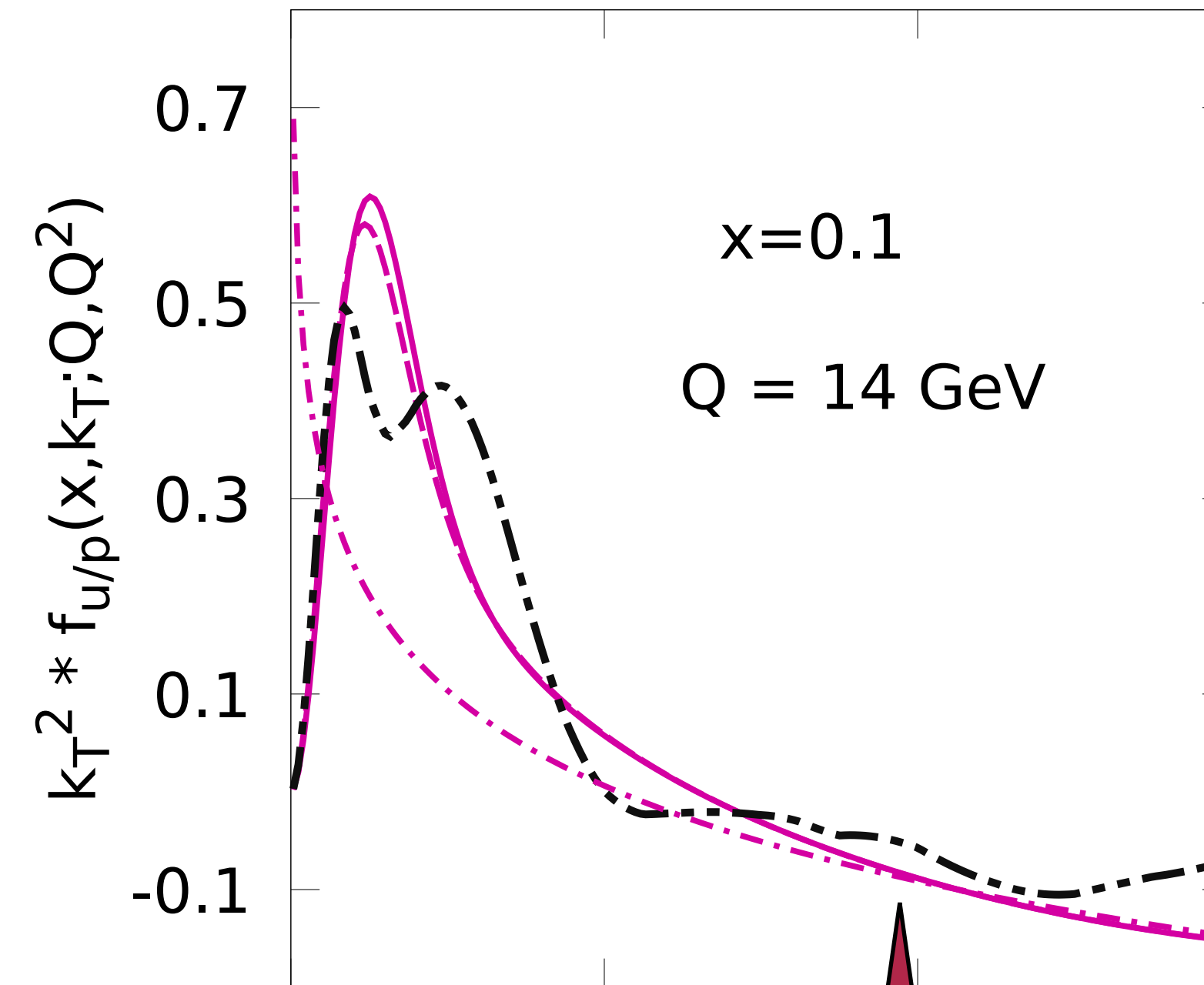
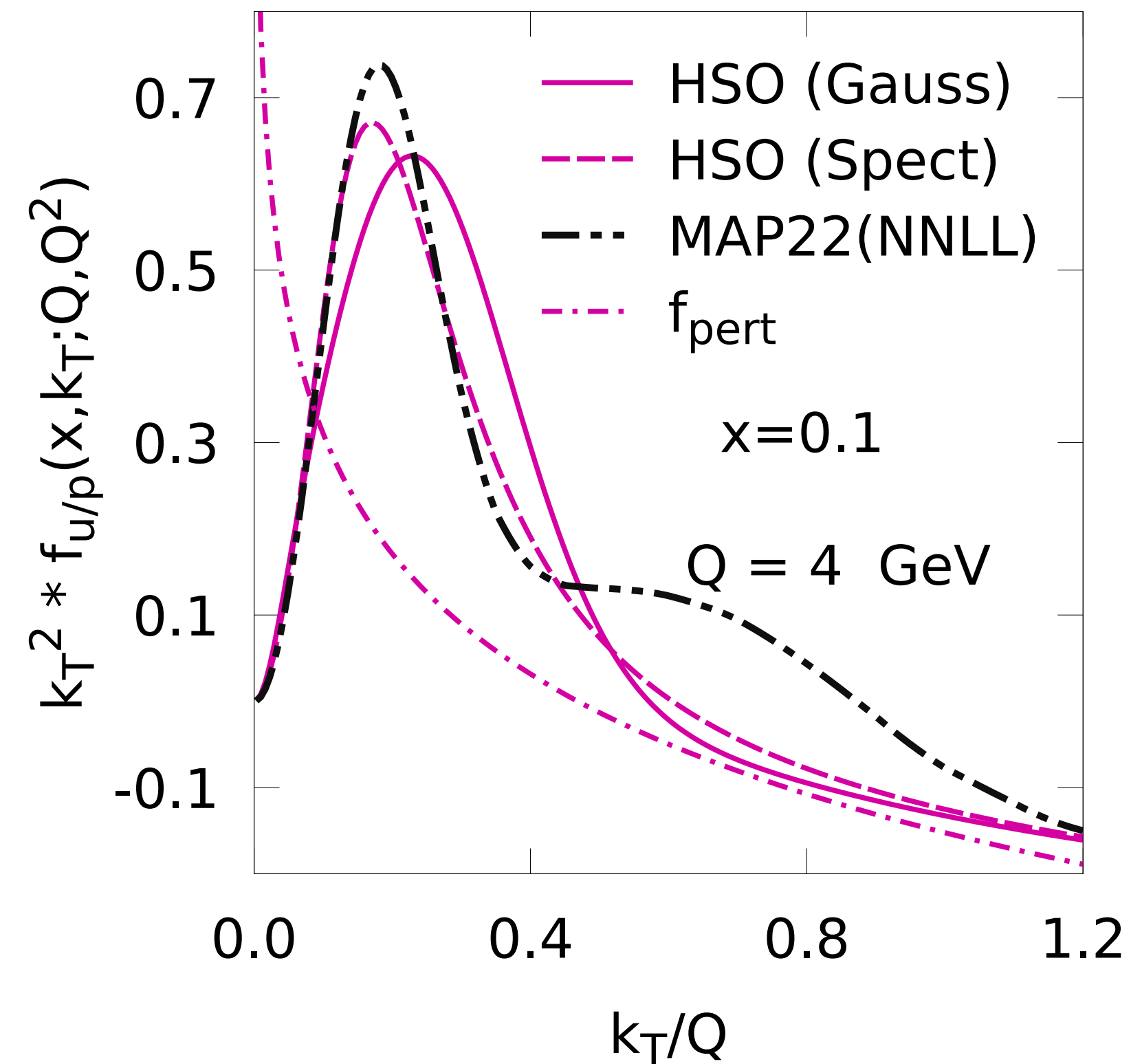
[Aslan, Boglione, Gonzalez-Hernandez, Rainaldi, Rogers, Simonelli, 2401.14266](#)



The k_T^2 weighing exposes the tails

The paper emphasizes the relevance of prescription choices and simultaneous TMD-PDF fit, but does not provide a fit to extended data sets.

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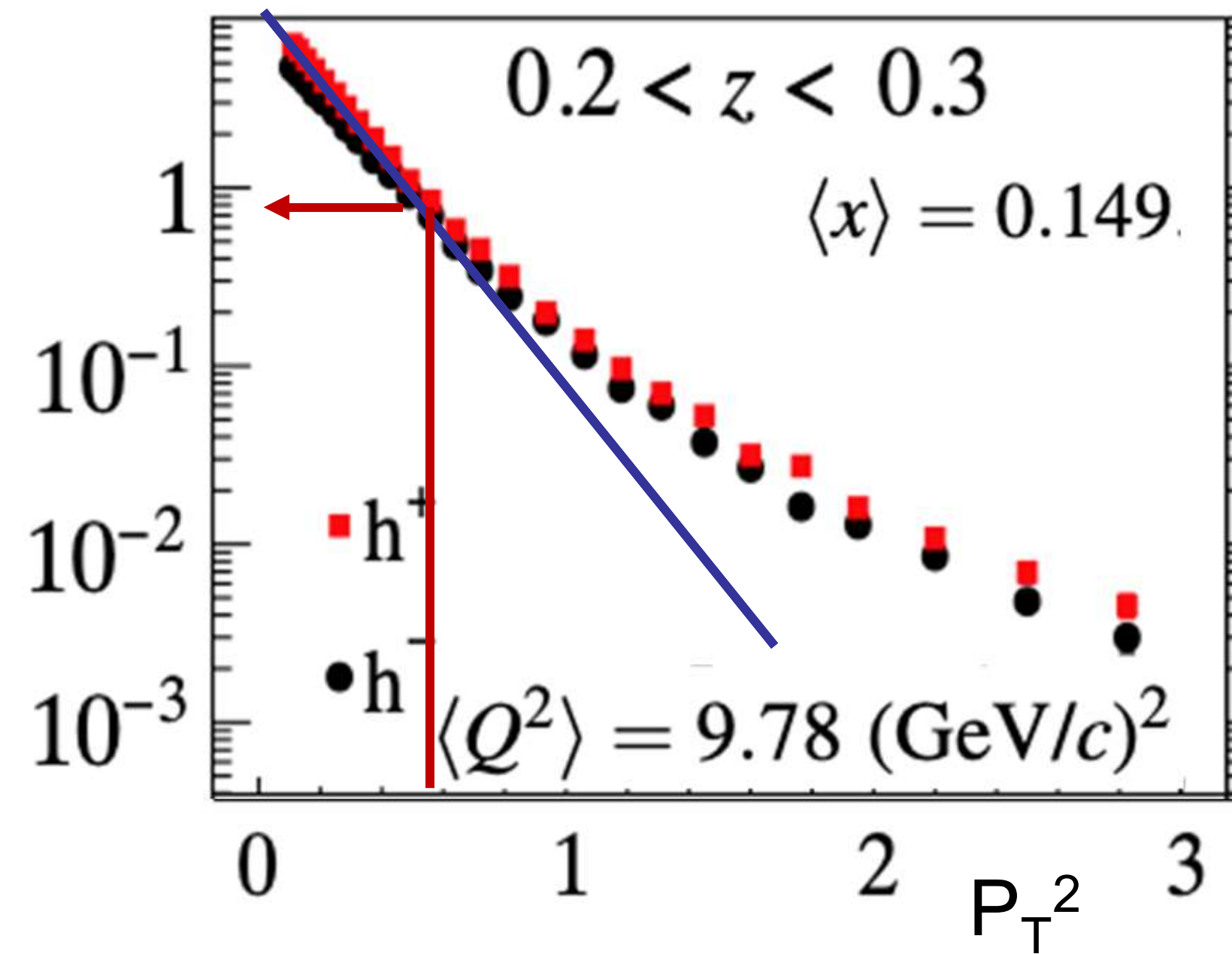
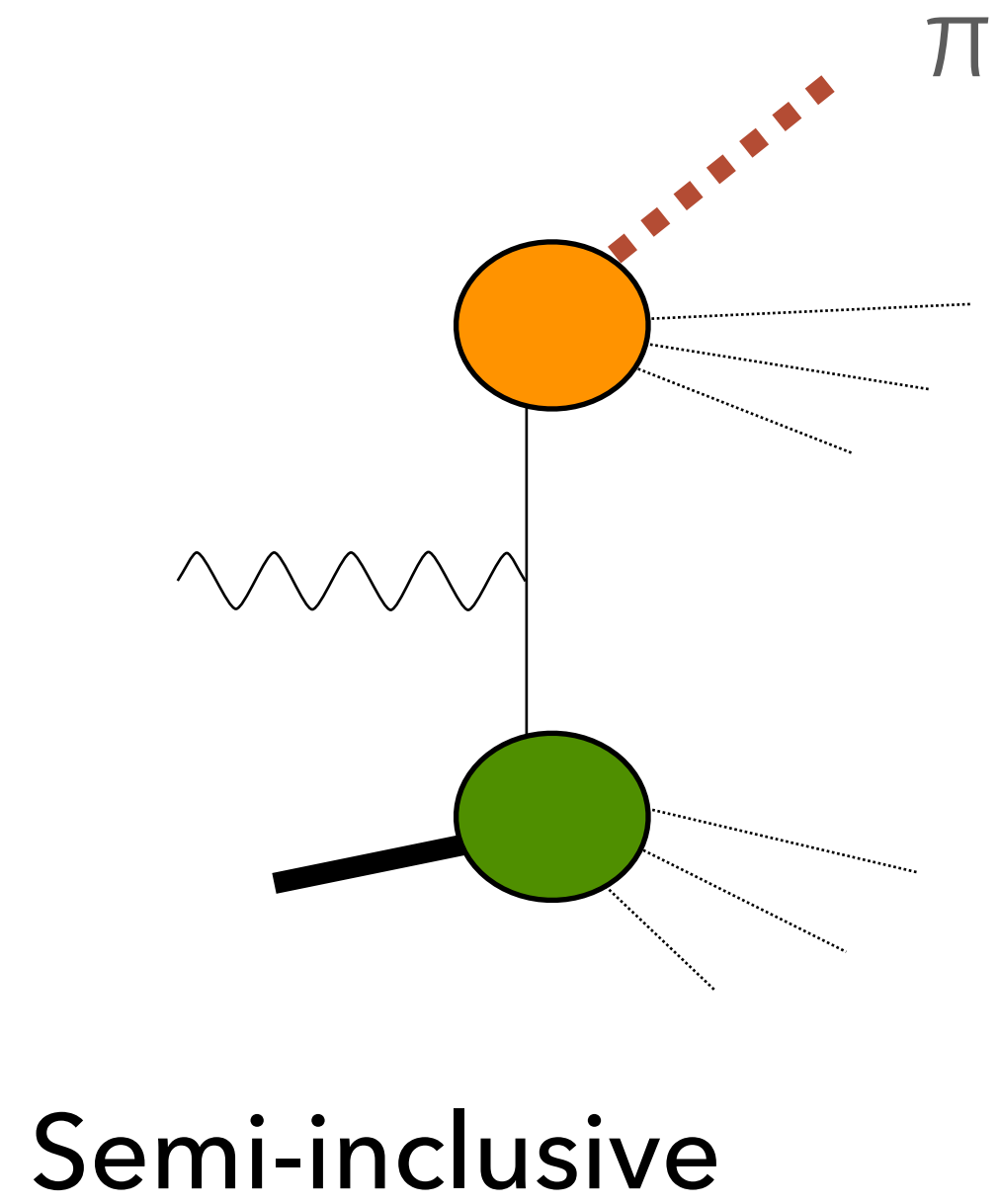


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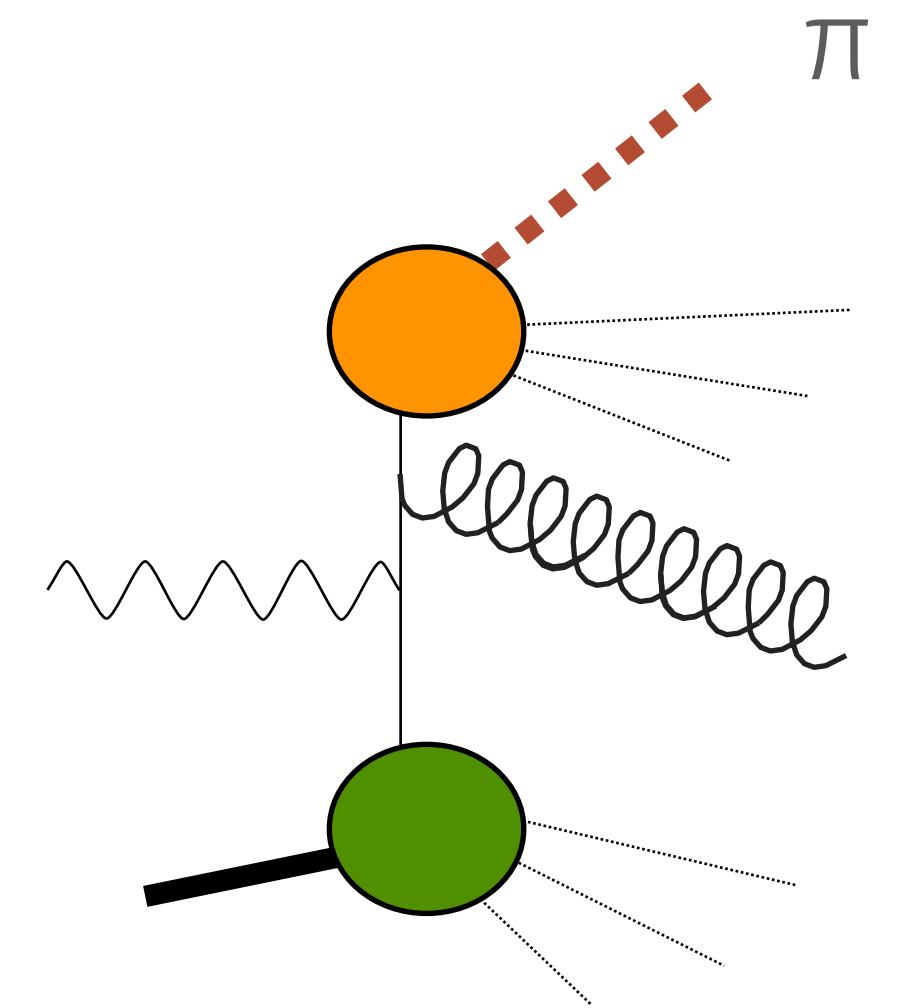
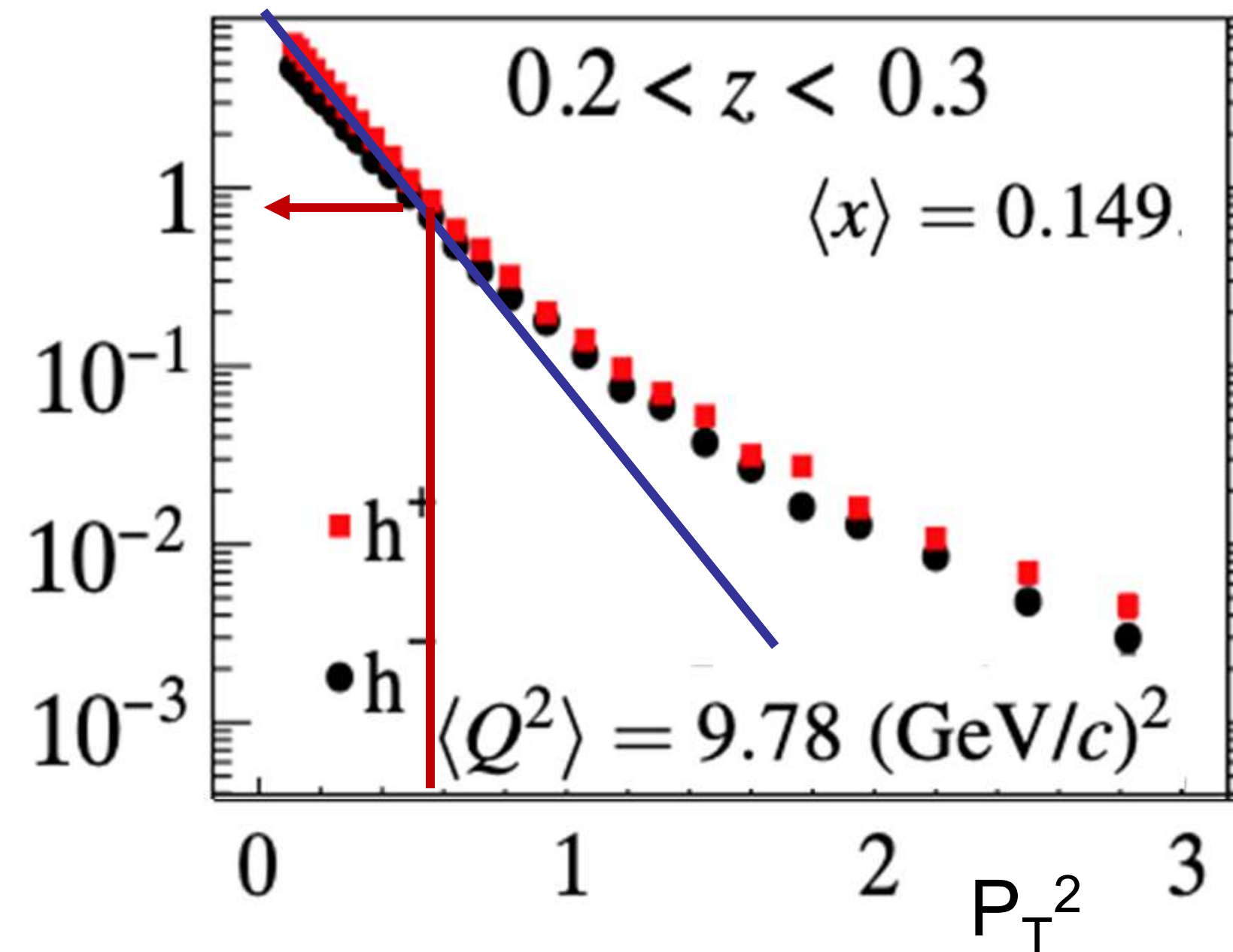
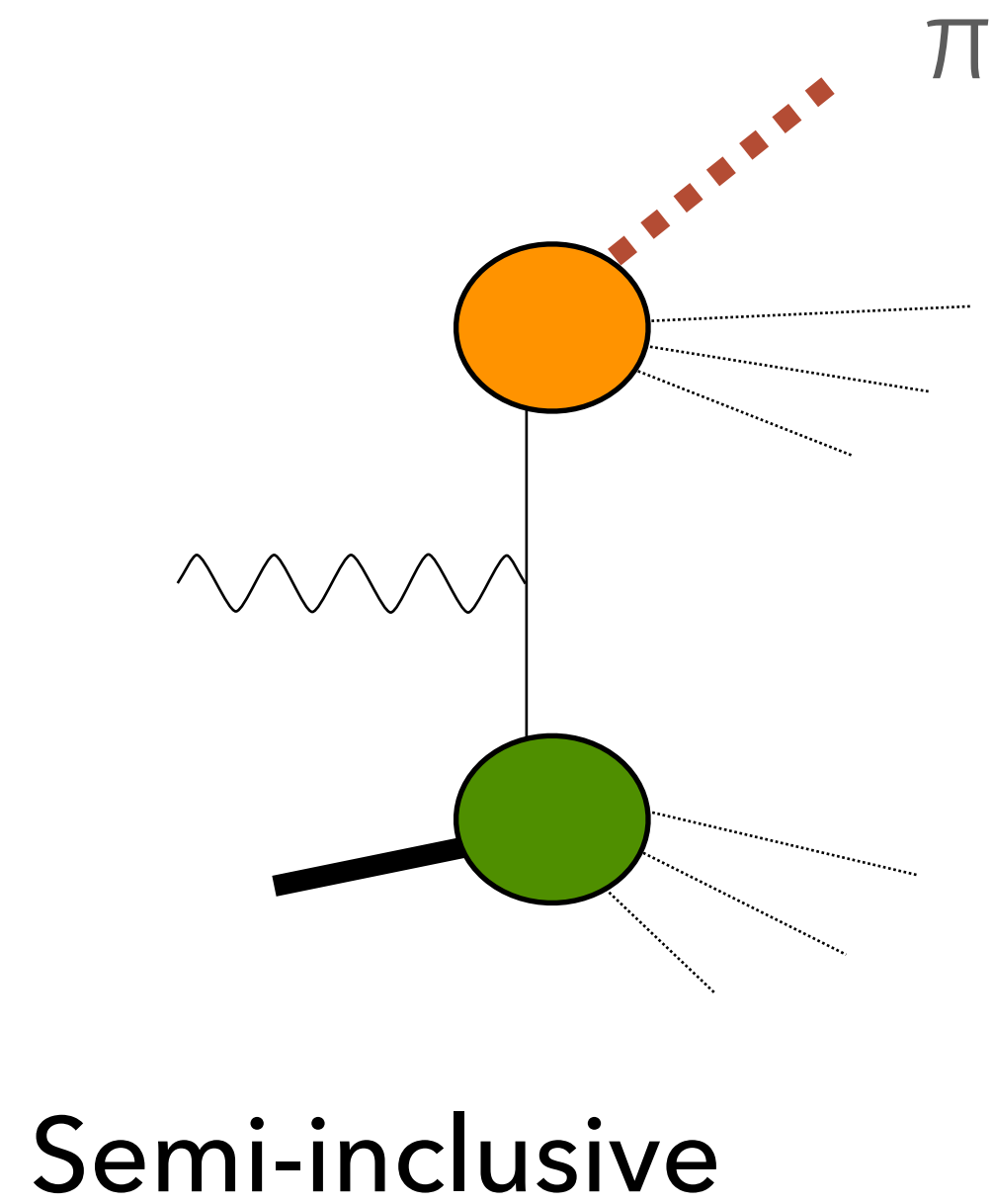
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See talk by Tommaso Rainaldi

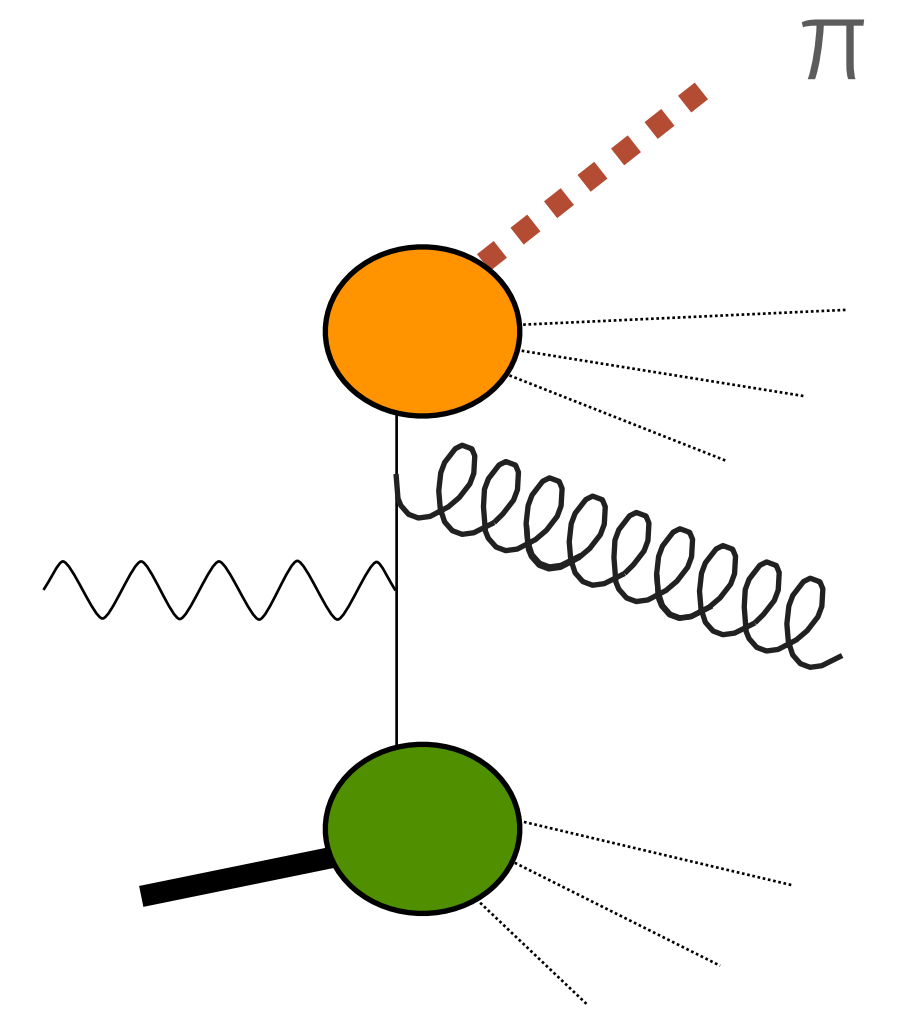
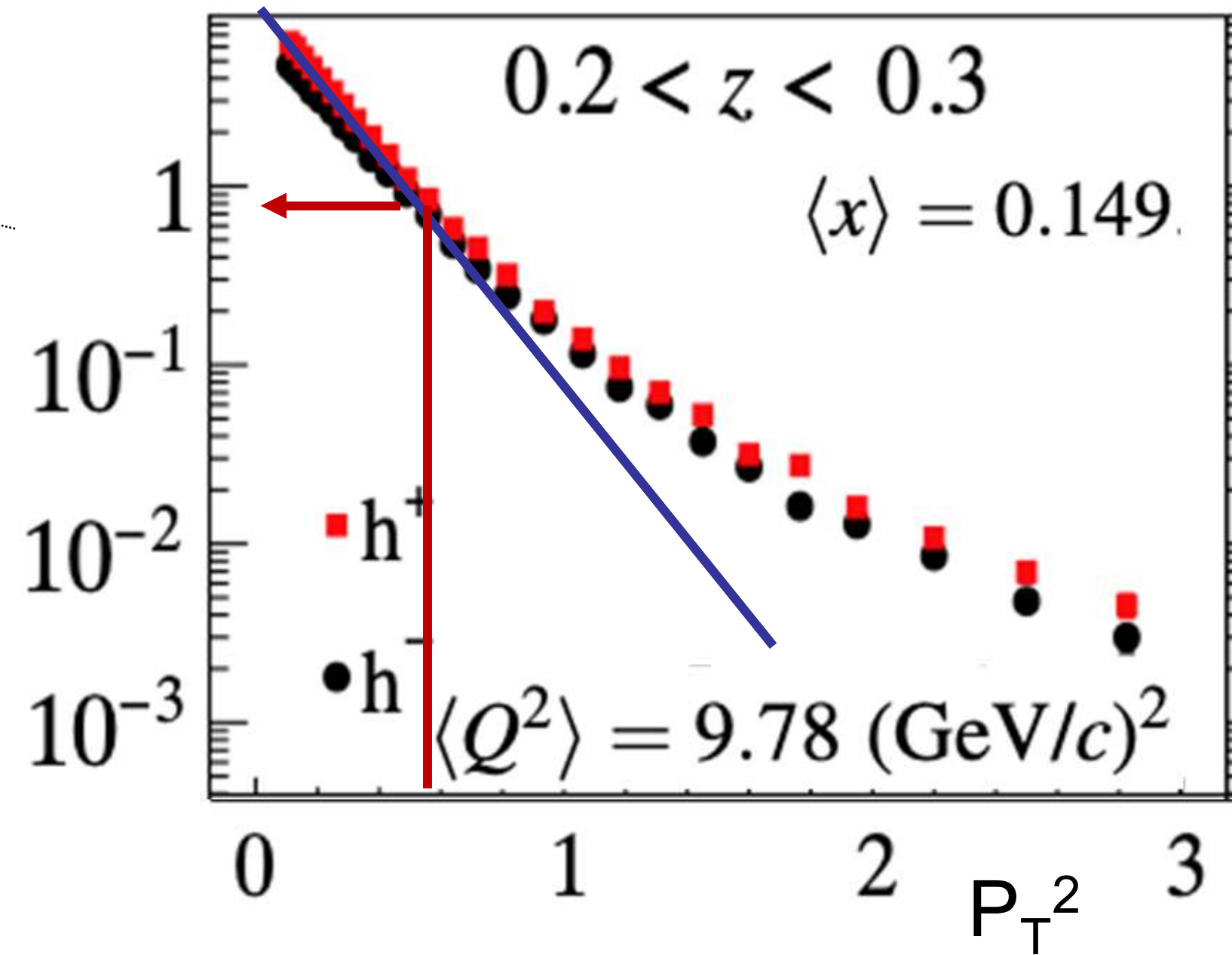
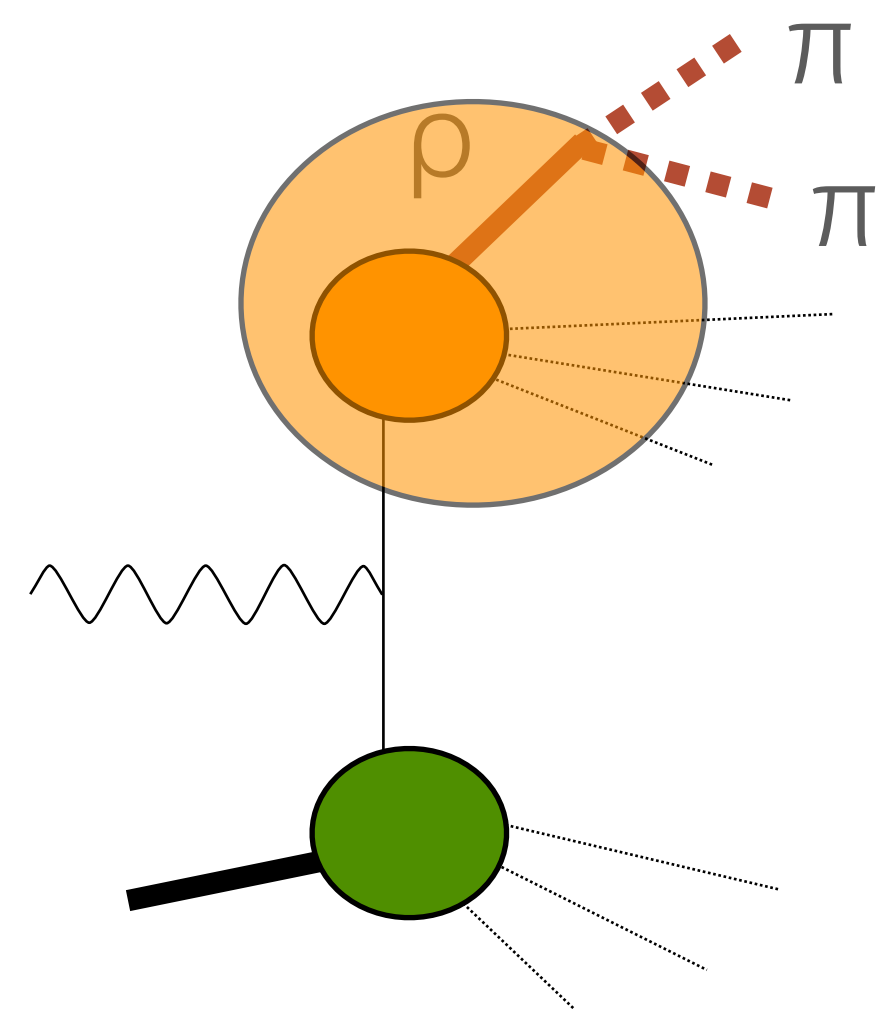
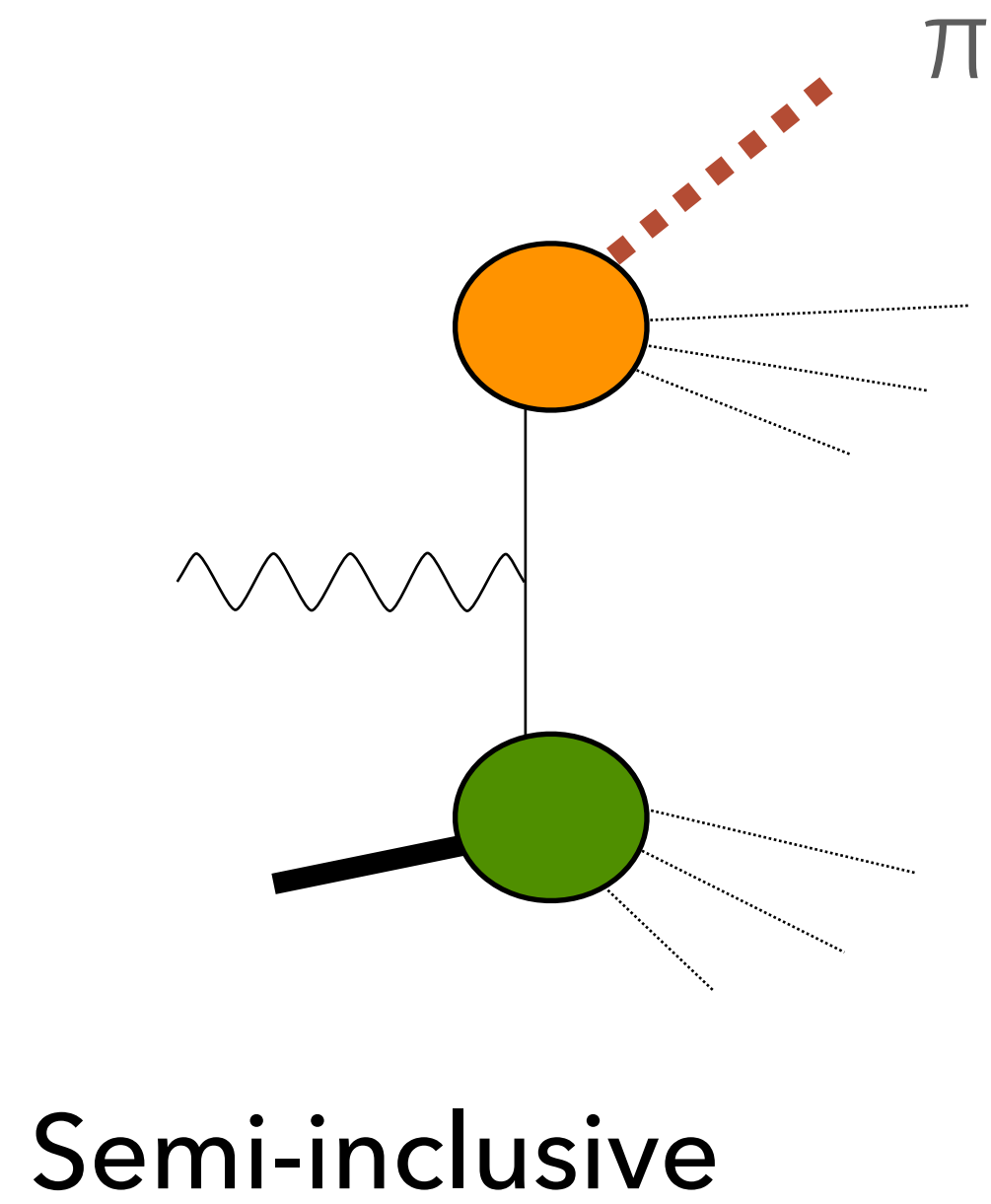
VECTOR MESON CONTAMINATIONS



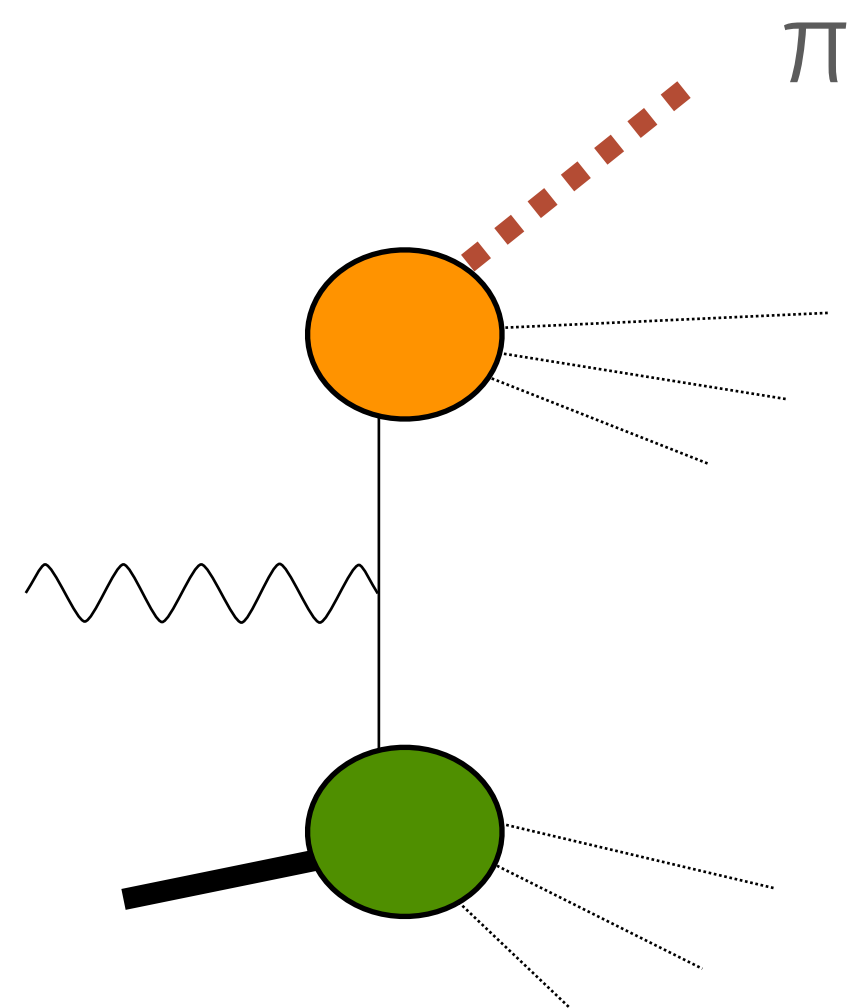
VECTOR MESON CONTAMINATIONS



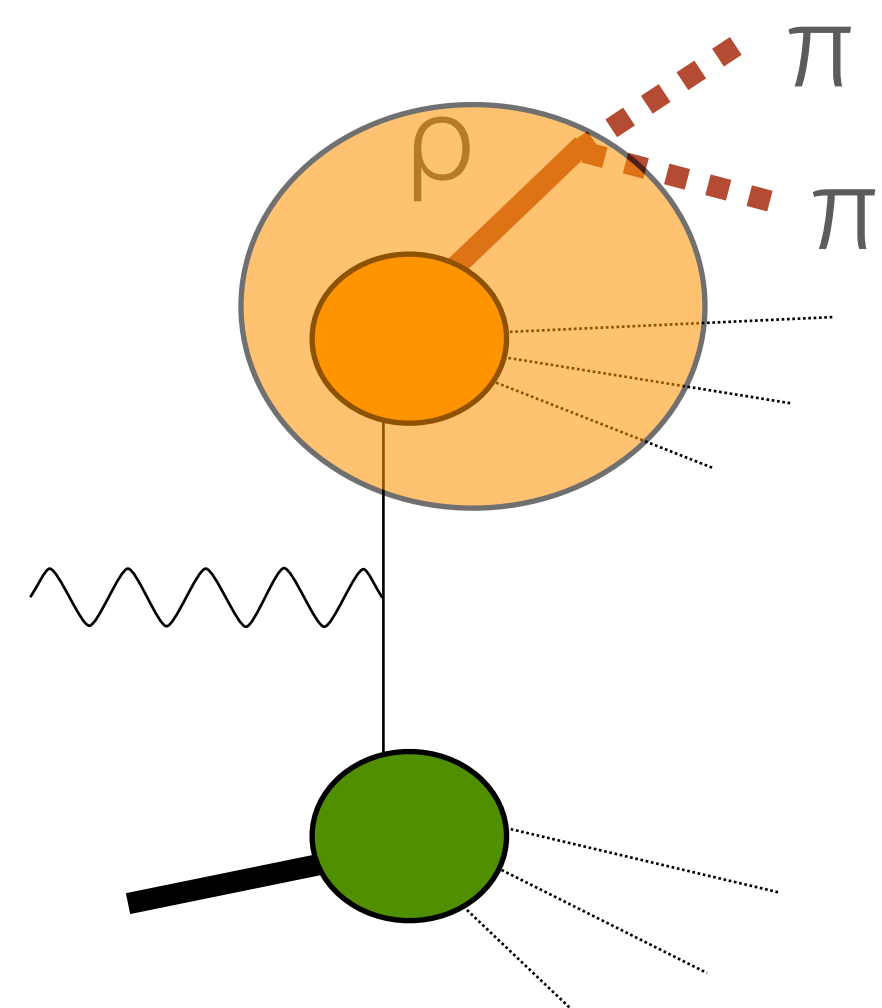
VECTOR MESON CONTAMINATIONS



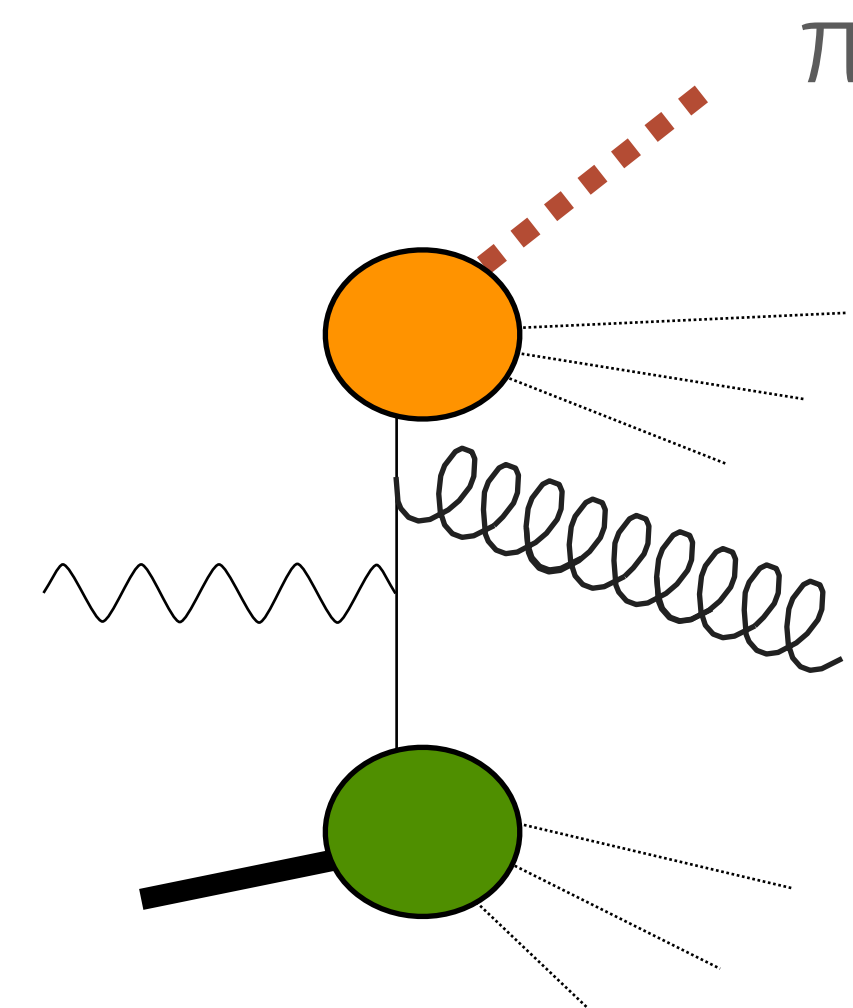
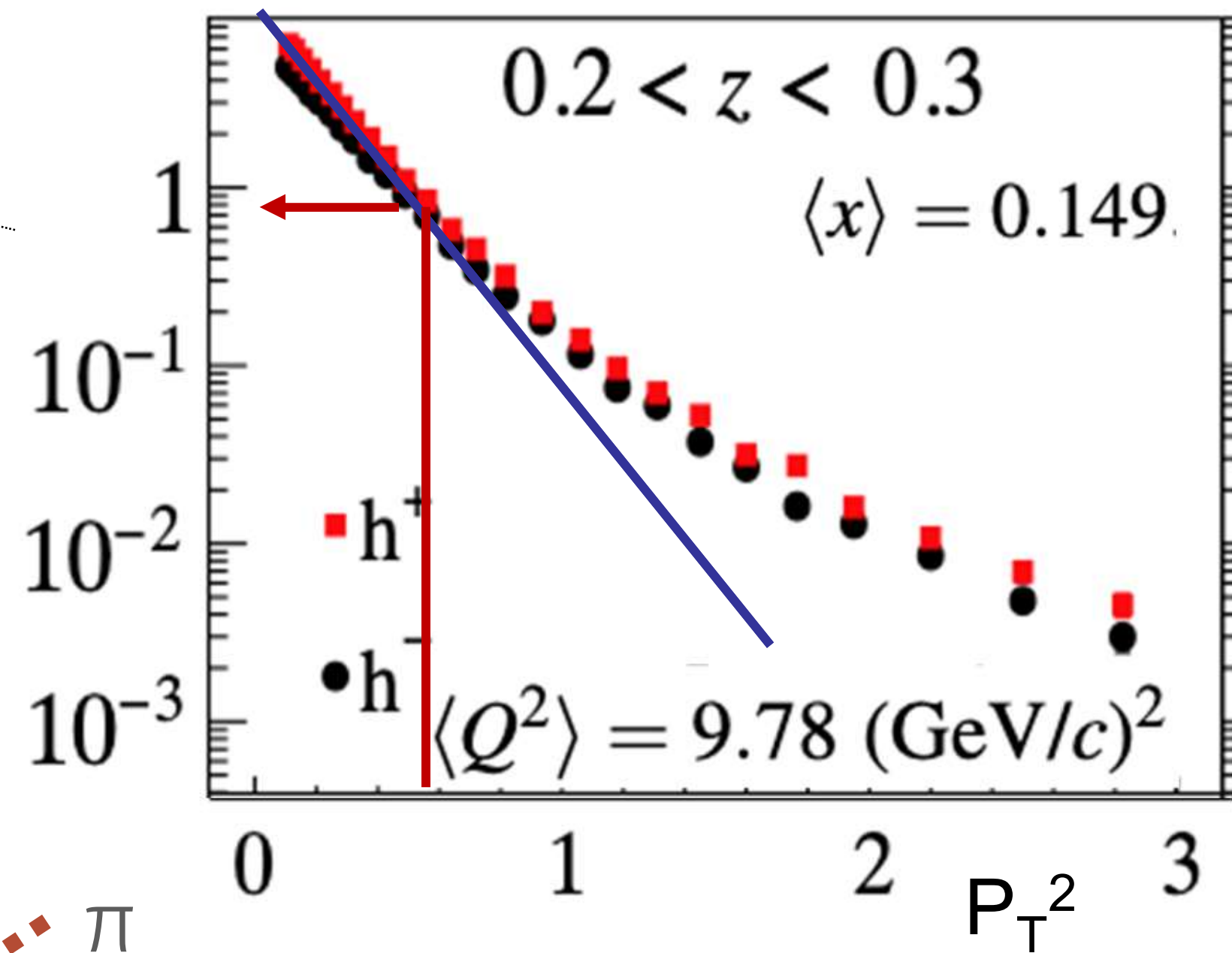
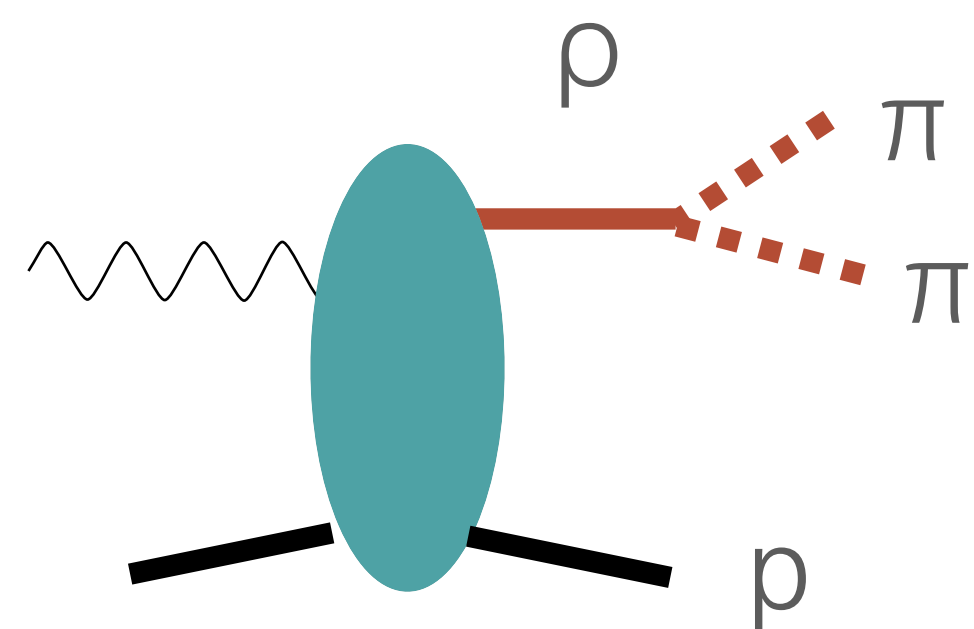
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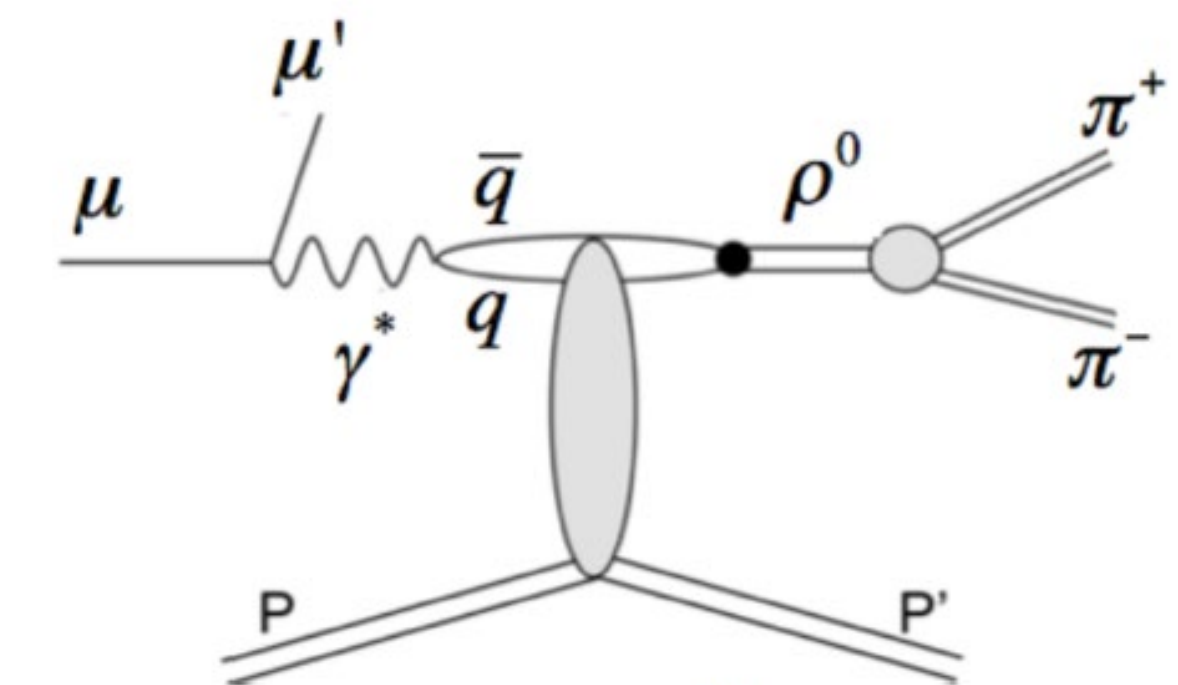
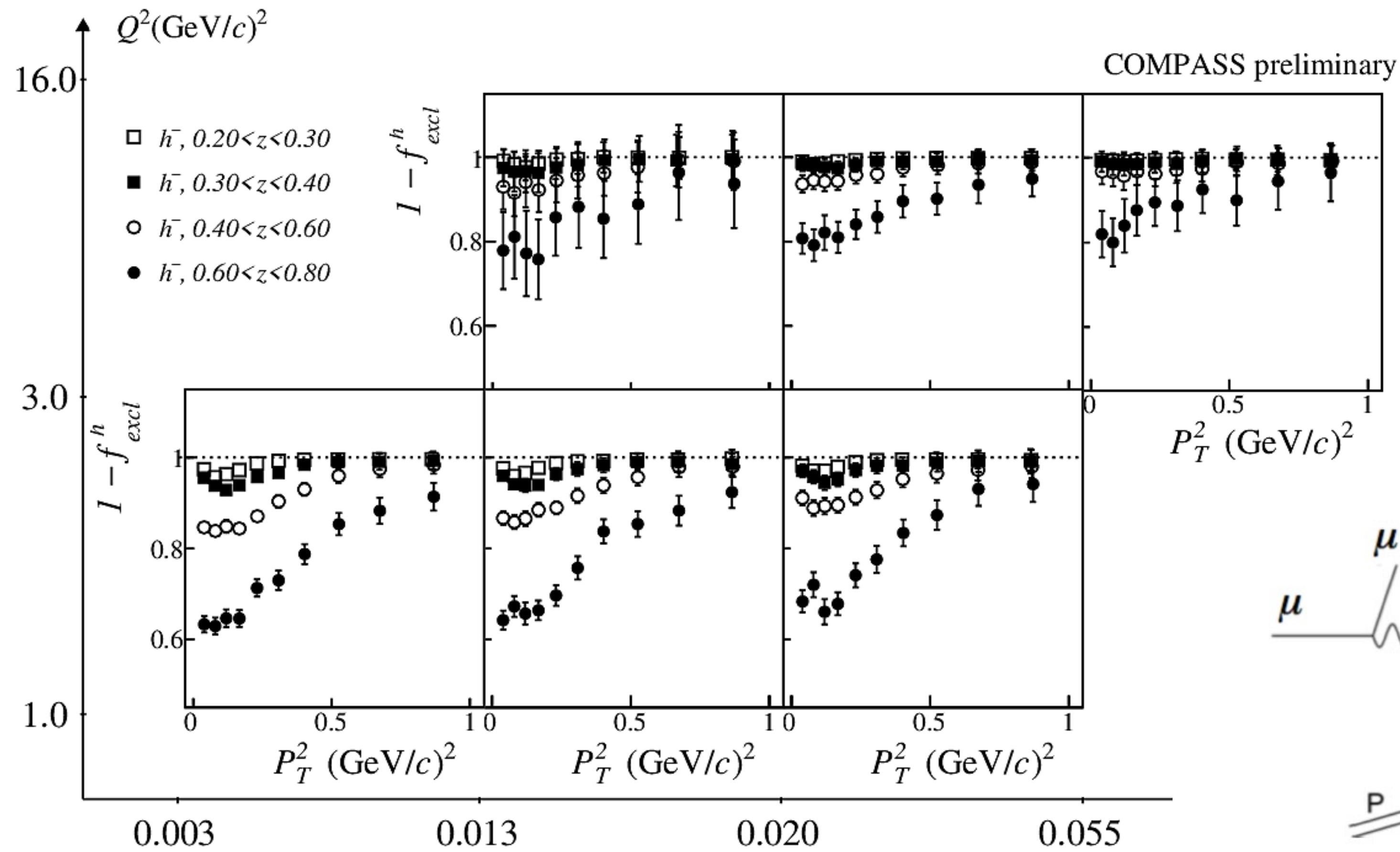
Semi-inclusive



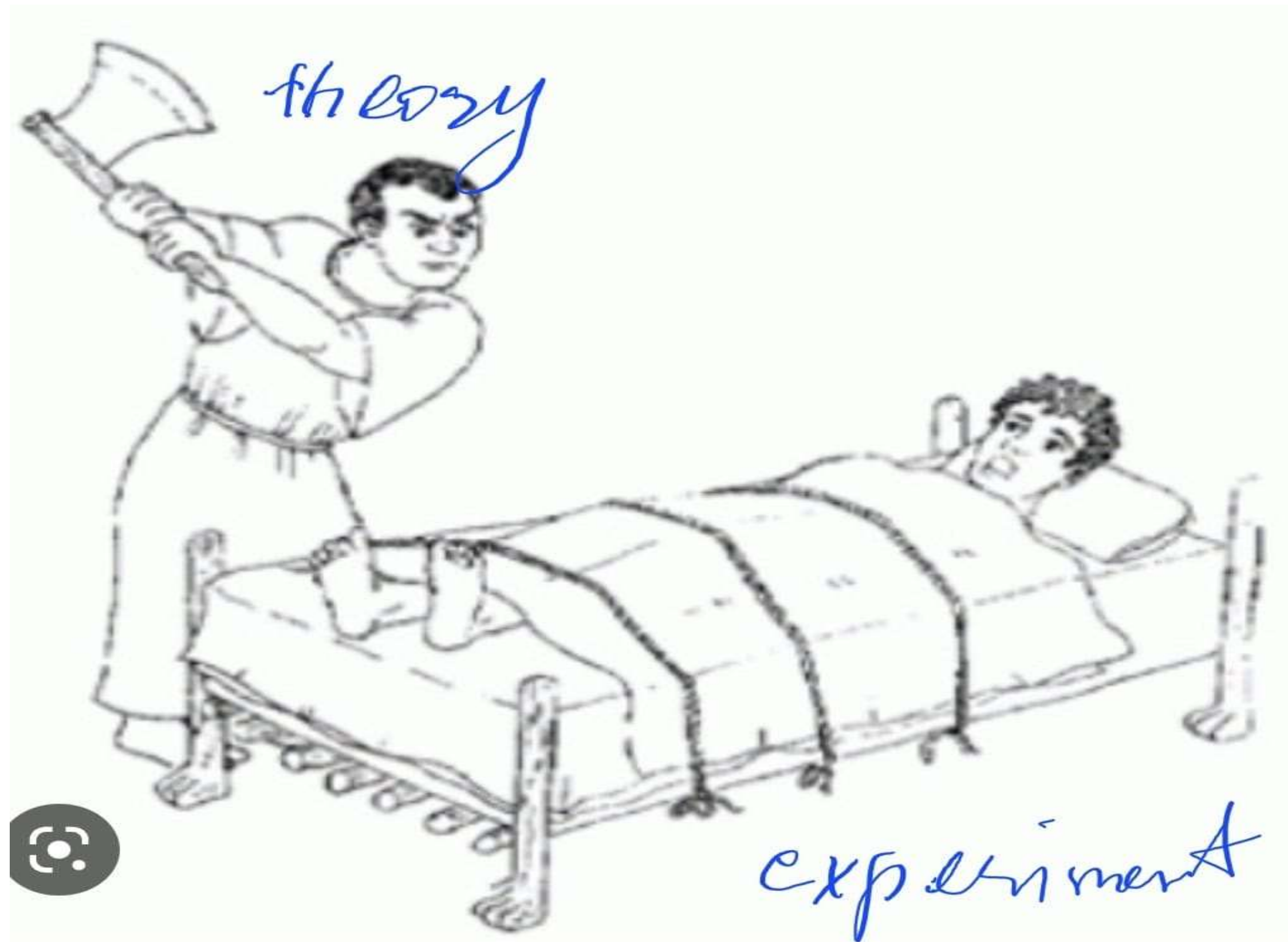
Exclusive
(in principle suppressed)



[A. Bressan's talk at Transversity 2024](#)

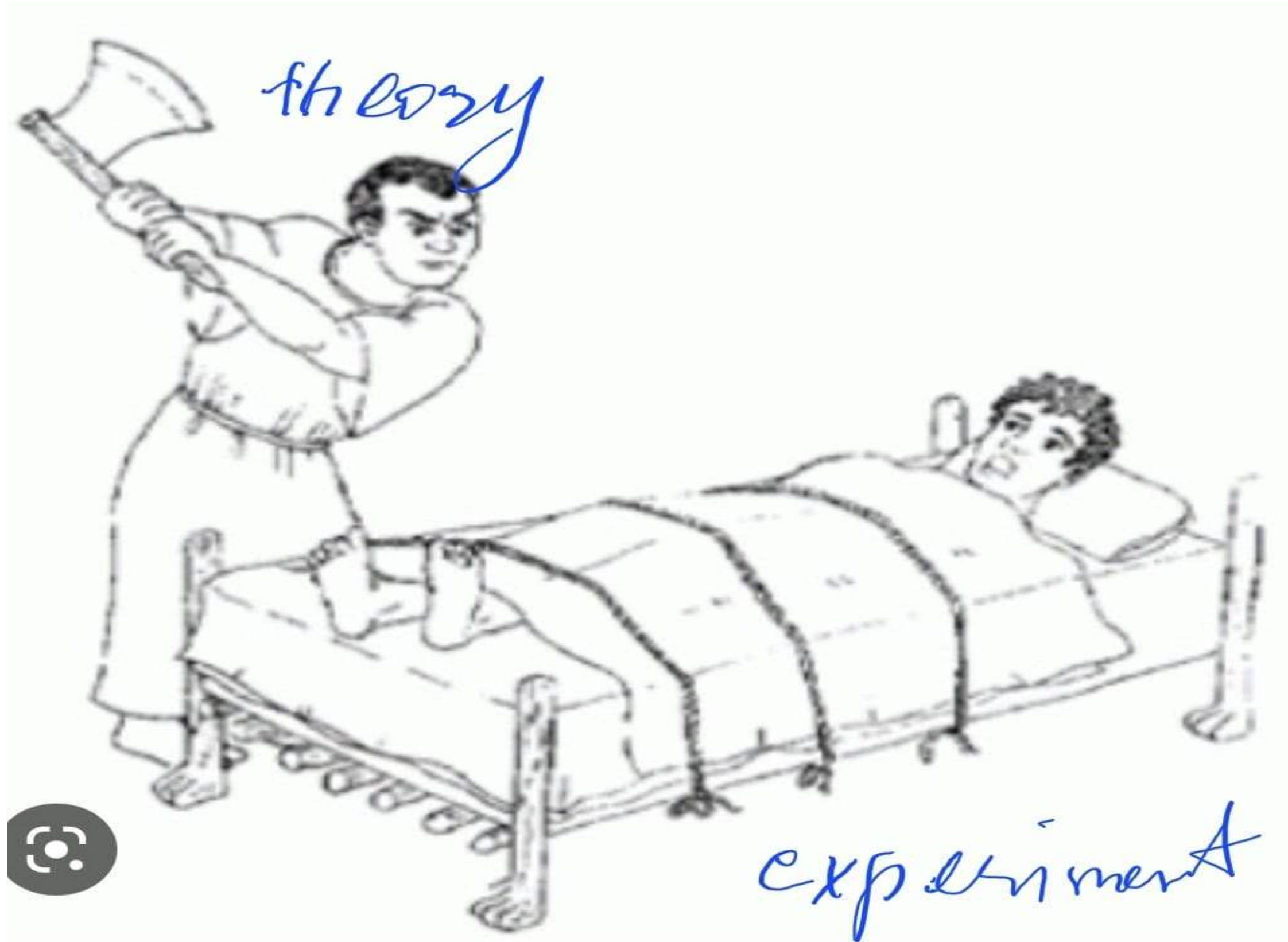


The diffractive ρ^0 production and decay.

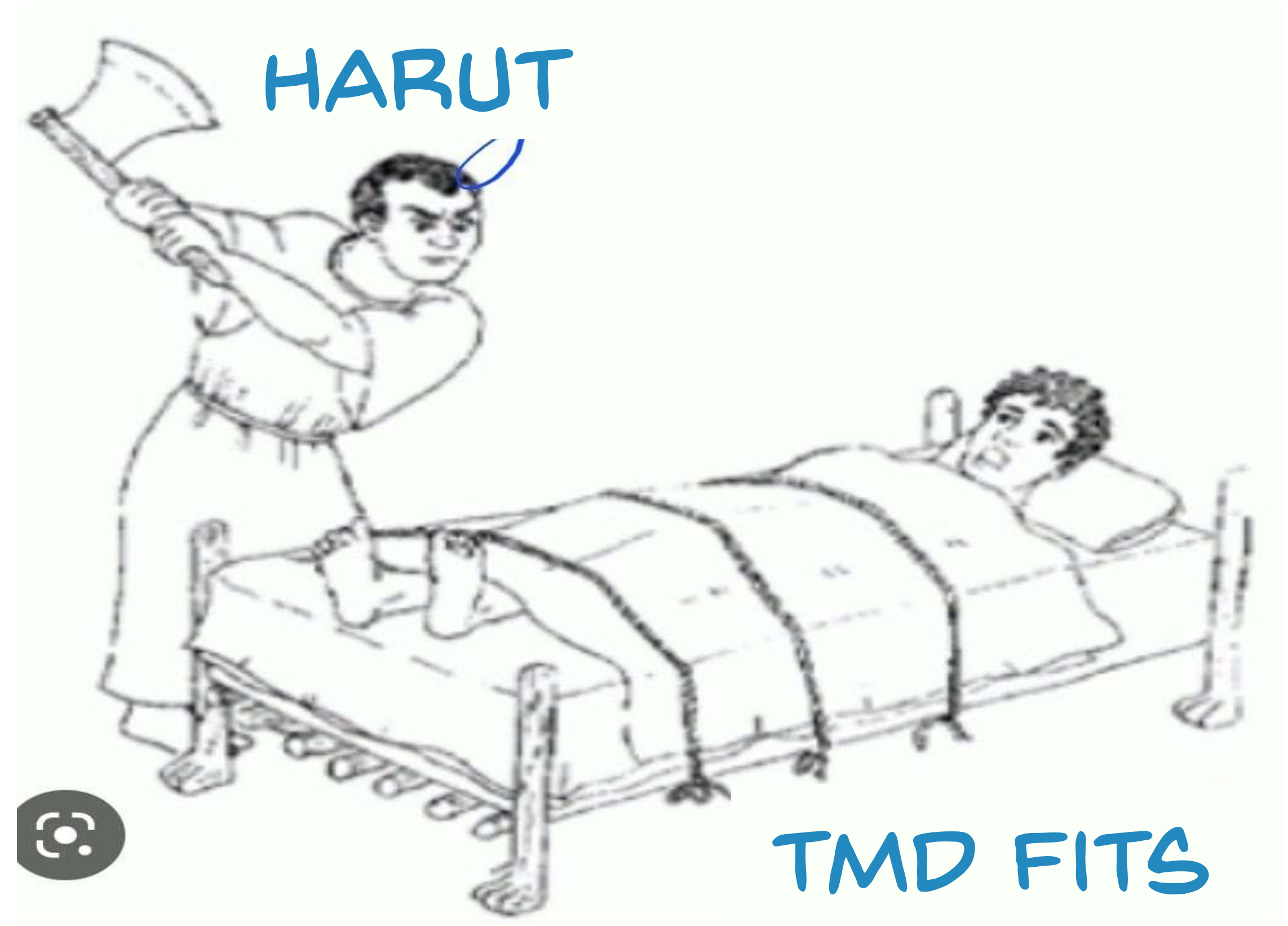


Procrustes - Greek Mythology

Modified version



Procrustes - Greek Mythology



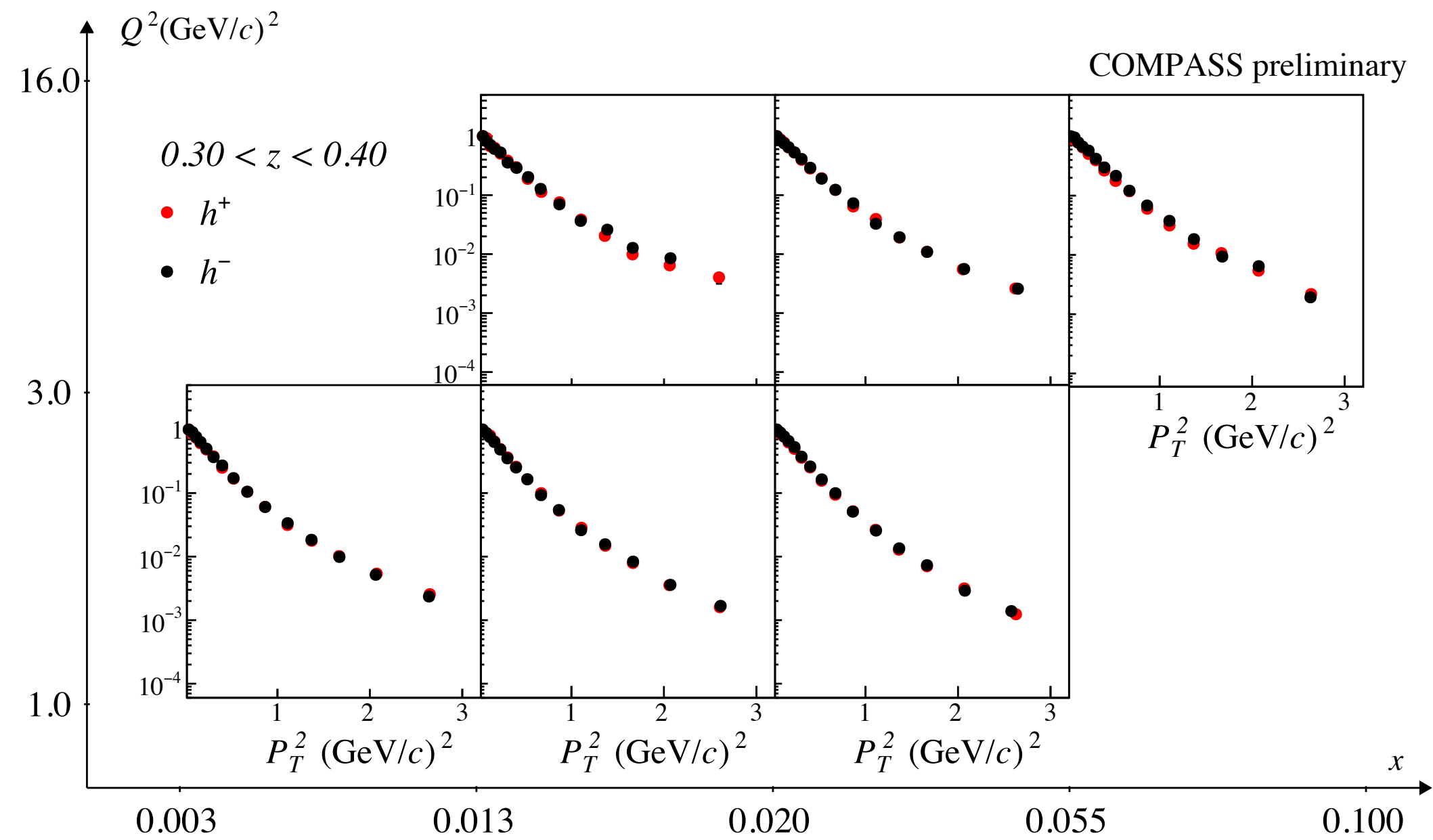
Procrustes - Greek Mythology

Let's hope that both data and fits can lie in comfortable beds





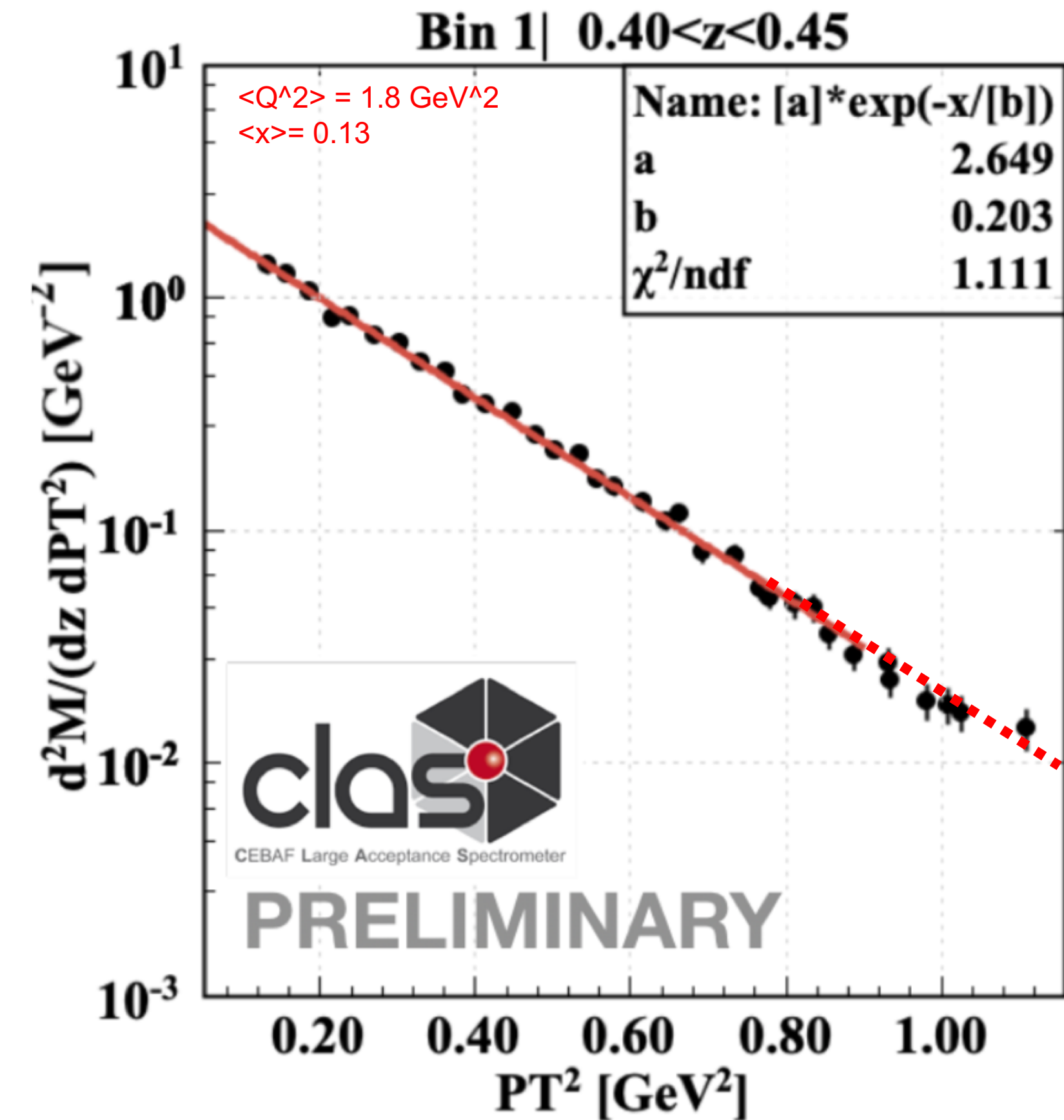
Compass multiplicities off protons



h^\pm distributions in $x : Q^2 : z : P_T^2$ (2nd z bin)

[A. Moretti (COMPASS), Proc. of ICNFP 2020]

[G. Angelini's talk at SarWors2021](#)



SOME LESSONS LEARNED

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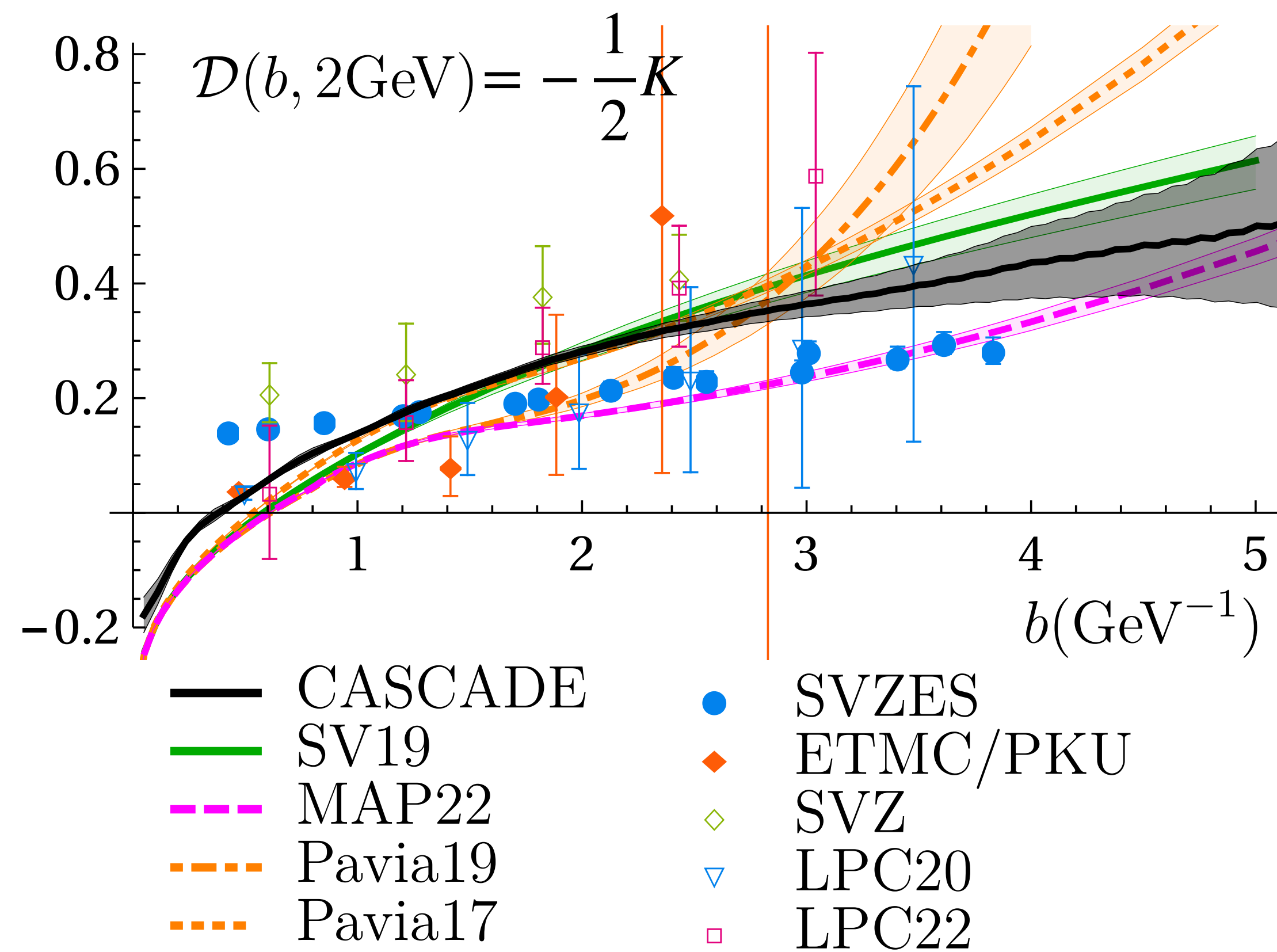
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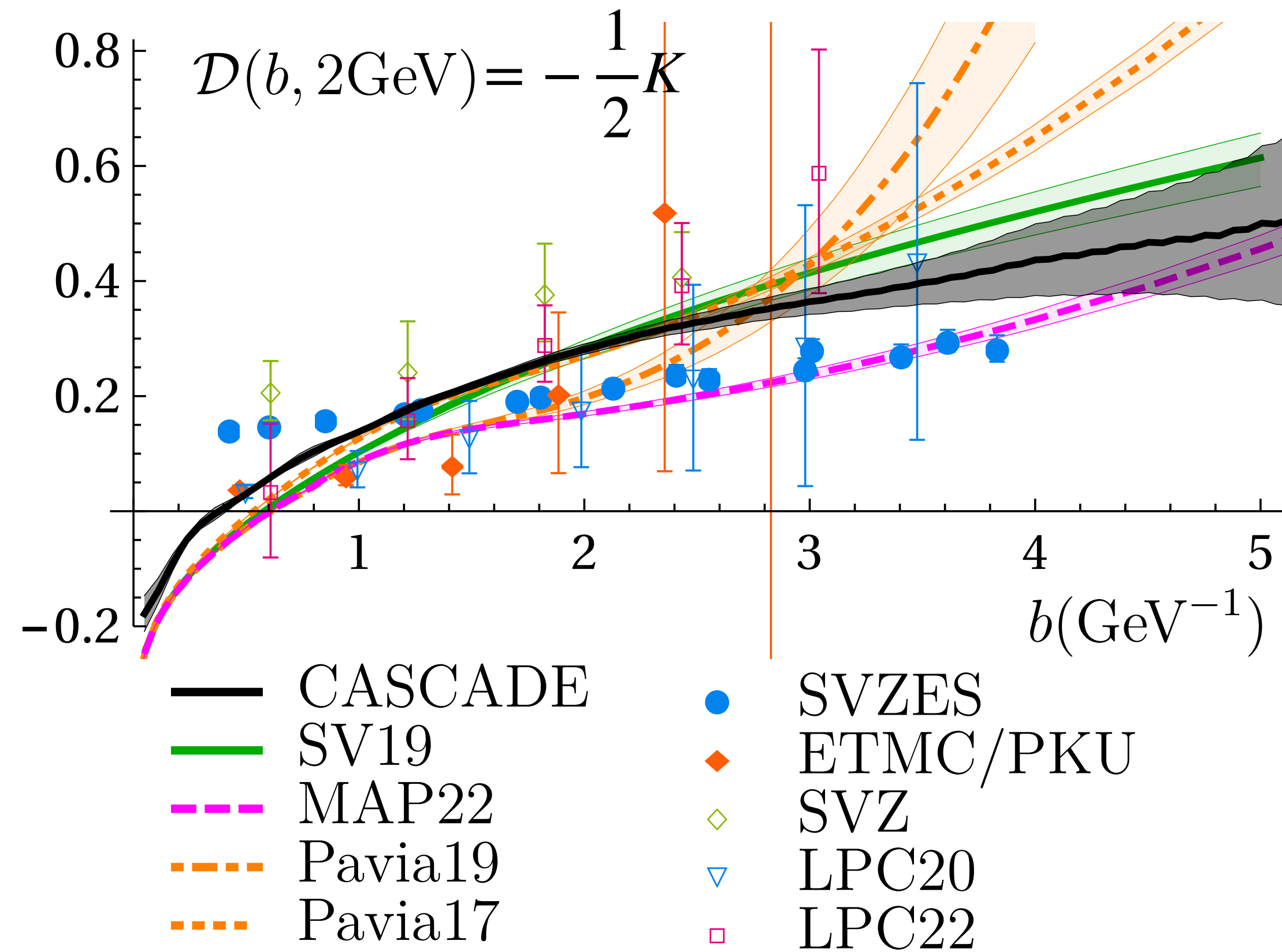
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**CONNECTIONS
WITH OTHER FIELDS**

[Bermudez Martinez, Vladimirov, arXiv:2206.01105](#)



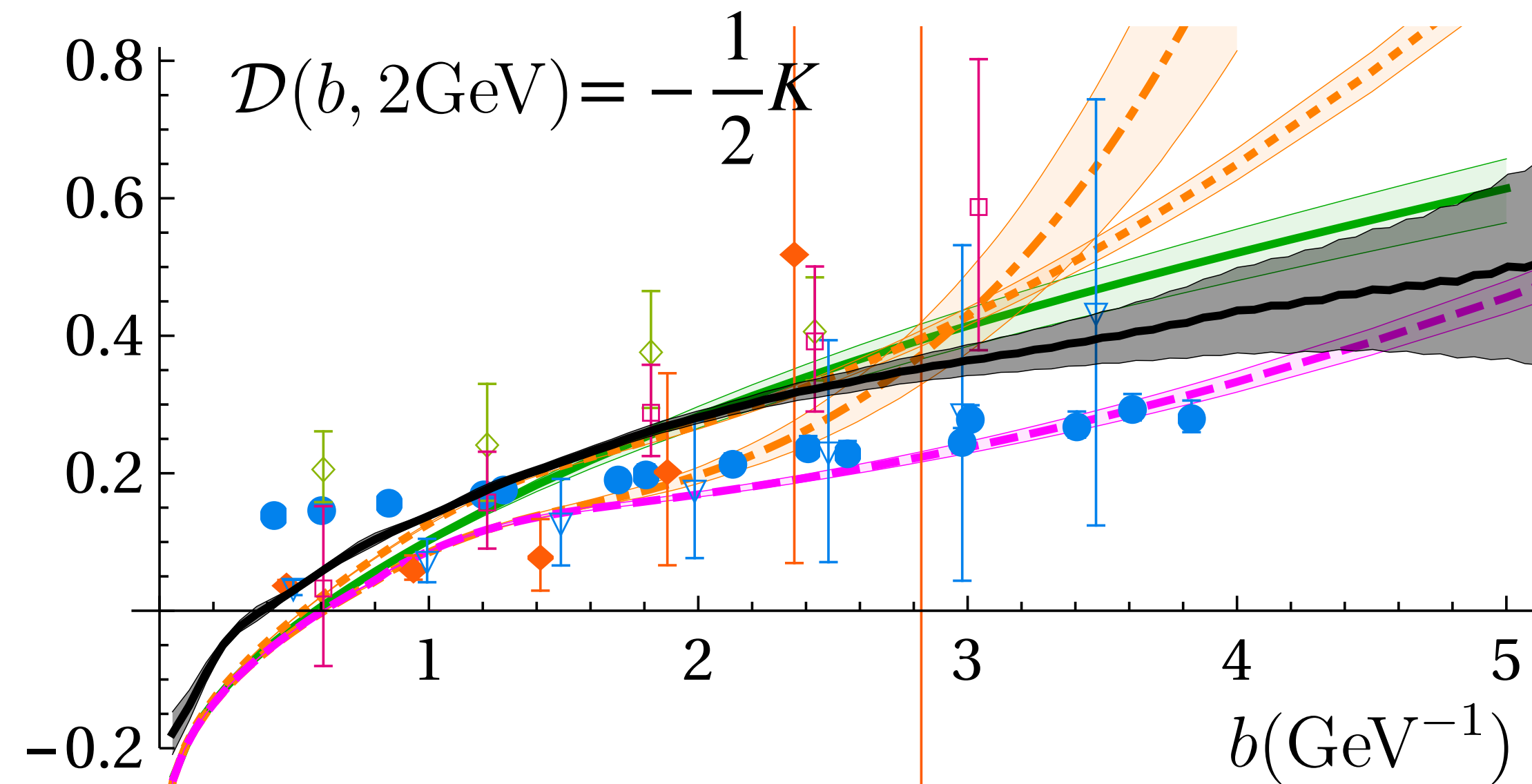
[Bermudez Martinez, Vladimirov, arXiv:2206.01105](#)



TMD phenomenology



Bermudez Martinez, Vladimirov, arXiv:2206.01105



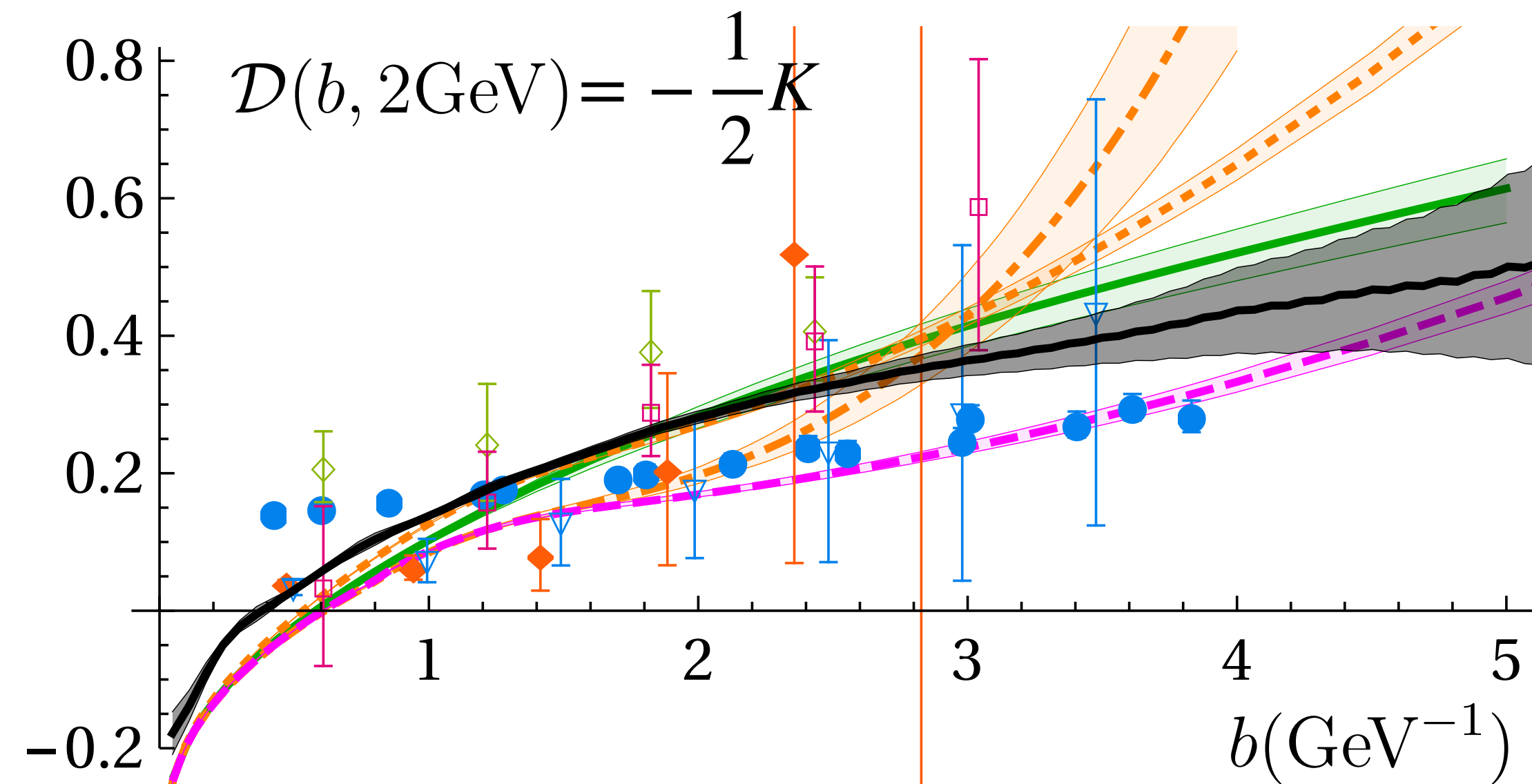
- CASCADE
- SV19
- - - MAP22
- - - Pavia19
- - - Pavia17
- SVZES
- ◆ ETMC/PKU
- ◇ SVZ
- ▽ LPC20
- LPC22

TMD phenomenology

Lattice QCD

[Bermudez Martinez, Vladimirov, arXiv:2206.01105](#)

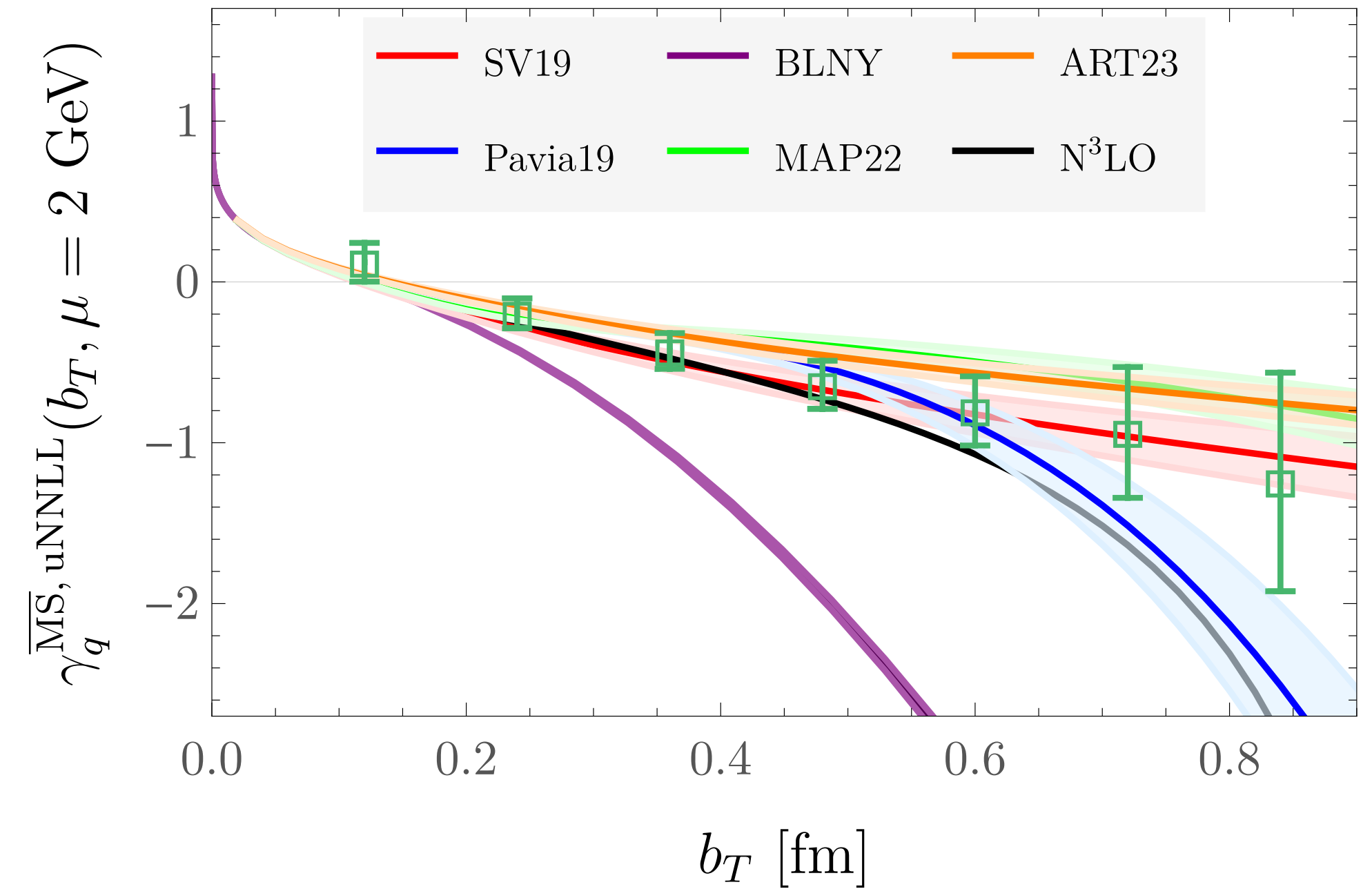
[Avkhadiev, Shanahan, Wagman, Zhao, arXiv:2307.12359](#)



- | | | | |
|-------|---------|---|----------|
| — | CASCADE | ● | SVZES |
| — | SV19 | ◆ | ETMC/PKU |
| - - - | MAP22 | ◇ | SVZ |
| - - - | Pavia19 | ▽ | LPC20 |
| - - - | Pavia17 | □ | LPC22 |

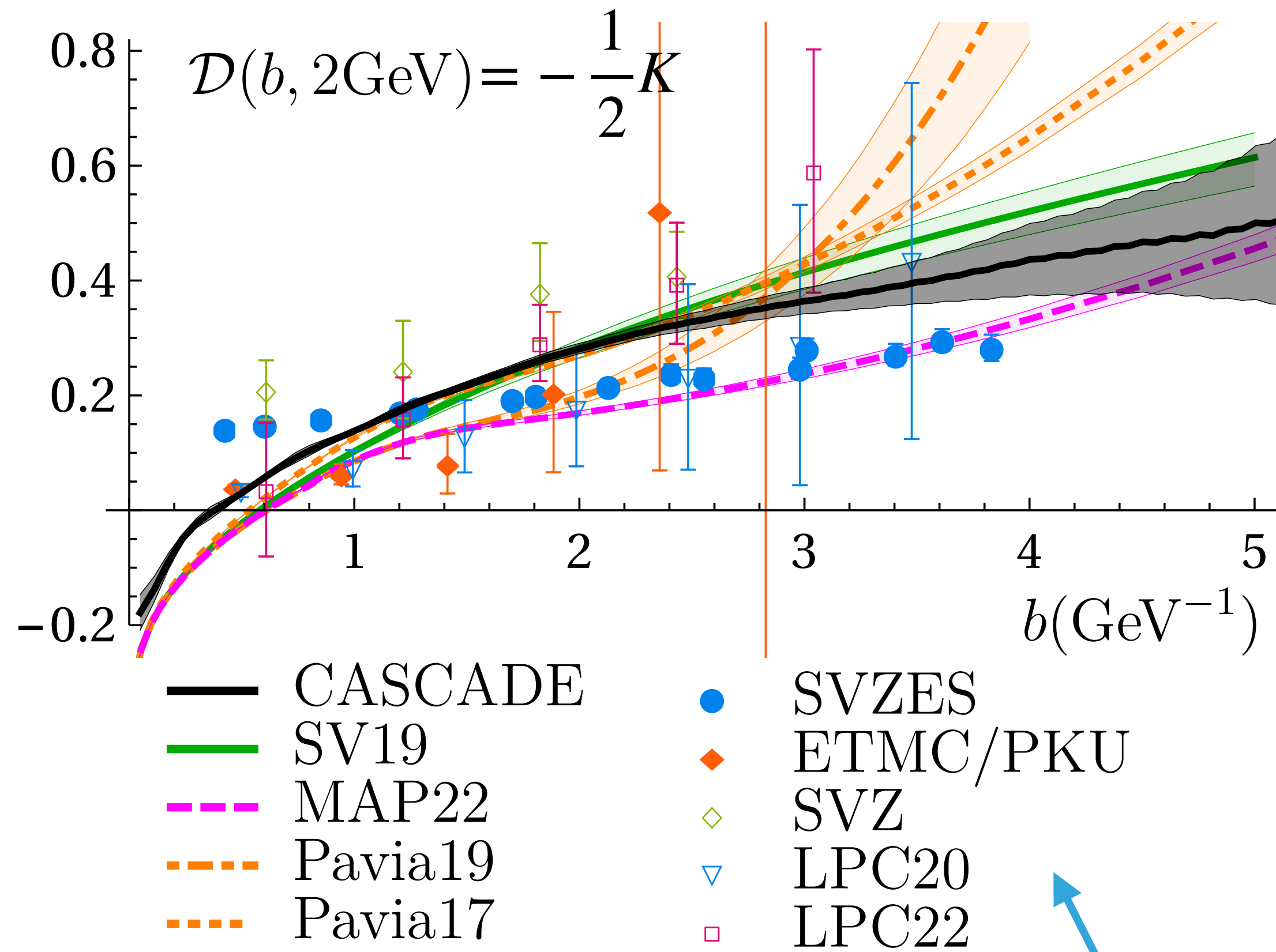
TMD phenomenology

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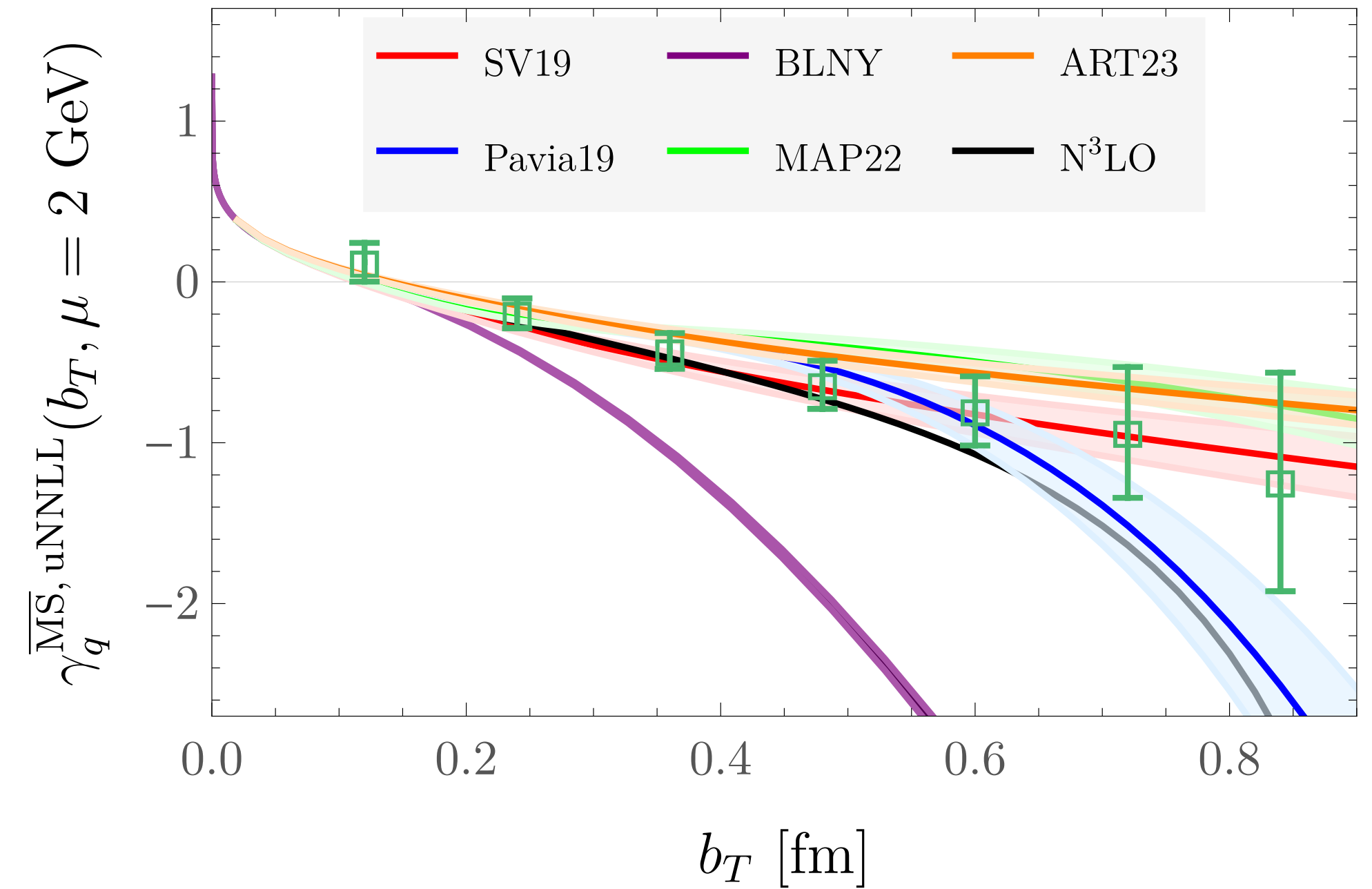
[Bermudez Martinez, Vladimirov, arXiv:2206.01105](#)

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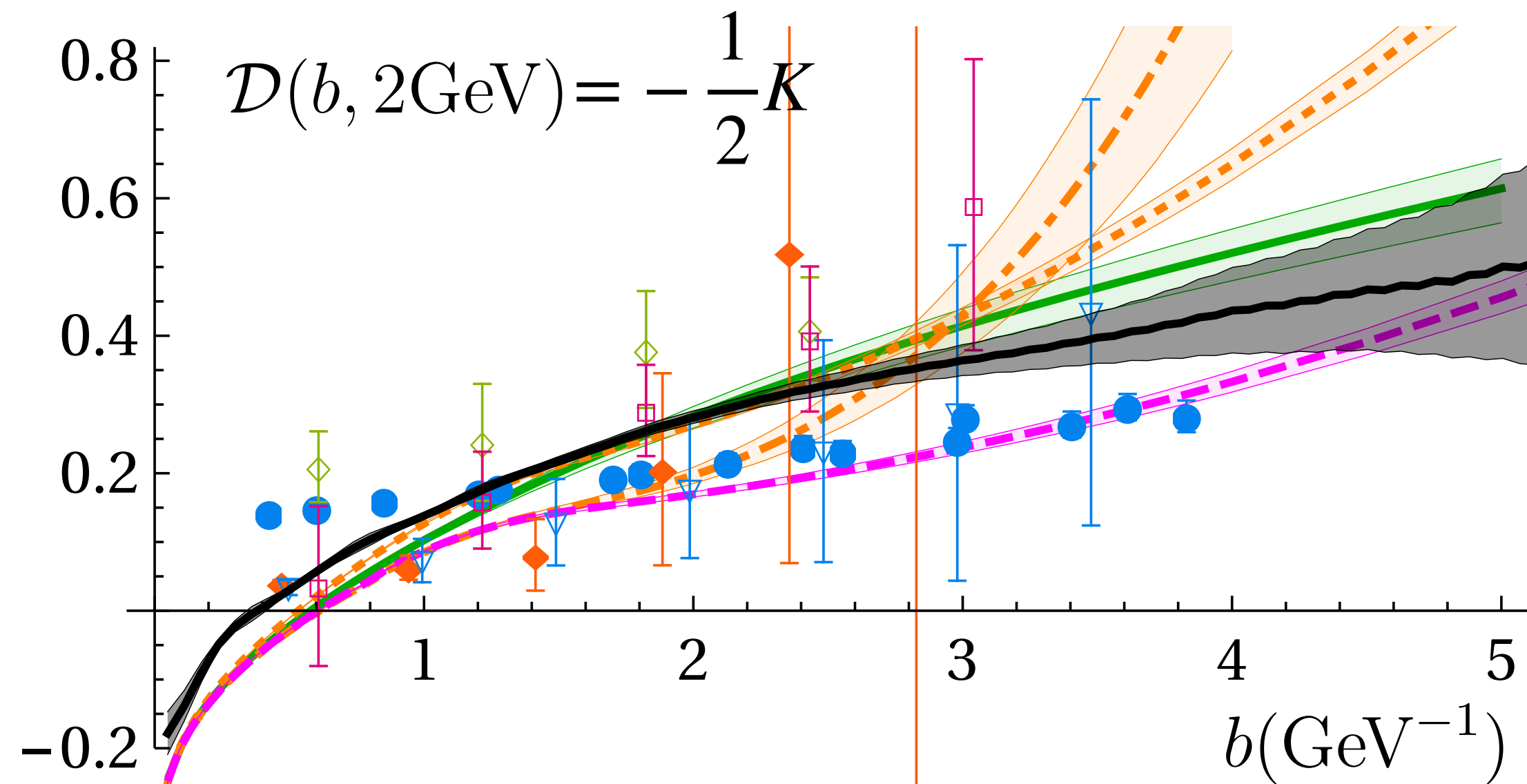
Lattice QCD



See talk by Patrizio Pucci

Bermudez Martinez, Vladimirov, arXiv:2206.01105

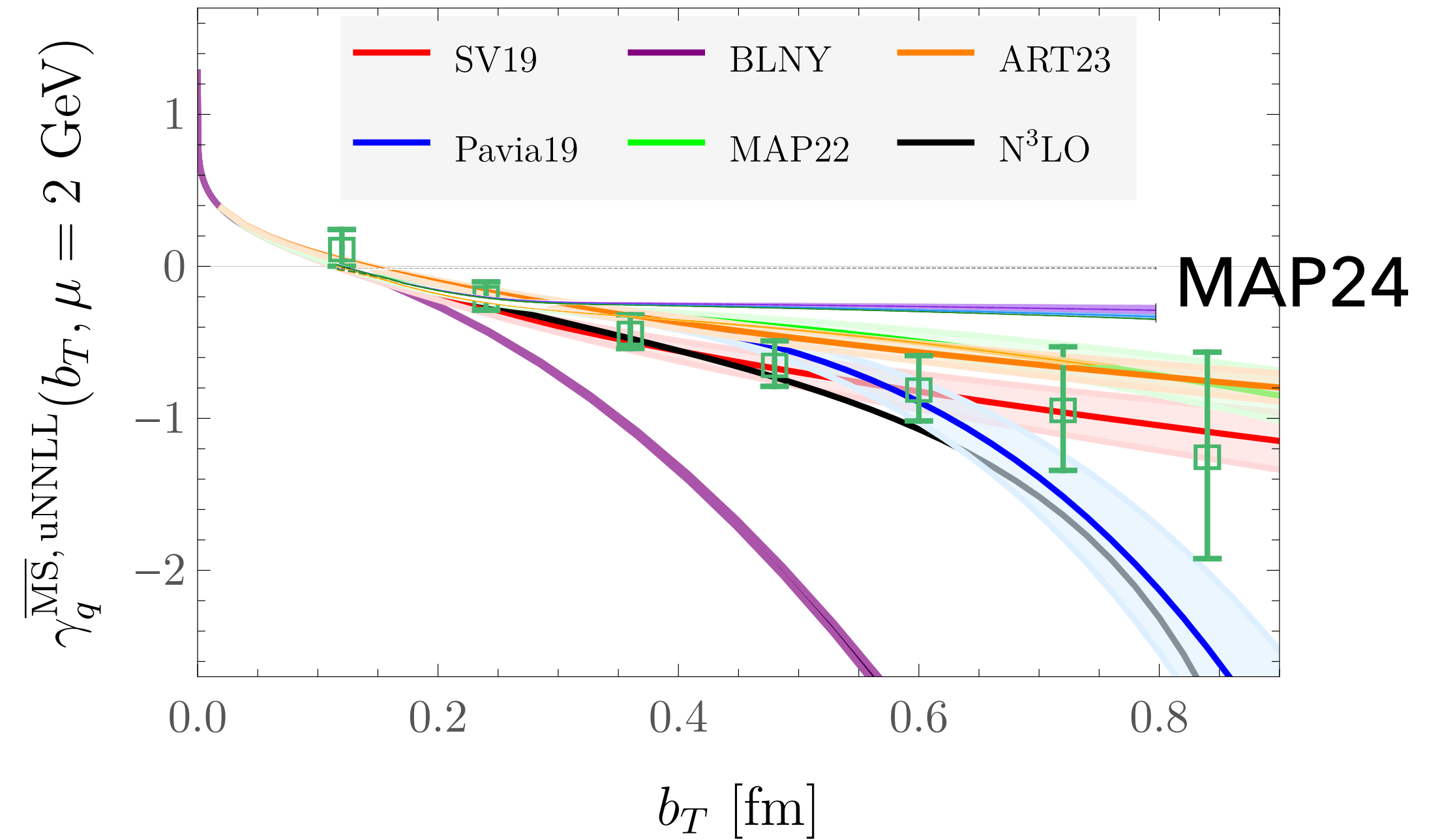
Avkhadiev, Shanahan, Wagman, Zhao, arXiv:2307.12359



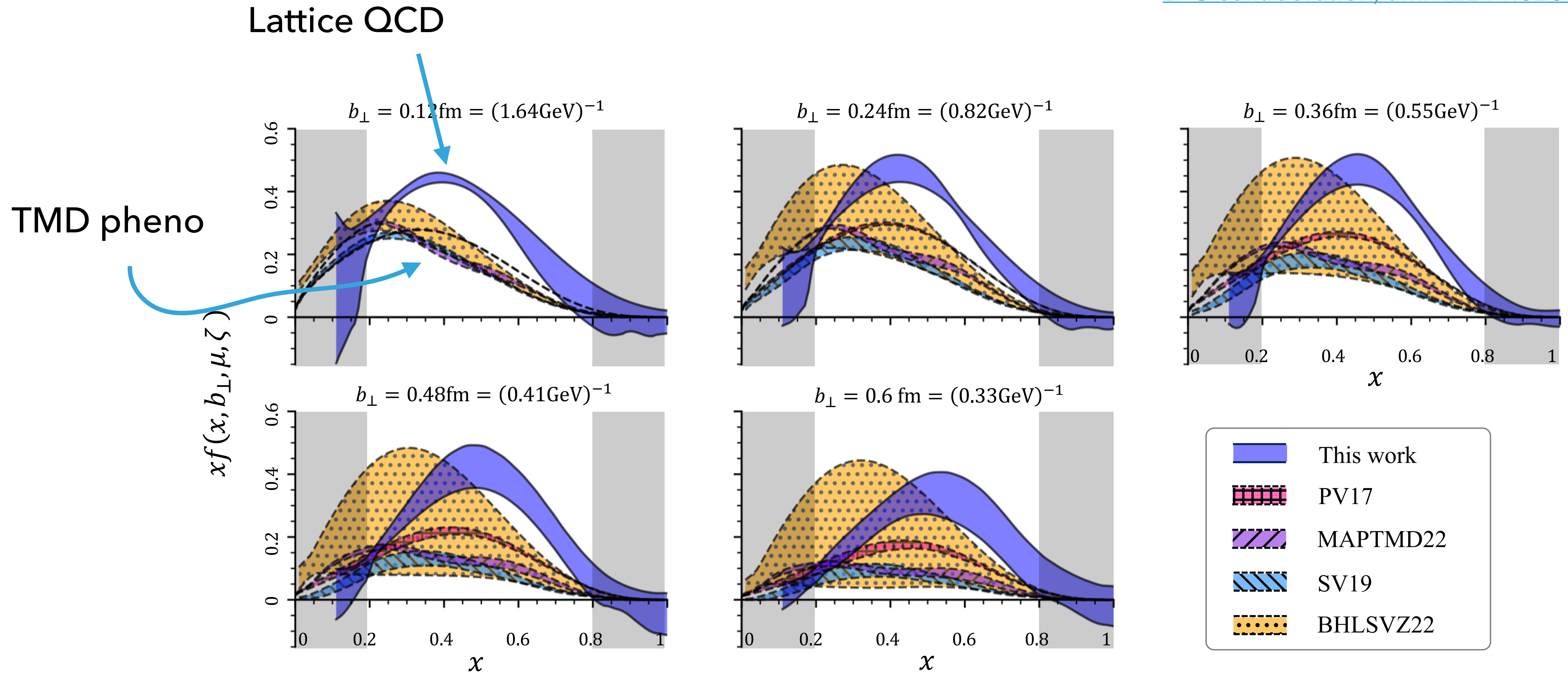
- | | | | |
|-------|---------|---|----------|
| — | CASCADE | ● | SVZES |
| — | SV19 | ◆ | ETMC/PKU |
| - - - | MAP22 | ◇ | SVZ |
| - - - | Pavia19 | ▽ | LPC20 |
| - - - | Pavia17 | □ | LPC22 |

TMD phenomenology

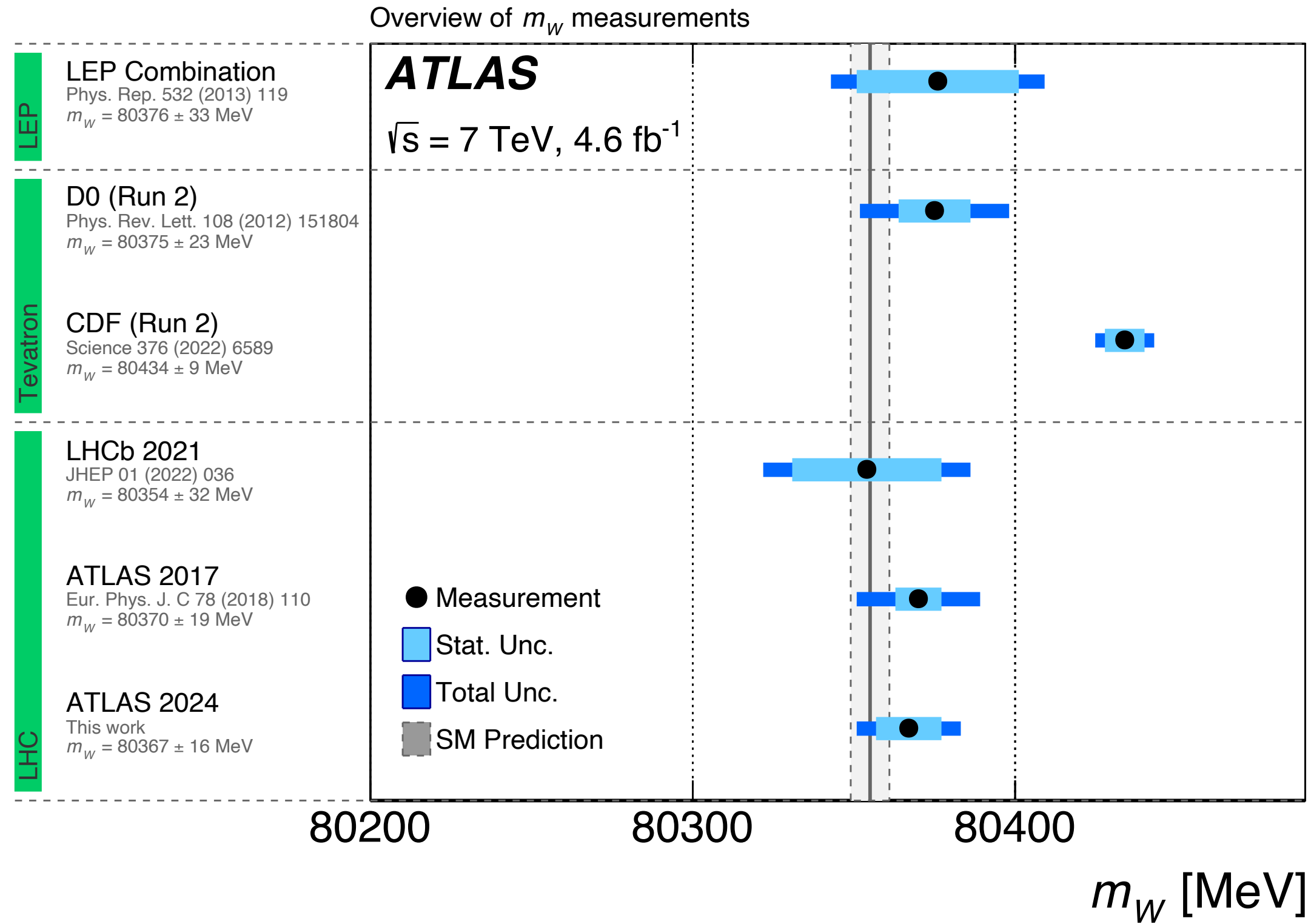
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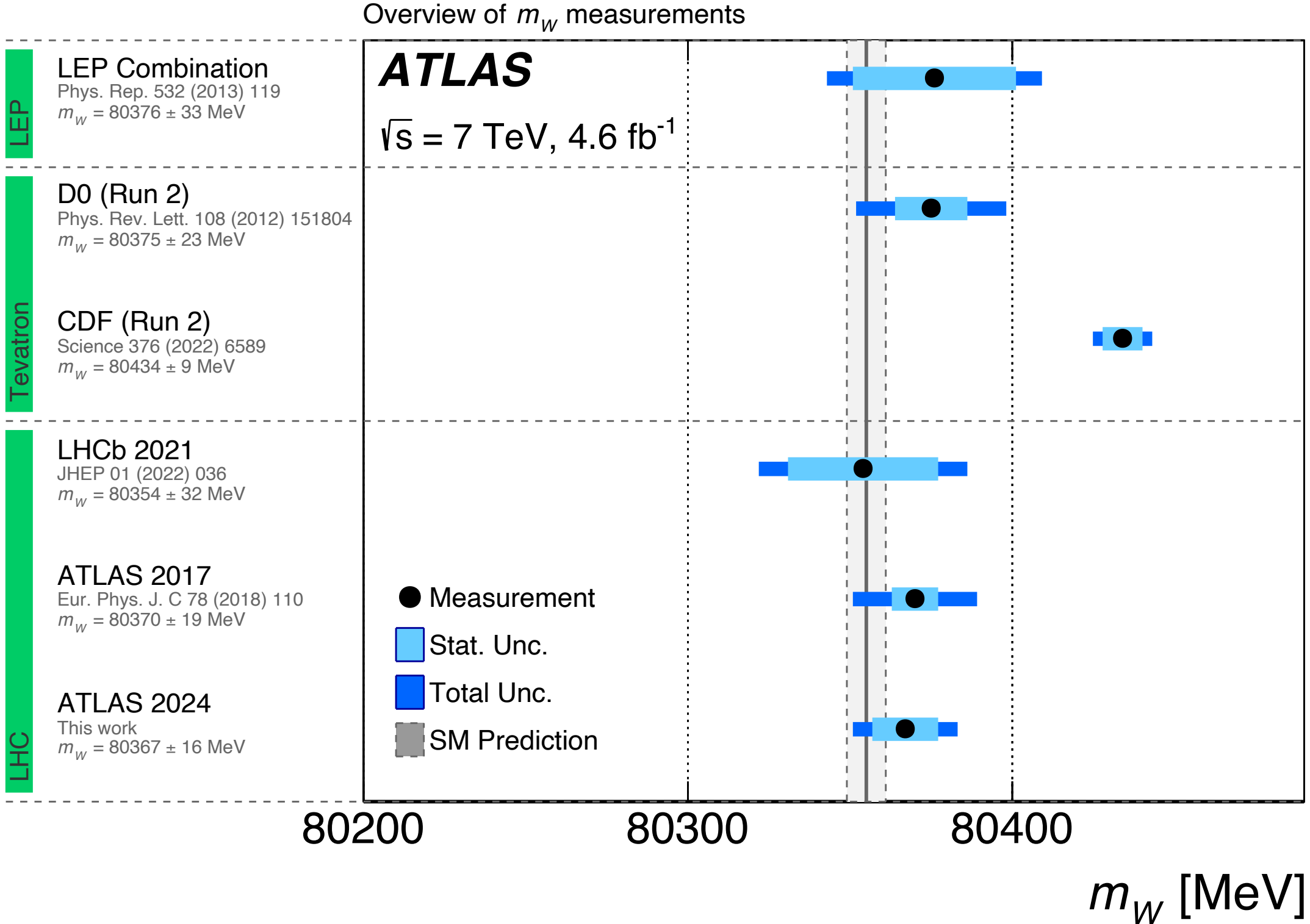


[ATLAS Collab. arXiv:2403.15085](https://arxiv.org/abs/2403.15085)



Unc. [MeV]	Total	Stat.	Syst.	PDF	A_i	Backg.	EW	e	μ	u_T	Lumi	Γ_W	PS
p_T^ℓ	16.2	11.1	11.8	4.9	3.5	1.7	5.6	5.9	5.4	0.9	1.1	0.1	1.5
m_T	24.4	11.4	21.6	11.7	4.7	4.1	4.9	6.7	6.0	11.4	2.5	0.2	7.0
Combined	15.9	9.8	12.5	5.7	3.7	2.0	5.4	6.0	5.4	2.3	1.3	0.1	2.3

[ATLAS Collab. arXiv:2403.15085](https://arxiv.org/abs/2403.15085)



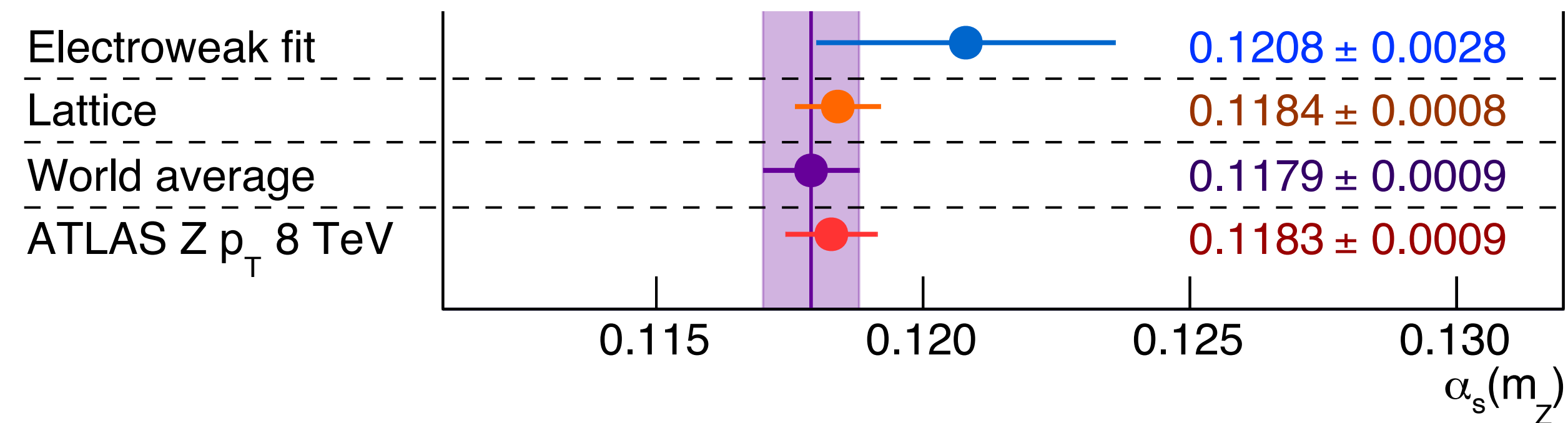
Not taking into account the flavor dependence of TMDs can lead to errors in the determination of the W mass, of the order of a few MeVs

[Bacchetta, Bozzi, Radici, Ritzmann, Signori, arXiv:1807.02101](https://arxiv.org/abs/1807.02101)

Unc. [MeV]	Total	Stat.	Syst.	PDF	A_i	Backg.	EW	e	μ	u_T	Lumi	Γ_W	PS
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The coupling constant of the strong force is determined from the transverse-momentum distribution of Z bosons produced in 8 TeV proton–proton collisions at the LHC and recorded by the ATLAS experiment.

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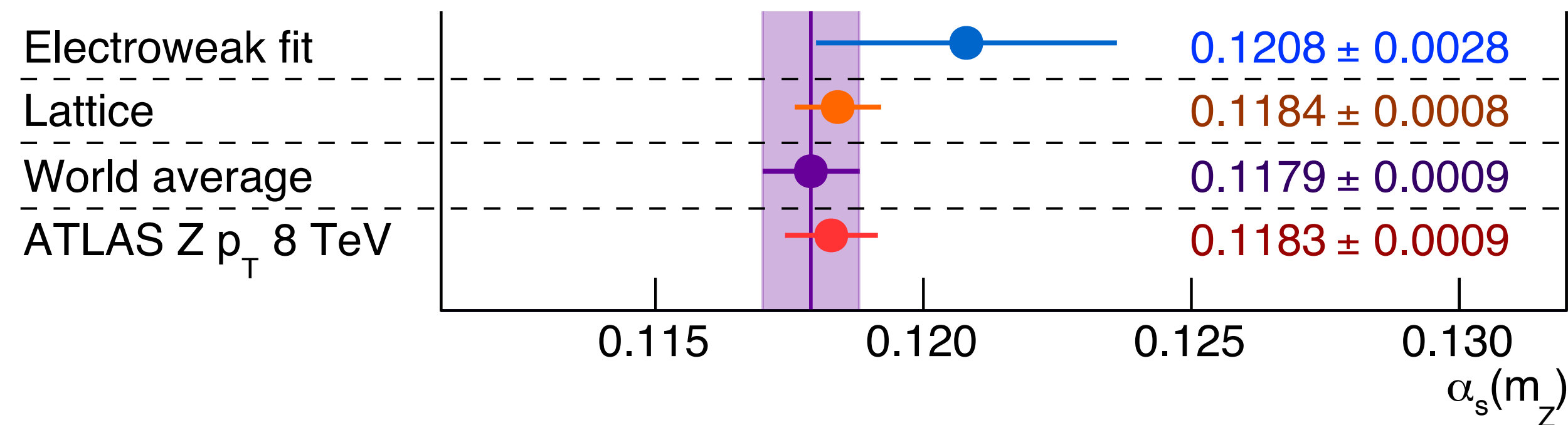
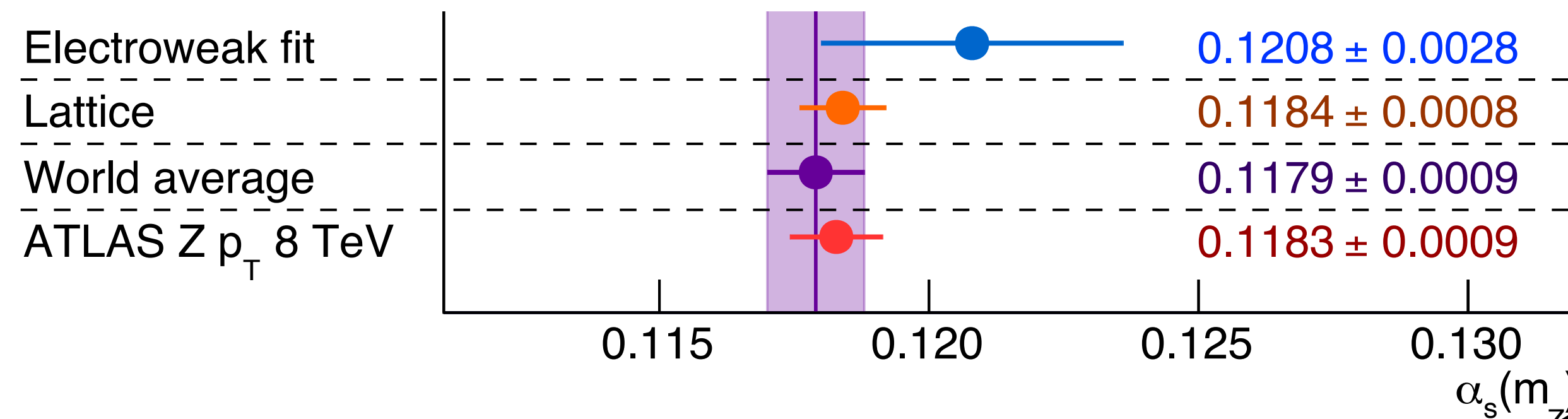


Table 1: Summary of the uncertainties in the determination of $\alpha_s(m_Z)$, in units of 10^{-3} .

Experimental uncertainty	±0.44	
PDF uncertainty	±0.51	
Scale variation uncertainties	±0.42	
Matching to fixed order	0	−0.08
Non-perturbative model	+0.12	−0.20
Flavour model	+0.40	−0.29
QED ISR	±0.14	
N^4 LL approximation	±0.04	
Total	+0.91	−0.88

The coupling constant of the strong force is determined from the transverse-momentum distribution of Z bosons produced in 8 TeV proton–proton collisions at the LHC and recorded by the ATLAS experiment.

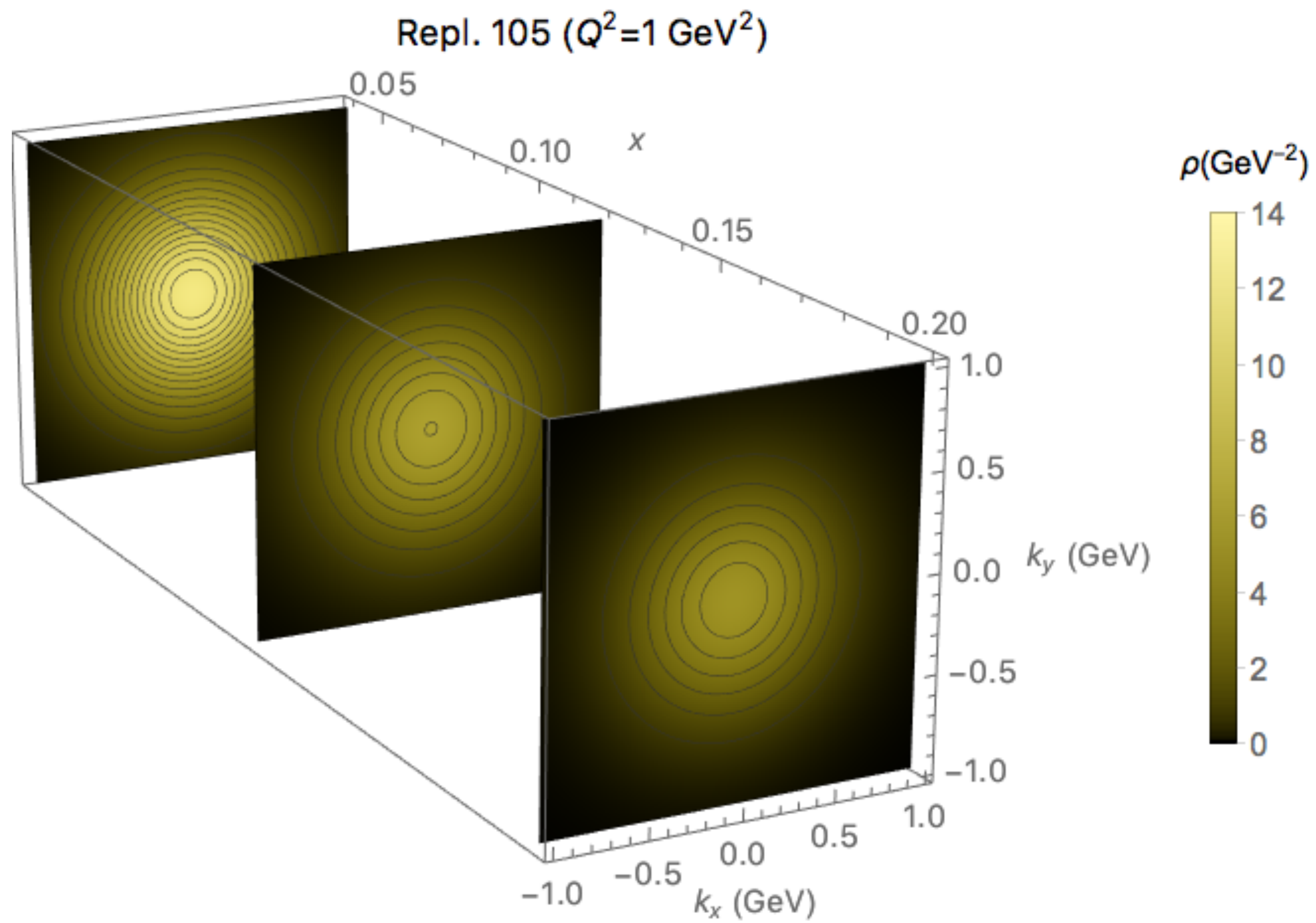


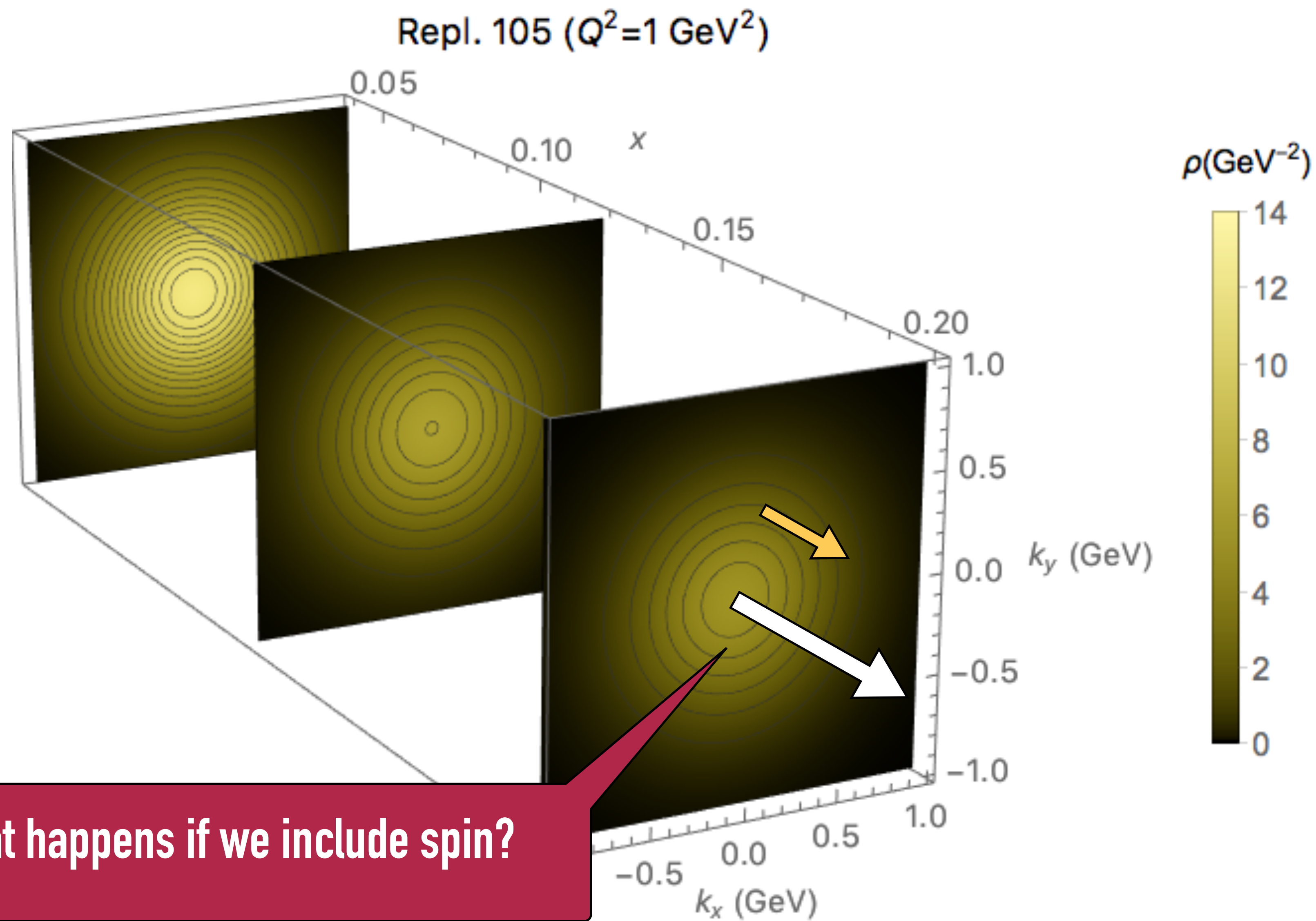
Estimate that does not take into account recent TMD results

Table 1: Summary of the uncertainties in the determination of $\alpha_s(m_Z)$, in units of 10^{-3} .

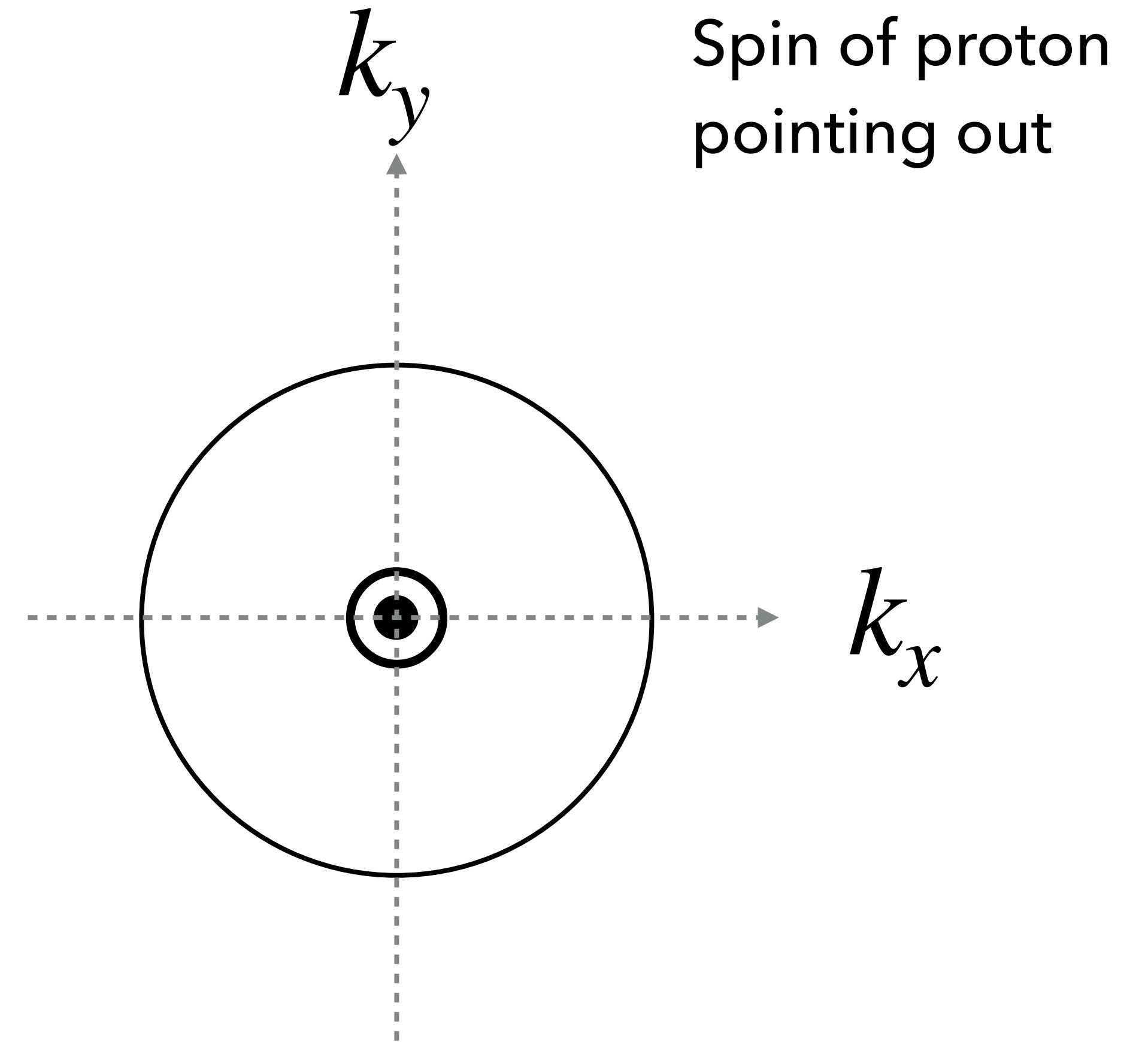
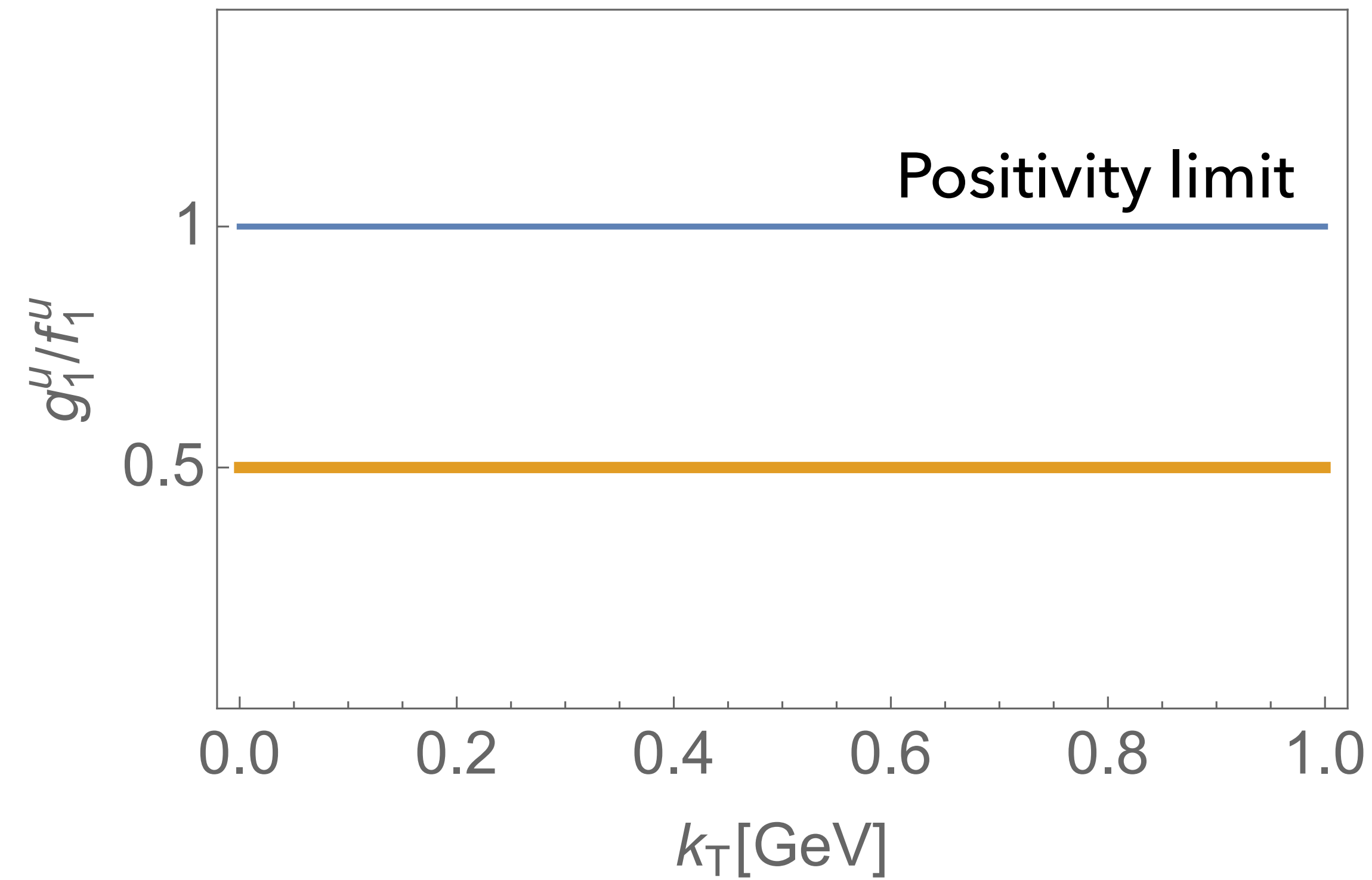
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HELICITY TMD

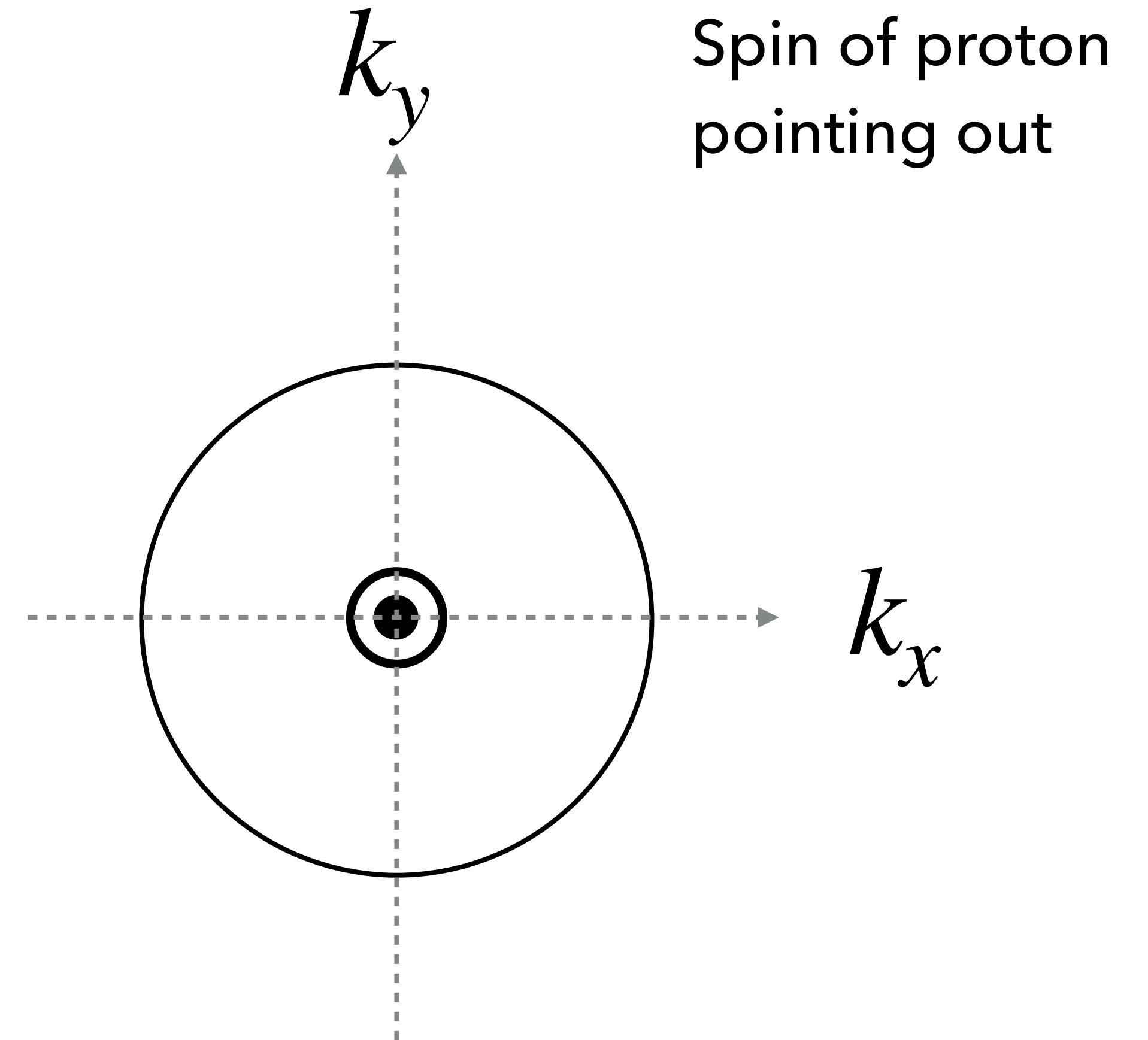
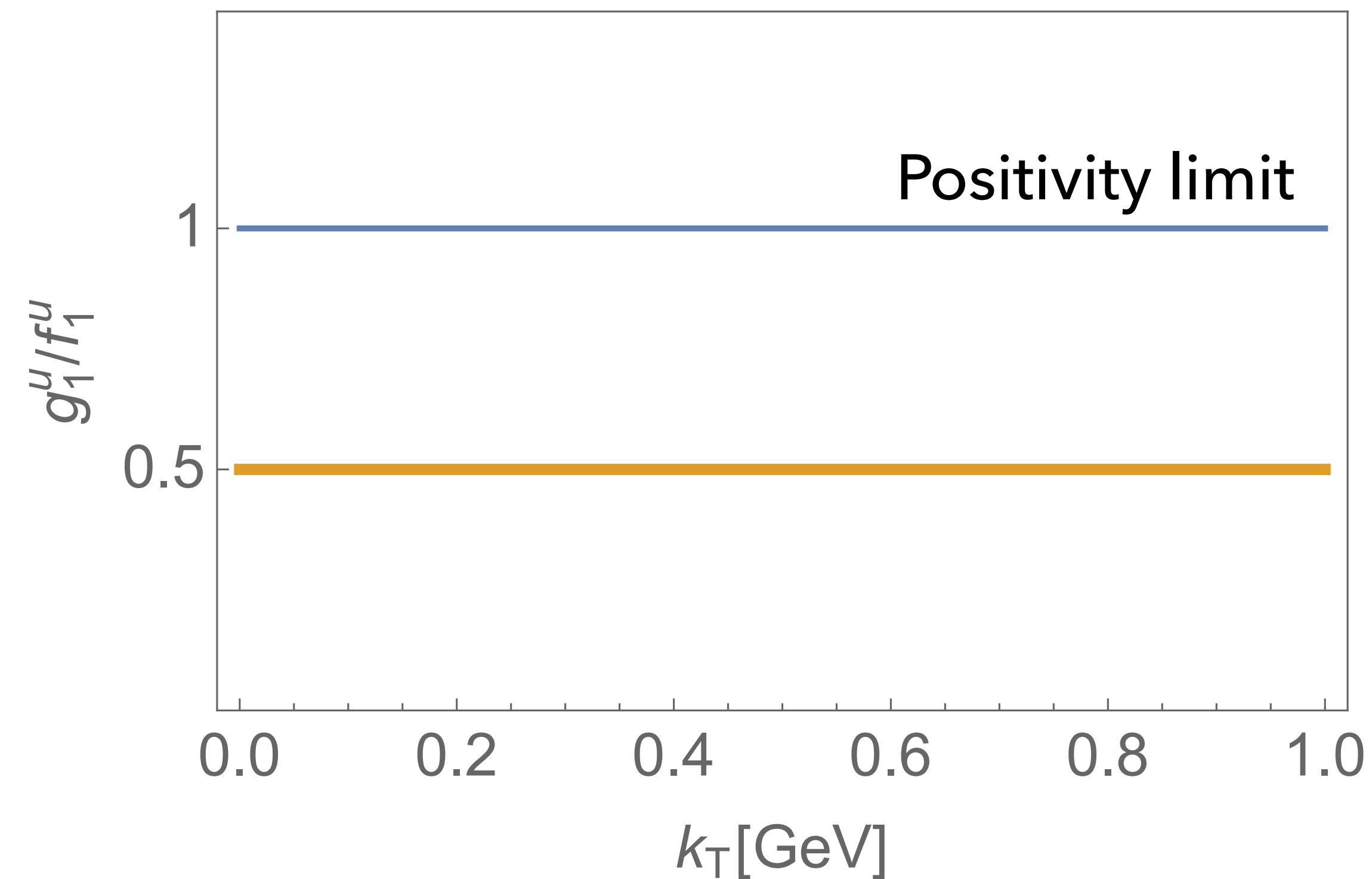




At a certain value of x

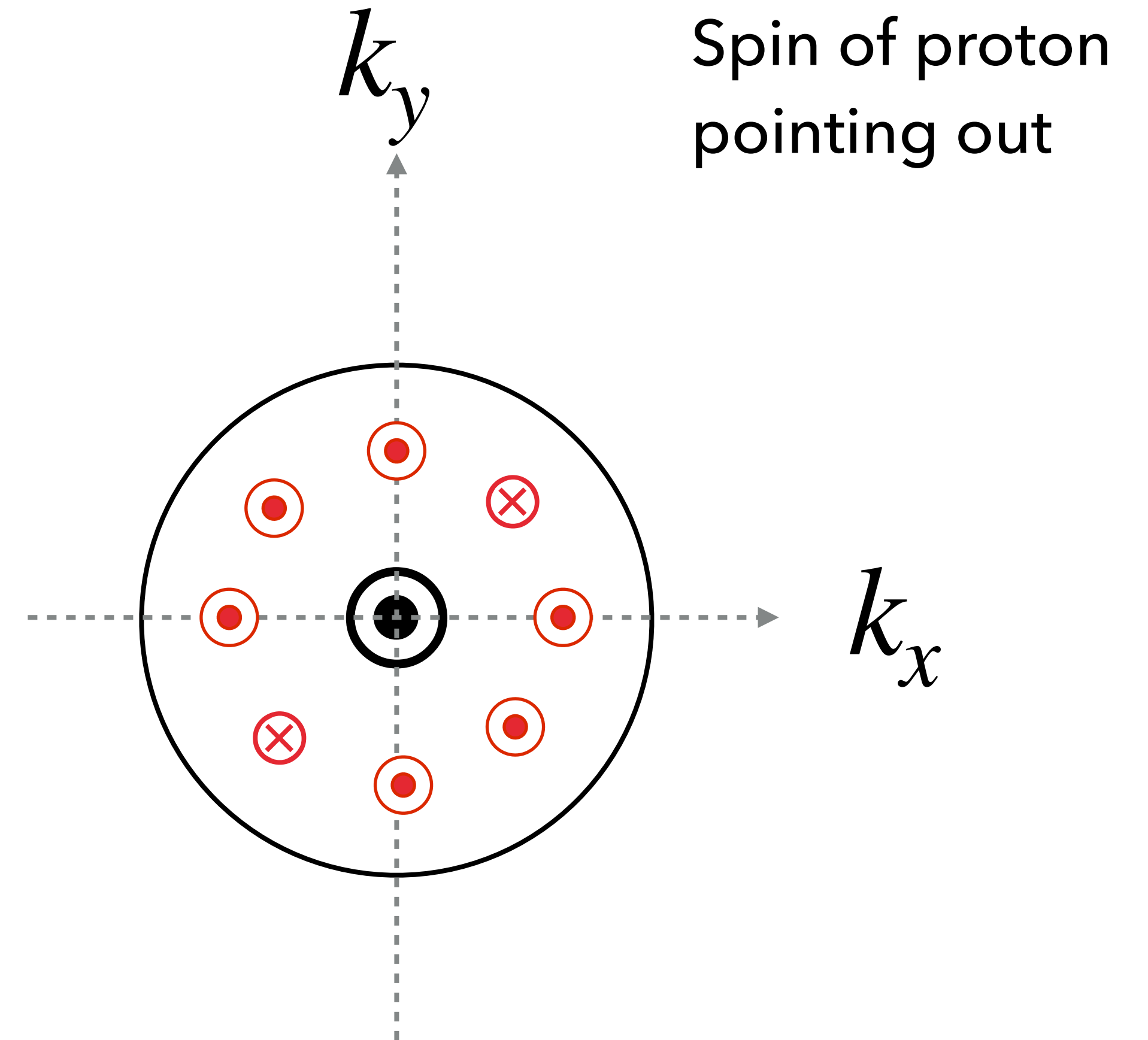
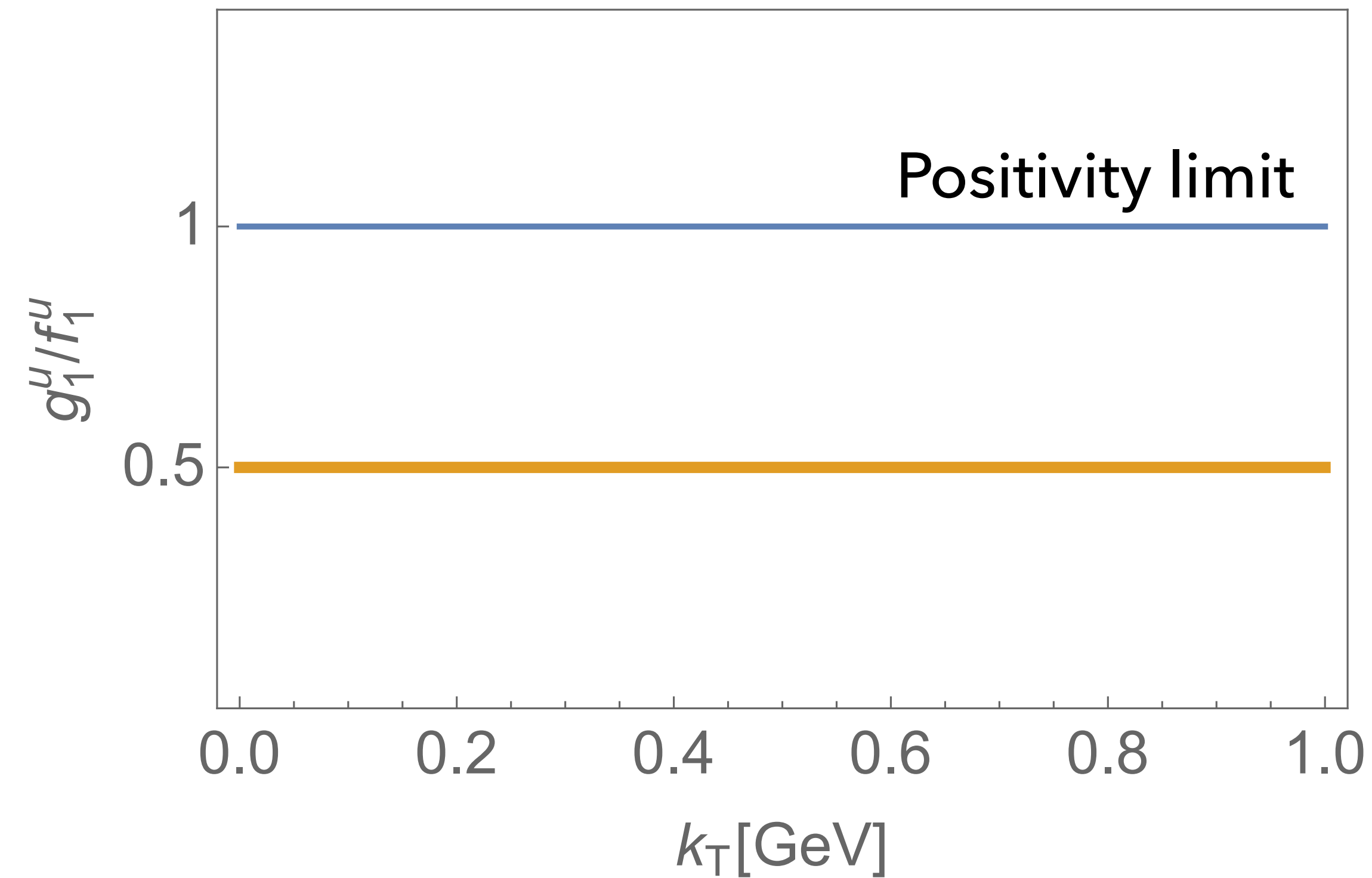


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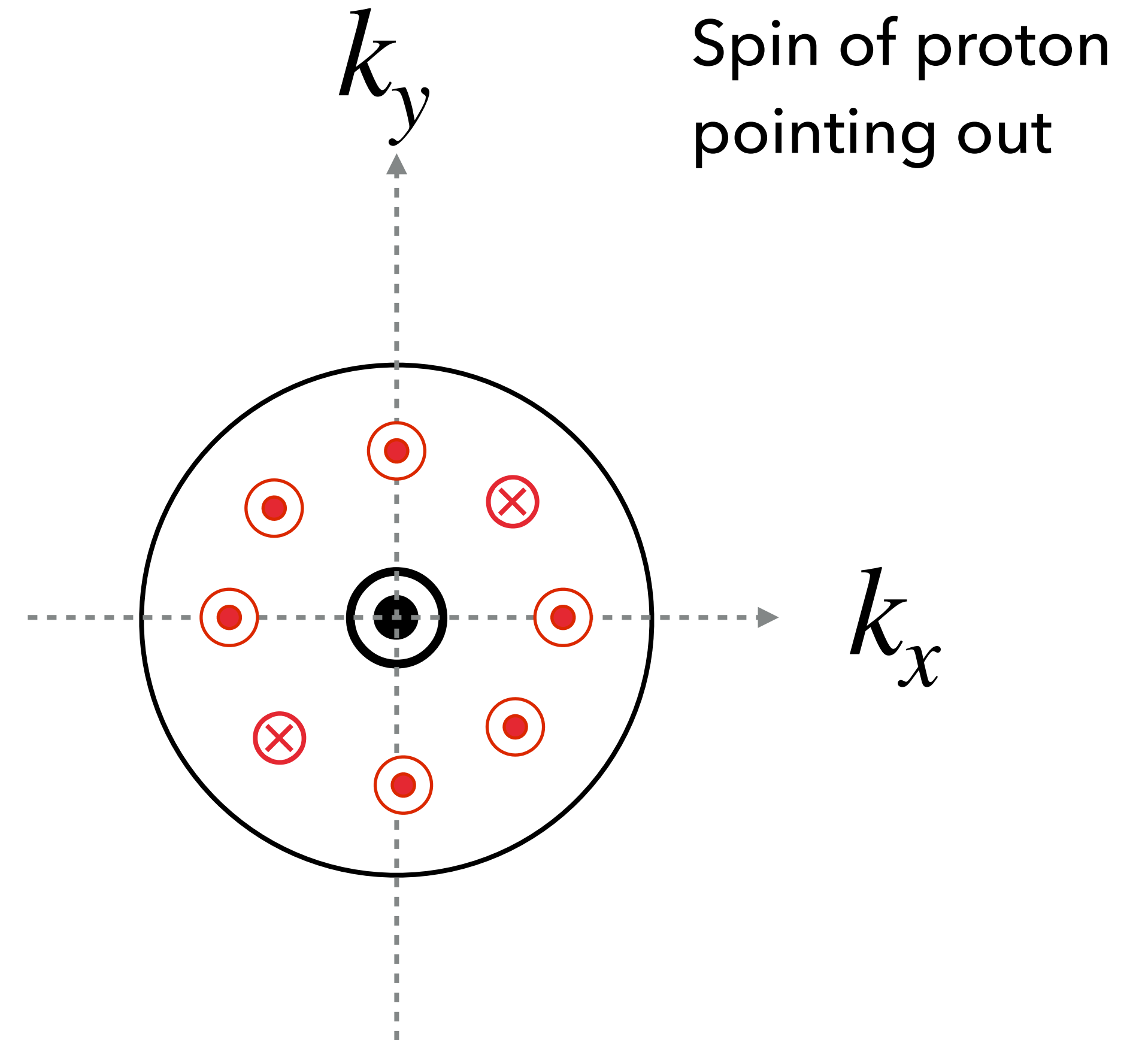
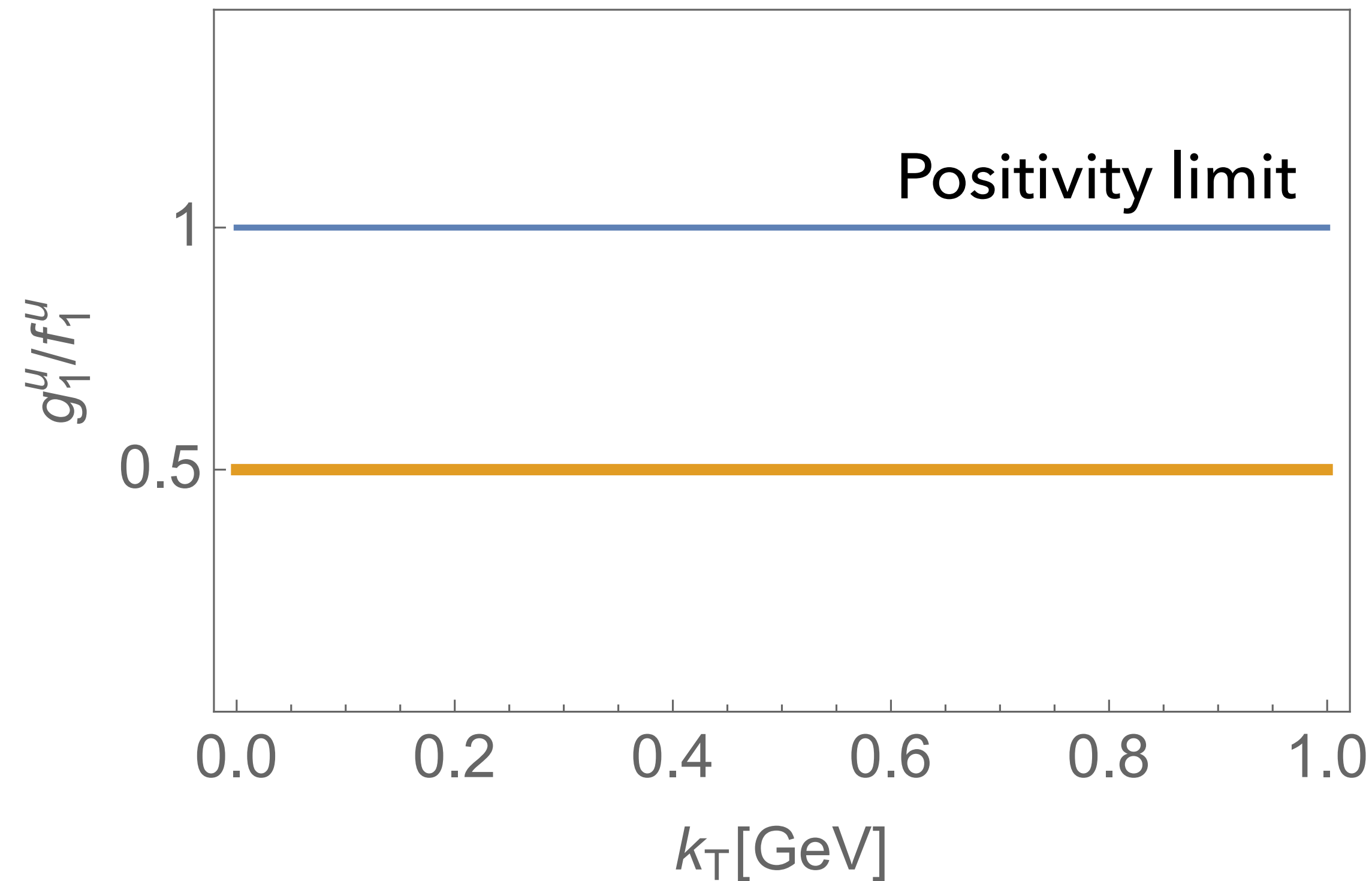
We try to always impose positivity limits. We prefer rigid and physical to flexible and unphysical

At a certain value of x



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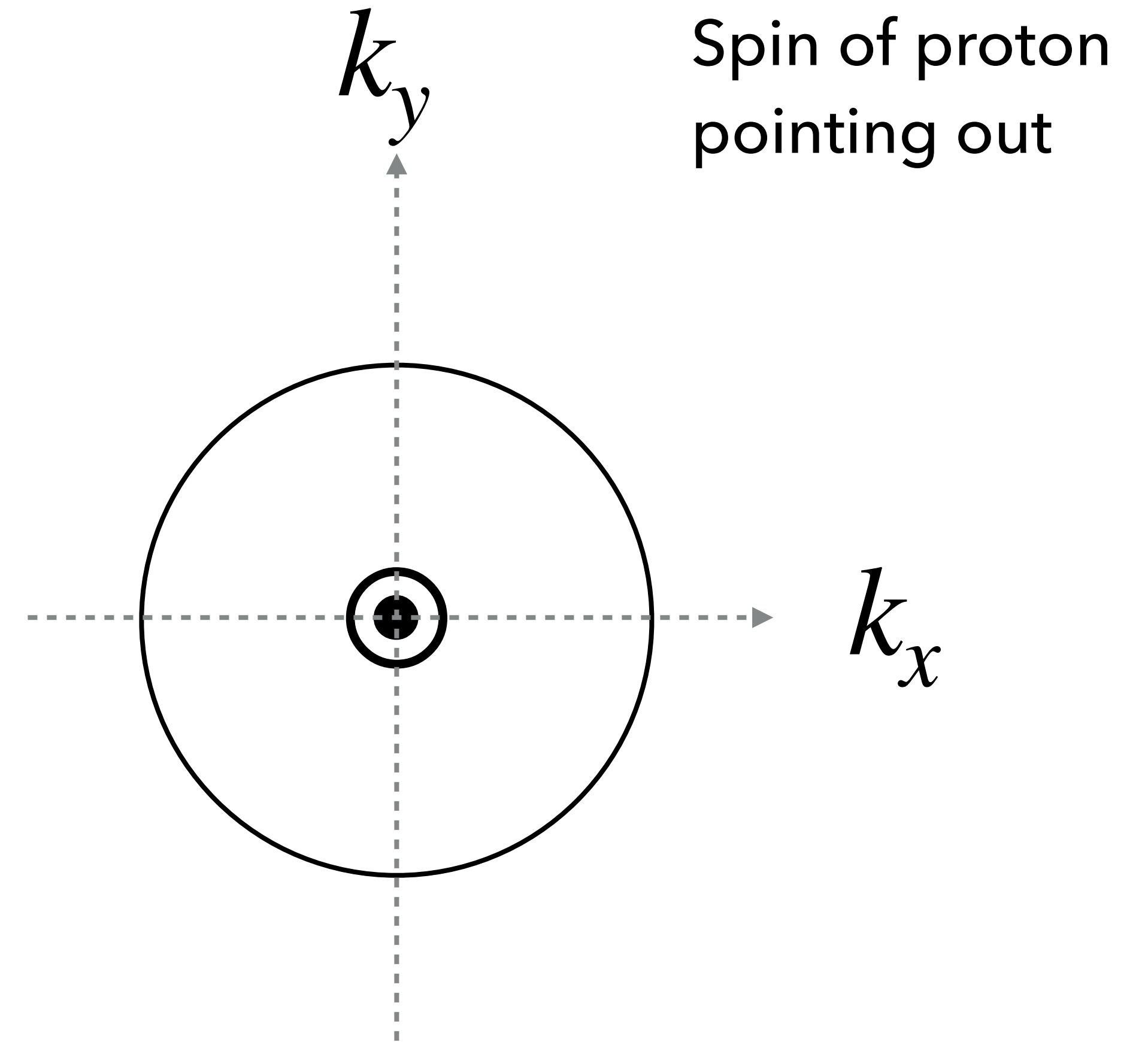
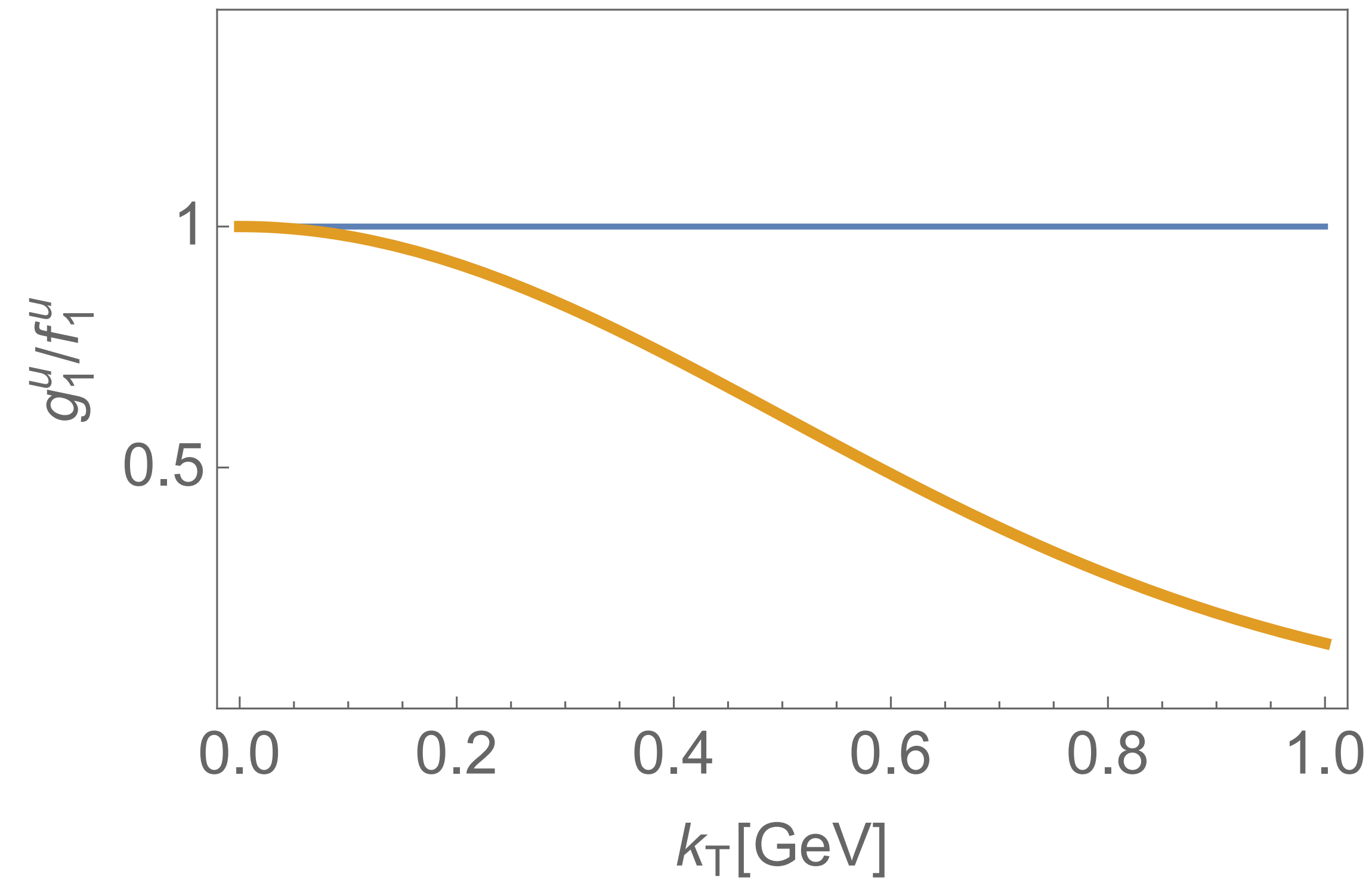
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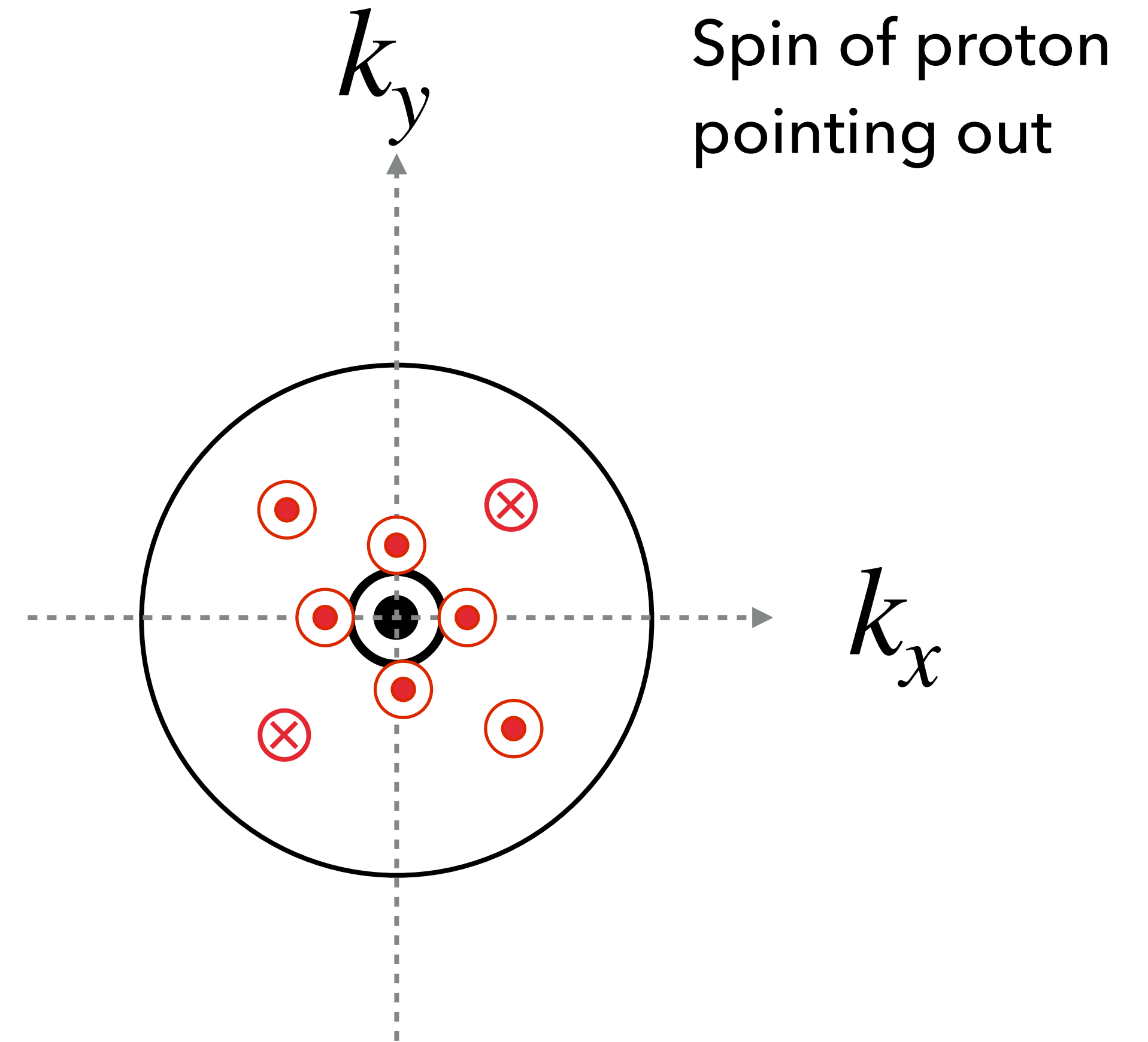
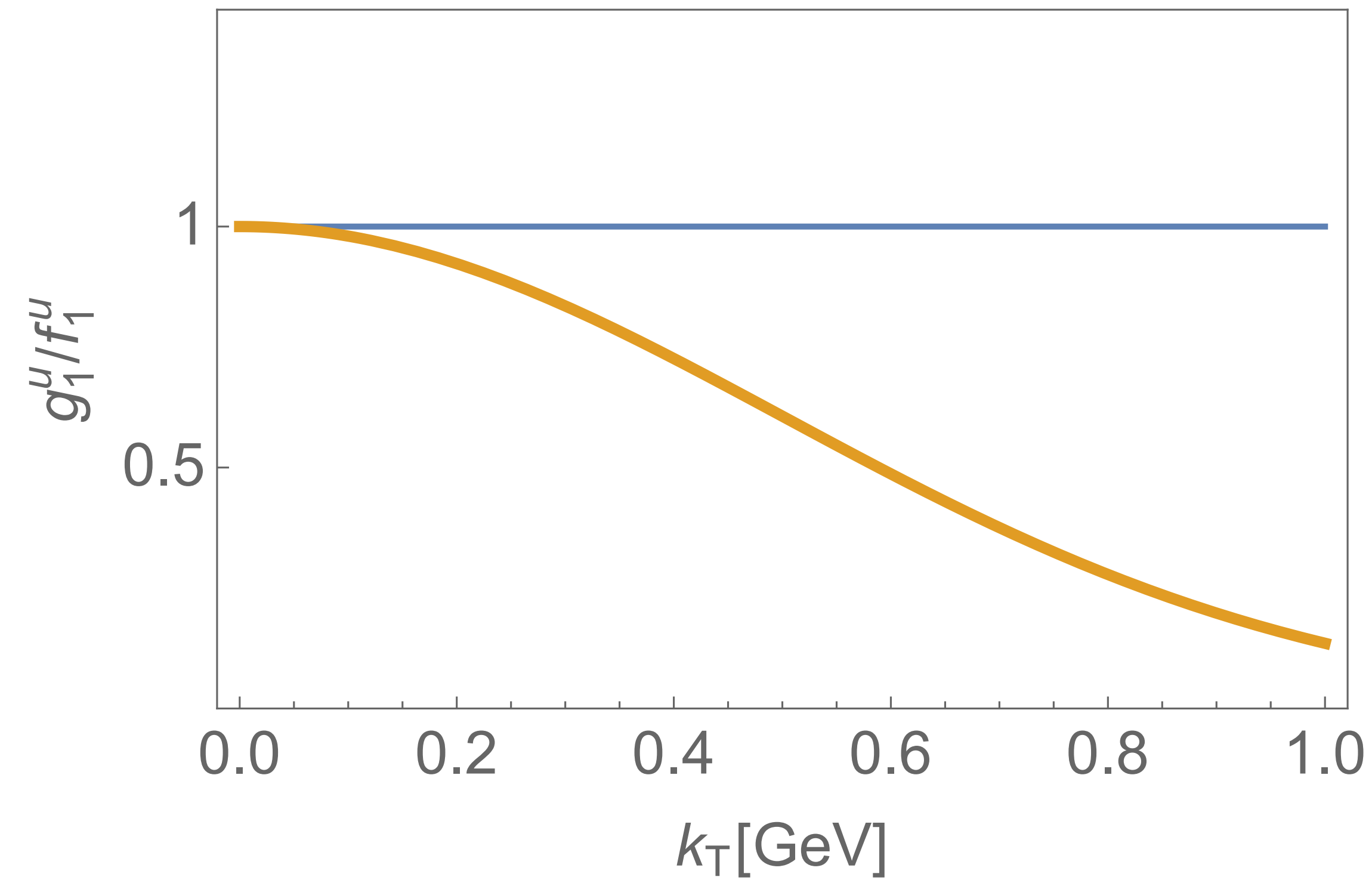
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The fraction of same/opposite helicities is the same at any transverse momentum

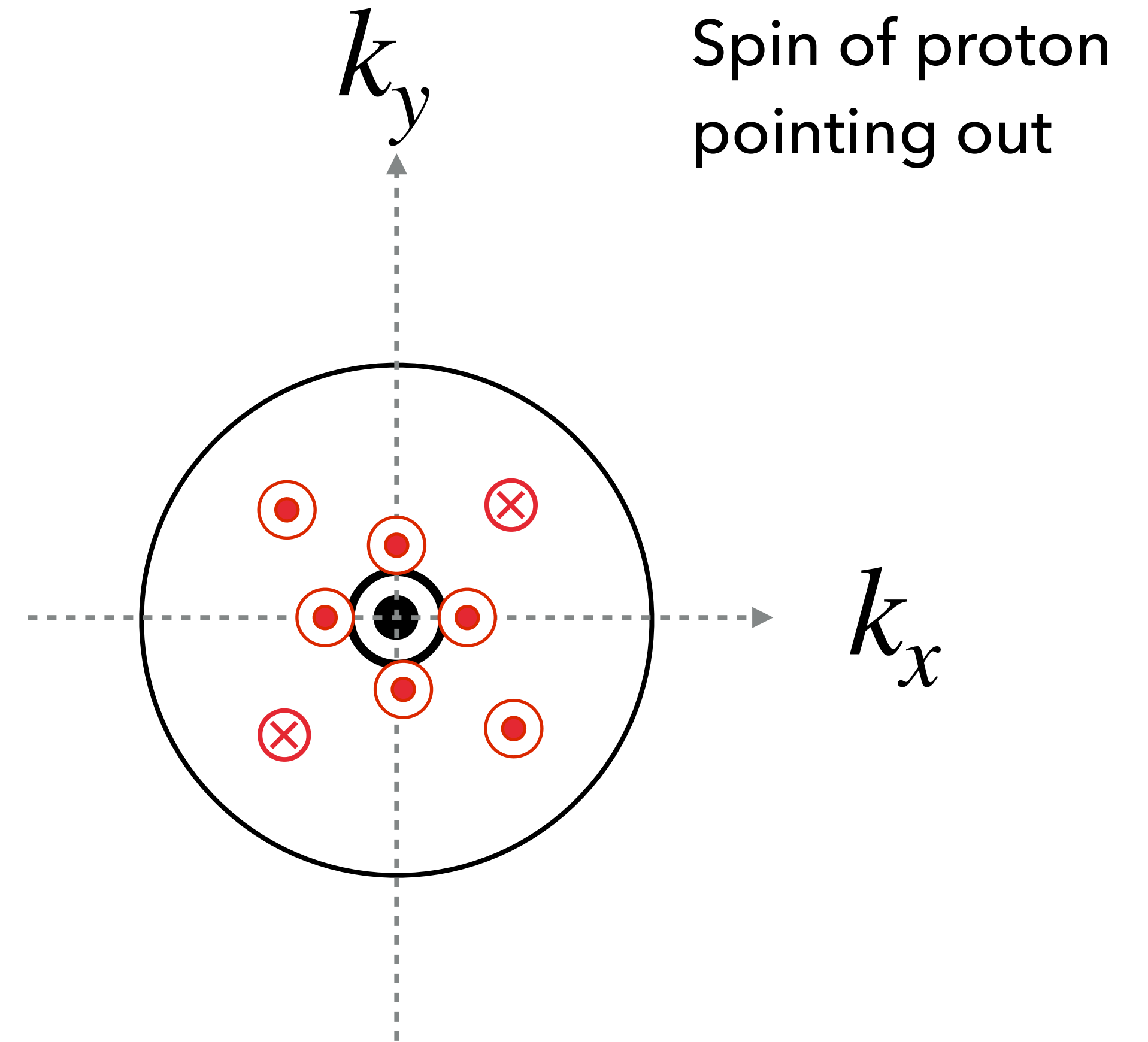
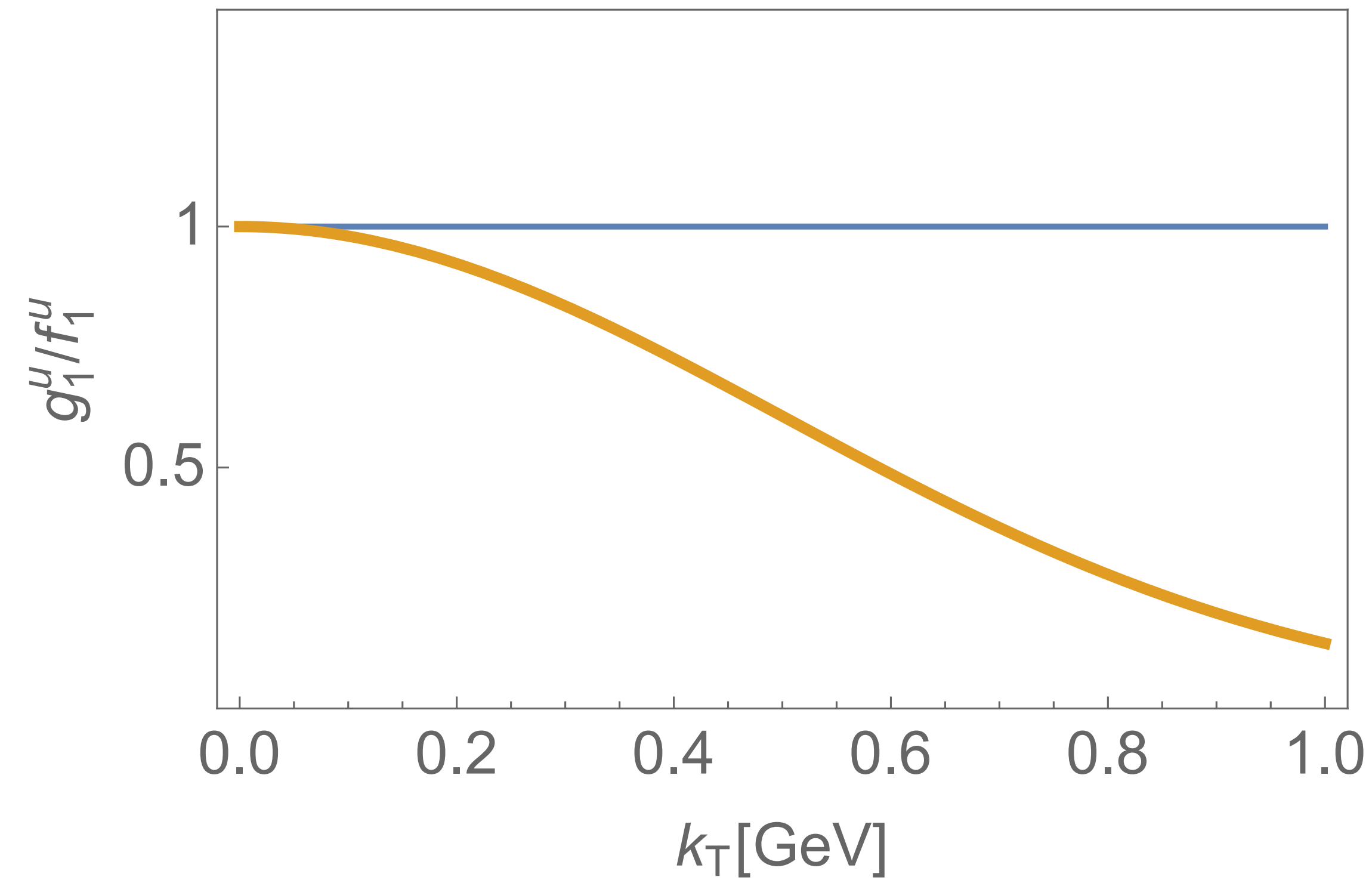
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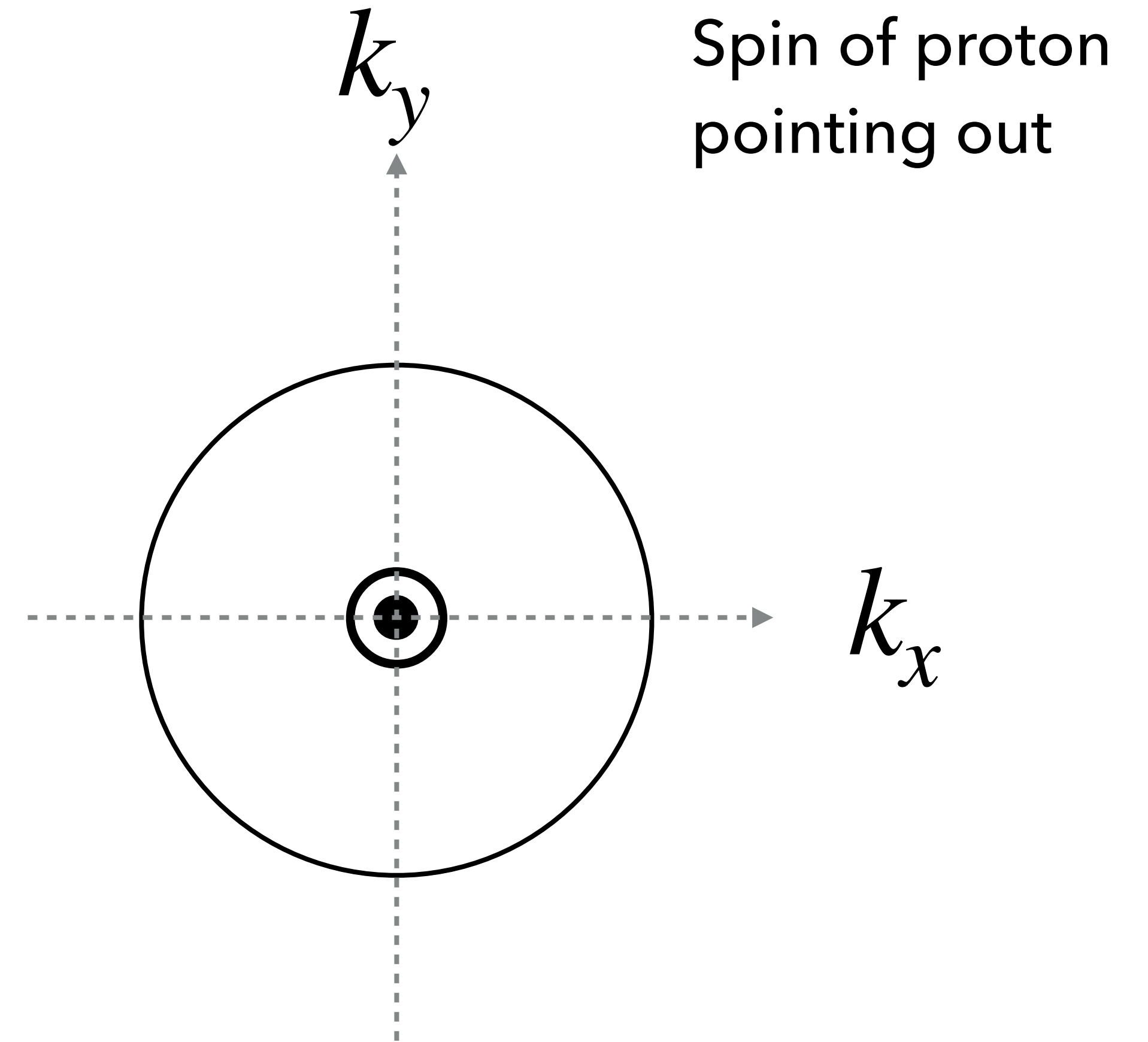
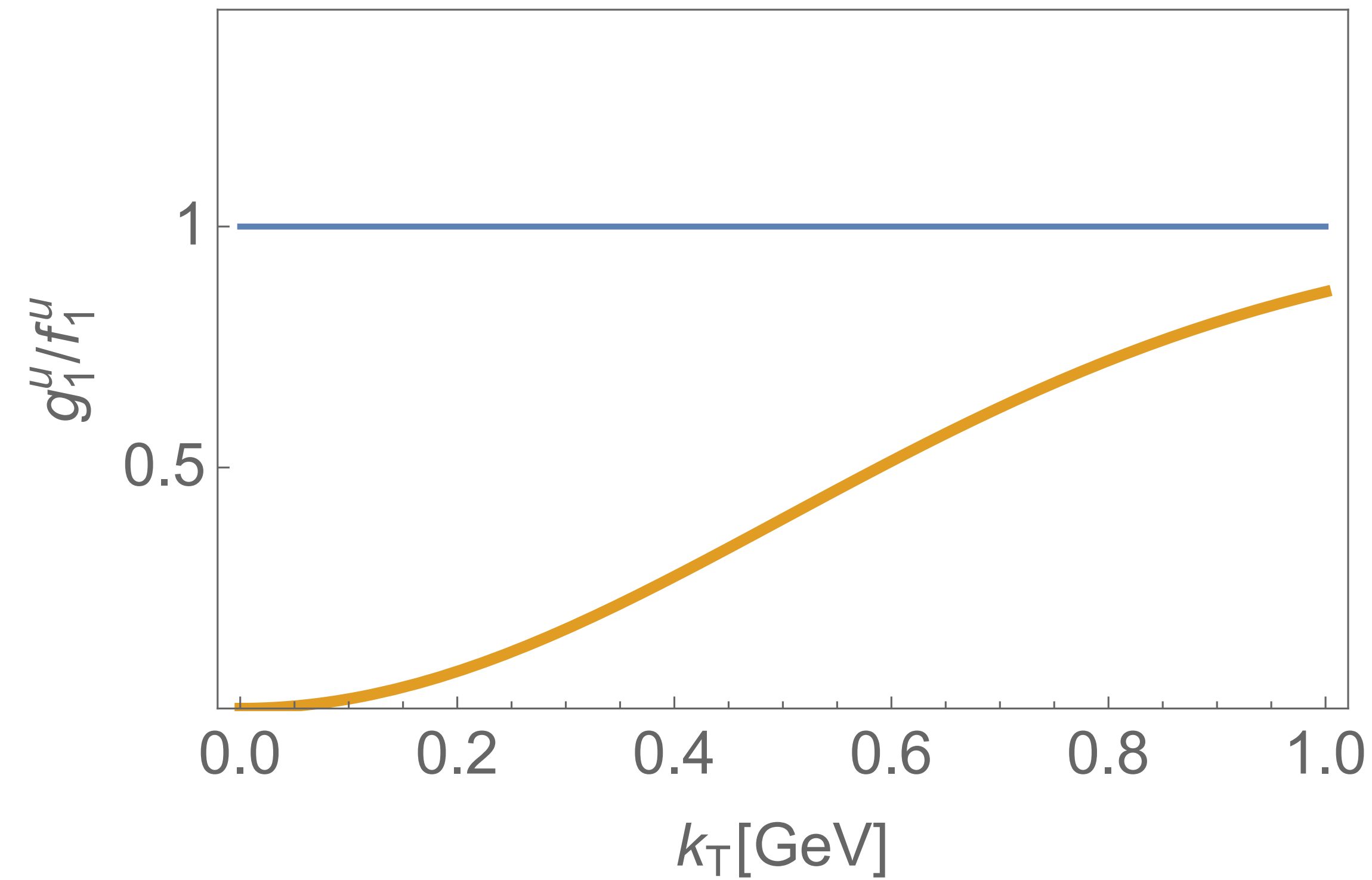


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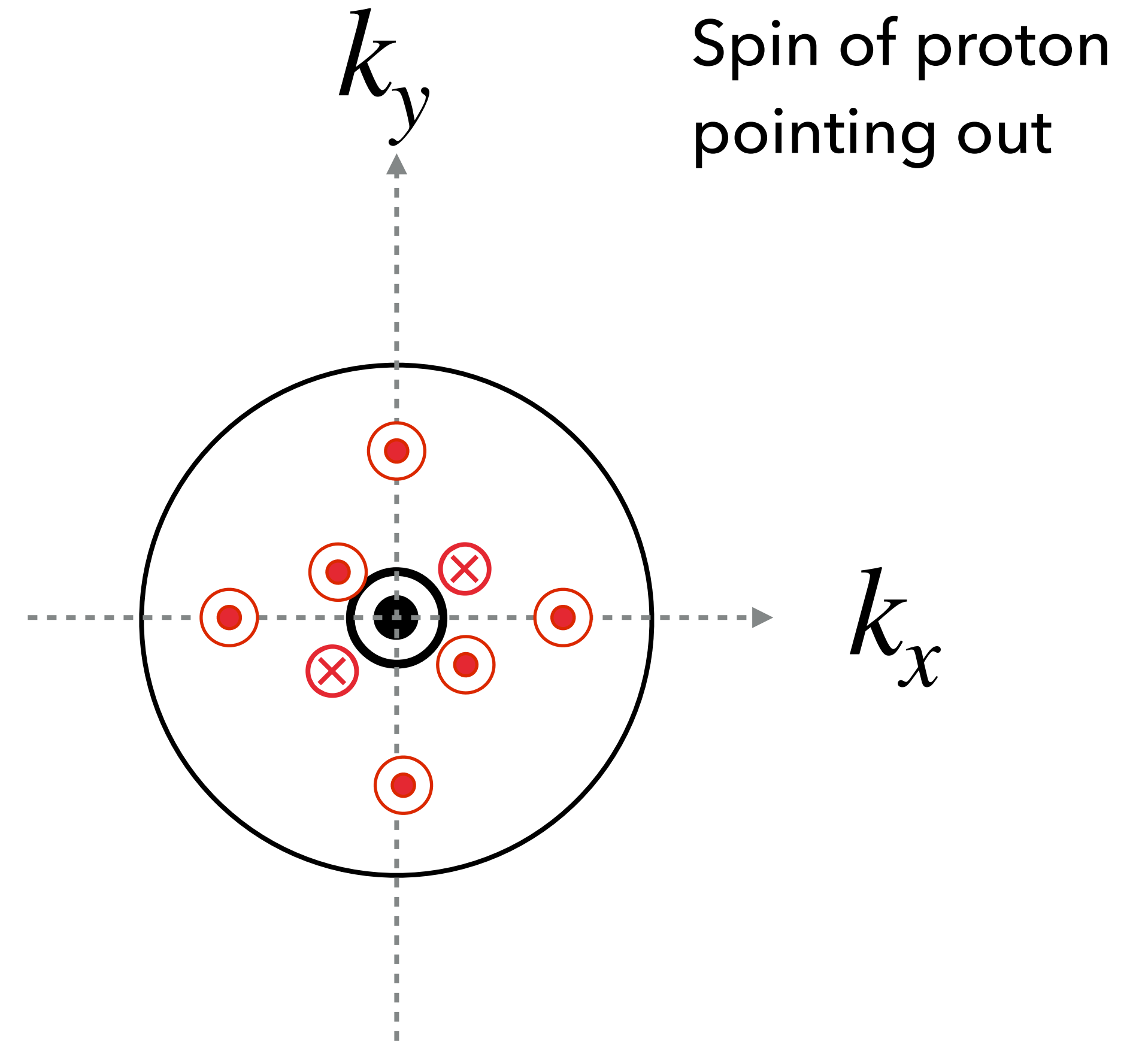
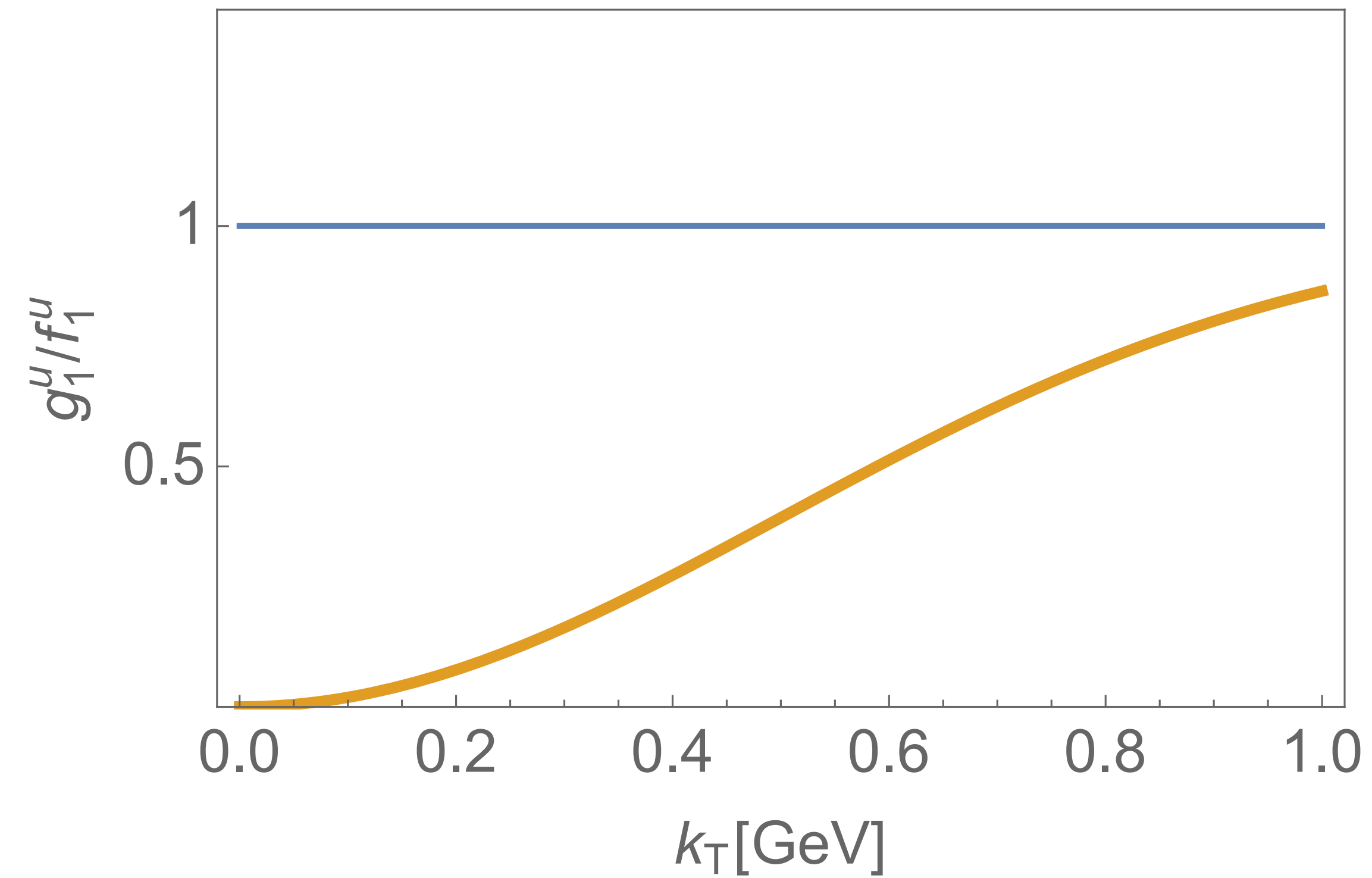


The quarks with the same helicity as the proton's have less transverse momentum

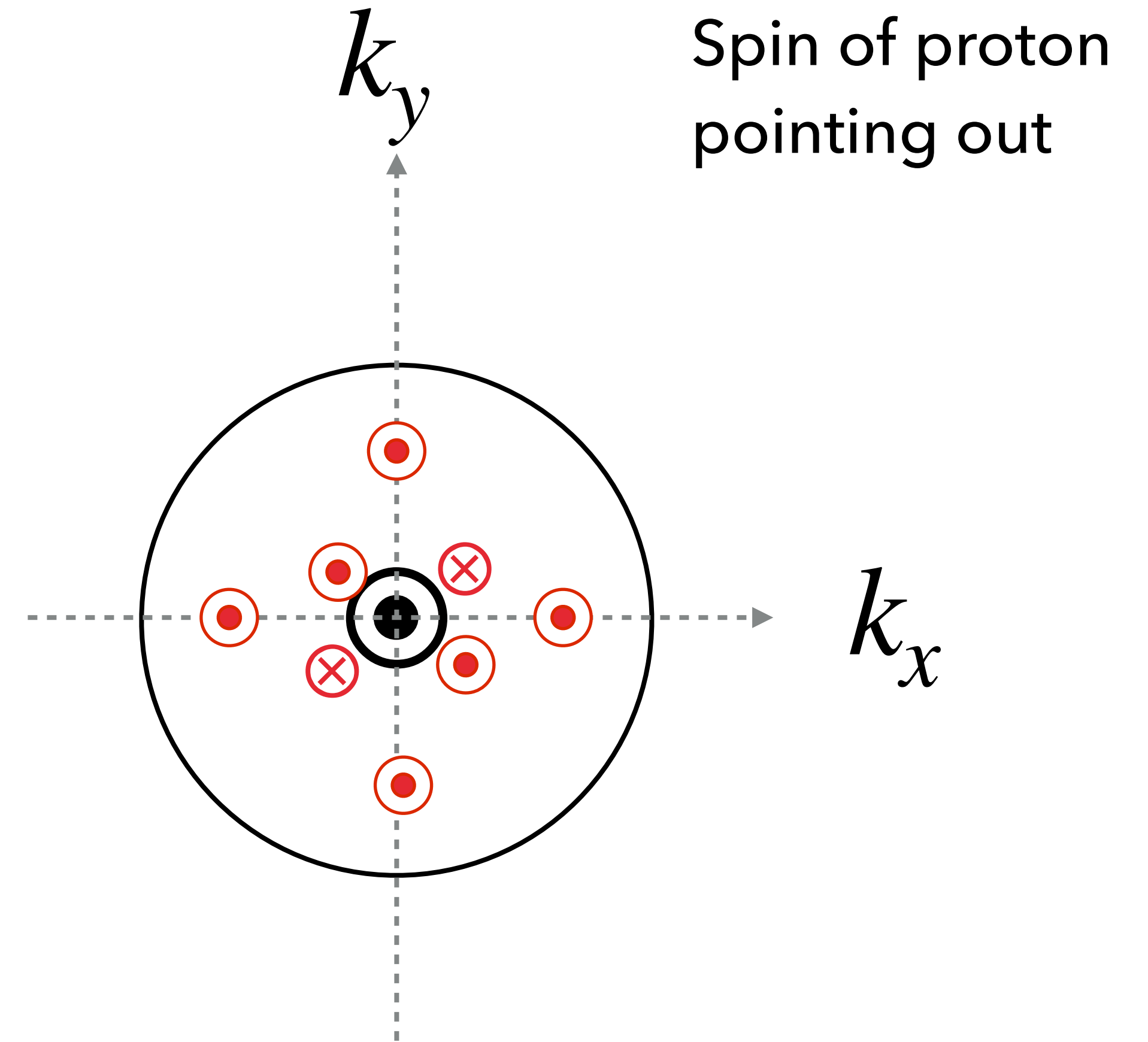
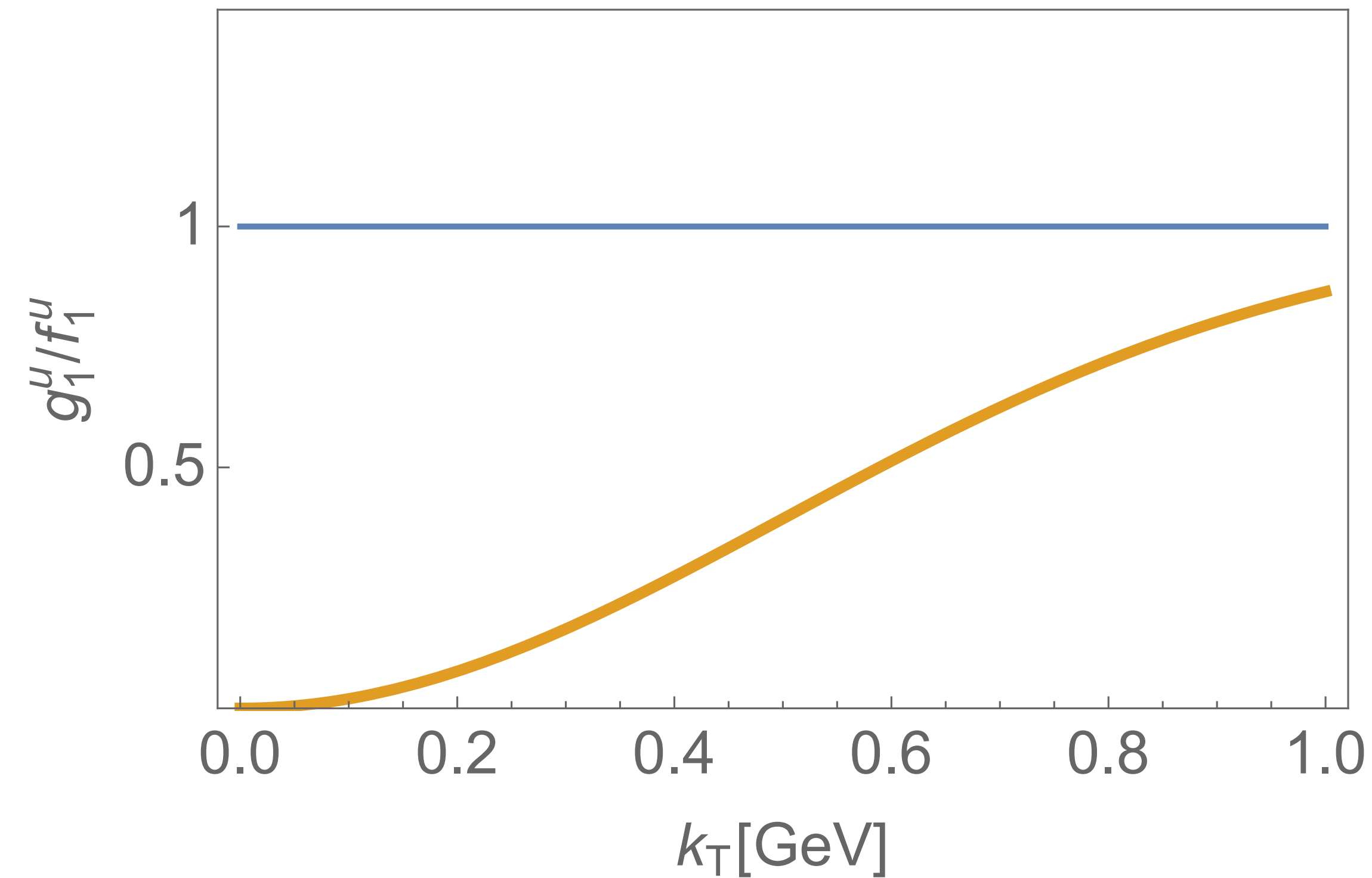
At a certain value of x



At a certain value of x



At a certain value of x

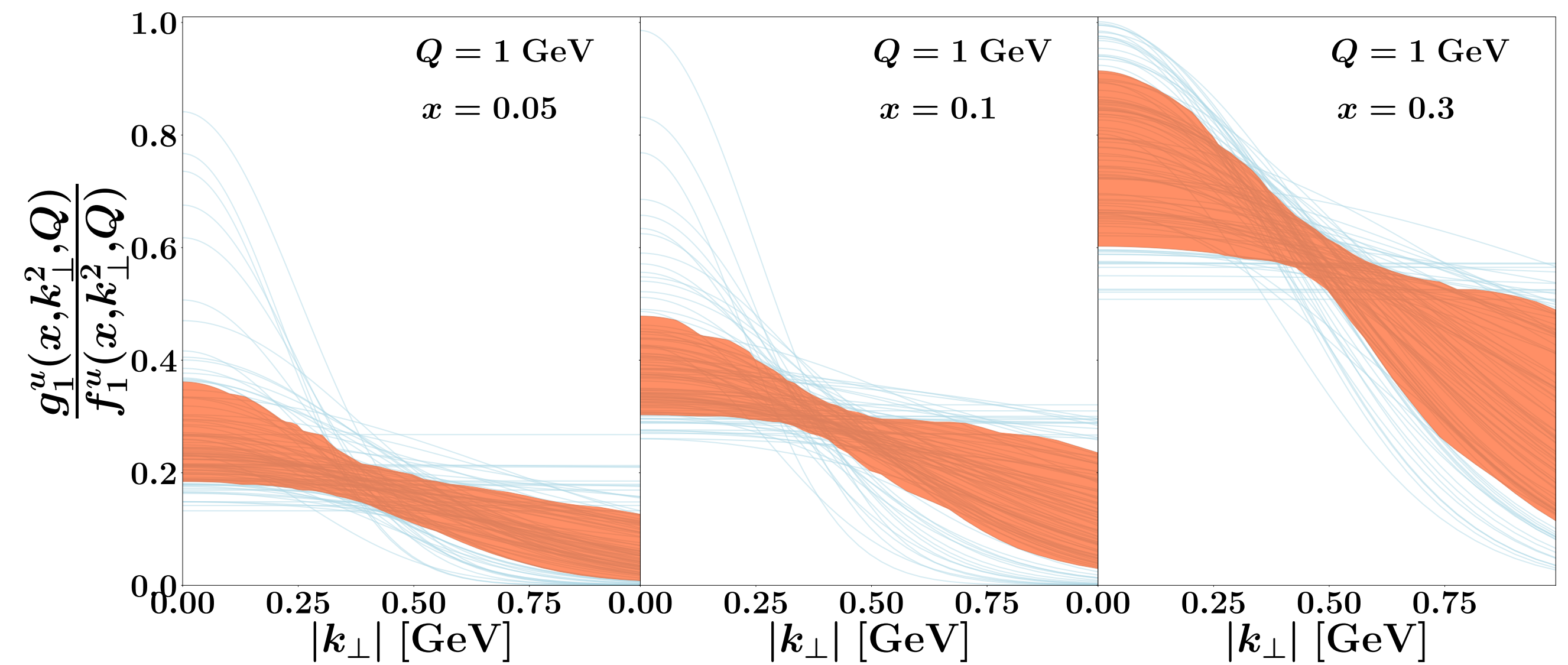
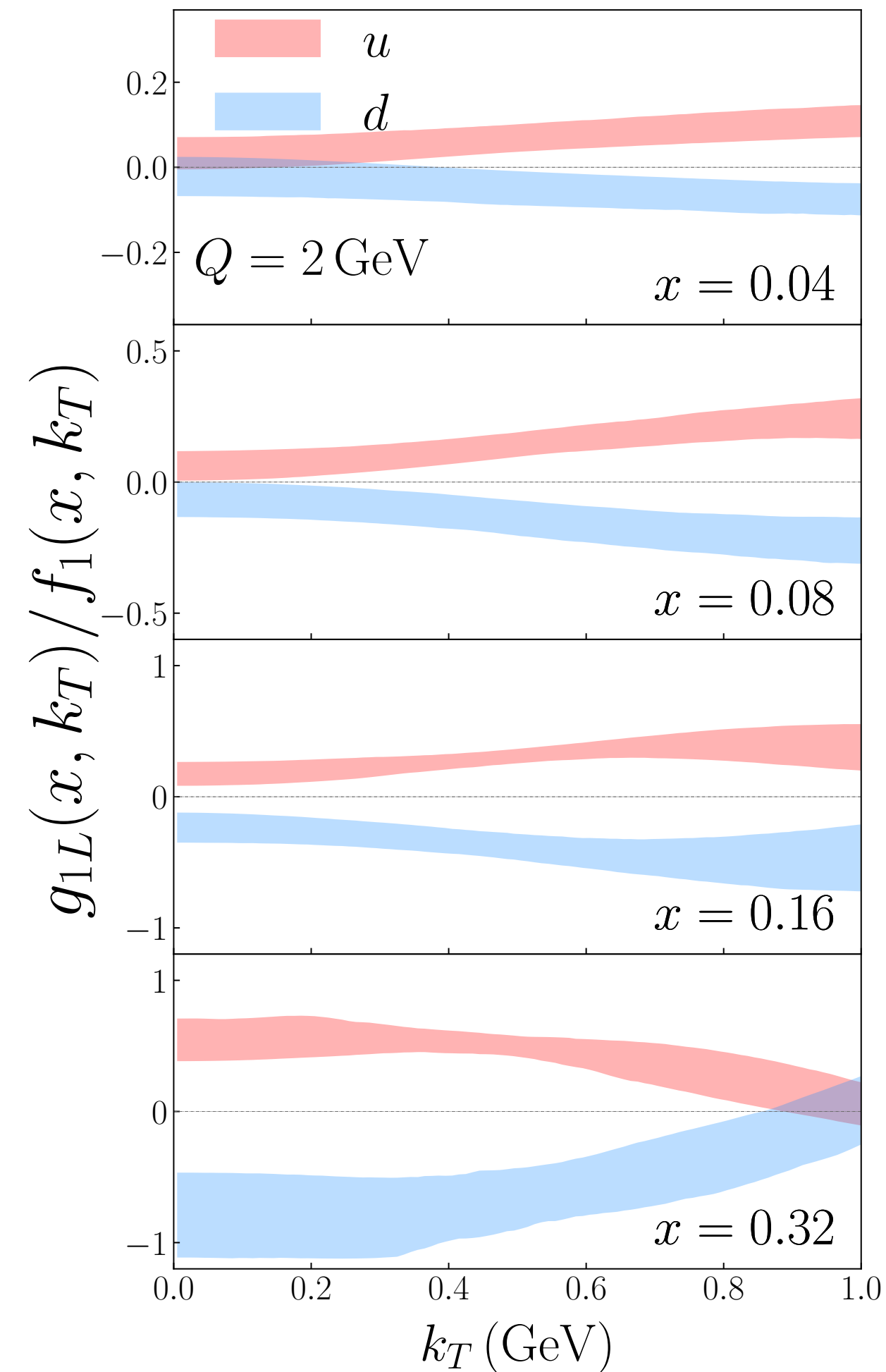


The quarks with the same helicity as the proton's have more transverse momentum

TWO (EXTREMELY) RECENT EXTRACTIONS

[Yang, Liu, Sun, Zhao, Ma, arXiv:2409.08110](#)

[MAP collaboration, arXiv:2409.18078](#)



[Yang, Liu, Sun, Zhao, Ma, arXiv:2409.08110](#)

Experiment	Process	Data points	χ^2/N
HERMES[82]	$e^\pm p \rightarrow e^\pm hX$	84 (160)	0.72
HERMES[82]	$e^\pm d \rightarrow e^\pm hX$	160 (317)	0.71
CLAS[83]	$e^- p \rightarrow e^- \pi^0 X$	9 (21)	1.43
Total		253 (498)	0.74

[MAP collaboration, arXiv:2409.18078](#)

Experiment	N_{dat}	$\chi^2_{\text{NLL}}/N_{\text{dat}}$	$\chi^2_{\text{NNLL}}/N_{\text{dat}}$
HERMES ($d \rightarrow \pi^+$)	47	1.34	1.30
HERMES ($d \rightarrow \pi^-$)	47	1.10	1.08
HERMES ($d \rightarrow K^+$)	46	1.26	1.25
HERMES ($d \rightarrow K^-$)	45	0.93	0.89
HERMES ($p \rightarrow \pi^+$)	53	1.17	1.21
HERMES ($p \rightarrow \pi^-$)	53	0.86	0.86
Total	291	1.11	1.09

[Yang, Liu, Sun, Zhao, Ma, arXiv:2409.08110](#)

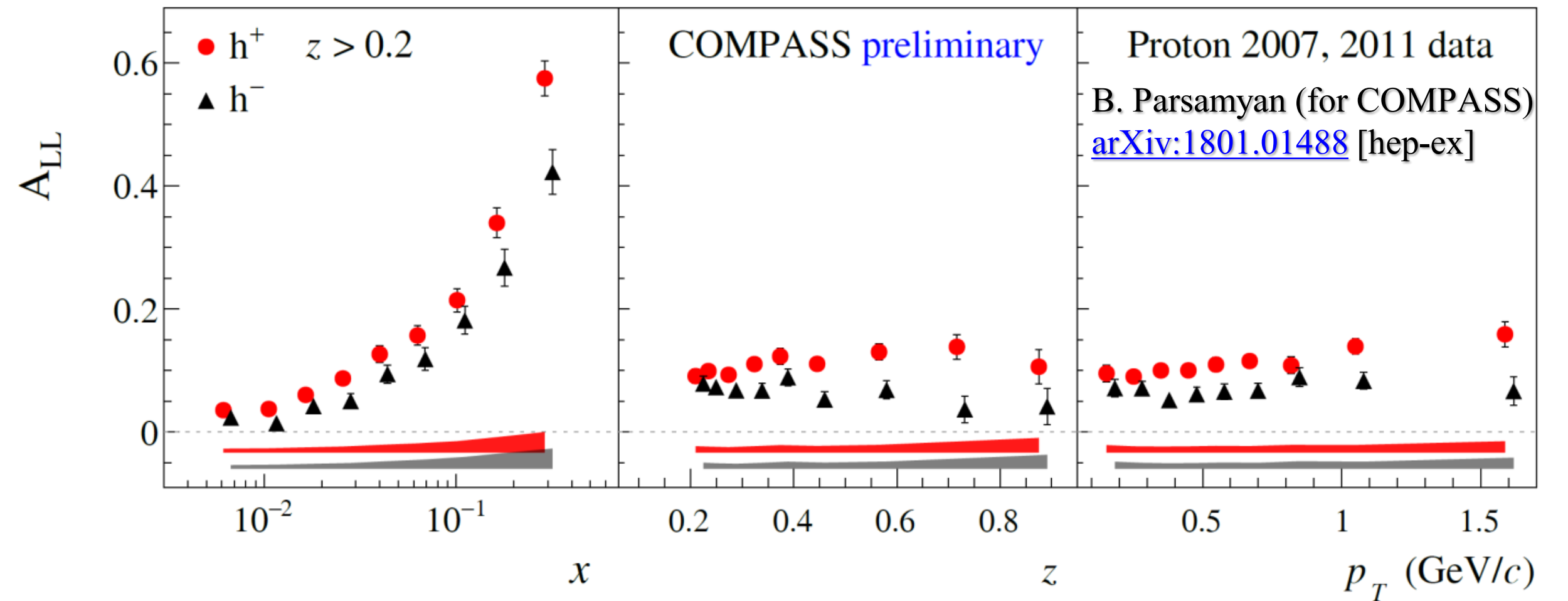
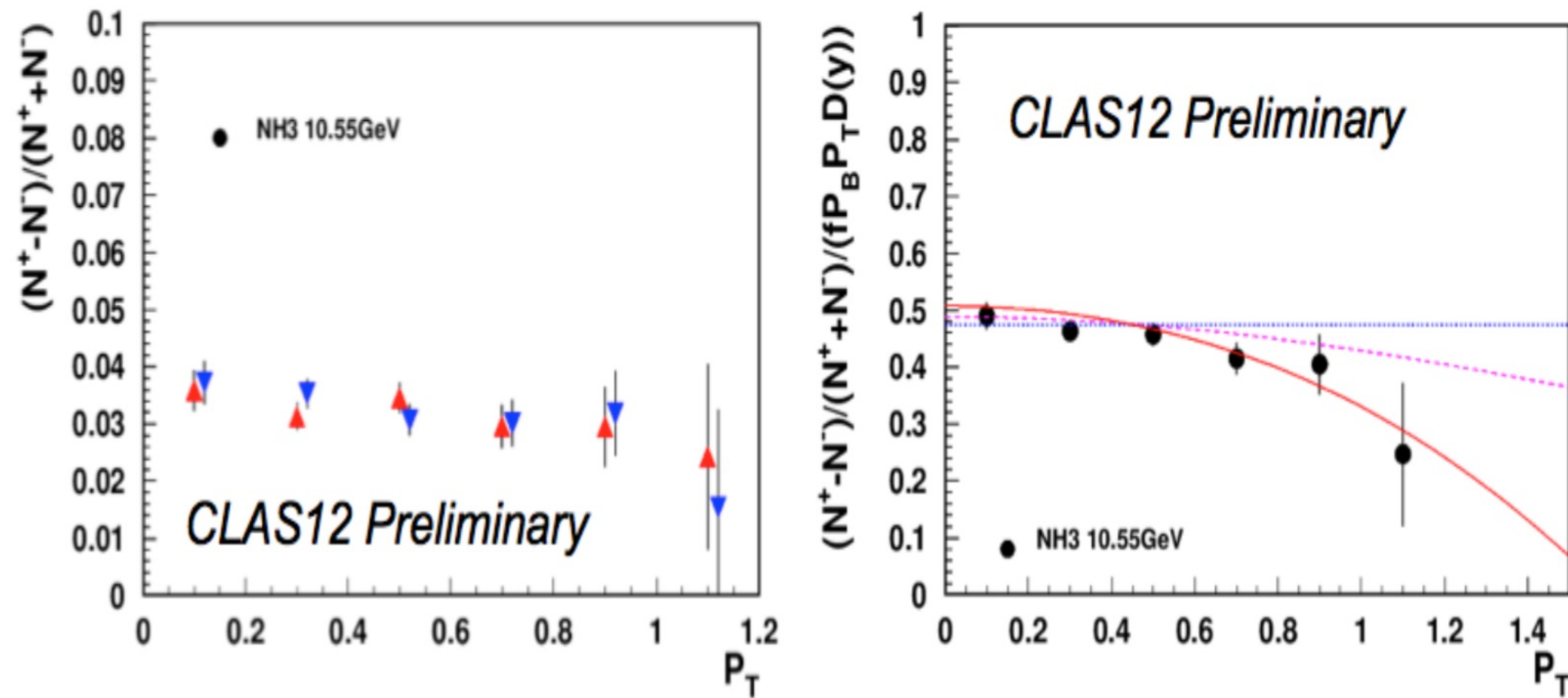
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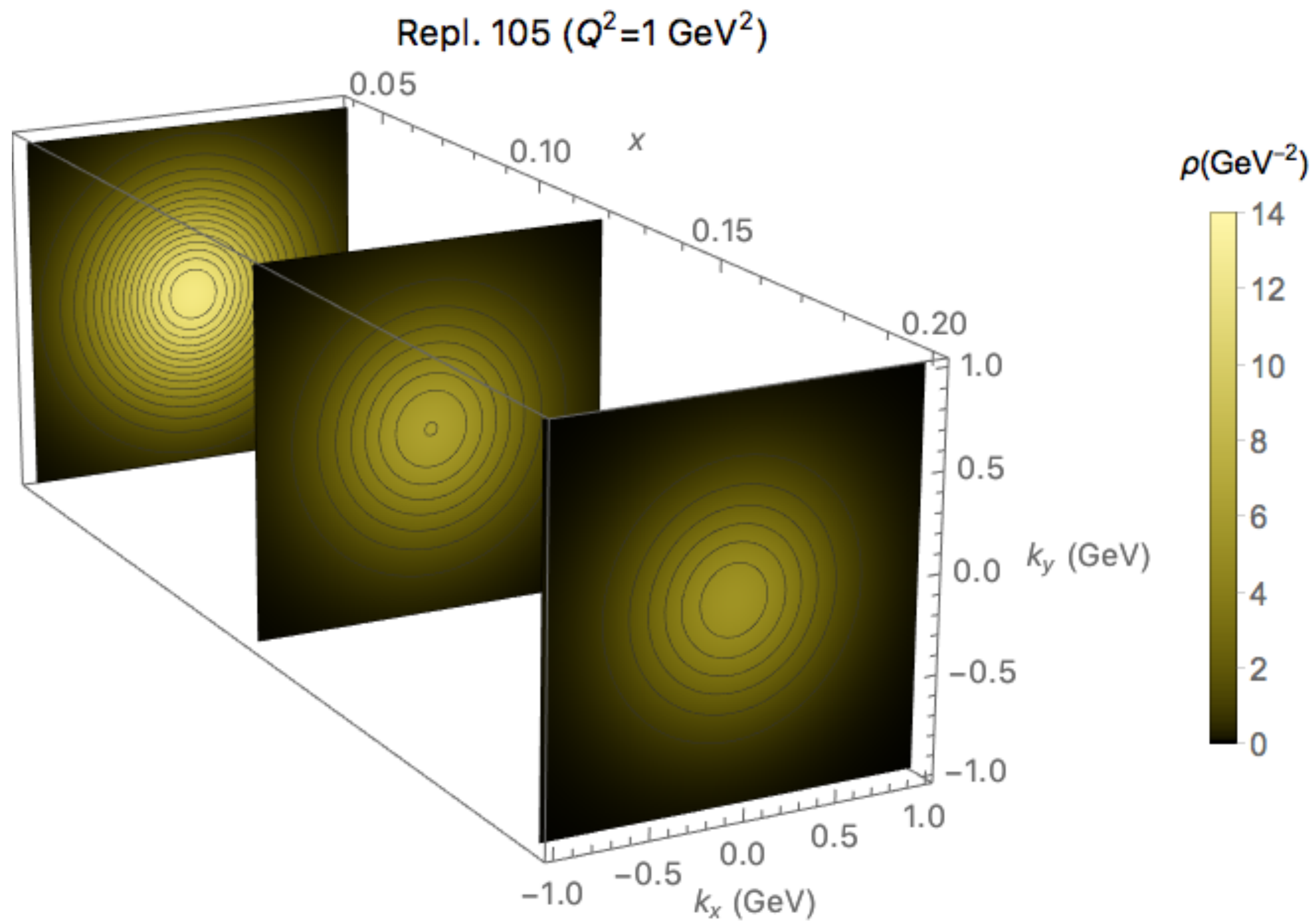
More data needed

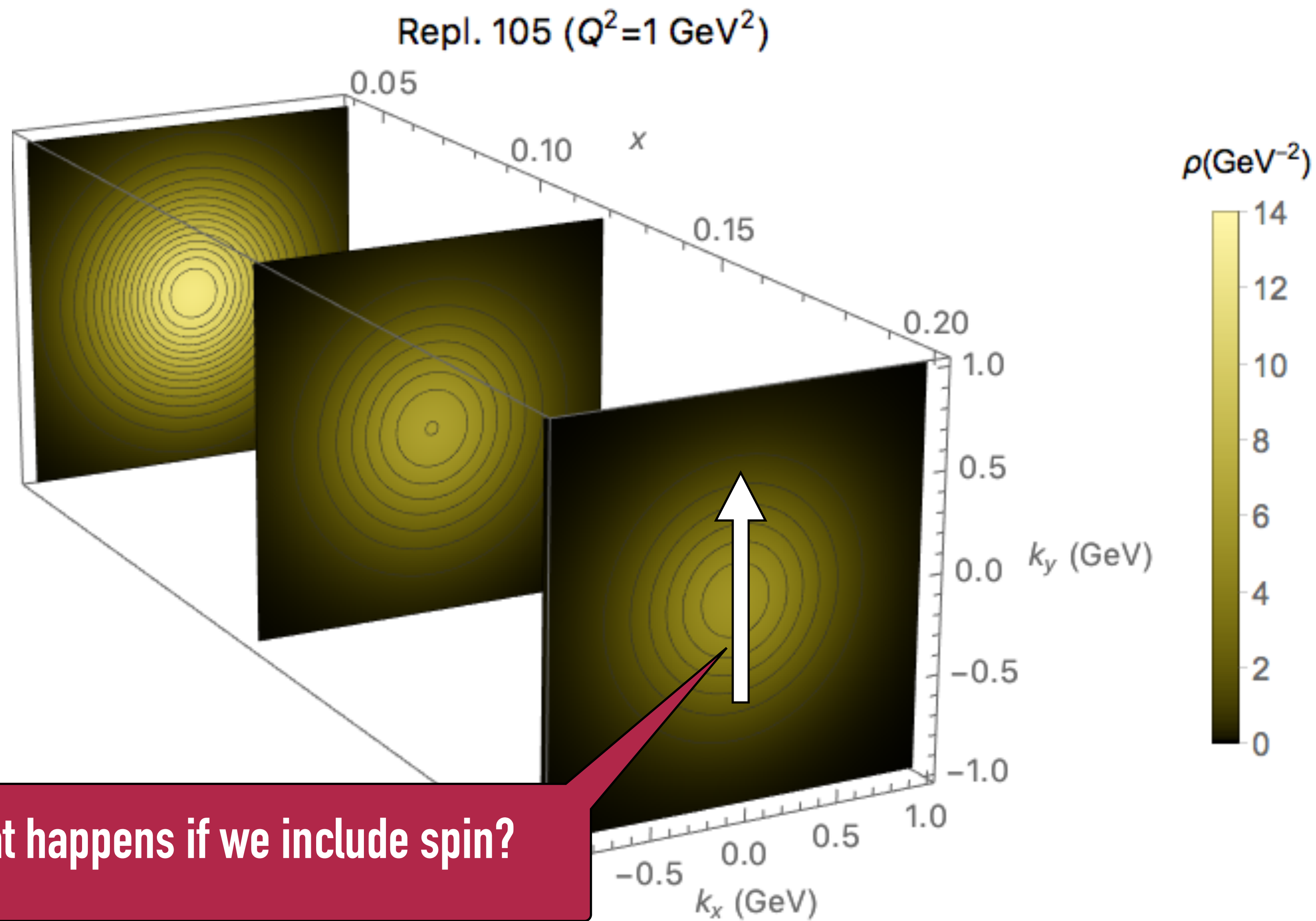
Multidimensional binning needed



See talks by Tim Hayward,
Bakur Parsamyan

SIVERS TMD





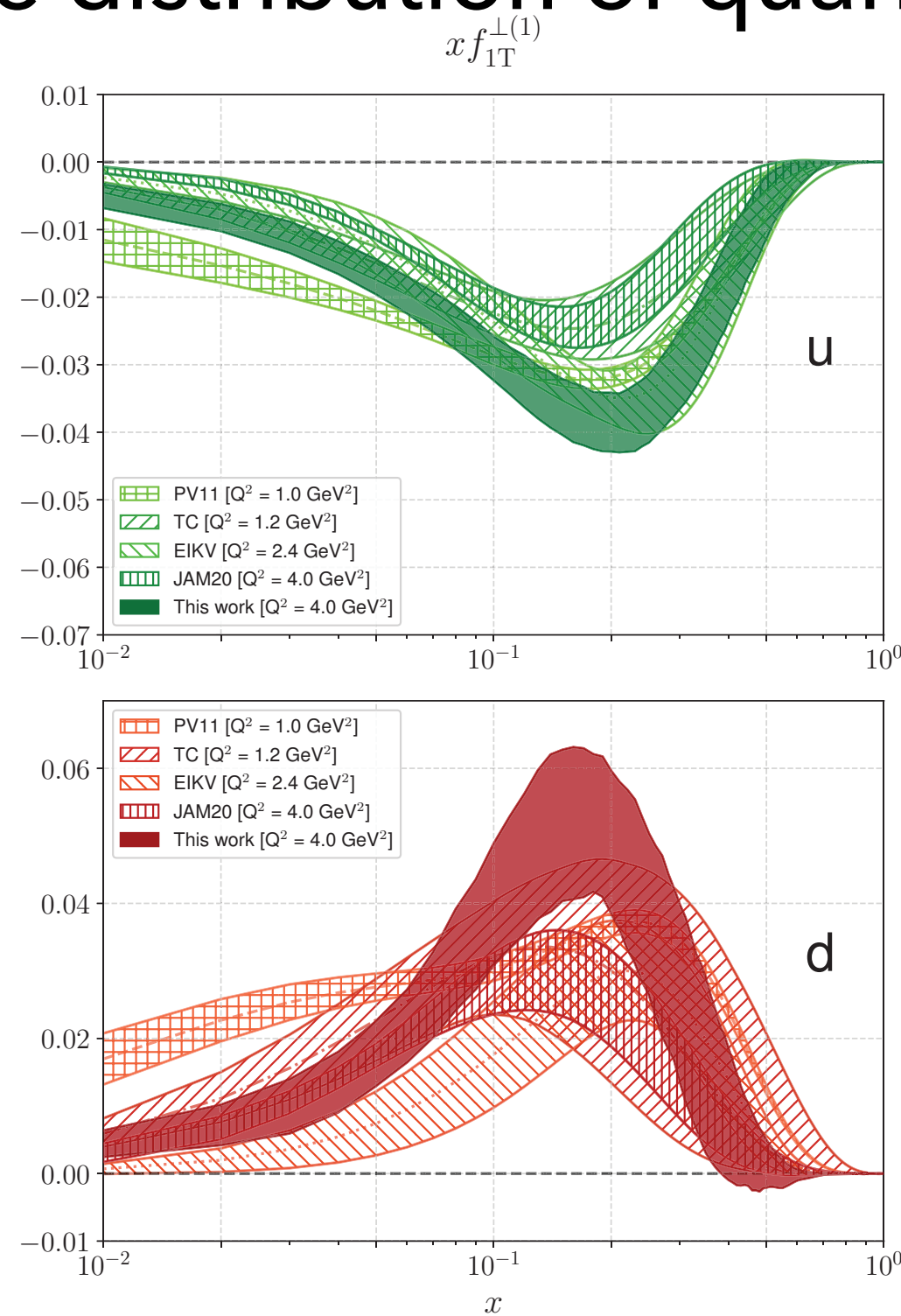
What happens if we include spin?

$$\rho_{N\uparrow}^q(x, k_x, k_y; Q^2) = f_1^q(x, k_T^2; Q^2) - \frac{k_x}{M} f_{1T}^{\perp q}(x, k_T^2; Q^2)$$

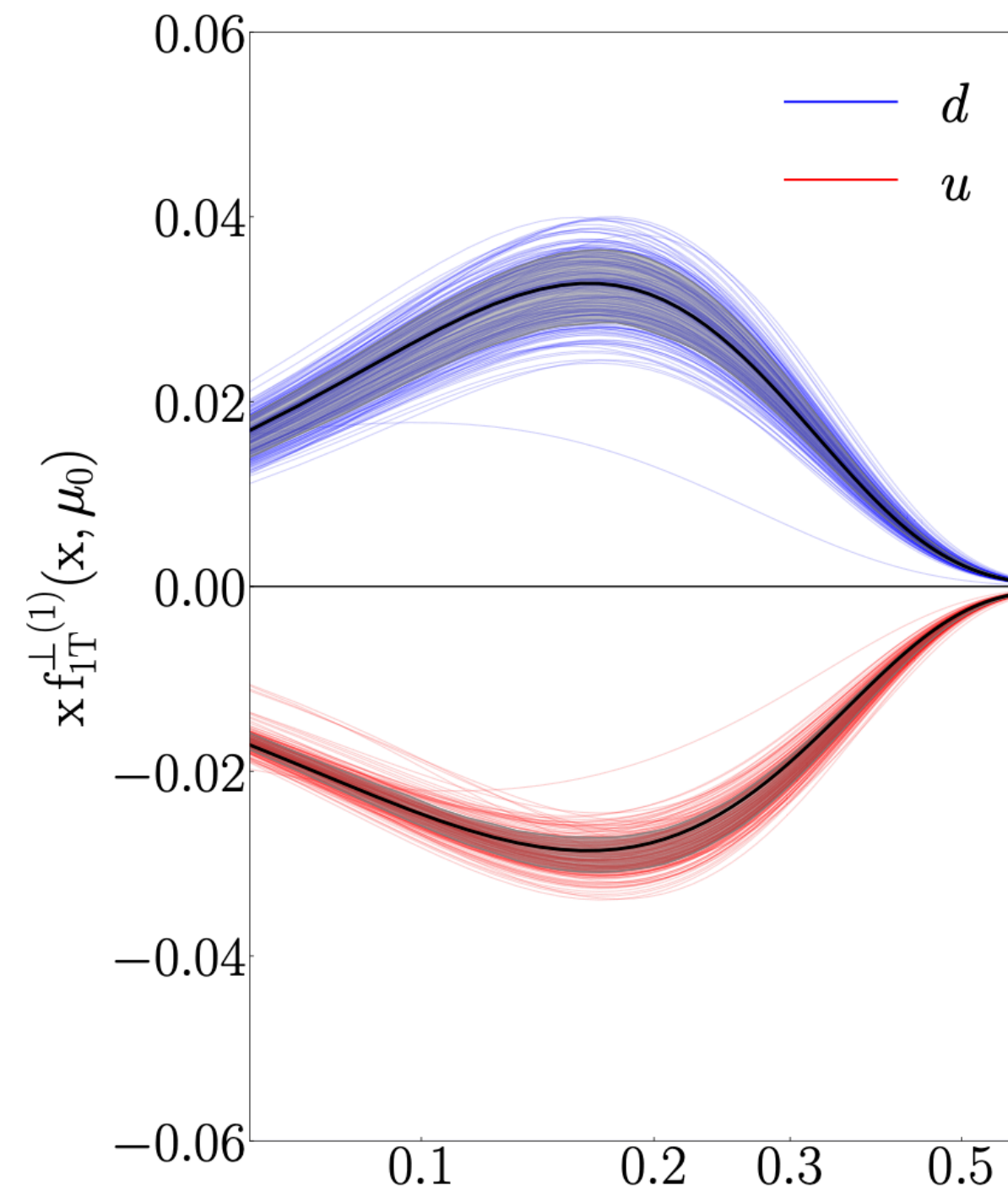
In a nucleon polarized in the +y direction,
the distribution of quarks can be distorted in the x direction

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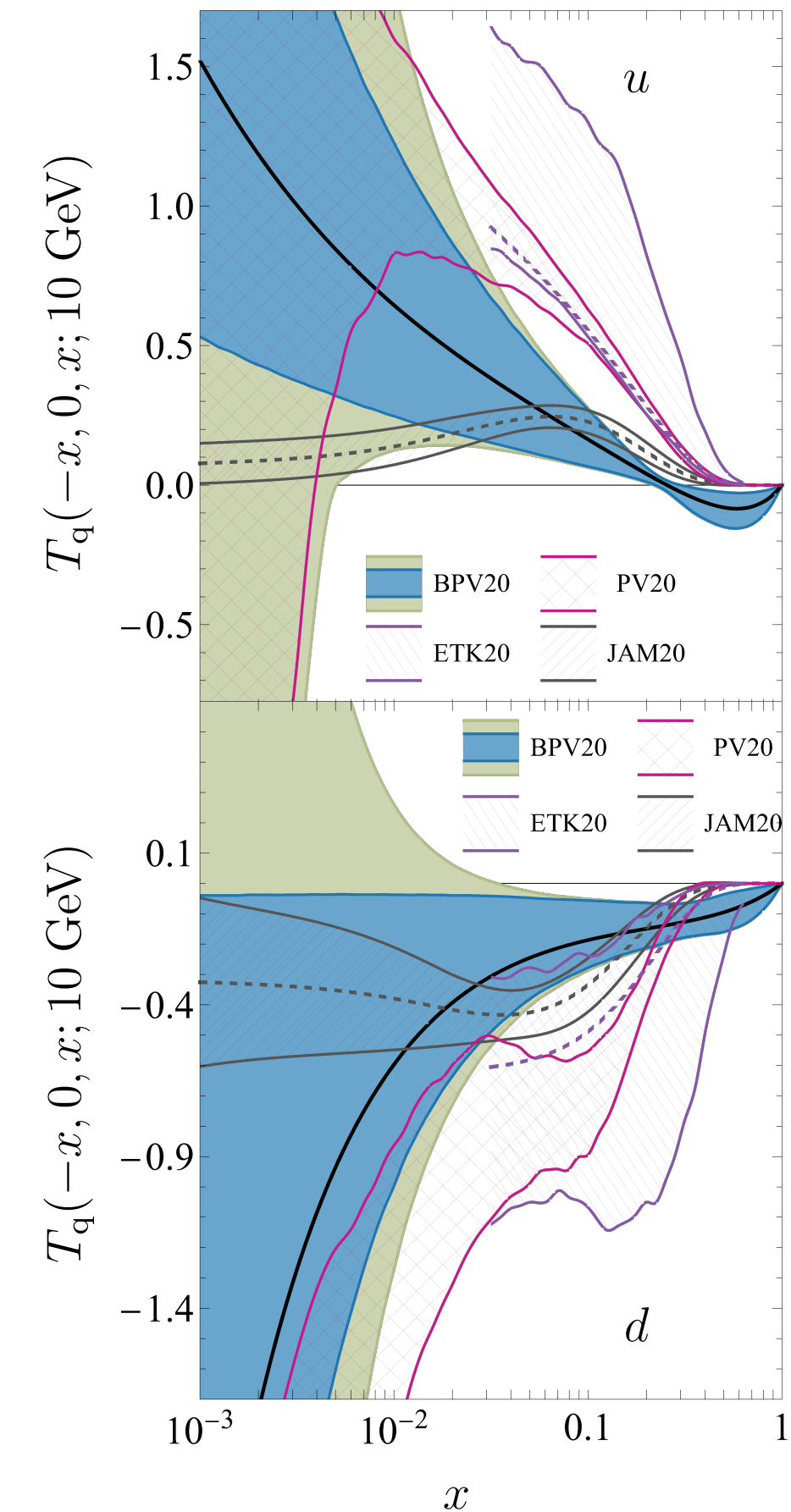
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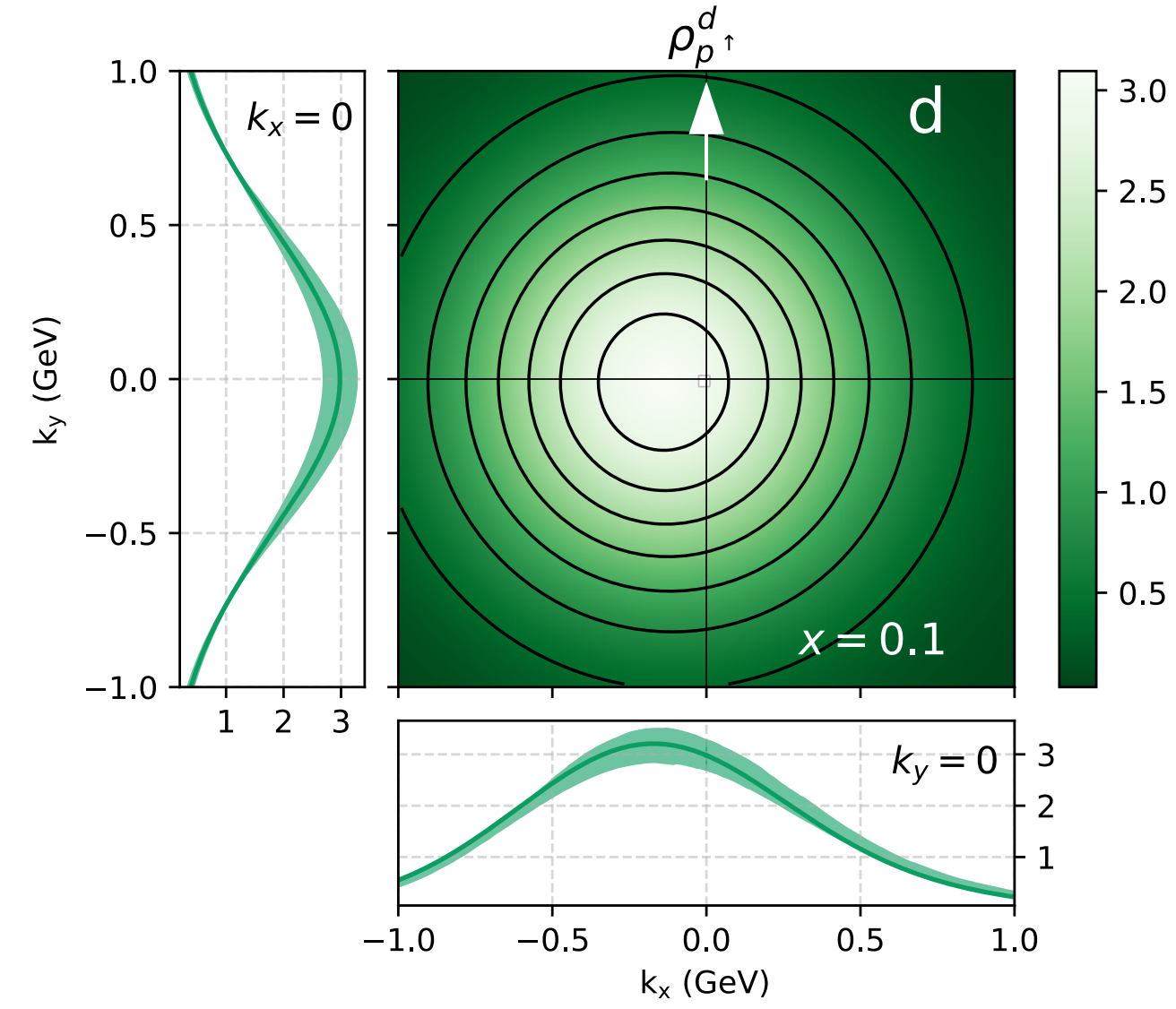
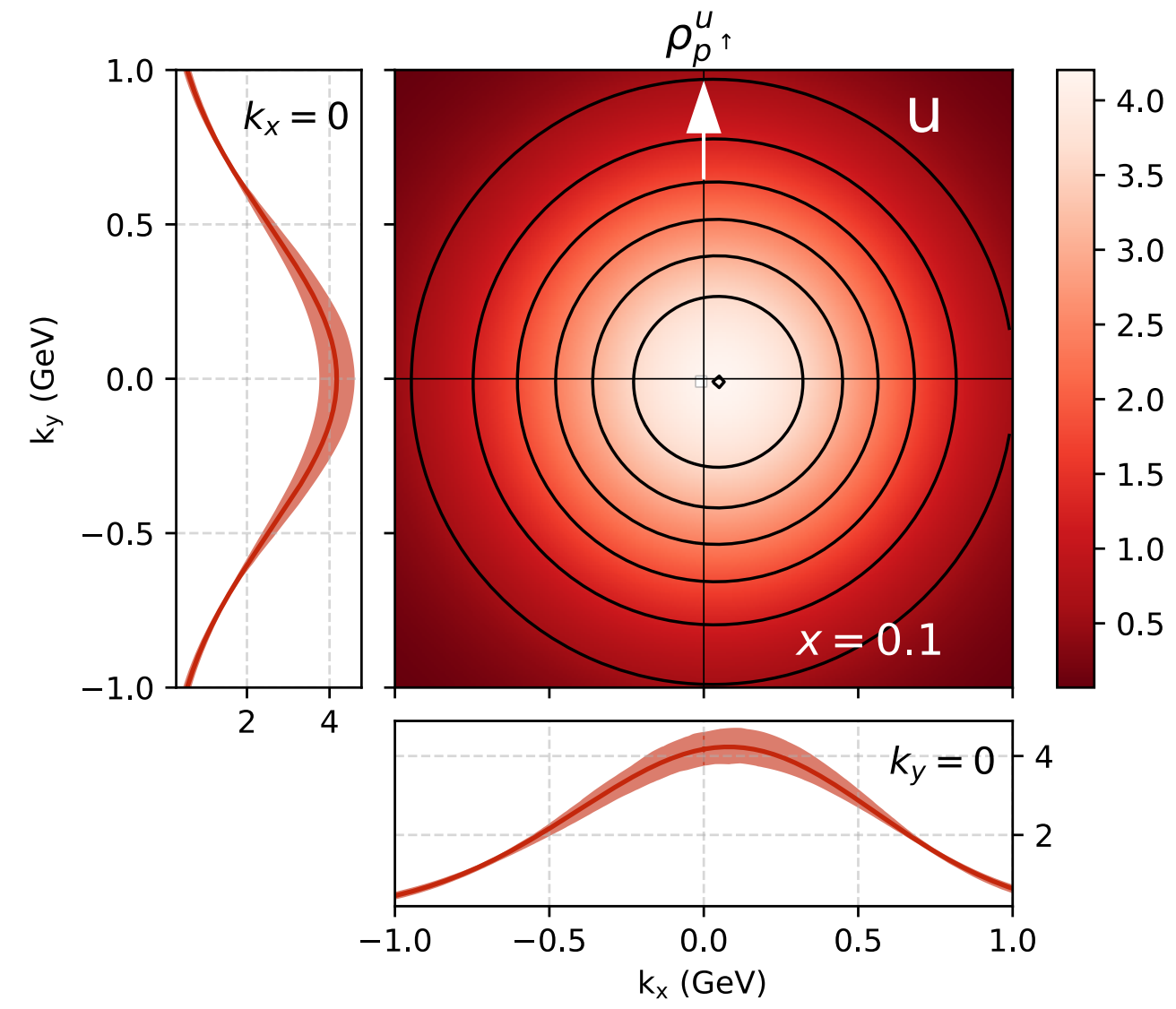
[Bacchetta, Delcarro, Pisano, Radici, arXiv:2004.14278](#)



[Echevarria, Kang, Terry, arXiv:2009.10710](#)



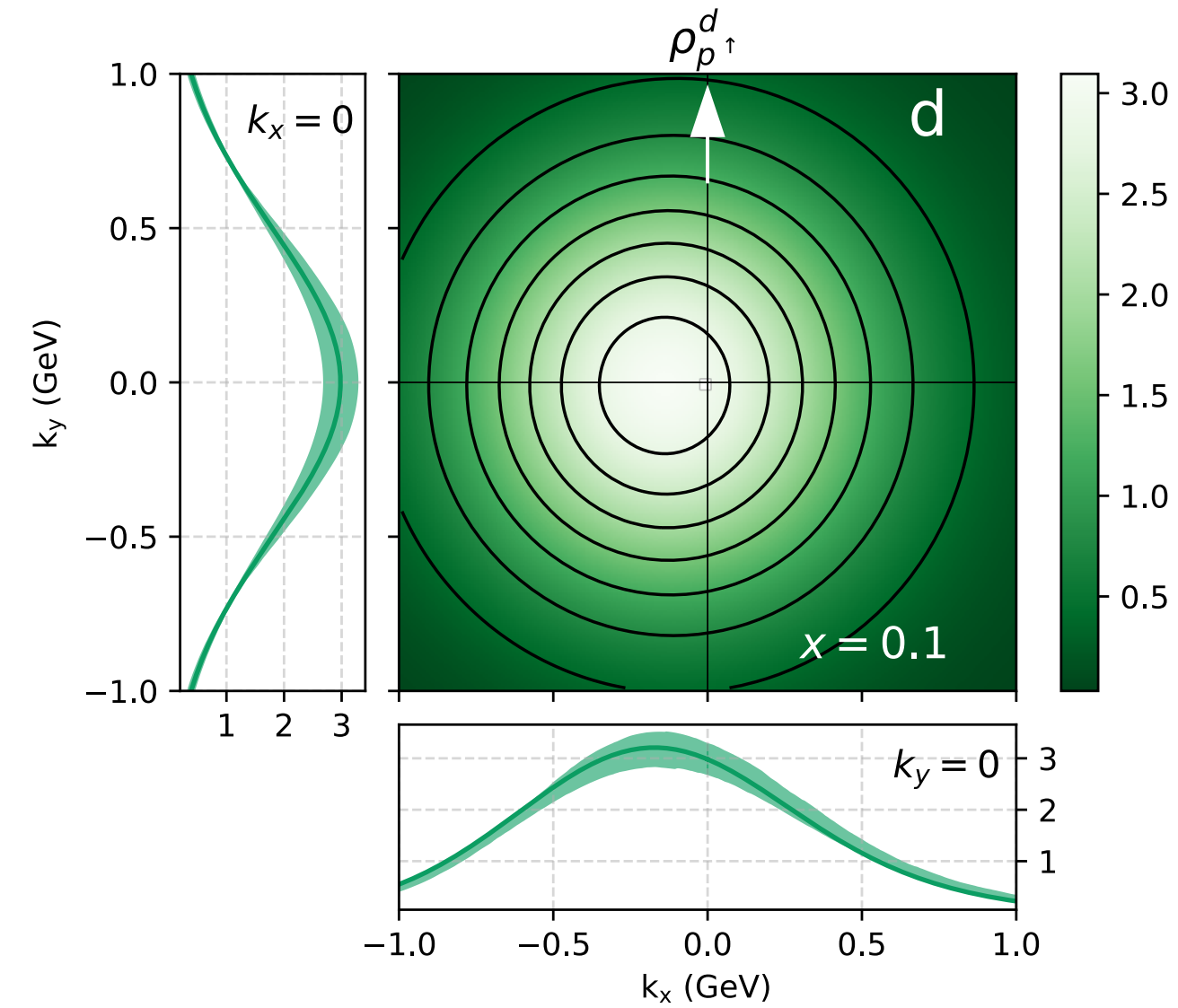
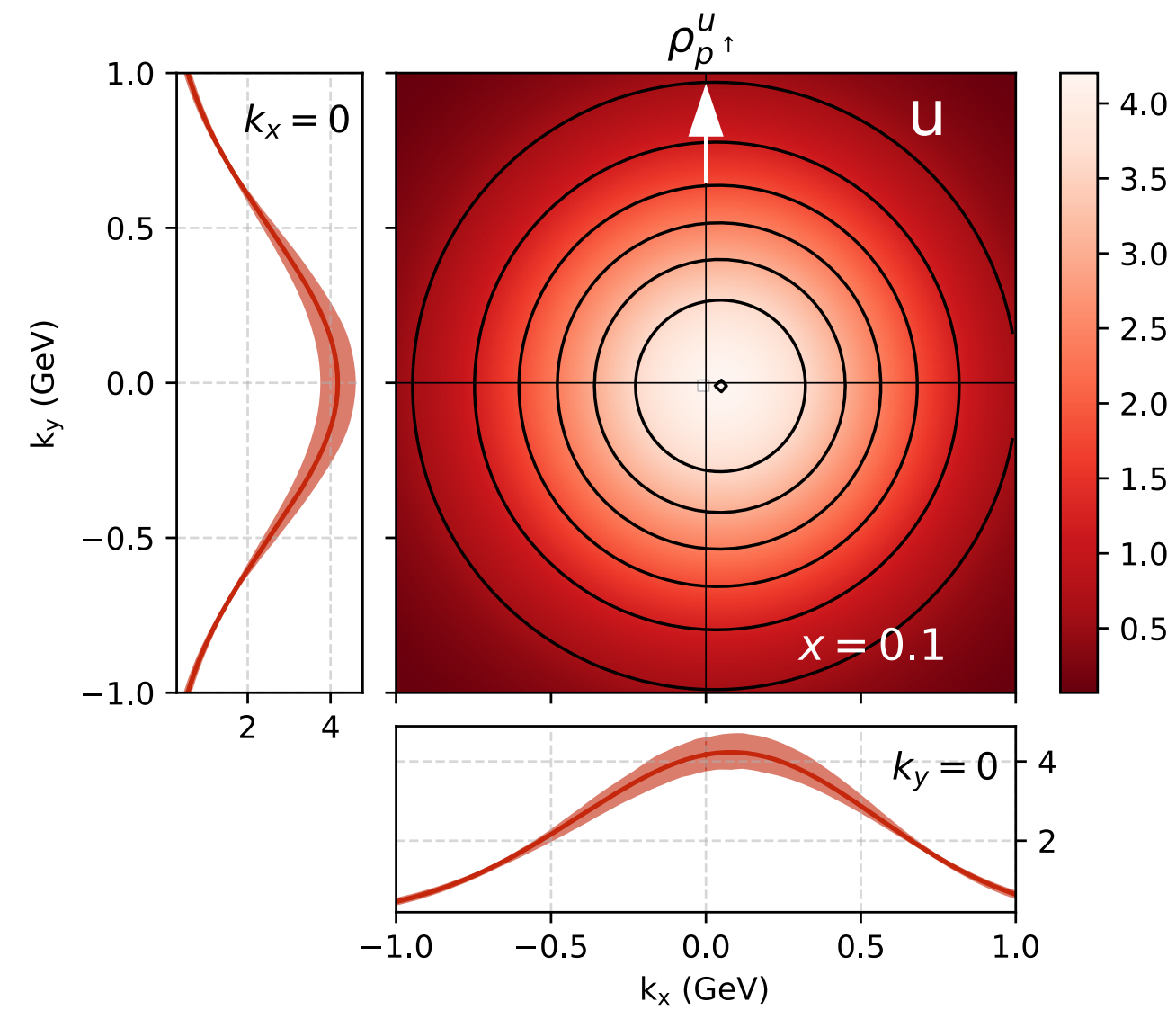
[Bury, Prokudin, Vladimirov, arXiv:2103.03270](#)



$Q = 2\text{GeV}$

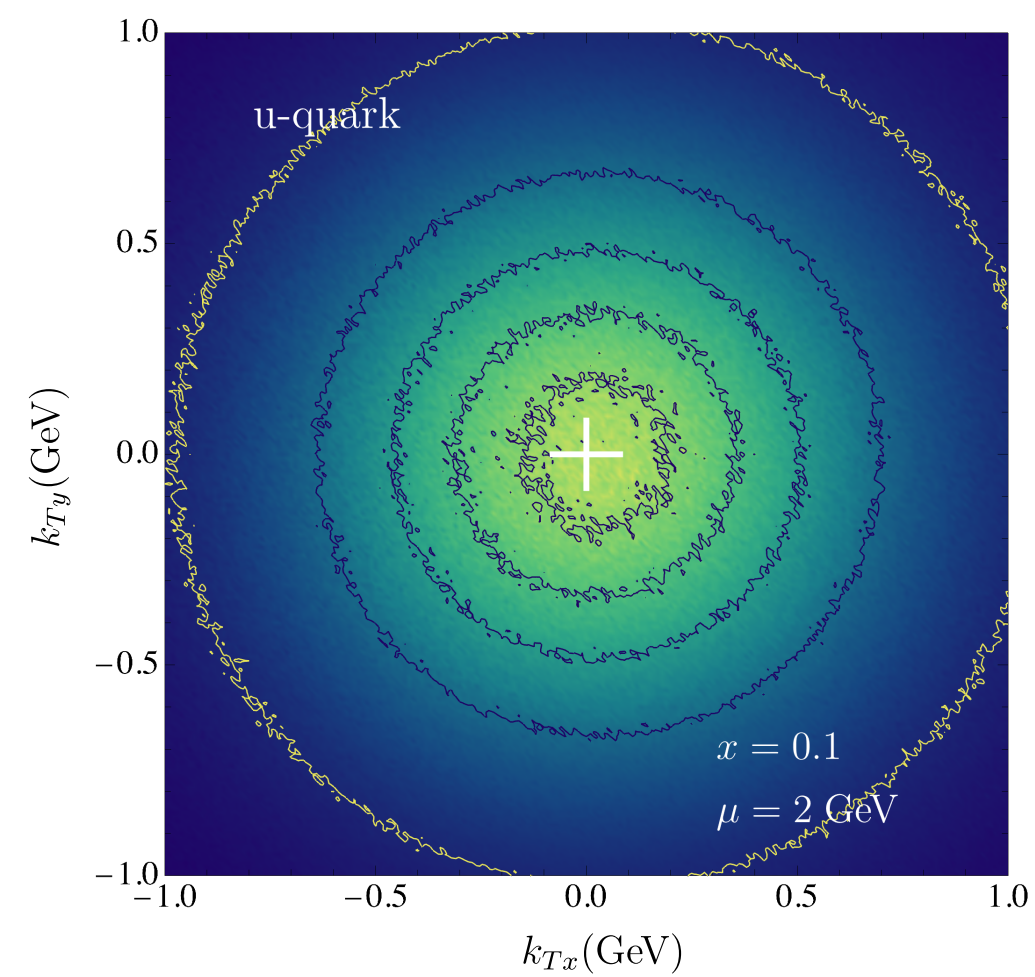
[Bacchetta, Delcarro,
Pisano, Radici,
arXiv:2004.14278](#)

3D STRUCTURE IN MOMENTUM SPACE

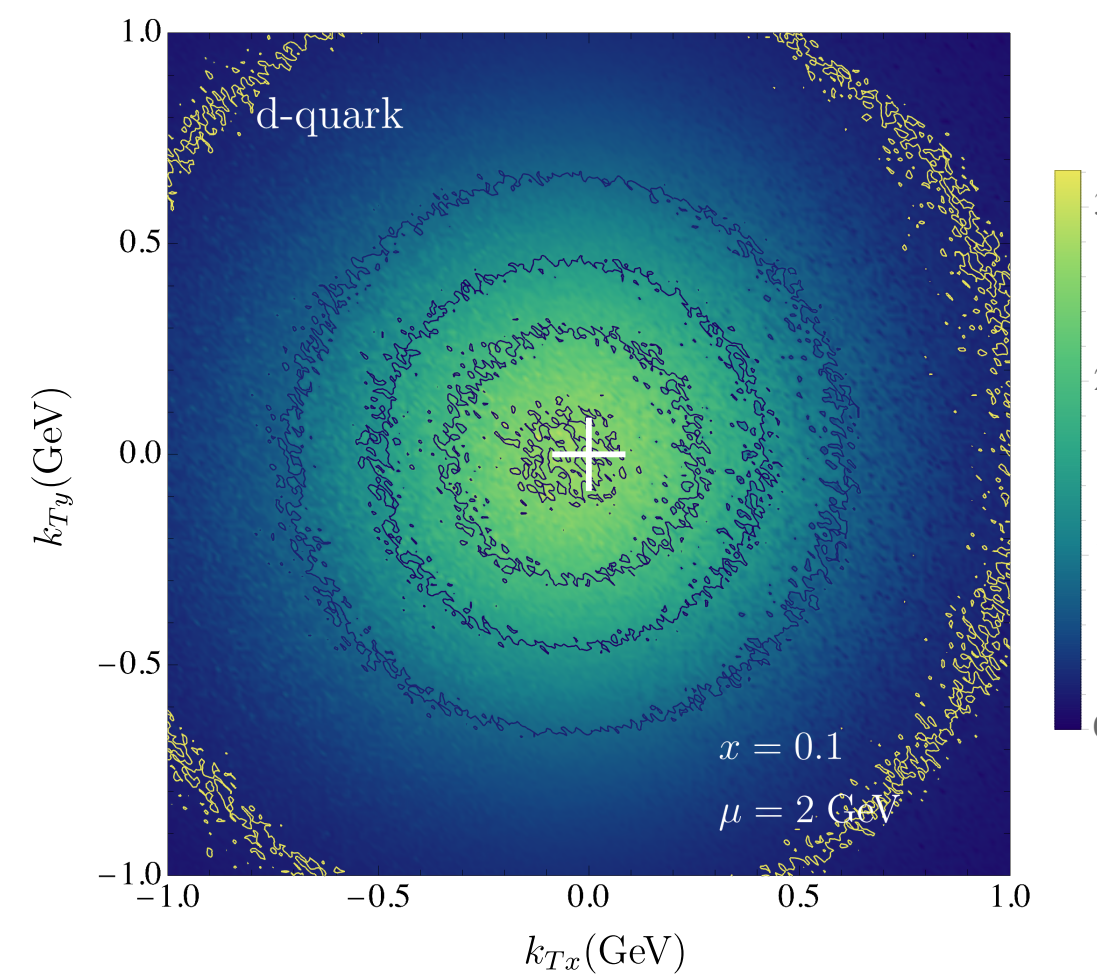


$Q = 2 \text{ GeV}$

[Bacchetta, Delcarro,
Pisano, Radici,
arXiv:2004.14278](#)

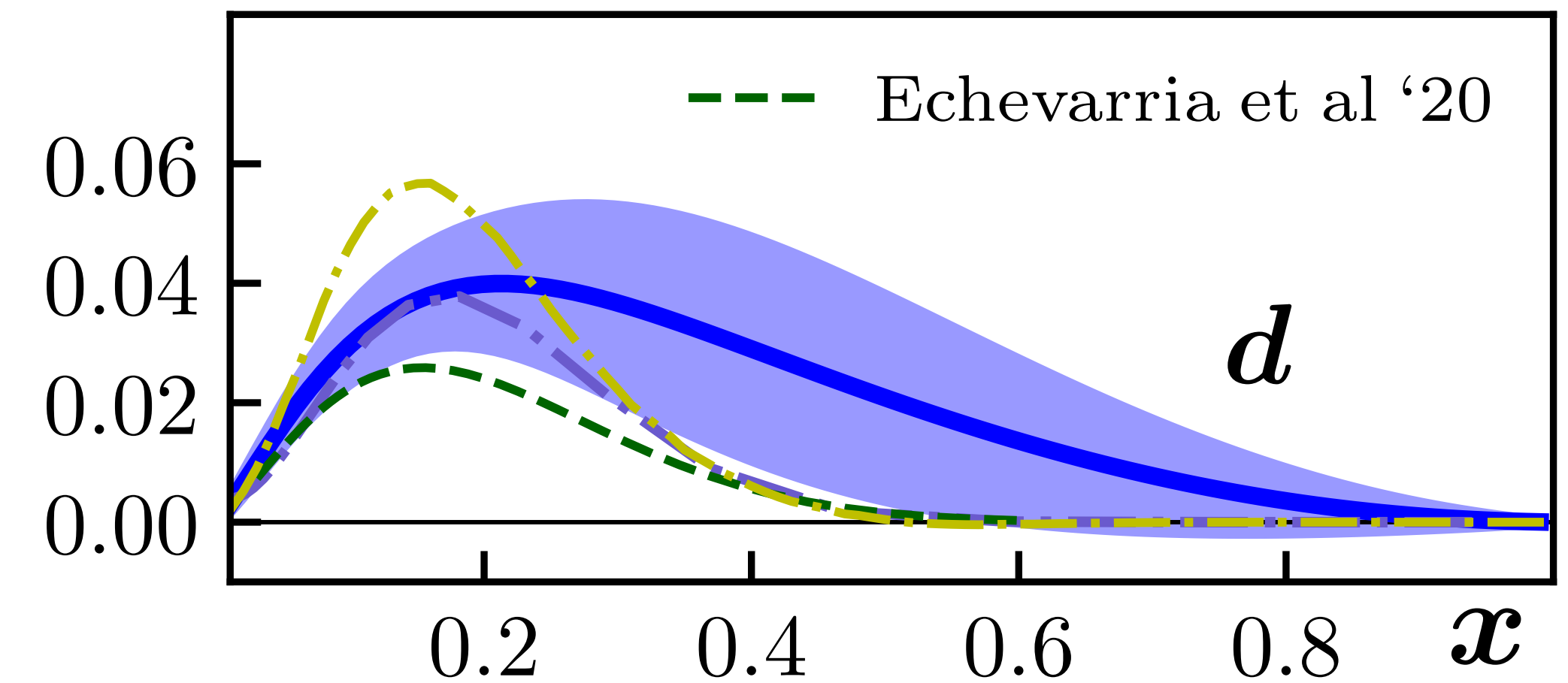
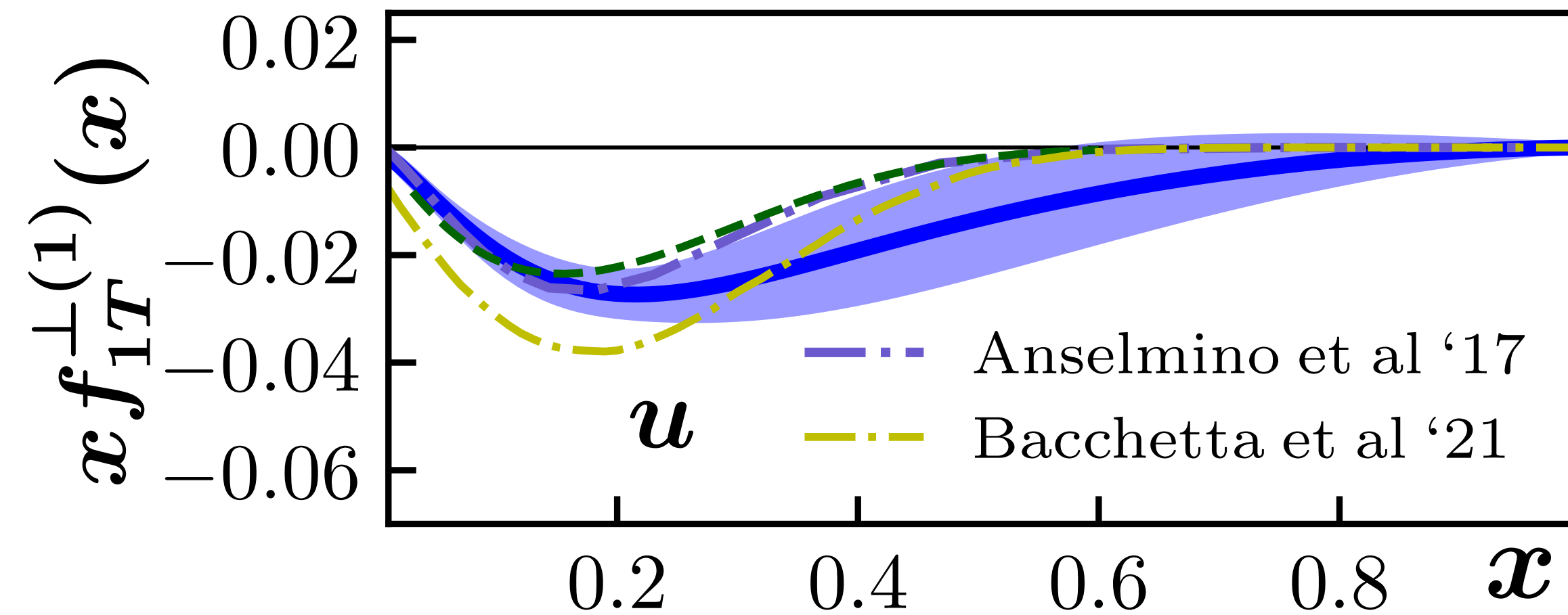


(a)

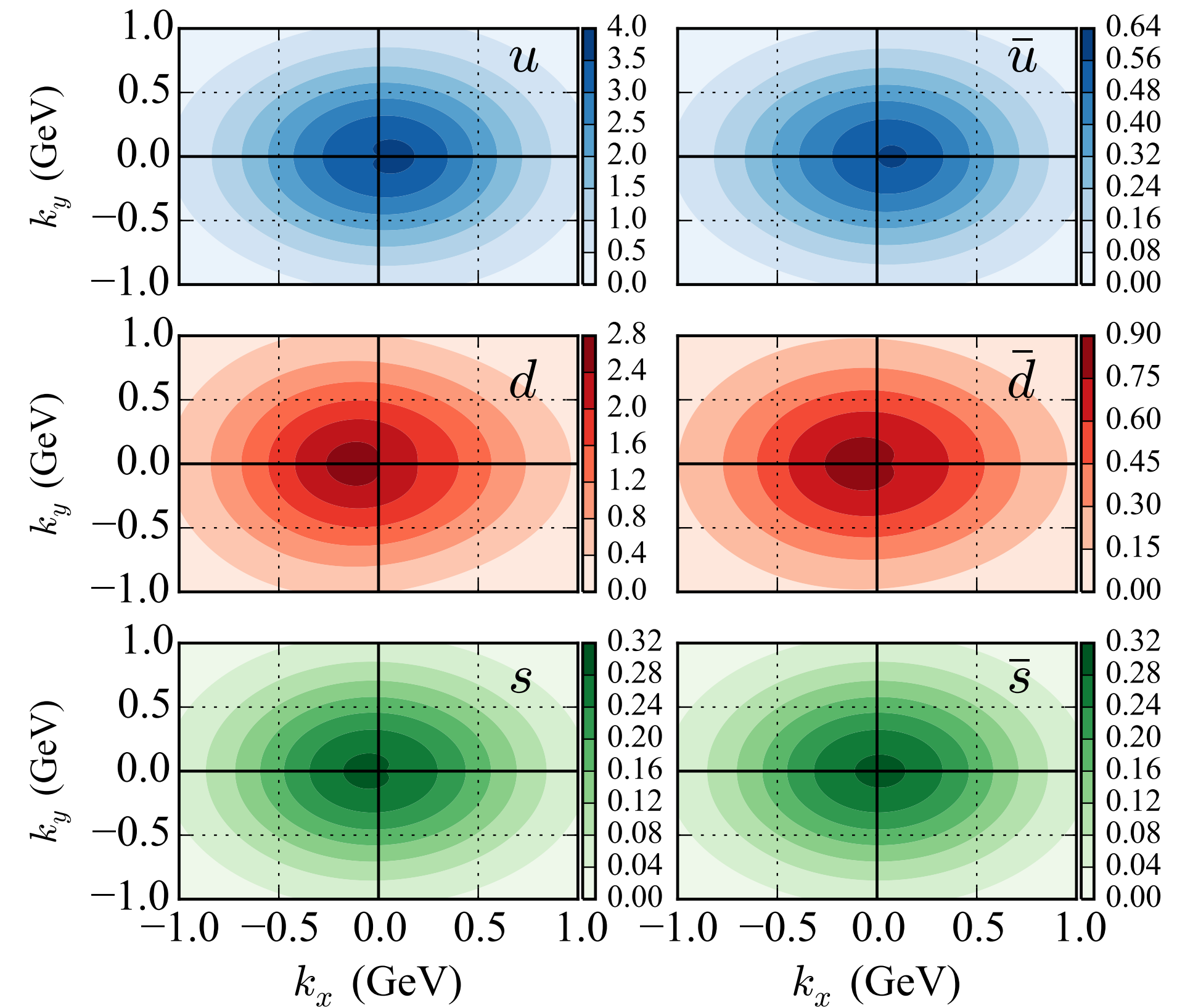
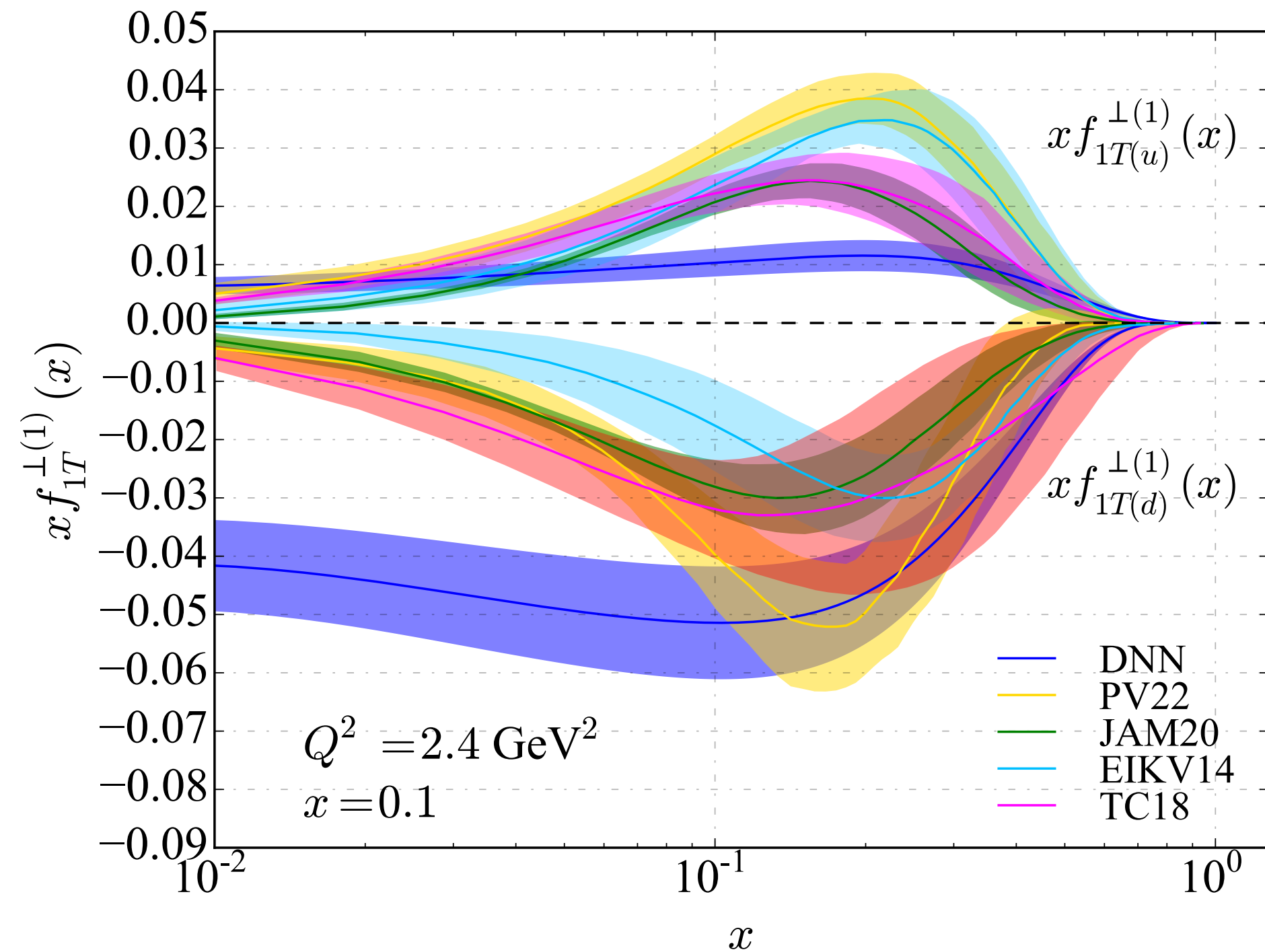


(b)

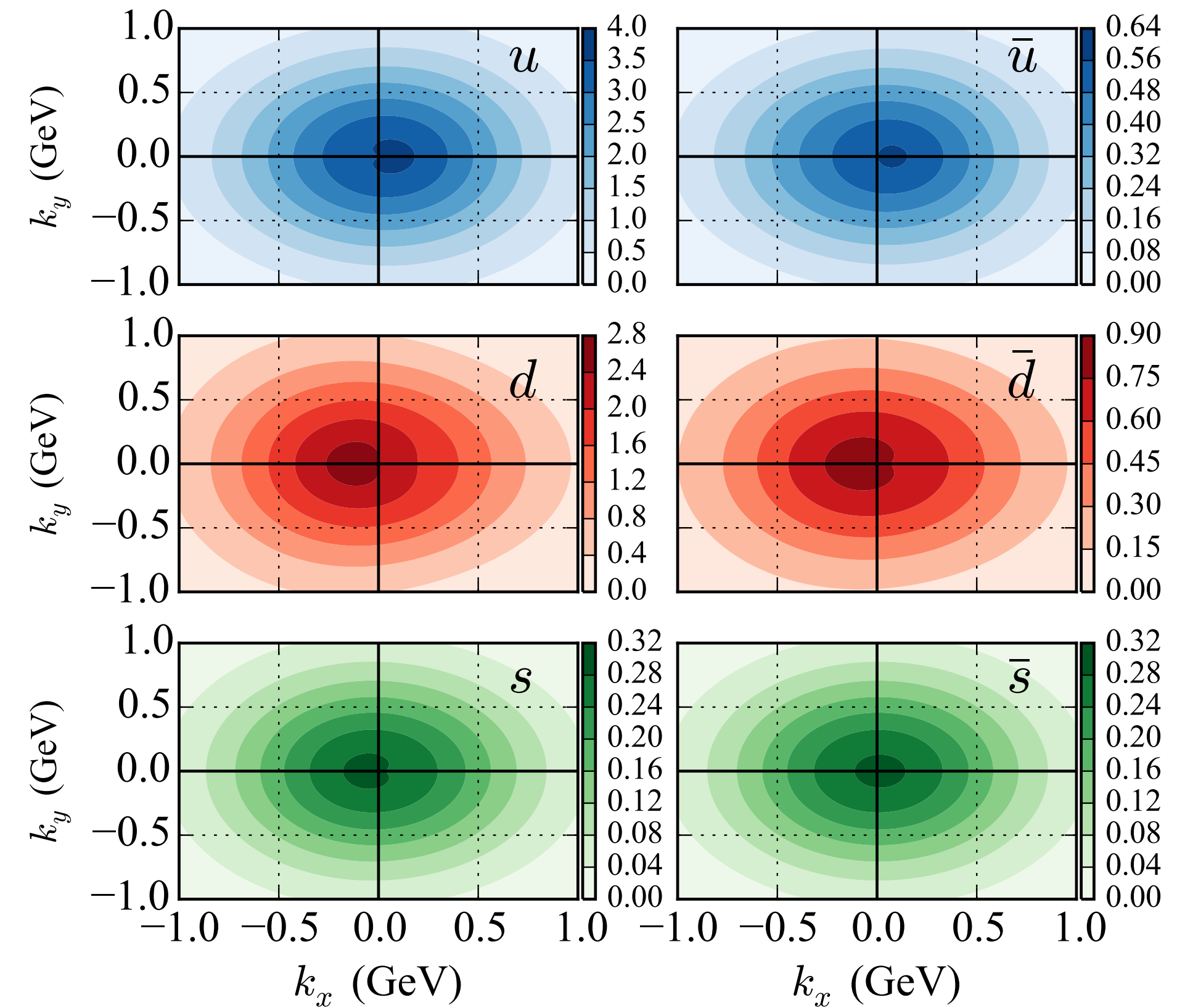
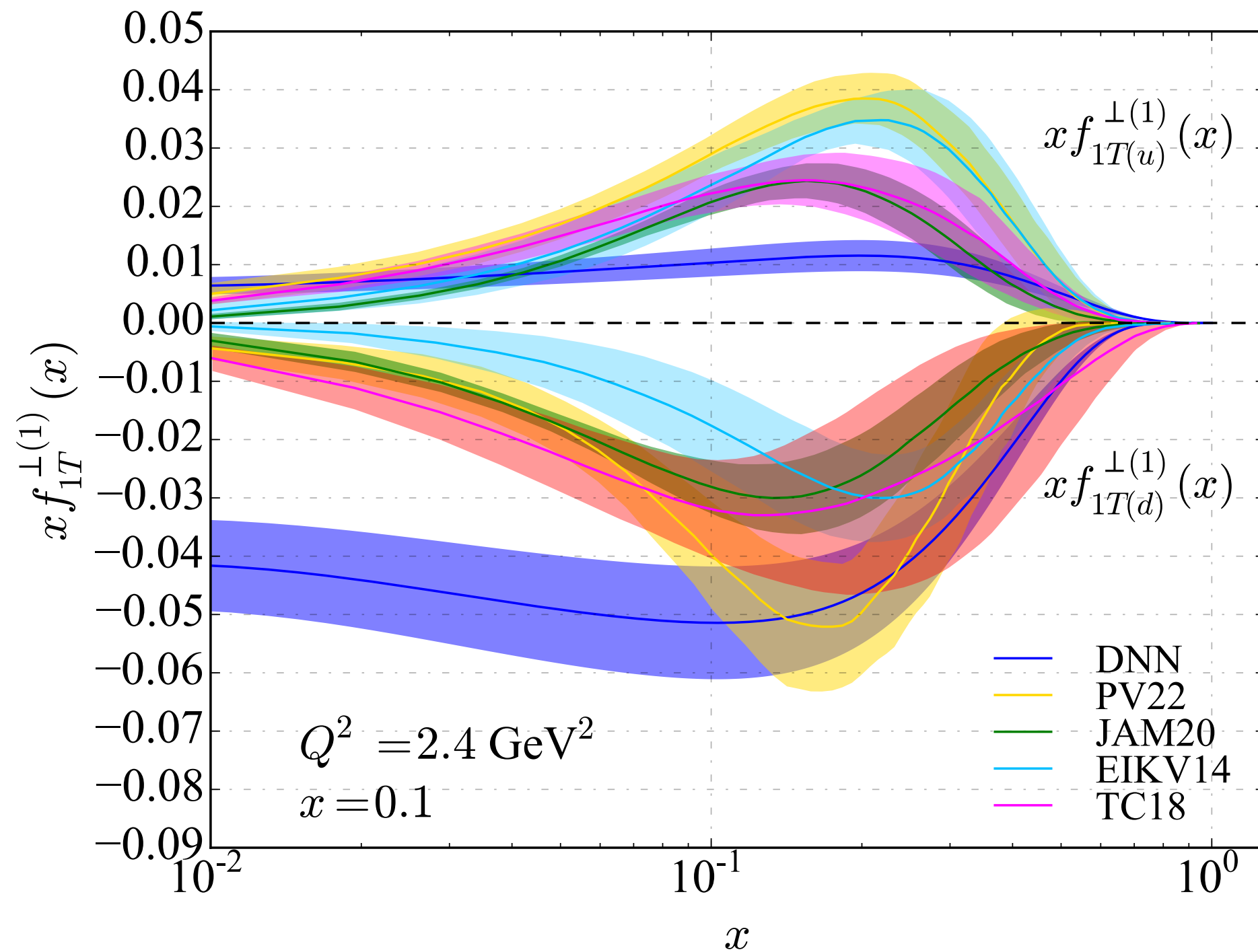
[Bury, Prokudin,
Vladimirov,
arXiv:2103.03270](#)



Interesting work from the point of view of simultaneous use of several measurements, but still limited from other perspectives (lack of TMD evolution and knowledge of the unpolarized function)

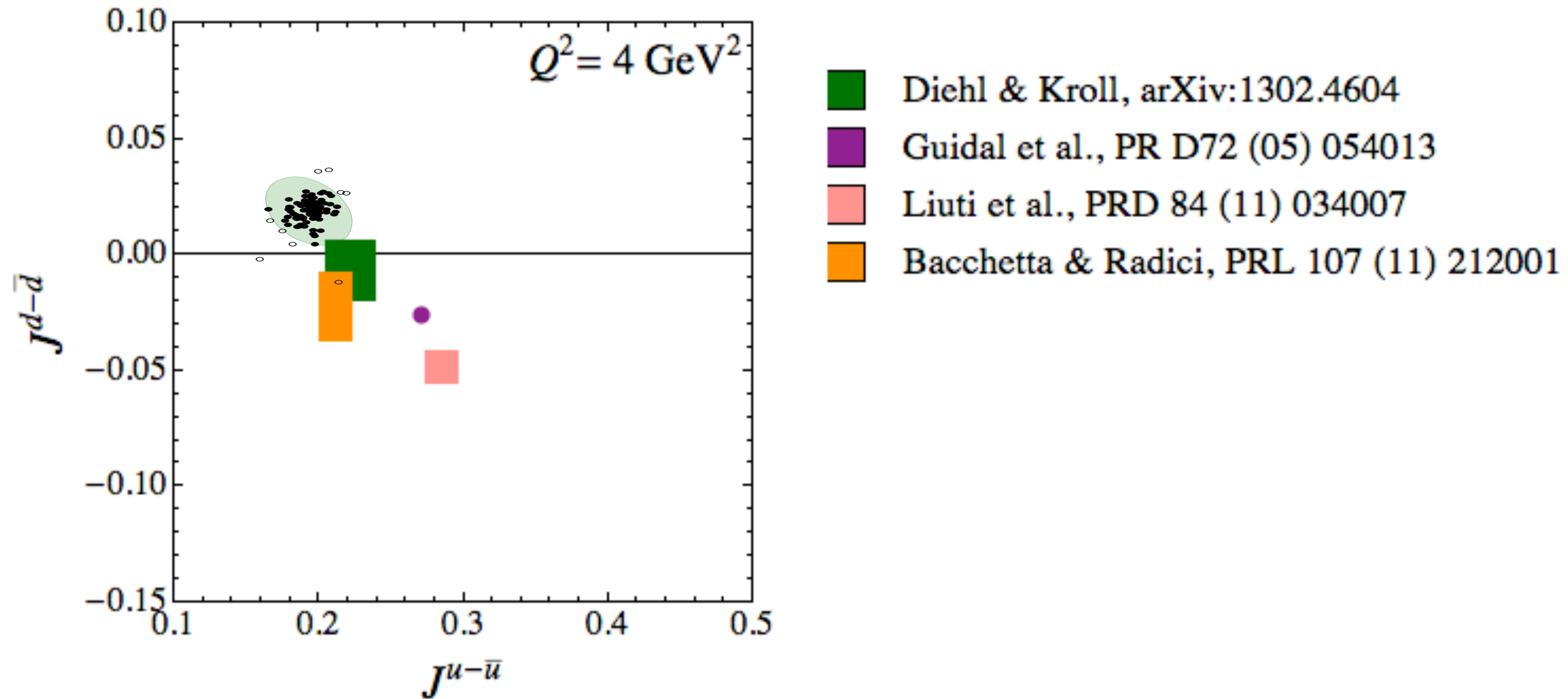


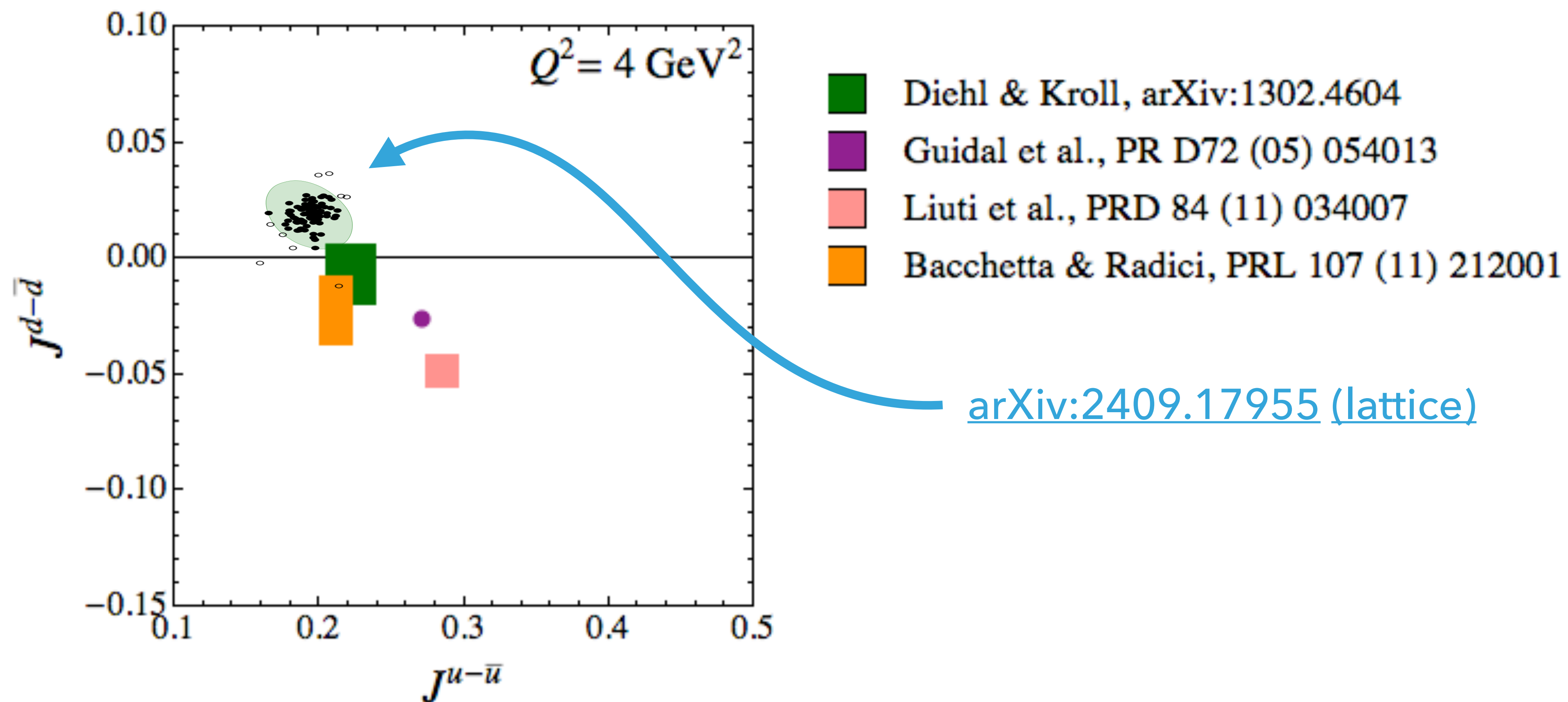
Interesting work from the point of view of the use of Neural Networks, but still limited from other perspectives (lack of TMD evolution and knowledge of the unpolarized function)

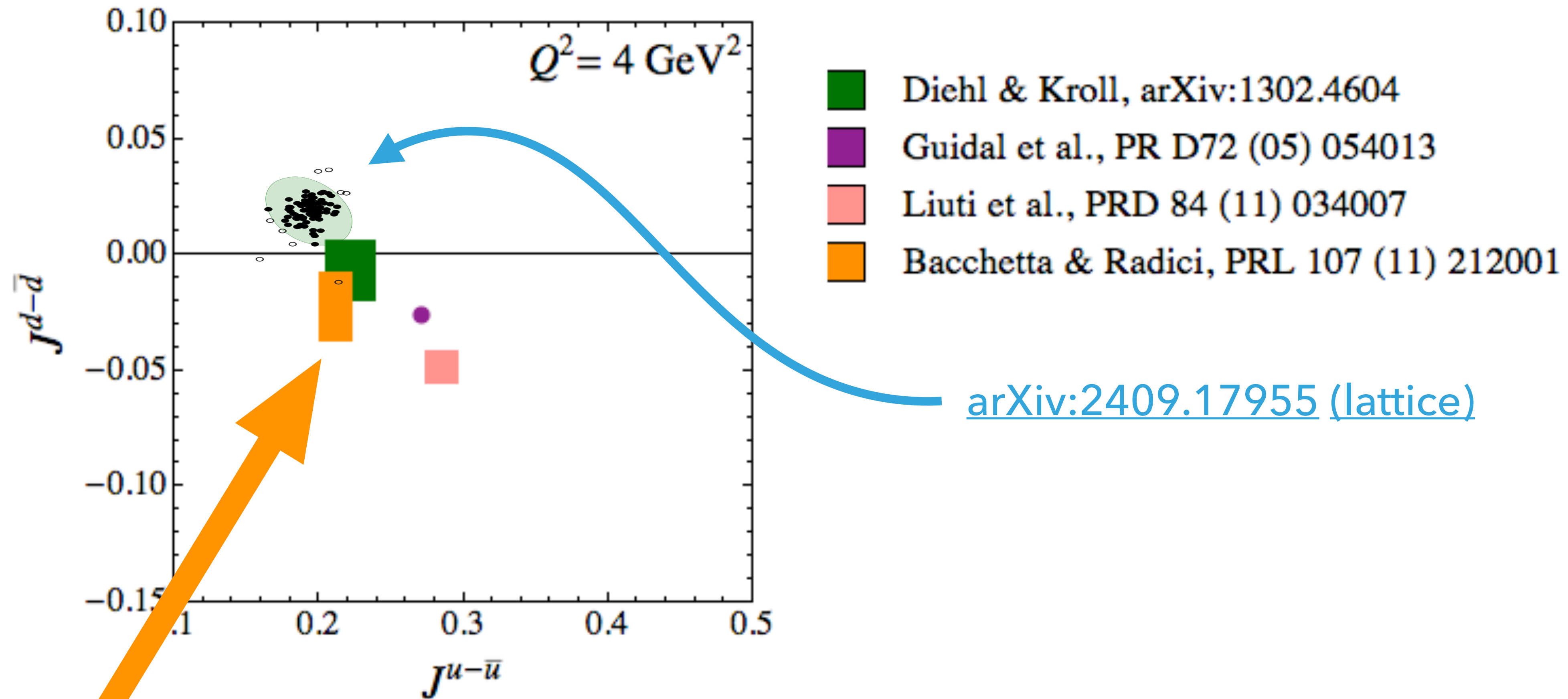


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See talks by Ishara Fernando







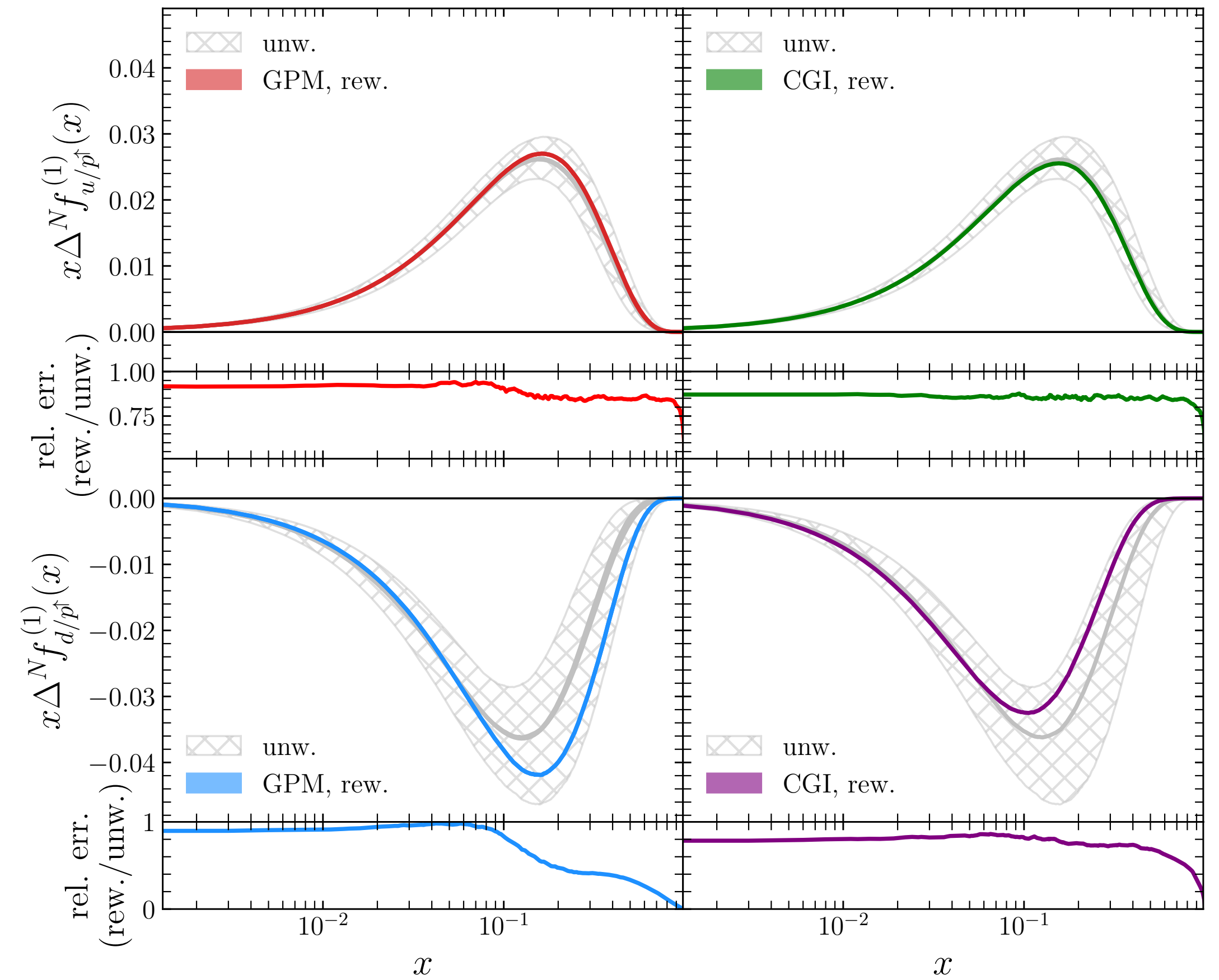
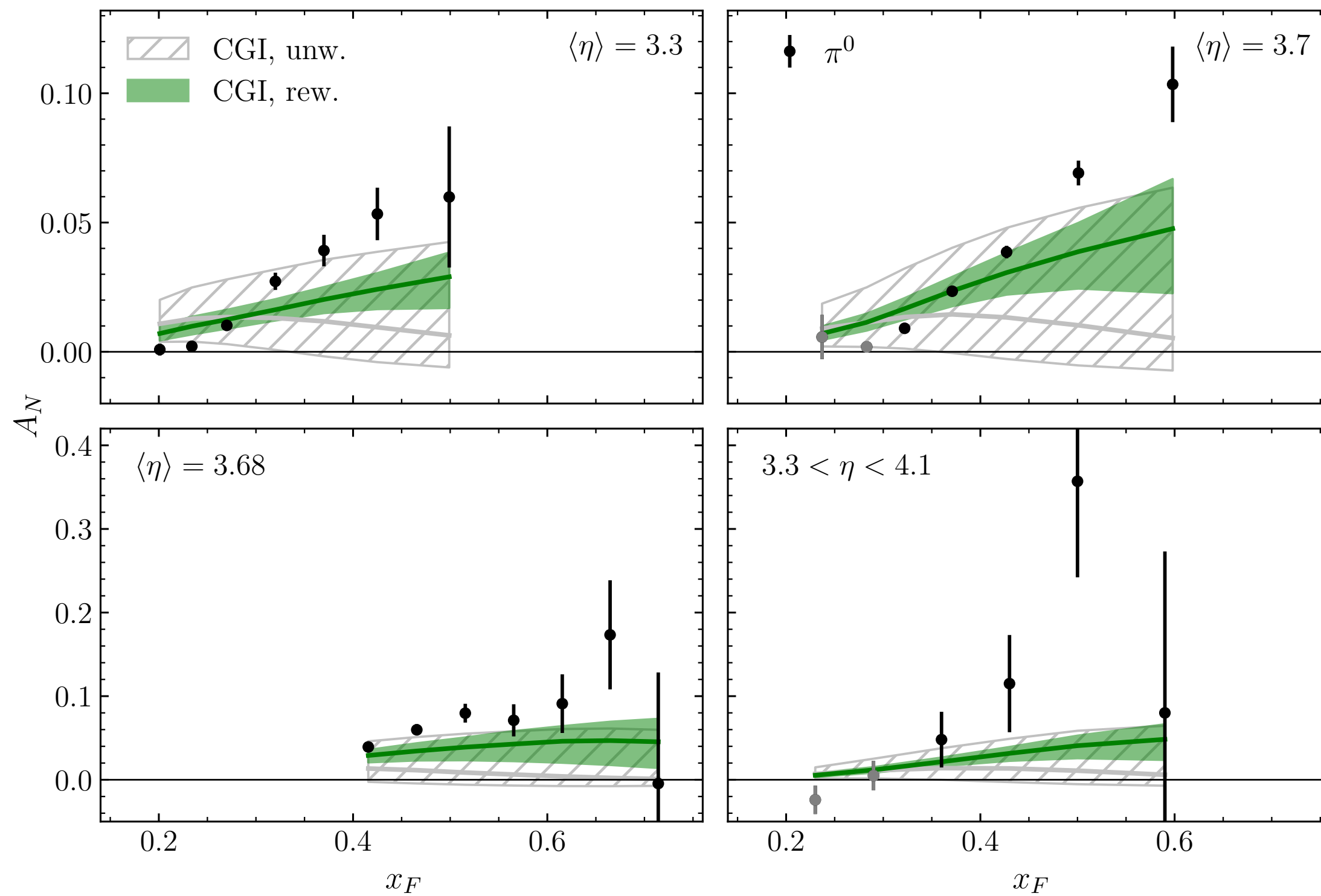
[arXiv:2409.17955 \(lattice\)](https://arxiv.org/abs/2409.17955)

 Estimate of angular momentum based on model assumptions + Sivers fit

[See also arXiv:1907.06960 for a critique](https://arxiv.org/abs/1907.06960)

Boglione, D'Alesio, Flore, Gonzalez-Hernandez, Murgia, Prokudin, [arXiv:2402.12322](https://arxiv.org/abs/2402.12322)

STAR, $p^\uparrow p \rightarrow \pi^0 X$, $\sqrt{s} = 200$ GeV



WHAT ABOUT PP → HADRONS DATA?

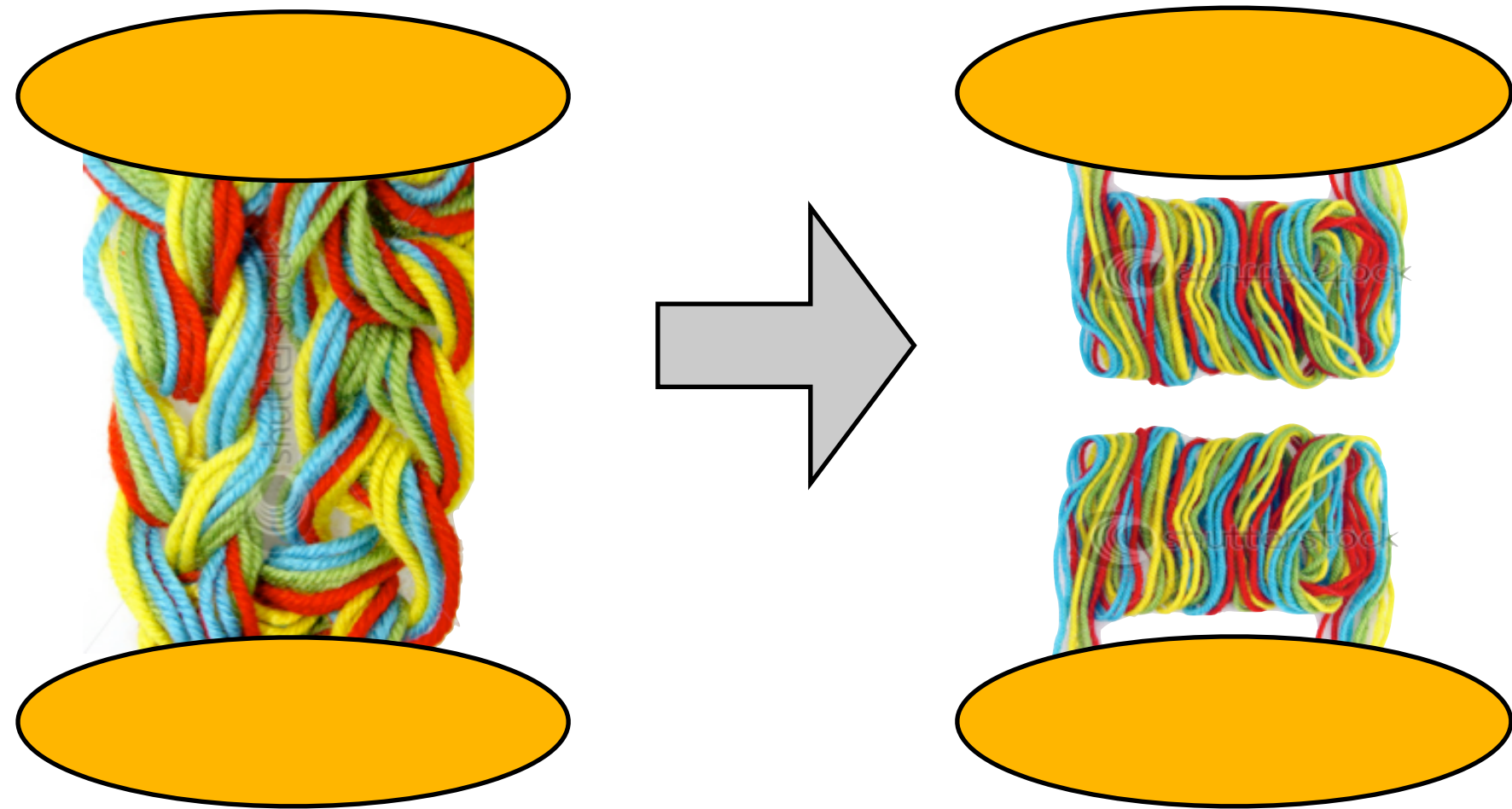
Artistic view of factorization

[*Mulders, Rogers, arXiv:1001.2977*](#)



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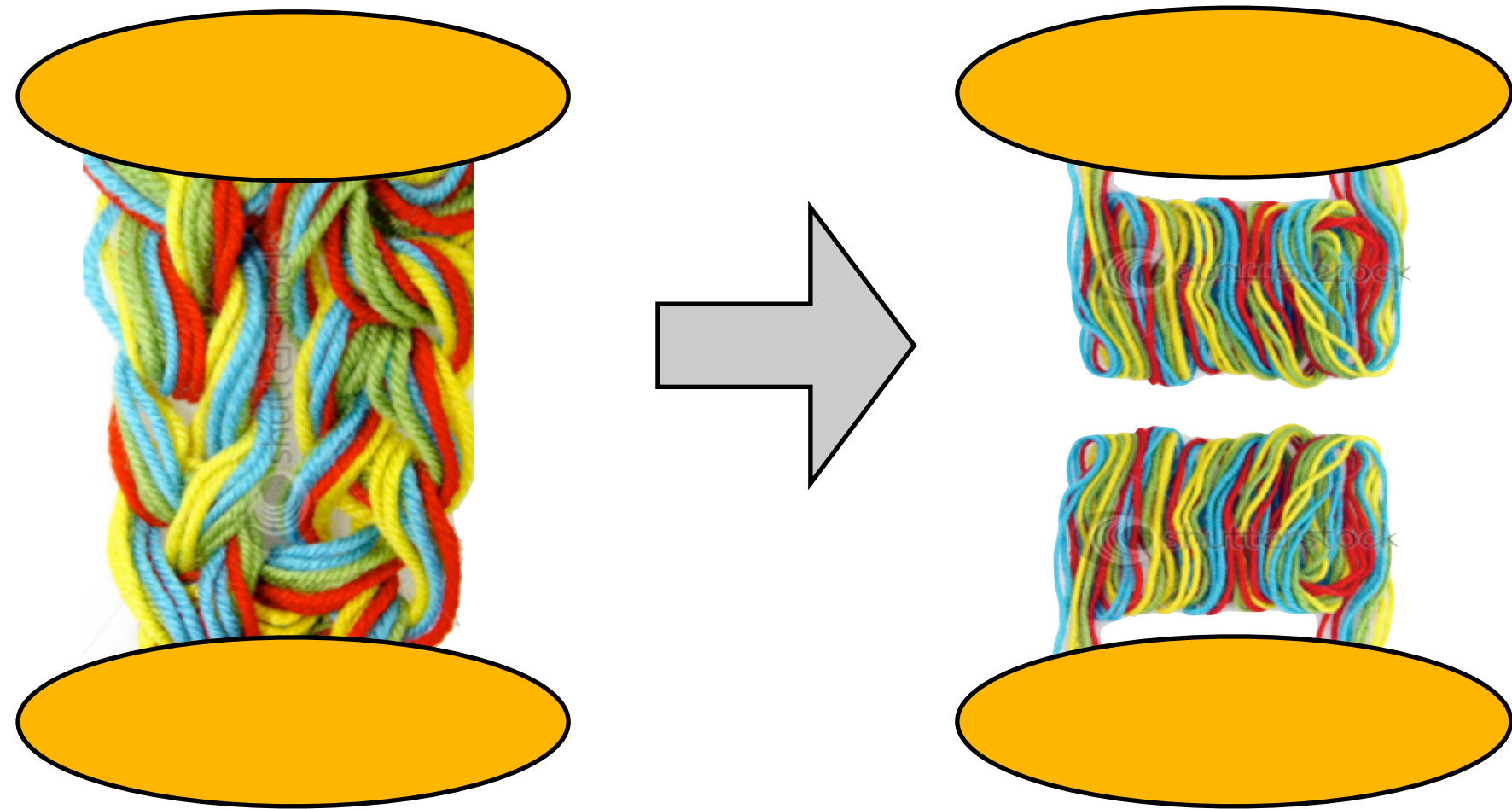


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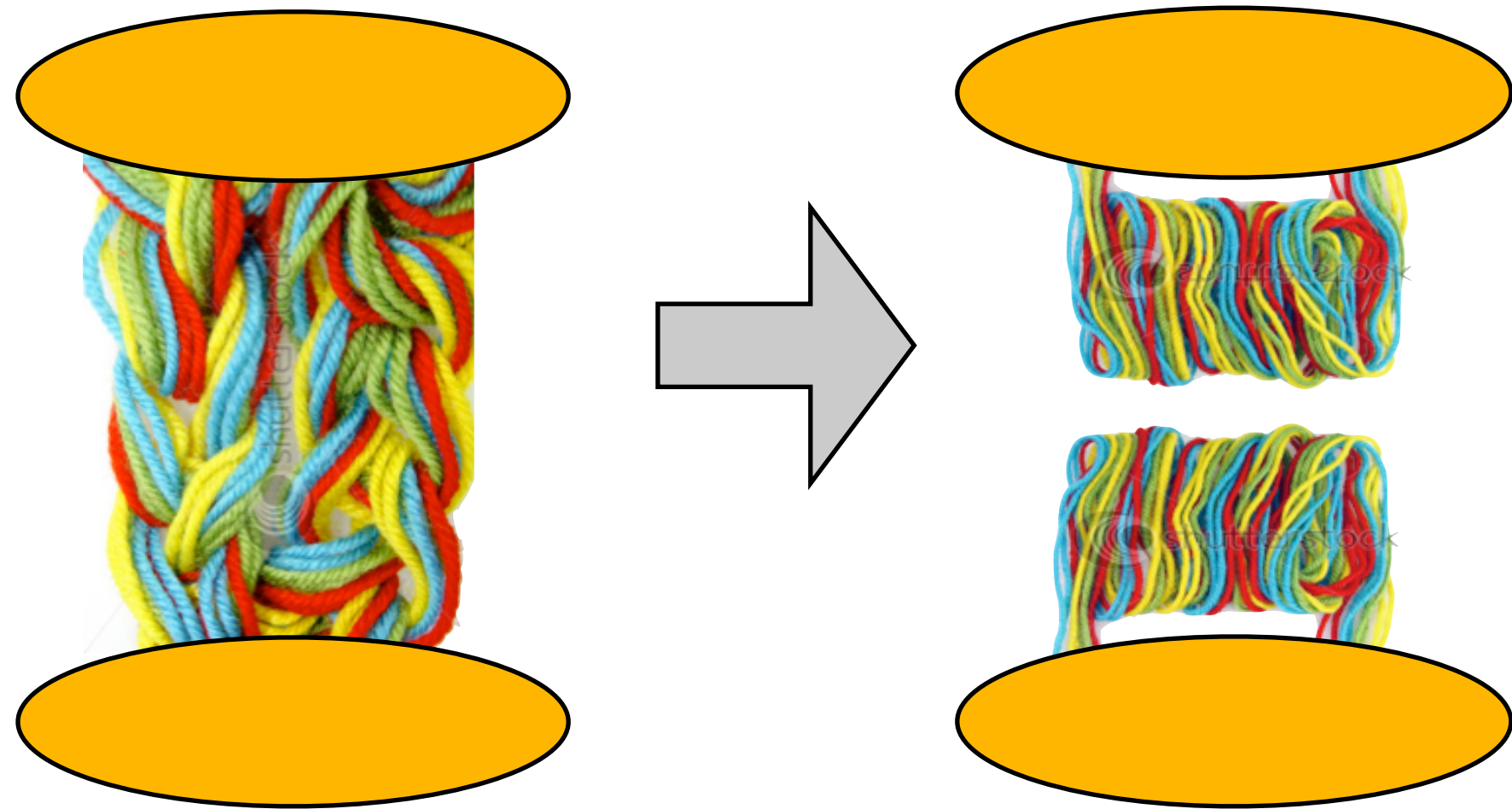


Works for SIDIS, Drell-Yan, e^-e^+ annihilation

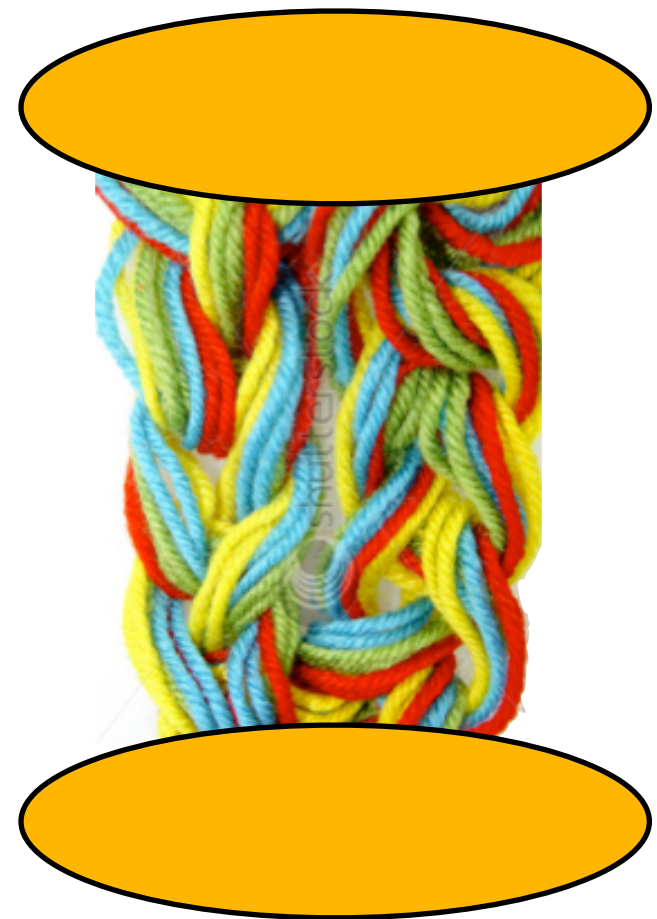
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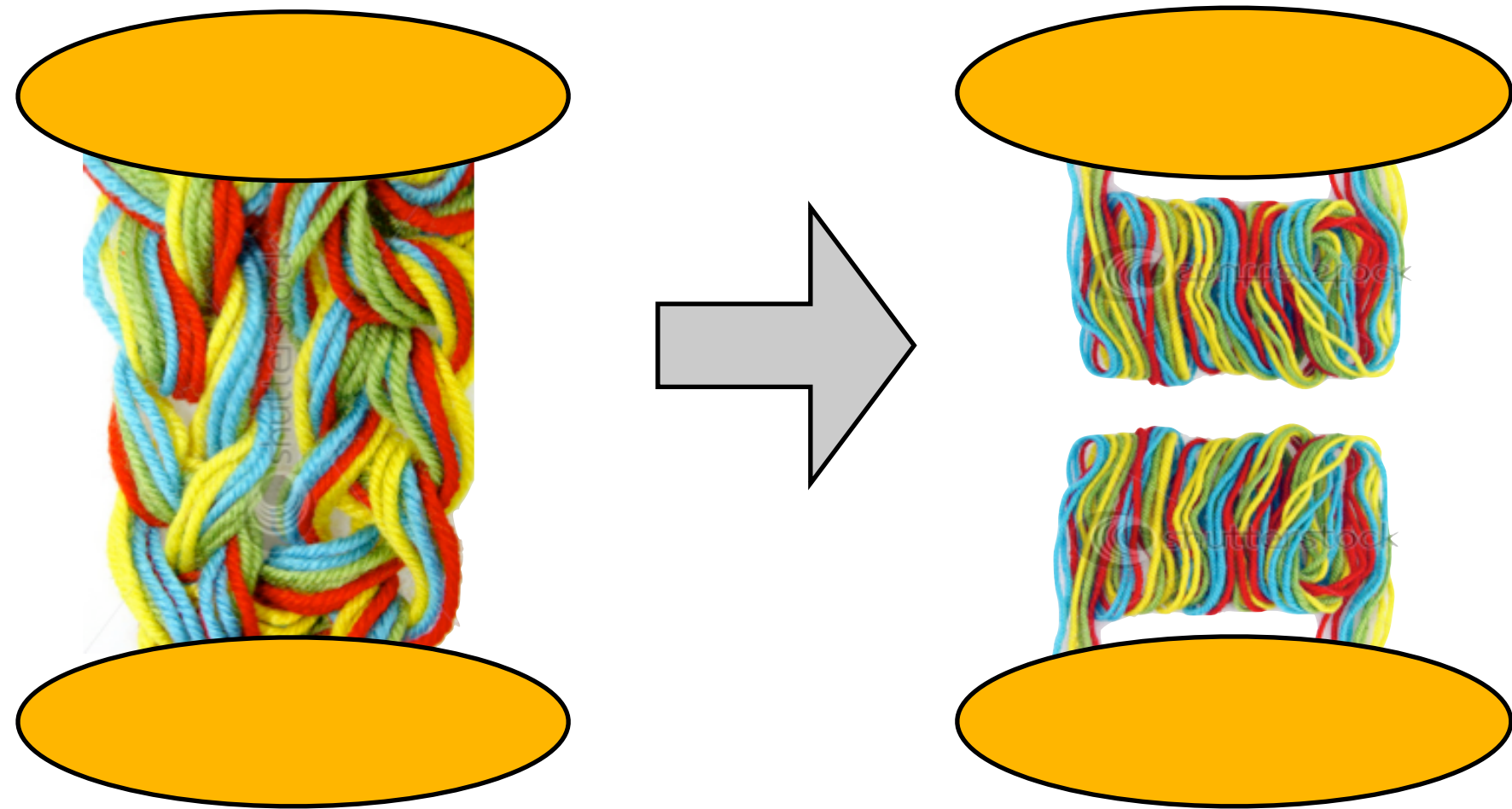
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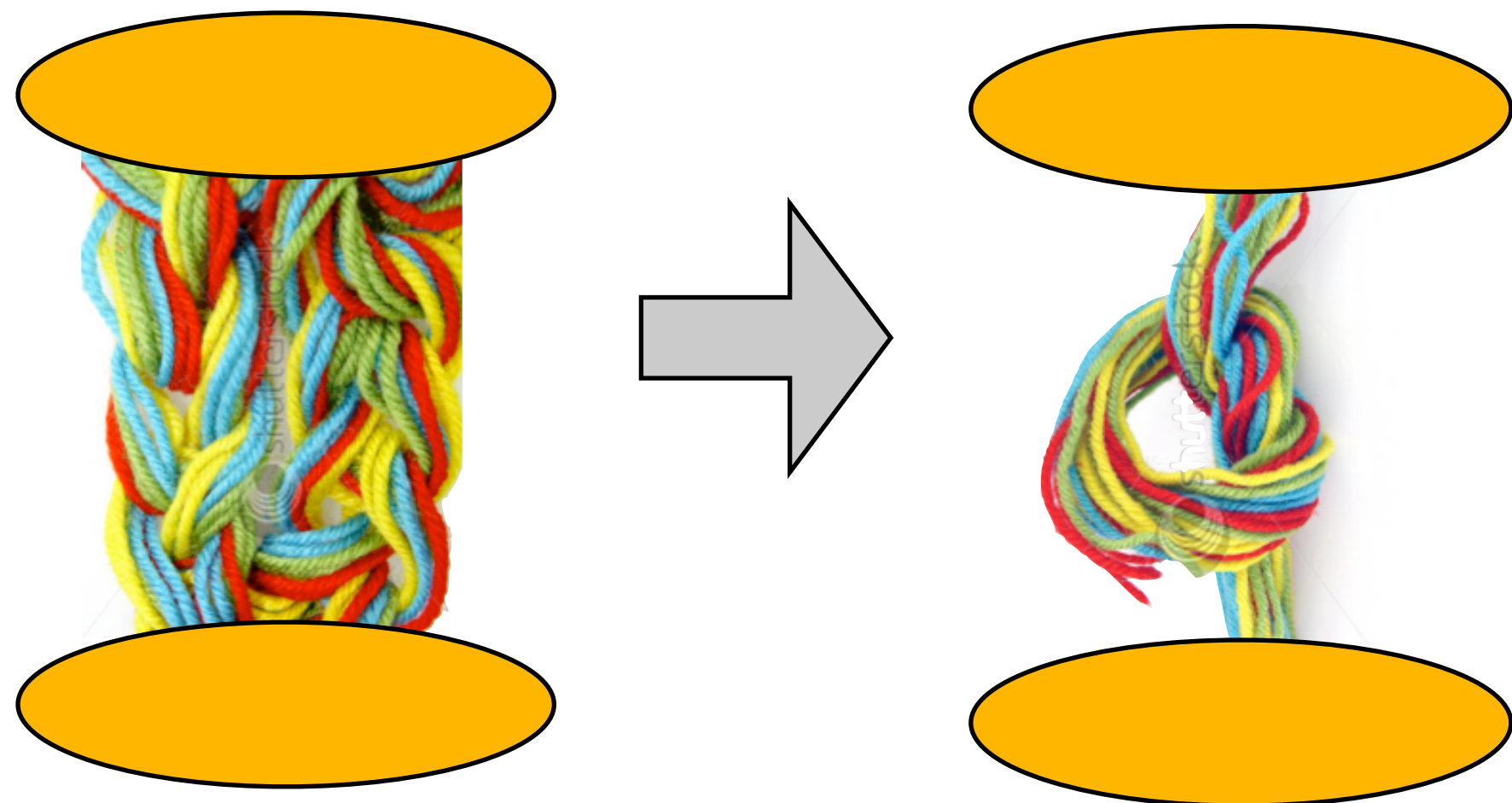
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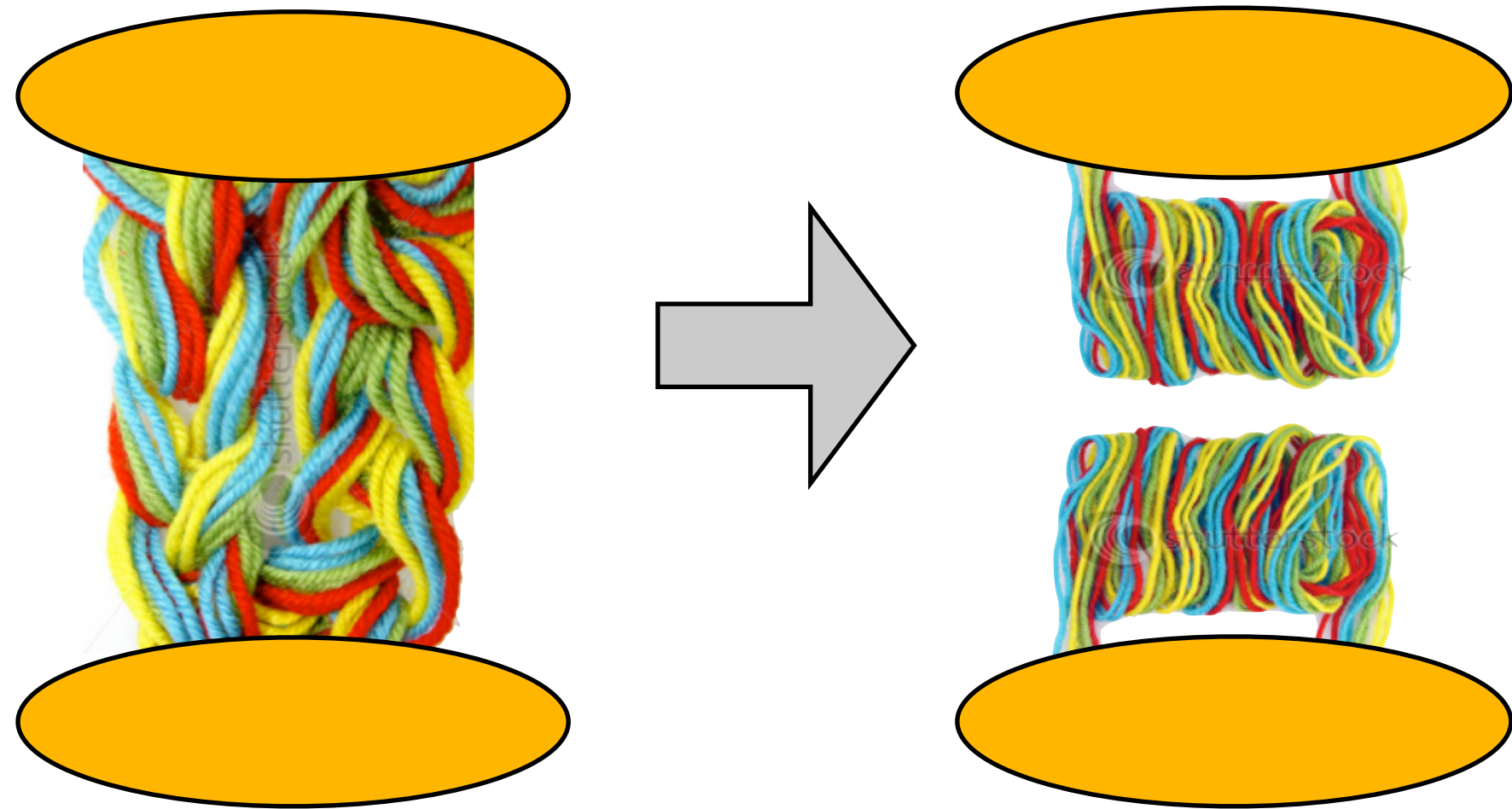
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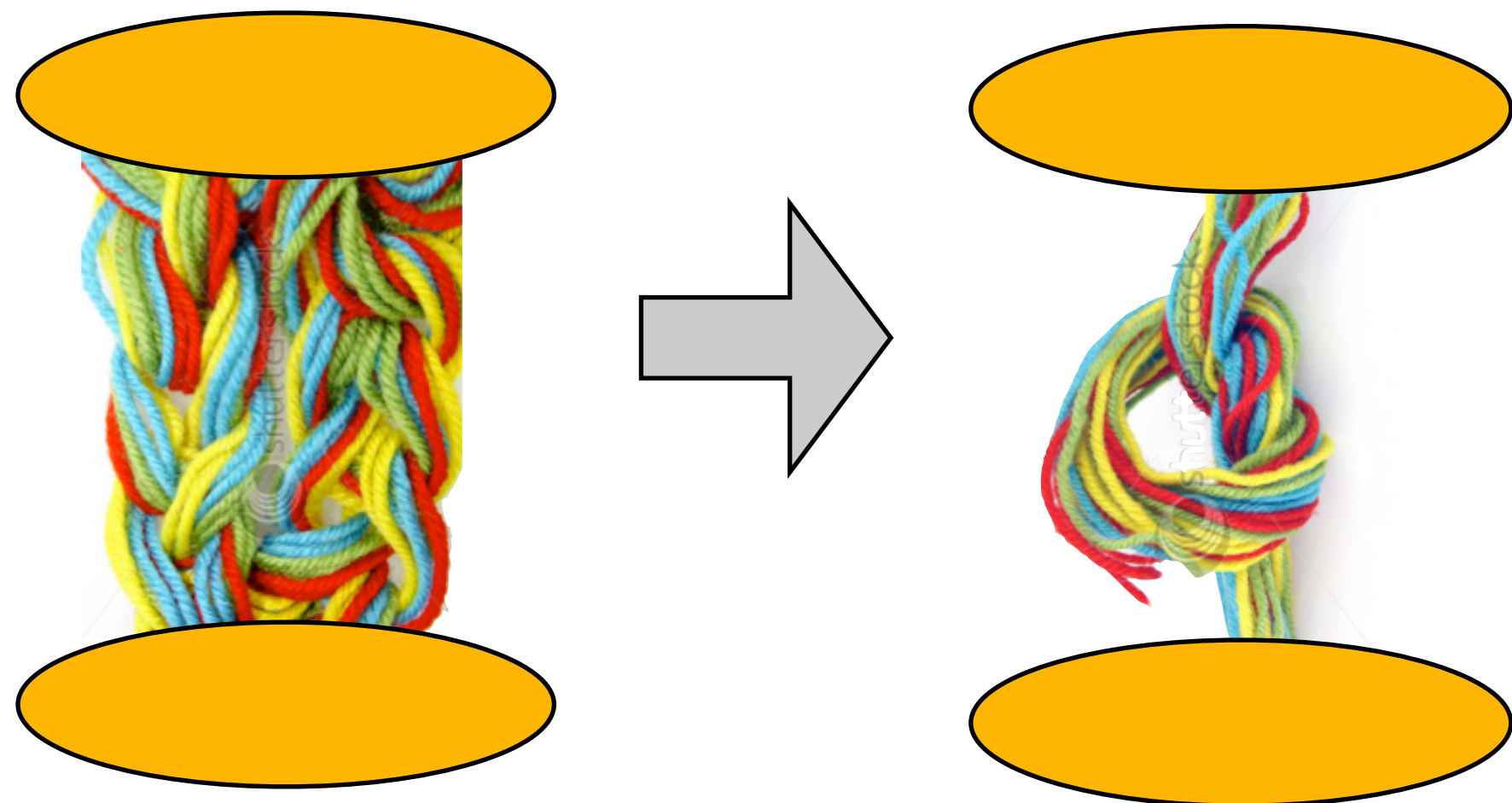
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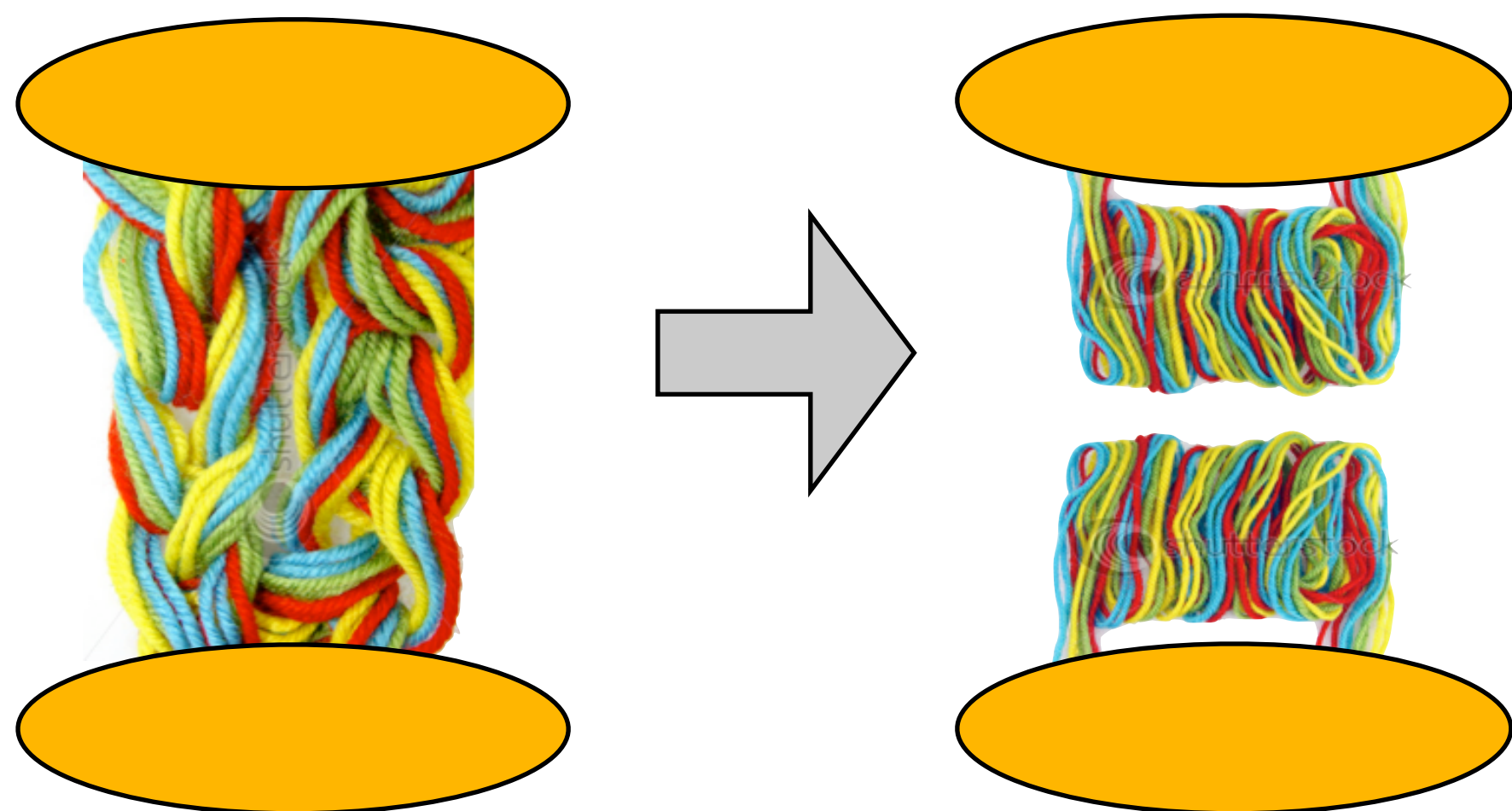
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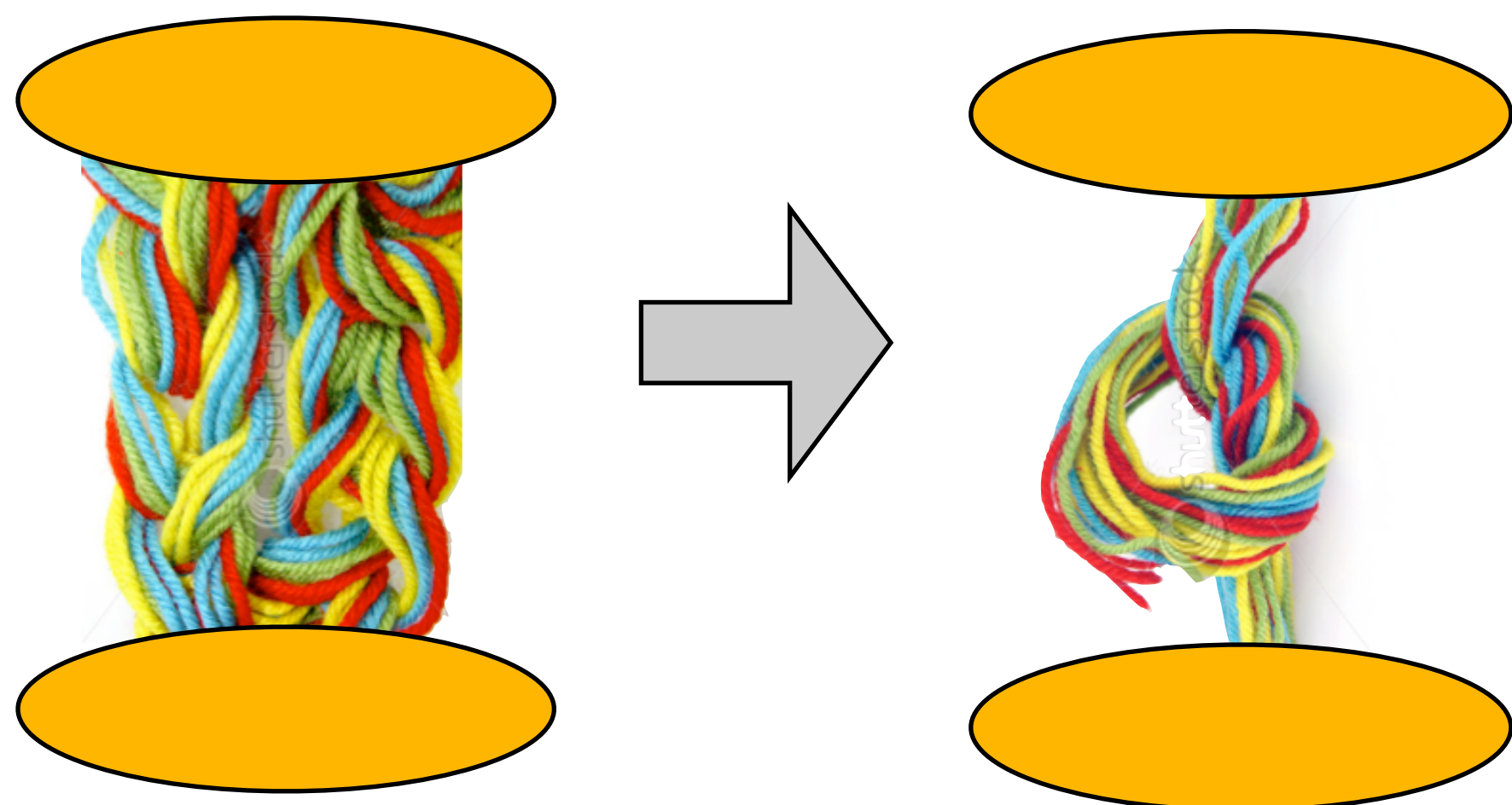
TMD factorization does not work for pp to hadrons

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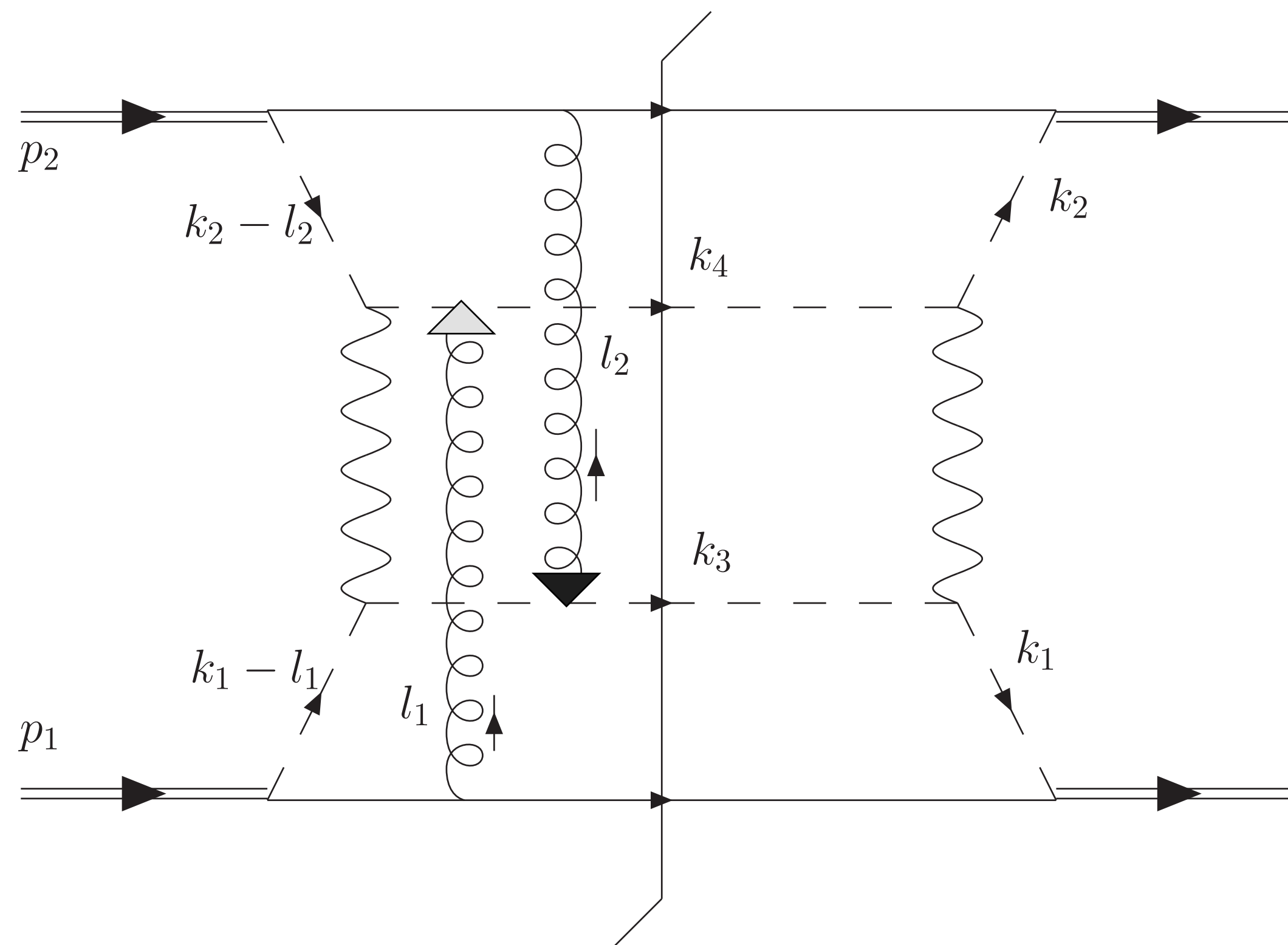


Works for SIDIS, Drell-Yan, e-e+ annihilation



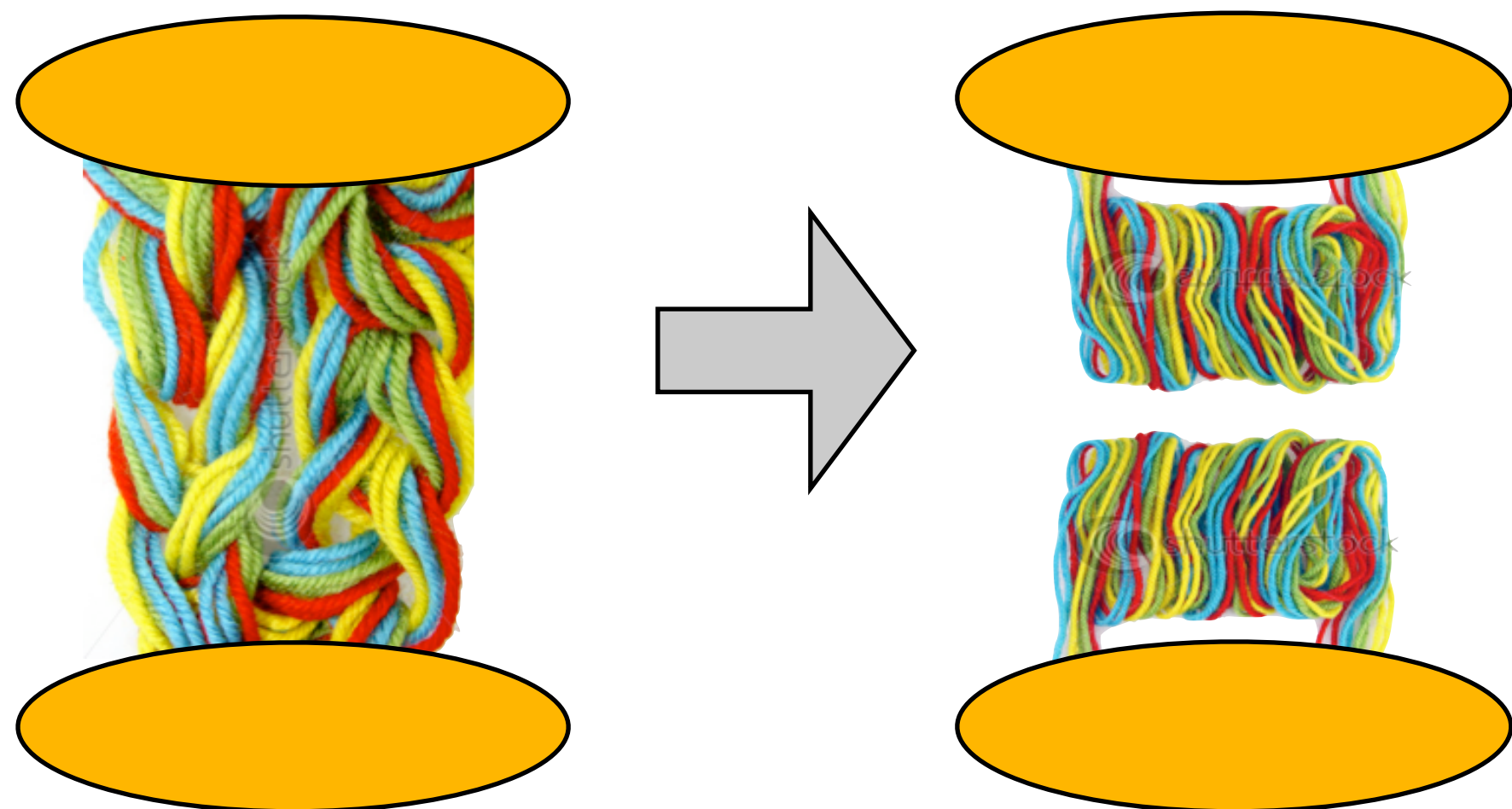
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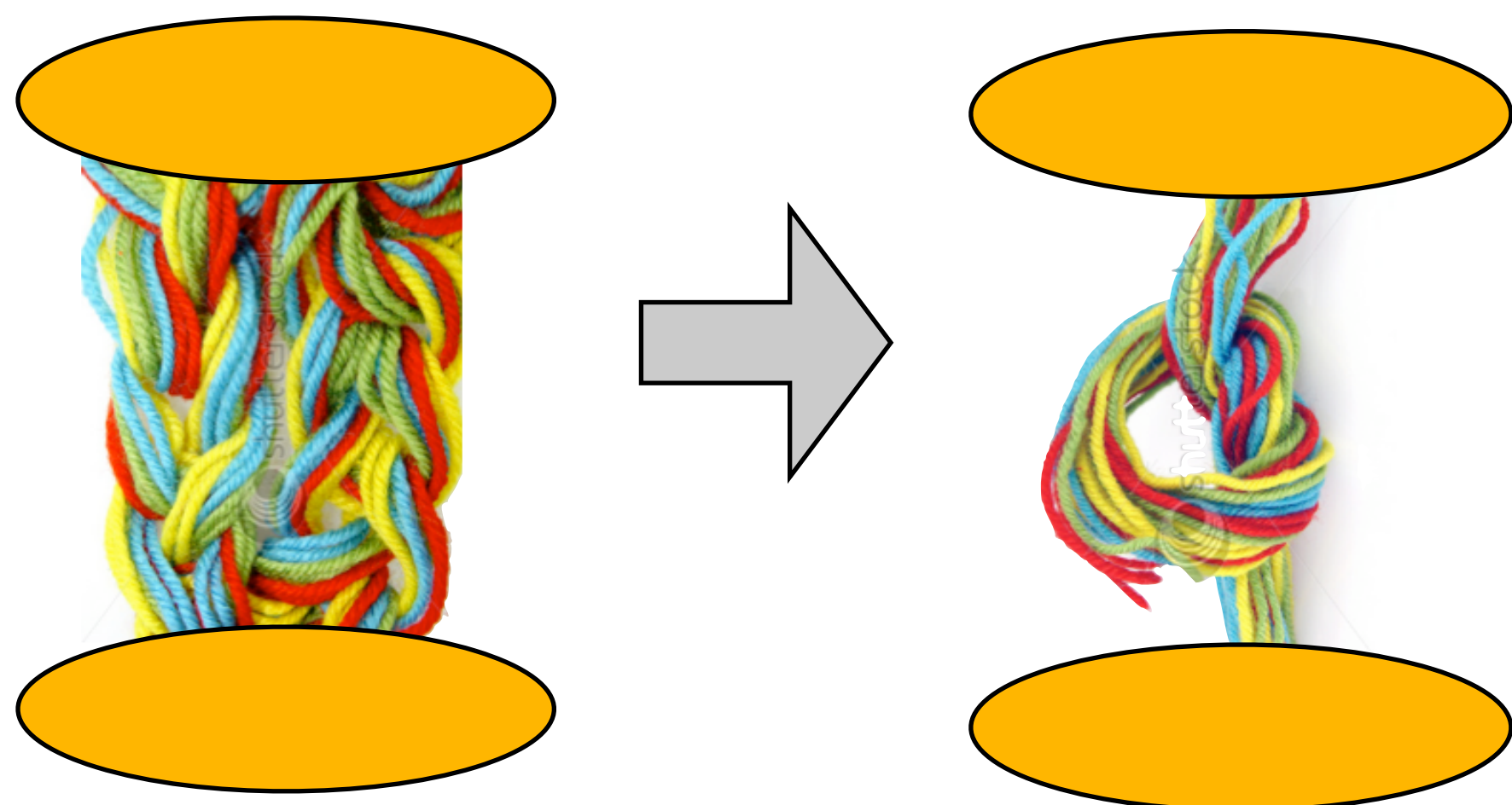


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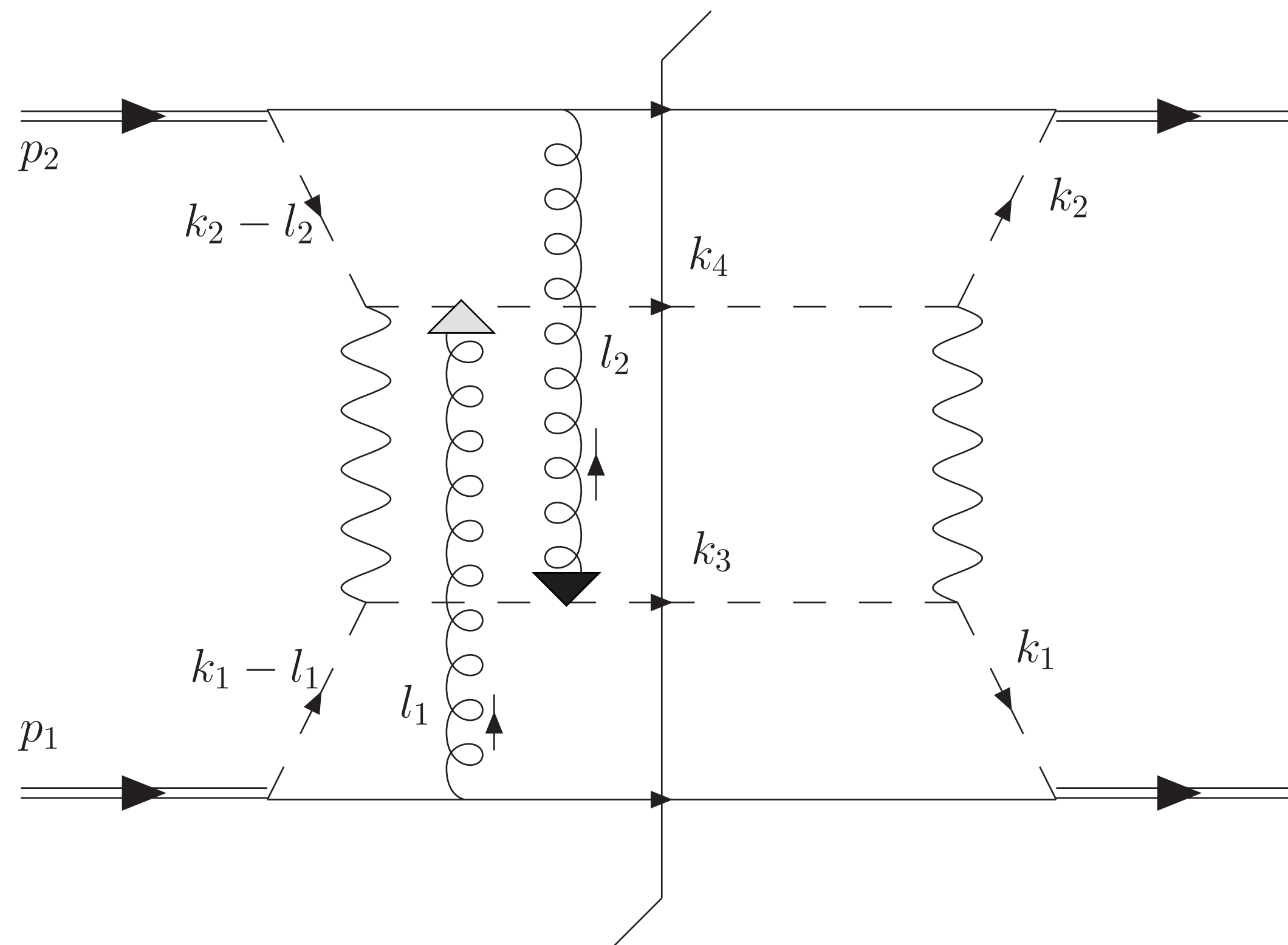


Works for SIDIS, Drell-Yan, e-e⁺ annihilation

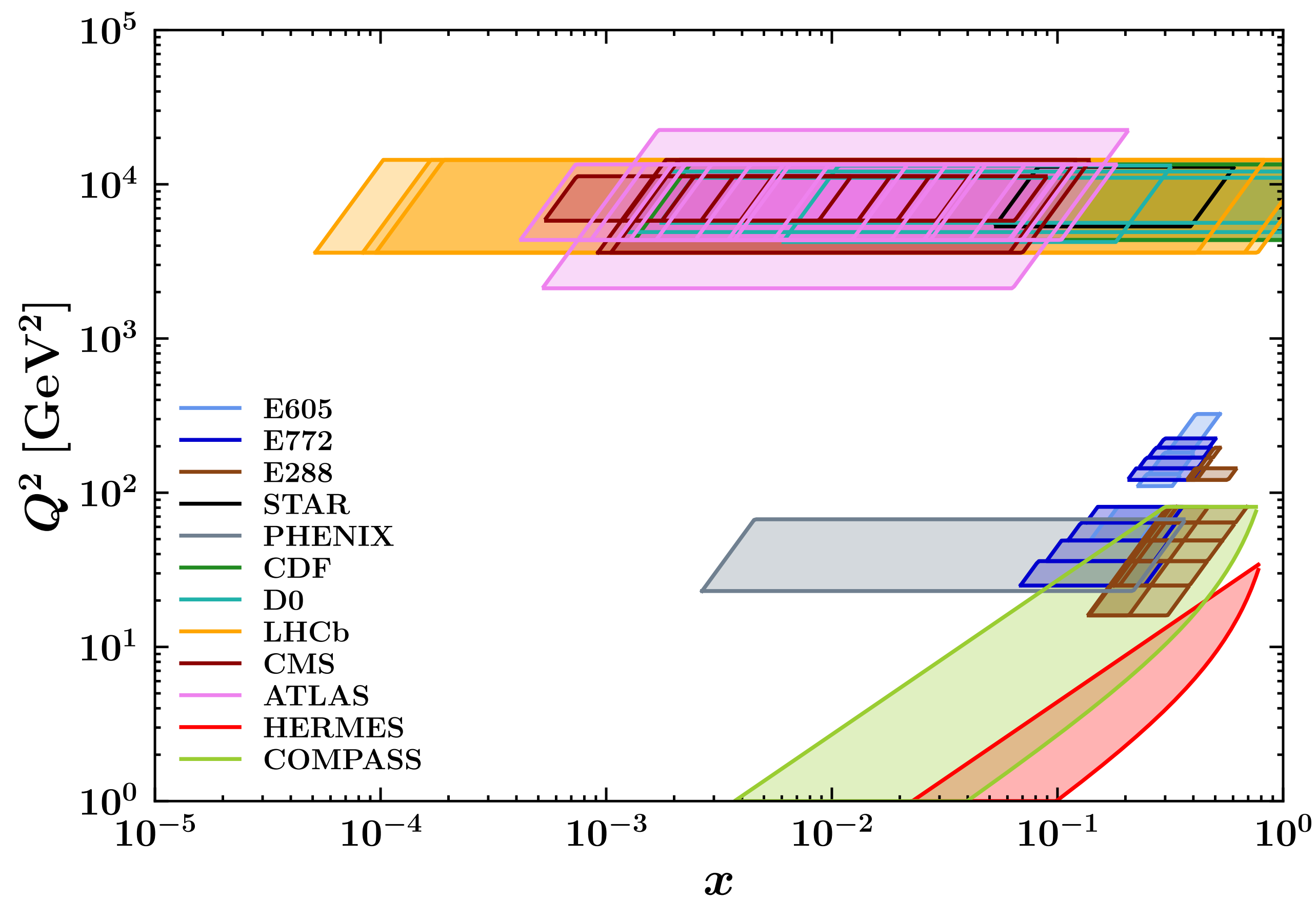


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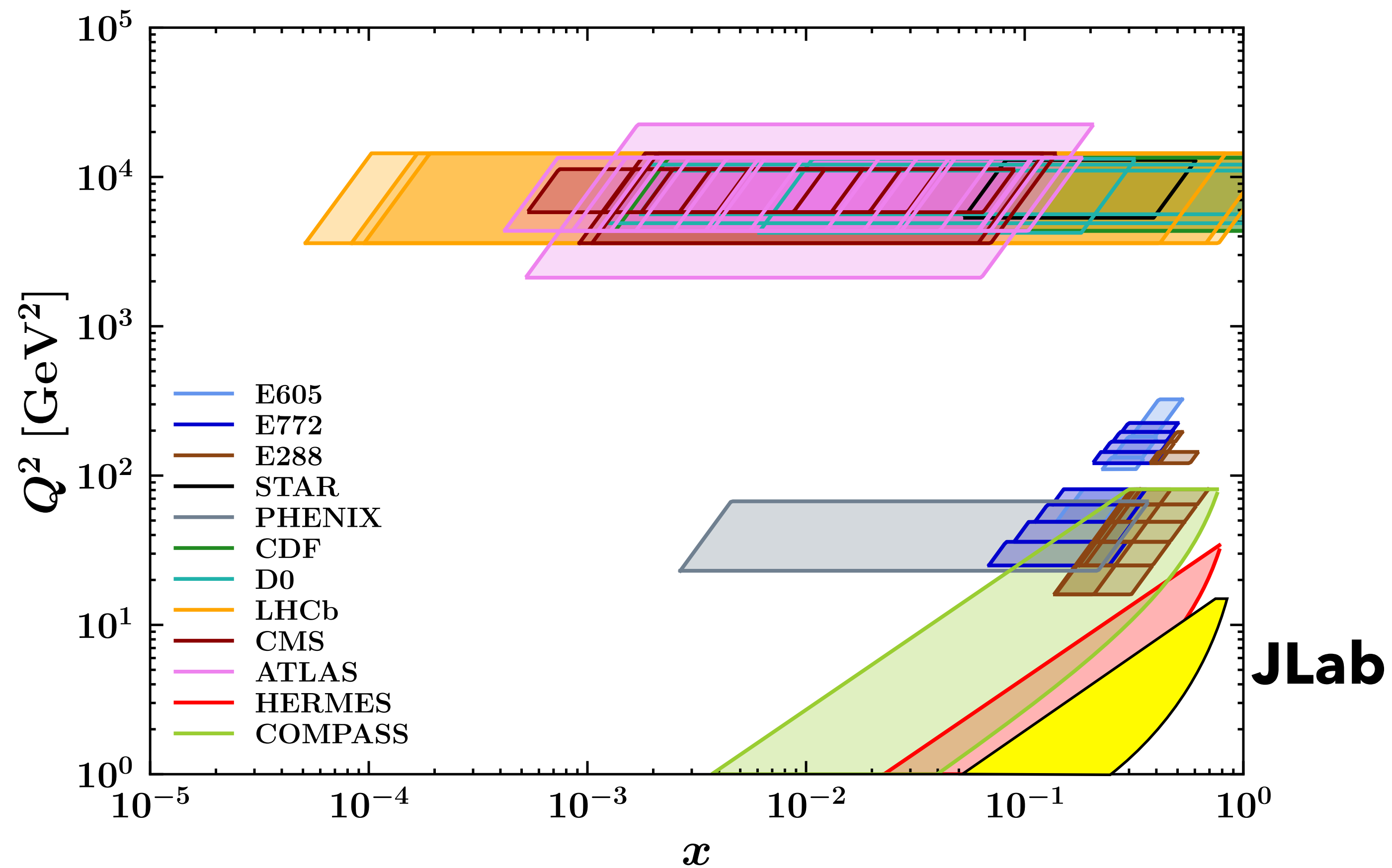
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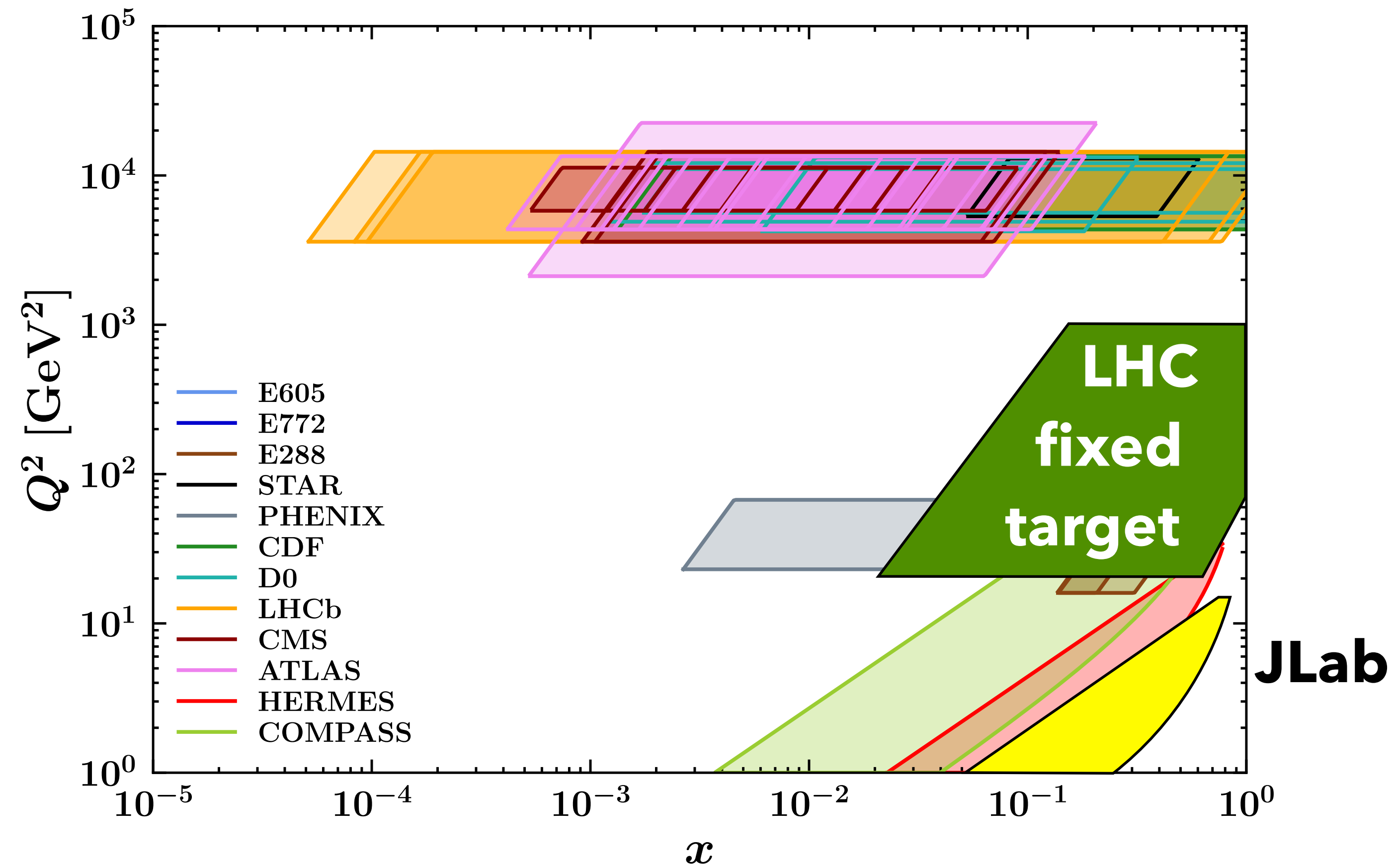
See talk by Oleg Eyser



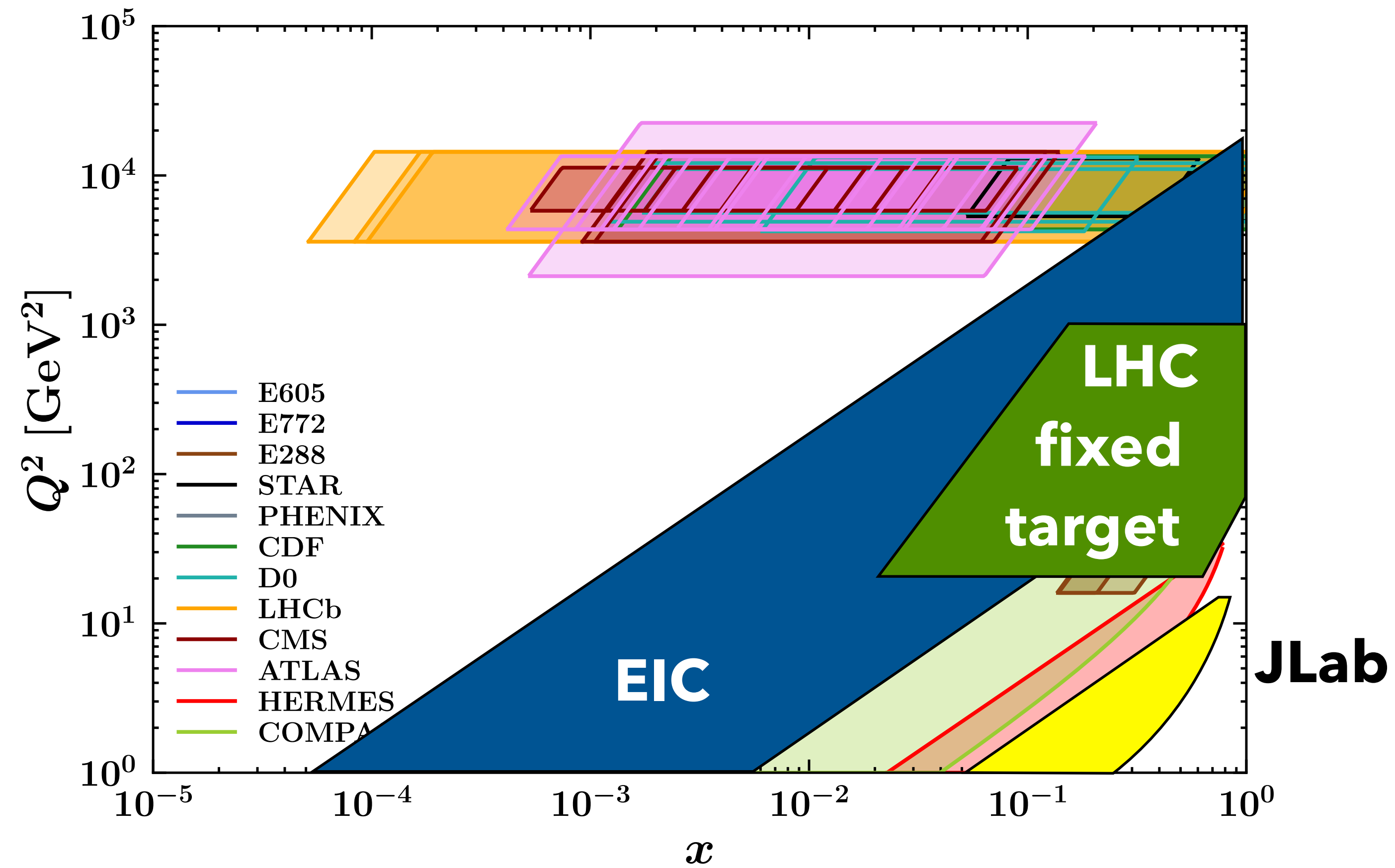
See talks by Abhay Deshpande, Pasquale Di Nezza, Patrizia Rossi



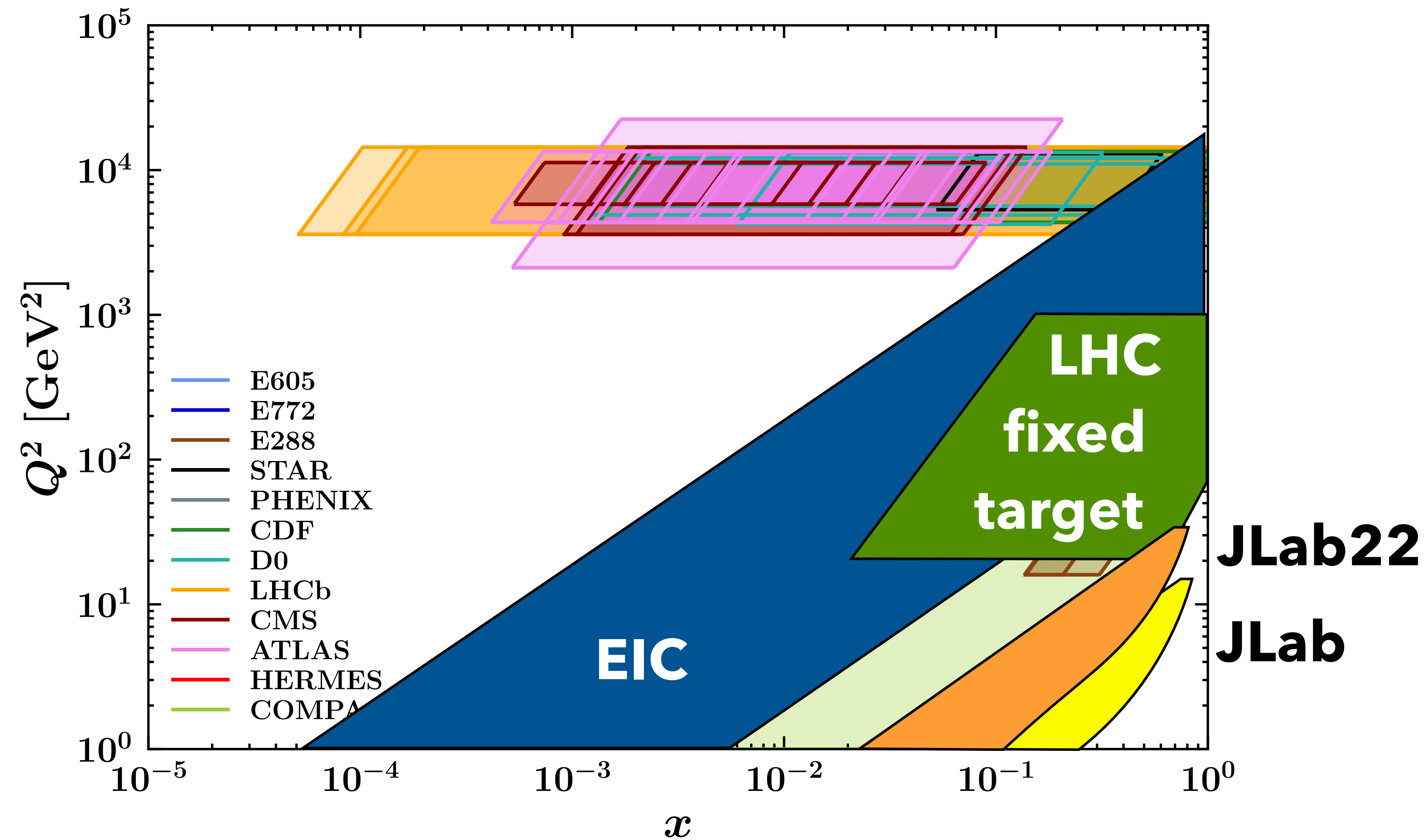
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In five to ten years

- TMD multiplicities for pions and kaons, off protons and deuterons, from COMPASS and JLab
- Drell-Yan and Z measurements from CERN, RHIC, FermiLab (COMPASS with pions)
- TMD multiplicities for pions and kaons in e^+e^- from BELLE and BES
- Better understanding and control of higher-order QCD corrections
- More flexible functional forms, flavour dependence, at least two or three alternative extractions
- Use TMDs for something else (W mass... comparison with lattice... Wigner distributions...)
- **READY TO USE EIC DATA**

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- ▶ Extractions of unpolarized TMDs are reaching a good level of sophistication, but there are still several open questions and new data are needed
- ▶ For other TMDs, the study has started and there is an increasing number of new results, but more data are needed