Non-perturbative aspects of Composite Higgs Models

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E. Bennett, J. Holligan, D.K. Hong, J.W. Lee, C.J.D. Lin, B.Lucini, MP, and D. Vadacchino, arXiv:2312.08465

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A. Athenodorou, E. Bennett, G. Bergner, D. Elander, C.J.D. Lin, B.Lucini, and MP, arXiv:1605.04258
J.W. Lee, B.Lucini, and MP, arXiv:1701.03228
D.K. Hong, J.W. Lee, B.Lucini, MP and D. Vadacchino, arXiv:1705.00286
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D.Elander, MP and J. Roughley, arXiv:2010.04100
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Outline

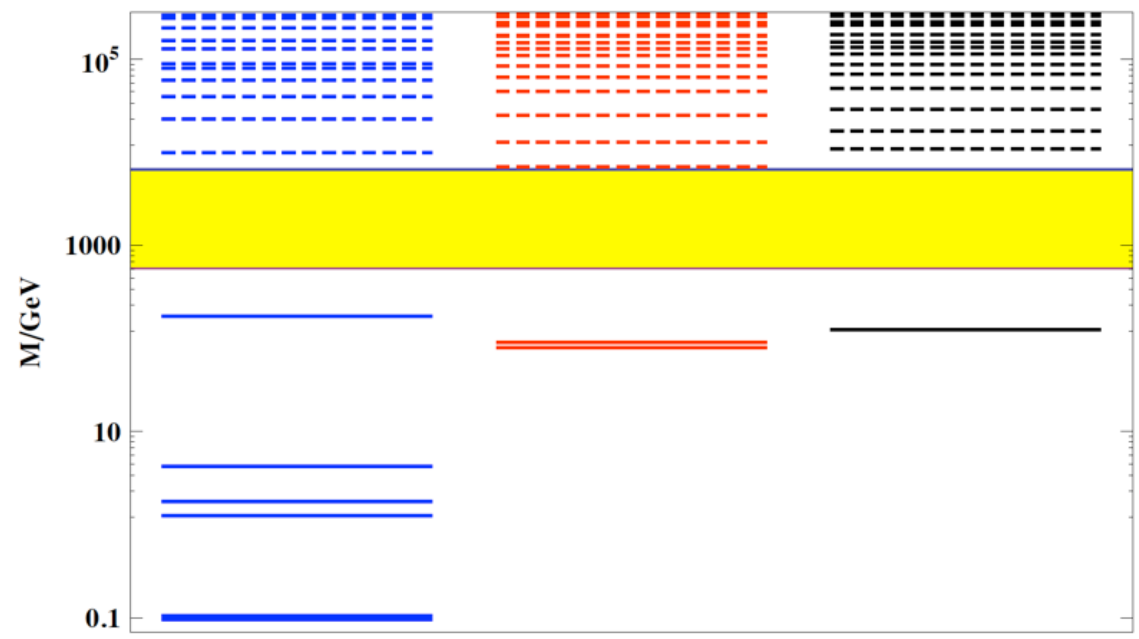
- Introduction and motivation
- Example 1: Dilaton EFT application to SU(3) lattice data (brief)
- Example 2: Dilaton in gauge-gravity dualities (brief)
- Example 3: Goldstone/Higgs from the lattice—Sp(2N) theories (long, and see other talks)
- Outlook

General message

- We are living in a golden age of numerical studies of non-perturbative field theories
  - Effective Field Theory
  - Gauge-gravity dualities
  - Lattice field theory
- Broad range of applications
  - Composite Higgs Models (Higgs as PNGB or Dilaton)
  - Top Partial Compositeness
  - Strongly Interacting Dark Matter
  - Gravitational Waves

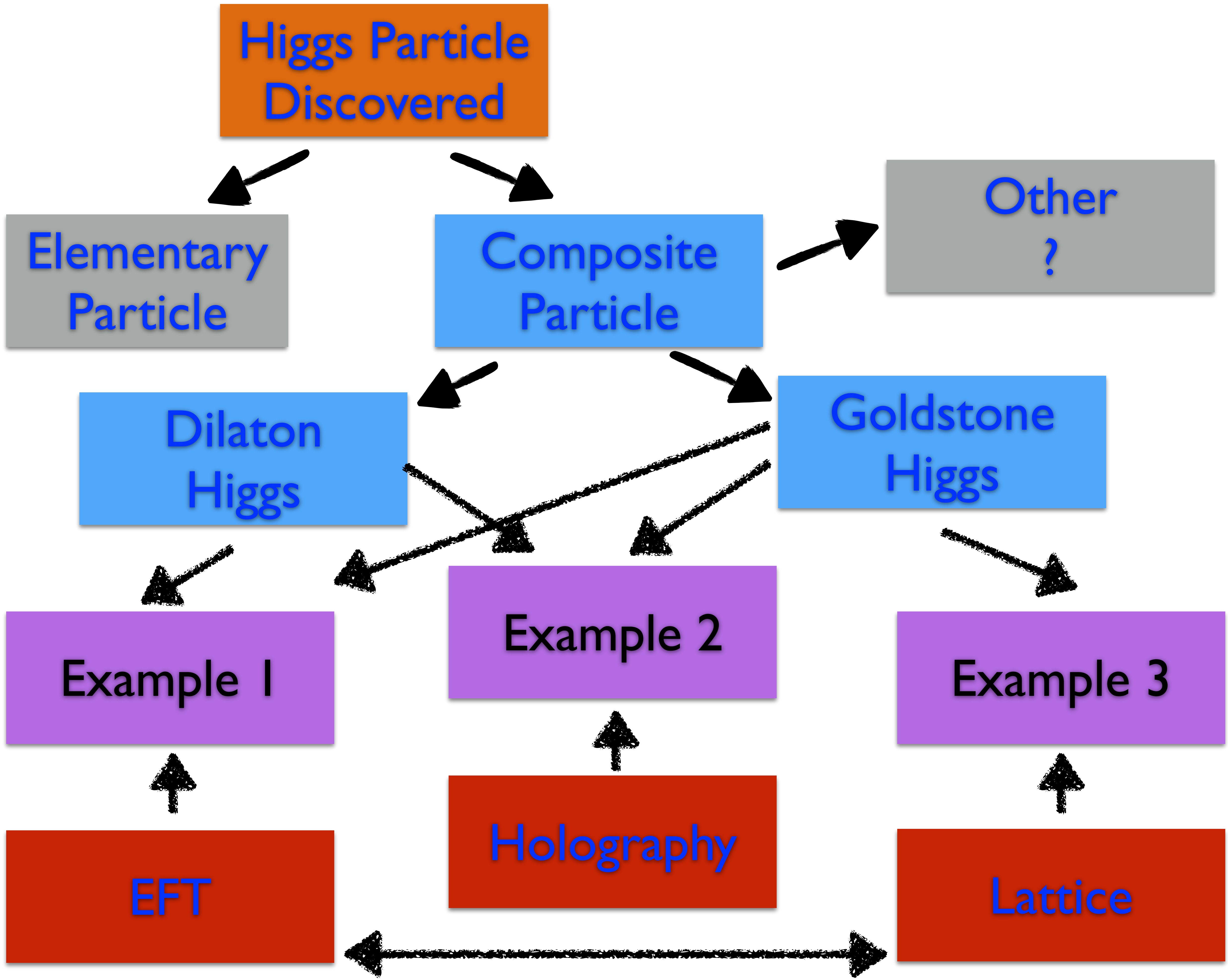
Statement of the Problem

- Weakly-coupled models of EWSB affected by BIG HIERARCHY PROBLEM
- Strong-coupling solution: replace Higgs sector with new interactions and fields, that UV-complete the theory (technicolor, composite Higgs, little Higgs…)
- NON-PERTURBATIVE INSTRUMENTS gauge-gravity dualities, lattice studies and applications of EFT
- LITTLE HIERARCHY PROBLEM



arXiv:1703.09205

Figure 1. *The mass spectrum of SM particles (continuous lines), compared to the range of current exclusions from LHC direct searches for exotics (shaded region) [2] and to the spectrum of a generic, hypothetical strongly-coupled new physics theory (dashed lines) with new states heavy enough to avoid current direct bounds. Fermions are rendered in blue, vectors in red and scalars in black.*


Higgs Particle Discovered
Elementary Particle
Composite Particle
Other ?
Dilaton Higgs
Goldstone Higgs
Example 1
Example 2
Example 3
EFT
Holography
Lattice

Three Examples

- Example I: Dilaton EFT description of SU(3) lattice data—application to Goldstone-Higgs models

- Example II: Dilaton and Goldstone-Higgs from top-down gauge-gravity dualities— maximal sugra in D=7 dimensions as example, SO(5)/SO(4).

- Example III: Goldstone-Higgs on the lattice—Sp(2N) theories

Dilaton EFT description of SU(3) lattice data—application to Goldstone-Higgs models

Lattice data on near conformal dynamics

- SU(3) theory, with either 8 fundamental (LatKMI, LSD Collaboration) or 2 symmetric (Dirac) fermions (Fodor et al.).
- Both theories expected to be close to edge of conformal window.
- Spectroscopy studied in some details, as a function of the quark mass, over large range of masses.
- Anomalously light scalar singlet bound state present.

T. Appelquist et al., arXiv:1601.04027;

A. D. Gasbarro and G. T. Fleming, arXiv:1702.00480

Y. Aoki et al. [LatKMI Collaboration], arXiv:1403.5000;

Y. Aoki et al. [LatKMI Collaboration], arXiv:1610.07011.

- What can it be? Could this be a light dilaton, and what can we learn about it?

Z. Fodor, K. Holland, J. Kuti, D. Nogradi, C. Schroeder and C. H. Wong, arXiv 1209.0391.

Z. Fodor, K. Holland, J. Kuti, S. Mondal, D. Nogradi and C. H. Wong, arXiv:1502.00028.

Z. Fodor, K. Holland, J. Kuti, S. Mondal, D. Nogradi and C. H. Wong, arXiv:1605.08750.

Z. Fodor, K. Holland, J. Kuti, D. Nogradi and C. H. Wong, arXiv:1712.08594.

Z. Fodor, K. Holland, J. Kuti, and C. H. Wong, arXiv:1901.06324.

Lattice data on near conformal dynamics

T. Appelquist et al., arXiv:1601.04027;

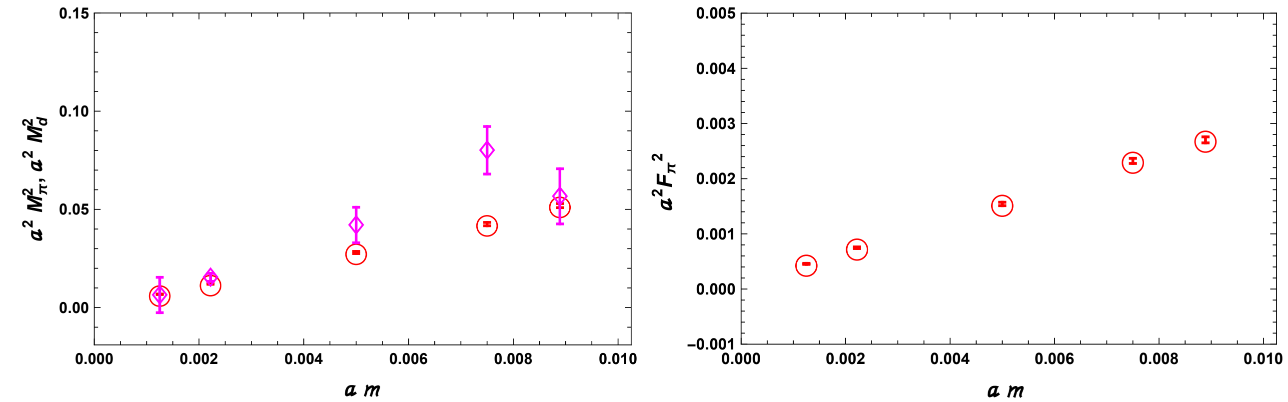


Figure 1. Lattice data from the LSD collaboration for the $SU(3)$ theory with $N_f = 8$ fundamentals [2]. Red circles represent the pseudoscalar data and their uncertainties are discussed in section 3.2.1. Pink diamonds represent the scalar data with uncertainties discussed in section 3.3. The lattice spacing is denoted by a.

Very different
from QCD

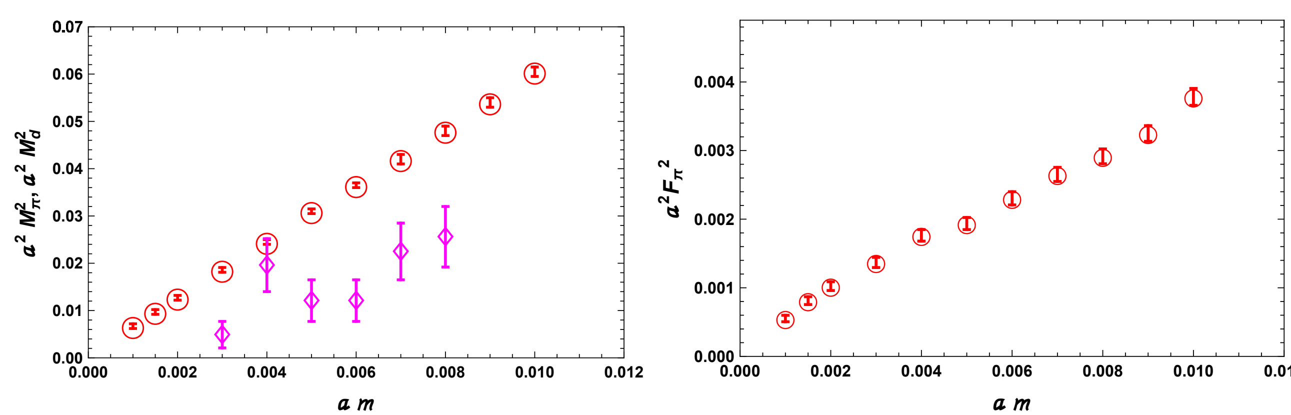


Figure 2. Lattice data extracted from plots in Refs. [5–7] for the $SU(3)$ theory with $N_f = 2$ sextets. Red circles represent the pseudoscalar data and pink diamonds represent the scalar. The lattice spacing is denoted by a. The errors are discussed in section 3.2.2.

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Z. Fodor, K. Holland, J. Kuti, and C. H. Wong, arXiv:1901.06324.

Dilaton EFT

T. Appelquist, J.Ingoldby and MP, arXiv:1702.04410
T. Appelquist, J.Ingoldby and MP, arXiv:1711.00067
T. Appelquist, J.Ingoldby and MP, arXiv:1908.00895

S. Coleman, "Aspects of Symmetry", Cambridge University Press;
W. D. Goldberger, B. Grinstein and W. Skiba, arXiv:0708.146
S. Matsuzaki and K. Yamawaki, arXiv:1311.3784;
M. Golterman and Y. Shamir, arXiv:1603.04575;
Kasai, K. i. Okumura and H. Suzuki, arXiv:1609.02264;
M. Golterman and Y. Shamir, arXiv:1610.01752;
M. Hansen, K. Langaeble and F. Sannino, arXiv:1610.02904;
M. Golterman and Y. Shamir, arXiv:1805.00198;
and more…3

- Weakly-coupled field theory, extends chiral Lagrangian by adding one scalar field.

$$\mathcal{L} = \frac{1}{2}\partial_\mu\chi\partial^\mu\chi + \mathcal{L}_\pi + \mathcal{L}_M - V(\chi)$$

$$\mathcal{L}_\pi = \frac{f_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^2 \mathrm{Tr}\left[\partial_\mu\Sigma(\partial^\mu\Sigma)^\dagger\right]$$

$$\mathcal{L}_M = \frac{m_\pi^2 f_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^y \mathrm{Tr}\left[\Sigma + \Sigma^\dagger\right]$$

- Minimum of V at f_d. If mass present, minimum: $\chi = F_d\,(\geq f_d)$

- Scaling relations, by varying quark mass one is exploring properties of the EFT.

$$\frac{F_\pi^2}{f_\pi^2} = \frac{F_d^2}{f_d^2}\,,$$

$$\frac{M_\pi^2}{m_\pi^2} = \left(\frac{F_d^2}{f_d^2}\right)^{y/2-1}$$

$$M_\pi^2 F_\pi^{2-y} = Cm$$

- One can measure scaling dimension y of chiral condensate.

- Possibly shape of the scalar potential, and possibly extrapolate towards massless limit.

$$\left.\frac{\partial V}{\partial\chi}\right|_{\chi=F_d} = \frac{yN_f m_\pi^2 f_\pi^2}{2f_d^y}F_d^{y-1} = \frac{yN_f f_\pi}{2f_d}M_\pi^2 F_\pi$$

Dilaton EFT

- Simplest Example: power-law potential $V \propto \chi^p$, second scaling relation:

$$M_\pi^2 F_\pi^{2-y} = Cm \qquad M_\pi^2 = B F_\pi^{p-2}$$

T. Appelquist, J.Ingoldby and MP, arXiv:1711.00067

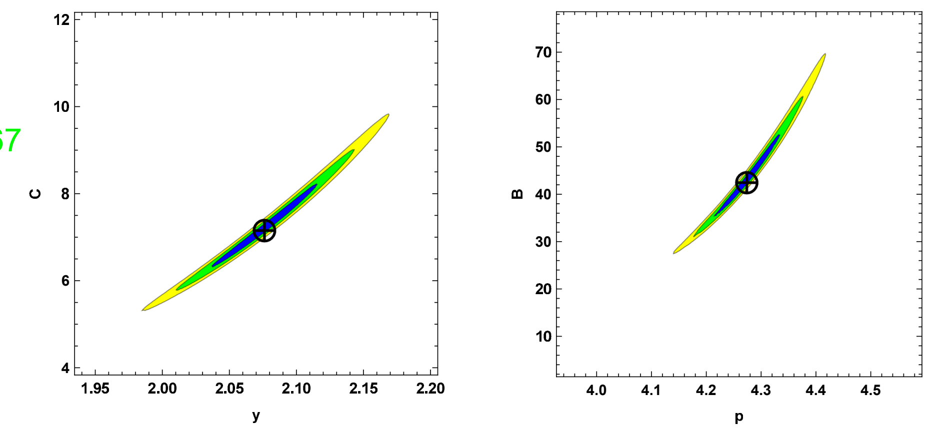


Figure 3. Contour plot from a 2-parameter fit based on Eq. (3.1) for the LSD data (left panel) and on Eq. (3.4), also for the LSD data (right panel). Contours correspond to 68.17% c.l. (blue), 95.45% c.l. (green) and 99.73% c.l., obtained for $\Delta\chi^2 = \{2.30\,,\, 6.18\,,\, 11.83\}$ respectively. The black crosses indicate the central values of the fit parameters.

T. Appelquist, J.Ingoldby and MP, arXiv:1711.00067

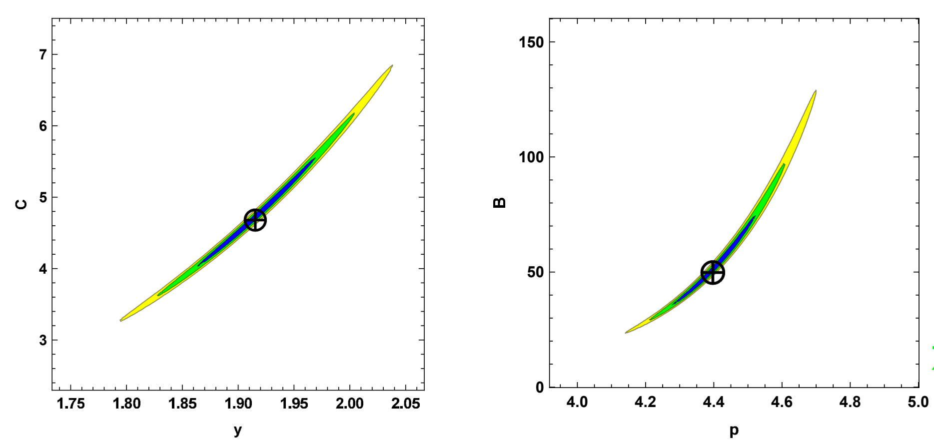


Figure 4. Contour plot from the 2-parameter fit based on Eq. (3.1) for the sextet data (left panel) and on Eq. (3.4) also for the sextet data (right panel). We show the contours corresponding to 68.17% c.l. (blue), 95.45% c.l. (green) and 99.73% c.l., obtained for $\Delta\chi^2 = \{2.30\,,\, 6.18\,,\, 11.83\}$ respectively. The black crosses indicate the central values of the fit parameters.

Both data sets suggest

y ~ 2, related to chiral condensate having dimension 2, not 3

See also:

Z. Fodor, K. Holland, J. Kuti, D. Nogradi and C. H. Wong, arXiv:1712.08594.

Z. Fodor, K. Holland, J. Kuti, and C. H. Wong, arXiv:1901.06324.

A Near-Conformal Composite Higgs Model

T. Appelquist, J.Ingoldby and MP, arXiv:2012.09698

L. Vecchi, arXiv:1506.00623
T. Ma and G. Cacciapaglia, arXiv:1508.07014

- Same SU(3) lattice theory can also be used to build a Nambu-Goldstone Composite Higgs Model, with coset SU(8)xSU(8)/SU(8)
- Field content in terms of SM and new SU(3) gauge theory:

Fermion	$\mathrm{SU}(2)_L$	$\mathrm{U}(1)_Y$	$SU(3)_c$	$SU(3)$
L_α	2	0	1	3
$R_{1,2}$	1	$\begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$	1	3
T	1	2/3	3	3
S	1	0	1	3

- Several special features: additional scalar singlet state in the spectrum, large anomalous dimensions, ordinary baryons act as composite top partners.
- Spectrum: one light scalar (Higgs boson) is admixture of a Goldstone mode and the scalar singlet, while all other scalars can be made heavier that 4 TeV (some are coloured, some have fractional charges).
- Highly non-trivial spectral information already available from lattice: masses of mesons and baryons for various values of mass term (explicit symmetry breaking), and anomalous dimensions—as seen.

Dilaton and Goldstone-Higgs from top-down gauge-gravity dualities— maximal sugra in D=7 dimensions

D.Elander, MP and J. Roughley, arXiv:2004.05656
D.Elander, MP and J. Roughley, arXiv:2010.04100
D.Elander, MP and J. Roughley, arXiv:2011.07049

Modelling Confinement in top-down holography

- Big questions: can one demonstrate that dilaton emerges from strong coupling models? Can one study microscopic dynamics of minimal SO(5)/SO(4) model? Se e.g. R. Contino, 1005.4269 [hep-ph] and references therein

    - In both cases, essential to be able to describe confinement reliably—-top-down holography may help.

- Basic idea borrowed from Witten: consider known AdS solution in a known supergravity, compactify one dimension on a circle and find new closely related background solution with smooth geometry in which one circle shrinks to zero size.

E. Witten, hep-th/9803131

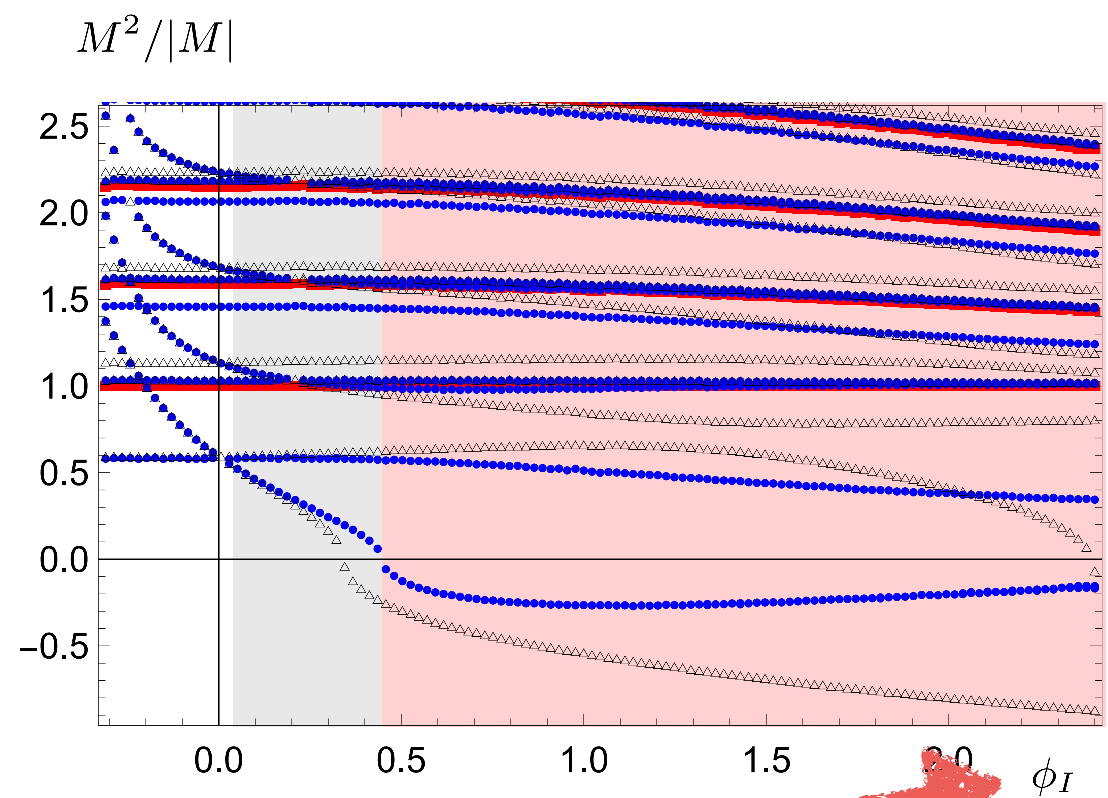
- Various realisations in the literature: smooth, stable solutions known, for theories that contain interplay of non-trivial flows in higher-dimensional field theory (dual scalars coupled to gravity) and confinement. Known examples both in half-maximal sugra in D=6 (Romans) and maximal sugra in D=7, smooth and regular (with caveats).

C. K. Wen, and HX Yang, hep-th/0404152
S. Kuperstein and J. Sonnenschein hep-th/0411009
D. Elander, A. Faedo, C. Hoyos, D. Mateos, MP, arXiv:1312.7160

- We found new solutions in both systems that are smooth and regular (with same caveats), that can be interpreted as dual to confining theories generalising the mechanism above, but that approach a classical instability—generalisation of suggestion by Kaplan et al. that physics in proximity of BZ bound is special.

D.B. Kaplan, J.W.Lee, D.T. Son, M.A. Stephanov, arXiv:0905.4752
V. Gorbenko, S. Rychkov, B. Za, arXiv:1807.11512
V. Gorbenko, S. Rychkov, B. Za, arXiv:1808.04380
A. Pomarol, O. Pujolas, L. Salas, arXiv:1905.02653

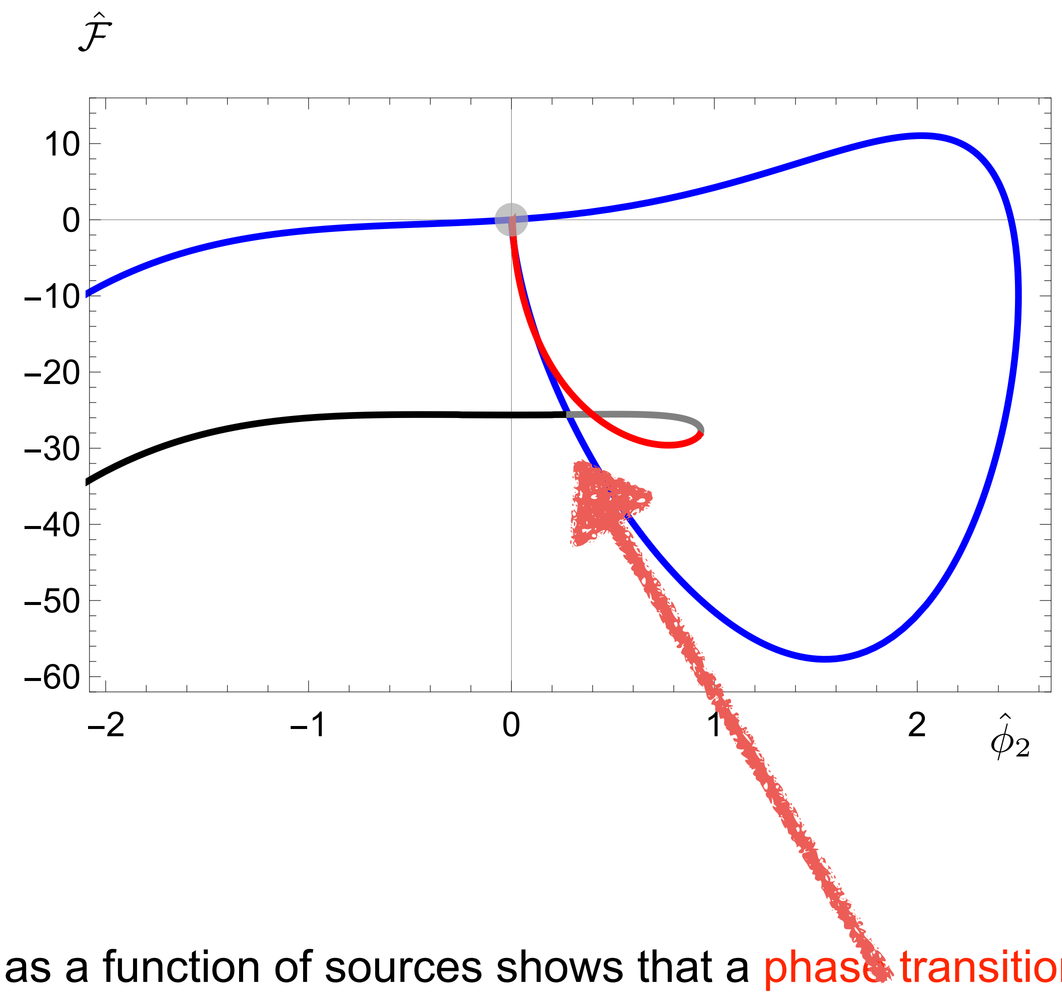
Modelling Confinement in holography (dilaton)



D.Elander, MP and J. Roughley, arXiv:2011.07049

- 1-parameter family of solutions in D=7 maximal sugra, dual to four-dimensional, confining theory.
- Spectrum of fluctuation interpreted as bound states. Blue: spin-0, red: spin-2,
    - black: again spin-0, but in probe approximation.
- Lightest scalar becomes massless, then tachyonic in a region of 1-parameter class of solutions
- Probe approximation fails when lighest scalar near massless—-state is an approximate dilaton
- But…

Modelling Confinement in holography (phase transition)



D.Elander, MP and J. Roughley, arXiv:2011.07049

- Free energy as a function of sources shows that a phase transition is taking place.
- The massless state emerges only "past" the phase transition. In physical region, lightest scalar not parameterically light.
- Open question: is this generic?
- Open question: can the phase transition be weak, in some special case?

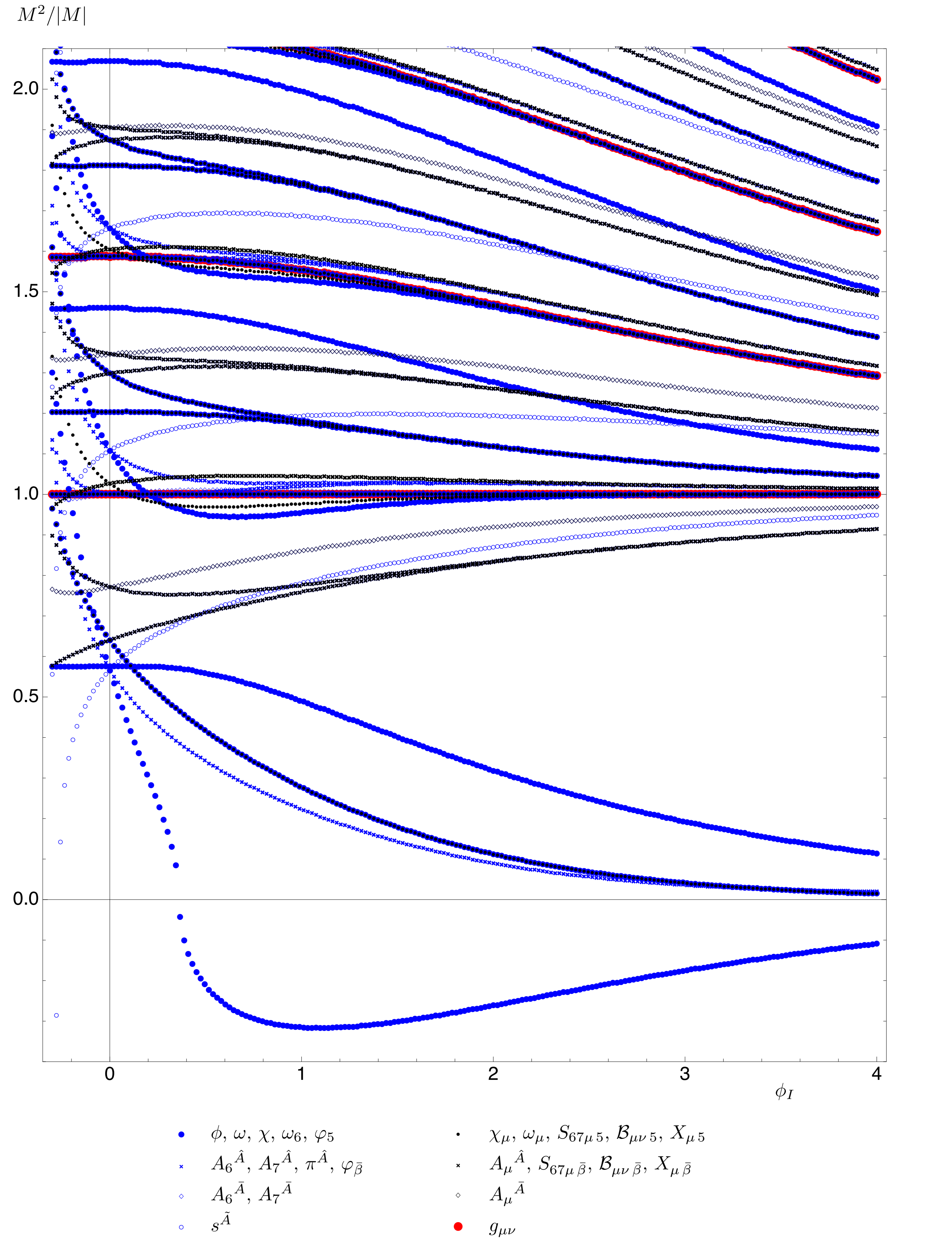
Modelling Confinement in holography (global symmetry)

D.Elander and MP, arXiv:2110.02945

- Same D=7 theory has SO(5) gauge symmetry, broken to SO(4).
- Same solutions (torus compactification) admit interpretation as four-dimensional confining theories, with global SO(5) broken to SO(4).
- This is a possible top-down completion for the minimal composite Goldstone-Higgs models, the SO(5)/SO(4) coset leads to 4 PNGBs interpreted as the SM Higgs doublet.
- Extensive literature within bottom-up holography exists.

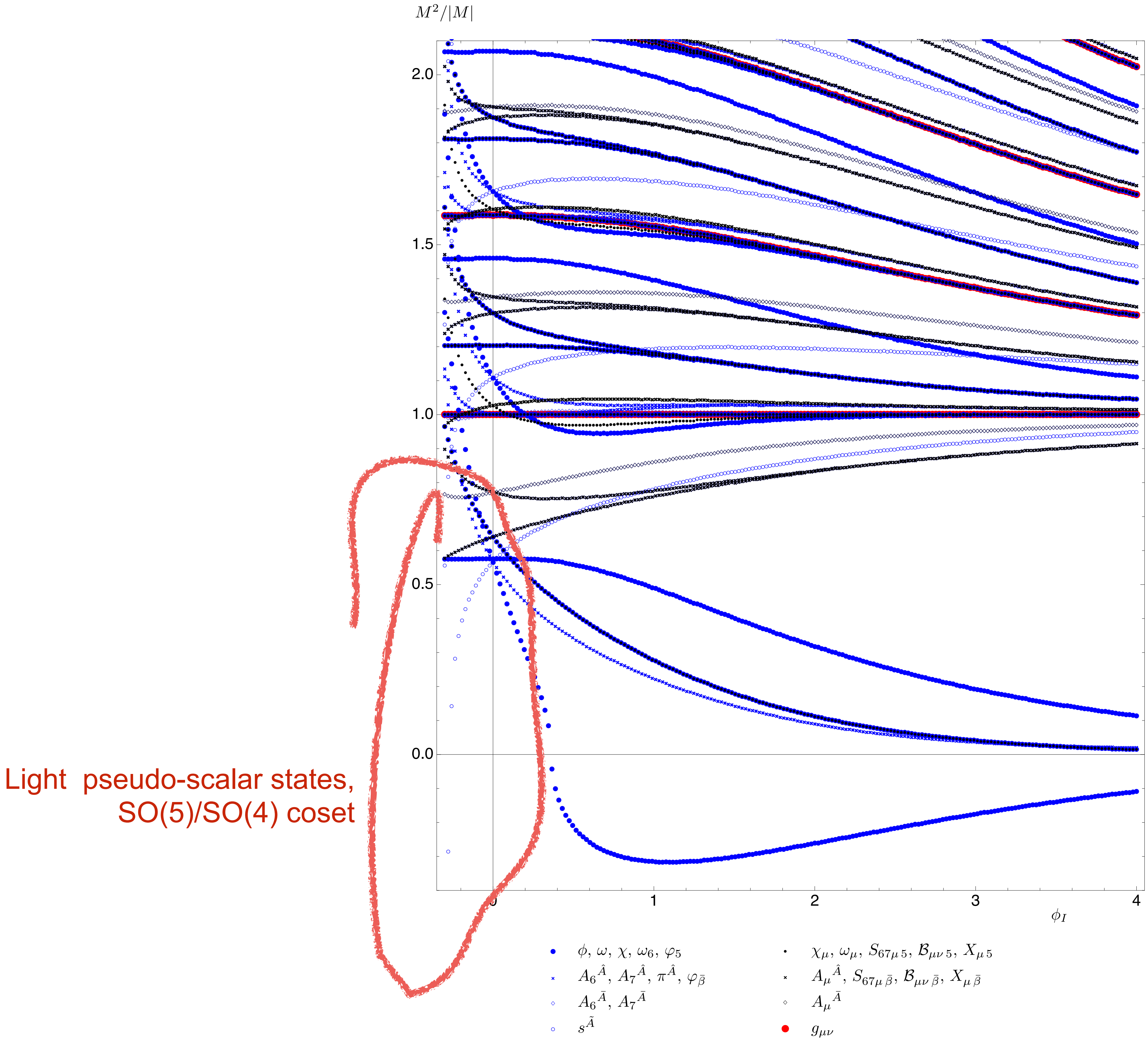
R. Contino, Y. Nomura, and A. Pomarol, arXiv:hep-ph/0306259
K. Agashe, R. Contino, and A. Pomarol, arXiv:hep-ph/0412089
K. Agashe and R. Contino, arXiv:hep-ph/0510164
K. Agashe, R. Contino, L. Da Rold, and A. Pomarol, arXiv:hep-ph/0605341
R. Contino, L. Da Rold, and A. Pomarol, arXiv:hep-ph/0612048

- We computed the spectrum of bound states along the 1-parameter family.
- Considered all bosonic fluctuations of the regular backgrounds.
- In a region of parameter space, pseudo-Goldstone bosons spanning SO(5)/SO(4) coset are parametrically light.



D.Elander and MP, arXiv:2110.02945

Figure 7. Detail of the combined mass spectrum $M^2 = -q^2$ of all the bosons, as a function of the parameter ϕ_I, normalised to the lightest tensor mode, and restricted to the low-mass region. In red we depict the spin-2 particles, in black the spin-1, in blue the spin-0. The markers are chosen to match the representations under $SO(4)$, while the legend refers back to the notation in Table 1. Numerical calculations use $\rho_1 = 10^{-10}$ and $\rho_2 = 10$.



D.Elander and MP, arXiv:2110.02945

Figure 7. Detail of the combined mass spectrum $M^2 = -q^2$ of all the bosons, as a function of the parameter ϕ_I, normalised to the lightest tensor mode, and restricted to the low-mass region. In red we depict the spin-2 particles, in black the spin-1, in blue the spin-0. The markers are chosen to match the representations under $SO(4)$, while the legend refers back to the notation in Table 1. Numerical calculations use $\rho_1 = 10^{-10}$ and $\rho_2 = 10$.

Goldstone/Higgs from the lattice—Sp(2N) theories

Statement of the Problem—cartoon

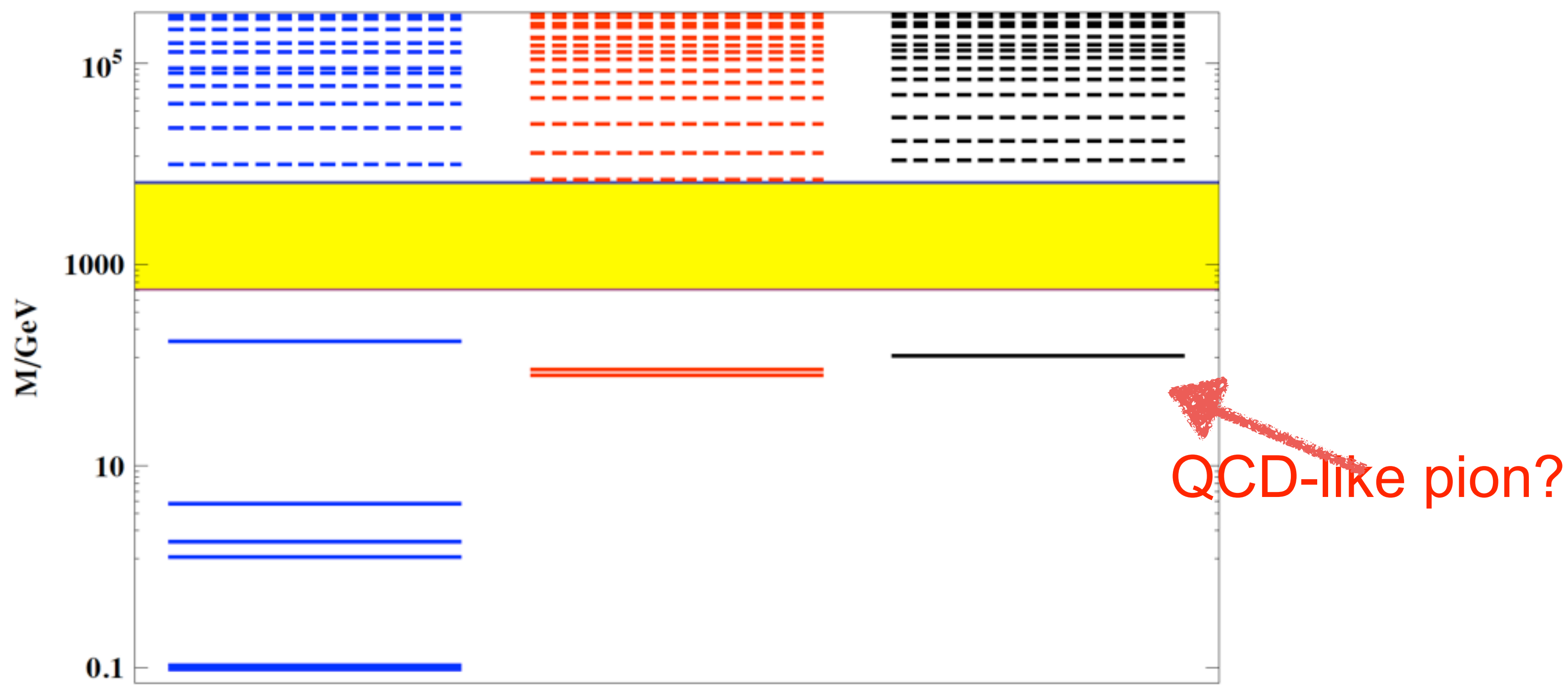


Figure 1. *The mass spectrum of SM particles (continuous lines), compared to the range of current exclusions from LHC direct searches for exotics (shaded region) [2] and to the spectrum of a generic, hypothetic strongly-coupled new physics theory (dashed lines) with new states heavy enough to avoid current direct bounds. Fermions are rendered in blue, vectors in red and scalars in black.*

Composite Higgs Models

- Consider strong-coupling dynamics of a new gauge theory.
- Dynamics leads to breaking of global symmetry G to subgroup H.
- Spectrum: massless pions in G/H coset, towers of massive states.
- Low-energy Effective Theory: describe pions by means of weakly-coupled scalar fields.
- Step 1: interpret four of the scalar/pion fields as the Higgs fields of the Standard Model.
- Step 2: embed the SM gauge symmetries into G, so that the Higgs/pions have correct quantum numbers.
- Step 3: top compositeness? Look for exotic fermion bound states.

Composite Higgs Models

D. B. Kaplan and H. Georgi, Phys. Lett. 136B, 183 (1984).

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G. Ferretti and D. Karateev, JHEP 1403, 077 (2014) [arXiv:1312.5330 [hep-ph]].

G. Cacciapaglia and F. Sannino, JHEP 1404, 111 (2014) [arXiv:1402.0233 [hep-ph]].

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Composite Higgs Models

T. Alanne, D. Buarque Franzosi and M. T. Frandsen, Phys. Rev. D 96, no. 9, 095012 (2017) [arXiv:1709.10473 [hep-ph]].

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Composite Higgs Models

II. UNDERLYING MODELS FOR A COMPOSITE HIGGS WITH TOP PARTIAL COMPOSITENESS

Coset	HC	ψ	χ	$-q_\chi/q_\psi$	Baryon	Name	Lattice
$\frac{SU(5)}{SO(5)} \times \frac{SU(6)}{SO(6)}$	SO(7)	$5 \times \mathbf{F}$	$6 \times \mathbf{Sp}$	5/6	$\psi\chi\chi$	M1	
	SO(9)	$5 \times \mathbf{F}$	$6 \times \mathbf{Sp}$	5/12	$\psi\chi\chi$	M2	
	SO(7)	$5 \times \mathbf{Sp}$	$6 \times F$	5/6	$\psi\psi\chi$	M3	
	SO(9)	$5 \times \mathbf{Sp}$	$6 \times F$	5/3	$\psi\psi\chi$	M4	
$\frac{SU(5)}{SO(5)} \times \frac{SU(6)}{Sp(6)}$	Sp(4)	$5 \times \mathbf{A}_2$	$6 \times \mathbf{F}$	5/3	$\psi\chi\chi$	M5	$\surd$
$\frac{SU(5)}{SO(5)} \times \frac{SU(3)^2}{SU(3)}$	SU(4)	$5 \times \mathbf{A}_2$	$3 \times (\mathbf{F}, \overline{\mathbf{F}})$	5/3	$\psi\chi\chi$	M6	$\surd$
	SO(10)	$5 \times \mathbf{F}$	$3 \times (\mathbf{Sp}, \overline{\mathbf{Sp}})$	5/12	$\psi\chi\chi$	M7	
$\frac{SU(4)}{Sp(4)} \times \frac{SU(6)}{SO(6)}$	Sp(4)	$4 \times \mathbf{F}$	$6 \times \mathbf{A}_2$	1/3	$\psi\psi\chi$	M8	$\surd$
	SO(11)	$4 \times \mathbf{Sp}$	$6 \times \mathbf{F}$	8/3	$\psi\psi\chi$	M9	
$\frac{SU(4)^2}{SU(4)} \times \frac{SU(6)}{SO(6)}$	SO(10)	$4 \times (\mathbf{Sp}, \overline{\mathbf{Sp}})$	$6 \times \mathbf{F}$	8/3	$\psi\psi\chi$	M10	
	SU(4)	$4 \times (\mathbf{F}, \overline{\mathbf{F}})$	$6 \times \mathbf{A}_2$	2/3	$\psi\psi\chi$	M11	$\surd$
$\frac{SU(4)^2}{SU(4)} \times \frac{SU(3)^2}{SU(3)}$	SU(5)	$4 \times (\mathbf{F}, \overline{\mathbf{F}})$	$3 \times (\mathbf{A}_2, \overline{\mathbf{A}_2})$	4/9	$\psi\psi\chi$	M12	

TABLE I. Model details. The first column shows the EW and QCD colour cosets, respectively, followed by the representations under the confining hypercolour (HC) gauge group of the EW sector fermions ψ and the QCD coloured ones χ. The $-q_\chi/q_\psi$ column indicates the ratio of charges of the fermions under the non-anomalous $U(1)$ combination, while "Baryon" indicate the typical top partner structure. The column "Name" contains the model nomenclature from Ref. [27], while the last column marks the models that are currently being considered on the lattice. Note that **Sp** indicates the spinorial representation of SO(N), while **F** and $\mathbf{A}_2$ stand for the fundamental and two-index anti-symmetric representations.

G. Cacciapaglia, G. Ferretti, T. Flacke, H. Serodio
arXiv:1902.06890

see also
G. Ferretti and D. Karateev. arXiv1312.5330

and references therein
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Composite Higgs Models

II. UNDERLYING MODELS FOR A COMPOSITE HIGGS WITH TOP PARTIAL COMPOSITENESS

Coset	HC	ψ	χ	$-q_\chi/q_\psi$	Baryon	Name	Lattice
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$\frac{SU(5)}{SO(5)} \times \frac{SU(6)}{Sp(6)}$	Sp(4)	$5 \times \mathbf{A}_2$	$6 \times \mathbf{F}$	5/3	$\psi\chi\chi$	M5	√
$\frac{SU(5)}{SO(5)} \times \frac{SU(3)^2}{SU(3)}$	SU(4)	$5 \times \mathbf{A}_2$	$3 \times (\mathbf{F}, \overline{\mathbf{F}})$	5/3	$\psi\chi\chi$	M6	√
	SO(10)	$5 \times \mathbf{F}$	$3 \times (\mathbf{Sp}, \overline{\mathbf{Sp}})$	5/12	$\psi\chi\chi$	M7	
$\frac{SU(4)}{Sp(4)} \times \frac{SU(6)}{SO(6)}$	Sp(4)	$4 \times \mathbf{F}$	$6 \times \mathbf{A}_2$	1/3	$\psi\psi\chi$	M8	√
	SO(11)	$4 \times \mathbf{Sp}$	$6 \times \mathbf{F}$	8/3	$\psi\psi\chi$	M9	
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	SU(4)	$4 \times (\mathbf{F}, \overline{\mathbf{F}})$	$6 \times \mathbf{A}_2$	2/3	$\psi\psi\chi$	M11	√
$\frac{SU(4)^2}{SU(4)} \times \frac{SU(3)^2}{SU(3)}$	SU(5)	$4 \times (\mathbf{F}, \overline{\mathbf{F}})$	$3 \times (\mathbf{A}_2, \overline{\mathbf{A}_2})$	4/9	$\psi\psi\chi$	M12	

TABLE I. Model details. The first column shows the EW and QCD colour cosets, respectively, followed by the representations under the confining hypercolour (HC) gauge group of the EW sector fermions ψ and the QCD coloured ones χ. The $-q_\chi/q_\psi$ column indicates the ratio of charges of the fermions under the non-anomalous $U(1)$ combination, while "Baryon" indicate the typical top partner structure. The column "Name" contains the model nomenclature from Ref. [27], while the last column marks the models that are currently being considered on the lattice. Note that **Sp** indicates the spinorial representation of SO(N), while **F** and $\mathbf{A}_2$ stand for the fundamental and two-index anti-symmetric representations.

G. Cacciapaglia, G. Ferretti, T. Flacke, H. Serodio
arXiv:1902.06890
and references therein
…

Sp(2N) on the lattice

Fields	$Sp(4)$	$SU(4)$	$SU(6)$
V_μ	10	1	1
q	4	4	1
ψ	6	1	6
Σ_6	1	6	1
M_6	1	$\bar{6} \sim 6$	1
Σ_{21}	1	1	21
M_{21}	1	1	$\overline{21}$

Table 1. Field content of the fundamental theory (V_μ, q, ψ) and of the low-energy EFT describing the PNGB's ($\Sigma_{6,21}$, $M_{6,21}$). $Sp(4)$ is the gauge group, while $SU(4)$ and $SU(6)$ are the global symmetries. The elementary fields V_μ are gauge bosons, while q and ψ are 2-component spinors. Σ_6 and Σ_{21} are composite scalar fields. They capture the long-distance dynamics of operators that are bi-linear in q and ψ, the VEV's of which are responsible for the breaking $SU(4) \to Sp(4)$ and $SU(6) \to SO(6)$, respectively. The mass matrices M_6 and M_{21} are treated as a scalar spurions, formally transforming as $\sim \bar{6} \sim 6$ of $SU(4)$, and $\sim \overline{21}$ of $SU(6)$, respectively.

J. Barnard, T. Gherghetta, and T. S. Ray, arXiv:1311.6562

see also
G. Ferretti and D. Karateev. arXiv1312.5330

- Sp(4) lattice gauge theory on the lattice.
- Wilson fermions: 2 Dirac fundamental and 3 Dirac 2-index Antisymmetric
- SU(4)/Sp(4) coset leads to Higgs Composite Model, SU(6)/SO(6) plays role in top compositeness.

Sp(2N) theory programme

- Higgs Compositeness: Sp(2N) origin of SU(4)/Sp(4) and SU(6)/SO(6) cosets.
- Lattice formulation for general Sp(2N), and general matter content.
- BSM spectroscopy: glueballs, mesons, (chimera) baryons, excited states.
- Effective Field Theory study of low energy data.
- Applications: Composite Higgs, top partial compositeness, dark matter, gravitational waves.
- Theory: new arena to test field theory ideas (large-N, topological susceptibility, effective string, gauge-gravity correspondences), and analysis techniques (e.g., spectral densities)

Sp(2N) theory progress

- Glueballs: uses variational approach and complete basis of states, continuum limit, extension from Sp(4) to Sp(2N), large-N extrapolation, comparison with other approaches…
- Flavored mesons, quenched: extensive studies of Sp(2N) with quenched matter (fundamental, antisymmetric, symmetric), including extrapolations to massless, continuum and large-N limits
- Flavored mesons, dynamical: results available for Sp(4) with fundamental fermions, in medium-to-large mass range, preliminary results for antisymmetric fermions, and for multiple representation fermions, work on spectral densities ongoing.
- Singlet mesons: study of singlet mesons using subtraction/smearing for SU(2) and Sp(4) available, multiple representations in progress
- Chimera baryons: proof of concept published (few lattice ensembles), systematic study and extrapolations for quenched Sp(4) completed, multiple representations in progress
- Topology: systematic study in Sp(2N), and extrapolations to large-N, for topology and topological susceptibility
- Phase transitions: Sp(4) at finite T, with LLR proof of concept complete, large volume in progress (Fugaku)

Sp(2N) theory scorecard

- Glueballs: ground states ok, compares well with SU(N) literature, excited states large error, topological freezing in large-N
- Flavored mesons, quenched: complete and extensive, some heavy states and excitations large error, massless extrapolation not rigorous, no anomalous dimensions (of course…)
- Flavored mesons, dynamical: fundamental fermions high statistics, preliminary results on multiple representations robust. Wilson fermions: no result below threshold
- Singlet mesons: (subtraction and) smearing+GEVP work, continuum massless extrapolation non trivial, especially for positive parity
- Chimera baryons: interesting results for Sp(4), more to come on dynamical multiple representations, low mass, and anomalous dimensions challenging
- Topology: topological susceptibility at large-N, topological freezing problematic
- Phase transitions: Sp(4) at finite T with LLR works, scalability under investigation

Highlight 1: Glueballs

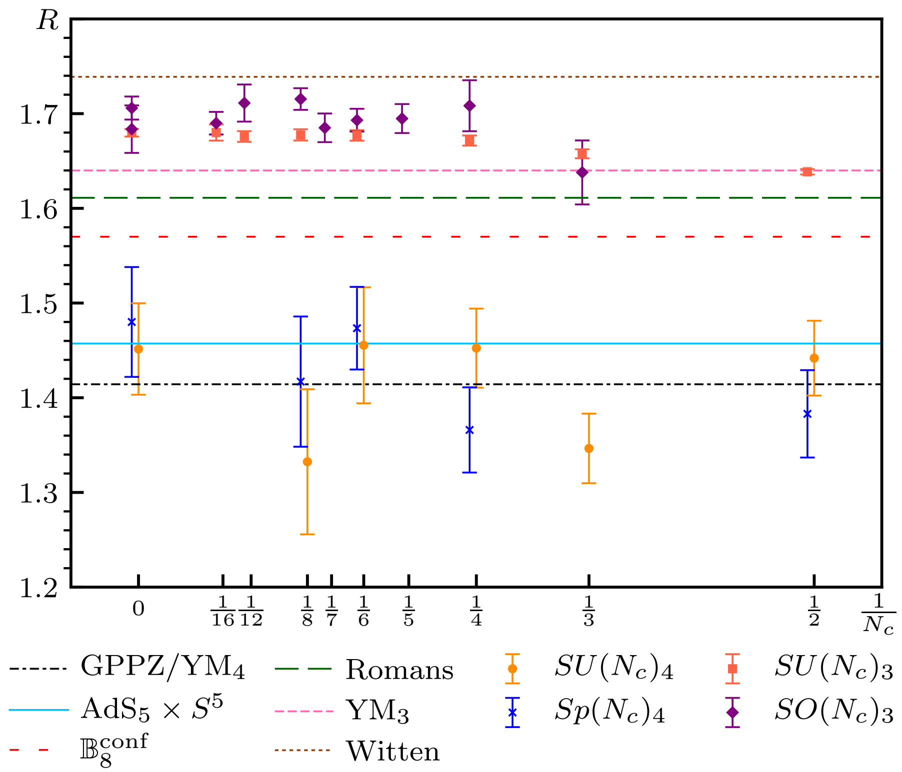


FIG. 1: Numerical and analytical results for the ratio R defined in Eq. (1). Different shaped markers denote the lattice measurements with continuum extrapolations in $D = 3 + 1$ dimensions for $Sp(N_c)$ and for $SU(N_c)$ [6], as well as in $D = 2 + 1$ dimensions for $SO(N_c)$ [7] and $SU(N_c)$ [8]. Extrapolations to the $N_c \to \infty$ limit are also included. Differently rendered lines at $R = \sqrt{2}, 1.46, 1.57, 1.61, 1.74$, are the holographic calculations in the GPPZ model [15], the circle reduction of AdS$_5 \times S^5$ [21, 22], the holographic model $\mathbb{B}_8^{\rm conf}$ in Ref. [37], the Witten model [21, 32], and the circle reduction of Romans supergravity [32, 34], respectively. With $R = \sqrt{2}, 1.64$ we report the field theoretical results from Refs. [19] and [43], for YM theories in $D = 3+1$ and $D = 2+1$ dimensions, respectively. More details can be found in the main text.

- Detailed study of continuum extrapolation of glueball spectra is Sp(2N). N=1,2,3,4
- All spins and parity
- Extrapolation to large N
- Evidence of universality in ratio of two lightest glueballs:

$$R \equiv \frac{m_{2^{++}}}{m_{0^{++}}}$$

E. Bennett, D.K. Hong, J.W. Lee, C.J.D. Lin, B.Lucini, MP and D. Vadacchino, arXiv:1712.04220
E. Bennett, J. Holligan, D.K. Hong, J.W. Lee, C.J.D. Lin, B.Lucini, MP and D. Vadacchino, arXiv:2004.11063
E. Bennett, J. Holligan, D.K. Hong, J.W. Lee, C.J.D. Lin, B.Lucini, MP and D. Vadacchino, arXiv:2010.15781

Highlight 2: Dynamical Fermions

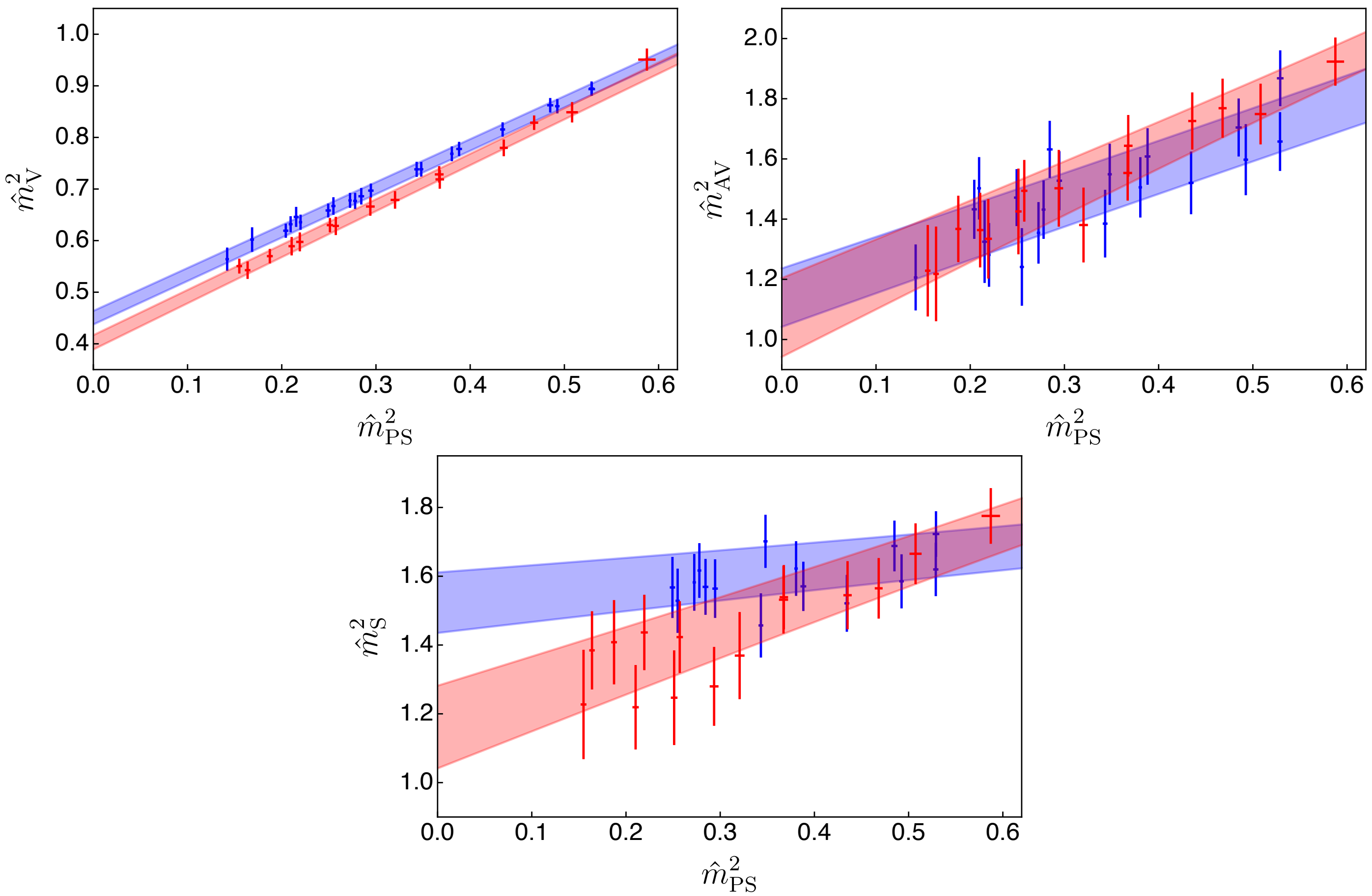
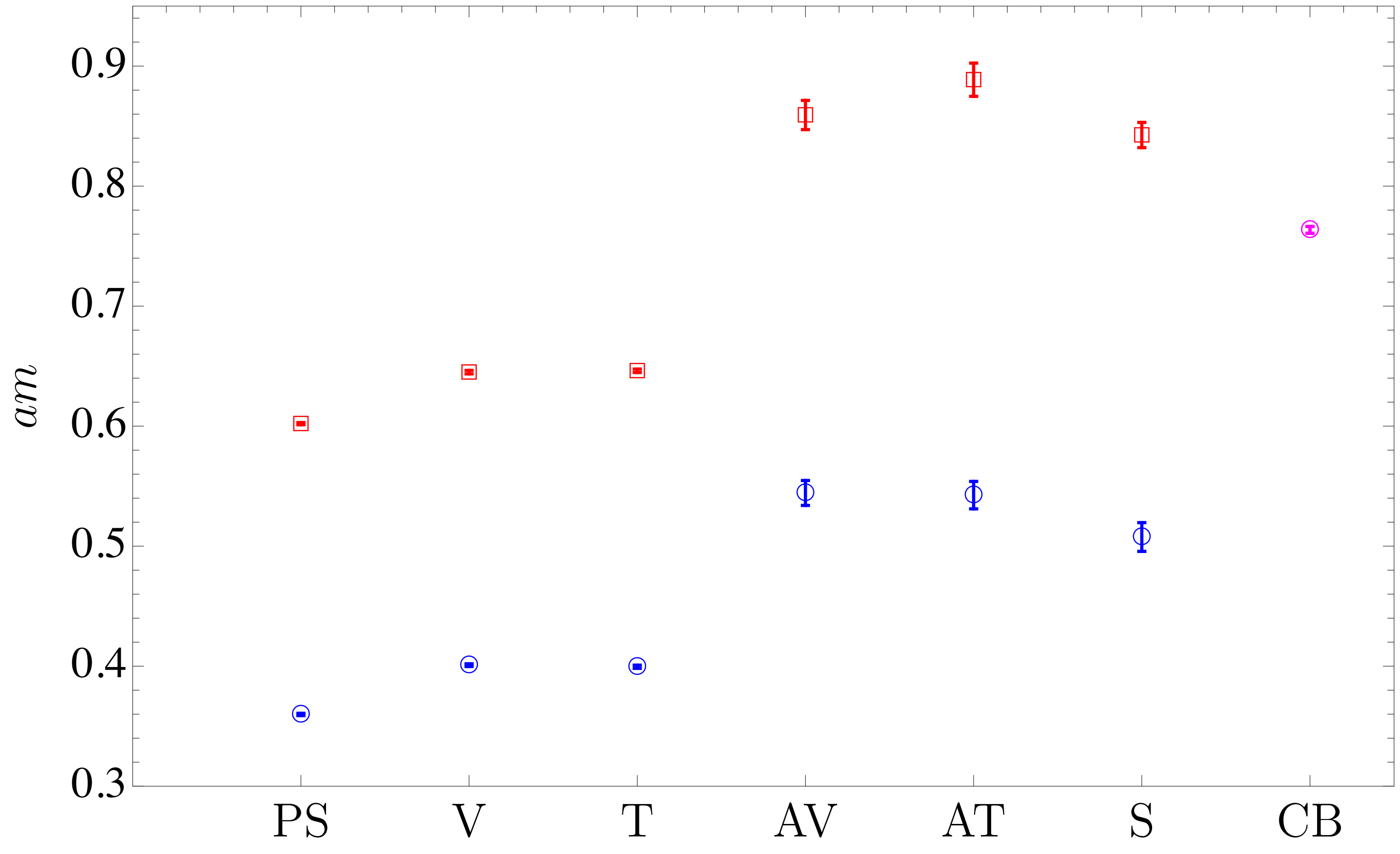


Figure 18: Meson masses squared from quenched (blue) and dynamical (red) calculations, in the continuum limit obtained by considering all the ensembles with $\hat{m}^2_{\rm PS} \lesssim 0.6$, as in Section 4.3. The coloured bands illustrate the fit of the measurements used in the massless extrapolations, with the width of the bands representing the statistical error in the fit.

- Sp(4) spectra computed both quenched and with 2 dynamical fundamental fermions
- Quenched approximation works well.
- But with caveats: large mass range, dependence of channel

E. Bennett, D.K. Hong, J.W. Lee, C.J.D. Lin, B.Lucini, MP and D. Vadacchino, arXiv:1712.04220
E. Bennett, D.K. Hong, J.W. Lee, C.J.D. Lin, B.Lucini, MP and D. Vadacchino, arXiv:1909.12662

Highlight 3: Multiple Representations

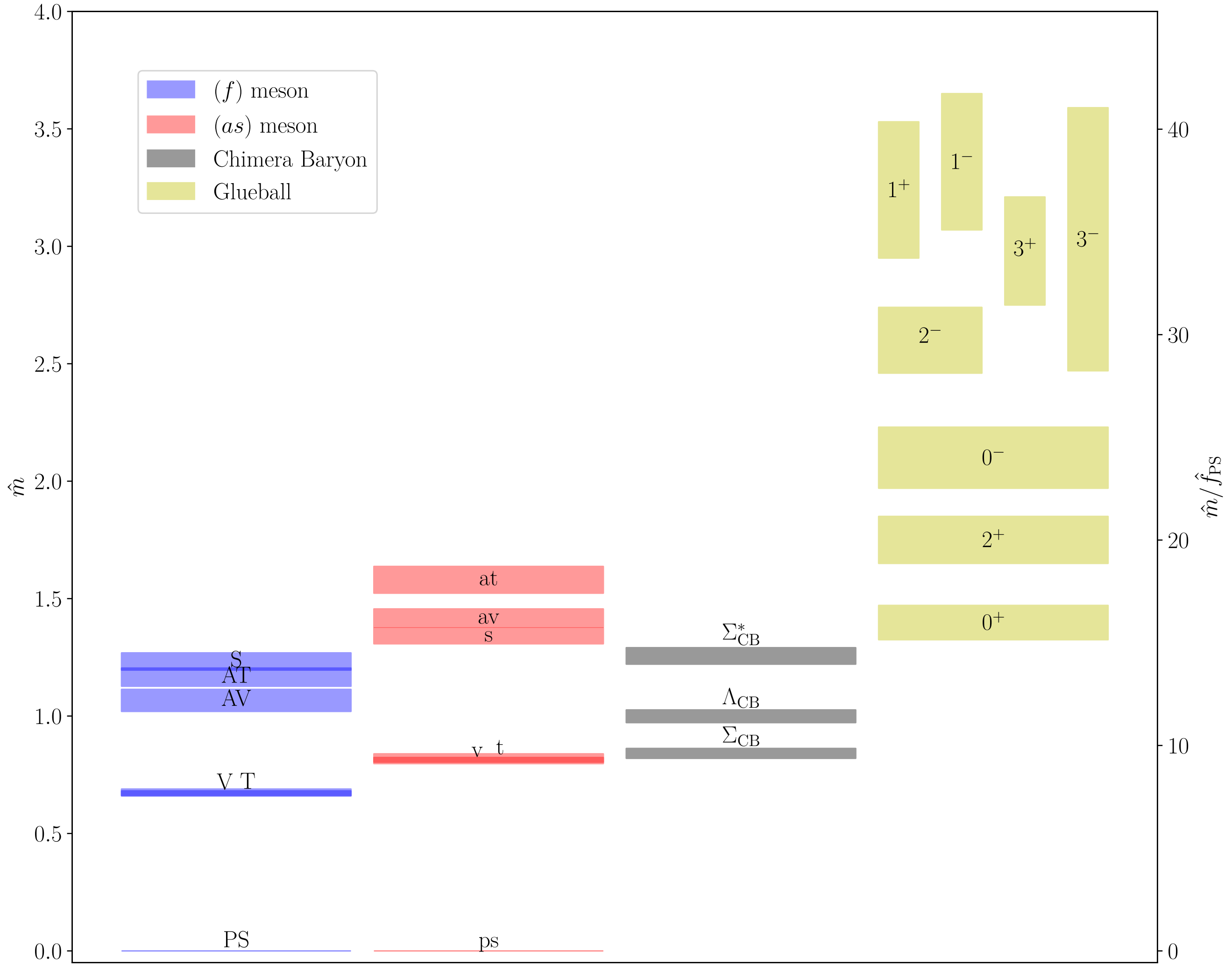


- Mixed representation calculations with dynamical fermions, one ensemble
- Quenched (flavored) spectrum for both fundamental, and well as antisymmetric matter
- Glueballs in all channels
- Chimera baryons
- Units: Wilson flow scale w

E. Bennett, D.K. Hong, J.W. Lee, C.J.D. Lin, B.Lucini, MP and D. Vadacchino, arXiv:1712.04220
E. Bennett, D.K. Hong, J.W. Lee, C.J.D. Lin, B.Lucini, M. Mesiti, MP, J. Rantatharju and D. Vadacchino, arXiv:1912.06505
E. Bennett, J. Holligan, D.K. Hong, J.W. Lee, C.J.D. Lin, B.Lucini, MP and D. Vadacchino, arXiv:2004.11063
E. Bennett, J. Holligan, D.K. Hong, J.W. Lee, C.J.D. Lin, B.Lucini, MP and D. Vadacchino, arXiv:2010.15781

E. Bennett, D.K. Hong, H. Hsiao, J.W. Lee, C.J.D. Lin, B.Lucini, M. Mesiti, MP, and D. Vadacchino, arXiv:2202.05516

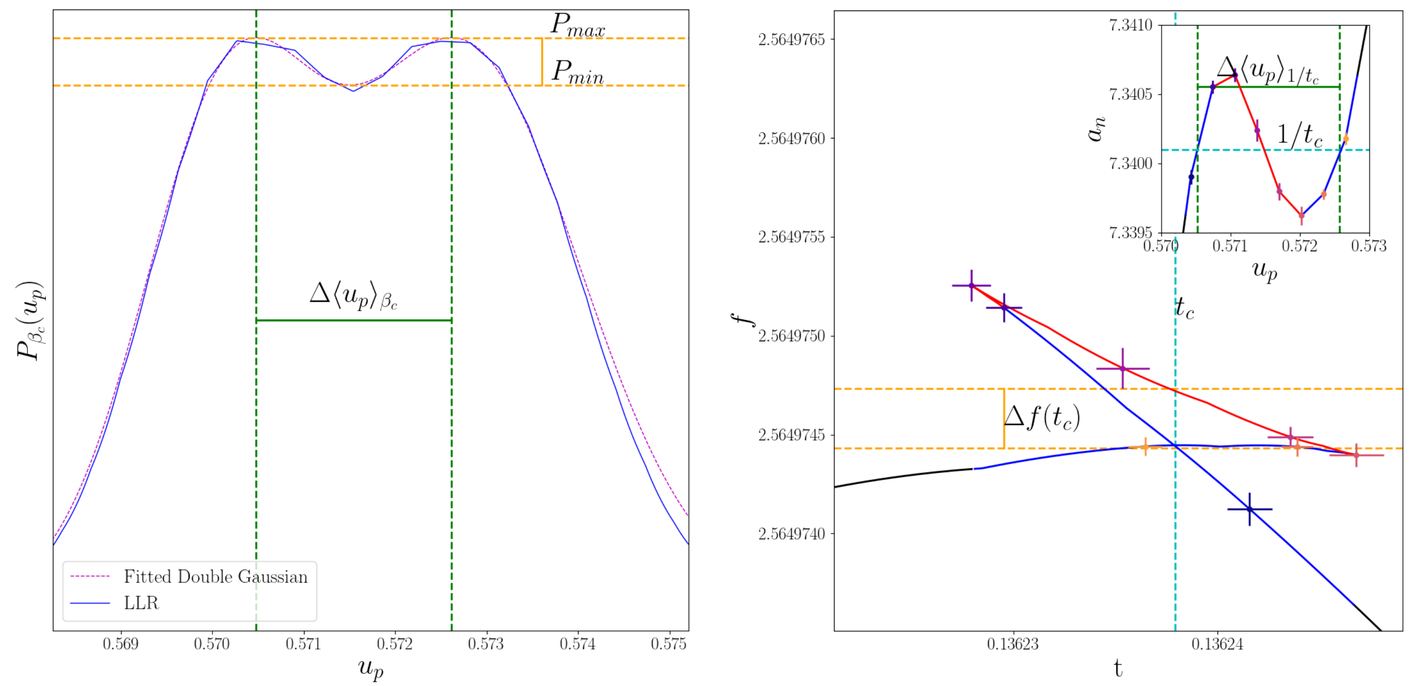
Highlight 4: Sp(4) quenched



- Quenched calculations, continuum and massless extrapolations
- Flavored spectrum for both fundamental, and well as antisymmetric matter
- Glueballs in all channels
- Chimera baryons
- Units: Wilson flow scale w or pion decay constant

E. Bennett, D.K. Hong, J.W. Lee, C.J.D. Lin, B.Lucini, MP and D. Vadacchino, arXiv:1712.04220
E. Bennett, D.K. Hong, J.W. Lee, C.J.D. Lin, B.Lucini, M. Mesiti, MP, J. Rantatharju and D. Vadacchino, arXiv:1912.06505
E. Bennett, J. Holligan, D.K. Hong, J.W. Lee, C.J.D. Lin, B.Lucini, MP and D. Vadacchino, arXiv:2004.11063
E. Bennett, J. Holligan, D.K. Hong, J.W. Lee, C.J.D. Lin, B.Lucini, MP and D. Vadacchino, arXiv:2010.15781
E. Bennett, D.K. Hong, H. Hsiao, J.W. Lee, C.J.D. Lin, B.Lucini, MP, and D. Vadacchino, arXiv:2311.14663

Highlight 5: Sp(4) at finite T



(a) **(b)**

Figure 2: The reconstructed plaquette distribution at the critical point (left) and the free-energy (right) for $Sp(4)$ pure gauge theory on a lattice of size 4×20^3, found using the LLR method with 48 intervals between plaquette values of $(u_p)_1 = 0.58$ and $(u_p)_{48} = 0.565$, with $\Delta_E a^4/6\tilde{V} = 0.0006$. The critical coupling in the left plot was found by tuning β until the two peaks of the distribution have equal height. The plaquette values corresponding to the peaks of the distribution are shown by the green dashed line. The height of the maxima, P_{max}, and minima, P_{min}, are shown by the orange dashed line. On the right panel the red, blue and black lines show the unstable, metastable and stable regions, respectively. The points in the inset match those of the main plot, showing the corresponding values of a_n and u_p. The critical coupling is shown by the dashed cyan line, the orange dashed line shows the free energy values when $t = t_c$ and the green dashed line on the inset show the plaquette values when $a_n = 1/t_c$.

- Yang-Mills Sp(4)
- LLR algorithm
- Phase transition confinement/ deconfinement
- One choice of lattice, varying Nt and Ns in progress

D. Mason, B. Lucini, M. Piai, E. Rinaldi and D. Vadacchino, arXiv:2310.02145 [hep-lat].

Outlook

Outlook

- Compositeness plays role in many new physics applications (CHM, TPC, SIMP, GW, …)
- Many possible proposals: illustrative examples in this talk, many more exist.
- Calculability always the main challenge: state of the art non-perturbative calculations needed.
- Complementarity/interplay of lattice studies, effective field theory and gauge-gravity dualities.
- Sp(2N) lattice programme reached a mature stage: spectroscopy, topology, large-N…
- Sp(2N), lattice programme: what next? Light masses, conformal window, anomalous dimensions, off-shell physics… more advanced techniques.

General message

- We are living in a golden age of numerical studies of non-perturbative field theories
  - Effective Field Theory
  - Gauge-gravity dualities
  - Lattice field theory
- Broad range of applications
  - Composite Higgs Models (Higgs as PNGB or Dilaton)
  - Top Partial Compositeness
  - Strongly Interacting Dark Matter
  - Gravitational Waves