Lattice Studies of
Conformal and Near-conformal Systems

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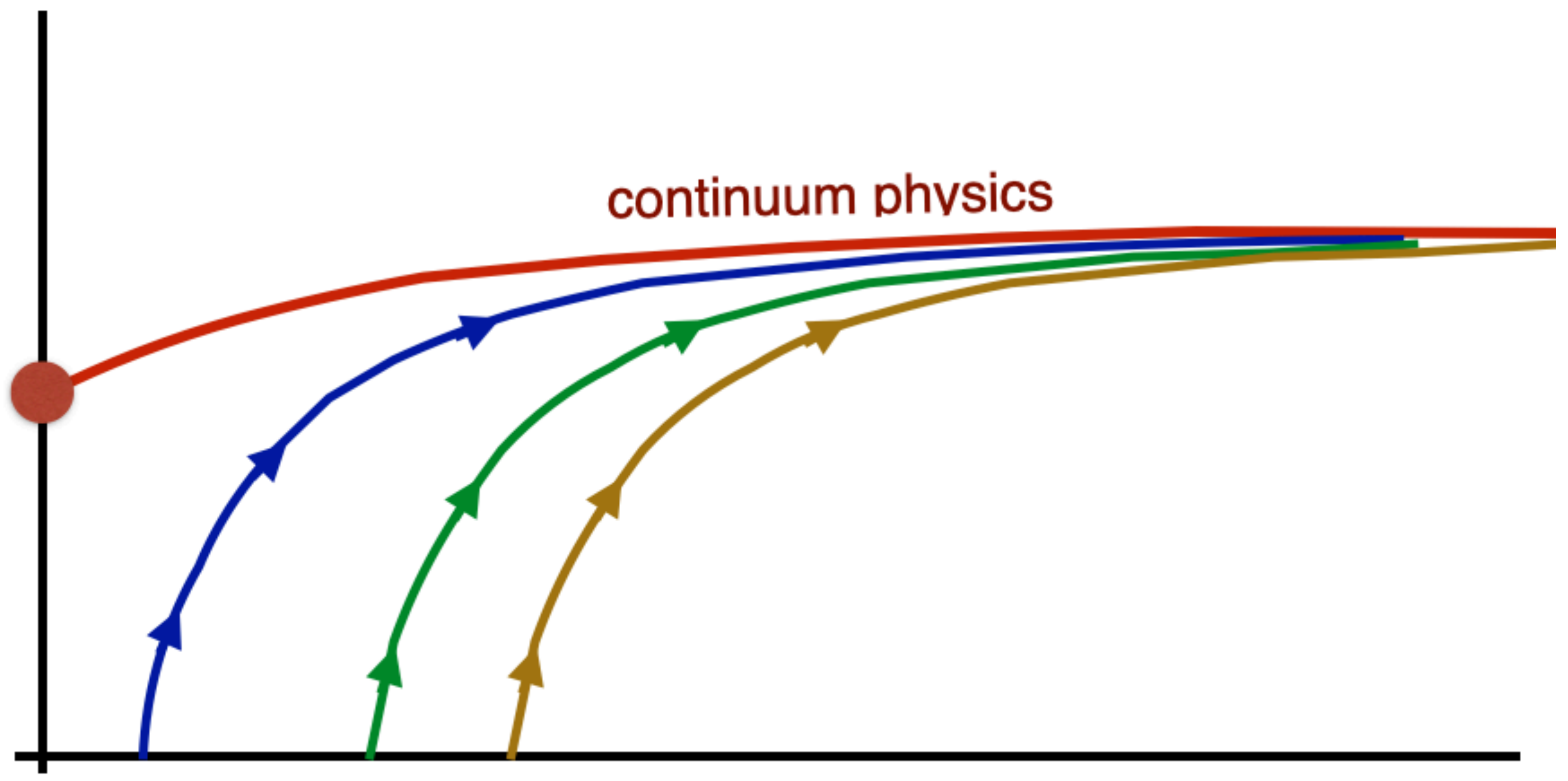
Composite Higgs - Busan Workshop

February 2024



Beyond the Standard Model

Composite Higgs models are attractive:
- describe EW symmetry breaking
- explain/predict Higgs mass
- BSM spectrum/dark matter/etc

Most (all?) feasible models require properties that only strongly coupled, near-conformal systems can satisfy

Lattice studies are well suited to
- identify suitable systems
- describe their nonperturbative properties

Composite Higgs models

Broadly, there are two (times two) possibilities:

Higgs:

(A) Higgs is the σ isosinglet scalar, dilaton of broken scale symmetry

- $f_{PS} = vev$ of standard model : predictive
- very long "walking scaling" is needed - does it exist?

(B) Higgs is pseudo Nambu-Goldstone boson : naturally light

- $f_{PS} = vev/\sin(\chi)$: less predictive

Fermion masses (two more):

(A) generated by $(\bar{\psi}\psi)(\bar{\Psi}\Psi)$ interaction: very long "walking scaling" is needed

(B) "partial compositeness" :generated by $(\psi)(\Psi\Psi\Psi)$: large anomalous dimension for $\Psi\Psi\Psi$ is needed

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Composite Higgs models

Questions for Lattice :

is the system conformal/near conformal? (RG β function)

(A) Higgs is the σ isosinglet scalar, dilaton of broken scale symmetry

- $f_{PS} = vev$ of standard model : predictive
- very long "walking scaling" is needed - what controls it?

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Composite Higgs models

Questions for Lattice :

What are the anomalous dimensions? (RG γ function)

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What is Near-Conformal : Phases of gauge-fermion systems

$SU(N_c)$ gauge with N_f fundamental flavors

$$\beta = \mu^2 \frac{dg^2}{d\mu^2} = b_0 g^4 + b_1 g^6 + \ldots$$

The coefficients of $\beta(g^2)$ are known perturbatively up to 5 loops

$$b_0 = \frac{1}{16\pi^2}\left(-\frac{11}{3}N_c + \frac{2}{3}N_f\right), \qquad b_1 = \frac{1}{(16\pi^2)}\left(-\frac{34}{3}N_c^2 + N_f\left(\frac{10}{3}N_c + \frac{N_c^2 - 1}{N_c}\right)\right)$$

$b_2, \quad b_3, \ldots$ depend on the RG scheme

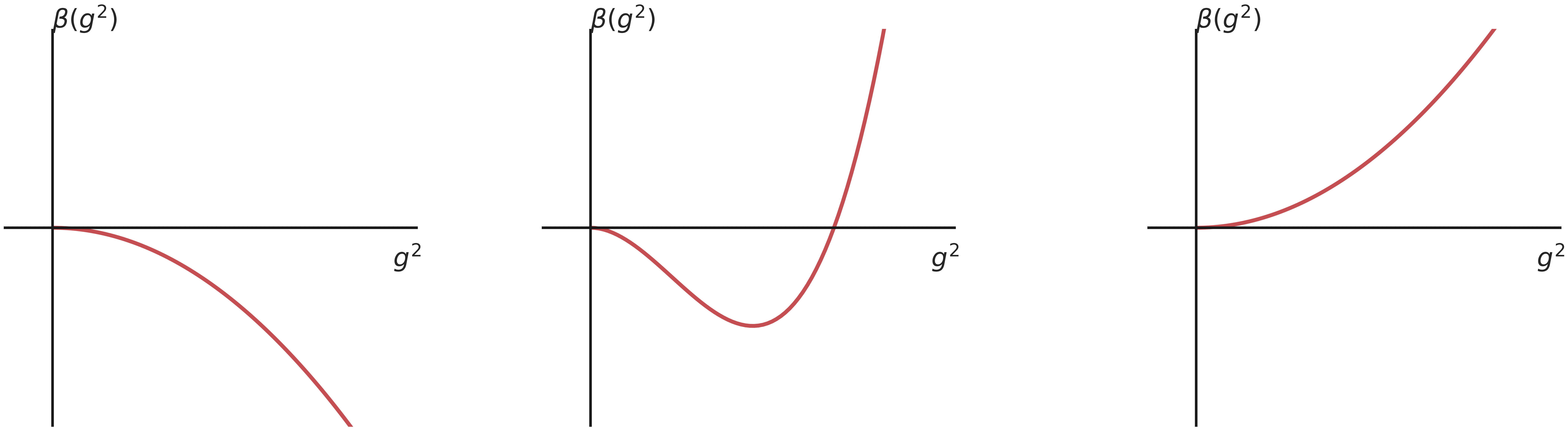
Conformal or chirally broken?

SU(3) gauge + N_f fermions

small N_f (<8) — Confining

$N^* < N_f < N^{IF}$ — Conformal

$N_f > N^{IF} = 16.5$ — Infrared free



Perturbatively: the IR fixed point emerges at $g_0^2 = \infty$ at $N_f = N^*$, moves to $g_0^2 = 0$ as $N_f \to N^{IF}$

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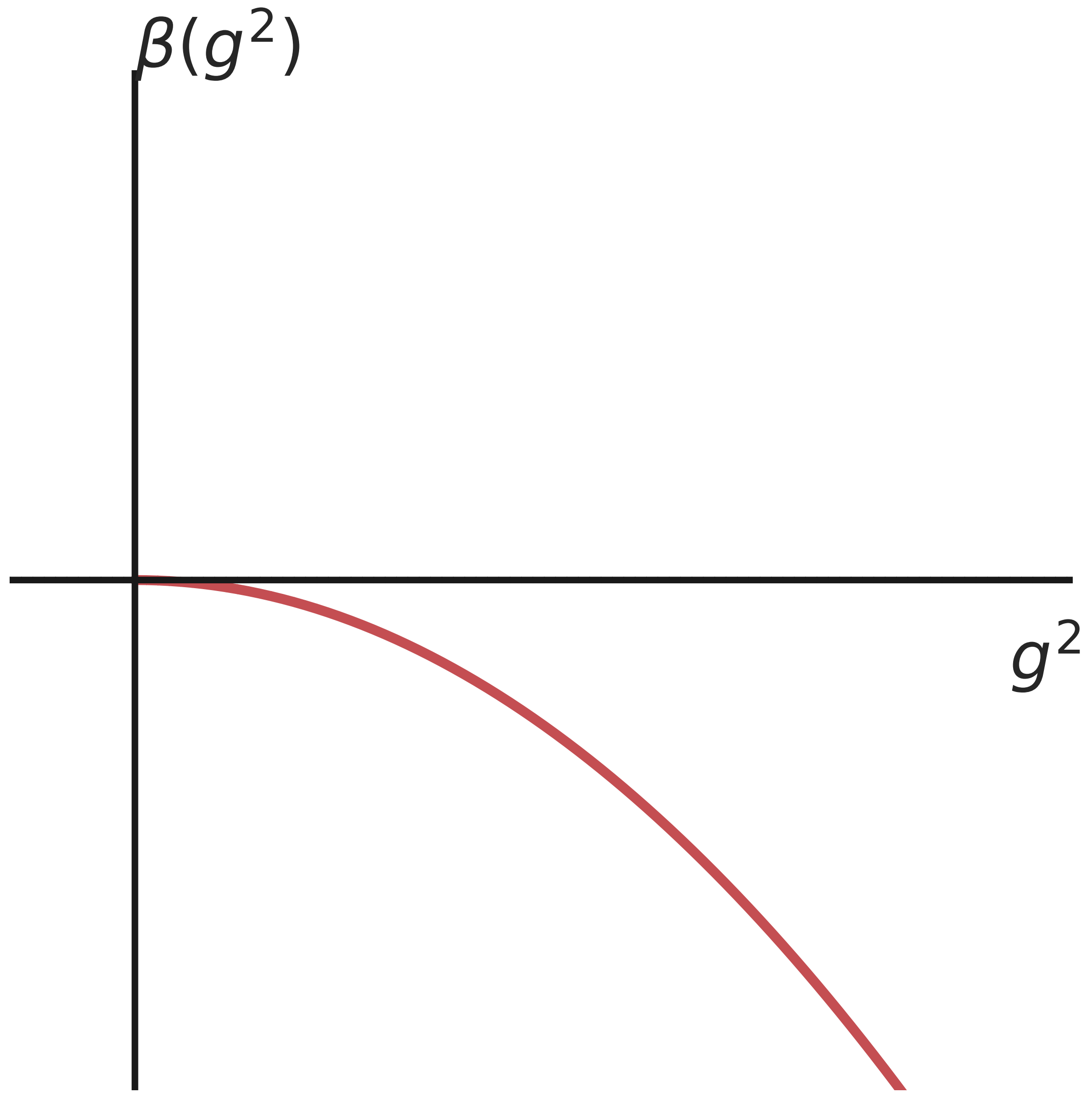
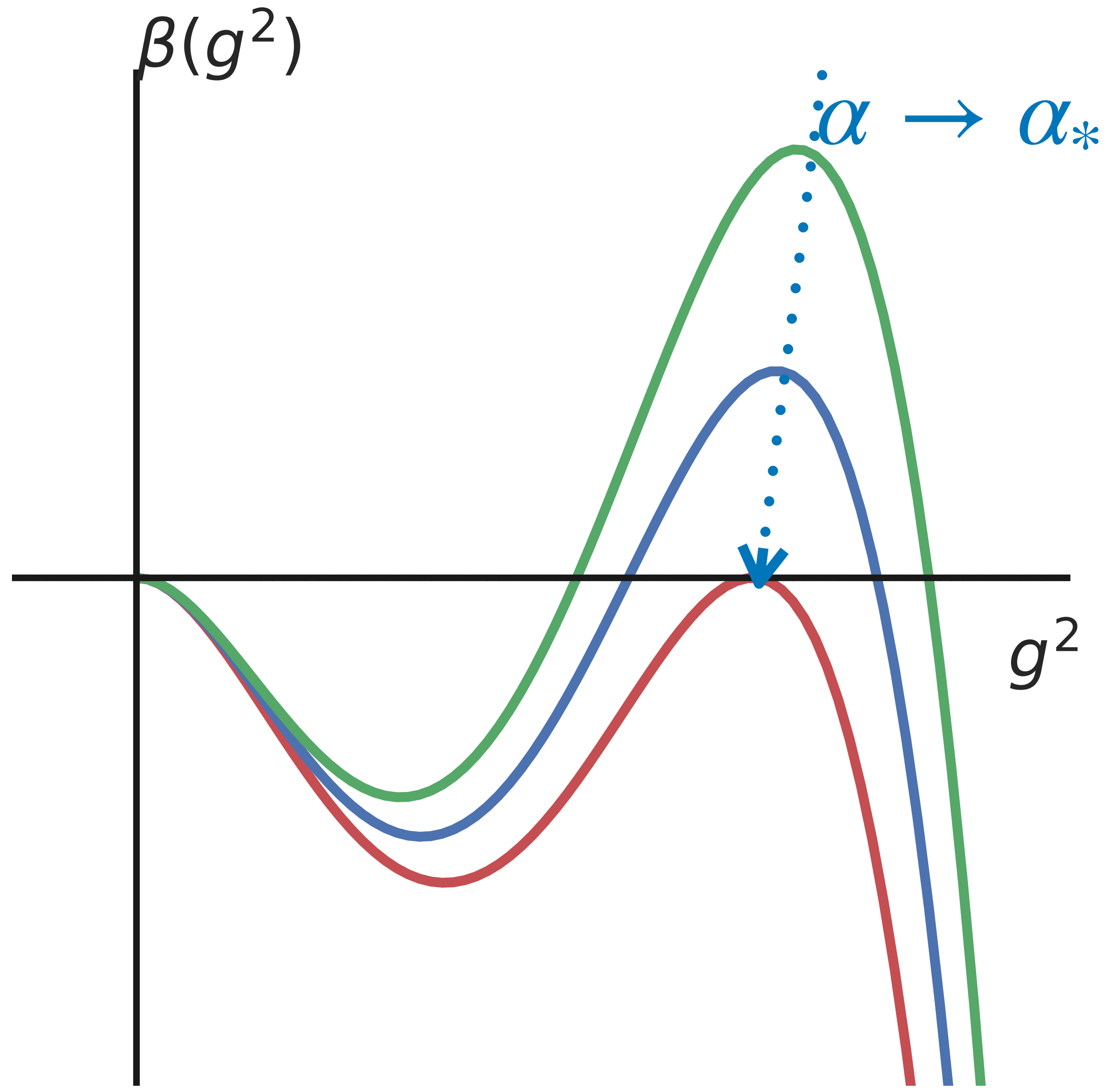


$N^* < N_f < N^{IF}$

Conformal



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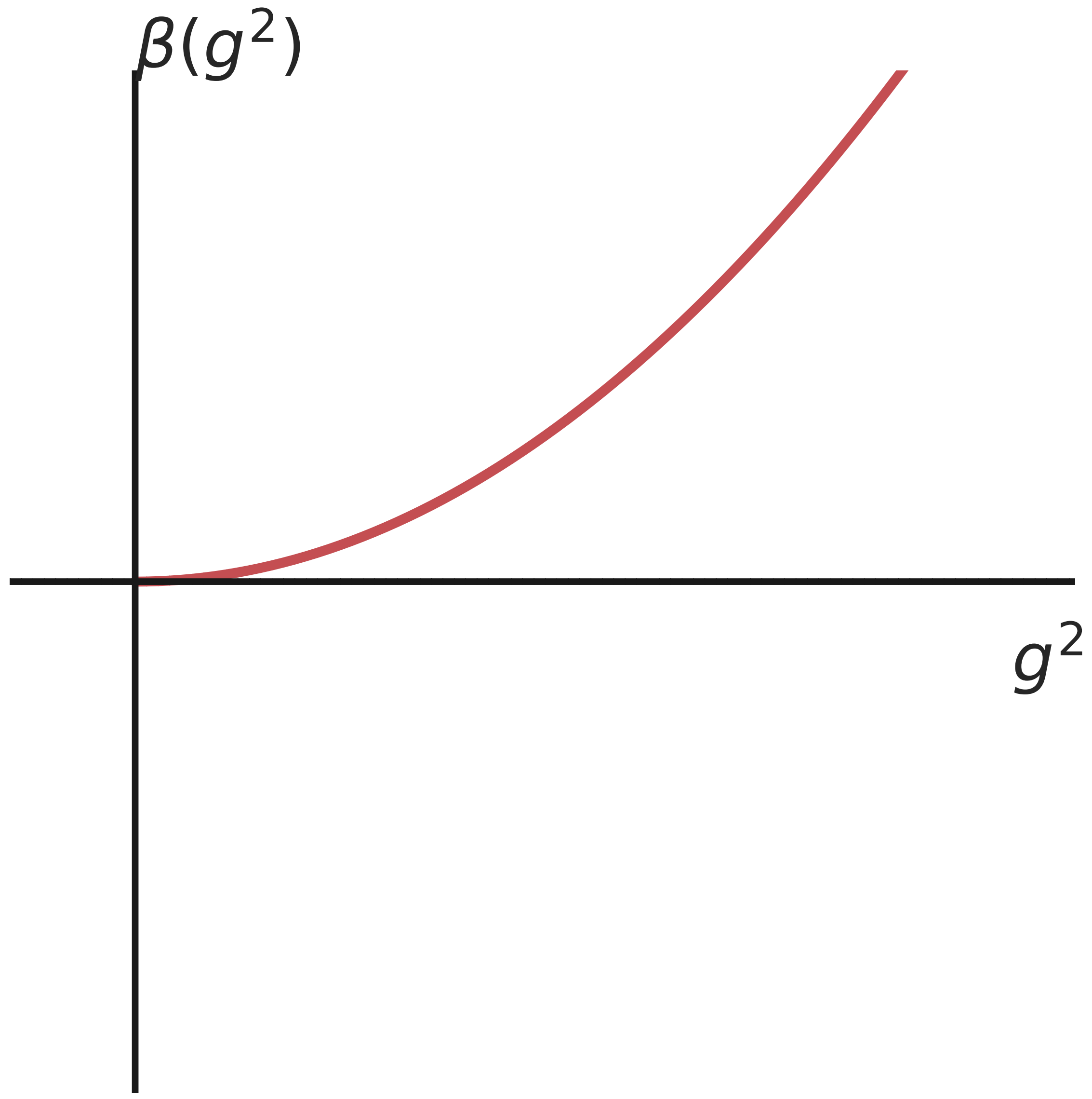

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Nonperturbatively: the IR fixed point could emerge at finite g_*^2 e.g.

$$\beta(g) \sim (\alpha - \alpha_*) - (g - g_*)^2$$

Kaplan et al PRD80,125005 (2009)
L. Vecchi PRD82, 045013 (2010)
Gorbenko et al JHEP10, 108 (2018)

Conformal or chirally broken?

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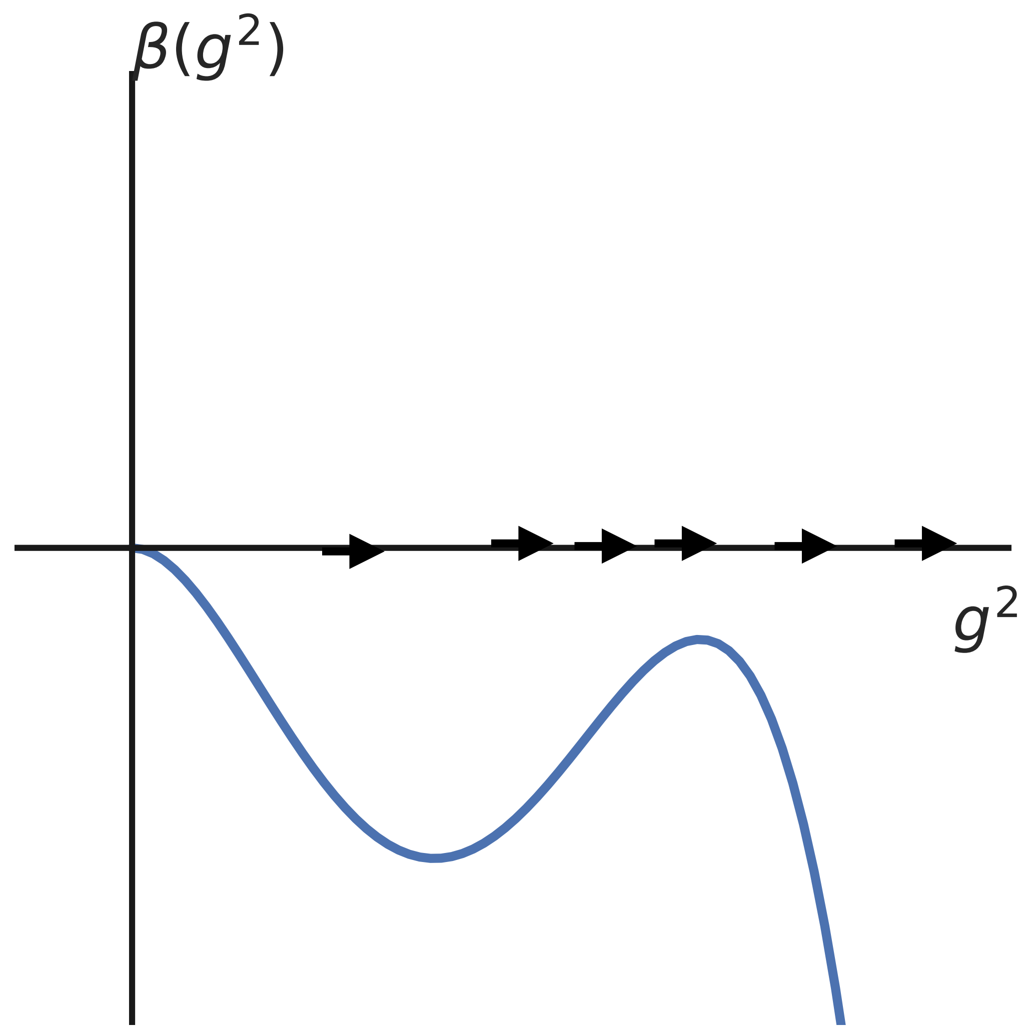
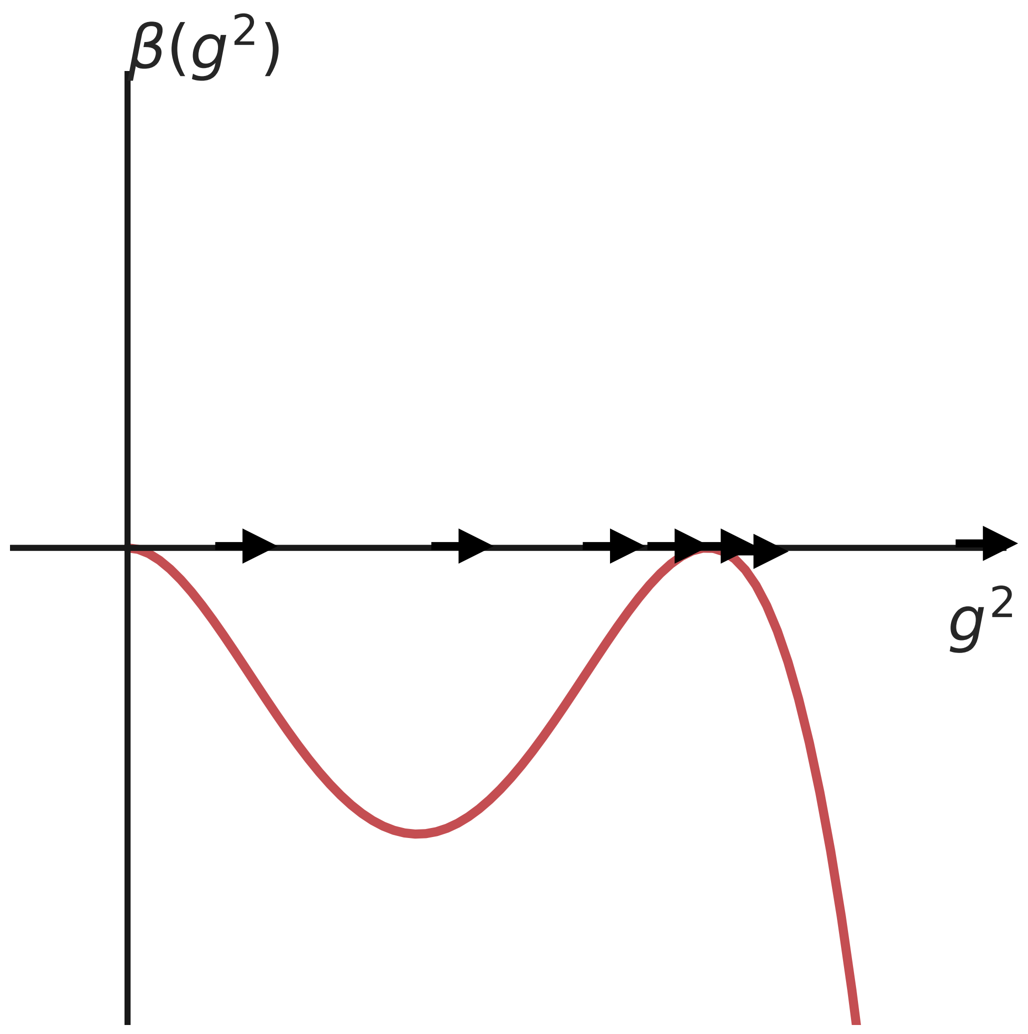
Walking



conformal sill



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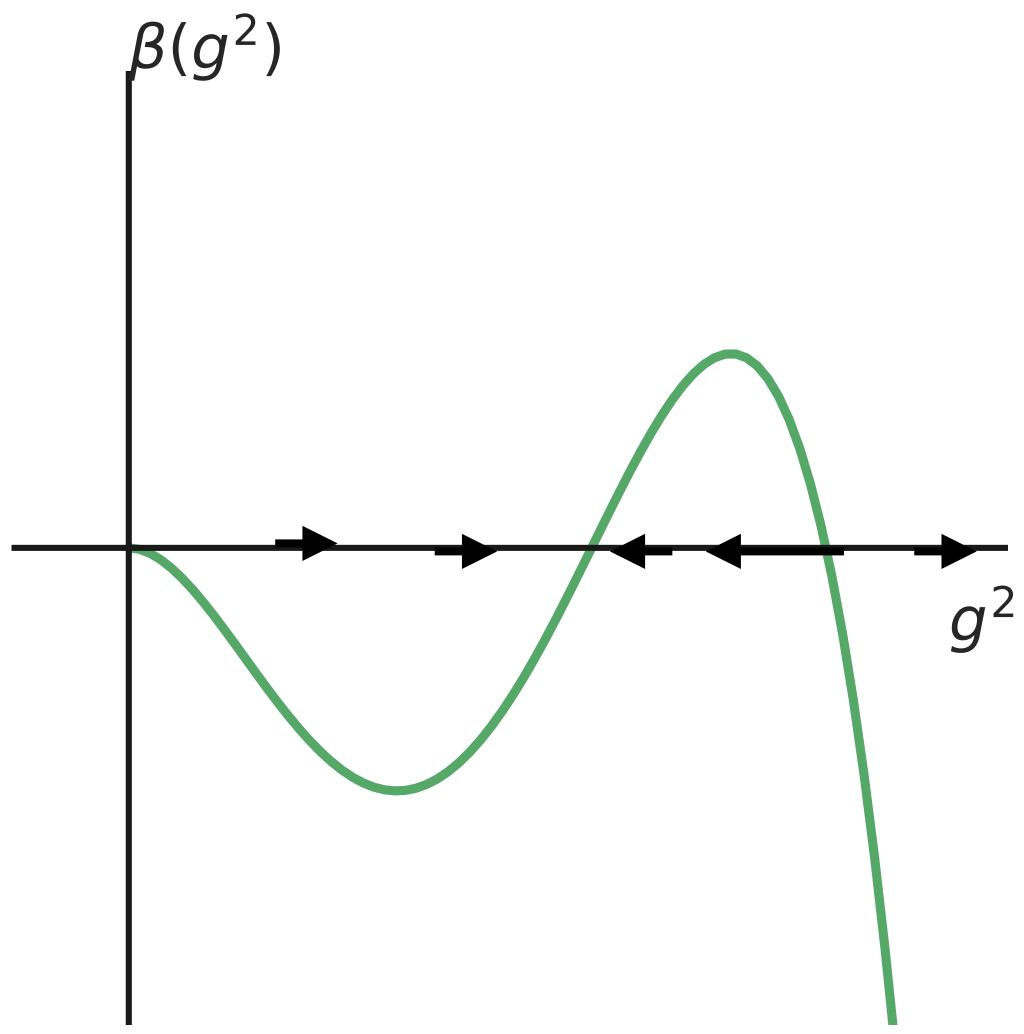


Walking:

Is it “walking” slow enough?

At the sill:

- Could be mass-split
- or use the strong coupling phase(?)

Conformal → mass-split

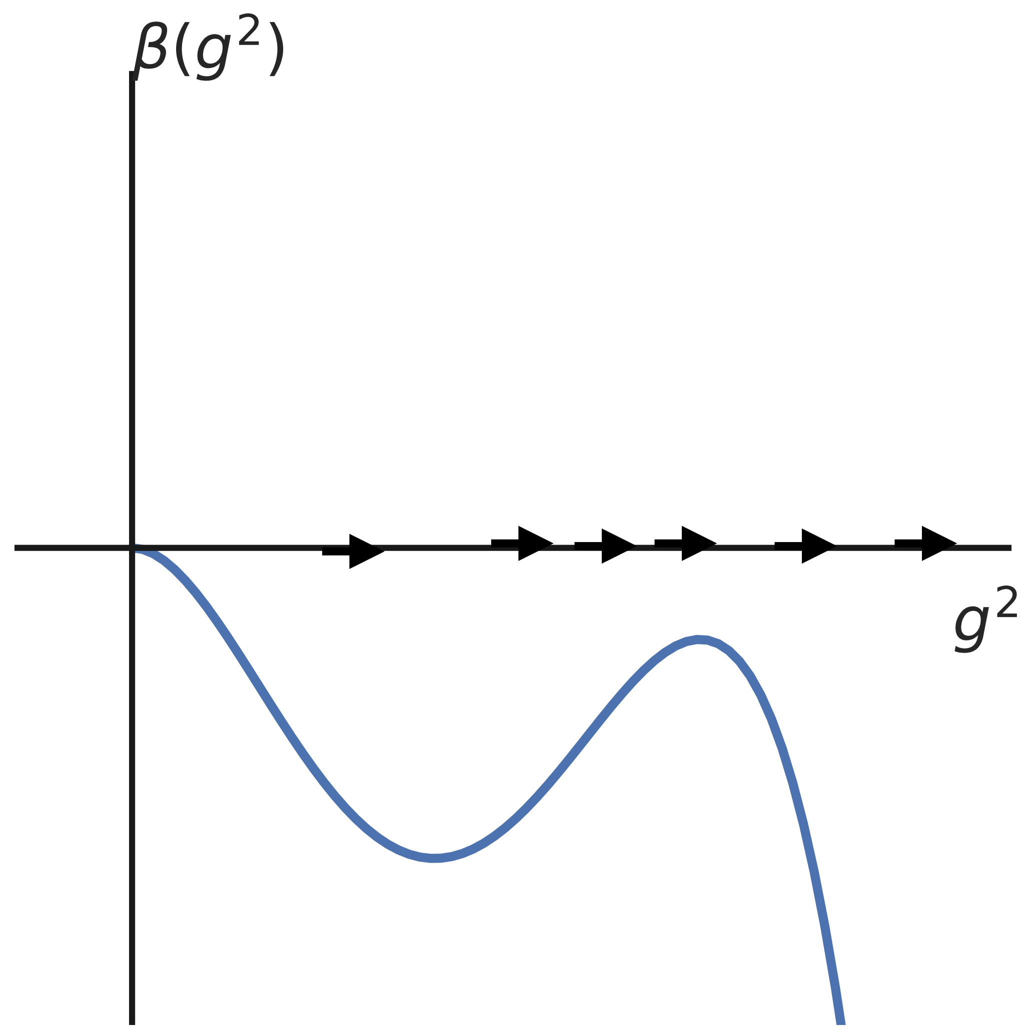
- Give mass to some flavors;
- When decouple, χSB
- Heavy mass controls “walking”

and continuum limit

Conformal or chirally broken?

SU(3) gauge + N_f fermions

Walking



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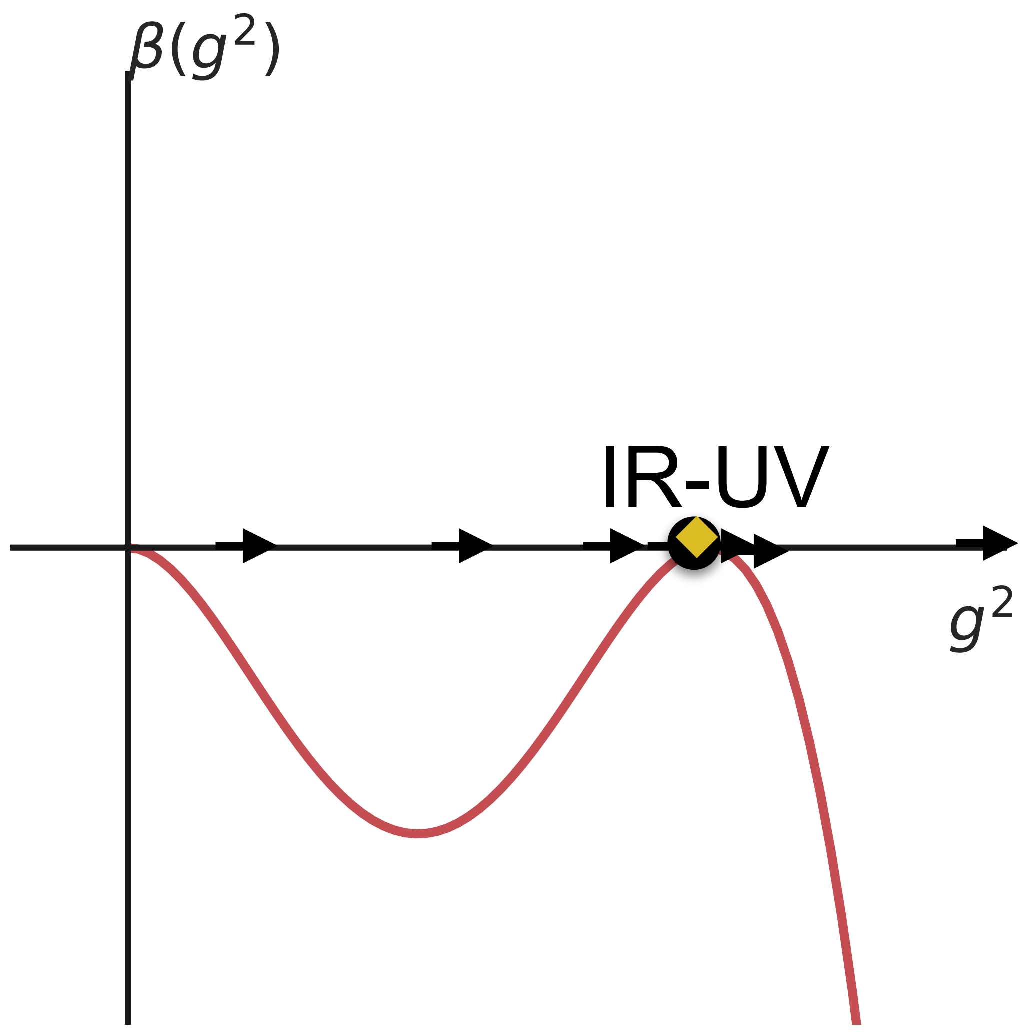

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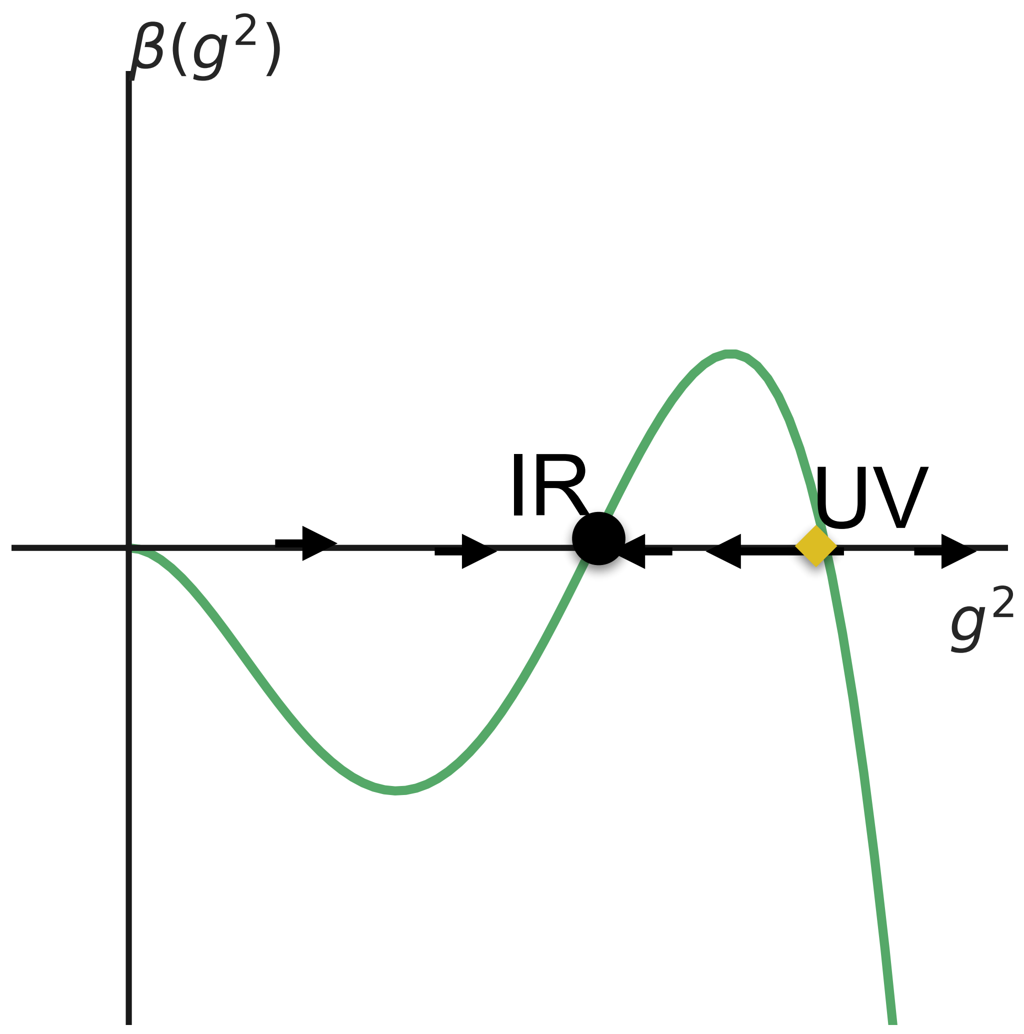

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Lattice studies:

We need to determine RG properties (β and γ functions) of the most promising models nonperturbatively

Simple enough,
yet after 15+ years the opening of the conformal window is still debated ….

But we are on the verge (perhaps even beyond) of success:
- improved action that reduce lattice artifacts (Essential!)
- we have better RG methods

Taming lattice artifacts with PV bosons

AH, Shamir, Svetitsky, PRD104, 074509 (2021)

$$S = \frac{6}{g_0^2}\sum_p ReTrV_\Box + \frac{1}{2}\sum_{n,\mu}\left(\bar{\psi}_n\gamma_\mu(n)U_\mu(n)\psi_{n+\mu} + cc\right) + am_f\sum_n \bar{\psi}_n\psi_n$$

Integrate out the fermions: an effective gauge action (hopping expansion)

$$S_{eff}^{(f)} = \frac{N_s}{(2am_f)^4}\sum_p ReTrV_\Box + c\frac{N_s}{(2am_f)^6}\sum_{6link} ReTrV_{\text{6-link}}\cdots$$

Bare gauge coupling $\beta = 6/g_0^2$ decreases to compensate, leading to rough gauge configurations, large cutoff effects

Taming lattice artifacts with PV bosons

AH, Shamir, Svetitsky, PRD104, 074509 (2021)

➡ Compensate with **heavy Pauli-Villars bosons**

-same interaction as fermions but with *bosonic statistics*

$$S_{eff}^{(PV)} = -\frac{N_s}{(2am_f)^4}\sum_p ReTrV_\square - c\frac{N_s}{(2am_f)^6}\sum_{6link} ReTrV_{\text{6-link}}\cdots$$

-$S_{eff}^{(PV)} < 0 \longrightarrow \beta = 6/g_0^2$ increases;

- Keep $am_{PV} \sim \mathcal{O}(1)$ fixed: in the IR $(a \to 0)$ the PV bosons decouple (does not change physics)

- range of effective gauge action is $\sim exp(-2am_{PV})$

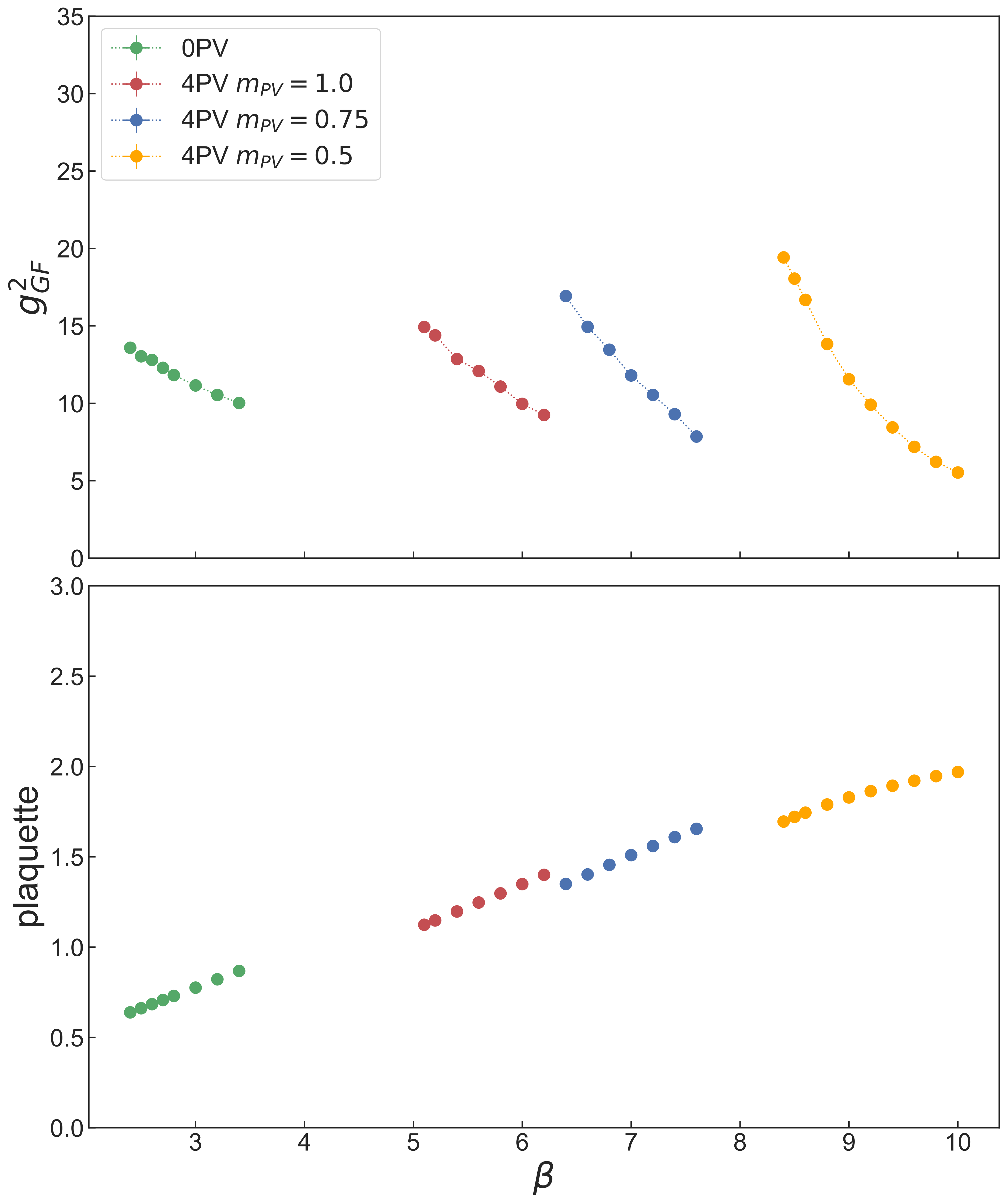
Similar to PV regulators in continuum

➡ The PV action is just an "improved gauge action" :

- add as many PV as you want
- use any lattice action that you want (ex. naive fermions)

Example: SU(3) with $N_f = 12$ fundamental flavors

AH, Shamir, Svetitsky, PRD104, 074509 (2021)



Compare different PV improvements
plaquette value signals UV fluctuations
at fixed physics (g^2)

With PV improvement UV fluctuation significantly decrease —> more reliable continuum limit

-plaquette is determined by thin link $\beta = 6/g^2$
- accessible parameter space in g^2 opens up

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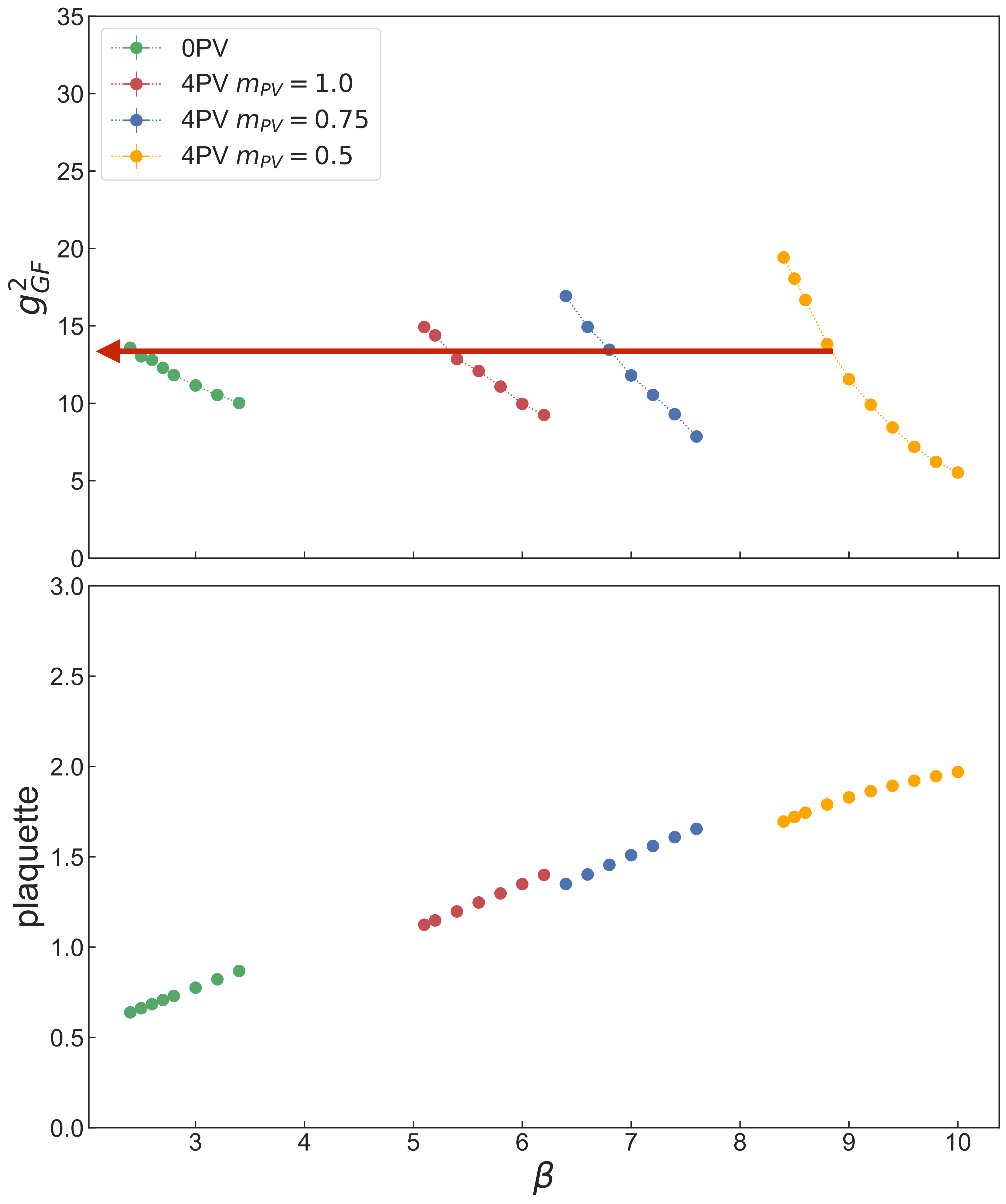

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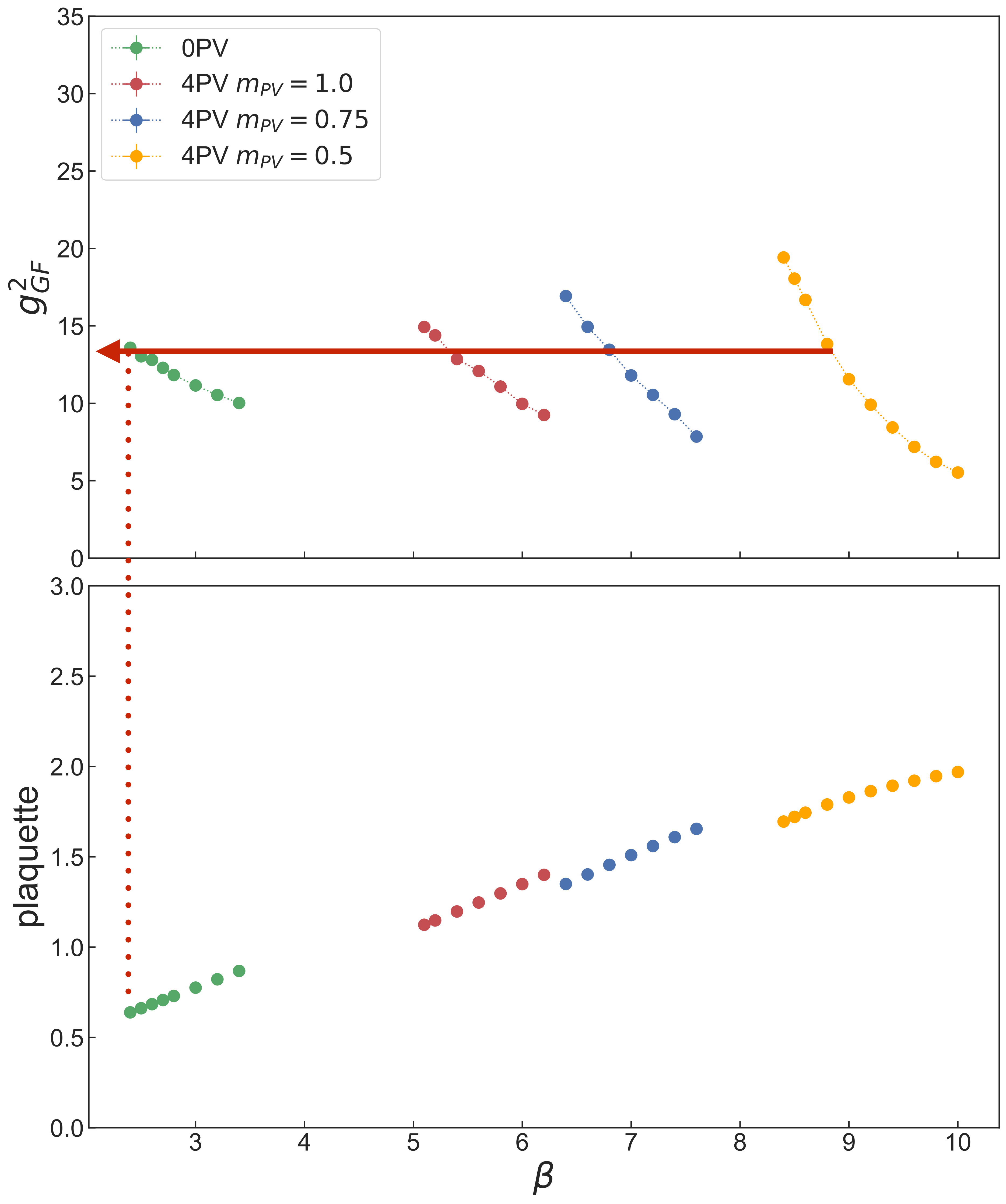

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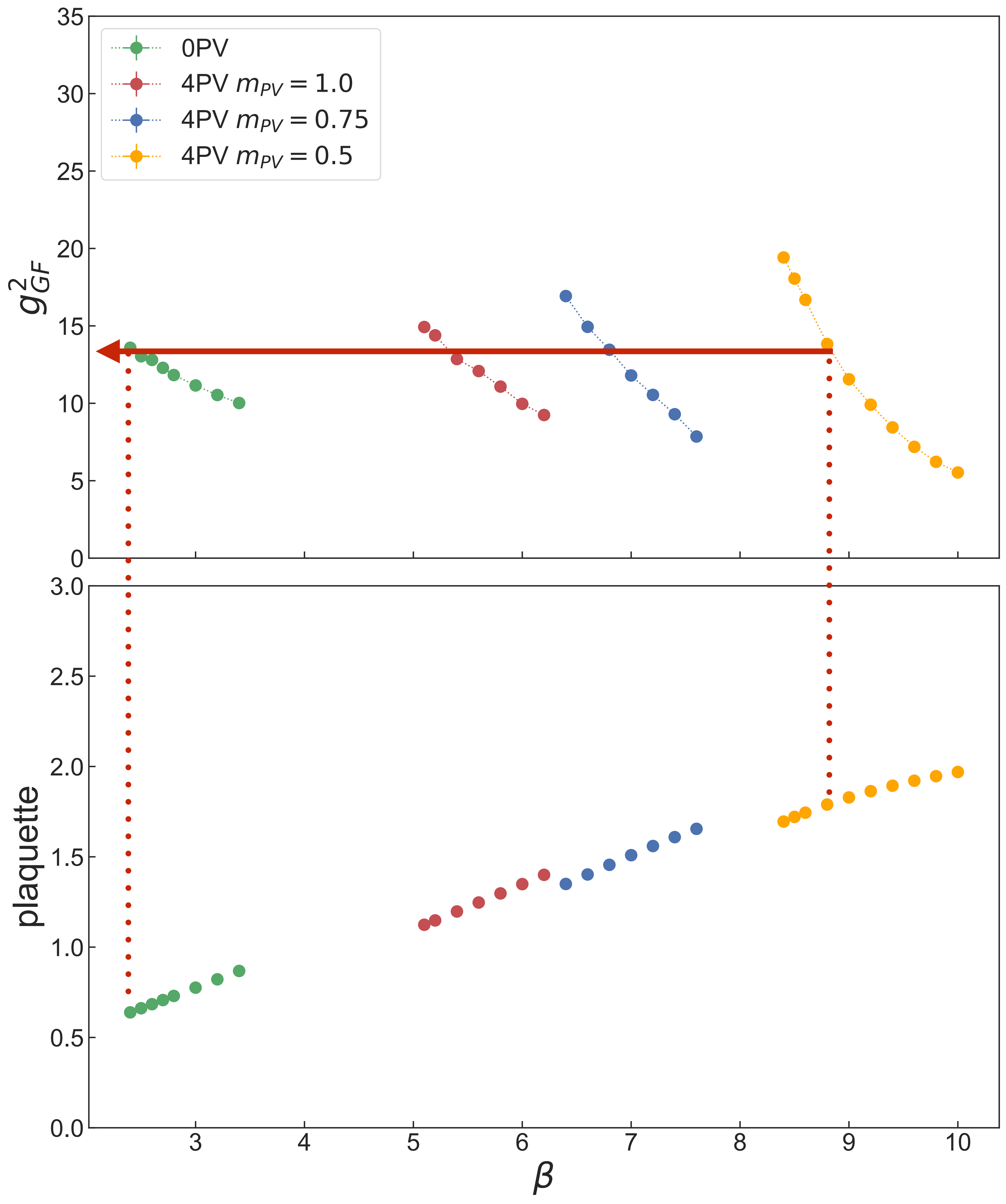

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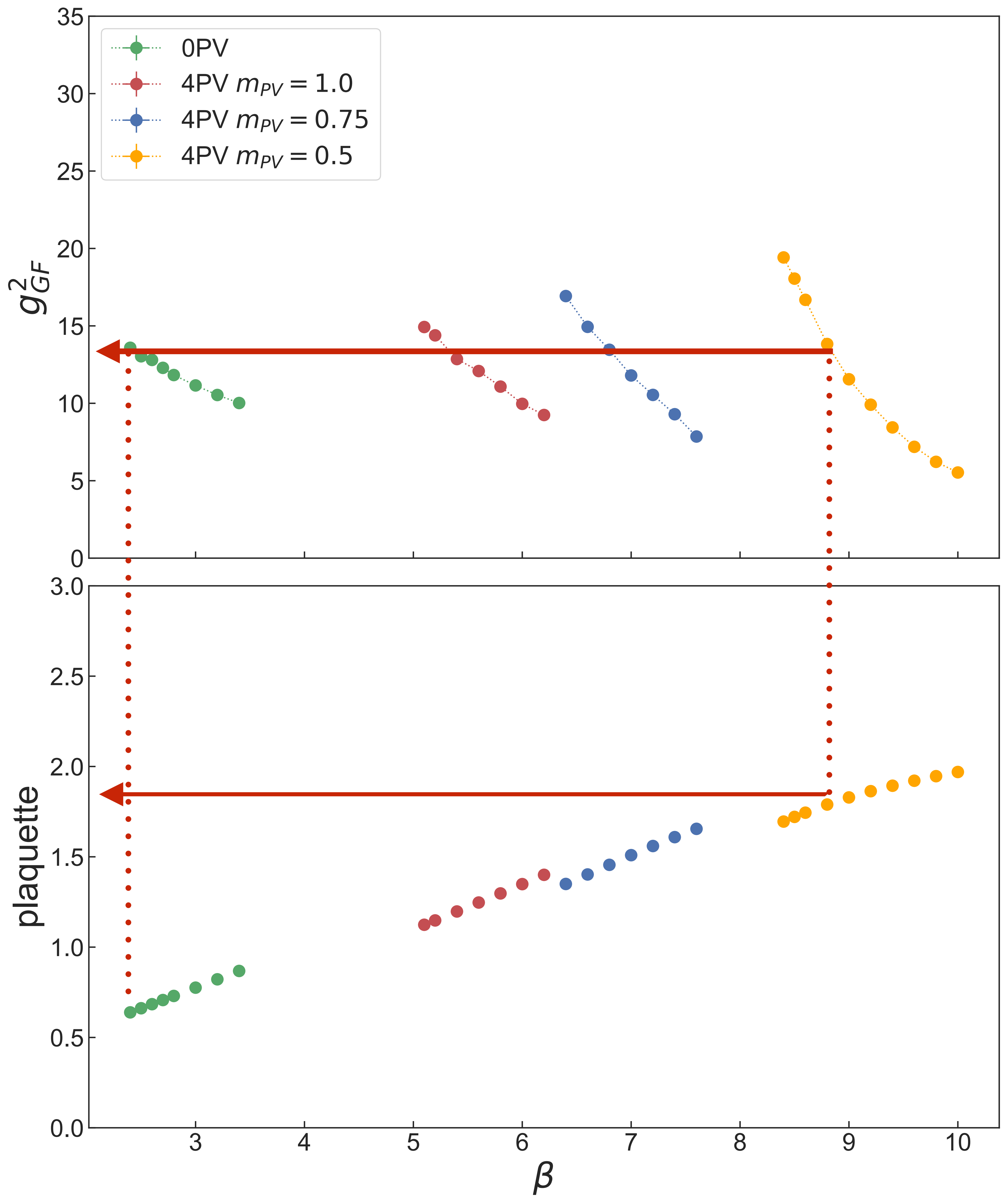

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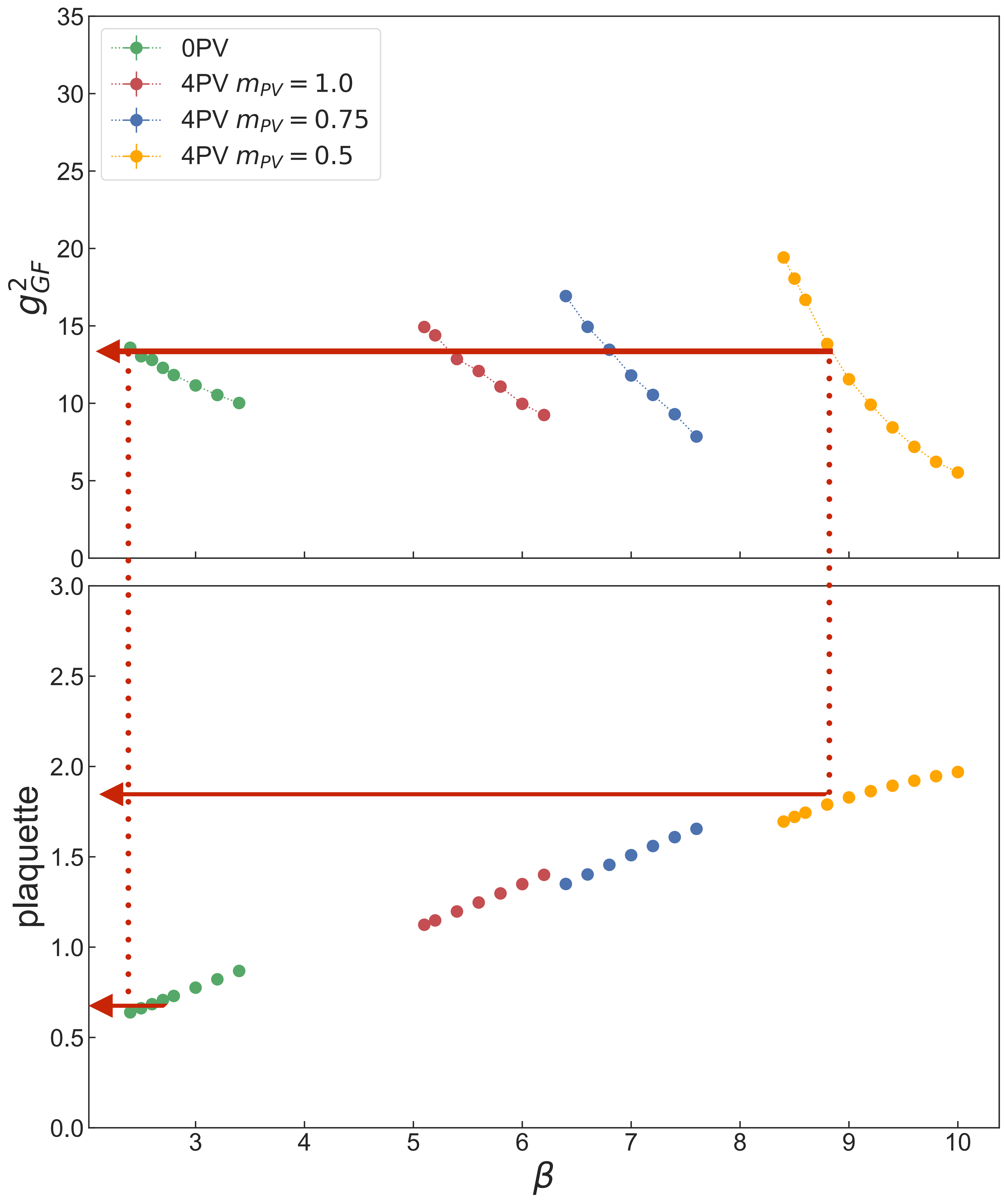

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RG Method: Gradient flow

Luscher *JHEP* 08 (2010) 071

Gradient flow (GF) is a continuous, invertible smoothing transformation

GF resembles RG block spin, but it is not an RG transformation

However

- in infinite volume
- for *local* operators

it can be *interpreted* as

continuous real space RG with $\mu \propto 1/\sqrt{8t}$

A. Carosso, AH, E. Neil, PRL 121,201601 (2018)

Sonoda, H., Suzuki, H. PTEP,023B05 (2021)

- $g^2_{GF} = \mathcal{N} t^2 < E(t) > \quad \Longrightarrow \quad \beta_{GF}(a; g^2_{GF}) = -t \frac{dg^2_{GF}(a;t)}{dt}$

- $\mathcal{O} = \bar{\psi}(x)\Gamma\psi(x)$ or $G_{\mathcal{O}}(x_4, t) = \langle\, \mathcal{O}(\bar{p} = 0, x_4; \mathbf{t})\ \mathcal{O}(\bar{p} = 0, 0; \mathbf{t} = \mathbf{0})\, \rangle$,

$$\Longrightarrow \quad t\frac{d\log G_{\mathcal{O}}(t, x_4)}{d\,t} = d_{\mathcal{O}} + \gamma_{\mathcal{O}}(t) + \eta_\psi(t)$$

- remove η_ψ by dividing with the vector correlator

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The continuous β function (CBF)

GF renormalized coupling: $g^2_{GF}(t) = \mathcal{N} t^2 \langle E(t) \rangle$

- $\langle E \rangle \propto (\Box U - 1)$ or (Clover) etc RG β function :

$$\beta(g_{GF}) = -t \frac{dg^2_{GF}}{dt}$$

AH, O. Witzel, *Phys.Rev.D* 101 (2020) 3
Fodor et al,EPJWeb Conf.175, 08027 (2018)

The RG picture is valid only

- in infinite volume limit : extrapolate in $(a/L)^4 \to 0$ while $\sqrt{8t} \ll L$
- in $am_f = 0$ chiral limit : extrapolate $am_f \to 0$ (only in confining regime)

Continuum limit :

- $t/a^2 \to \infty$ while keeping g^2_{GF} (or t) fixed

Same approach as $N_f = 0{,}2$

AH,C.Peterson, O.Witzel, J.VanSickle
Phys.Rev.D 108 (2023) 1

Lattice results

Some recent results:

- SU(3) $N_f = 12$ fundamental flavors with staggered fermions - β function
- SU(3) $N_f = 10$ fundamental flavors with Wilson fermions - β and γ_m functions
- SU(4) 4+4 sextet+fundamental flavors, Wilson fermions - β and γ_m and $\gamma_{chimera}$
- SU(3) $N_f = 8$ fundamental flavors with staggered fermions -
- could that be the opening of the conformal window?

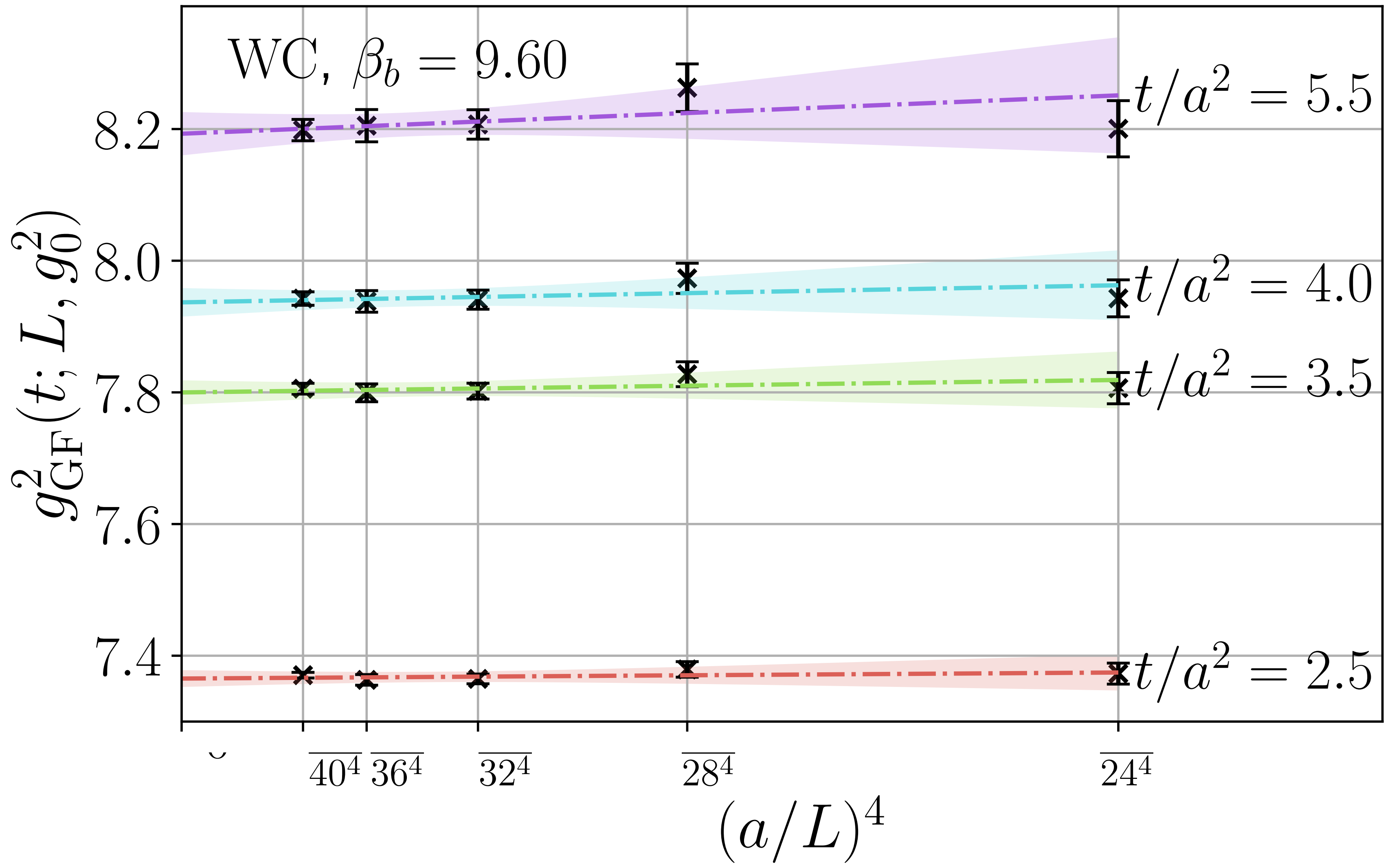
$N_f = 12$ fundamental flavors

A.H., C. Peterson, in preparation

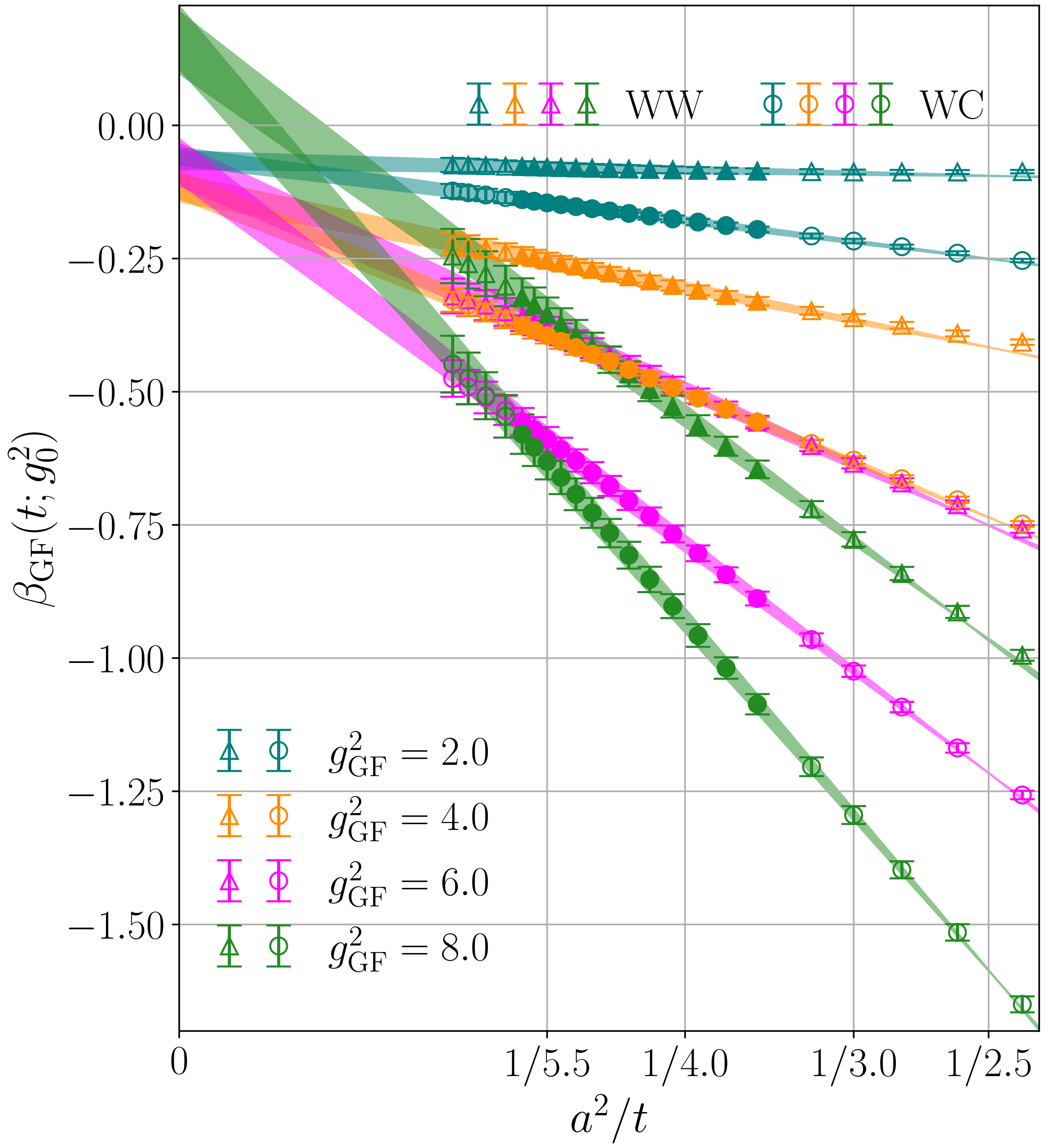
New simulations with PV action :

small cutoff effects

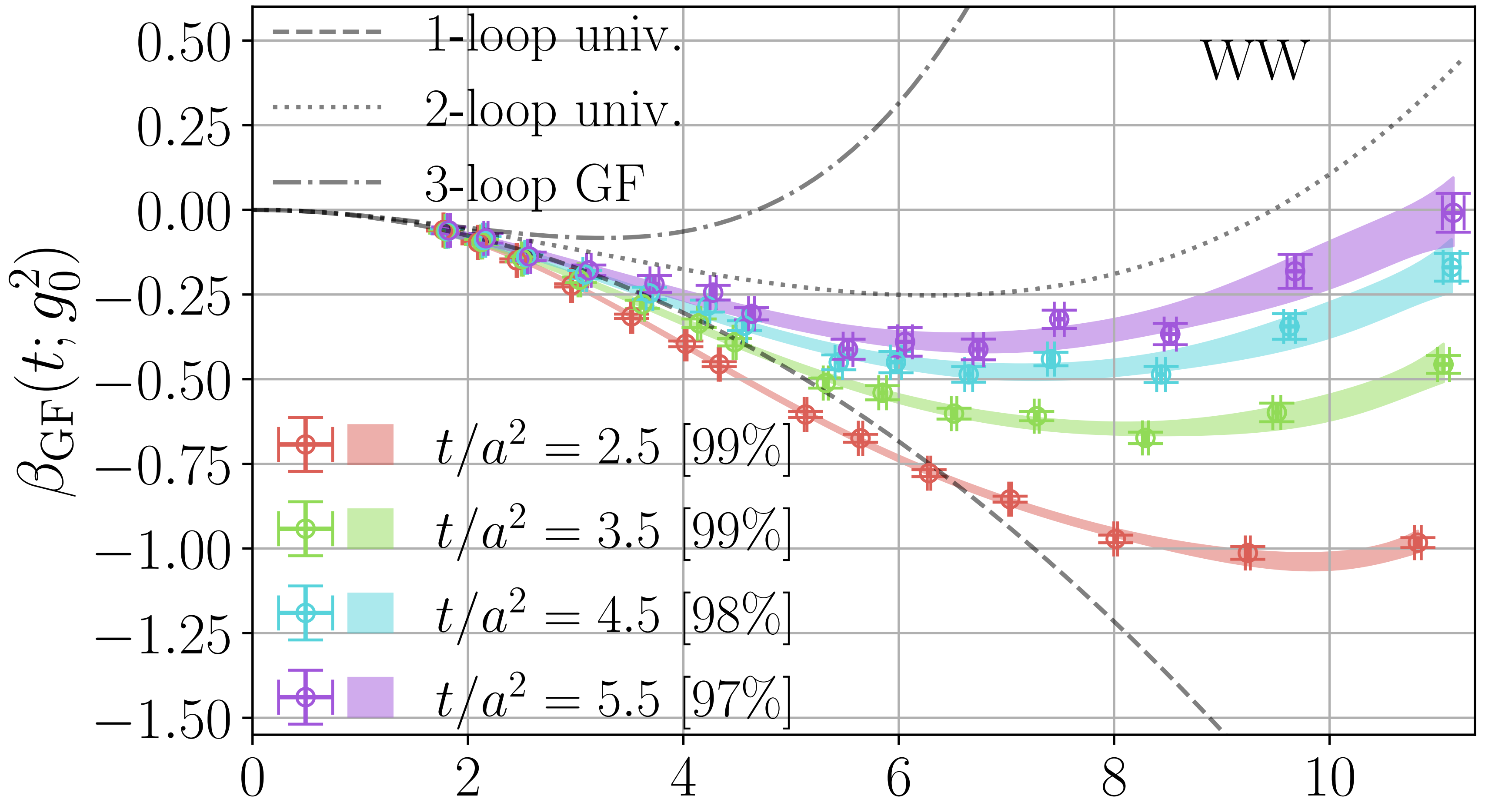
stable extrapolations



Volume up to L=40; mild dependence



$a^2/t \to 0$ continuum limit



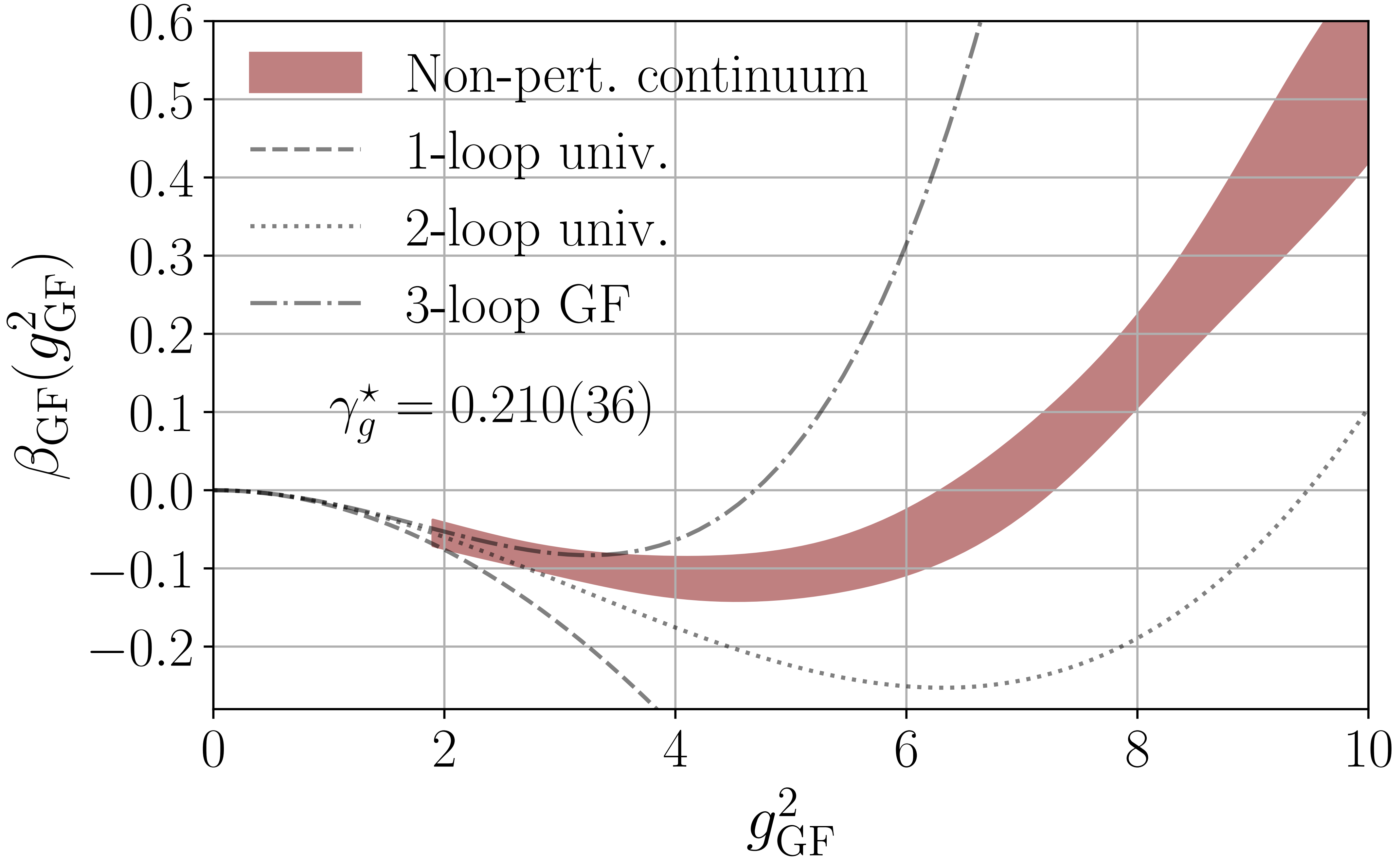
$\beta(g^2)$ interpolation

$N_f = 12$ fundamental flavors (staggered)

A.H., C. Peterson, in preparation

New simulations with PV action :

- weak coupling matches 2-loop/3-loop GF prediction
- stable IRFP consistent with old (no PV, step scaling) result
- slope $\gamma^*_{IRFP} = 0.210(36)$ is consistent, with old resut, close to perturbative prediction



$g^2_{IRFP} \approx 7$: not even strongly coupled

Interpret it

either:

- PV action has the same IR as no PV

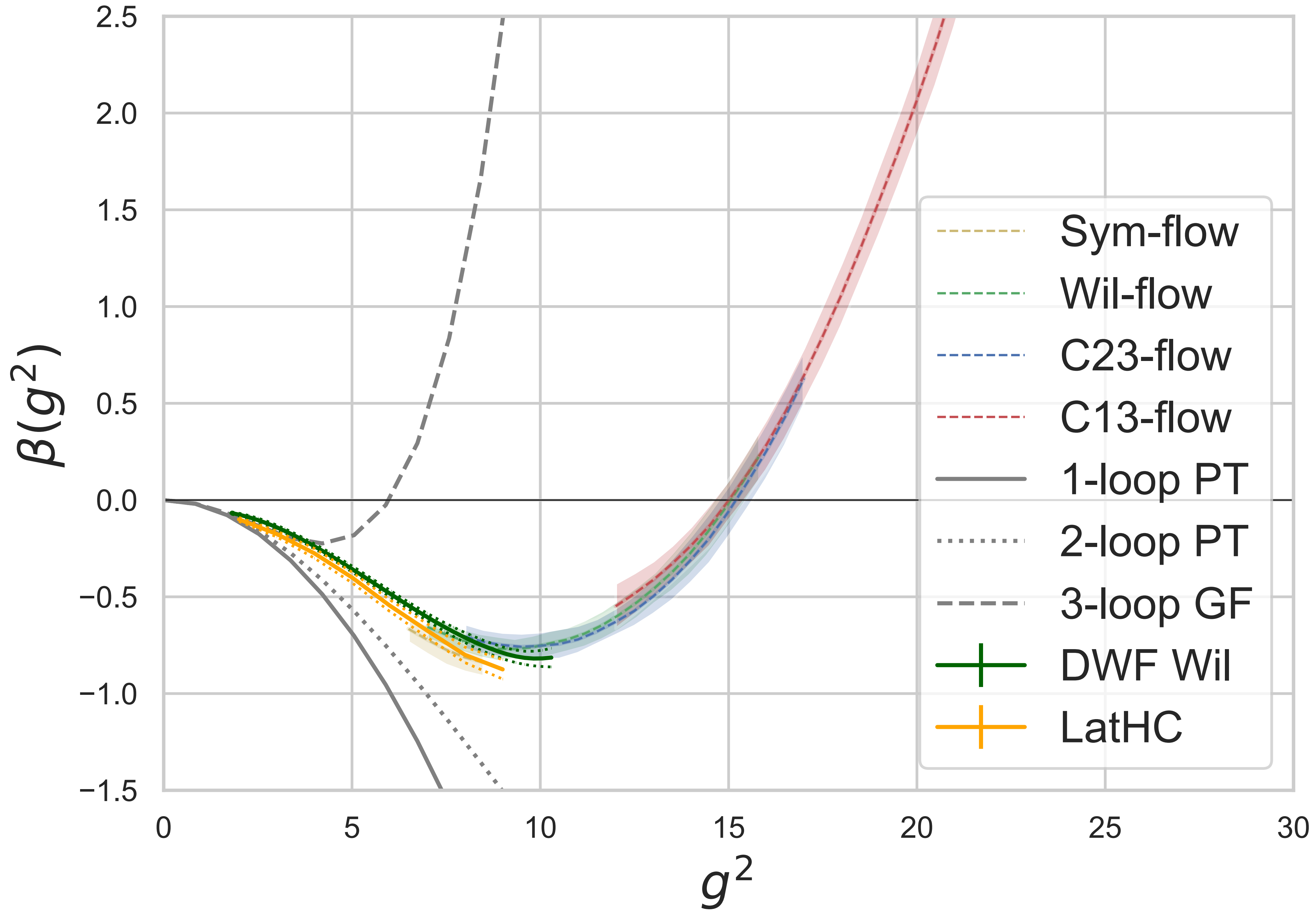
or:

- old simulations and analysis were correct

A.H., D. Schaich, *JHEP* 02 (2018) 132

$N_f = 10$ fundamental flavors (Wilson fermions)

A.H.,Neil, Shamir, Svetitsky, Witzel,
Phys.Rev.D 108 (2023) 7



Prior simulations:

-staggered (LatHC)

-domain wall (Boulder)

limited to $g^2 \lesssim 10$

New simulations

-add PV bosons : opens parameter space from $g^2 \approx 10$ to $g^2 \gtrsim 25$

-use several gradient flow actions: find RT close to simulation action (but Gaussian FP to IRFP is *universal*)

$g^2_{IRFP} \approx 15$: getting strongly coupled

$N_f = 10$ fundamental flavors (Wilson fermions)

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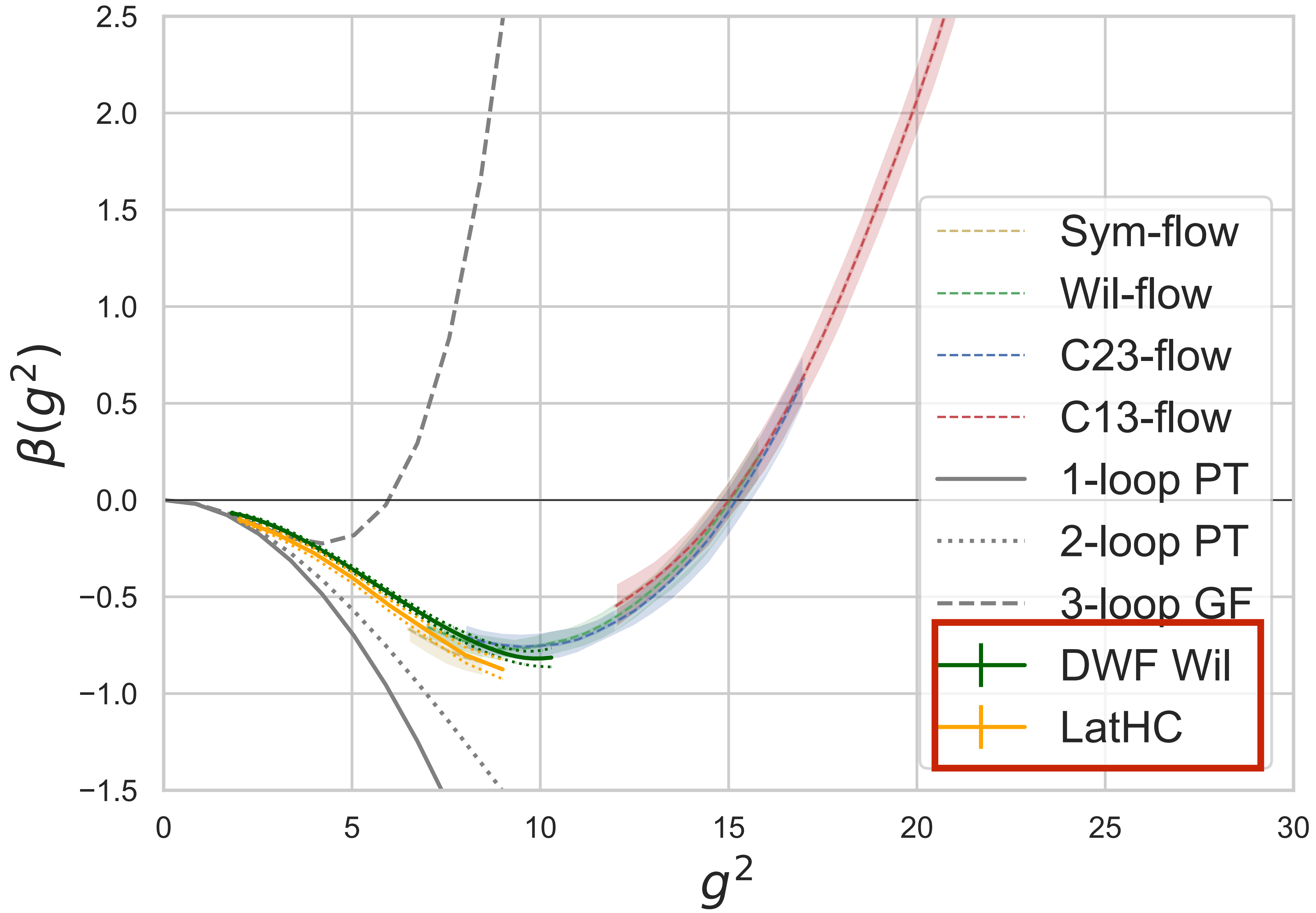

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$N_f = 10$ fundamental flavors - anomalous

A.H.,Neil, Shamir, Svetitsky, Witzel,
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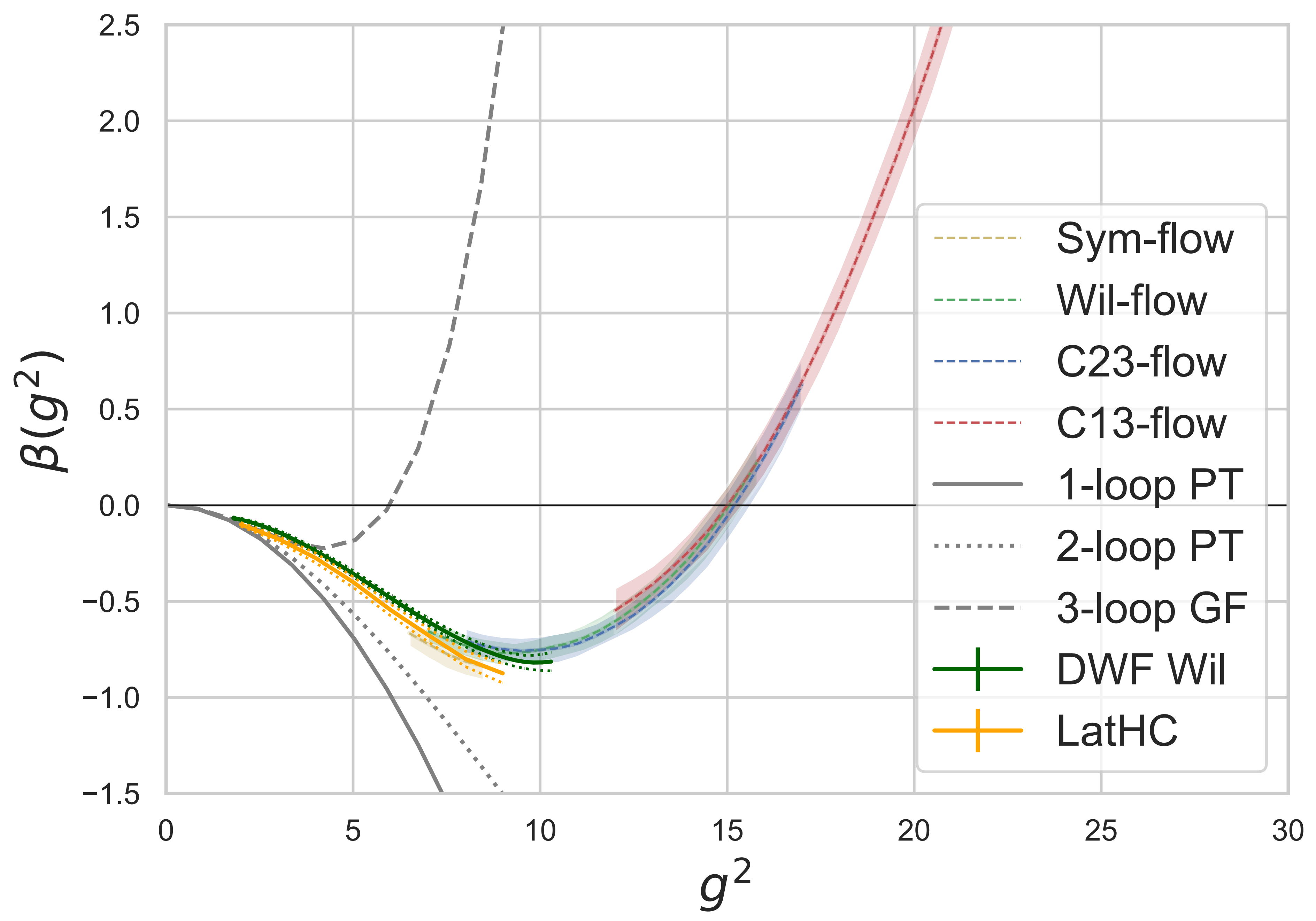

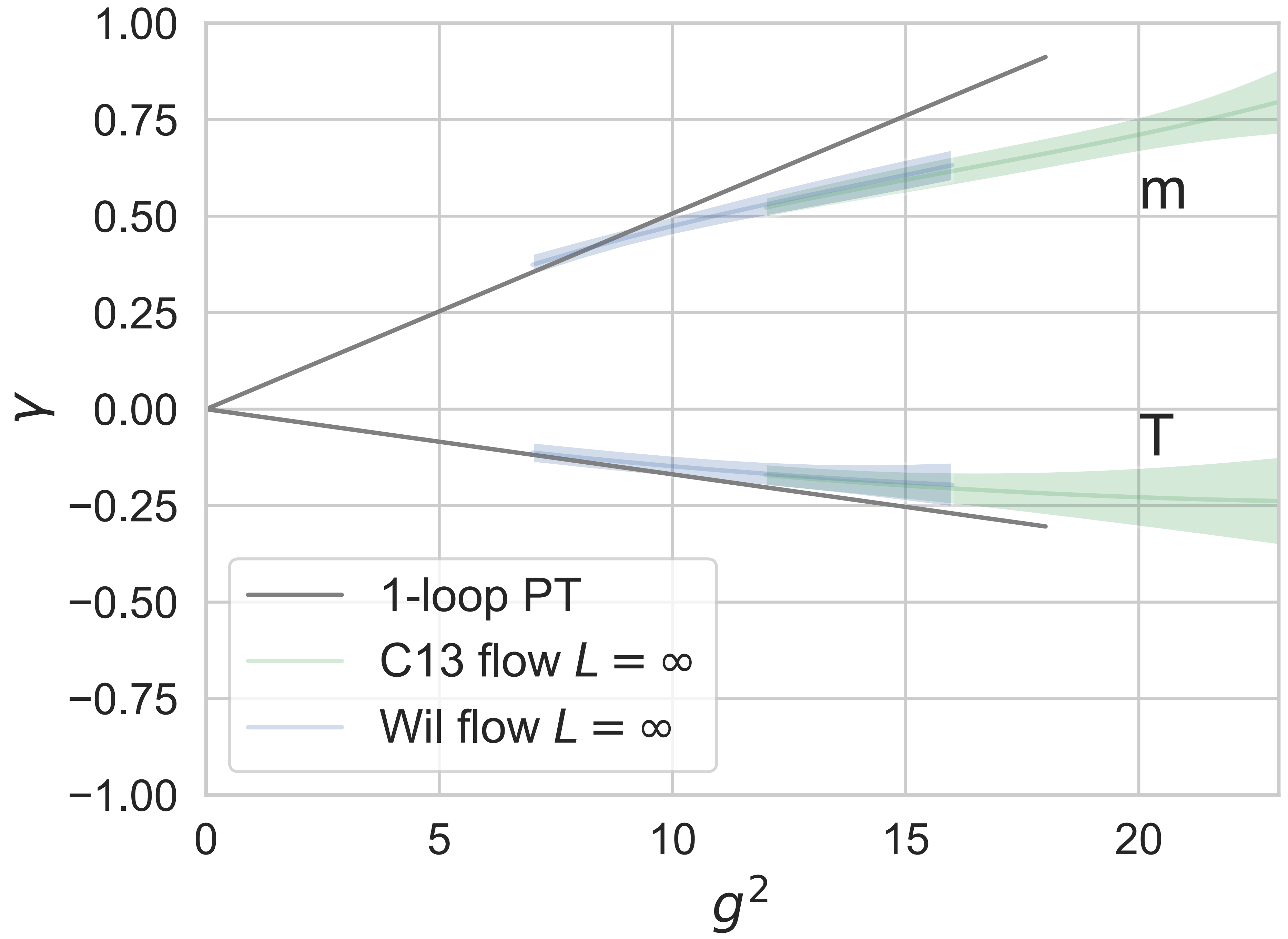

IRFP at $g^2 \simeq 15$

Anomalous dimension $\gamma_m^* \simeq 0.60$
(not even close to the conformal sill)

$N_f = 10$ fundamental flavors - anomalous

A.H.,Neil, Shamir, Svetitsky, Witzel,
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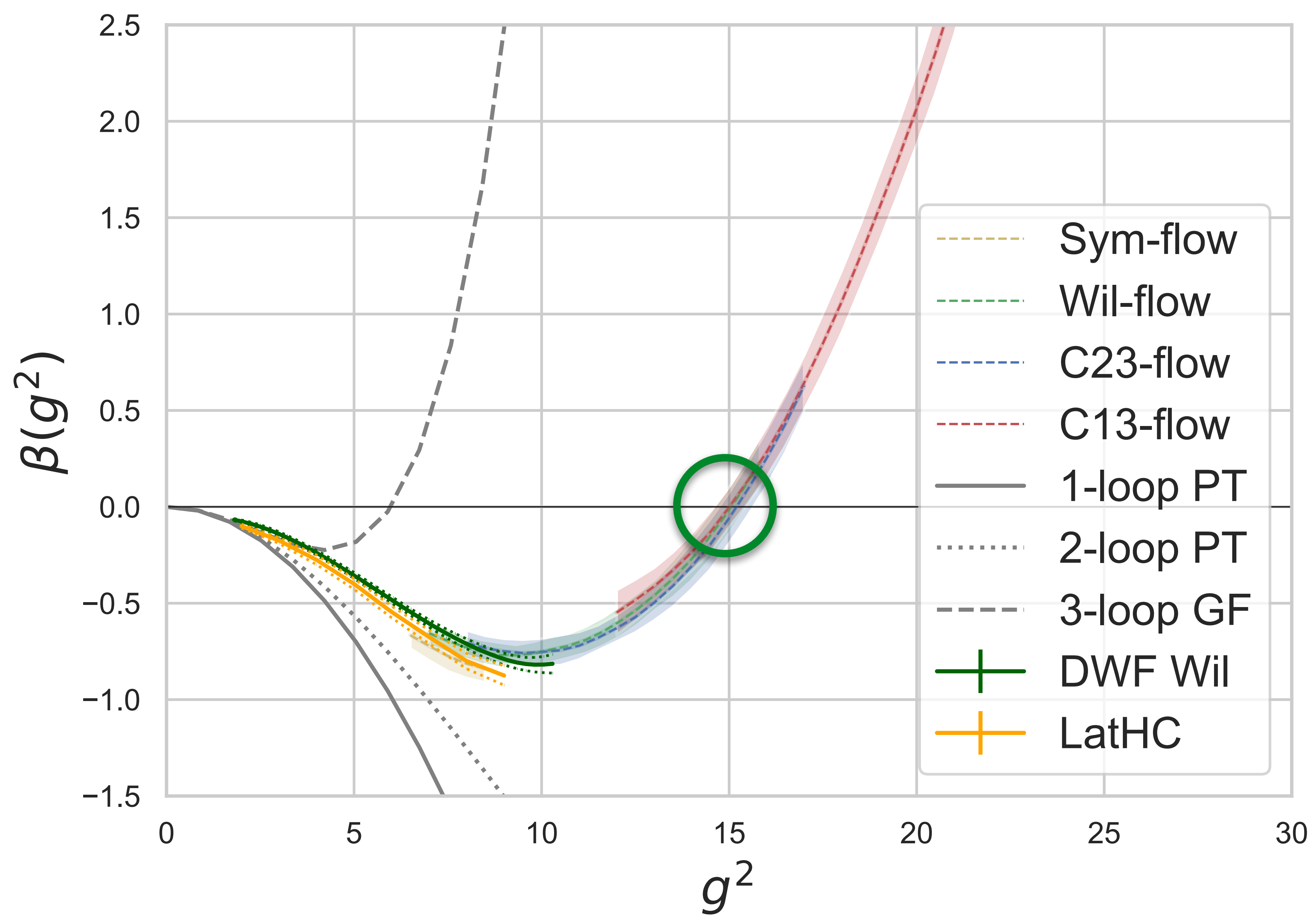

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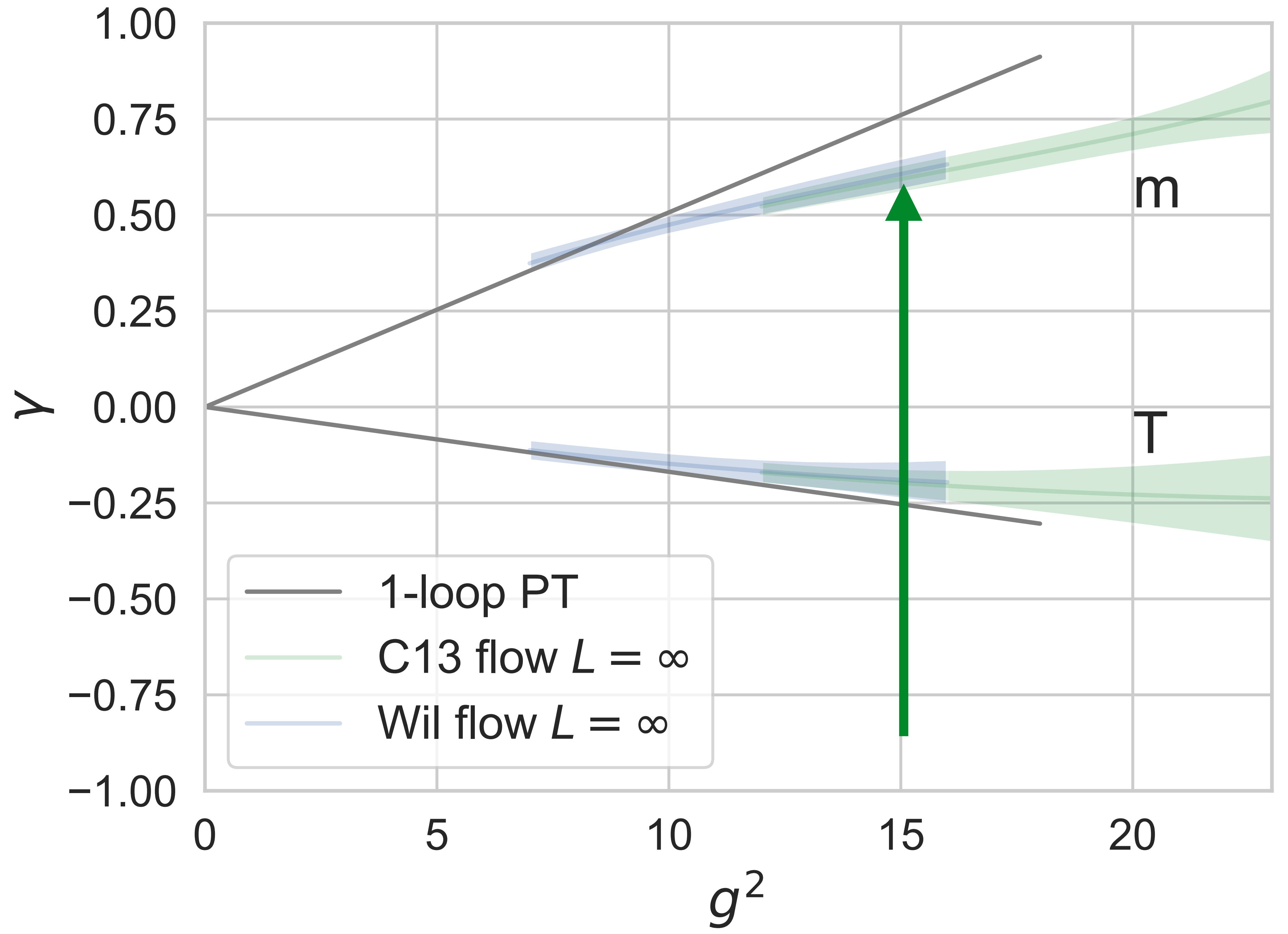

Anomalous dimension $\gamma_m^* \simeq 0.60$
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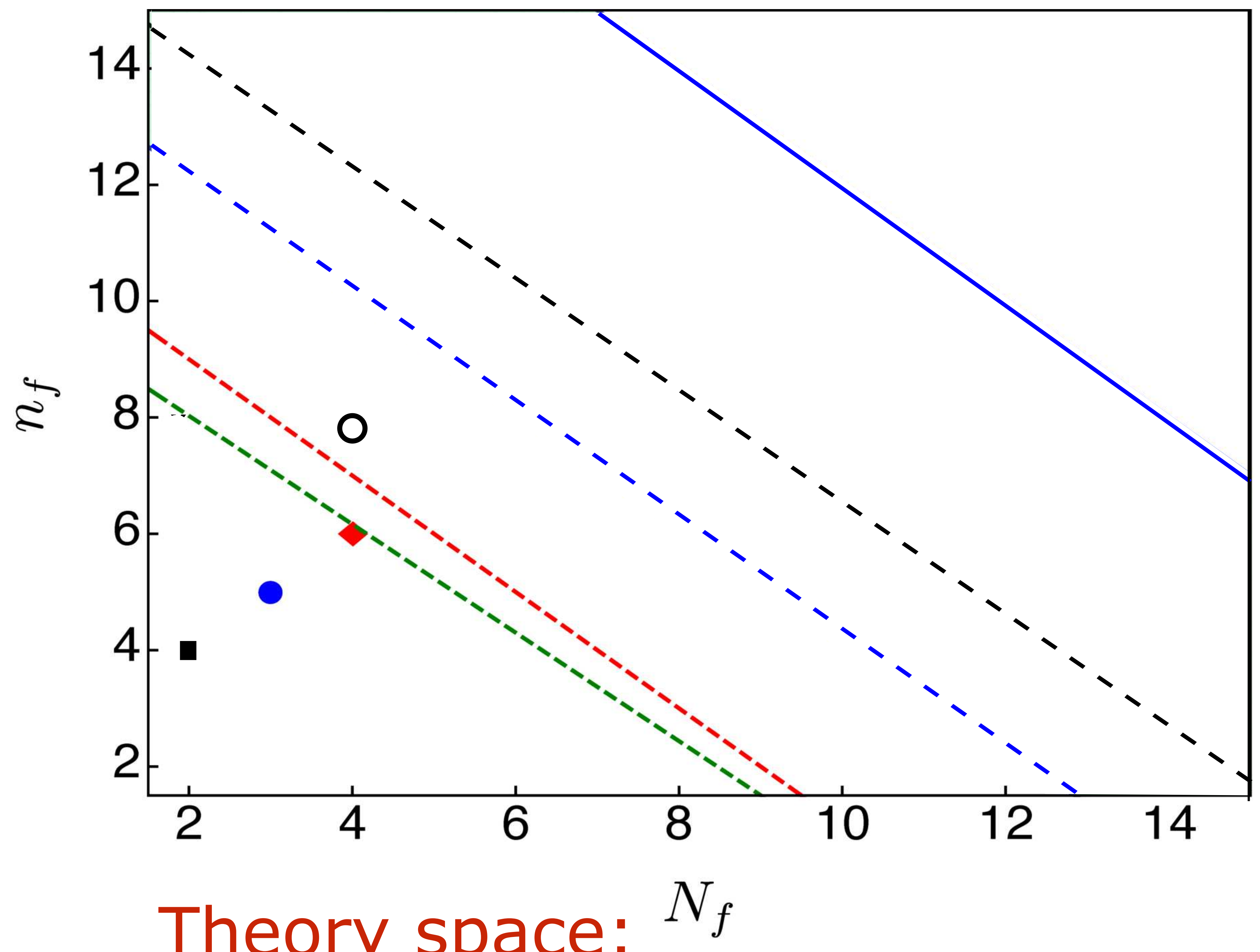
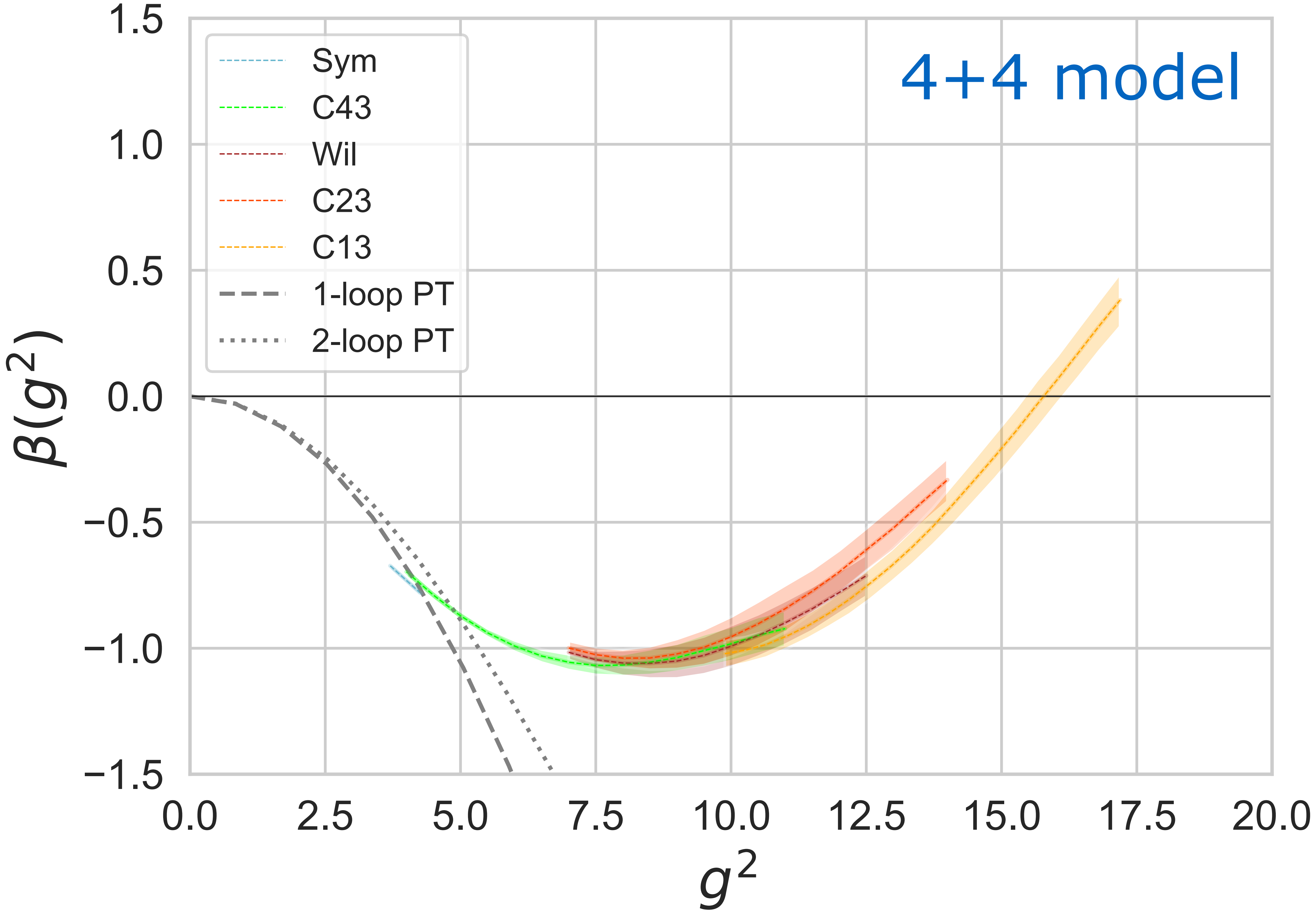
Composite Higgs+Partial composite top in SU(4) 2-rep model

A.H.,Neil, Shamir, Svetitsky, Witzel,
Phys.Rev.D 107 (2023) 11, 114504



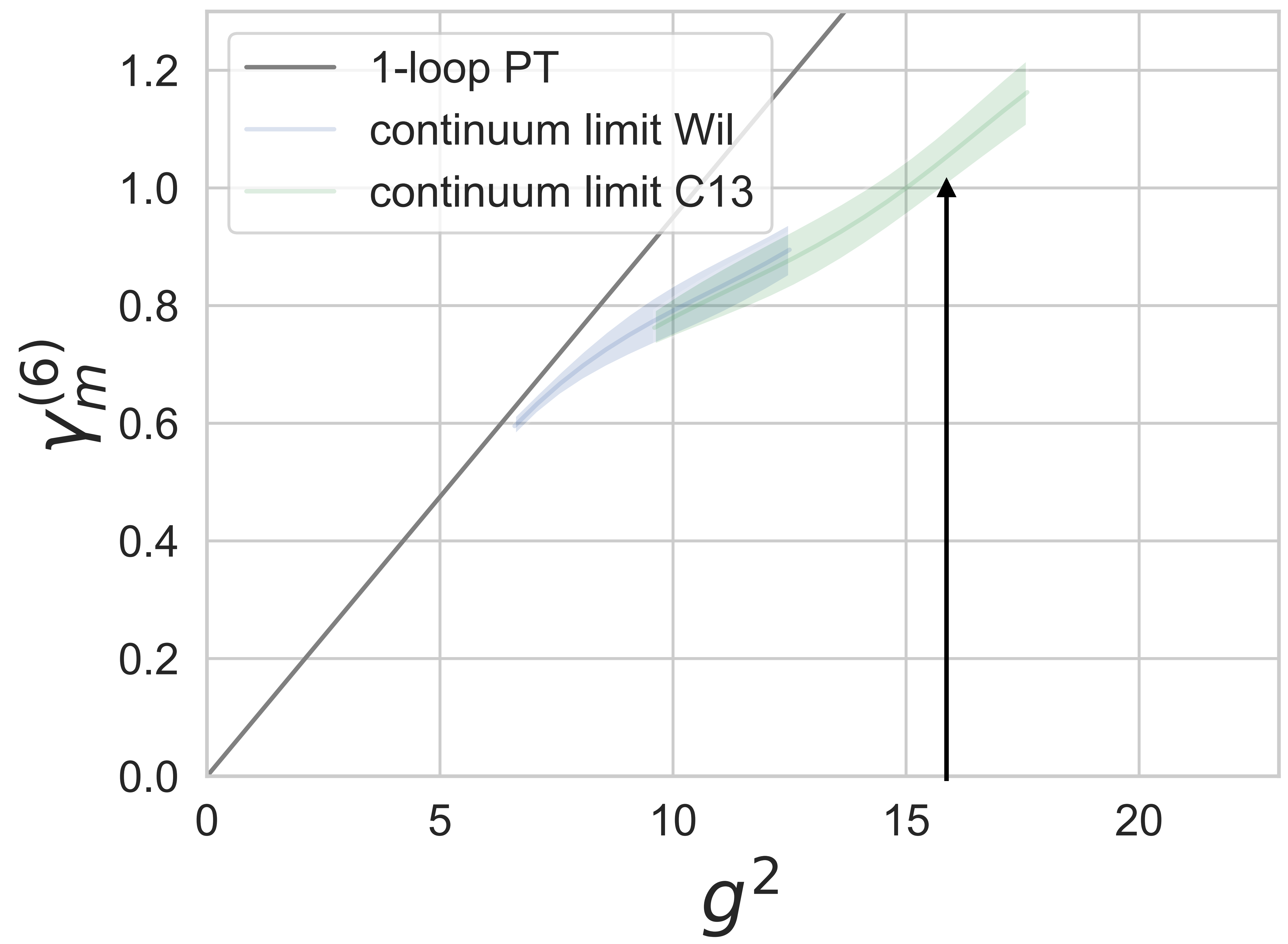
Theory space:
sextet (composite Higgs)
+fundamental(chimera baryon)
black square: 2+2 model :old
open circle: 4+4 model : new

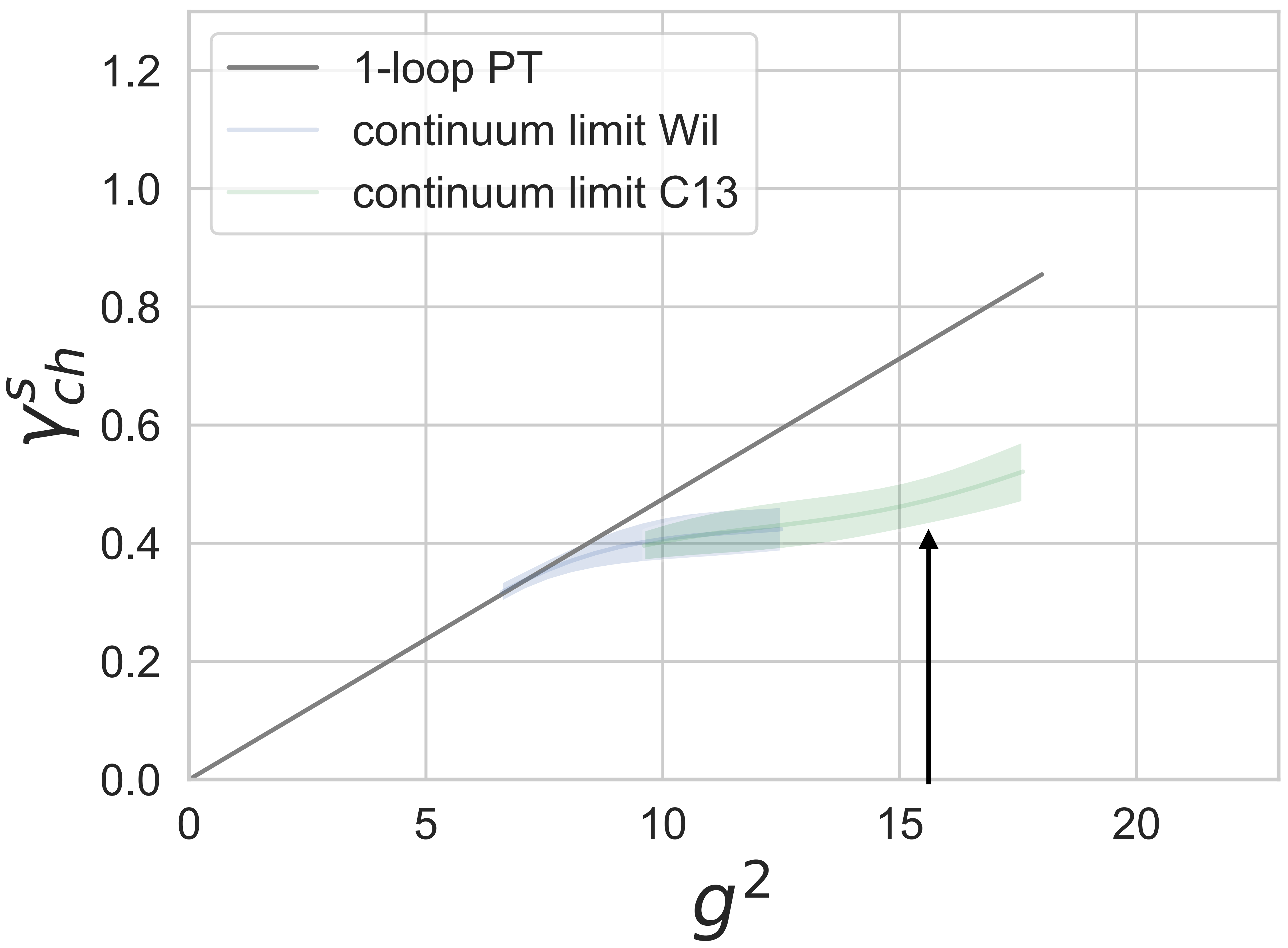


Simulations: Wilson fermions + PV boson and several GF action
IRFP at $g^2 \simeq 16$

Composite Higgs+Partial composite top in a 2-rep model

A.H.,Neil, Shamir, Svetitsky, Witzel, *Phys.Rev.D* 107 (2023) 11, 114504





Mass anomalous dimension:
not far from the conformal sill

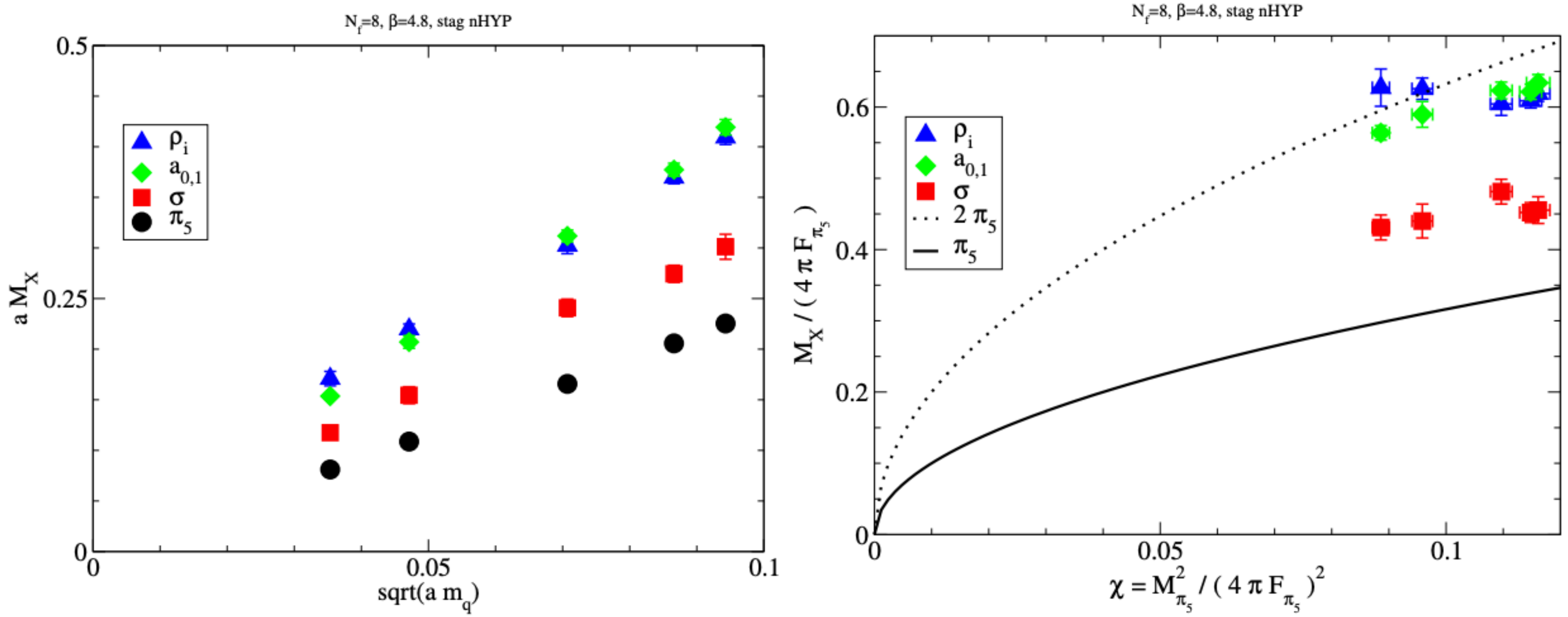
Chimera anomalous dimension:
but partial compositeness is not supported

$N_f = 8$ fundamental - staggered fermions

A.H. PRD 106 (2022) 014513

Despite of "common knowledge" (belief?), there is no evidence that SU(3) with 8 fundamental fermions is chirally broken

LSD Collaboration
e-Print: 2306.06095,
Phys.Rev.D 108 (2023) 9



Compare to Maurizio's plot yesterday

FIG. 11. Two different presentations of the spectrum from Tab. IX. On the left, in units of the lattice spacing a *vs.* a chiral expansion parameter assuming conformal symmetry and $\gamma^* \approx 1$. On the right, in units of the chiral breaking scale $4\pi\widehat{F}_{\pi_5}$ *vs.* a chiral expansion parameter assuming spontaneous chiral symmetry breaking. The dotted line on the right indicates the energy threshold for decays to two pions.

$N_f = 8$ fundamental - staggered fermions

A.H. PRD 106 (2022) 014513

Despite of "common knowledge" (belief?), there is no evidence that SU(3) with 8 fundamental fermions is chirally broken

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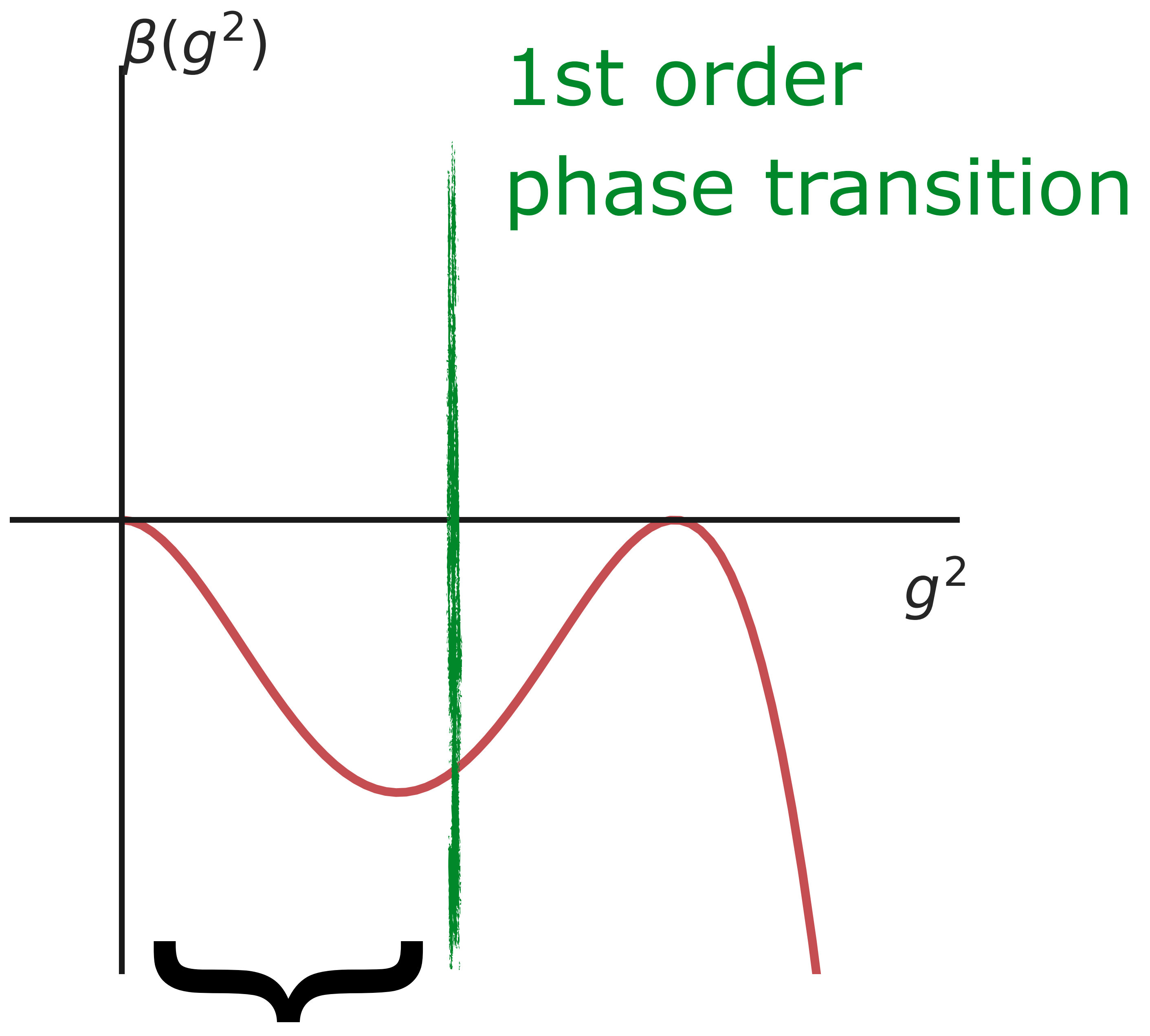
Phys.Rev.D 108 (2023) 9

Most simulations are limited by a lattice first-order bulk transition



Simulations probe only weak coupling regime
Properties of IRFP/walking is not observable

$N_f = 8$ fundamental - staggered fermions

A.H. PRD 106 (2022) 014513

Despite of "common knowledge" (belief?), there is no evidence that SU(3) with 8 fundamental fermions is chirally broken

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e-Print: 2306.06095,
Phys.Rev.D 108 (2023) 9

Most simulations are limited by a lattice first-order bulk transition
PV improved actions reach stronger couplings and show a different picture

$N_f = 8$ with staggered fermions (Dirac-Kaehler!) is special:

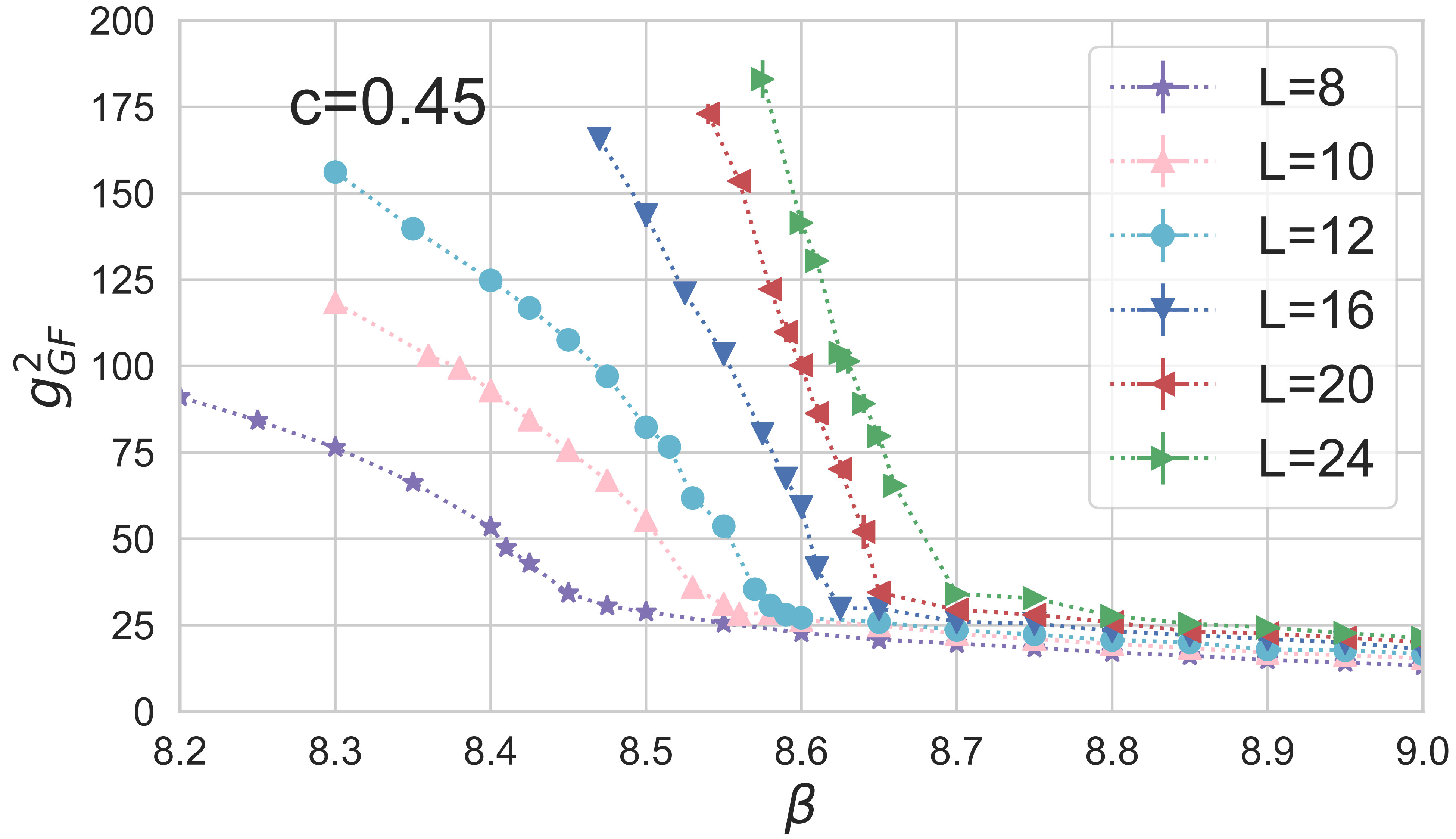
- free of all ’t Hooft anomalies
- does not have to satisfy anomaly matching
  —> no spontaneous chiral symmetry breaking necessary

Catterall et al PRD104,014503 (2021)
Catterall PRD107,014501 (2022)
Catterall 2311.02487
(D. Tong in continuum+ lots of stat. mech)

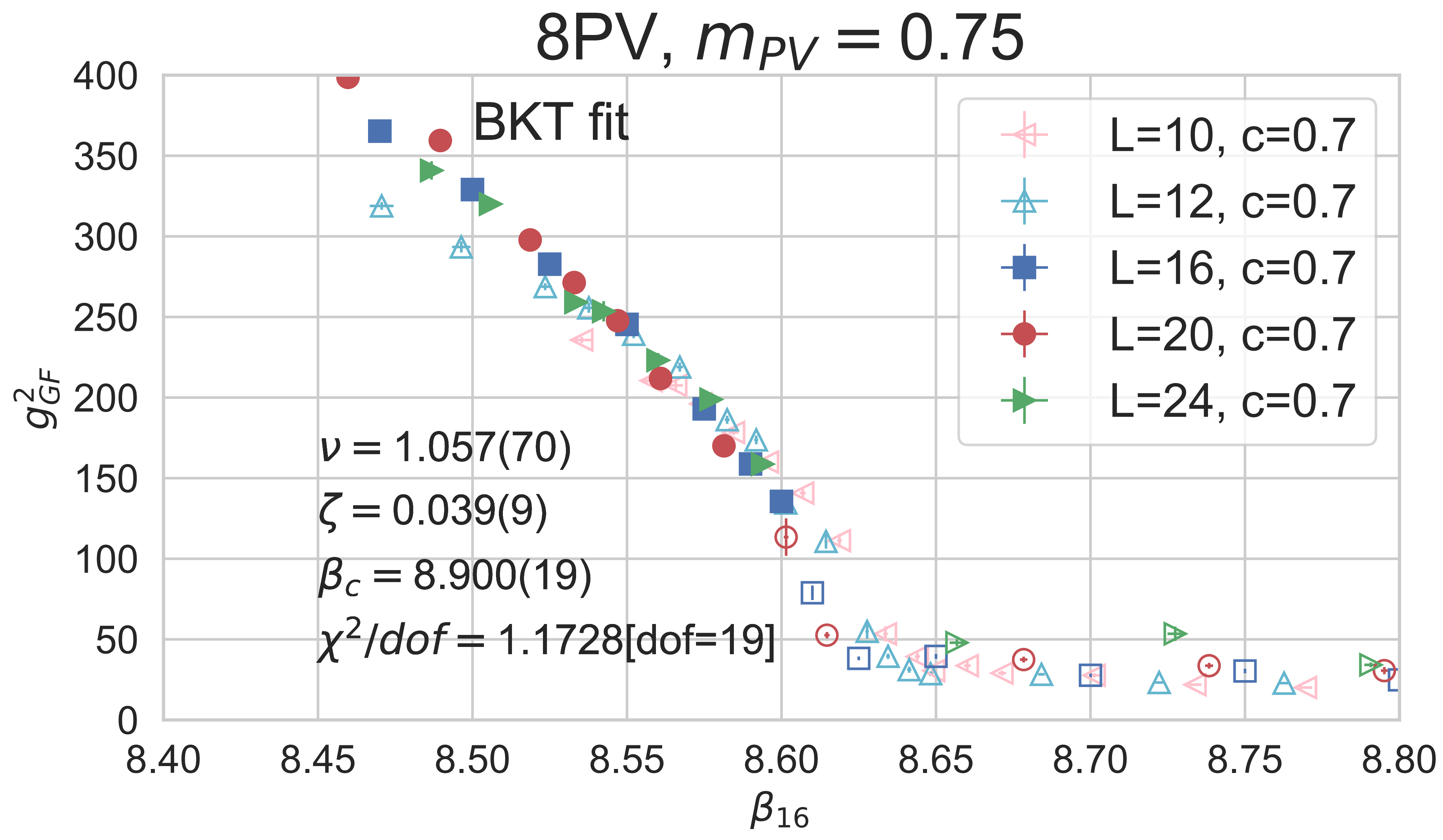
$N_f = 8$: order of phase transition

A.H. PRD 106 (2022) 014513

- Simulations with improved gauge action show a phase transition with 8 flavors
- Finite size scaling from strong coupling might suggest BKT* transition: $\xi \propto e^{-\zeta(\beta-\beta_c)^{-\nu}}$



renormalized coupling at $\mu = c/L$



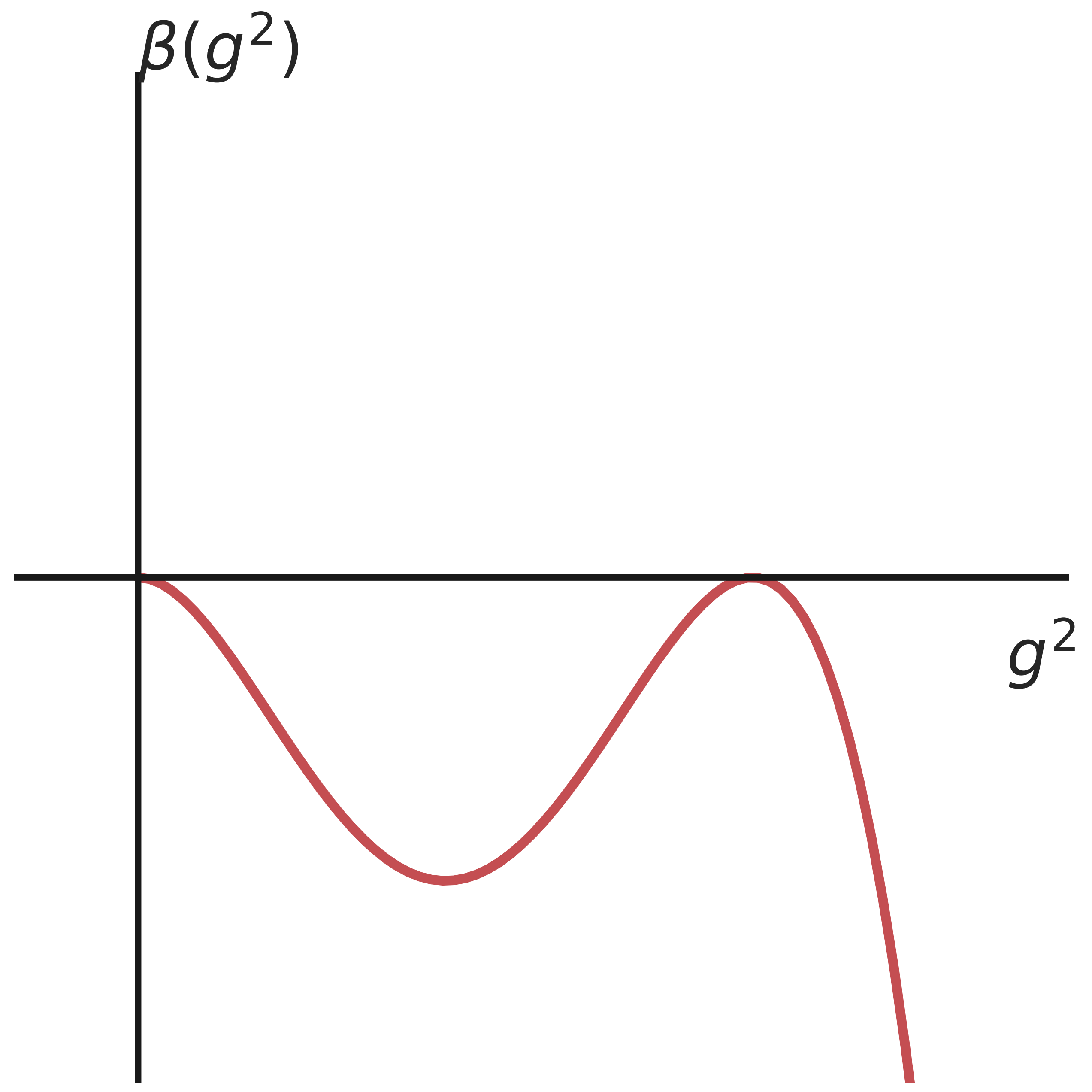
Finite size scaling/curve collapse of renormalized coupling

*Berezinsky, Kosterlitz, Thouless

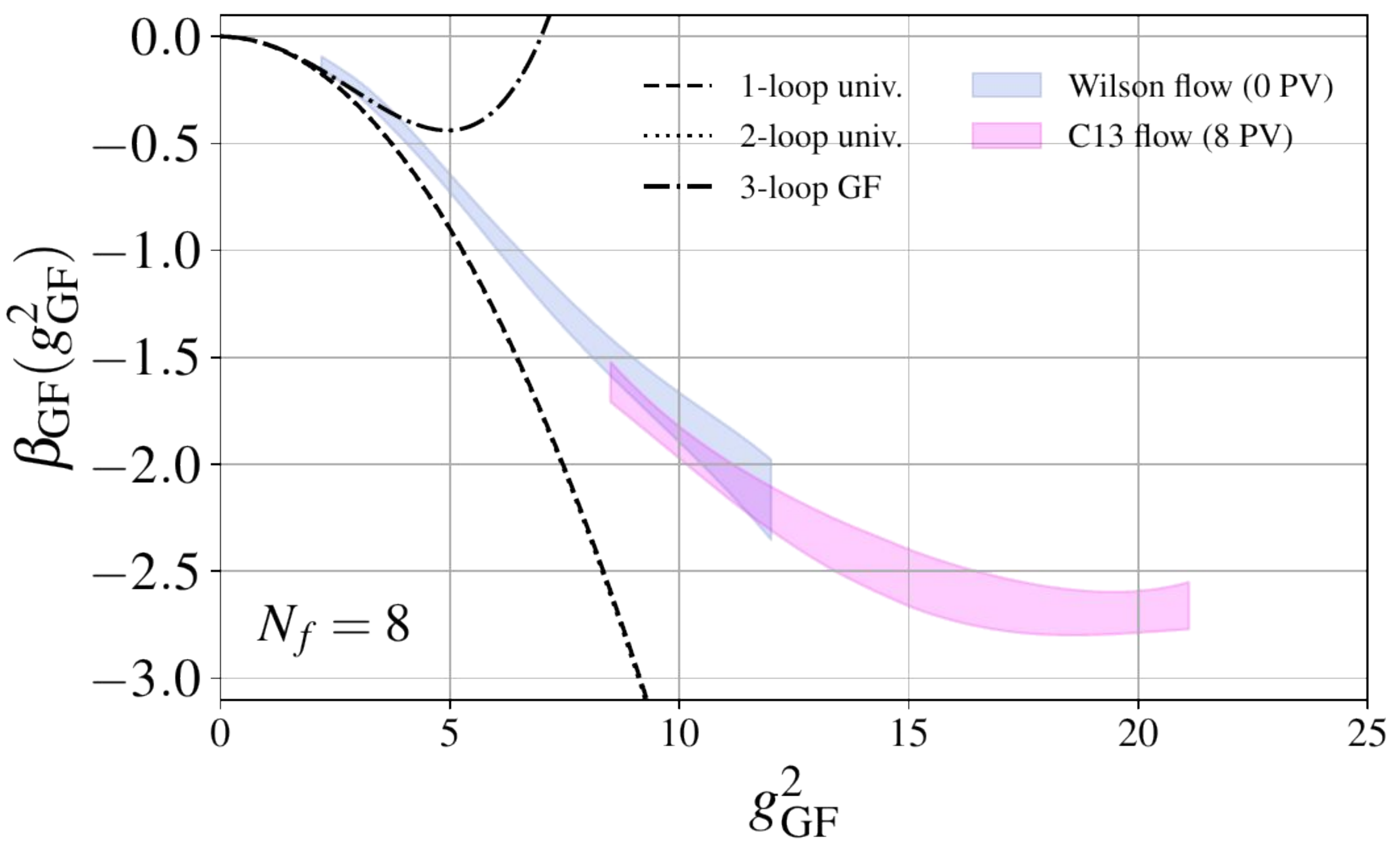
$N_f = 8$: β function

A.H., C. Peterson, in prep

If the phase transition is BKT, this could indicate the opening of the conformal window



If at the sill



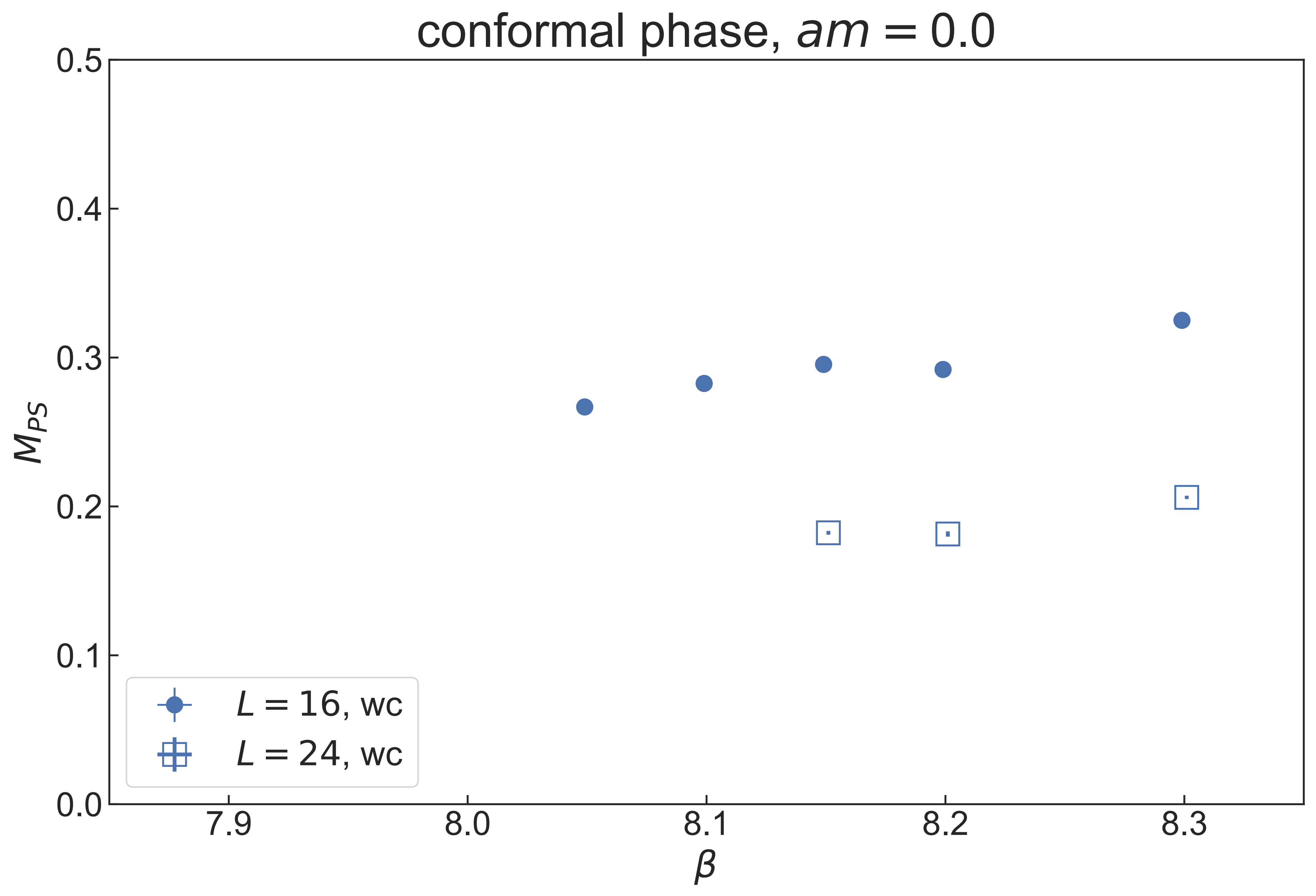
Preliminary numerical result
(blue: no PV)

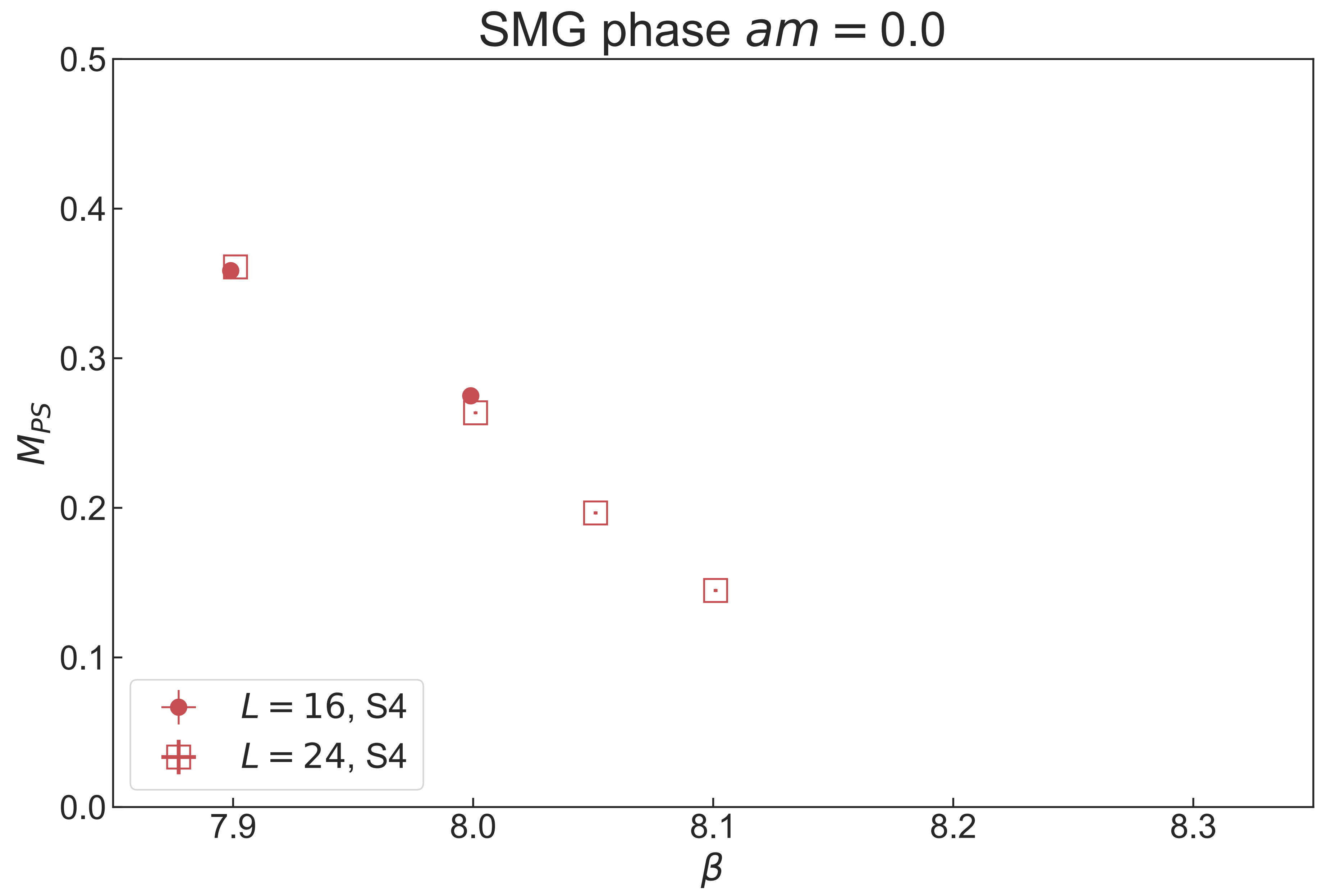
$N_f = 8$: spectrum

Two phases: weak coupling: conformal
strong coupling: chirally symmetric but gapped

Cheng et al *Phys.Rev.D* 85 (2012)
A.H. PRD 106 (2022) 014513

pseudo scalar mass at $m_f = 0$:





Weak coupling: $M_H \propto 1/L \to 0$ as $L \to \infty$
(conformal)

SMG: Volume independent
PS is massive even when $L \to \infty$

Symmetric mass generation

SMG is a new paradigm:

SMG phase is confining, gapped, but chirally symmetric
- spectrum is parity doubled
- possible only without ’t Hooft anomalies
- $N_f = 8$ continuum or 2 sets of staggered fields are anomaly free - could be SMG

Ayyar, Chandrasekharan
PRD91,065035 (2015)
Catterall et al PRD104,014503 (2021)
Catterall PRD107,014501 (2022)
A.H. PRD 106 (2022) 014513
D. Tong,JHEP 007(2022)001
Wu, Young,

Summary: Composite Higgs and (near-)conformal systems

Lattice simulations have come a long way:

-gauge action improvement: Pauli-Villars fields
-renormalization group β and γ functions paint a consistent picture

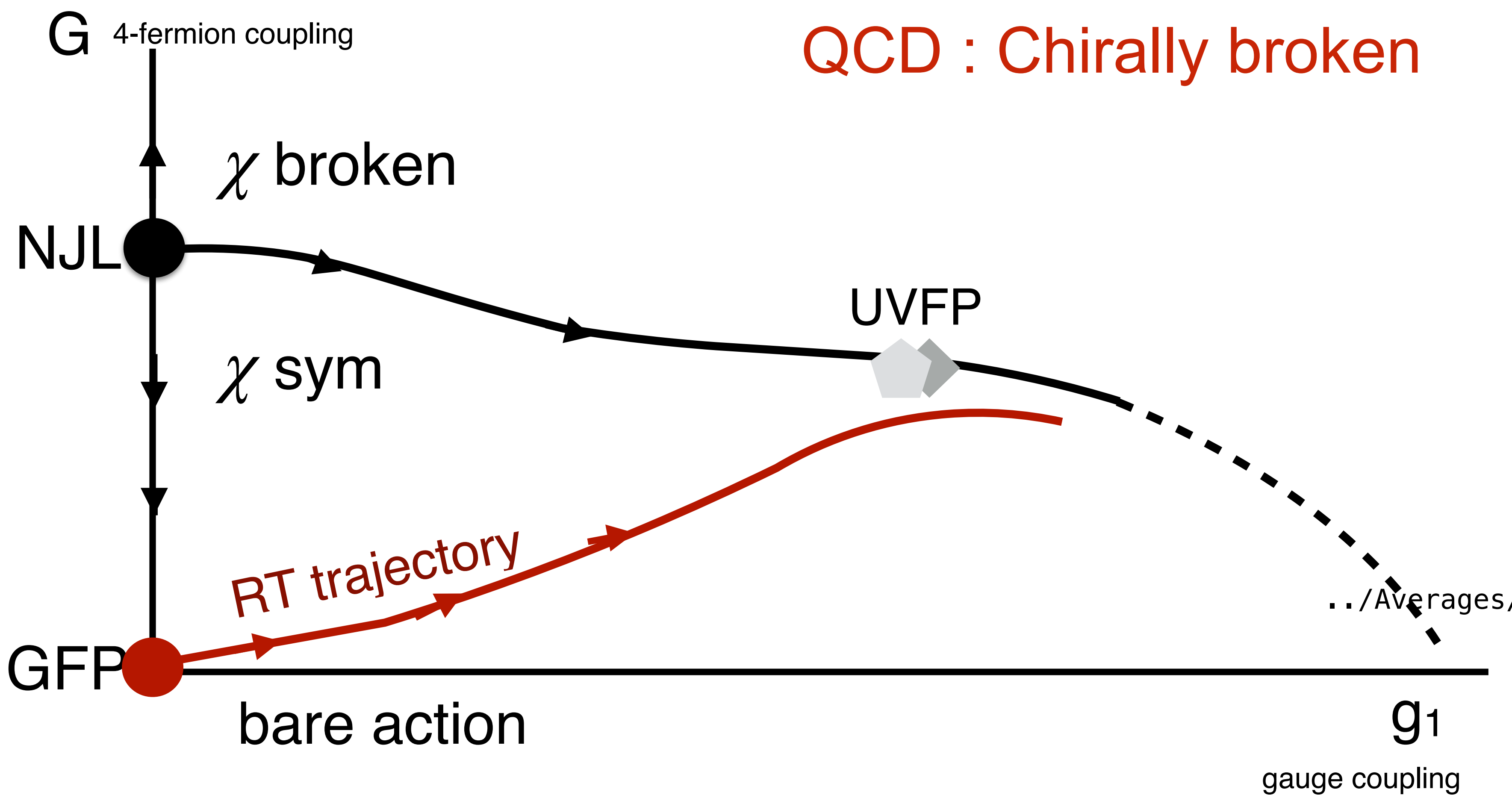
Theoretical developments - SMG - point beyond the lattice

EXTRA SLIDES

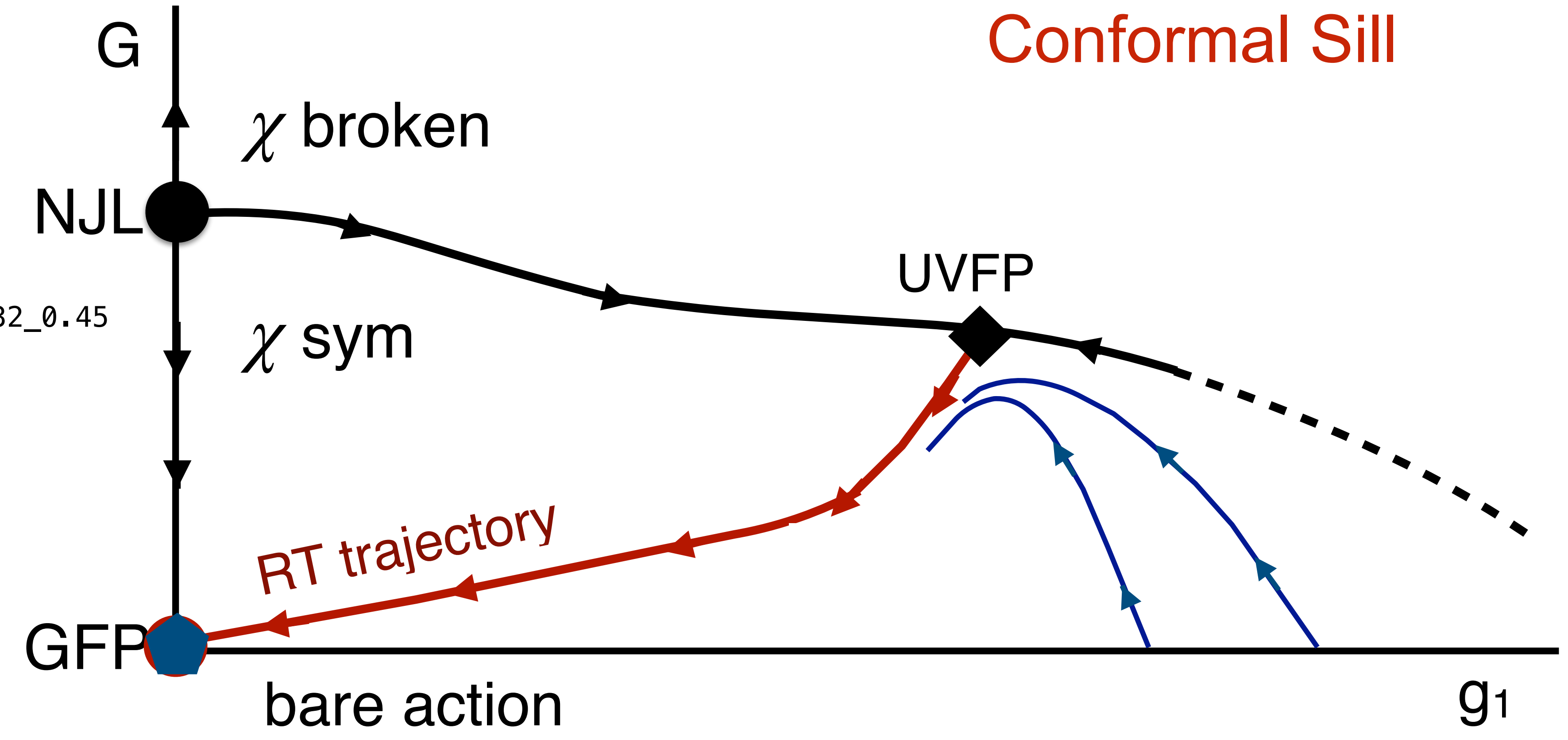
Gauge-fermion systems with 4-fermion interaction

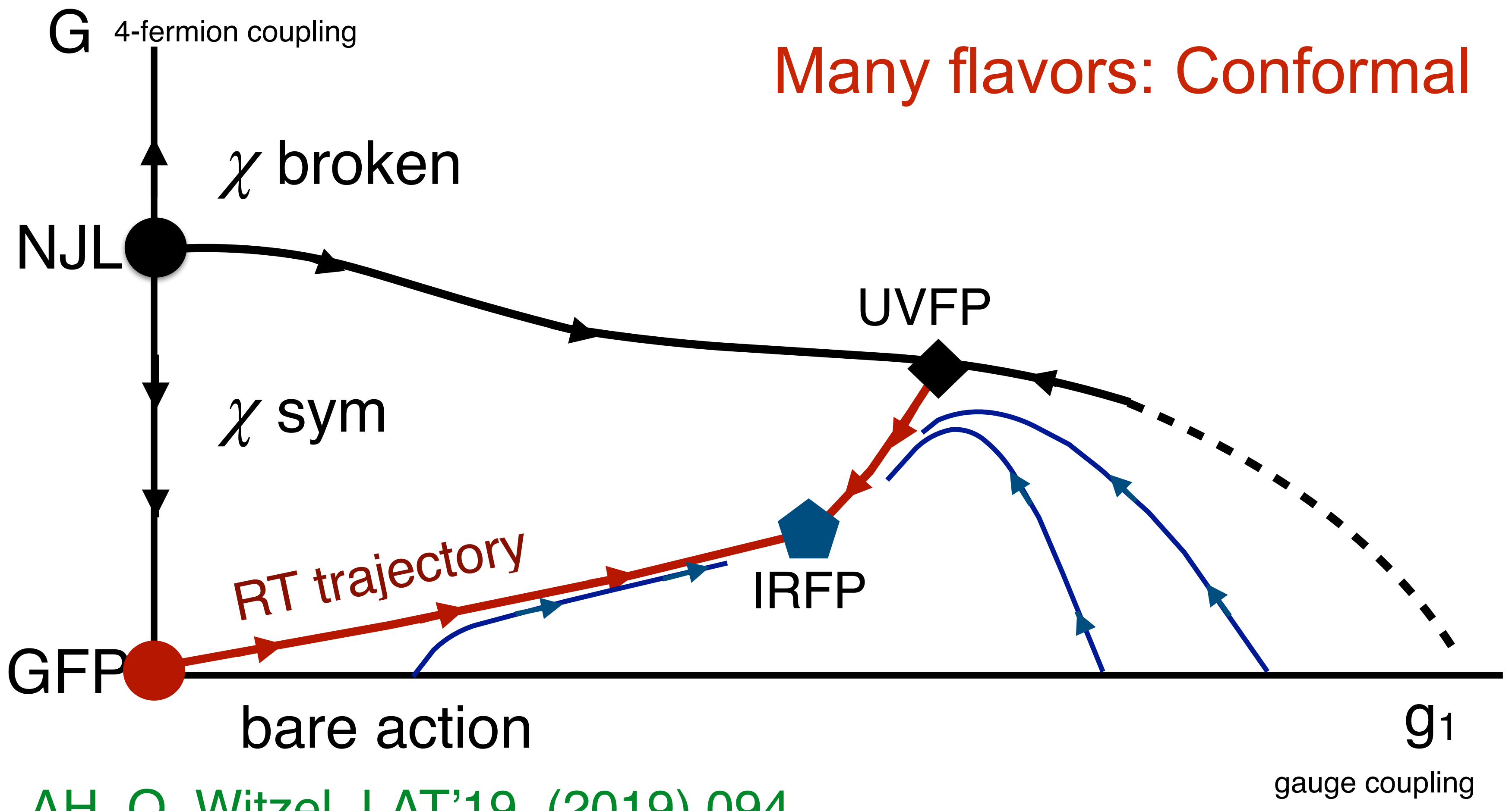
- Quantum effects generate new interaction
- Conjectured phase diagram in the extended parameter space

Kaplan et al, *Phys.Rev.D* 80 (2009) 125005
Gorbenko et al, *JHEP* 10 (2018) 108



../Averages/aver_g2_32_0.45





AH, O. Witzel, LAT'19. (2019) 094

Staying within gauge-fermion systems we can explore the RT up to a possible IRFP

- RG $\beta(g^2)$ function
- anomalous dimension $\gamma_{\mathcal{O}}(g^2)$

Staggered fermions

Becher, Joos 1982

are Kaehler-Dirac fermions distributed in a 2^4 hypercube

$$S = \frac{1}{2}\sum_{n,\mu}(\bar{\chi}_n \alpha_\mu(n) U_\mu(n) \chi_{n+\mu} + cc) + m \sum_n \bar{\chi}_n \chi_n \,, \qquad \alpha_\mu(n) = (-1)^{n_0 + \dots n_{\mu-1}}$$

χ : 1-component fermion

1 set of staggered fermions ≡ 4 Dirac flavors in flat space, $g_0^2 = 0$

2 sets of massless staggered fermions ≡ 4 sets of reduced staggered
≡ 16 Weyl fermions

Catterall et al 2101.01026

Massless staggered fermions suffer from Z_4 gauge anomaly - cancelled when 2 staggered species are present

—> 2 staggered species could exhibit symmetric mass generation : mass without spontaneous symmetry breaking

S4 phase gapped, chiral symmetric

Zero momentum correlators $C(t) = \sum_{\bar{x},\bar{y}} \langle O_S(\bar{x}, t=0) O_S(\bar{y}, t) \rangle$

"Pion states" : spin $\otimes$ taste in terms of 1-component fields

pseudoscalar : $P1 = \gamma_5 \otimes \gamma_5$: $\mathcal{O}_S = \sum_{\bar{x}} \bar{q}(\bar{x})\, q(\bar{x})\, (-1)^{x_1+x_2+x_3}$

scalar : $S1 = \gamma_0\gamma_5 \otimes \gamma_0\gamma_5$: $\mathcal{O}_S = \sum_{\bar{x}} \bar{q}(\bar{x})\, q(\bar{x})$

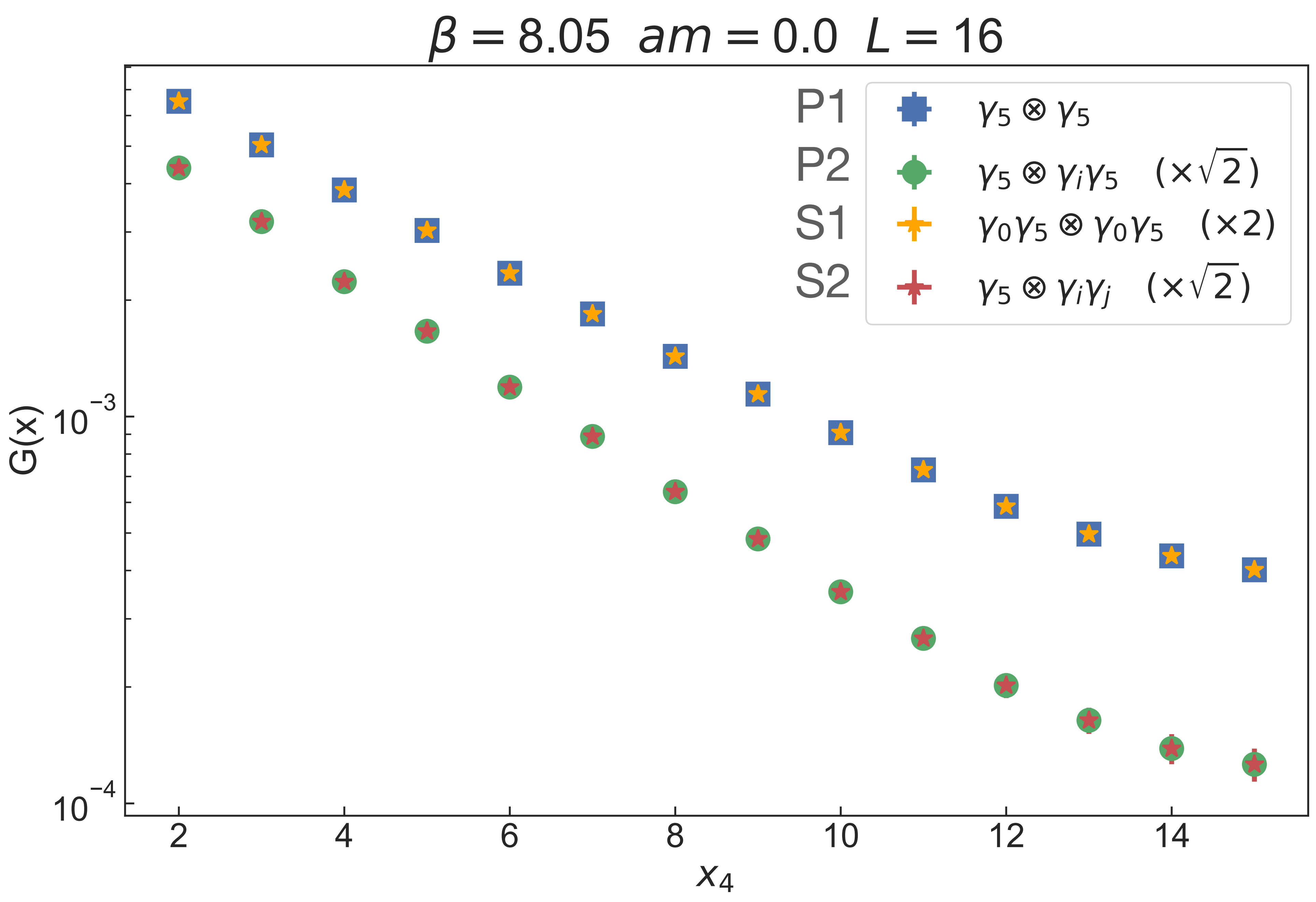
pseudoscalar : $P2 = \gamma_5 \otimes \gamma_i\gamma_5$: $\mathcal{O}_S = \sum_{\bar{x}} \bar{q}(\bar{x}) U_i(\bar{x}) q(\bar{x}+i)(-1)^{x_1+x_2+x_3}$

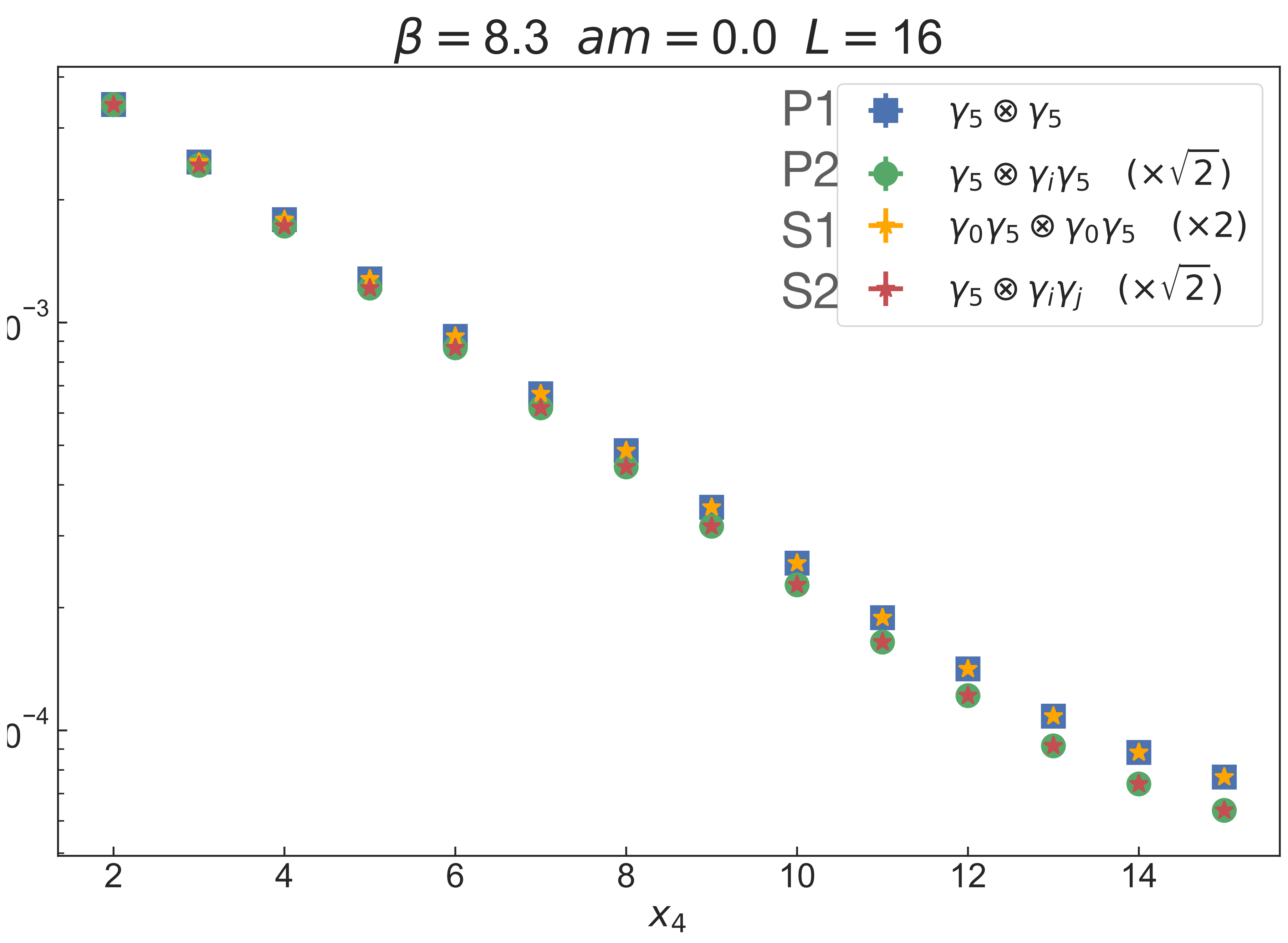
scalar : $S2 = \gamma_0\gamma_5 \otimes \gamma_0\gamma_i\gamma_5$: $\mathcal{O}_S = \sum_{\bar{x}} \bar{q}(\bar{x}) U_i(\bar{x}) q(\bar{x}+i)$

(all four operators couple to scalar and pseudoscalar, but mostly to one only)

S4 phase chiral symmetric

"Pion" correlators





S4 phase

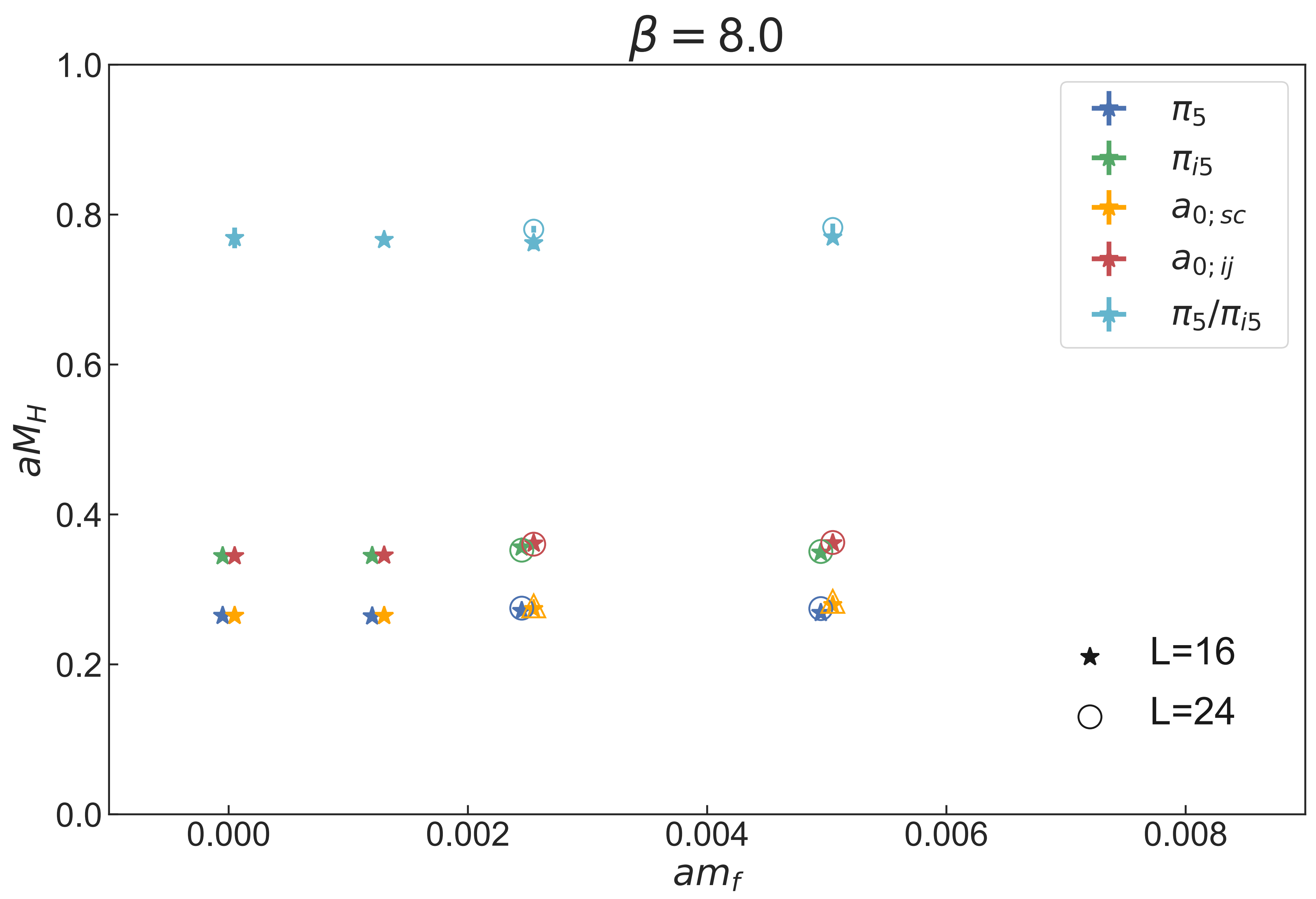
- chirally symmetric (P = S)
- P1-P2, S1-S2 are broken

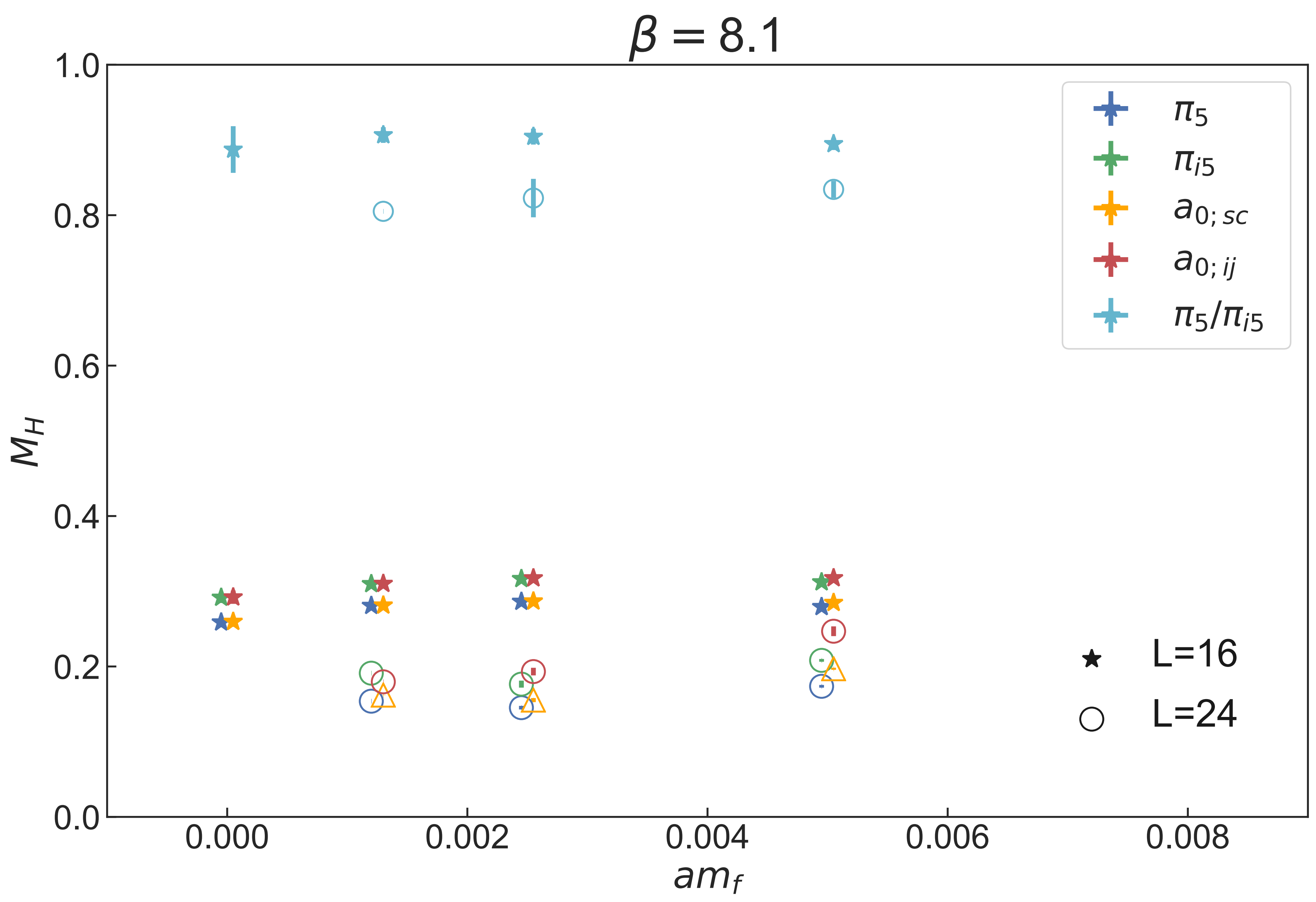
Conformal phase

- chirally symmetric (P = S)
- P1,P2, S1,S2 are nearly degenerate
  (good taste symmetry)

S4 phase gapped

"Pion" masses





S4 phase :mesons are massive

- nearly constant in fermion mass
- nearly independent of volume

Conformal phase :mesons are massive

- due to finite volume!
- all masses vanish in the infinite volume chiral limit