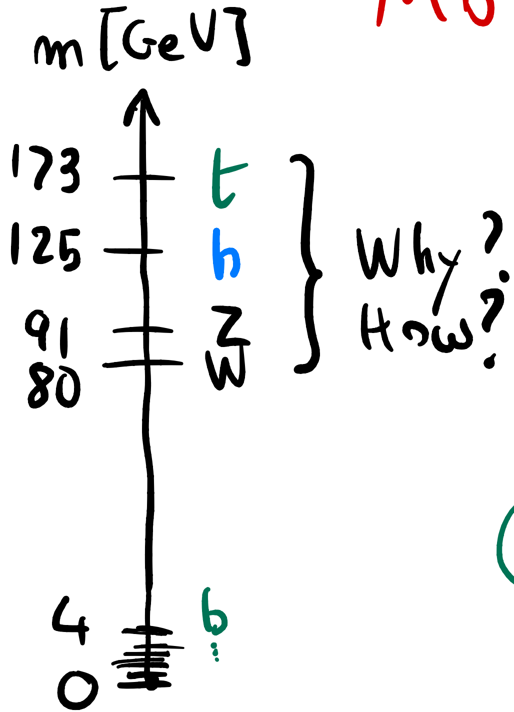
VACUUM MISALIGNMENT for ELECTROWEAK SYMMETRY BREAKING

(Partly based on 2302.11598, PRD 107 (2023) with Avik Banerjee)

부산 2024

MOTIVATION



FLAVOR & **HIERARCHY**

Kaplan

"t" mixes with a composite state

(PARTIAL COMPOSITNESS)

Georgi & Kaplan

"h" as a pNGB

$f_\pi \sim 100\,\text{MeV} \to f_h \sim 1\,\text{TeV}$

$\langle \pi^0 \rangle = 0 \qquad \langle h \rangle \neq 0$

(COMPOSITE HIGGS.)

Combine in a 4D framework.

Led to a nice interplay between LATTICE and PHENOMENOLOGY.

Idea:

$$\mathcal{L}_{\text{COMP.}} + \mathcal{L}_{\text{SM}_0} + \mathcal{L}_{\text{INT.}} \longrightarrow \mathcal{L}_{\text{SM}}$$

$$\Lambda_{HC} \simeq 10 \text{ TeV}.$$
$$\simeq 4\pi f_h.$$

Typical resonance masses $\sim \Lambda_{HC}$

pNGB masses $\ll \Lambda_{HC}$

$$\mathcal{L}_{\text{COMP.}} + \mathcal{L}_{\text{SM}_0} + \mathcal{L}_{\text{INT.}} \longrightarrow \mathcal{L}_{\text{SM}}$$

UNDERLYING 4D STRONGLY COUPLED GAUGE THEORY WITH FERMIONIC MATTER.

"HYPERCOLOR"

$$\mathcal{L}_{\text{COMP}} = -\frac{1}{4} F_{HC}^2 + i\,\bar{\psi}\,\not{D}\,\psi + i\,\bar{\chi}\,\not{D}\,\chi + \ldots$$

$$\mathcal{L}_{\text{COMP.}} + \mathcal{L}_{\text{SM}_0} + \mathcal{L}_{\text{INT.}} \longrightarrow \mathcal{L}_{\text{SM}}$$

"HIGGSLESS" STANDARD MODEL

$$\mathcal{L}_{\text{SM}_0} = -\frac{1}{4}\sum_{F=G,W,B} F^2 + i\sum_{f=QudLe} \bar{f}\,\not{D} f$$

$$\mathcal{L}_{COMP.} + \mathcal{L}_{SM_0} + \mathcal{L}_{INT.} \longrightarrow \mathcal{L}_{SM}$$

INTERACTION LAGRANGIAN

(TYPICALLY 4-FERMI INTERACTIONS COUPLING HYPERFERMIONS & QudLe)

$$\mathcal{L}_{INT.} \sim f^2 \psi^2 \text{ and } f \psi^3$$

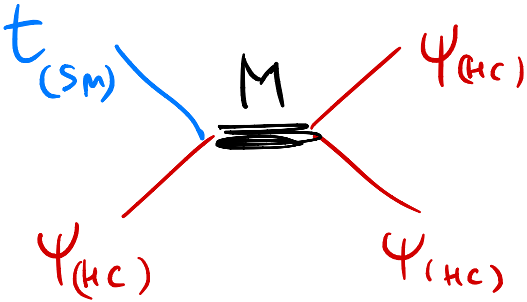
(various chirality structures...)

$$\boxed{\mathcal{L}_{\text{COMP.}}} + \mathcal{L}_{\text{SM}_0} + \mathcal{L}_{\text{INT.}} \longrightarrow \mathcal{L}_{\text{SM}}$$

CHALLENGES for the LATTICE.

- SPECTRUM of BOUND STATES. $m_{\text{Res.}}$
- ANOMALOUS DIMENSIONS γ^* of COMPOSITE OPERATORS
- MATRIX ELEMENTS a of COMPOSITE OPERATORS.

Required for the top mass.



$$\sim \frac{1}{M^2}\, t\, \psi^3 \longrightarrow \frac{1}{\Lambda_{HC}^2}\left(\frac{\Lambda_{HC}}{M}\right)^{2-\gamma^*} t\, \psi^3$$

$$\longrightarrow \frac{1}{\Lambda_{HC}^2}\left(\frac{\Lambda_{HC}}{M}\right)^{2-\gamma^*} a\, f_h^3\, t\, T$$

Ideally, $a \gg 1$ & $\gamma^* \simeq 2$

(I'll come back to this ...).

$$\mathcal{L}_{\text{COMP.}} + \mathcal{L}_{\text{SM}_0} + \boxed{\mathcal{L}_{\text{INT.}}} \longrightarrow \mathcal{L}_{\text{SM}}$$

CHALLENGES for PHENOMENOLOGY

- DERIVE $\mathcal{L}_{\text{INT.}}$ from a UV theory $\mathcal{L}_{UV}$
- TRIGGER VACUUM MISALIGNMENT
- OBTAIN REALISTIC SPECTRUM & CKM MIX.
- AVOID EWPT & FCNC CONSTRAINTS.

Before moving to discuss VACUUM MISALIGNMENT, let me review some basic aspects of the construction and connect with some recent lattice results.

CONSTRUCTING $\mathcal{L}_{comp.}$

We require: (1312.5330)

- GLOBAL SYMMETRY BREAKING G/H COMPATIBLE WITH CUSTODIAL SYMMETRY:

$$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X \subset H$$

$$\text{Higgs: } (\mathbf{1}, \mathbf{2}, \mathbf{2})_0 \in G/H$$

- ASYMPTOTIC FREEDOM OUTSIDE THE CONFORMAL WINDOW

- EXISTENCE OF TOP PARTNER BOUND STATES.

Solutions to these requirements are, e.g.

$$G_{HC} = Sp(4) \text{ with hyperfermions } \left.\begin{array}{l} \in \mathbf{4}, \mathbf{5} \\ \in \mathbf{4}, \mathbf{6} \end{array}\right\} \begin{smallmatrix}\square\\\square\end{smallmatrix}$$

$$= SU(4) \quad \text{"} \qquad \text{"}$$

$$= SO(7) \quad \text{"} \qquad \text{"} \qquad \in \mathbf{8}, \mathbf{7}$$

$\vdots$

In fact, since $Sp(4) \simeq SO(5)$ & $SU(4) \simeq SO(6)$

They all fall into the sequence

$SO(N)$ with
- N_F Fundamental
- N_S Spin ($\overline{\text{Spin}}$)

CONDENSATES: "Fund^2" "Spin^2" TOP PARTNERS: "Spin^2 Fund"

Examples

G_{HC} : Sp(4) SU(4) SO(7) . . .

Sp(4)

$N_{\mathbf{4}}$	$N_{\mathbf{5}}$
4	6

(1311.6562)

SU(4)

$N_{\mathbf{4}}$	$N_{\mathbf{6}}$
3+3	5

(1404.7137)

SO(7)

$N_{\mathbf{8}}$	$N_{\mathbf{7}}$
5	6

Number of <u>WEYL</u> spinors relevant to phenomenology.

G_{HC} : $\underbrace{Sp(4) \qquad SU(4)}_{\text{Studied on the lattice by some of the leading lattice collaborations.}} \qquad SO(7) \; \ldots$

Nothing is known.

G_{HC} : Sp(4) SU(4) SO(7) ...

2304.11729 : Anomalous dimension of top partners in a CONFORMAL : $(N_4 = 4+4,\ N_6 = 8)$ theory very close to strongly coupled edge of the conformal window :

But! $\gamma^* \simeq 0.5$ (far from $\gamma^* \simeq 2$)

G_{HC} : $Sp(4)$ $SU(4)$ $SO(7)$ ...

1812.02727: Matrix element for the CONFINING $(N_4 = 2+2,\ N_6 = 4)$ theory

$$\langle 0 | \chi\psi\chi | P=0, \sigma, T \rangle \sim \underbrace{0.35\, f_h^3}_{a} \cdot u_\sigma(0)$$

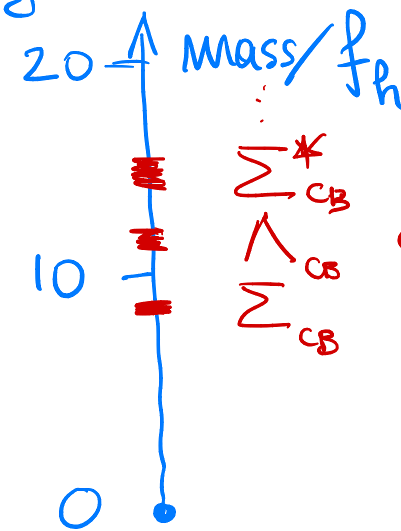
Quite small! (With various caveats).

Note that in QCD $a \sim 7$. $(P \to e^+ \pi^0)$

G_{HC} : Sp(4)   SU(4)   SO(7) . . .

The spectrum is known in detail
(2311.14663 & 2401.05637) $(N_4 = 4,\ N_5 = 6)$

Among other things, for the top PARTNERS:



$\left.\begin{array}{l} \Lambda_{CB} \in \mathbf{5} \\ \Sigma_{CB} \in \mathbf{10} \end{array}\right\}$ of the global Sp(4)

$$\left(G/H = \frac{SU(4)}{Sp(4)} \right)$$

G_{HC} : Sp(4) SU(4) SO(7) ...

IT WOULD BE INTERESTING TO:

- Compute the MATRIX ELEMENTS for Σ_{CB}, Λ_{CB}

$$\langle 0 | \psi^\dagger \chi^\dagger \psi | \Sigma_{CB} \rangle , \quad \langle 0 | \psi^\dagger \chi^\dagger \psi | \Lambda_{CB} \rangle$$

- Go inside the CONFORMAL WINDOW and compute the ANOMALOUS DIMENSIONS for the various $\psi^\dagger \chi^\dagger \psi$, $\psi \chi \psi$

($N_4 = 4$ and $N_5 = 8$ MIGHT BE ENOUGH)

(2 Dirac $\square$ and 4 Dirac $\boxminus$)

VACUUM MISALIGNMENT

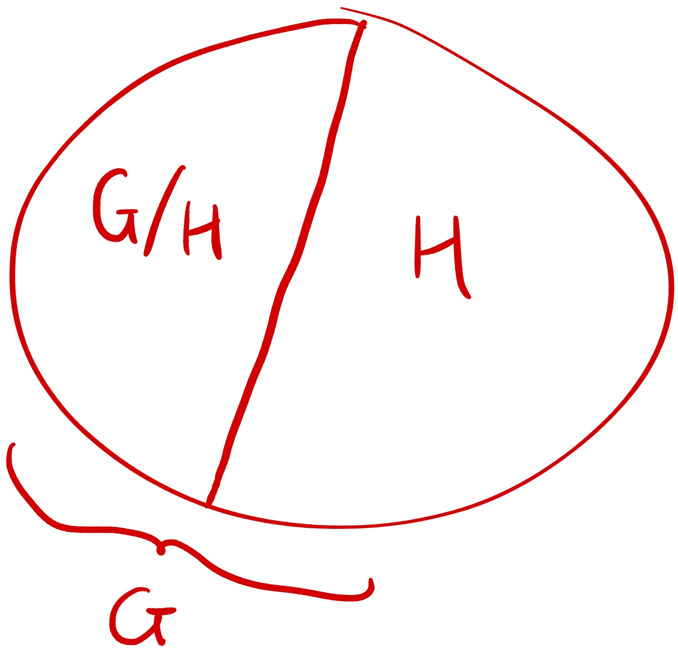
Let's start with the strongly coupled theory $\mathcal{L}_{\text{COMP.}}$ with GLOBAL SYMMETRY G. SPONTANEOUSLY BROKEN $\longrightarrow H$.

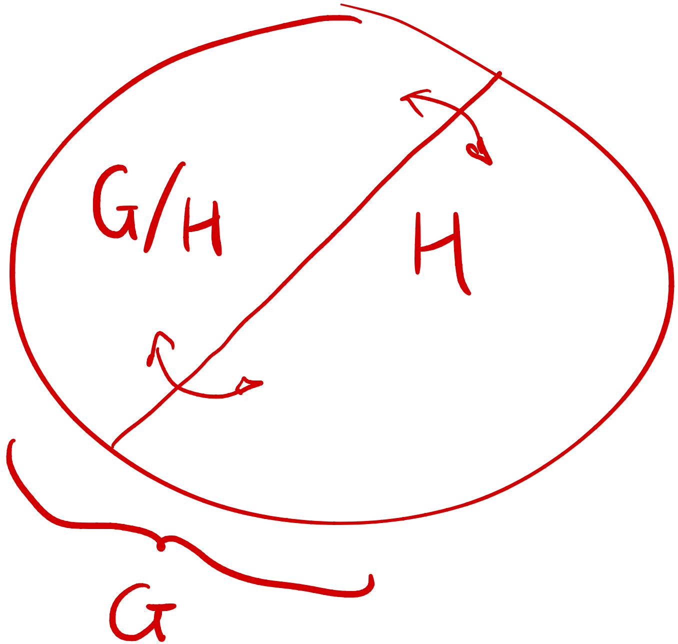
$$\underset{G}{Q^A} \equiv \underset{H}{Q^a}, \quad \underset{G/H}{Q^{\hat{a}}}$$

We can pick a reference vacuum $|vac\rangle_0$

$$Q^a |vac\rangle_0 = 0, \quad {}_0\langle vac| \varphi^i \varphi^j |vac\rangle_0 = -B\, I_0^{ij}$$

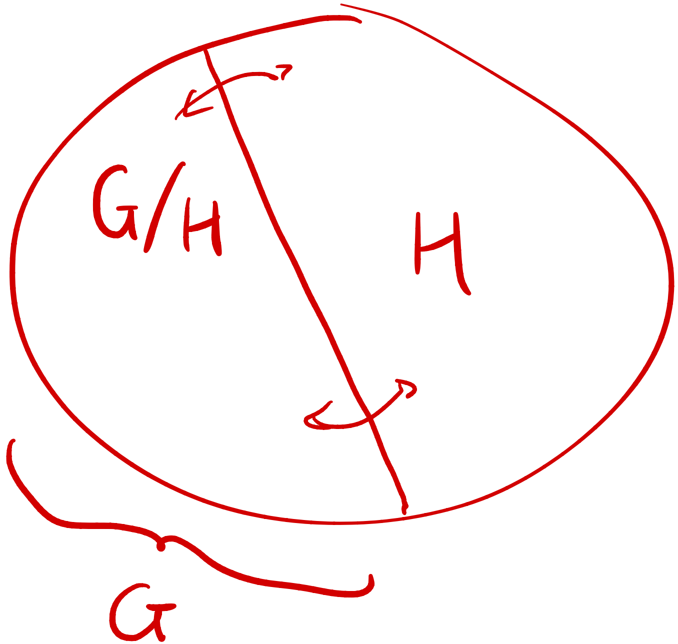
H invariant.


G/H
H
G



But of course, as long as the theory is G-invariant $[Q^A, \mathcal{H}_G] = 0$, I could use any vacuum

$$|vac\rangle_\pi = e^{i\pi^{\hat{a}} Q^{\hat{a}}} |vac\rangle_0$$



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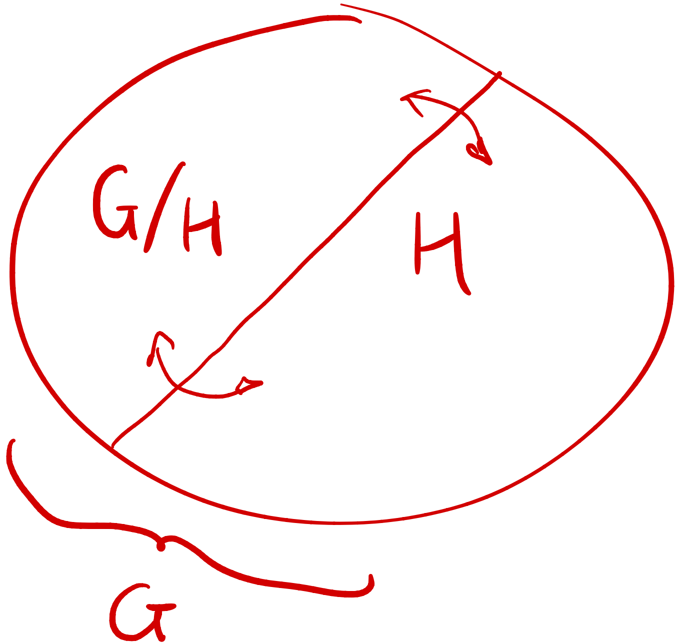

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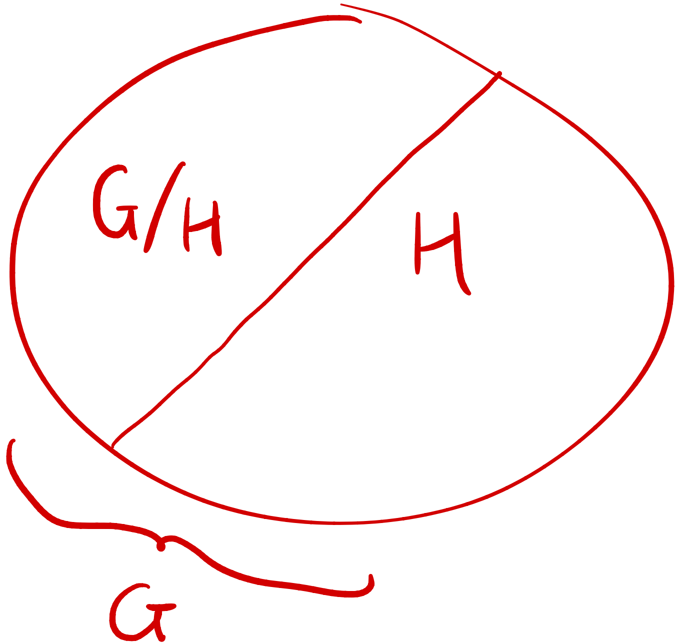


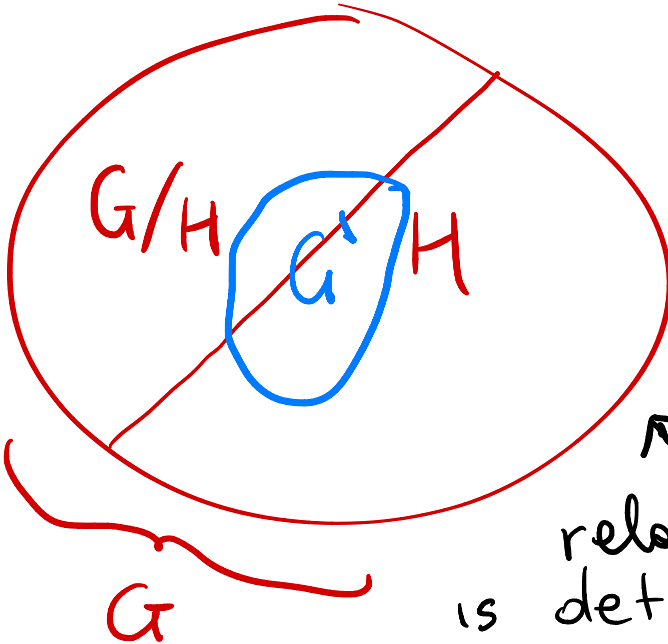
But of course, as long as the theory is G-invariant

$$[Q^A, \mathcal{H}_G] = 0,$$

I could use any vacuum

$$|vac\rangle_\pi = e^{i\pi^{\hat{a}} Q^{\hat{a}}} |vac\rangle_0$$

$$V(\pi) = {}_\pi\langle vac| \mathcal{H}_G |vac\rangle_\pi = {}_0\langle vac| \mathcal{H}_G |vac\rangle_0 = \text{const.}$$



Now suppose I turn on an interaction $\mathcal{H}'$ that only preserves a SUBGROUP G' of G.

Generically I could have, but now the relative aligment of G' and H is determined dynamically.

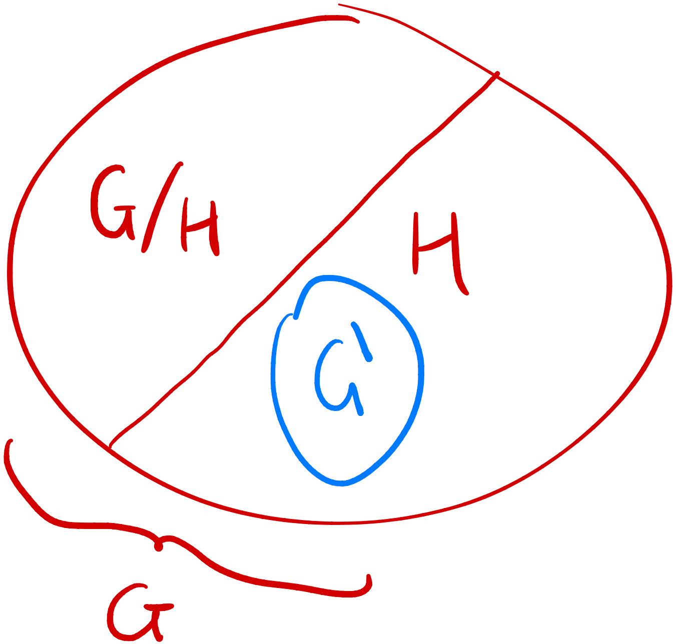
$$V(\pi) = {}_{\pi}\langle \mathrm{vac} | \mathcal{H}' | \mathrm{vac} \rangle_{\pi} = \mathrm{const} - i\pi^{\hat{a}} \, {}_{0}\langle \mathrm{vac} | [Q^{\hat{a}}, \mathcal{H}'] | \mathrm{vac} \rangle_{0}$$

$$- \frac{1}{2} \pi^{\hat{a}} \pi^{\hat{b}} \, {}_{0}\langle \mathrm{vac} | [Q^{\hat{a}}, [Q^{\hat{b}}, \mathcal{H}']] | \mathrm{vac} \rangle_{0} + \cdots$$

$${}_0\langle \mathrm{vac} | [\hat{Q}^a, \mathcal{H}'] | \mathrm{vac} \rangle_0 = 0$$ "no tadpole"

$${}_0\langle \mathrm{vac} | [\hat{Q}^a, [\hat{Q}^b, \mathcal{H}']] | \mathrm{vac} \rangle_0 > 0$$ "no tachyon"

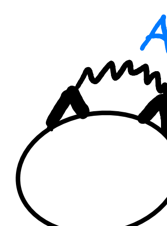
Conditions for $|\mathrm{vac}\rangle_0$ to remain stable after turning on $\mathcal{H}'$.



This is particularly relevant when G' is weakly gauged. In this case it is well known that $\mathcal{H}_{gauge}$ DOES NOT misalign the vacuum.

$\{\underline{a},\underline{b}\ldots\} \subset \{a,b..\}$ UNBROKEN.

$$\mathcal{H}_{gauge} = -\frac{i}{2} g^2 \int d^4x \, \underbrace{A^{\mu \underline{a}}(x)\, A^{\nu \underline{b}}(0)}_{= \Delta^{\mu\nu}(x)\, \delta^{\underline{a}\underline{b}}} \, T\left(J_\mu^{\underline{a}}(x)\, J_\nu^{\underline{b}}(0)\right)$$



$J = \bar{\psi}\gamma^\mu\psi$

$$\langle vac | [\hat{Q}, [\hat{Q}, \cdots [\hat{Q}, \mathcal{H}] \cdots] | vac \rangle$$

CAT GRAPH (Kaplan and Georgi)

$\langle vac|[\hat{Q},[\hat{Q},\cdots[\hat{Q},\mathcal{H}]\cdots]]|vac\rangle$

CAT GRAPH (Kaplan and Georgi)

1984

Of course, The correct name is:

COSMIC CAT

(Dr. Snuggles)

1974

GRAPH

$$\langle vac | [\hat{Q}, [\hat{Q}, \cdots [\hat{Q}, \mathcal{H}] \cdots] | vac \rangle$$

CAT GRAPH (Kaplan and Georgi)

Very schematically:

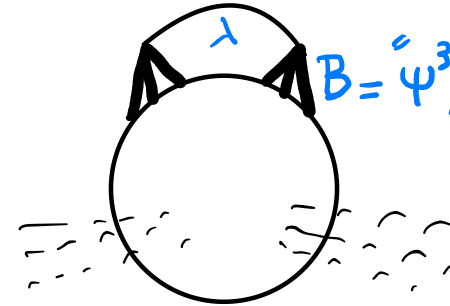
$$\mathcal{H}_{gauge} \sim \int \Delta_{\mu\nu} J^\mu J^\nu$$

broken unbroken

$$\langle [\hat{Q}, \mathcal{H}_{gauge}] \rangle \sim \int \Delta_{\mu\nu} \langle \overset{\text{broken}}{\hat{J}^\mu} \overset{\text{unbroken}}{J^\nu} \rangle = 0$$

$$\langle [\hat{Q}, [\hat{Q}, \mathcal{H}_{gauge}]] \rangle \sim \underbrace{\int \Delta_{\mu\nu} \left(\langle J^\mu J^\nu \rangle - \langle \hat{J}^\mu \hat{J}^\nu \rangle \right)}_{\text{shown to be positive definite (Witten)}}$$

VACUUM MISALIGNMENT via PARTIAL COMPOSITENESS



$$\mathcal{L}_{PC} = y\, P^{\dagger}_{R i}\, \lambda^{i\alpha} B^{R}_{\alpha} + h.c.$$

$$\mathcal{H}_{PC} = -\frac{i}{2} y^2 \int d^4x\, \underbrace{\lambda^{i\alpha}(x)\, \lambda^{\dagger\dot{\alpha}}_{j}(0)}_{=\Delta^{\dot{\alpha}\alpha}(x)\,\cdot\,\delta^{i}_{j}}\, T\left(P^{\dagger}_{R i} B^{R}_{\alpha}(x)\, B^{\dagger}_{Q\dot{\alpha}}(0)\, P^{Qj} + h.c.\right)$$

Proj. $\mathrm{Rep}(\lambda) \longrightarrow R$

$$[Q^A, B^R_\alpha] = -(T^A)^R{}_S\, B^S_\alpha$$

IRREP R of G

The composite fermions $B^{R}_{\alpha} \sim$ "$\psi^{3}_{H.F.}$"
transform under an irrep R of G
but their $\langle B B^{\dagger} \rangle$ must be
decomposed under H.

$G \longrightarrow$	H
$SU(N)$	$SO(N)$
Adj	$\mathrm{Adj} \oplus \mathrm{Symm}_2$
Symm_2	$1 \oplus \mathrm{Symm}_2$
$SU(2N)$	$Sp(2N)$
Adj	$\mathrm{Adj} \oplus \mathrm{Antisym}_2$
$\mathrm{Antisym}_2$	$1 \oplus \mathrm{Antisym}_2$
$SU(N) \times SU(N)$	$SU(N)$
$(\mathrm{Fund}, \mathrm{Fund})$	$\mathrm{Antisym}_2 \oplus \mathrm{Symm}_2$
$(\mathrm{Fund}, \overline{\mathrm{Fund}})$	$1 \oplus \mathrm{Adj}$

Cases where $\quad R \longrightarrow r \oplus r'$

$$B^{R}_{\alpha} = B^{r}_{\alpha} \oplus B^{r'}_{\alpha}$$

$$\langle B^{r}_{\alpha}(x)\, B^{\dagger}_{q\dot{\alpha}}(0) \rangle_{0} = G_{\alpha\dot{\alpha}}(x)\, \delta^{r}_{q}$$

$$\langle B^{r'}_{\alpha}(x)\, B^{\dagger}_{q'\dot{\alpha}}(0) \rangle_{0} = G'_{\alpha\dot{\alpha}}(x)\, \delta^{r'}_{q'}$$

$$\langle B^{r'}_{\alpha}(x)\, B^{\dagger}_{r\dot{\alpha}}(0) \rangle_{0} = 0.$$

Dropping $\alpha, \dot\alpha$

$$\mathcal{H}_{pc} = -\frac{i}{2} y^2 \int dx\, \Delta(x)\, P^{\dagger}_{Ri}\, B^R(x)\, B^{\dagger}_Q(0)\, P^{Qi}$$

Tadpole condition:

$$\langle [Q^{\hat{Q}}, \mathcal{H}_{pc}] \rangle = -\frac{i}{2} y^2 \int dx\, \Delta(x) \Big(-P^{\dagger}_{Ri}\, T^{\hat{Q}R}_{s}\, \langle B^{s}(x) B^{\dagger}_Q(0) \rangle P^{Qi} + P^{\dagger}_{Ri} \langle B^R(x) B^{\dagger}_s(0) \rangle T^{\hat{Q}s}_{Q}\, P^{Qi} \Big)$$

$$R \longrightarrow r, \; r'$$

$$\langle B^R(x) B^{\dagger}_s(0) \rangle \longrightarrow G(x)\, \delta^r_s \oplus G'(x)\, \delta^{r'}_{s'}$$

Dropping $\alpha, \dot{\alpha}$

$$\mathcal{H}_{pc} = -\frac{i}{2} y^2 \int dx\, \Delta(x)\, P^{\dagger}_{Ri}\, B^{R}(x)\, B^{\dagger}_{Q}(0)\, P^{Qi}$$

Tadpole condition:

$$\langle [Q^{\hat{a}}, \mathcal{H}_{pc}] \rangle = -\frac{i}{2} y^2 \int dx\, \Delta(x) \Big(-P^{\dagger}_{Ri}\, T^{\hat{a}R}{}_{S} \langle B^{S}(x) B^{\dagger}_{Q}(0) \rangle P^{Qi} + P^{\dagger}_{Ri} \langle B^{R}(x) B^{\dagger}_{S}(0) \rangle T^{\hat{a}S}{}_{Q}\, P^{Qi} \Big)$$

$$= -\frac{i}{2} y^2 \int dx\, \Delta(x) \Big(-P^{\dagger}_{Ri}\, T^{\hat{a}R}{}_{\alpha}\, P^{\alpha i}\, G(x) - P^{\dagger}_{Ri}\, T^{\hat{a}R}{}_{s'}\, P^{s'i}\, G'(x) + P^{\dagger}_{ri}\, T^{\hat{a}r}{}_{Q}\, P^{Qi}\, G(x) + P^{\dagger}_{r'i}\, T^{\hat{a}r'}{}_{Q}\, P^{Qi}\, G'(x) \Big) =$$

$$= -\frac{i}{2} y^2 \int dx\, \Delta(x) \Big(-P^{\dagger}_{Ri} T^{\hat{a} R}_{\Delta} P^{\Delta i} G(x), - P^{\dagger}_{Ri} T^{\hat{a} R}_{s'} P^{s' i} G'(x)$$

$$+ P^{\dagger}_{ri} T^{\hat{a} r}_{Q} P^{Q i} G(x) + P^{\dagger}_{r' i} T^{\hat{a} r'}_{Q} P^{Q i} G'(x) \Big) =$$

$$= -\frac{i}{2} y^2 \int dx\, \Delta(x) \Big(-P^{\dagger}_{ri} T^{\hat{a} r}_{s} P^{s i} G(x) - P^{\dagger}_{r' i} T^{\hat{a} r'}_{s} P^{s i} G(x)$$

$$- P^{\dagger}_{ri} T^{\hat{a} r}_{s'} P^{s' i} G'(x) - P^{\dagger}_{r' i} T^{\hat{a} r'}_{s'} P^{s' i} G'(x)$$

$$+ P^{\dagger}_{ri} T^{\hat{a} r}_{q} P^{q i} G(x) + P^{\dagger}_{ri} T^{\hat{a} r}_{q'} P^{q' i} G(x)$$

$$+ P^{\dagger}_{r' i} T^{\hat{a} r'}_{q} P^{q i} G'(x) + P^{\dagger}_{r' i} T^{\hat{a} r'}_{q'} P^{q' i} G'(x) \Big)$$

$$= -\frac{i}{2} y^2 \int dx\, \Delta(x) \Big(- P^\dagger_{Ri} T^{\hat{a}R}_{\Delta} P^{\Delta i} G(x), - P^\dagger_{Ri} T^{\hat{a}R}_{s'} P^{s'i} G'(x)$$

$$+ P^\dagger_{ri} T^{\hat{a}r}_{Q} P^{Qi} G(x) + P^\dagger_{r'i} T^{\hat{a}r'}_{Q} P^{Qi} G'(x) \Big) =$$

$$= -\frac{i}{2} y^2 \int dx\, \Delta(x) \Big(\cancel{- P^\dagger_{ri} T^{\hat{a}r}_{s} P^{si}} G(x) - P^\dagger_{r'i} T^{\hat{a}r'}_{s} P^{si} G(x)$$

$$- P^\dagger_{ri} T^{\hat{a}r}_{s'} P^{s'i} G'(x) \cancel{- P^\dagger_{r'i} T^{\hat{a}r'}_{s'} P^{s'i} G'(x)}$$

$$+ \cancel{P^\dagger_{ri} T^{\hat{a}r}_{q} P^{qi}} G(x) + P^\dagger_{ri} T^{\hat{a}r}_{q'} P^{q'i} G(x)$$

$$+ P^\dagger_{r'i} T^{\hat{a}r'}_{q} P^{q'i} G'(x) + \cancel{P^\dagger_{r'i} T^{\hat{a}r'}_{q'} P^{q'i} G'(x)} \Big)$$

$$= -\frac{i}{2} y^2 \int dx\, \Delta(x) \Big(\cancel{-P^{\dagger}_{ri} T^{\hat{a} r}_{s} P^{si} G(x)} - P^{\dagger}_{r'i} T^{\hat{a} r'}_{s} P^{si} G(x)$$

$$- P^{\dagger}_{ri} T^{\hat{a} r}_{s'} P^{s'i} G'(x) \cancel{- P^{\dagger}_{r'i} T^{\hat{a} r'}_{s'} P^{s'i} G'(x)}$$

$$\cancel{+ P^{\dagger}_{ri} T^{\hat{a} r}_{q} P^{qi} G(x)} + P^{\dagger}_{ri} T^{\hat{a} r}_{q'} P^{q'i} G(x)$$

$$+ P^{\dagger}_{r'i} T^{\hat{a} r'}_{q} P^{qi} G'(x) \cancel{+ P^{\dagger}_{r'i} T^{\hat{a} r'}_{q'} P^{q'i} G'(x)} \Big)$$

$$= -\frac{i}{2} y^2 \int dx\, \Delta(x) \big(G(x) - G'(x) \big) \big(P^{\dagger}_{ri} T^{\hat{a} r}_{q'} P^{q'i} - P^{\dagger}_{r'i} T^{\hat{a} r'}_{s} P^{si} \big)$$

VANISHES IF λ is embedded into r OR r' ONLY (ie P^{ri} OR $P^{r'i} = 0$)

A symilar calculation for $\langle [Q^{\hat a}[Q^{\hat b}, \mathcal{H}_{pc}]]\rangle$ yields, in the absence of tadpoles:

$$M^{\hat a\hat b} = \frac{y^2}{f^2}\, \mathbf{K} \left(P^{t}_{ri}\left(T^{\hat b r}_{q'} T^{\hat a q'}_{s} + T^{\hat a r}_{q'} T^{\hat b q'}_{s}\right) P^{si} - P^{t}_{r'i}\left(T^{\hat b r'}_{q} T^{\hat a q}_{s'} + T^{\hat a r'}_{q} T^{\hat b q}_{s'}\right) P^{s'i} \right)$$

where:

$$\mathbf{K} = i\int d^4x\, \Delta^{\dot\alpha\alpha}(x)\left(G_{\alpha\dot\alpha}(x) - G'_{\alpha\dot\alpha}(x)\right) = 2\int \frac{d^4k_E}{(2\pi)^4}\int d\mu^2\, \frac{\rho(\mu^2) - \rho'(\mu^2)}{k_E^2 + \mu^2}$$

$$\left(\int d^4x\, e^{ikx} G_{\alpha\dot\alpha}(x) = i k^\mu \sigma_{\mu\alpha\dot\alpha} \int d\mu^2\, \frac{\rho(\mu)}{k^2 - \mu^2 + i\epsilon} \right)$$

A symilar calculation for $\langle [Q^{\hat a}[Q^{\hat b}, \mathcal{H}_{pc}]]\rangle$ yields, in the absence of tadpoles:

$$M^{\hat a \hat b} = \frac{y^2}{f^2} K \left(\underbrace{P^t_{ri}\left(T^{\hat b r}_{q'} T^{\hat a q'}_{s} + T^{\hat a r}_{q'} T^{\hat b q'}_{s}\right)}_{} P^{si} - \underbrace{P^t_{r'i}\left(T^{\hat b r'}_{q} T^{\hat a q}_{s'} + T^{\hat a r'}_{q} T^{\hat b q}_{s'}\right)}_{} P^{s'i} \right)$$

POSITIVE MATRICES

where:

$$K = i\int d^4x\, \Delta^{\dot\alpha\alpha}(x)\left(G_{\alpha\dot\alpha}(x) - G'_{\alpha\dot\alpha}(x)\right) = 2\int \frac{d^4k_E}{(2\pi)^4}\int d\mu^2\, \frac{\rho(\mu^2) - \rho'(\mu^2)}{k_E^2 + \mu^2}$$

$\Rightarrow$ For either sign of K, one irrep tends to misalign.

The sign of

$$K = 2\int\frac{d^4k_E}{(2\pi)^4}\int d\mu^2\,\frac{\rho(\mu^2)-\rho'(\mu^2)}{k_E^2+\mu^2}$$

depends on the spectral densities in the channels r and r' and determines which coupling of λ will tend to misalign.

$K>0 \Longrightarrow r'$ misalign

$K<0 \Longrightarrow r$ misalign

$$K = 2\int \frac{d^4k_E}{(2\pi)^4} \int d\mu^2 \, \frac{\rho(\mu^2) - \rho'(\mu^2)}{k_E^2 + \mu^2}$$

- We expect the full symmetry to be restored in the UV:

$$\int d\mu^2 \rho(\mu^2) = \int d\mu^2 \rho'(\mu^2)$$

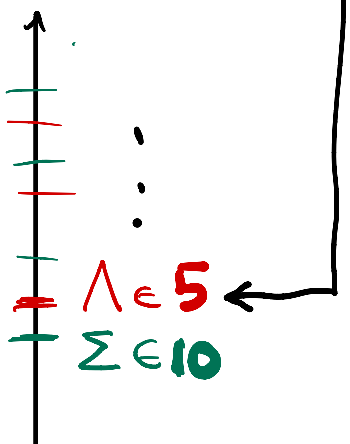
- Furthermore, assuming the dominance of the leading resonance:

$$\rho(\mu^2) = Z\delta(\mu^2 - m^2) \qquad \rho'(\mu^2) = Z\delta(\mu^2 - m'^2)$$

$$\text{sign}(K) = \text{sign}(m'^2 - m^2)$$

If you buy this approximation, then it is the NEXT-TO-LIGHTEST COMPOSITE FERMION that decides which irrep misaligns...

e.g. 2401.05637



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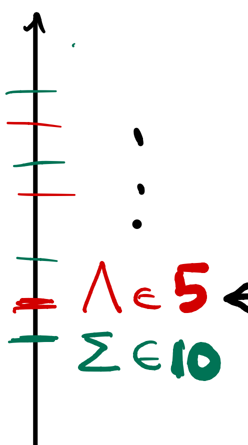

This may seem weird...

On the other hand:

In the model 1311.6562

	$G_{HC} = Sp(4)$	$G = SU(4) \longrightarrow H = Sp(4)$
ψ	$\square$	$\mathbf{4}$
χ	$\begin{array}{|c|}\hline \\ \hline \\ \hline\end{array}$	/
$B \sim \psi^\dagger \chi^\dagger \psi$	/	$\mathbf{15} \longrightarrow \mathbf{5} + \mathbf{10}$
$\hat{B} \sim \psi \chi \psi$	/	$\mathbf{6} \longrightarrow \mathbf{1} + \mathbf{5}$

$$B \in \mathbf{15} \left\{ \begin{array}{l} \mathbf{5} \\ \mathbf{10} \end{array} \right.$$

In the model 1311.6562

	$G_{HC} = Sp(4)$	$G = SU(4) \longrightarrow H = Sp(4)$
ψ	$\square$	4
χ	$\boxminus$	/
$B \sim \psi^\dagger \chi^\dagger \psi$	/	$15 \longrightarrow 5 + 10$
$\hat{B} \sim \psi \chi \psi$	/	$6 \longrightarrow 1 + 5$

$B \in 15 \left\{ \begin{matrix} 5 \\ 10 \end{matrix} \right.$

$\hat{B} \in 6 \left\{ \begin{matrix} 5 \\ 1 \end{matrix} \right.$

CONJECTURE!?!

Seems more reasonable.

CONCLUSIONS:

- 4D MODELS of CH+PC are a lovely time-machine forcing us to revisit the ideas and the papers of the golden age of particle physics.

- They lead to a very satisfying interaction between LATTICE and MODEL BUILDING that is enriching both fields

- There are NOT yet fully satisfactory models. The LATTICE has already shown that some constructions are not promising
- From the MODEL BUILDING side, I talked about VACUUM MISALIGNMENT but the big issue that really needs to be addressed is FLAVOR

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- From the MODEL BUILDING side, I talked about VACUUM MISALIGNMENT but the big issue that really needs to be addressed is FLAVOR

Thank you for your attention!