Composite Dark Matter from Sp(2N) gauge theory

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Busan, February 20, 2024

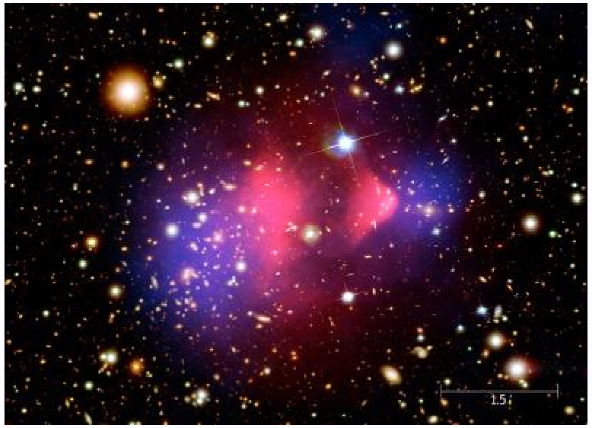
PNU workshop on Composite Higgs Physics

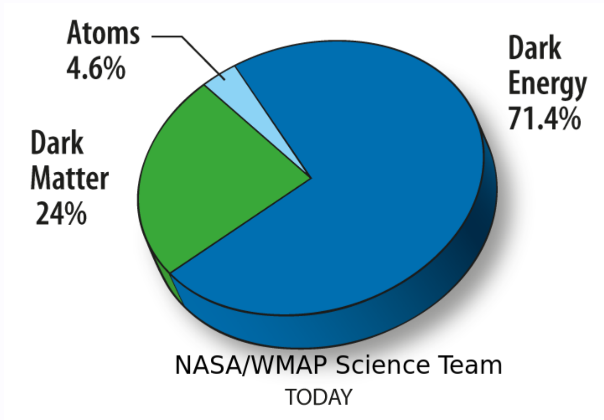
1. QCD inspired Dark Matter models

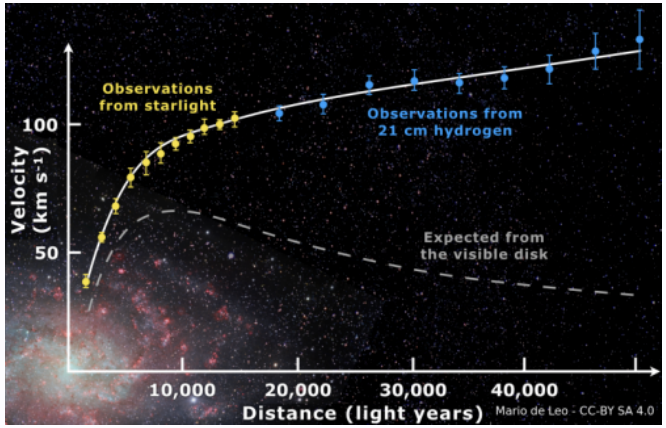
[1] e.g. Bertone, Hooper, Silk. [hep-ph/0404175] [2] e.g. PDG review and Famaey, McGaugh [1112.3960]

Dark Matter - Why?

- Strong observational evidence at many scales! [1]
- Modified Gravity [2] is a potential alternative
- New particles beyond the Standard Model (BSM) promising!



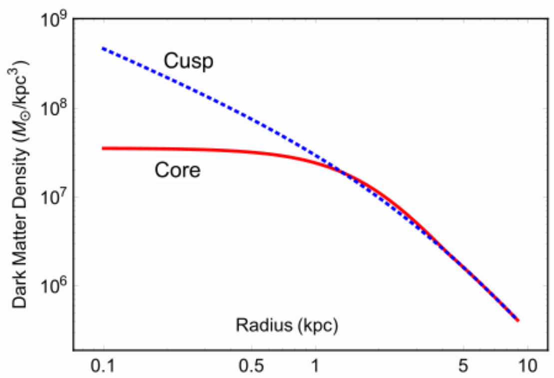


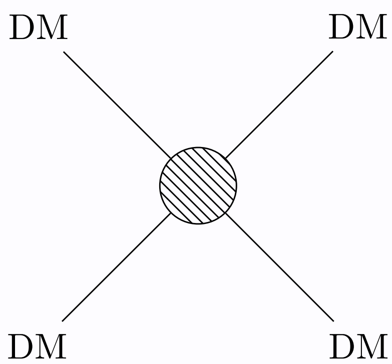


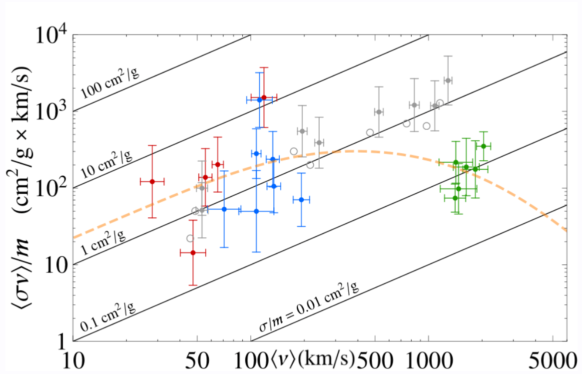
[1] see e.g. Bullock,Boylan-Kolchin [1707.04256], Tulin, Yu [1705.02358]

Dark Matter properties

- DM self-interaction phenomenologically allowed[1] and potentially relevant for small-scale structure problems
  - non-vanishing scattering cross-sections $\sigma_{2\mathrm{DM}\to 2\mathrm{DM}}$
  - velocity dependence of $\sigma_{2\mathrm{DM}\to 2\mathrm{DM}}$ preferred



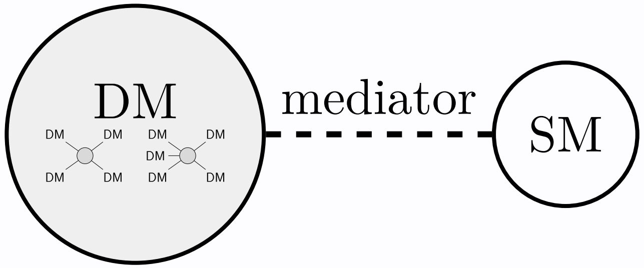




QCD-like Dark Matter can those provide self-interactions!

Strongly Interacting Gauge Theories in DM Models

- With fermions: Global symmetries make DM stable
- With mediator: Dark sector coupled to SM



$$\mathcal{L}_{\mathrm{DM}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}_f (i \not{D} + m_f) \psi_f$$

- Non-vanishing self-scattering cross-section arise

$$\langle v \sigma_{\pi\pi \to \pi\pi} \rangle \neq 0$$

- Relic density driven by strong processes

Dark meson scattering: Determine DM relic density

- Any model must predict the current density of DM correctly
  - number density n can be calculated using Boltzmann equations

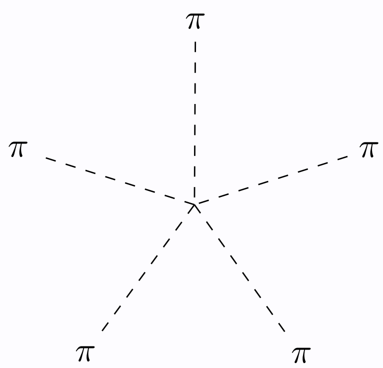
$$\partial_t n + 3Hn = f\left(\langle v\sigma_{\mathrm{number\ changing}}\rangle\right)$$

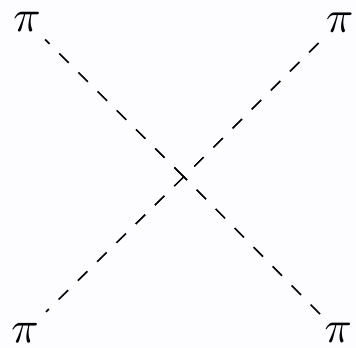
- Cross-sections $\langle \sigma v\rangle$ are input for Boltzmann equations
  - describe non-equilibrium dynamics
  - H is the Hubble rate

[1] Hochberg et. al. [1402.5143] [2] Tulin, Yu [1705.02358], [3] Kondo et.al. [2205.08088] [4] Chu et.al. [2401.12283]

Relevant pion scattering channels

- $3\pi \to 2\pi$ (semi-annihilation) [1]
- $2\pi \to 2\pi$ (self-scattering)
  - self-scattering among DM [2]
  - resonant enhancements [3]
- $2n\pi \to 2\pi$ (multi-hadron bound states) [4]





[1] Hochberg et. al. [1402.5143], [2] Wess, Zumino(Phys. Lett. B 1971), Witten (Nucl. Phys. B 1983)

Strongly Interacting Massive Particles (SIMPs)

- Depletion via $3\mathrm{DM} \to 2\mathrm{DM}$ [1], i.e. $3\pi \to 2\pi$
  - same as $KK \to 3\pi$ in QCD [2]
  - Early universe: $\mathrm{SM} \rightleftharpoons \mathrm{DM}$ equilibrium
  - Dark matter depletion process: ***freeze-out***
- LO ChiPT matches relic density at

$$m_\pi \approx \mathcal{O}(100)\mathrm{MeV} - \mathcal{O}(1)\mathrm{GeV}$$

`Dark Matter with 3DM → 2DM depletion and self-interactions`

Other mass scales than QCD are relevant!

- Lagrangian has two free parameters: g^2 and m_f
  - one overall energy scale
  - one intrinsic strong scale
- Overall scale should allow sufficiently heavy DM
- m_f should lead to parametrically light m_π
  - both scales can deviate strongly from QCD!

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Lattice investigations of a larger parameter space are useful!
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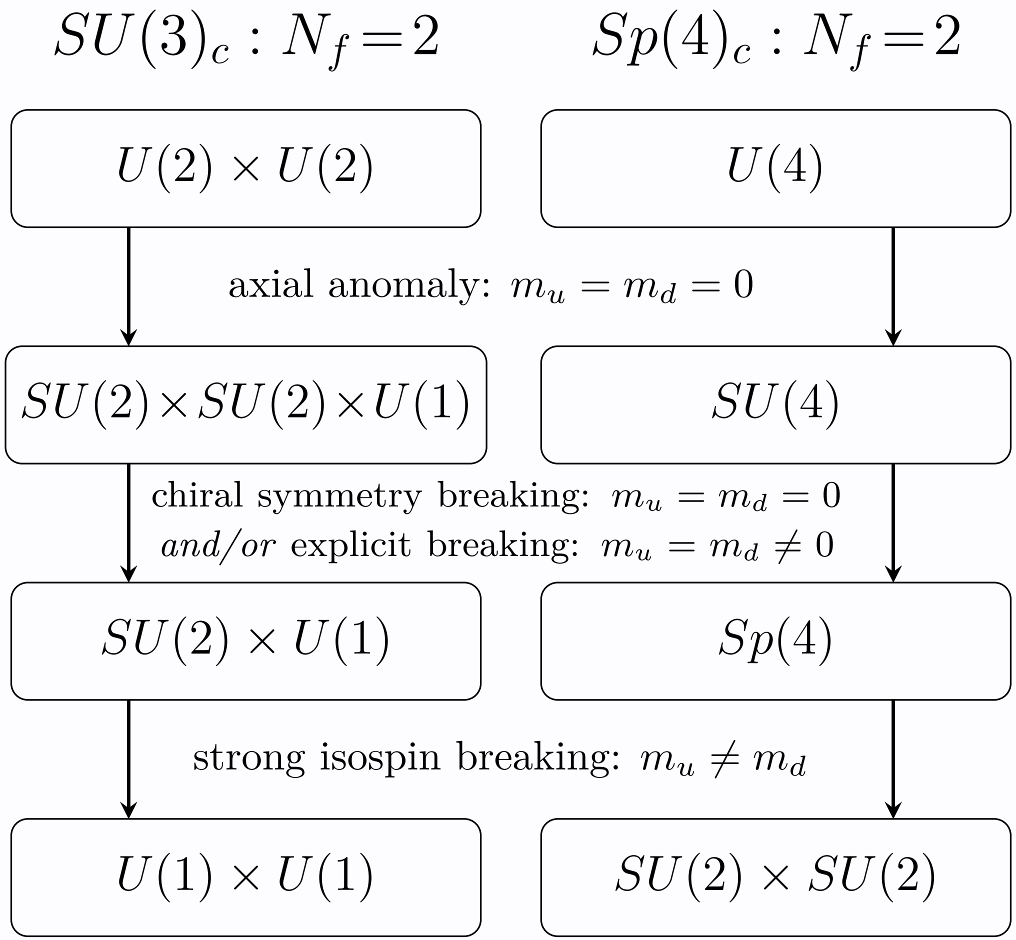
[1] Hochberg et. al. [1411.3727] [1512.07917] [2] Choi et.al. [1801.07726] Bernreuther et.al. [2311.17157]
[3] Kulkarni et.al. [2202.05191]
see also Hansen et.al. [1507.01590]

Other relevant channels

- decay to Standard Model: $2\pi \to SM$ [1]
- involvement of vector mesons: $\pi\pi \to \pi\rho$, $3\pi \to \pi\rho$ [2]
- influence of light singlets: $\eta'\eta' \to \pi\pi, \pi\pi \to \eta'\pi, \ldots$ [3]

- The relevance depends on the spectrum
  - investigation of the meson spectrum important
  - lattice investigations inform EFT construction



[1] Kosower (Phys.Lett.B. 1984)

[2] Hochberg et. al. [1411.3727] [1512.07917]

SIMPs from $Sp(4)$ gauge theory

- Pseudo-real representation: [1]
  $\Rightarrow$ more pseudo-Goldstones
  $\Rightarrow$ no fermionic bound states
- $N_f = 2$: exactly 5 Goldstones
  - Allows $3\mathrm{DM} \to 2\mathrm{DM}$ [2]

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Sp(4) with two fermions is
a minimal SIMP DM realisation
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Lagrangian of $Sp(4)_c$ with fermions

$$\mathcal{L}_{Sp(4)} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \sum_{f=u,d} \bar{\psi}_f (i\not{D} + m_f) \psi_f$$

- Higher symmetry than QCD-like theories

$$\Psi = \begin{pmatrix} u_L \\ d_L \\ -SCu_R^* \\ -SCd_R^* \end{pmatrix} = \begin{pmatrix} u_L \\ d_L \\ \tilde{u}_R \\ \tilde{d}_R \end{pmatrix}$$

C ... charge conj.
S ... colour matrix

$$\mathcal{L}_{Sp(4)} = i\bar{\Psi}\not{D}\Psi - \frac{1}{2}\left(\Psi^T SCM\Psi + h.c.\right) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

- generators τ_a in fundamental repr. : $S\tau_a S = -\tau_a^T$

[1] Ryttov, Sannino [0809.0713] [2] Bennett et. al. [1912.06505]

Meson multiplets of $Sp(4)_c$ with $N_f = 2$

- Extra gauge invariant states: $q^T \dots q$ and $\bar{q} \dots \bar{q}^T$
- $Sp(2N_f)$ flavour symmetry

$$Sp(4)_F : \qquad 4 \otimes 4 = 1 \oplus 5 \oplus 10$$

- Different multiplets depending on J^P number

$$5\pi,\ 10\rho,\ 5a_0,\ 5a_1,\ 1\eta',\ 1\sigma,\ 0\omega$$

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The global symmetries lead to a richer meson multiplet structure!
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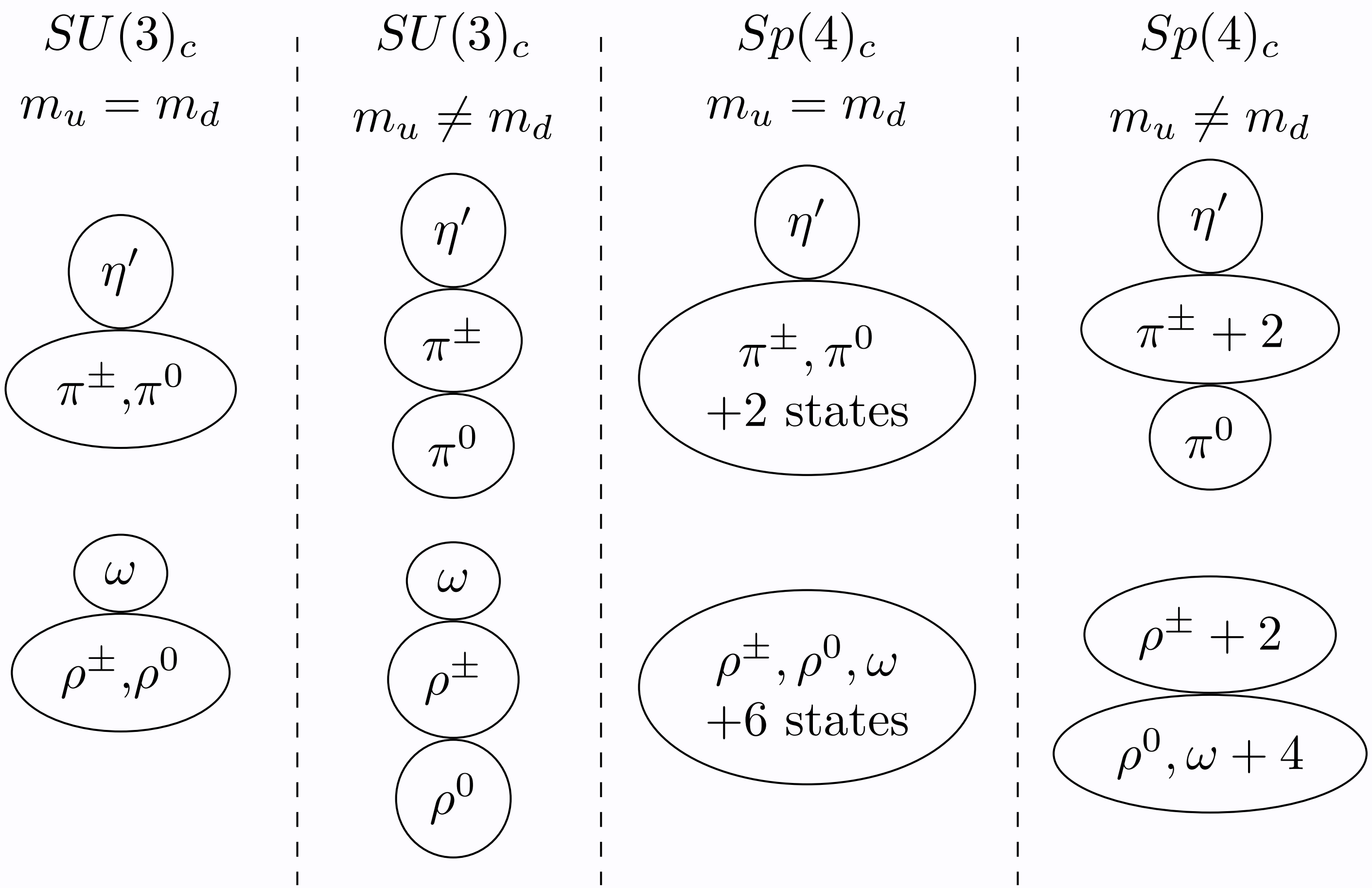
Pseudoscalar (PS) and vector (V) multiplets

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The same patterns persist for other channels.
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BSM wishlist from the lattice

1. Masses and decay constants of dark hadrons
    - Non-singlet and singlet mesons, glueballs
2. Scattering of dark pions
    - $2\pi \rightarrow 2\pi$ for self-interaction crossection
    - $3\pi \rightarrow 2\pi$ for SIMP semi-annihilation
3. Applicability of χPT and related EFTs

[1]Nogradi,Szikszai[2107.05996][2]Bennett et.al.[2010.15781][3]Bennett et.al.[2202.05516],Drach et.al.[2107.09974]
[4] e.g. Boz. et.al. [1912.10975], [5] Mason et.al. [2310.02145]

Applications of $Sp(2N)$ gauge theory beyond SIMP DM

- Generic features of non-Abelian confining gauge thoeries
  - Hadron masses as functions of N_f and N_c [1]
  - large N_c limit [2]
- Higgs compositeness, partial top compositeness [3]
  - Mixed fermion representations: near conformal behaviour?
- Model theory for finite density calculations (no sign problem) [4]
- Finite temperature behaviour: Deconfinemt and chiral symmetry
  - Potential first oder phase transitions? [5]

Composite Higgs studies can be repurposed

- Coset spaces for Higgs physics are large enough for SIMP DM
  - applies also to different fermion reps. (e.g. $Sp(4), N_f^{as} \geq 2$)
  - or mixed representation theories
- Particle spectrum determines relevant hadronic states
- Scattering studies in the context of WW scattering

Lattice Investigations:

Quantitative Insights

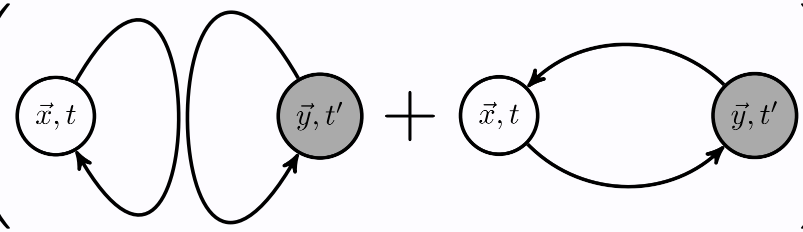
Lattice spectroscopy: Getting meson masses

- Construct operator with same quantum numbers
- Energy levels from Euclidean correlator

$$C_{\mathcal{O}}(t) = \sum_n \frac{1}{2E_n} \langle 0|\mathcal{O}|n\rangle^* \langle n|\mathcal{O}|0\rangle e^{-E_n t}.$$

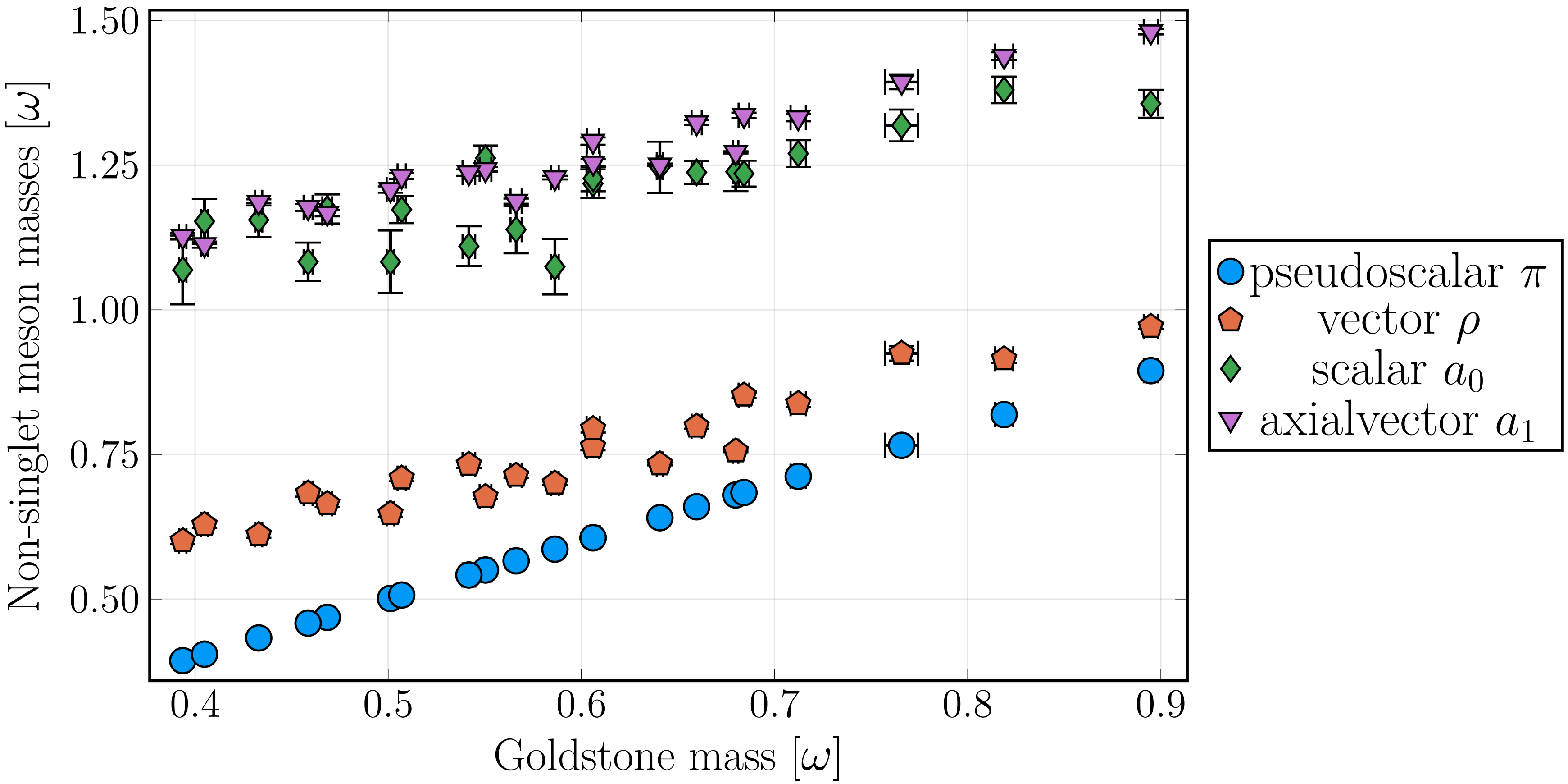
- For mesons a generic correlator

$$C(t-t') = \sum_{\vec{x},\vec{y}} \Big(\; \cdots \; \Big) + \underbrace{\text{const.}}_{=|\langle 0|O|0\rangle|^2}$$

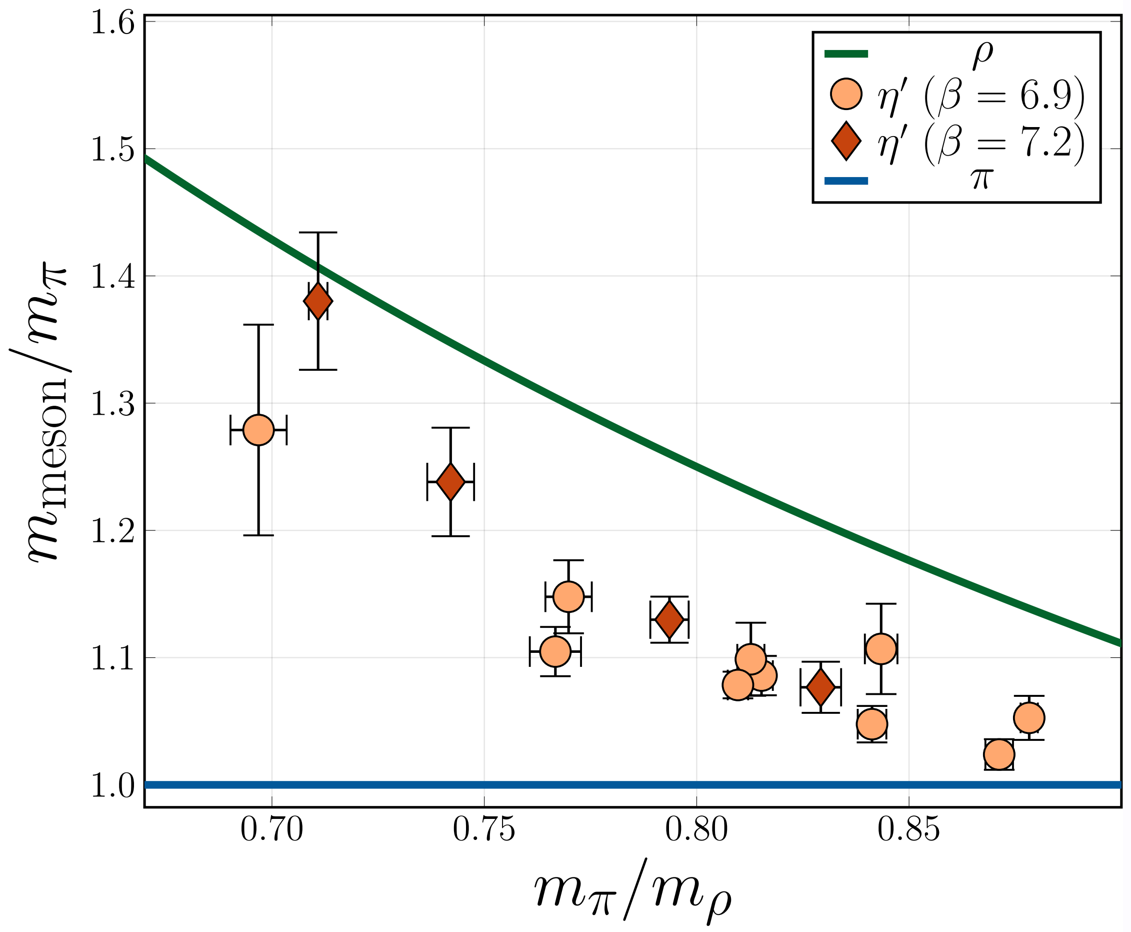


Non-singlet spectrum

Bennett et. al. [1909.12662]



The pseudoscalar and vector mesons are the lightest non-singlets.



[1] Bennett et. al. [2304.07191]

The pseudoscalar singlet η' is surprisingly light!

- Phenomenologically relevant:
  - $m_\rho > m_{\eta'}$ different from QCD
  - relevant low-energy dof
  - η' relevant for $\pi\pi$ scattering
  - more accessible channels for decays into SM

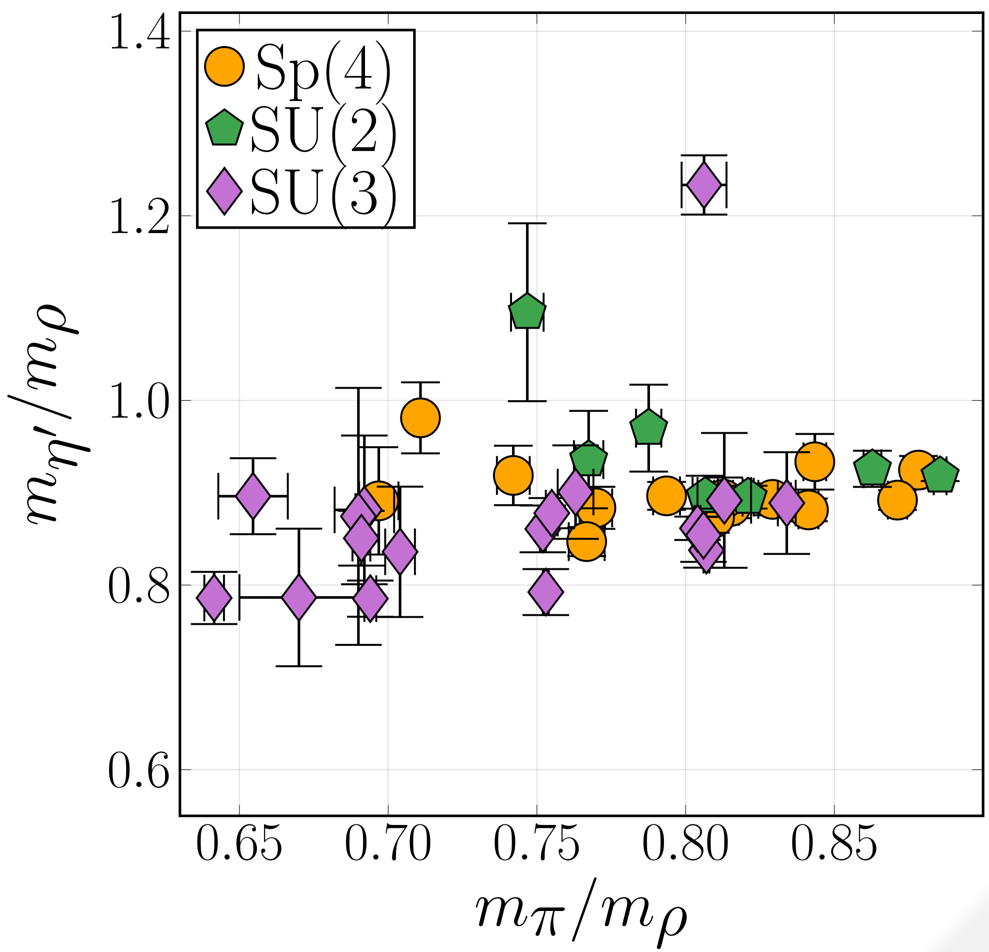
Interesting! Is this surprising?

Bennett et. al. [2304.07191]

Consider different theories:

- Large N_c: $m_{\eta'} - m_\pi \propto N_f / N_c$
  - $N_f = 2$ could be "small"
  - $N_c = 4$ could be "large"

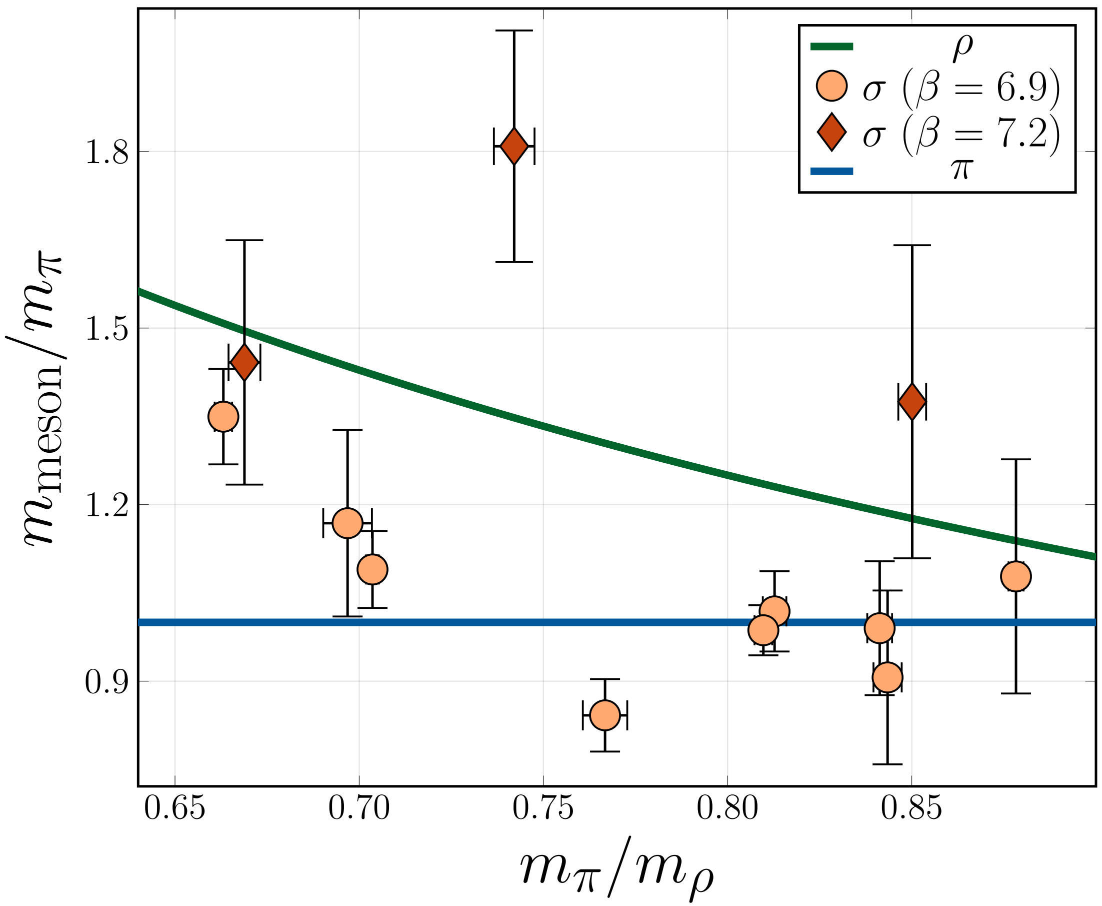
SU(2) and SU(3) comparison:



- Similarities:generic $N_f = 2$ feature?
- QCD: strong N_f dependence
- Differences may arise $m_\pi / m_\rho \to 0$

mass driven by flavour content!

[1] Bennett et. al. [2304.07191]



The scalar singlet σ

- Strong finite spacing effects!
- Potentially light state below $2m_\pi$ threshold
- Unclear systematics

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Overall, inconclusive results.
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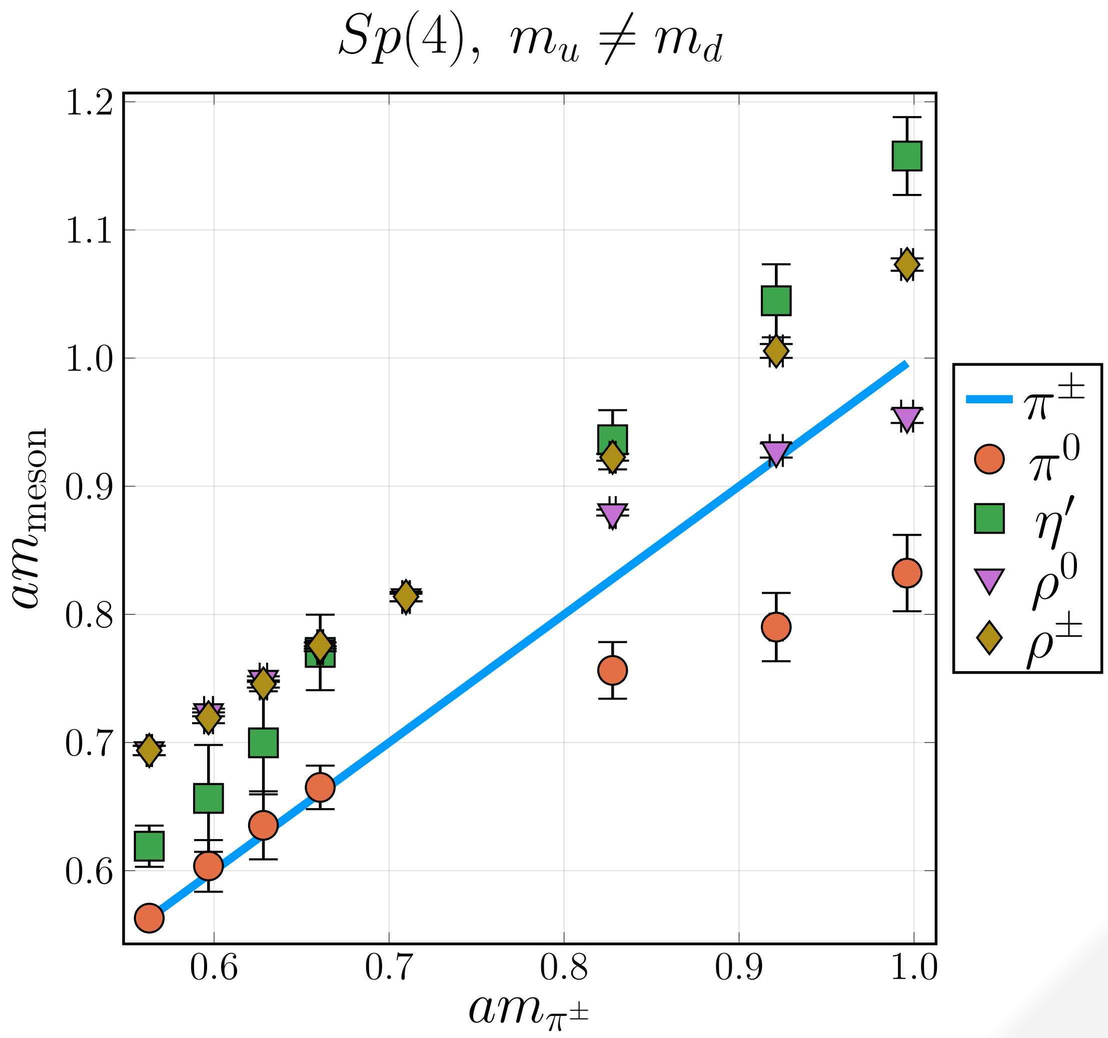
[1] Bennett et. al. [2304.07191]

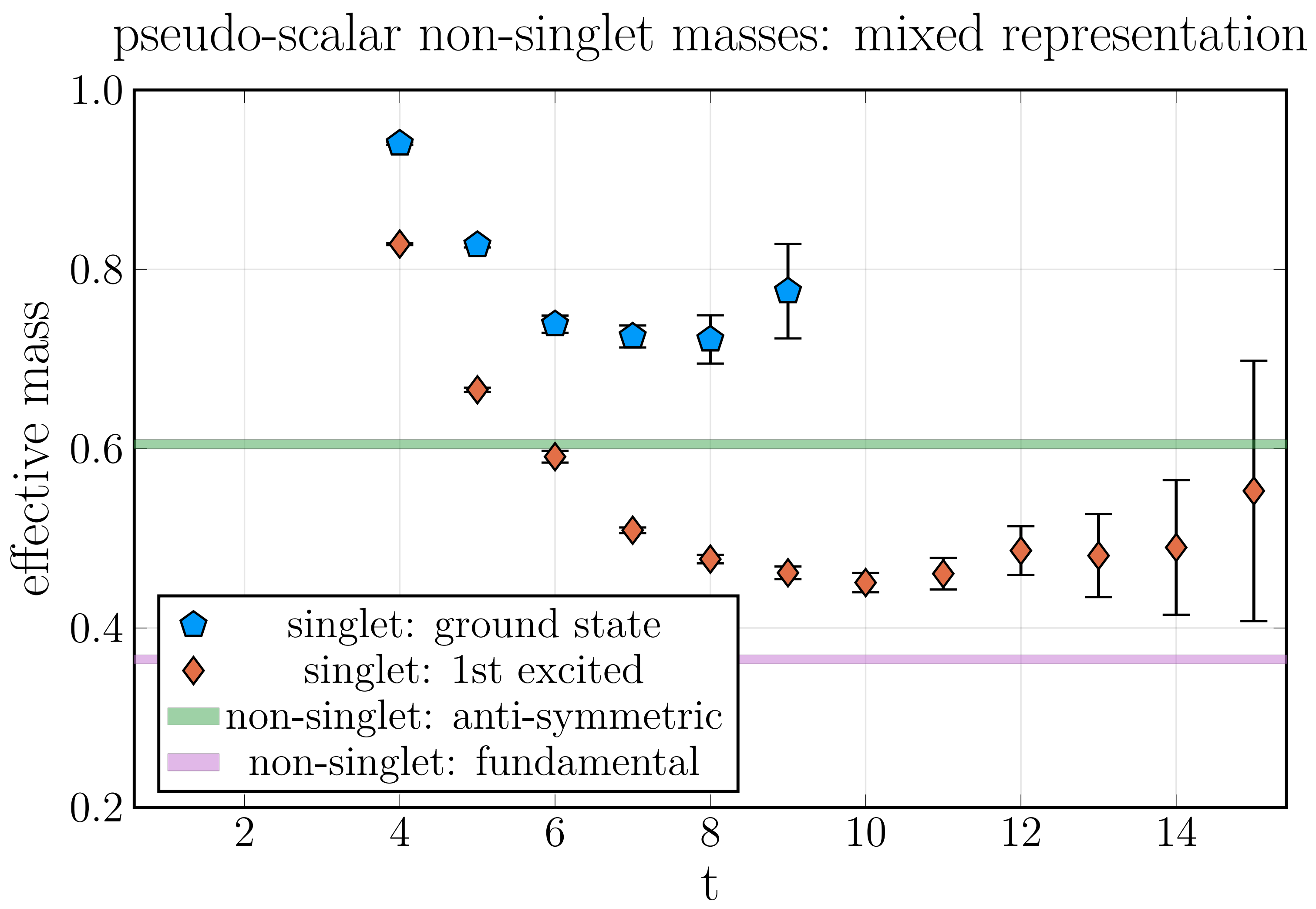
Kulkarni al. [2202.05191]

Non-degenerate fermions

- Different mass hierarchies
- Transition from a flavour-symmetric theory to a heavy-light system
- One light and one heavy pseudoscalar flavour-singlet



Pseudoscalar (non-)singlets: mixed representation



Kulkarni al. [2202.05191]

Consequences for Dark Matter

- Mass hierarchies: limit χPT validity
  - inclusion of other states than π required, e.g. η' and ρ
  - additional tests needed (fermions are too heavy)
- Light unprotected states η', π^0 allow decays into SM
  - no protection from symmetry

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Are these fermion masses phenomenologically relevant?
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Dark Matter Scattering on the Lattice

- Pions are in the 5-dimensional representations
- A two pion scattering is in one of three irreps

$$5 \times 5 = 14 \oplus 10 \oplus 1$$

- Corresponds to the usual QCD channels
  - $14 \Leftrightarrow$ isospin $I = 2$ in QCD, e.g. $\pi^+\pi^+$
  - $10 \Leftrightarrow$ isospin $I = 1$ in QCD, e.g. $\pi\pi \to \rho$
  - $0 \Leftrightarrow$ isospin $I = 0$ in QCD, e.g. $\pi\pi \to \sigma/f_0$

[1] M. Lüscher (Nucl.Phys.B 1991)

Scattering information from the lattice

- Scattering phase shift $\delta_0(p)$ from finite volume energy

$$\tan(\delta_0(q)) = \frac{\pi^{\frac{3}{2}} q}{\mathcal{Z}_{00}^{\vec{0}}(1, q^2)}, \quad q = p^* \frac{L}{2\pi}$$

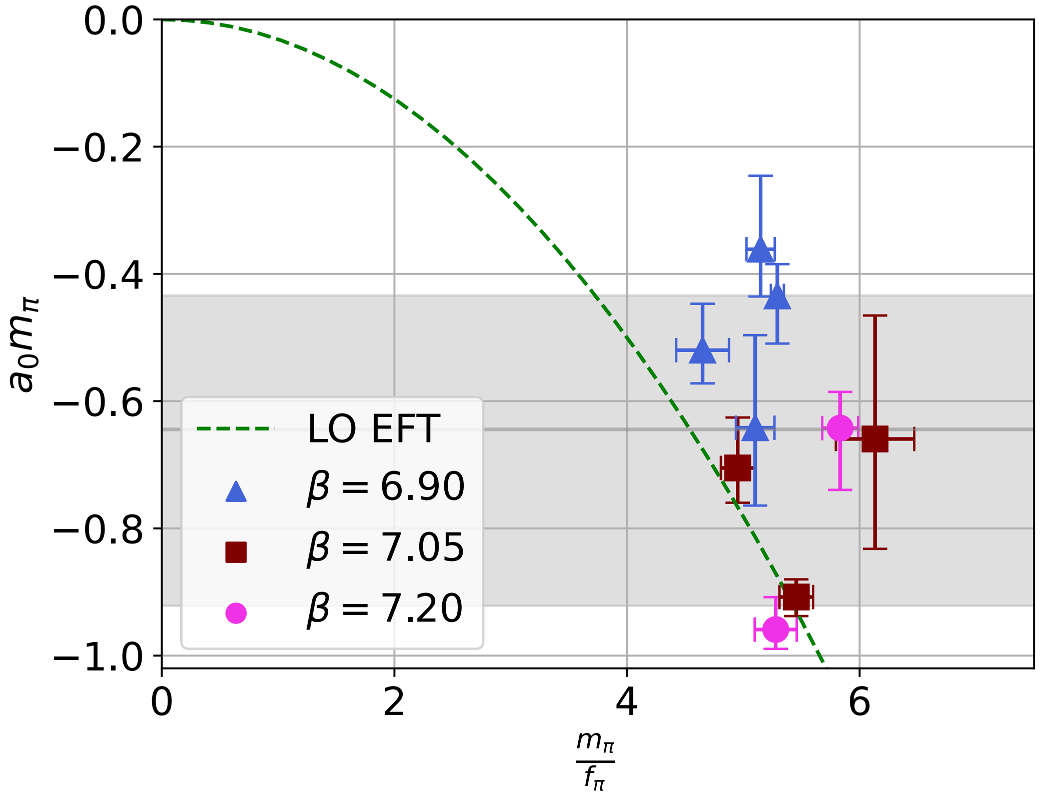
$$\cosh\left(\frac{E_{\pi\pi}}{2}\right) = \cosh(m_{\pi\pi}) + 2\sin\left(\frac{p^*}{2}\right)^2$$

- Low-velocity behaviour: Scattering length

$\Rightarrow$ relation between $\pi\pi$ energy $E_{\pi\pi}$ and m_π on a lattice [1]

$$\frac{\delta E_{\pi\pi}}{m_\pi} = \frac{4\pi m_\pi a_0}{(m_\pi L)^3}\left(1 + c_1 \frac{m_\pi a_0}{m_\pi L} + c_2 \left(\frac{m_\pi a_0}{m_\pi L}\right)^2\right)$$

[1] Dengler et.al. [2311.18549]



First investigation of isospin-2 scattering

- repulsive $\pi\pi$ interaction
- few lattice energy levels available $\Rightarrow$ systematics
- finite volume effects present
- roughly matches ChiPT

Summary

- Full light hadron spectrum of two-flavour $Sp(4)$
  - surprisingly light η', potentially light σ
  - input for EFTs: masses and decay constants
  - first determination of isospin-2 $\pi\pi$ scattering

Outlook

- Full scattering analysis of $2\pi \to 2\pi$ and $3\pi \to 2\pi$
- Better understanding of singlets and scattering states: Reduce lattice artefacts
- Spectroscopy closer to the chiral limit

Thank you