Lattice investigations of the chimera baryon spectrum in the Sp(4) gauge theory

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Outline

- Introduction:
  - ~~Sp(4) gauge theory: A Composite Higgs model~~
  - ~~Lattice method~~
  - Chimera baryon
- Results
  - Projections
  - Mass hierarchy of chimera baryons
  - Chiral EFT and AIC
- Summary and Outlook

Our choice of model

- Sp(4) gauge theory with 2F+3AS Dirac fermions

  ↓ 4F+6AS 2-component Weyl fermions

- Breaking pattern:

$$G/H = SU(4)\times SU(6) \,/\, Sp(4)\times SO(6)$$

Enhanced global symmetry due to the (pseudo-) reality

- $SU(4)/Sp(4)$ gives 5 goldstone bosons.
  - 4: SM Higgs doublet
  - 1: made heavy in model building

- SU(3) embedded in antisymmetric representation:

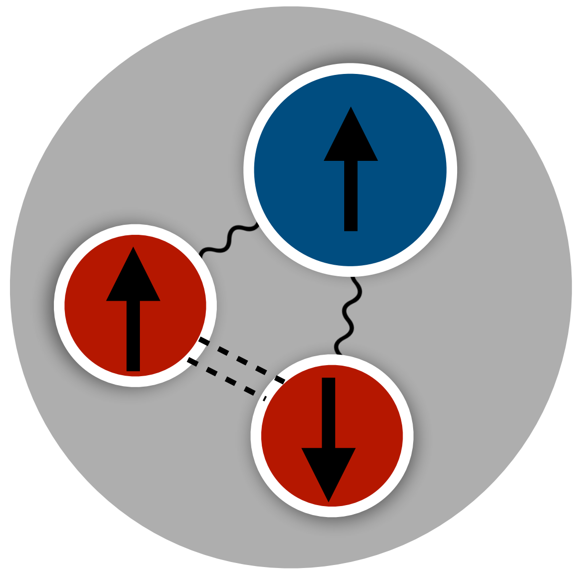
$$SU(6) \rightarrow SO(6) \supset SU(3)$$

↳ QCD colour SU(3)

Chimera Baryon

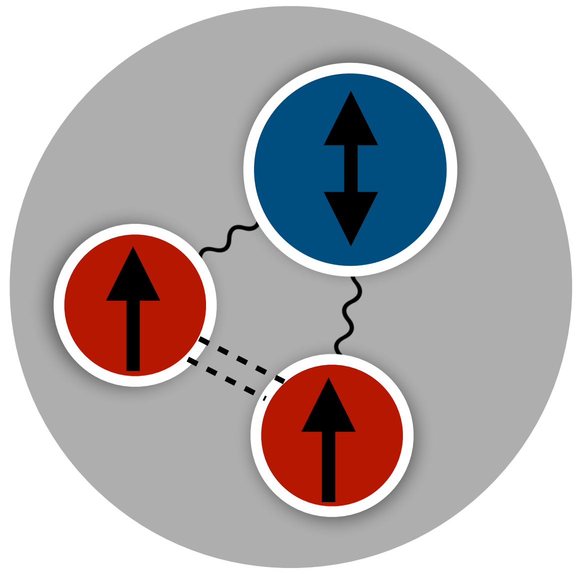
a, b, c: hypercolour
Ω: 4×4 symplectic matrix
J: spin
R: irreducible rep. of the fundamental sector

- Interpolating operators
  - Λ type: $\mathcal{O}_{\mathrm{CB},\gamma^5} = \left(\bar{\psi}^{1\,a}\gamma^5\psi^{2\,b}\right)\Omega_{bc}\chi^{k\,ca}$

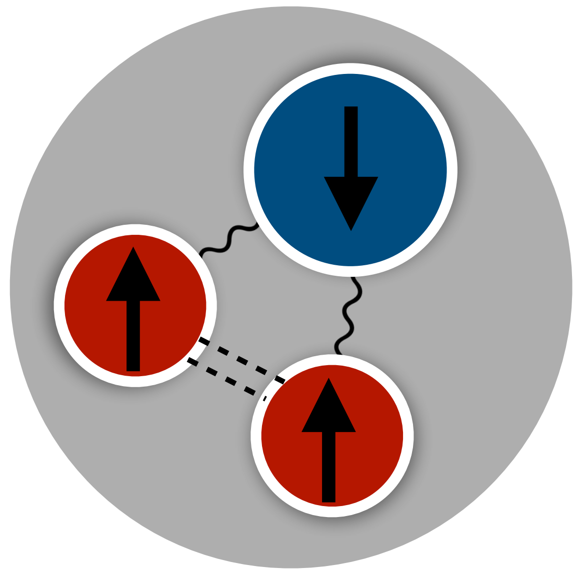


$(J, R) = (1/2, 5)$
*top partner

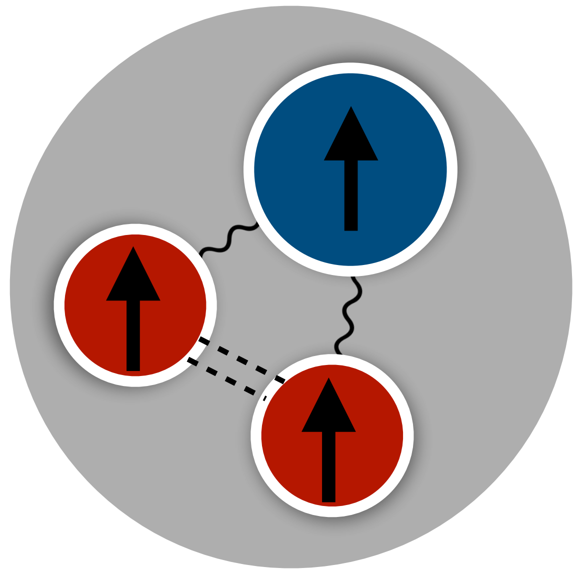
  - Σ type: $\mathcal{O}_{\mathrm{CB},\gamma^\mu} = \left(\bar{\psi}^{1\,a}\gamma^\mu\psi^{2\,b}\right)\Omega_{bc}\chi^{k\,ca}$



Spin projection



Σ: $(J, R) = (1/2, 10)$
*top partner



Σ^*: $(J, R) = (3/2, 10)$

Results

Quenched approximation

- Projections
- Mass hierarchy of chimera baryons
- Chiral EFT and AIC

$\hat{m}_{\rm PS}$: fundamental
$\hat{m}_{\rm ps}$: Antisymmetric

$\hat{a} \equiv a/\omega_0$ and $\hat{m} \equiv \omega_0 m$

Ensemble	β	$N_t \times N_s^3$	$\langle P \rangle$	ω_0/a
QB1	7.62	48×24^3	0.60192	1.448(3)
QB2	7.7	60×48^3	0.608795	1.6070(19)
QB3	7.85	60×48^3	0.620381	1.944(3)
QB4	8.0	60×48^3	0.630740	2.3149(12)
QB5	8.2	60×48^3	0.643228	2.8812(21)

Results

Projection-CB two-point function

- Interpolating operator

$$\mathcal{O}^{\gamma}_{\rm CB}(x) \equiv \left(Q^{i\,a}{}_{\alpha}(x) \Gamma^{1\,\alpha\beta} Q^{j\,b}{}_{\beta}(x) \right) \Omega_{ad}\Omega_{bc}\Gamma^{2\,\delta\gamma}\Psi^{k\,cd}{}_{\gamma}(x)$$

- two-point function

$$C^{\gamma\gamma'}(t) \equiv \sum_{\vec{x}} \langle \mathcal{O}^{\gamma}_{\rm CB}(x) \overline{\mathcal{O}^{\gamma'}_{\rm CB}}(0) \rangle$$

$$= -\sum_{\vec{x}} \left(\Gamma^2 S^{k\,cd}_{\Psi}{}_{c'd'}(x,0) \overline{\Gamma^2} \right)_{\gamma\gamma'} \Omega_{cb}\Omega^{b'c'}\Omega_{ad}\Omega^{d'a'}$$

$$\times \mathrm{Tr}\left[\Gamma^1 S^{b}_{Q}{}_{b'}(x,0) \overline{\Gamma^1} S^{a}_{Q}{}_{a'}(x,0) \right]$$

At large Euclidean time

$$\rightarrow P_e \left[c_e e^{-m_e t} + c_o e^{-m_o(T-t)} \right] - P_o \left[c_o e^{-m_o t} + c_e e^{-m_e(T-t)} \right]$$

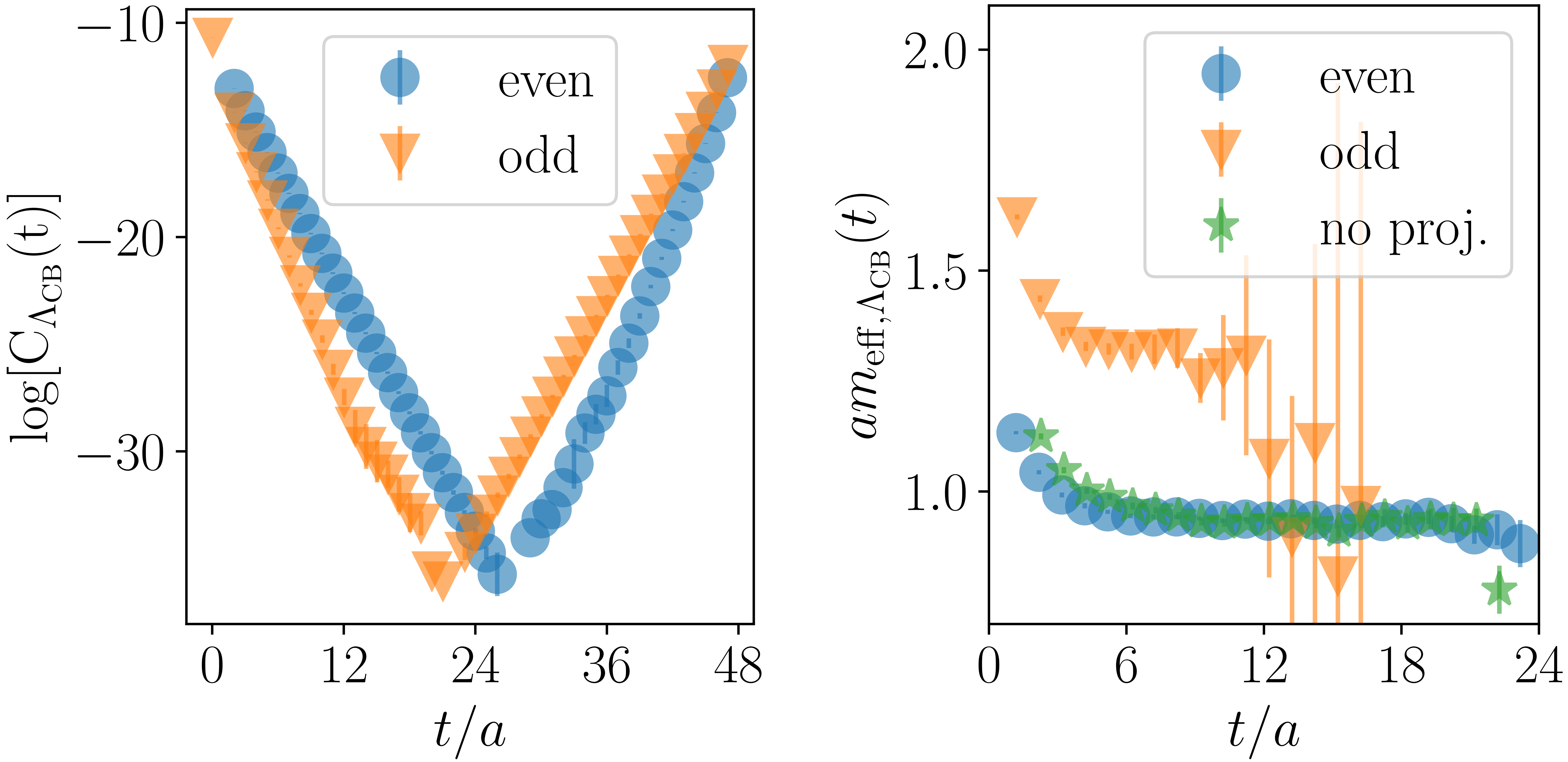
$$P_e \equiv \frac{1}{2}(1+\gamma^0) \text{ and } P_o \equiv \frac{1}{2}(1-\gamma^0)$$

Results

Projection-Parity

$$C_{\mathrm{CB}}(t) \to P_e\left[c_e e^{-m_e t} + c_o e^{-m_o(T-t)}\right] - P_o\left[c_o e^{-m_o t} + c_e e^{-m_e(T-t)}\right]$$

- The log plot of the chimera baryon correlators (left) and their effective mass plot (right) with the parity projection.



Chimera Baryon

- Spin projector for Σ-type baryon:

$$\left(P^{3/2}\right)^{ij} = \delta^{ij} - \frac{1}{3}\gamma^i\gamma^j$$

$$\left(P^{1/2}\right)^{ij} = \frac{1}{3}\gamma^i\gamma^j$$

- Two-point function

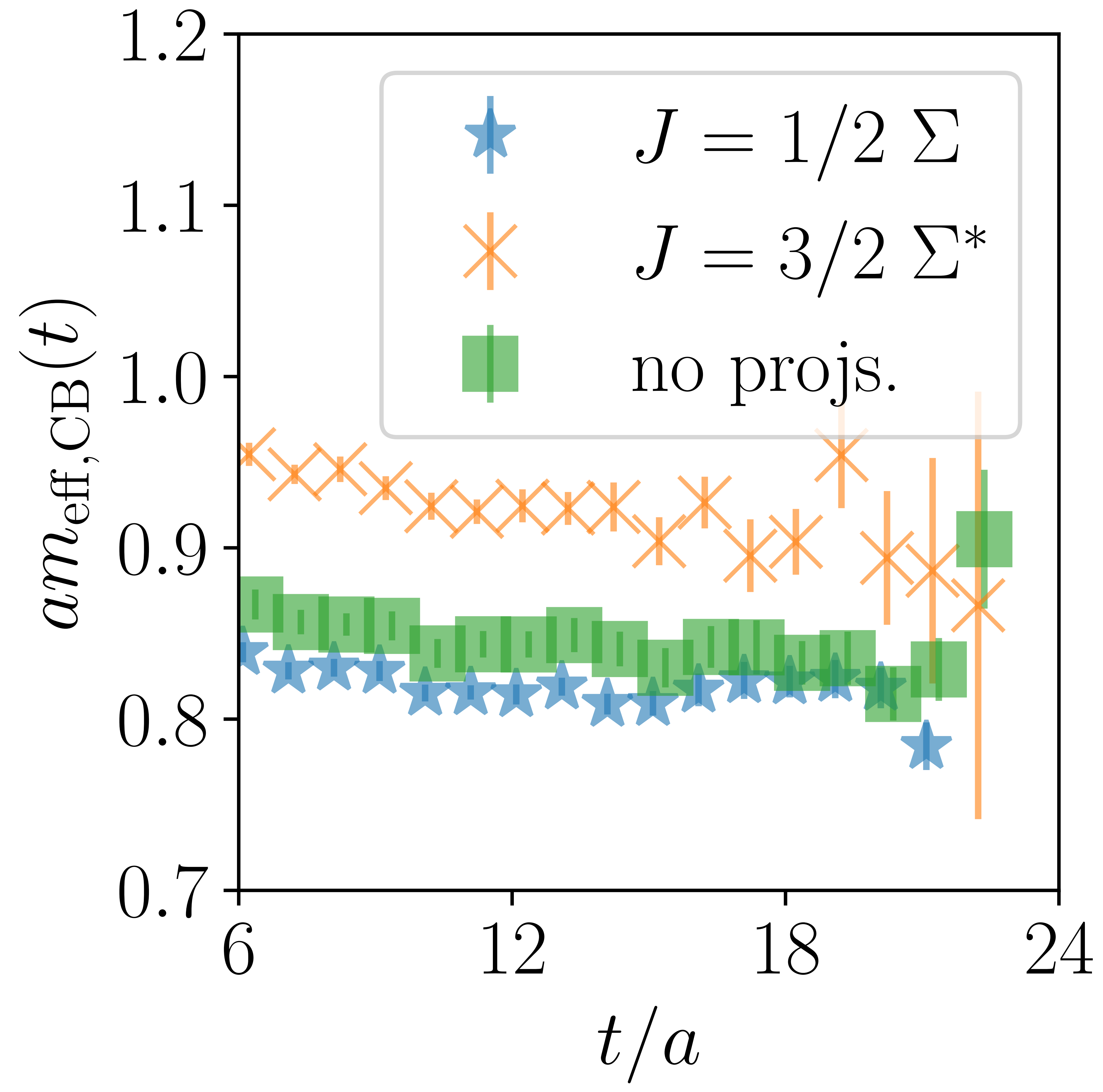
$$C_{ij}(t) = \sum_{\vec{x}} \left\langle \mathcal{O}^i{}_{\mathrm{CB}}(x)\bar{\mathcal{O}}^j{}_{\mathrm{CB}}(0) \right\rangle \text{ with } \mathcal{O}^i_{CB} = \left(\bar{\psi}\gamma^i\psi\right)\chi$$

$$\rightarrow C_\Sigma^{1/2}(t) = \mathrm{Tr}\left[\left(P^{1/2}\right)^{ij} C_{jk}(t)\right]$$

Results

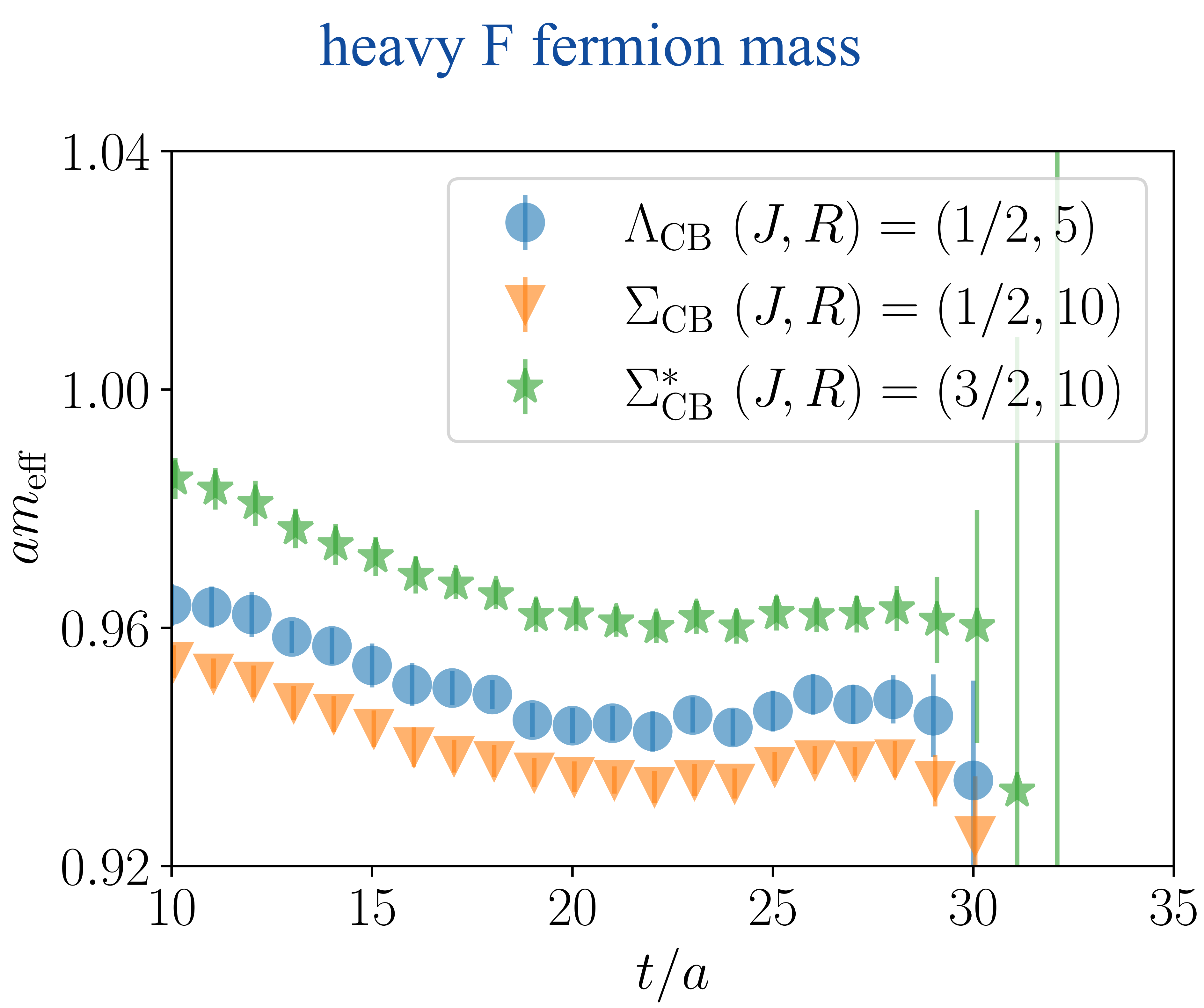
Projection-Spin

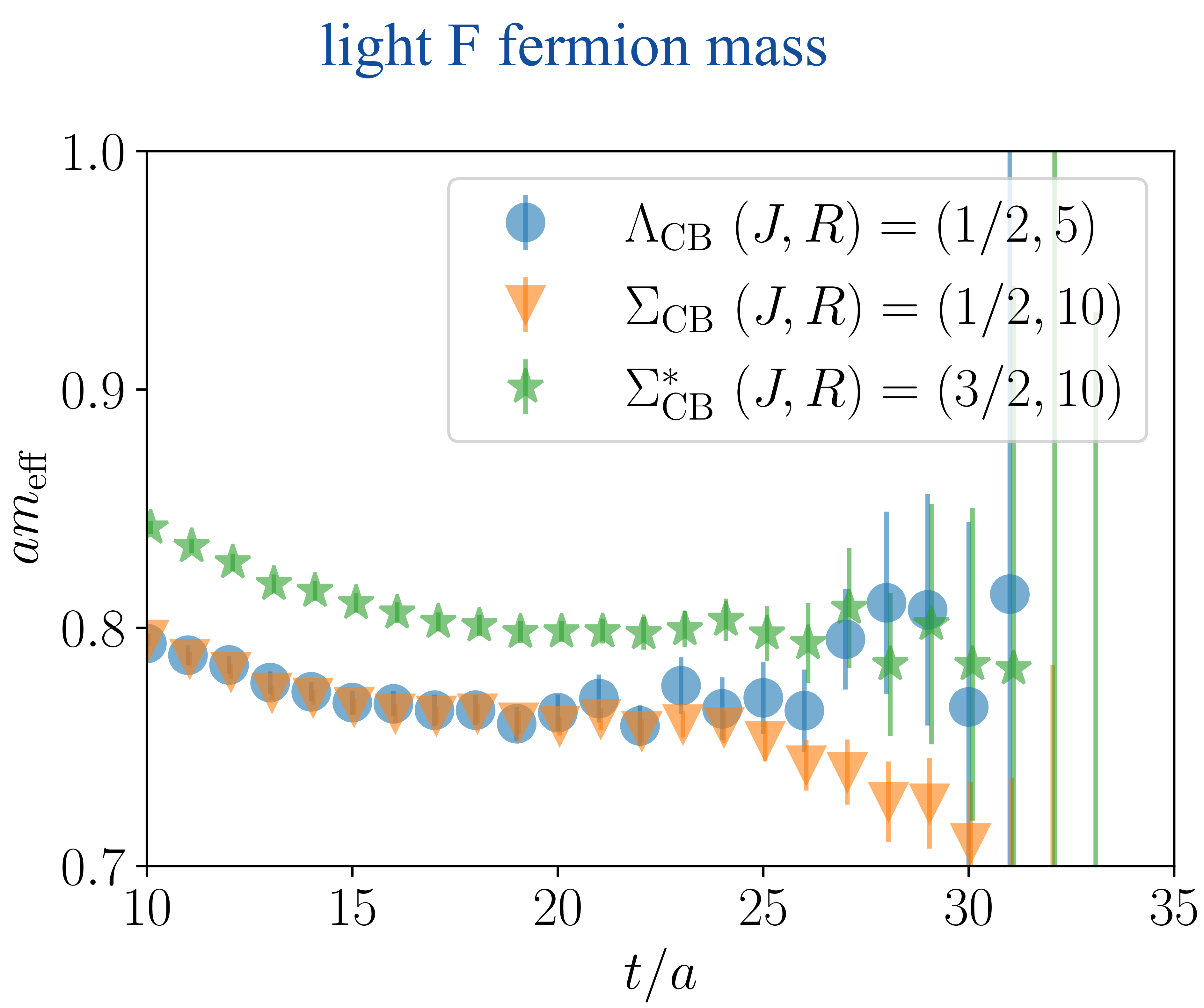
- Comparison of effective mass plot between two spin projected states and the state without spin projection.



Results

Mass hierarchy

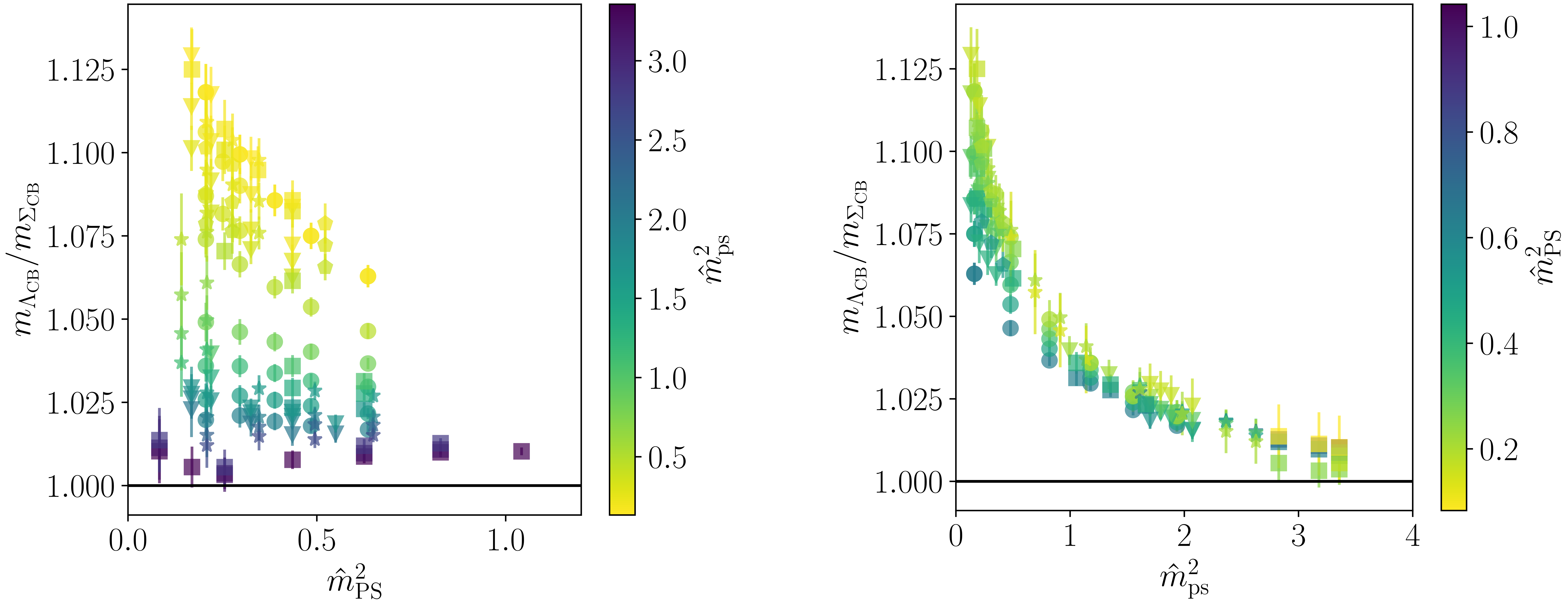




Effective mass plot of chimera baryons calculated with different F fermion masses, at fixed AS fermion mass. The lattice size is 60×48^3 with $\beta = 8.0$.

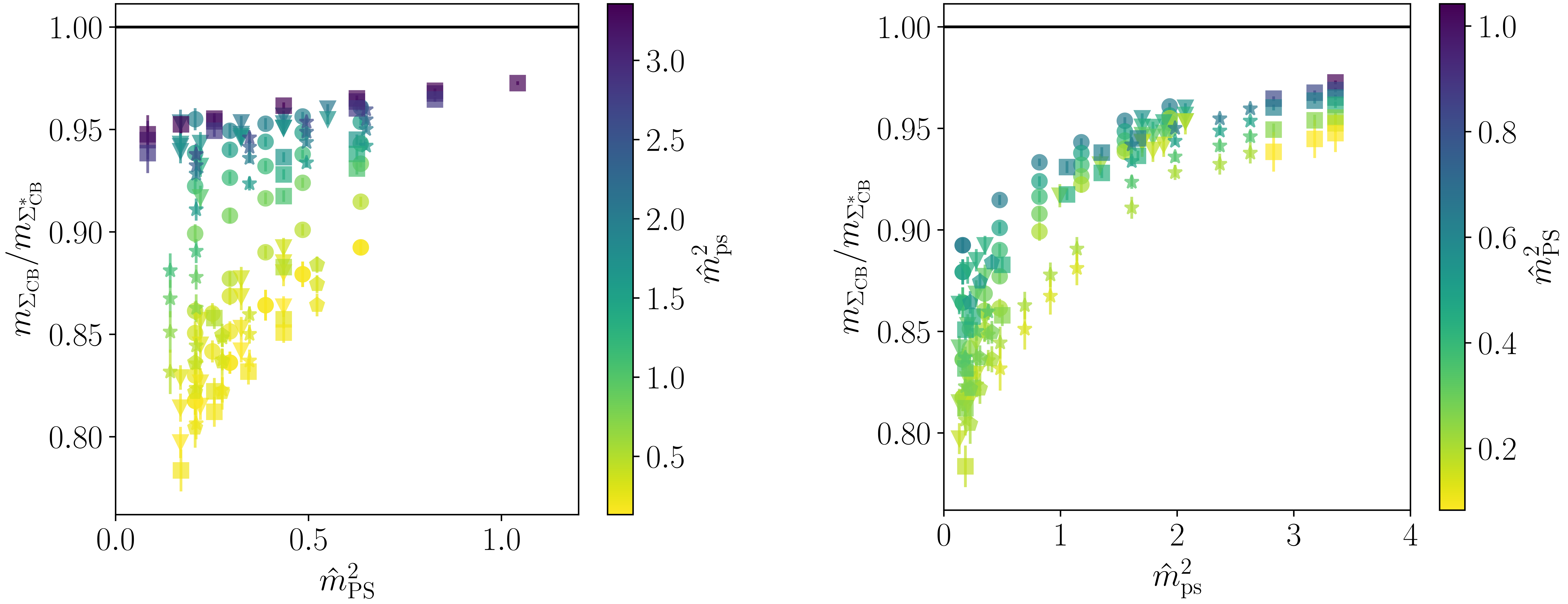
Results

Mass hierarchy



Results

Mass hierarchy



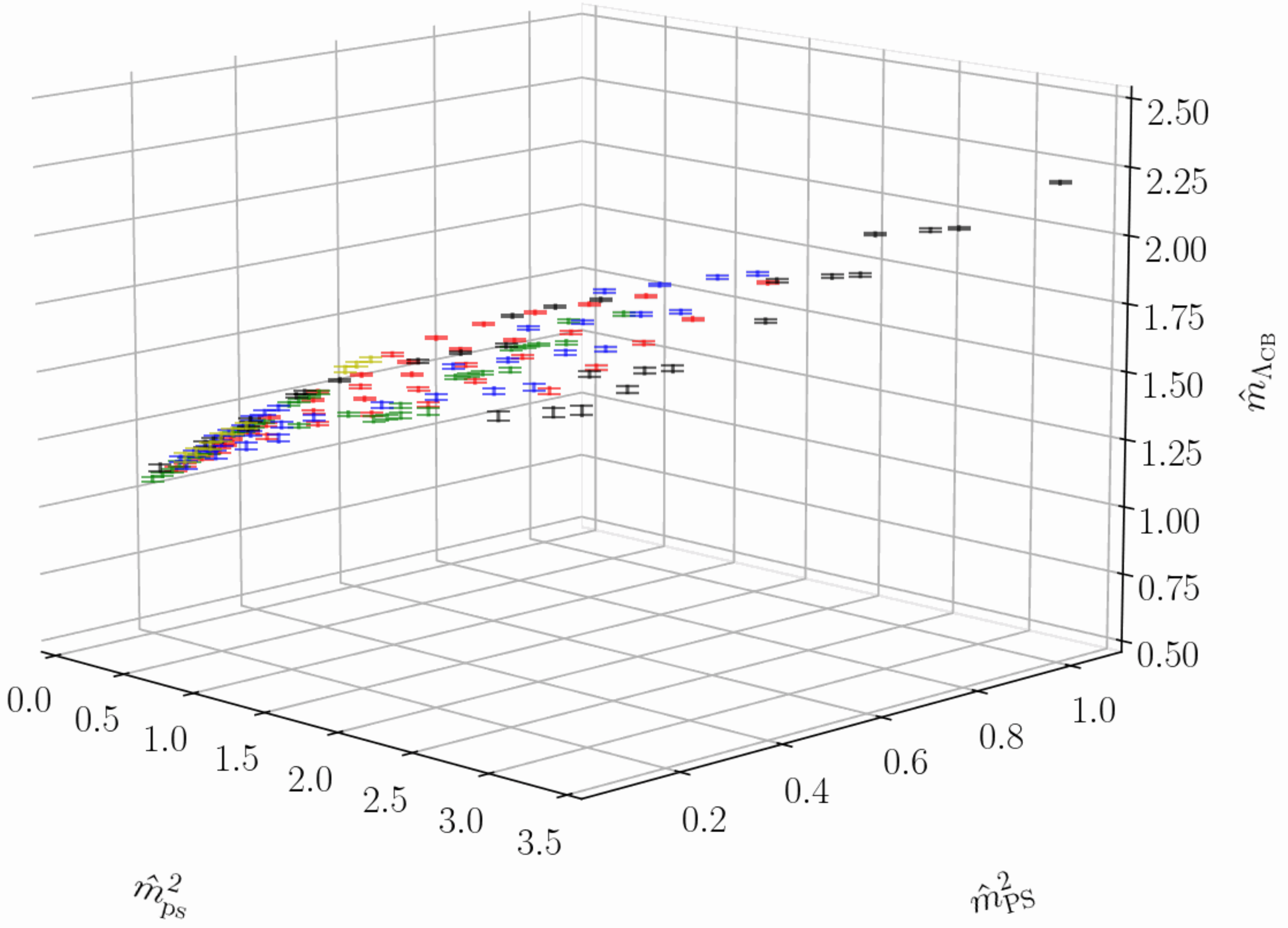
Results

Fitting

- Apply tree-level baryon chiral perturbation theory

$$m_{\mathrm{CB}} = m_{CB}^{\chi} + F_2 \hat{m}_{\mathrm{PS}}^2 + A_2 \hat{m}_{\mathrm{ps}}{}^2 + L_1 \hat{a}$$
$$+ F_3 \hat{m}_{\mathrm{PS}}^3 + A_3 \hat{m}_{\mathrm{ps}}^3 + L_{2F} \hat{a} \hat{m}_{\mathrm{PS}}^2 + L_{2A} \hat{a} \hat{m}_{\mathrm{ps}}^2$$
$$+ F_4 \hat{m}_{\mathrm{PS}}^4 + A_4 \hat{m}_{\mathrm{ps}}^4 + C_4 \hat{m}_{\mathrm{PS}}^2 \hat{m}_{\mathrm{ps}}^2$$

Returning large $\chi^2/N_{\mathrm{d.o.f.}}$



Results

Optimal search

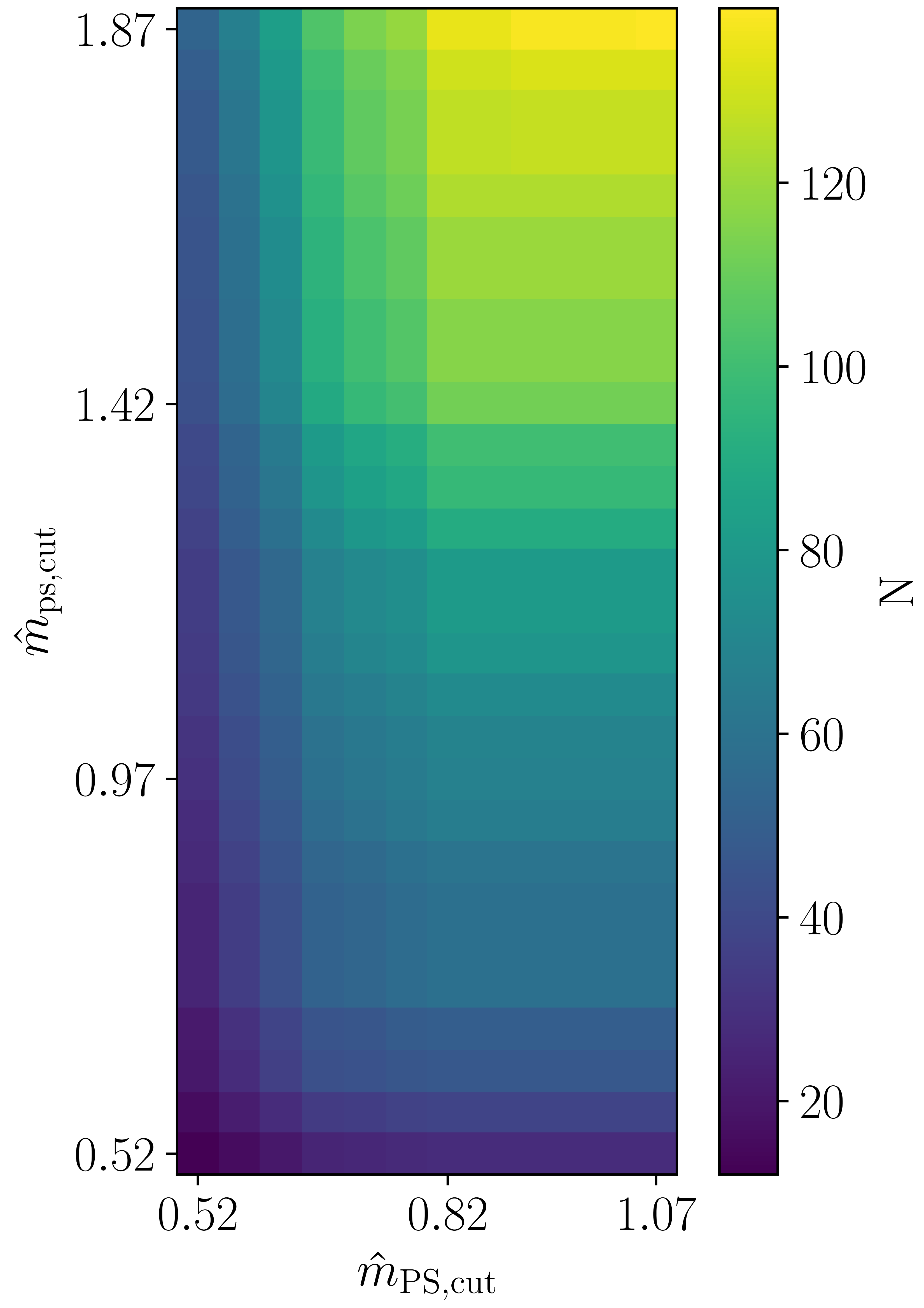
- Try including different order of corrections
- Calculate AICs for each data set, and scan through all the possible cuts:
  - ➡ Fix the cut value for $\hat{m}_{\rm PS}$ and vary $\hat{m}_{\rm ps}$
  - ➡ Increase the fixed value of $\hat{m}_{\rm PS}$
- Goodness of a fit: Akaike information criterion (AIC)

William I. Jay and Ethan T. Neil [2008.01069]

$$\mathrm{AIC}(\mathrm{M}, N_{\rm cut}) \equiv \chi^2 + 2k + 2N_{\rm cut}$$

- Probability weight

$$W(\mathrm{M}, N_{\rm cut}) = \frac{1}{\mathcal{N}} \exp\left[-\frac{1}{2}\mathrm{AIC}(\mathrm{M}, N_{\rm cut})\right]$$



Results

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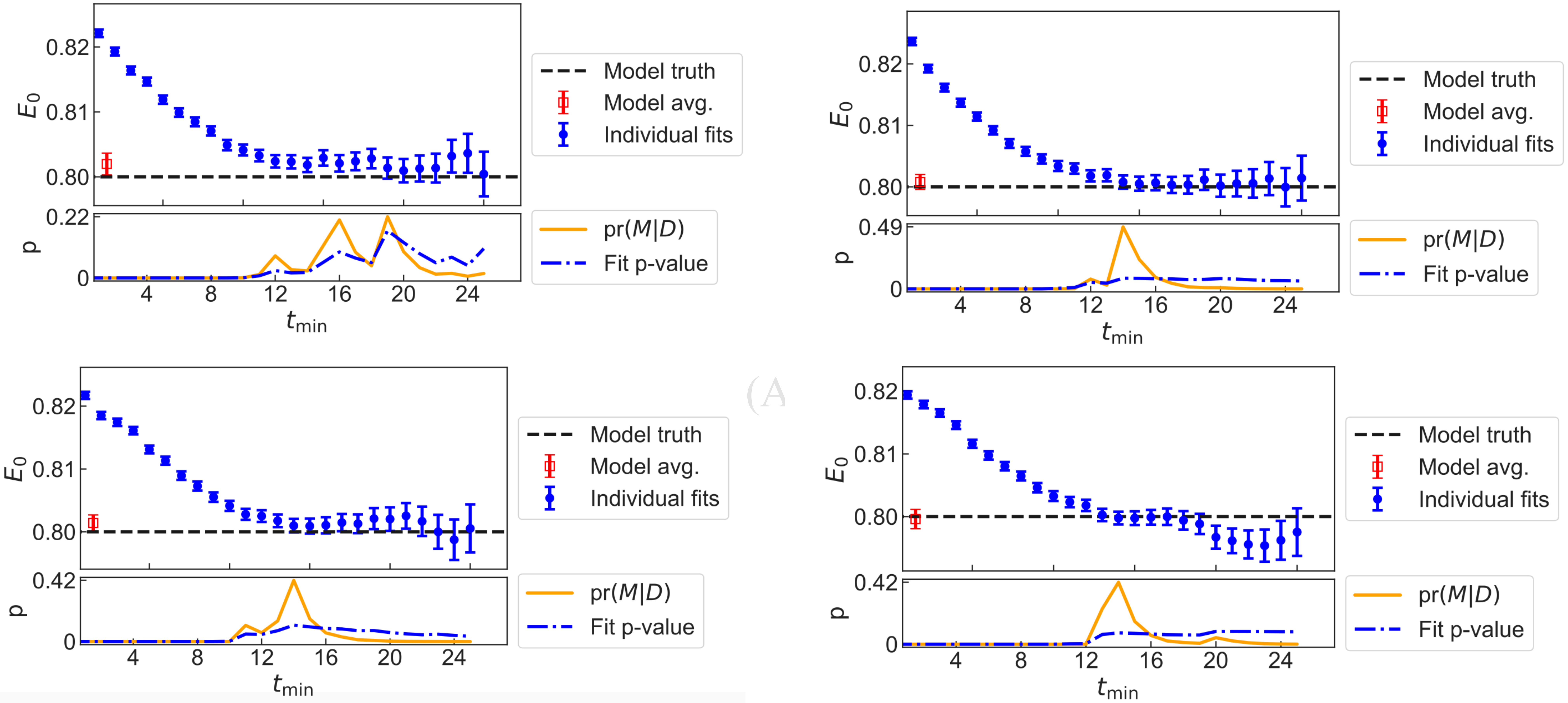

Results

Optimal search

- Try including different order of corrections
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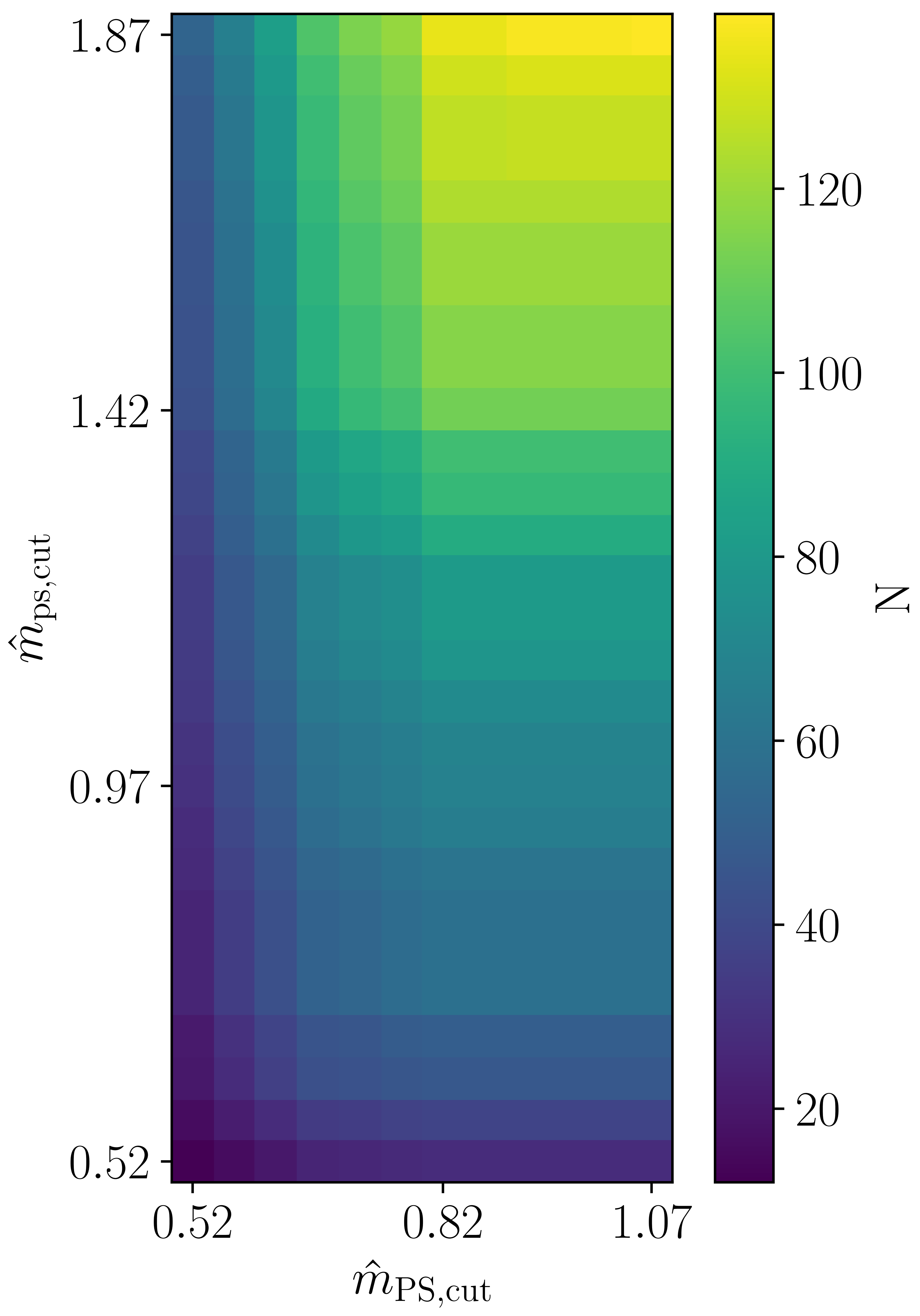


Results

Fitting

- Apply tree-level baryon chiral perturbation theory

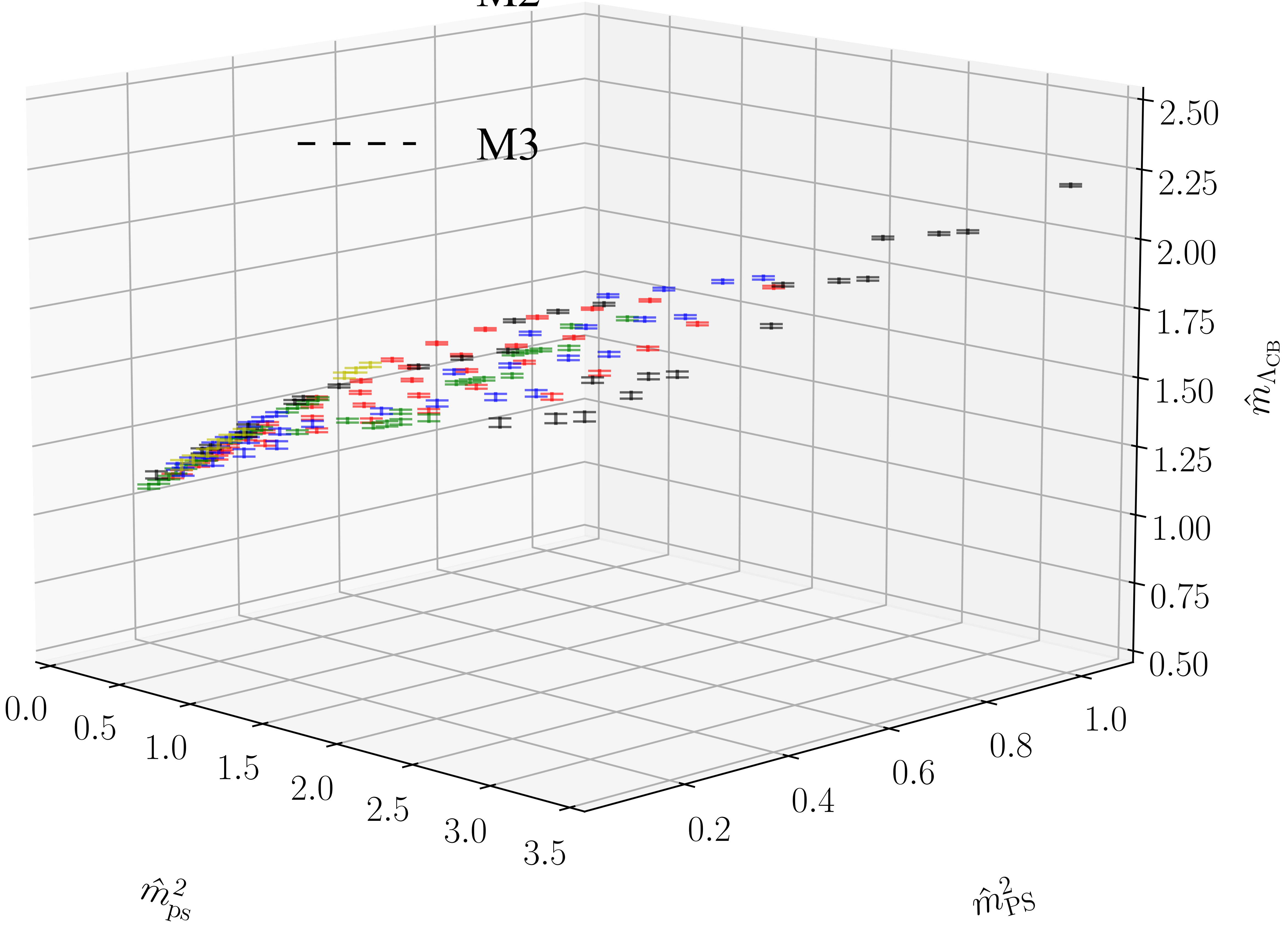
$$m_{\rm CB} = m_{CB}^{\chi} + F_2\hat{m}_{\rm PS}^2 + A_2\hat{m}_{\rm ps}{}^2 + L_1\hat{a}$$
$$+ F_3\hat{m}_{\rm PS}^3 + A_3\hat{m}_{\rm ps}^3 + L_{2F}\hat{a}\hat{m}_{\rm PS}^2 + L_{2A}\hat{a}\hat{m}_{\rm ps}^2$$
$$+ F_4\hat{m}_{\rm PS}^4 + A_4\hat{m}_{\rm ps}^4 + C_4\hat{m}_{\rm PS}^2\hat{m}_{\rm ps}^2$$

MF4 MA4 MC4

M2

M3

Still returning large $\chi^2/N_{\rm d.o.f.}$



Results

Fittings of Λ_{CB}

- Apply tree level baryon chiral perturbation theory

M2 $$m_{CB} = m^{\chi}_{CB} + F_2 \hat{m}^2_{PS} + A_2 \hat{m}_{ps}{}^2 + L_1 \hat{a}$$

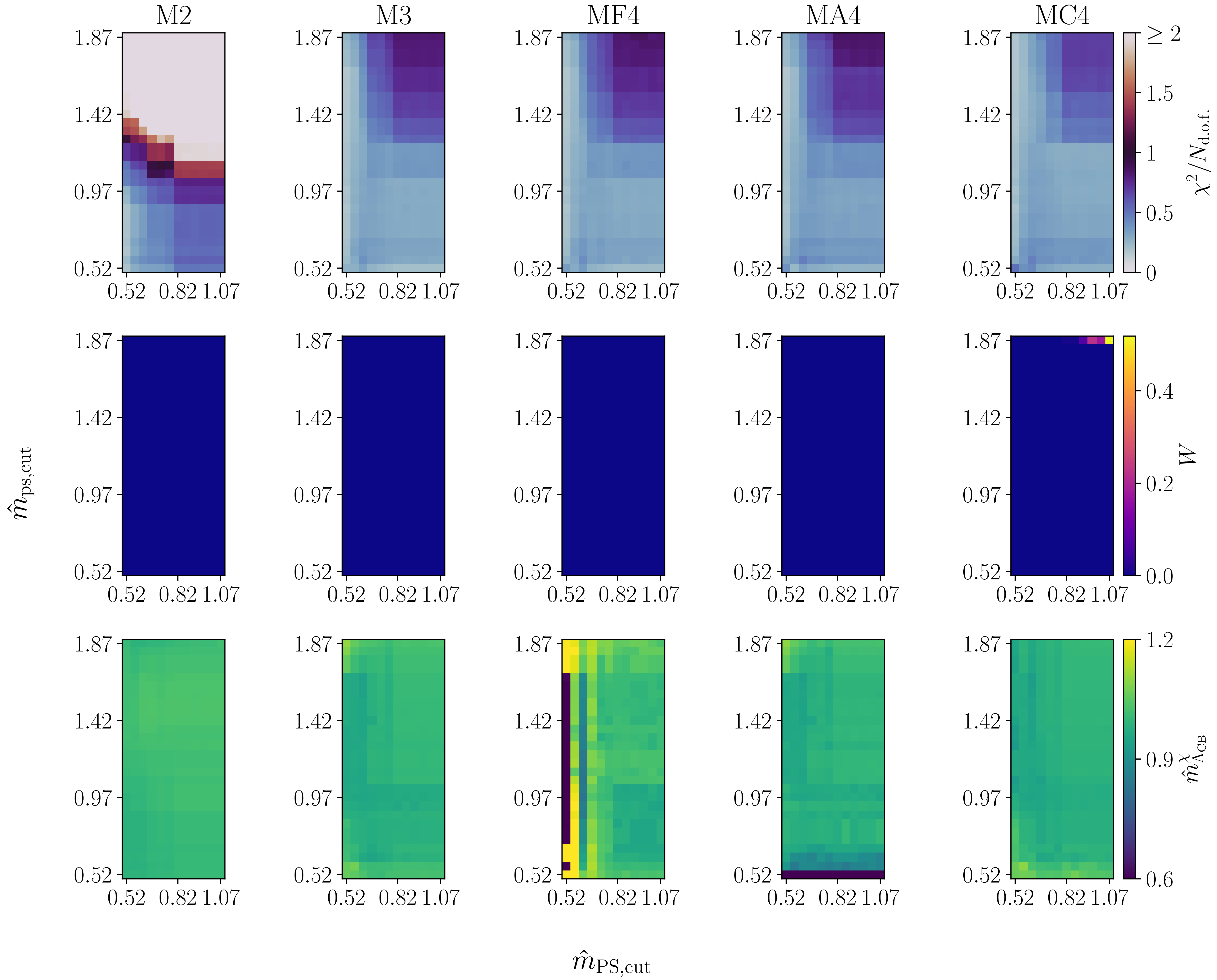
M3 $$+F_3 \hat{m}^3_{PS} + A_3 \hat{m}^3_{ps} + L_{2F} \hat{a} \hat{m}^2_{PS} + L_{2A} \hat{a} \hat{m}^2_{ps}$$

$$+F_4 \hat{m}^4_{PS} + A_4 \hat{m}^4_{ps} + C_4 \hat{m}^2_{PS} \hat{m}^2_{ps}$$

MF4 MA4 M2C

- probability weight

$$W(\mathrm{M}, N_{\mathrm{cut}}) = \frac{1}{\mathcal{N}} \exp\left[-\frac{1}{2}\mathrm{AIC}(\mathrm{M}, N_{\mathrm{cut}})\right]$$



Results

Fittings of Σ_{CB}

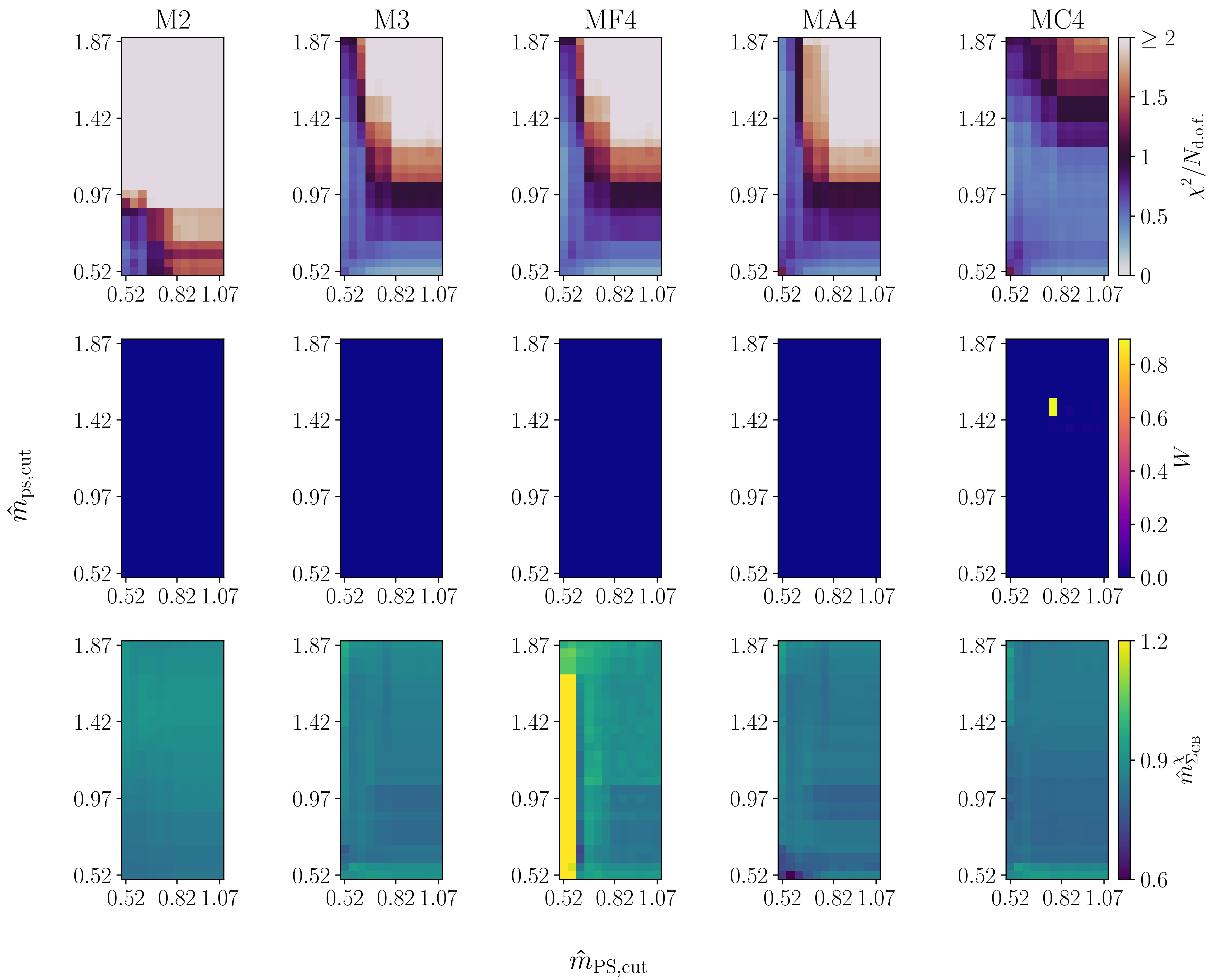
- Apply tree level baryon chiral perturbation theory

M2 $$m_{\text{CB}} = m^{\chi}_{CB} + F_2\hat{m}^2_{\text{PS}} + A_2\hat{m}_{\text{ps}}{}^2 + L_1\hat{a}$$

M3 $$+F_3\hat{m}^3_{\text{PS}} + A_3\hat{m}^3_{\text{ps}} + L_{2F}\hat{a}\hat{m}^2_{\text{PS}} + L_{2A}\hat{a}\hat{m}^2_{\text{ps}}$$

$$+F_4\hat{m}^4_{\text{PS}} + A_4\hat{m}^4_{\text{ps}} + C_4\hat{m}^2_{\text{PS}}\hat{m}^2_{\text{ps}}$$

MF4 MA4 M2C



Results

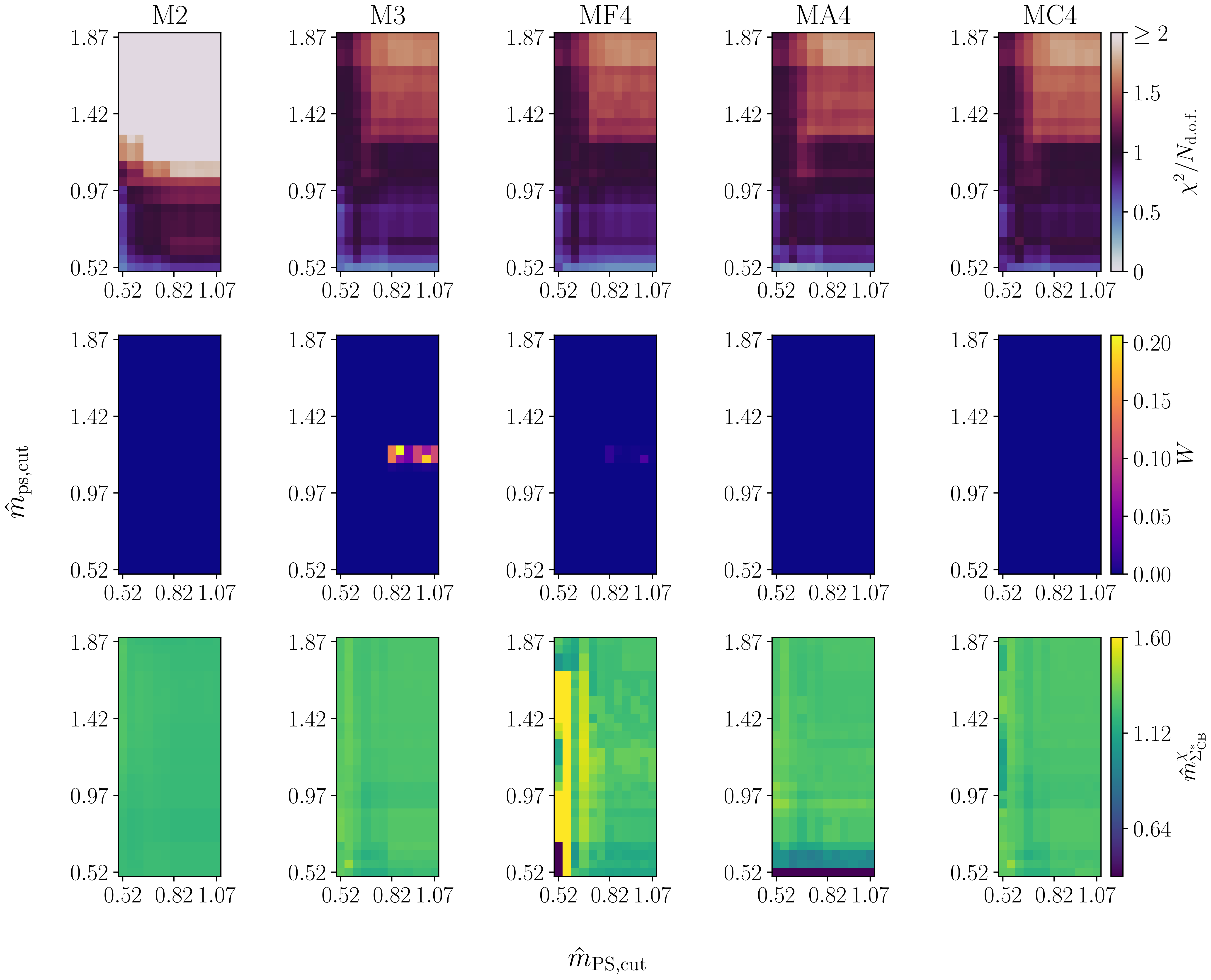
Fittings of $\Sigma^*_{\rm CB}$

- Apply tree level baryon chiral perturbation theory

M2 $$m_{\rm CB} = m^{\chi}_{CB} + F_2\hat{m}_{\rm PS}^2 + A_2\hat{m}_{\rm ps}{}^2 + L_1\hat{a}$$

M3 $$+F_3\hat{m}_{\rm PS}^3 + A_3\hat{m}_{\rm ps}^3 + L_{2F}\hat{a}\hat{m}_{\rm PS}^2 + L_{2A}\hat{a}\hat{m}_{\rm ps}^2$$

$$\underbrace{+F_4\hat{m}_{\rm PS}^4}_{\rm MF4}\ \underbrace{+A_4\hat{m}_{\rm ps}^4}_{\rm MA4}\ \underbrace{+C_4\hat{m}_{\rm PS}^2\hat{m}_{\rm ps}^2}_{\rm M2C}$$



Results

Cross check

- Apply tree-level baryon chiral perturbation theory

$$
\begin{aligned}
m_{\mathrm{CB}} = m_{CB}^{\chi} &+ F_2 \hat{m}_{\mathrm{PS}}^2 + A_2 \hat{m}_{\mathrm{ps}}{}^2 + L_1 \hat{a} \\
&+ F_3 \hat{m}_{\mathrm{PS}}^3 + A_3 \hat{m}_{\mathrm{ps}}^3 + L_{2F} \hat{a} \hat{m}_{\mathrm{PS}}^2 + L_{2A} \hat{a} \hat{m}_{\mathrm{ps}}^2 \\
&+ F_4 \hat{m}_{\mathrm{PS}}^4 + A_4 \hat{m}_{\mathrm{ps}}^4 + C_4 \hat{m}_{\mathrm{PS}}^2 \hat{m}_{\mathrm{ps}}^2
\end{aligned}
$$

Results

Cross check

- At a fixed $\hat{m}_{\mathrm{PS}}^{as}$, the fitting function becomes

$$
\begin{aligned}
m_{\mathrm{CB}} = m_{CB}^{\chi} &+ F_2 \hat{m}_{\mathrm{PS}}^2 + A_2 \hat{m}_{\mathrm{ps}}^2 + L_1 \hat{a} \\
&+ F_3 \hat{m}_{\mathrm{PS}}^3 + A_3 \hat{m}_{\mathrm{ps}}^3 + L_{2F} \hat{a} \hat{m}_{\mathrm{PS}}^2 + L_{2A} \hat{a} \hat{m}_{\mathrm{ps}}^2 \\
&+ F_4 \hat{m}_{\mathrm{PS}}^4 + A_4 \hat{m}_{\mathrm{ps}}^4 + C_4 \hat{m}_{\mathrm{PS}}^2 \hat{m}_{\mathrm{ps}}^2
\end{aligned}
$$

Results

Cross check

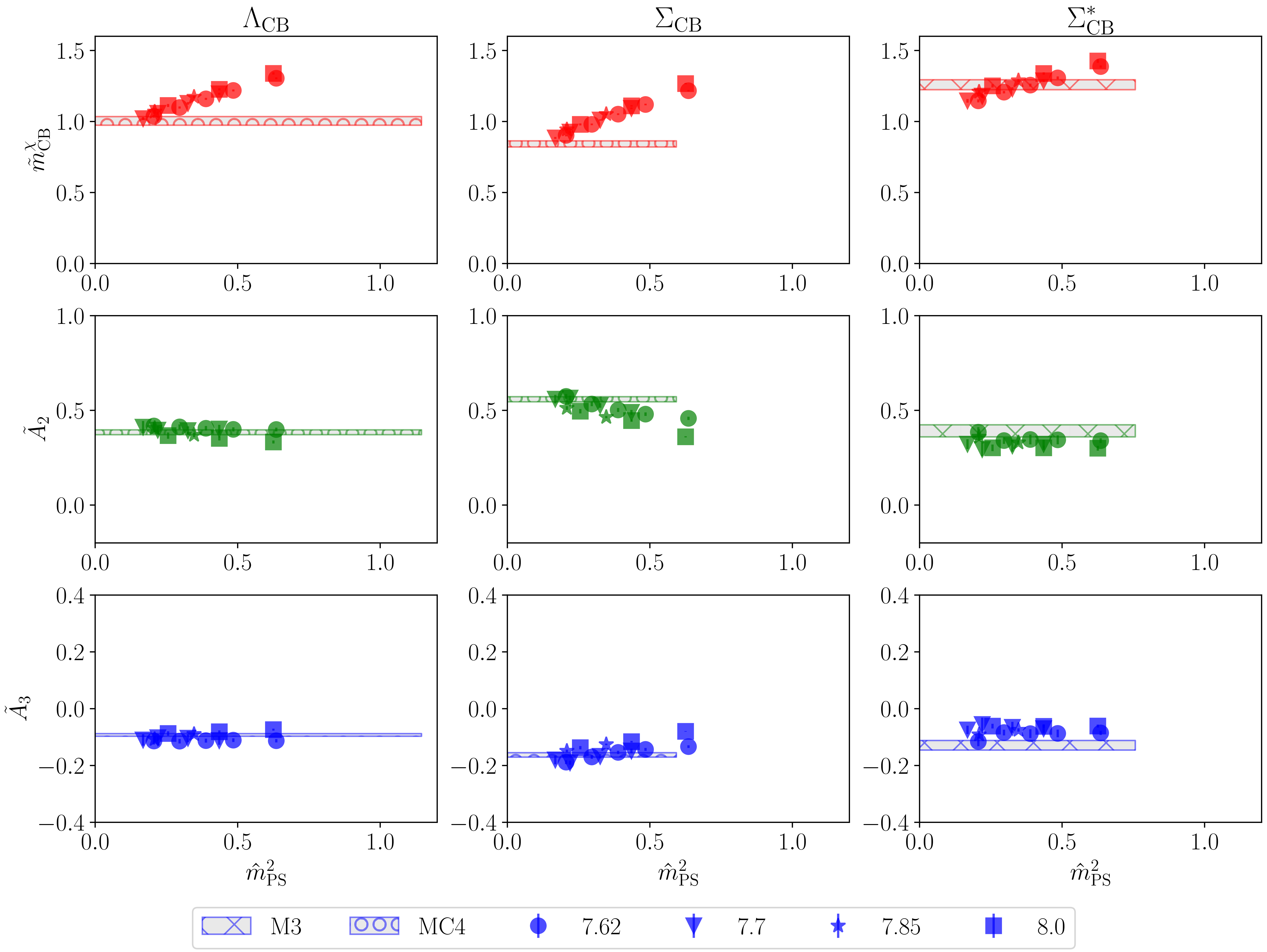
- At a fixed $\hat{m}_{\text{PS}}^{as}$, the fitting function becomes

$$
\begin{aligned}
m_{\text{CB}} = m_{CB}^{\chi} &+ A_2 {\hat{m}_{\text{PS}}^{as}}^2 + L_1 \hat{a} + A_3 {\hat{m}_{\text{PS}}^{as}}^3 + L_{2A} \hat{a} {\hat{m}_{\text{PS}}^{as}}^2 + A_4 {\hat{m}_{\text{PS}}^{as}}^4 \\
&+ F_2 {\hat{m}_{\text{PS}}^{f}}^2 + C_4 {\hat{m}_{\text{PS}}^{f}}^2 {\hat{m}_{\text{PS}}^{as}}^2 + L_{2F} \hat{a} {\hat{m}_{\text{PS}}^{f}}^2 \\
&+ F_3 {\hat{m}_{\text{PS}}^{f}}^3 + F_4 {\hat{m}_{\text{PS}}^{f}}^4
\end{aligned}
$$

$$
\Rightarrow \tilde{m}_{CB}^{\chi}(\hat{m}_{\text{ps}}, A, L, \hat{a}) + \tilde{F}_2(\hat{m}_{\text{ps}}, C, L, \hat{a}) \hat{m}_{\text{PS}}^2 + \tilde{F}_3 \hat{m}_{\text{PS}}^3 + F_4 {\hat{m}_{\text{PS}}^{f}}^4
$$

Results

Cross check

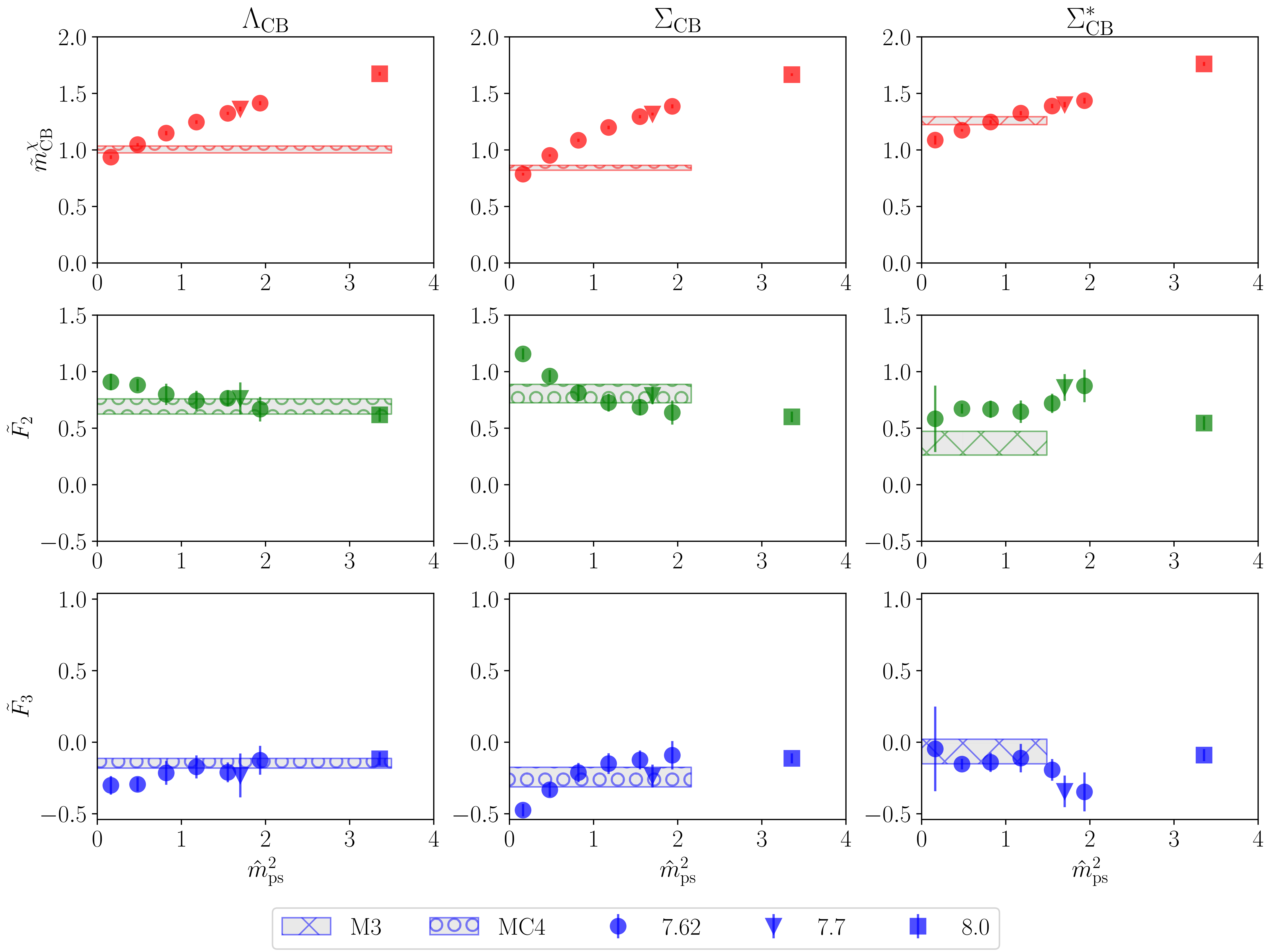


- At a fixed $\hat{m}_{ps}$

$$m_{CB} = \tilde{m}^{\chi}_{CB}(\hat{m}_{ps}, A, L, \hat{a}) + \tilde{F}_2(\hat{m}_{ps}, C, L, \hat{a})\hat{m}^2_{PS} + \tilde{F}_3\hat{m}^3_{PS}$$

Results

Cross check

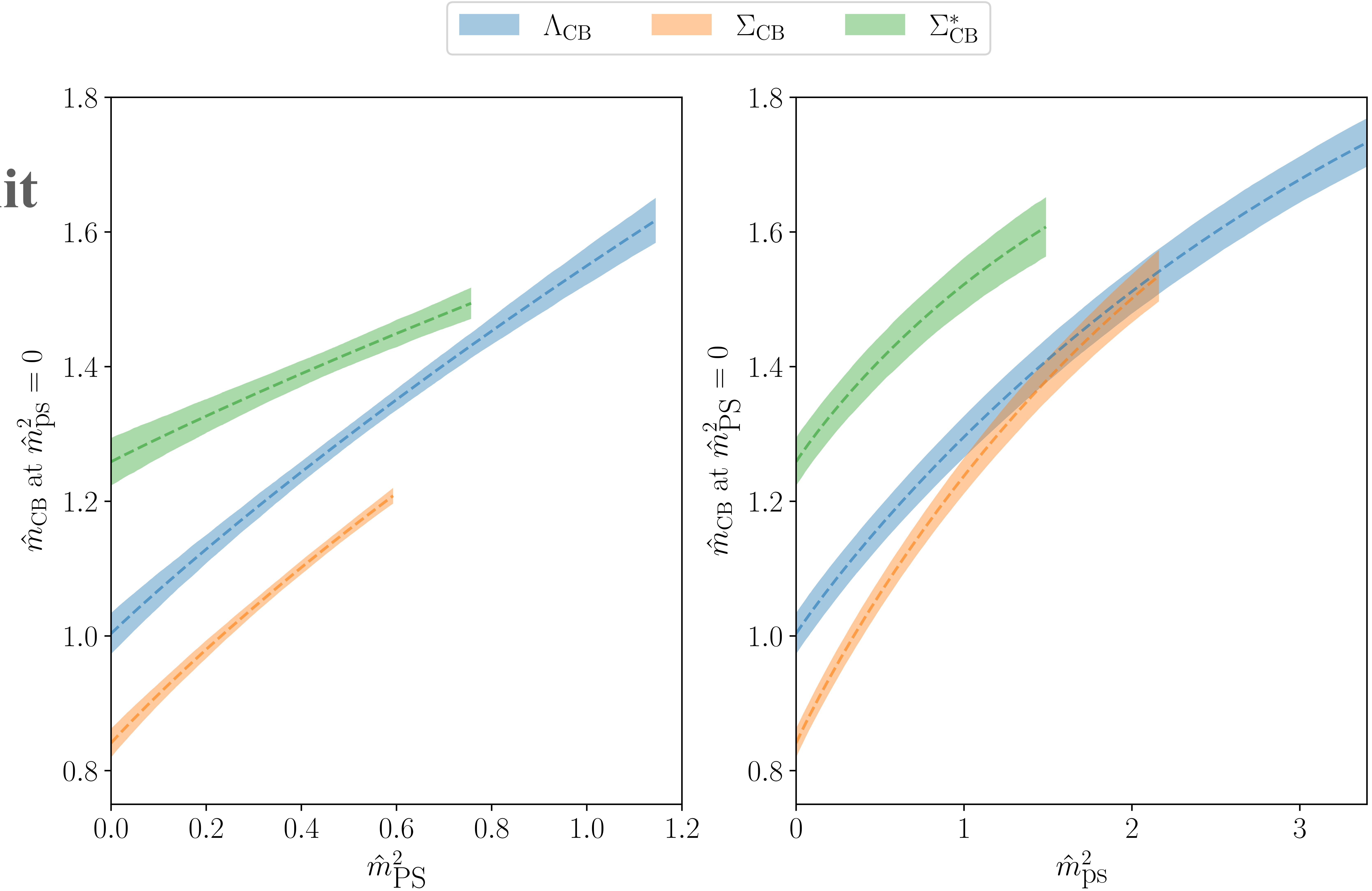


- At a fixed $\hat{m}_{PS}$

$$m_{CB} = \tilde{m}^{\chi}_{CB}(\hat{m}_{PS}, F, L, \hat{a}) + \tilde{A}_2(\hat{m}_{PS}, C, L, \hat{a})\hat{m}^2_{ps} + \tilde{A}_3\hat{m}^3_{ps}$$

Results

Massless-continuum limit

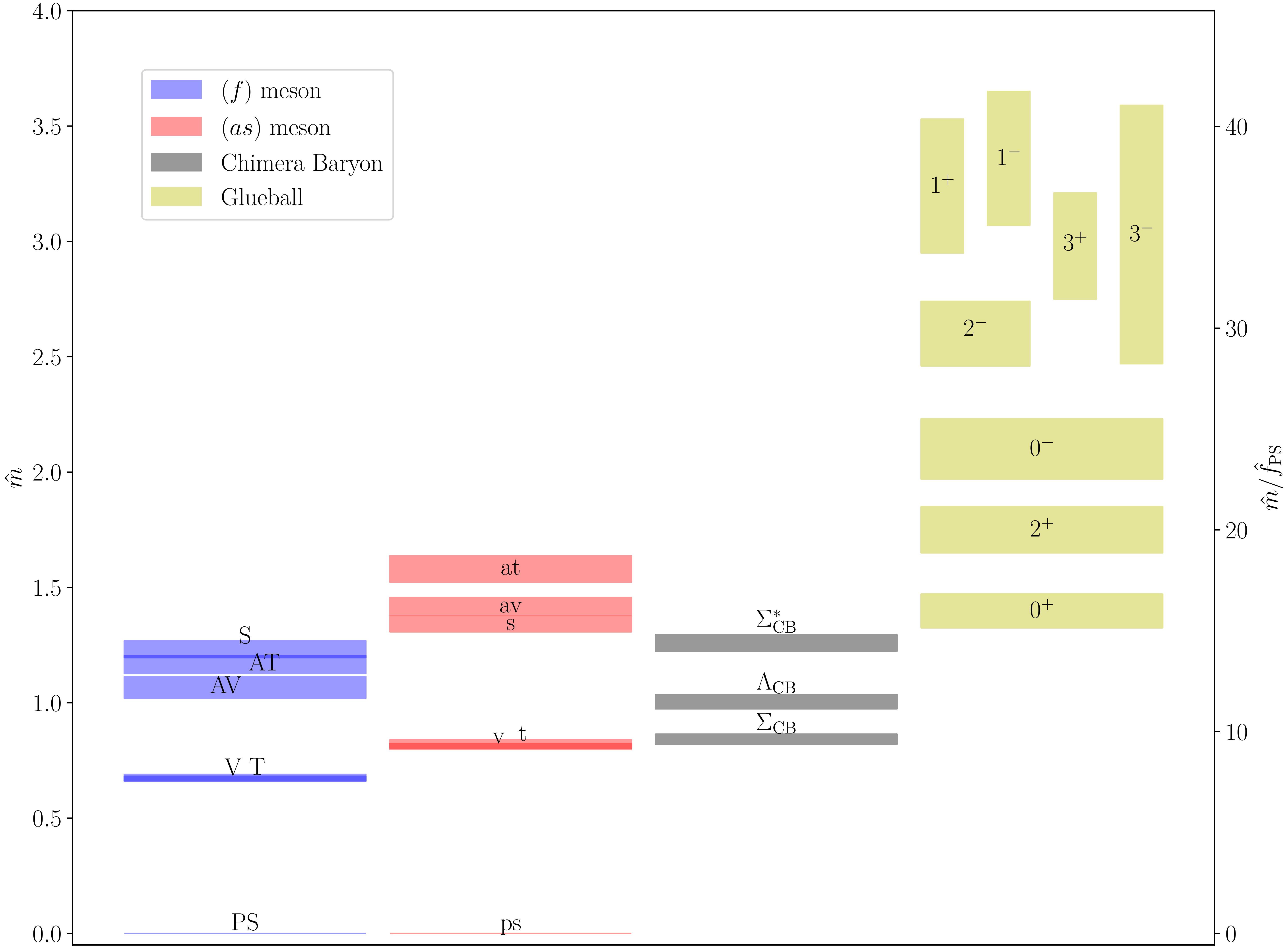


CB	Ansatz	$\hat{m}^{\chi}_{\rm CB}$	F_2	A_2	L_1	F_3	A_3	L_{2F}	L_{2A}	C_4
$\Lambda_{\rm CB}$	MC4	1.004(30)	0.692(67)	0.384(12)	-0.14(46)	-0.14(33)	-0.092(46)	0.091(76)	0.003(13)	-0.024(60)
$\Sigma_{\rm CB}$	MC4	0.842(21)	0.806(81)	0.558(13)	-0.14(33)	-0.24(68)	-0.162(77)	0.193(62)	-0.01(16)	-0.079(62)
$\Sigma^*_{\rm CB}$	M3	1.258(35)	0.36(10)	0.391(31)	-0.33(53)	-0.06(85)	-0.12(16)	0.335(86)	0.006(30)	-

Results

Massless-continuum limit

Comparison with masses of mesons in quenched approximation for fermions in the fundamental (blue bands) and antisymmetric (red bands) representation of $Sp(4)$, and glueballs (yellow at massless-continuum limit.



Summary and Outlook

- ☑ Chimera baryons
  - Λ and Σ: Top partner candidates in our model
  - Σ^* with spin-3/2
- ☑ Projection (Spin and Parity)
- ☑ The mass hierarchy of chimera baryons
- ☑ Chiral effective field theory

END

Thank you