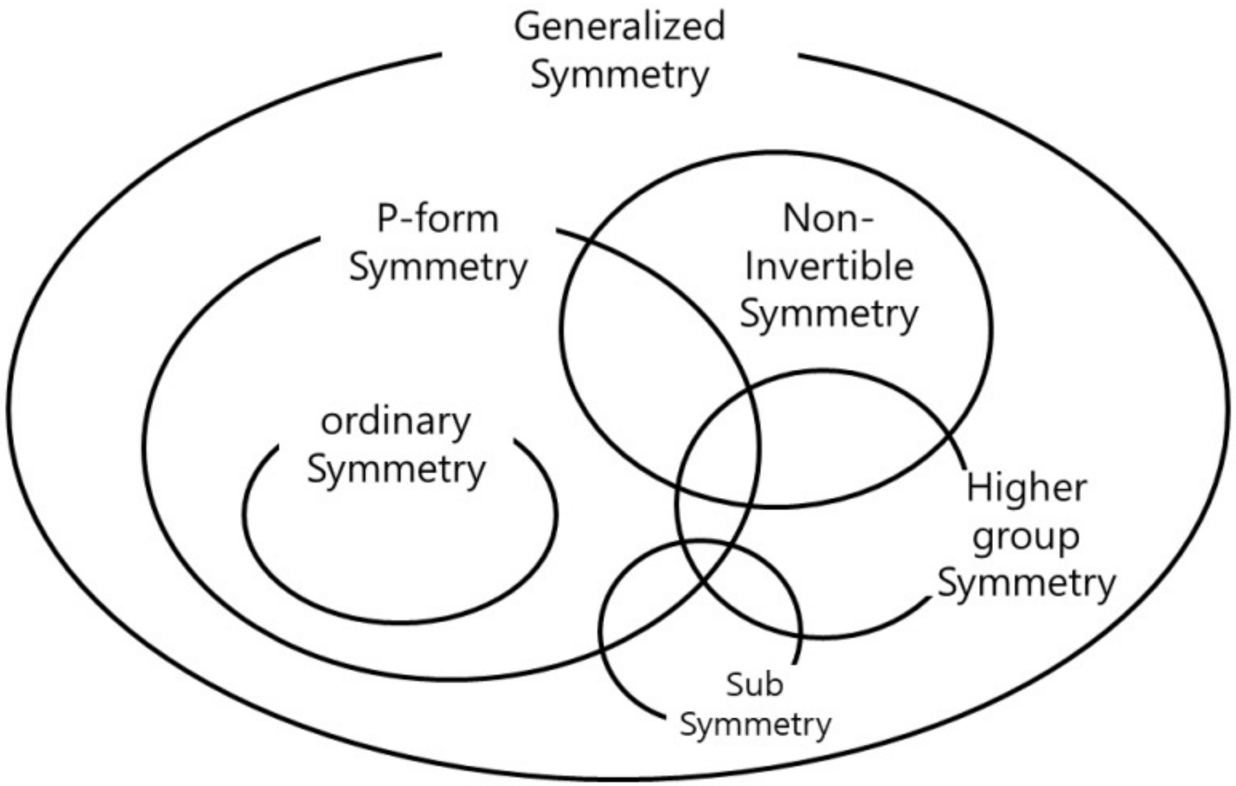
Generalized Symmetries

Jeonghak Han

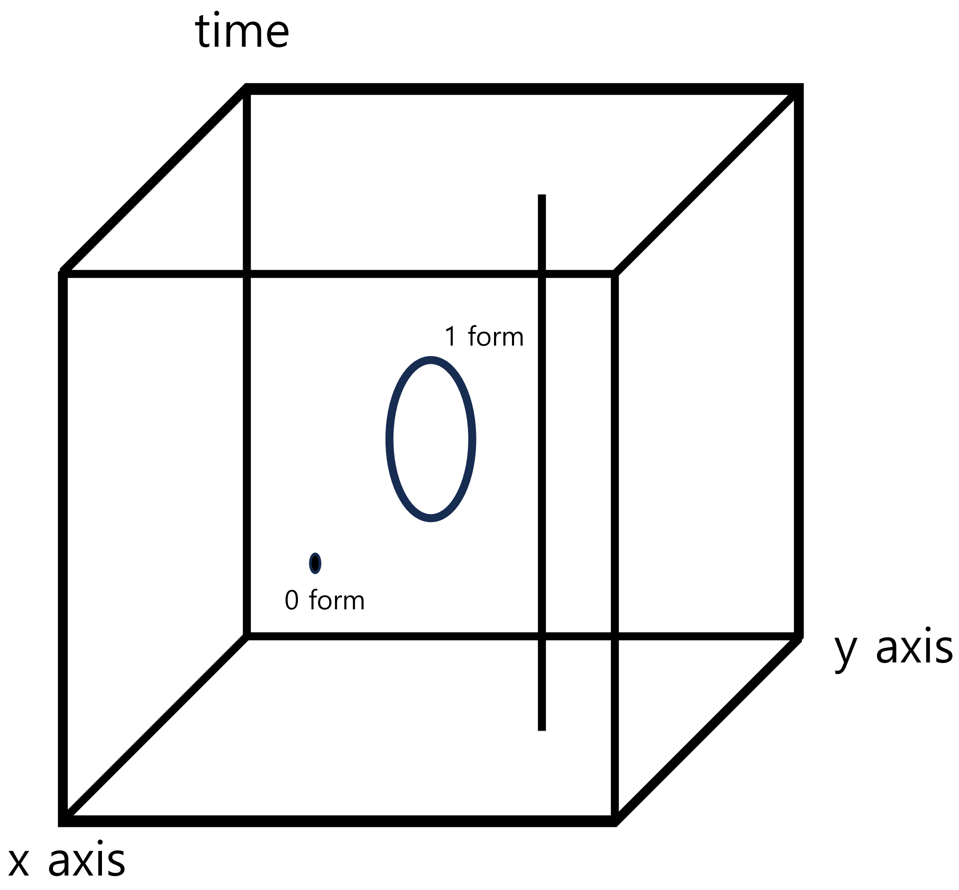
1. What is Generalized Symmetry?
2. Form and SDO
3. How does SDO act?
4. Example: Pure Maxwell theory

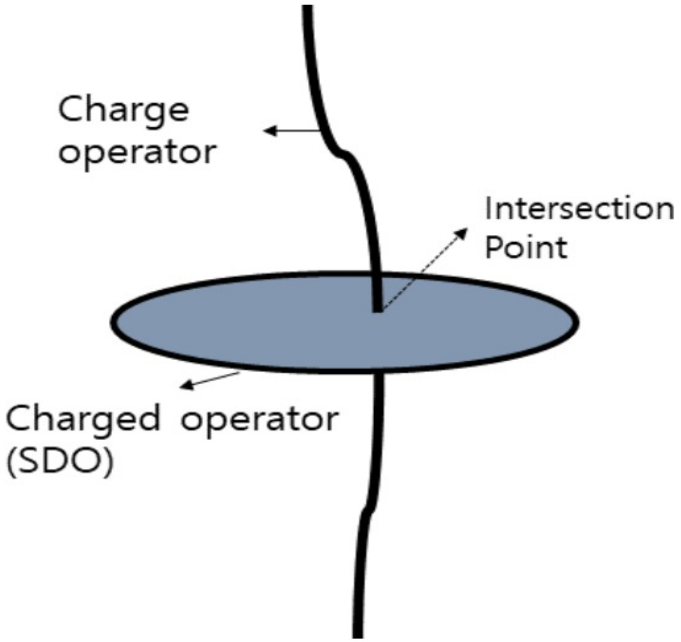
What is Generalized Symmetry?



Topological Symmetry defect operator exists → There exists Symmetry!

Form and SDO



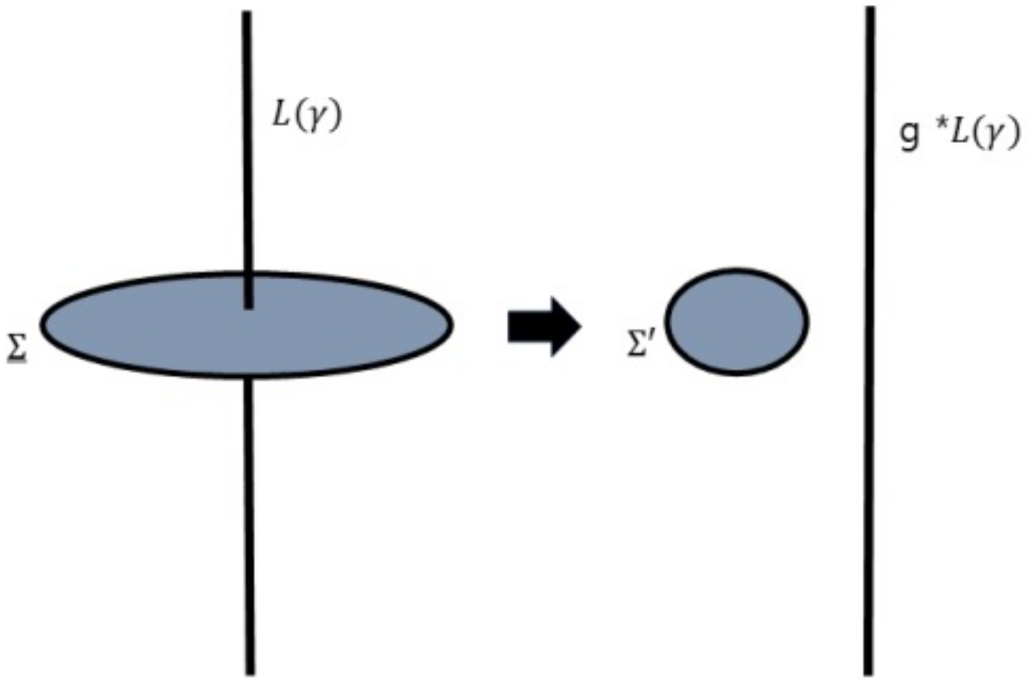


$$\partial_\mu J^{[\mu\nu]}(x) = 0 \quad \text{or} \quad d * J_2 = 0$$

$$S = \ldots + i \int B_{\mu\nu} J^{\mu\nu} d^d x = \ldots + i \int B_2 \wedge * J_2$$

$$d * J_2(x)\, L_q(\gamma) = q\, \delta^{(d-1)}(x \in \gamma)\, L_q(\gamma)$$

How does SDO act?



$$\langle U_g(\Sigma_{d-2})\, L_q(\gamma) \rangle = e^{iq\lambda Link(\Sigma_{d-2},\gamma)} \langle L_q(\gamma)\, U_g(\Sigma'_{d-2}) \rangle$$

Example: Pure Maxwell theory

$$S = \frac{1}{2g^2}\int F \wedge * F = -\frac{1}{4g^2}\int F_{\mu\nu}F^{\mu\nu}$$

2 types of current are exist.

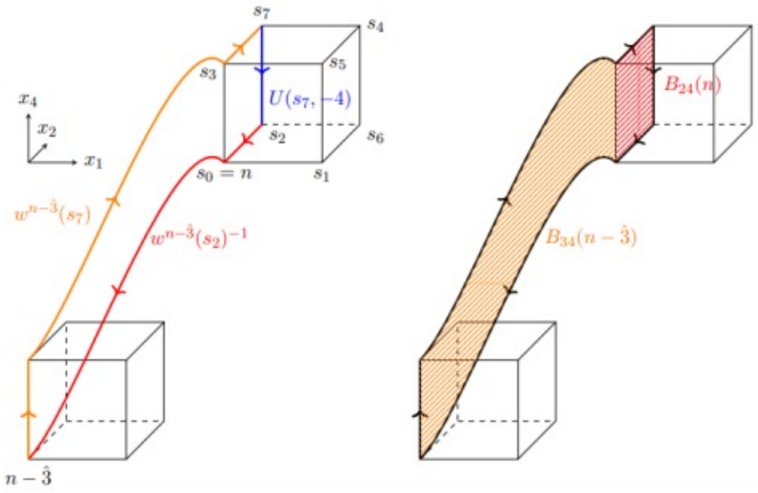
$$d * F = 0 \qquad\qquad dF = d * (* F) = 0$$

$$W(q,\gamma) = e^{iq\int_\gamma A}, \quad q \in \mathbb{Z}\,. \qquad T_1(m,\gamma) = e^{im\int_\gamma \tilde{A}}$$

$$U_g^e(\Sigma_2) = \exp\left(i\lambda \oint_{\Sigma_2} * J_2^e\right) \qquad U_g^e(\Sigma_2) = \exp\left(i\lambda \oint_{\Sigma_2} * J_2^m\right)$$

$$S = \frac{1}{2g^2}\int (F - B_2^e) \wedge * (F - B_2^e) + \frac{i}{2\pi}\int B_2^m \wedge (F - B_2^e)$$

$$S_{\text{inflow}} = -\frac{i}{2\pi}\int_{N_5} B_2^m \wedge dB_2^e$$



Higgs inflation with non-minimal coupling
(Review of Higgs inflation)

CAU HEP center workshop

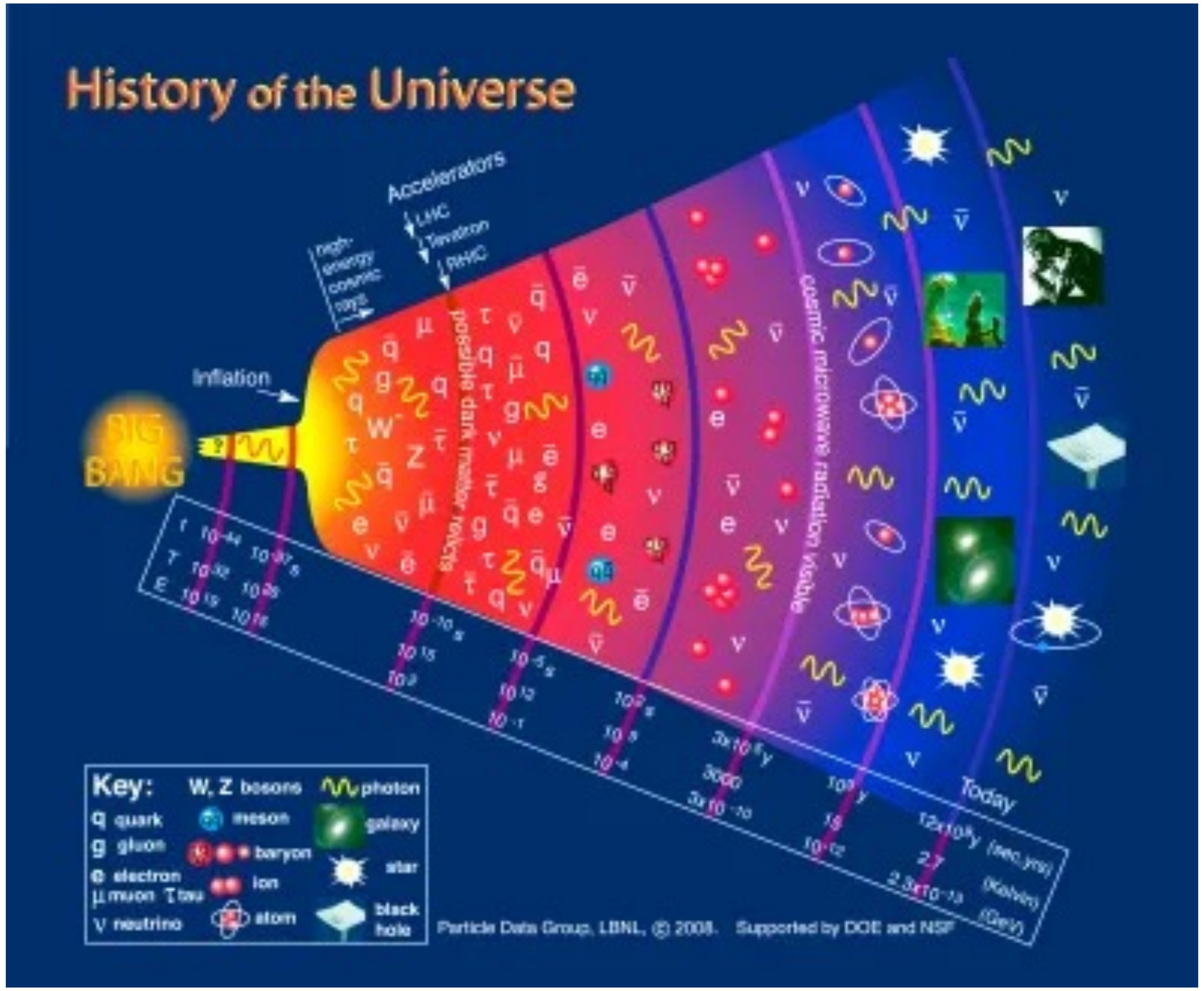
23.12.27

JoonSik Yu
THEP group undergraduate research student
yuyuyu7020@gmail.com

Abstract

In the early universe,
there is a exponential expansion of space called inflation.
To explain this process, we need to modify ordinary gravity theory, f(R) gravity.
There are several inflation model based on f(R) gravity. One of most proper inflation model is Higgs inflation since Higgs inflation fits in observational result.
So we can regard Higgs field as inflaton field which involves non-minimal coupling between gravity and inflaton.
This paper shows that prediction of Higgs inflation.

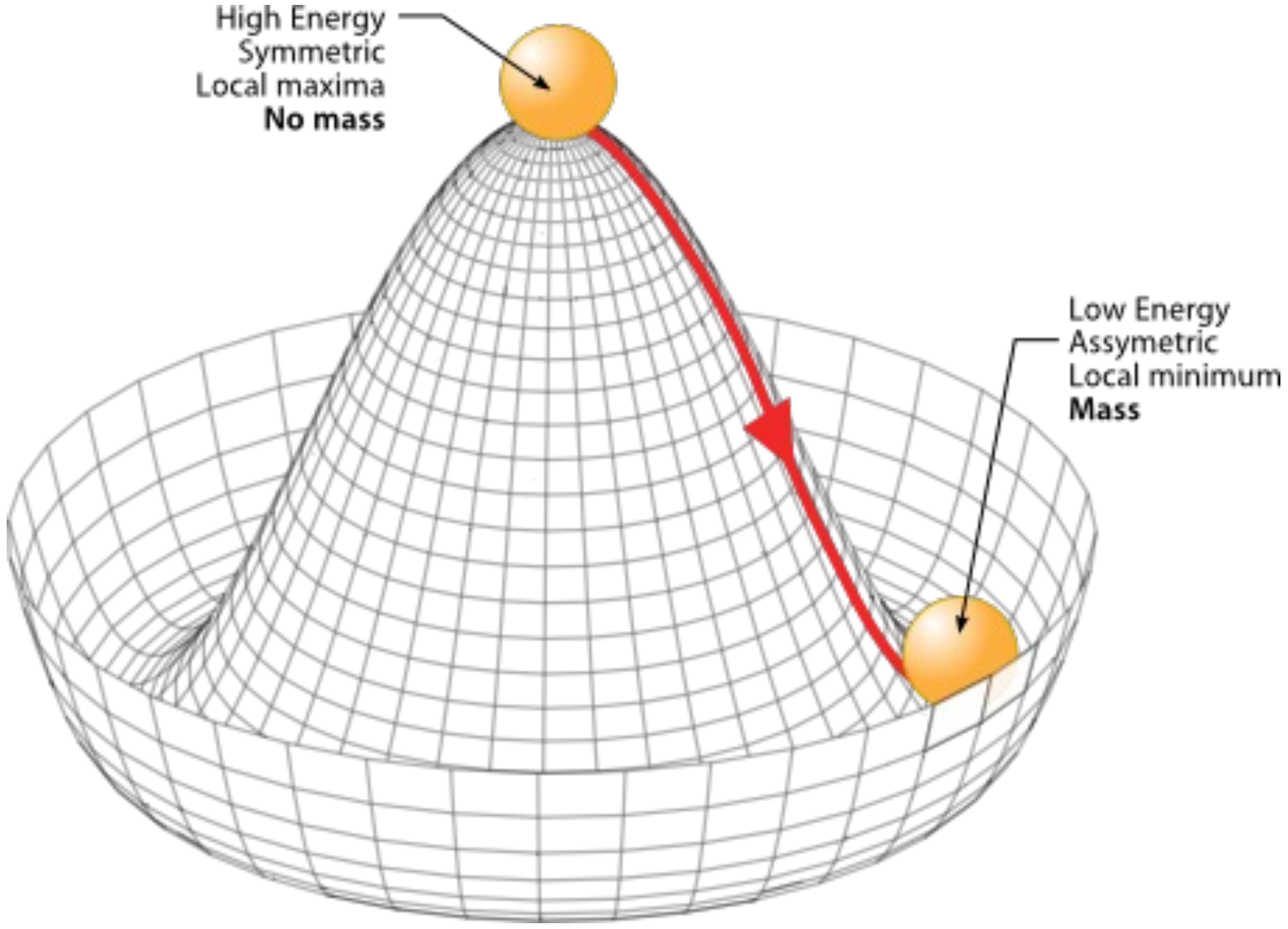
Inflationary comology



Exponential expansion in the early universe

Higgs mechanism

$$V(\phi) = -\frac{1}{2}\mu^2\phi^2 - \frac{\lambda}{4!}\phi^4$$



Spontaneous Symmetry Breaking

f(R) gravity

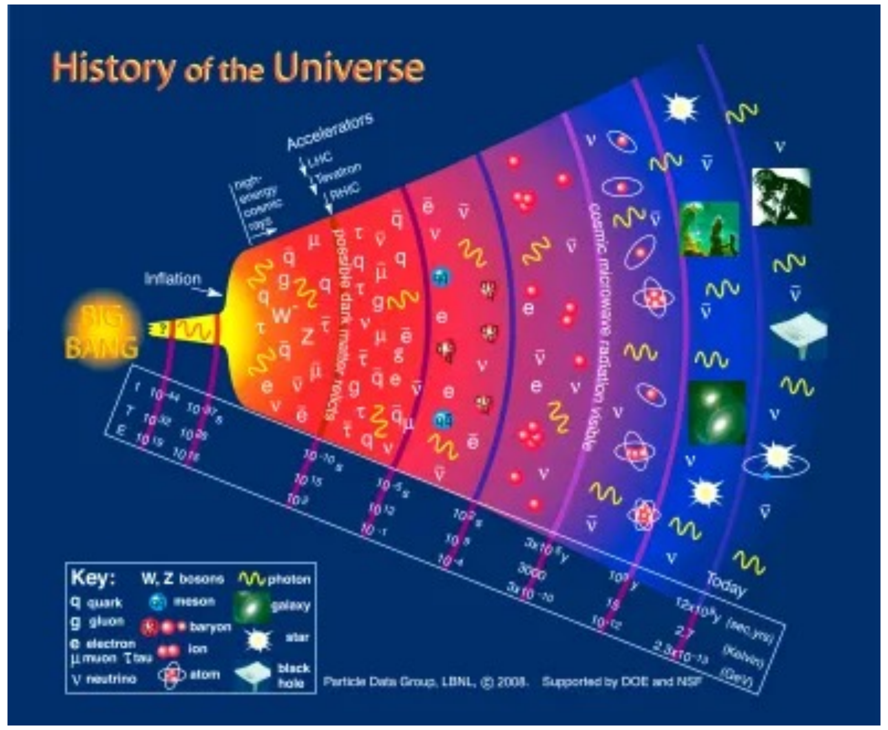
How we describe the inflation?

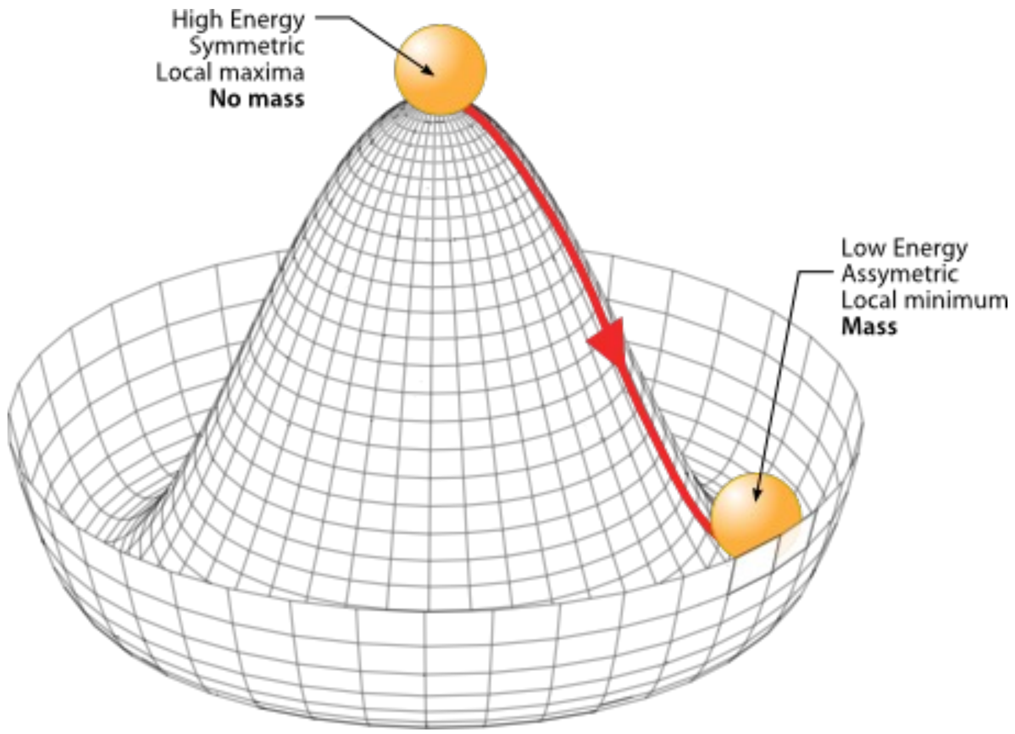
To explain the era of inflation, we need to modify the ordinary gravity theory

$$S_{EH} = \int (\frac{M_{pl}^2}{2} R + L_{matter}) \sqrt{-g} d^4 x. \longrightarrow S = \int (\frac{M_{pl}^2}{2} f(R) + L_{matter}) \sqrt{-g} d^4 x.$$

Inflation model

We can regard Higgs field as inflaton field





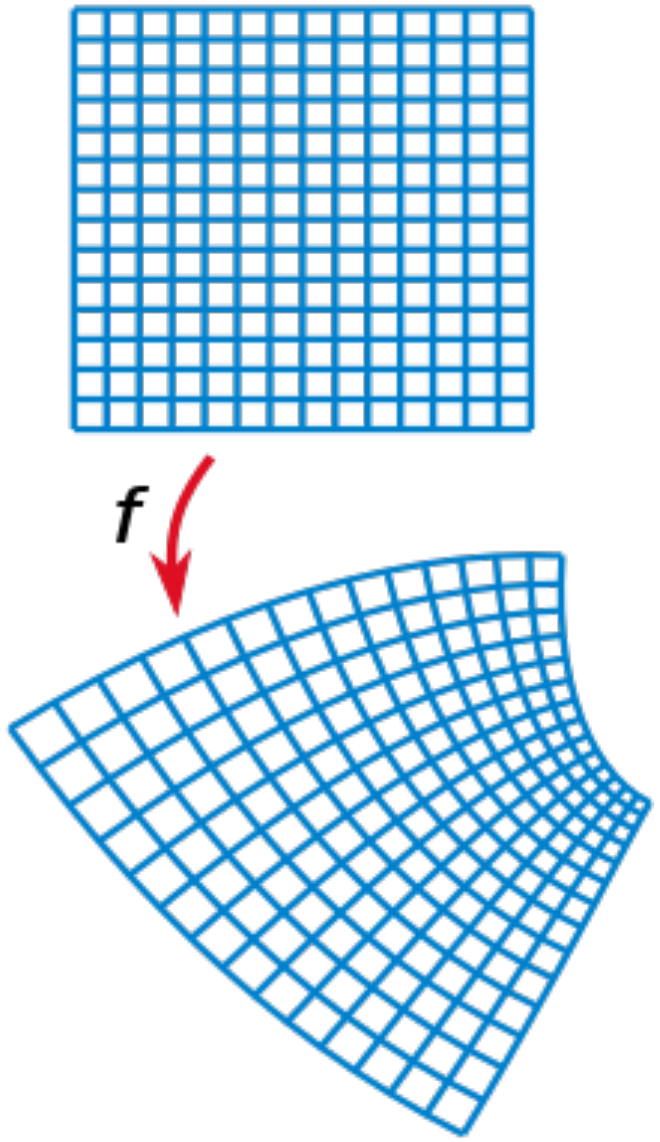
Higgs Inflation

Action that leads to inflation :

$$S = \int \left[\frac{M_{pl}^2}{2} \left(1 + \frac{\xi}{M_{pl}^2} \phi^2\right) R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{\lambda}{4} \phi^4 \right] \sqrt{-g} d^4 x.$$

non-minimal coupling term between gravity and scalar field

Conformal transformation



$$g_{\hat{\mu\nu}} = \Omega^2 g_{\mu\nu},\ g^{\hat{\mu}\nu} = \frac{1}{\Omega^2} g^{\mu\nu},\ \sqrt{-\hat{g}} = \Omega^4 \sqrt{-g}$$

$$\Gamma^{\hat{\mu}}_{\alpha\beta} = \Gamma^{\mu}_{\alpha\beta} + \frac{1}{\Omega}(\delta^{\mu}_{\beta}\partial_{\alpha}\Omega + \delta^{\mu}_{\alpha}\partial_{\beta}\Omega - g_{\alpha\beta}\partial^{\mu}\Omega)$$

$$\hat{R}^{\rho}_{\sigma\mu\nu} = \partial_{\mu}\hat{\Gamma}^{\rho}_{\nu\sigma} - \partial_{\nu}\hat{\Gamma}^{\rho}_{\mu\sigma} + \hat{\Gamma}^{\rho}_{\mu\lambda}\hat{\Gamma}^{\lambda}_{\nu\sigma} - \hat{\Gamma}^{\rho}_{\nu\lambda}\hat{\Gamma}^{\lambda}_{\mu\sigma}$$

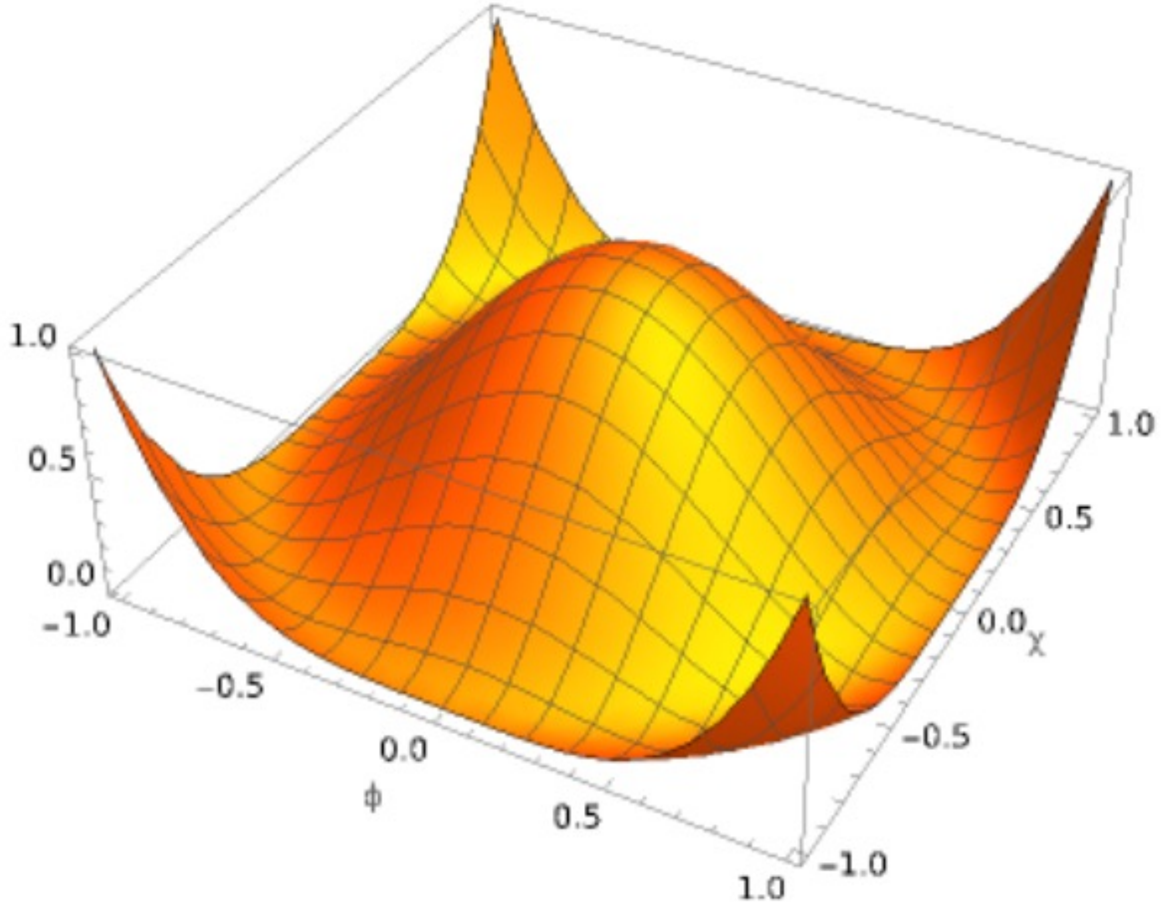
Conformal transformation

$$S = \int (\frac{1}{2}M_{pl}^2 f(\phi)R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi))\sqrt{-g}d^4x$$: Jordan frame

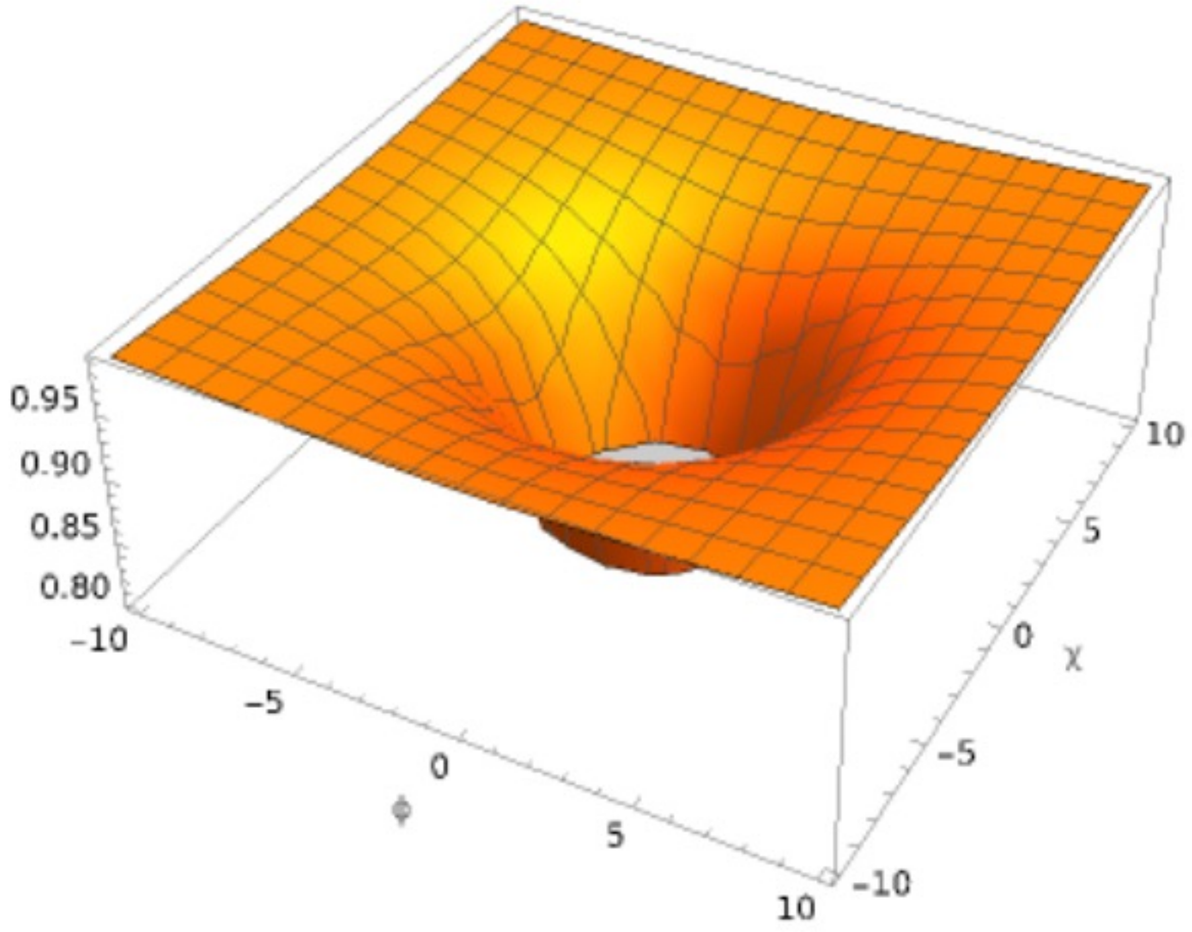
$$S = \int (\frac{1}{2}M_{pl}^2 \hat{R} - \frac{1}{2}g^{\hat{\mu}\nu}\partial_\mu\hat{\phi}\partial_\nu\hat{\phi} - V\hat{(}\phi))\sqrt{-\hat{g}}d^4x$$: Einstein frame

With, $\frac{\partial\hat{\phi}}{\partial\phi} \equiv \sqrt{\frac{6M_{pl}^2}{\Omega^2}(\frac{\partial\Omega}{\partial\phi})^2 + \frac{1}{\Omega^2}}, V\hat{(\phi)} \equiv \frac{V(\phi)}{\Omega^4}$

Conformal transformation



Potential in Jordan frame



Potential in Einstein frame

Cosmological parameter

From potential

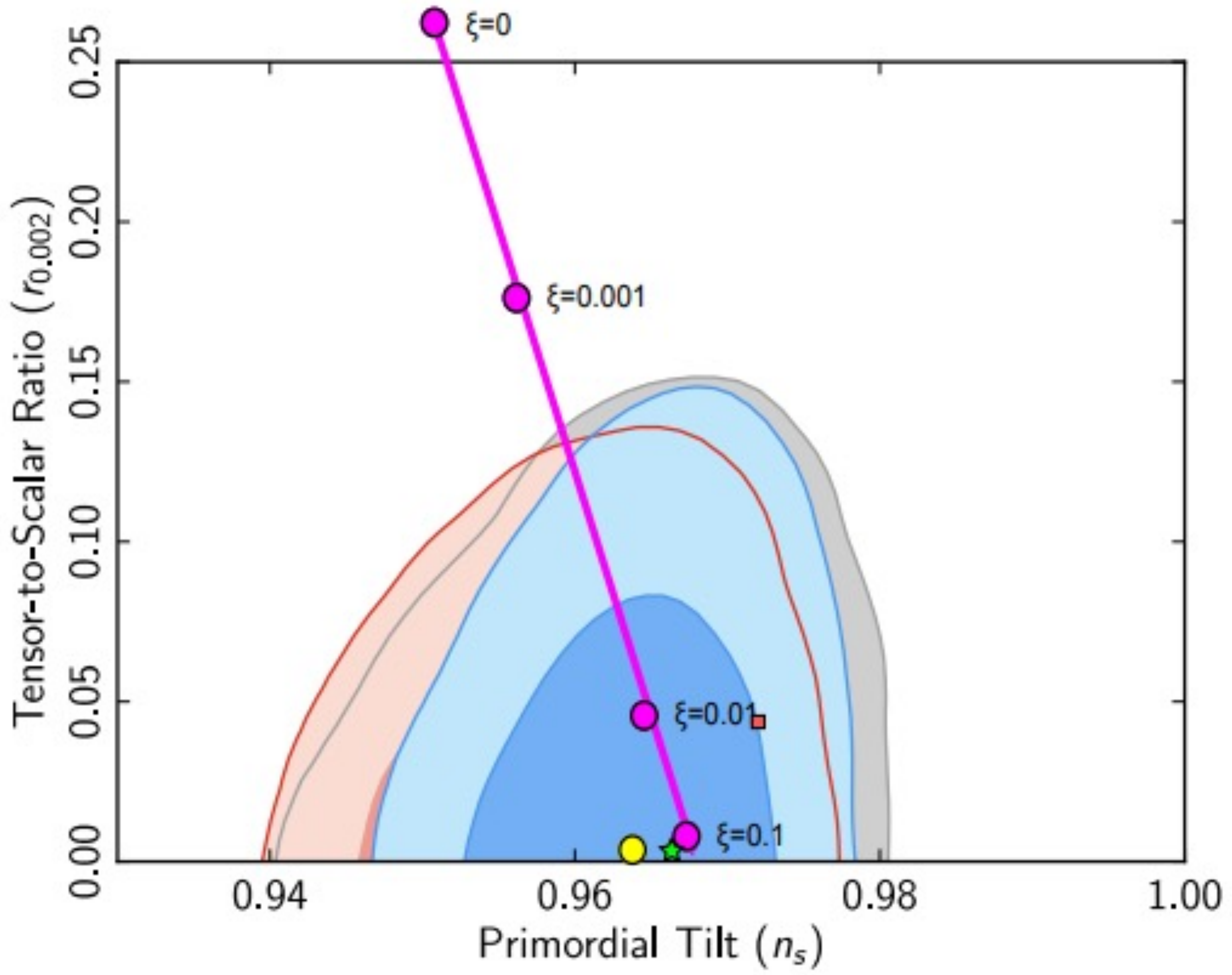
V

$$r = 16\varepsilon_{\nu} = \frac{12}{N^2}, n_s = 1 + 2\eta_{\nu} - 6\varepsilon_{\nu} = 1 - \frac{2}{N} - \frac{9}{2N^2}$$

Inflation condition requires that number of e-folds N=60

$$r = 0.0033,\ n_s \approx 0.967$$

Observational constraint



This figure is predictions from the inflationary models and the Planck satellite observed bounds. We can check that Higgs inflation(green star) fits in observational constraints.

Conclusion

I was able to check that Higgs inflation fits in observational constraint from CMB measurement successfully.

Further discussion

That action really gives the inflationary solution?
Gauge invariance of observational quantitiy
etc.

Reference

[1] Dhong Yeon Cheong, Sung Mook Lee, and Seong Chan Park. Progress in higgs inflation. *Journal of the Korean Physical Society*, 78(10):897–906, feb 2021.

[2] Fedor Bezrukov. The higgs field as an inflaton. *Classical and Quantum Gravity*, 30(21):214001, oct 2013.

[3] J. M. Fernández Cristóbal. Weyl invariance in metric $f(R)$ gravity. *Rev. Mex. Fis.*, 64(2):181–186, 2018.

[4] David I. Kaiser. Conformal transformations with multiple scalar fields. *Physical Review D*, 81(8), apr 2010.

[5] Erik Schildt. Higgs inflation, 2018.

[6] Ross N. Greenwood, David I. Kaiser, and Evangelos I. Sfakianakis. Multifield dynamics of higgs inflation. *Physical Review D*, 87(6), March 2013.

PQ Inflation

CAU HEP Workshop
THEP
MyeongJung Seong
27 Dec 2023

Contents

- Inflation
- Axion
- PQ inflation

Peccei-Quinn Inflation at the Pole and Axion Kinetic Misalignment

arXiv: 2310.17710

Inflation

- To solve cosmological problems
ex) Horizon problem
Flatness problem

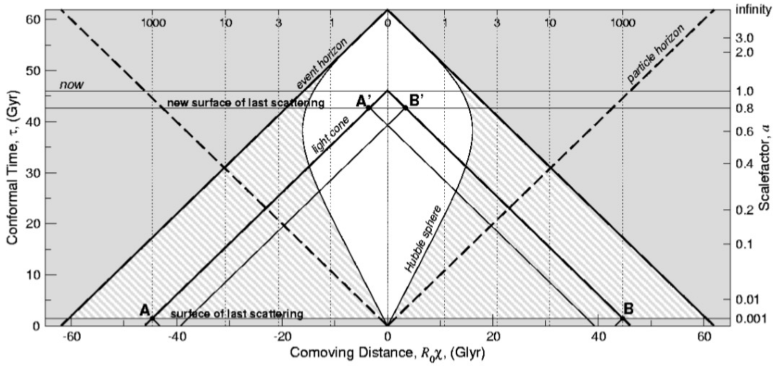
- Universe expands very fast

- Inhomogenity also can be explained by quantum fluctuation.

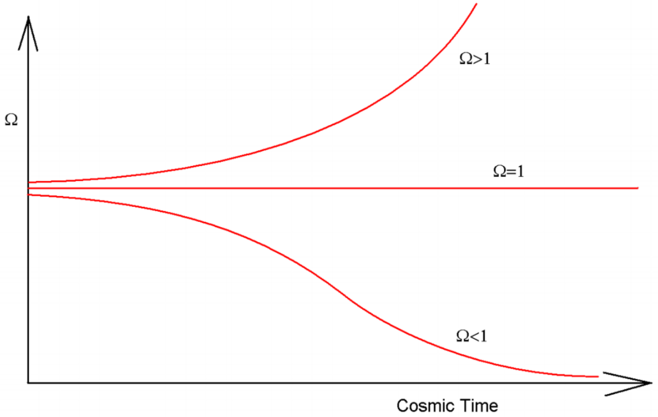
- First introduced by Alan Guth (1979)

- Need enough time for inflation

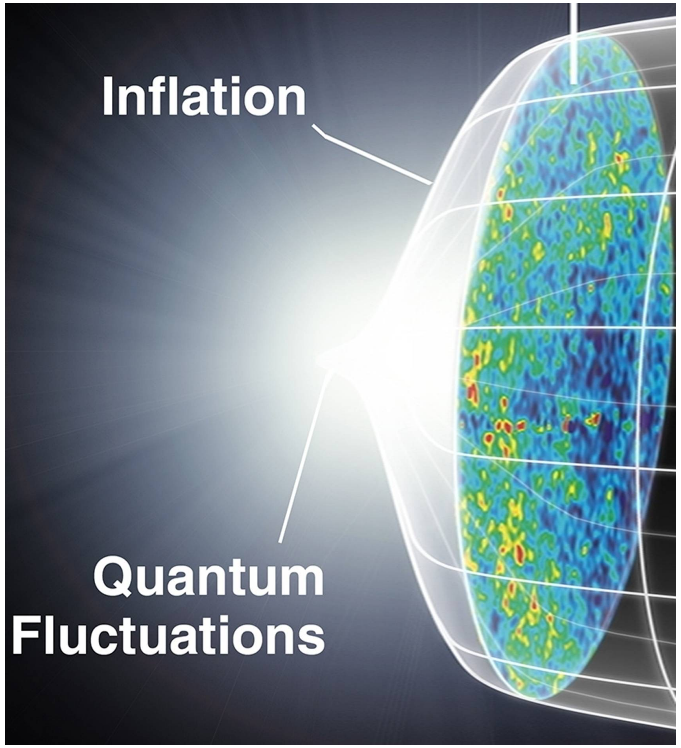
- Observational result (Planck)



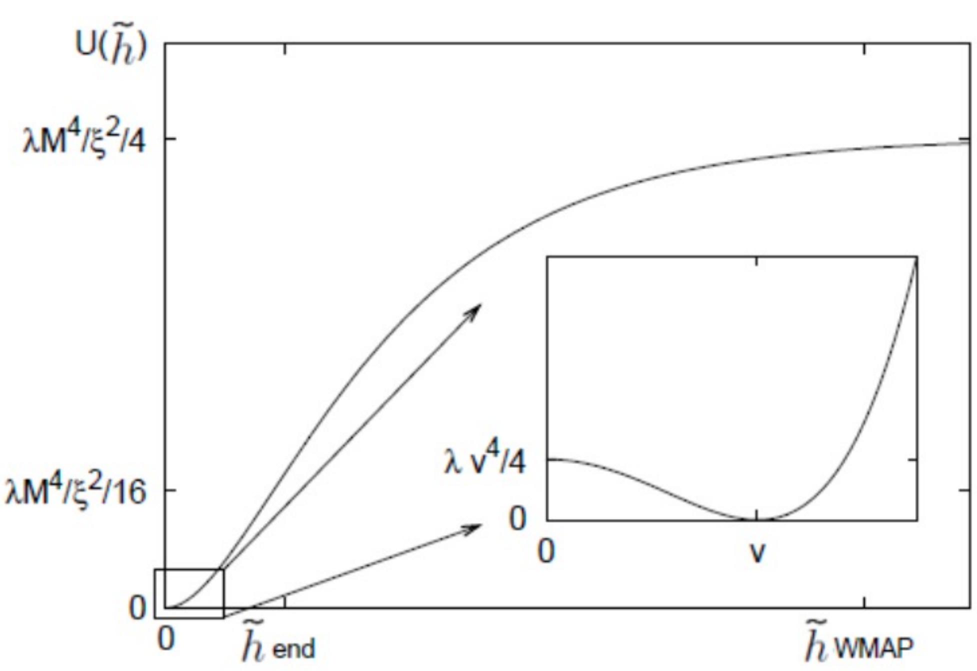
Horizon problem



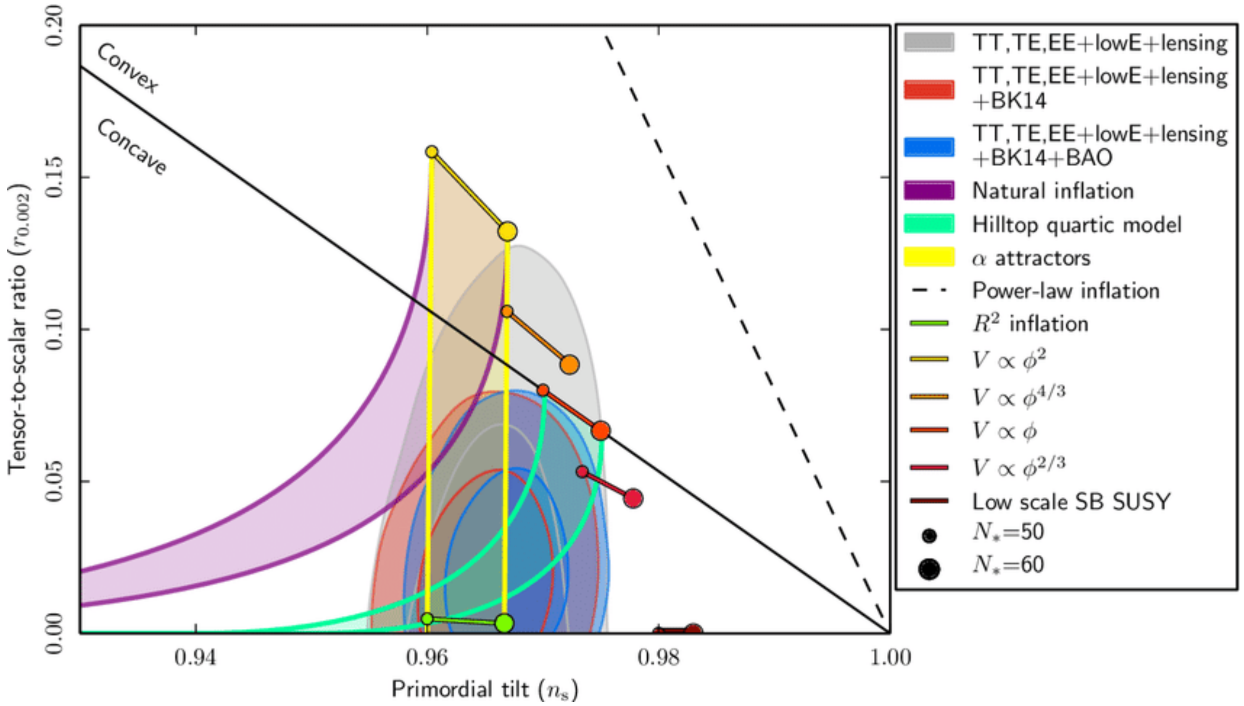
Flatness problem



Inflation



Slow roll for enough inflation



PLANCK constraints

Axion

- QCD Lagrangian has two independent CP violating source

- Sum of two constants should be very small by Nedm experiment

- Introduce new dynamical scalar

- PQ symmetry: axion

- First Introduced by Peccei & Quinn (1977)

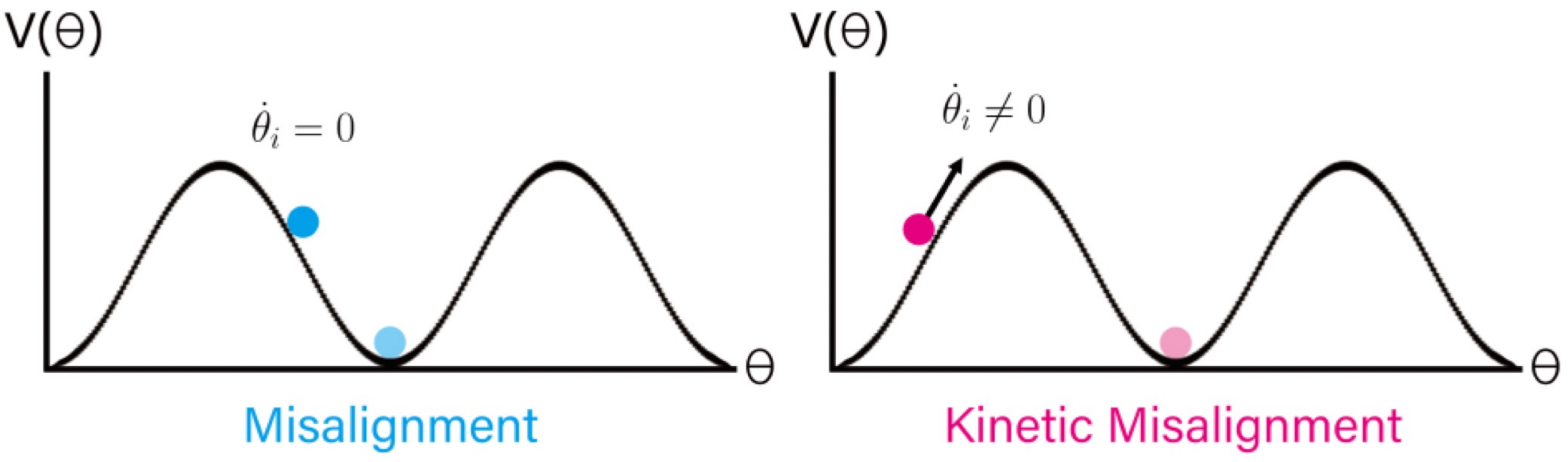
- Non-zero initial velocity to enhance abundance: Kinetic

$$\mathcal{L}_{QCD} = -\frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \bar{q}(iD)q - \bar{q}me^{-\theta_Y\gamma^5}q + \theta_{QCD}\frac{g^2}{16\pi^2}G\tilde{G}$$

CP violation

Neutron Electric Dipole Moment: $\theta = \theta_{QCD} + \theta_Y \leq 10^{-10}$

New scalar field a: Axion - $\bar{\theta} = \theta + \frac{a}{f} \rightarrow 0$



PQ Inflation

- Higgs pole inflation with PQ symmetry

- Radial motion, PQ conserving
-> inflaton (Black)

- Angular motion, PQ violating
-> axion (Blue)

- Explain inflation and Strong CP

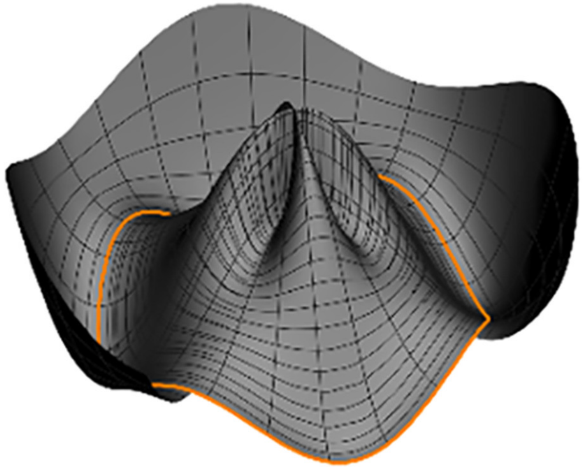
- Reheating after inflation also can be explained

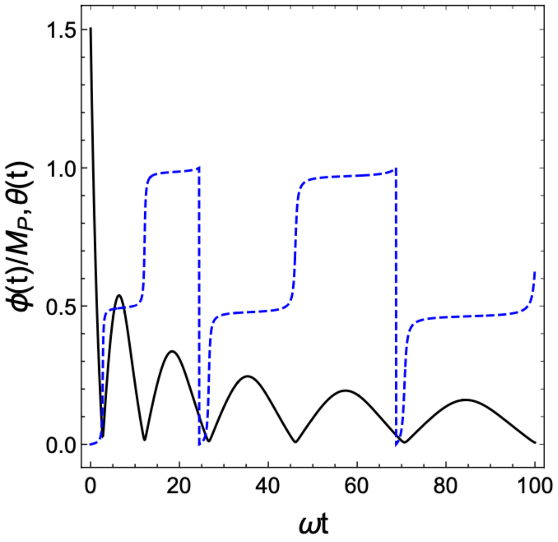
$$\frac{\mathcal{L}_E}{\sqrt{-g_E}} = -\frac{1}{2}M_P^2 R + \frac{1}{2}(\partial_\mu \phi)^2 + 3M_P^2 \sinh^2\left(\frac{\phi}{\sqrt{6}M_P}\right)(\partial_\mu \theta)^2 - V_E(\phi,\theta)$$

$$V_E(\phi,\theta) = V_{\rm PQ}(\phi) + V_{\rm PQV}(\rho,\theta)$$

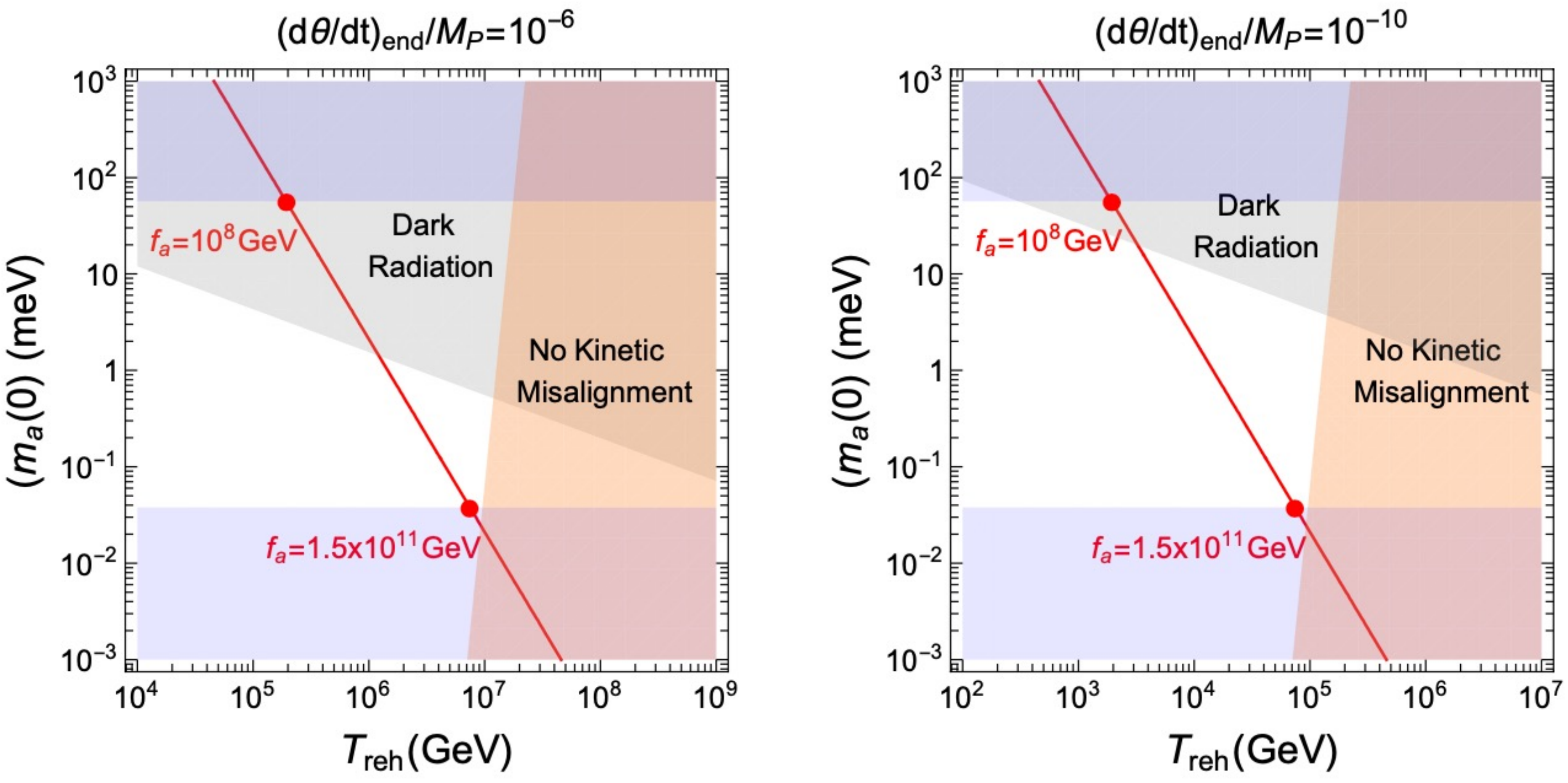
$$V_{\rm PQ}(\phi) = V_0 + \frac{1}{4}\lambda_\Phi\left(6M_P^2\tanh^2\left(\frac{\phi}{\sqrt{6}M_P}\right) - f_a^2\right)^2,$$

$$V_{\rm PQV}(\rho,\theta) = 3^{n/2}M_P^4\tanh^n\left(\frac{\phi}{\sqrt{6}M_P}\right)\sum_{k=0}^{[n/2]}|c_k|\cos\big((n-2k)\theta + A_k\big)$$





PQ Inflation



Muon g-2 and proton decay in the minimal SU(5) GUT with split particle masses

Theoretical High Energy Physics Group
SungBo Sim

CAU HEP Center Workshop, Dec 27, 2023

Outline

- Muon g-2
- SUSY
  - Gauge mediation
- Proton decay
- Gauge coupling unification

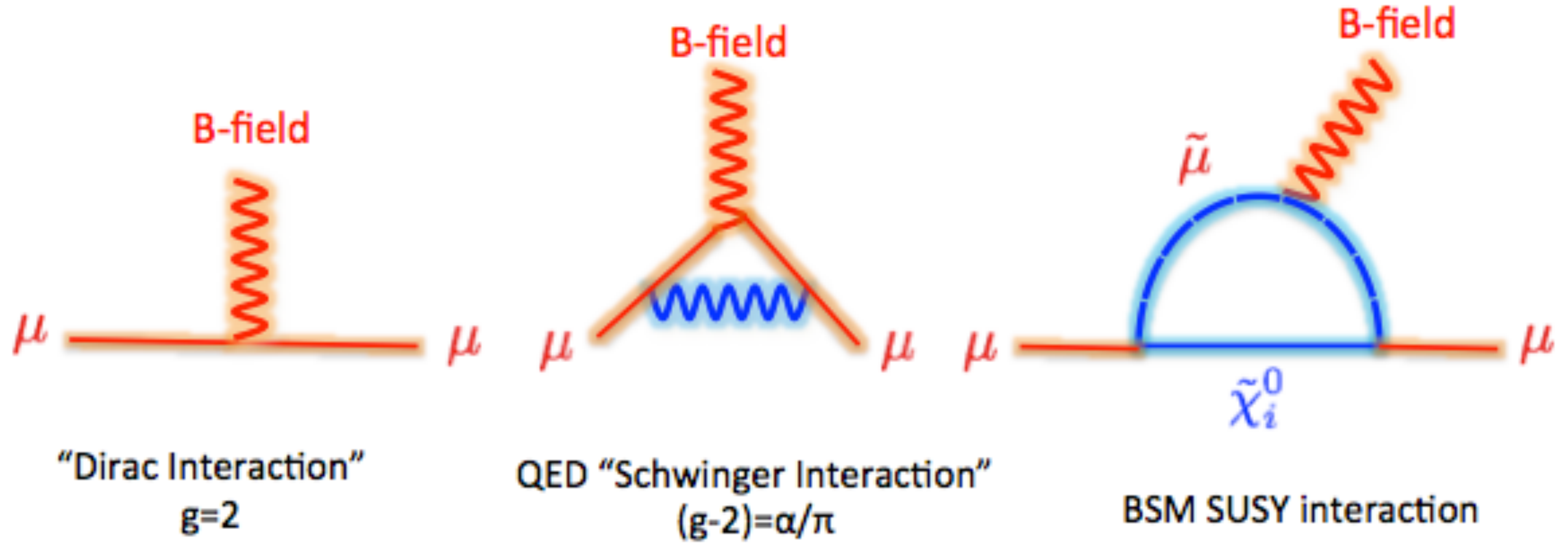
Muon g-2

- In the non-relativistic limit, the Dirac equation in the presence of an external magnetic field produces a Hamiltonian,

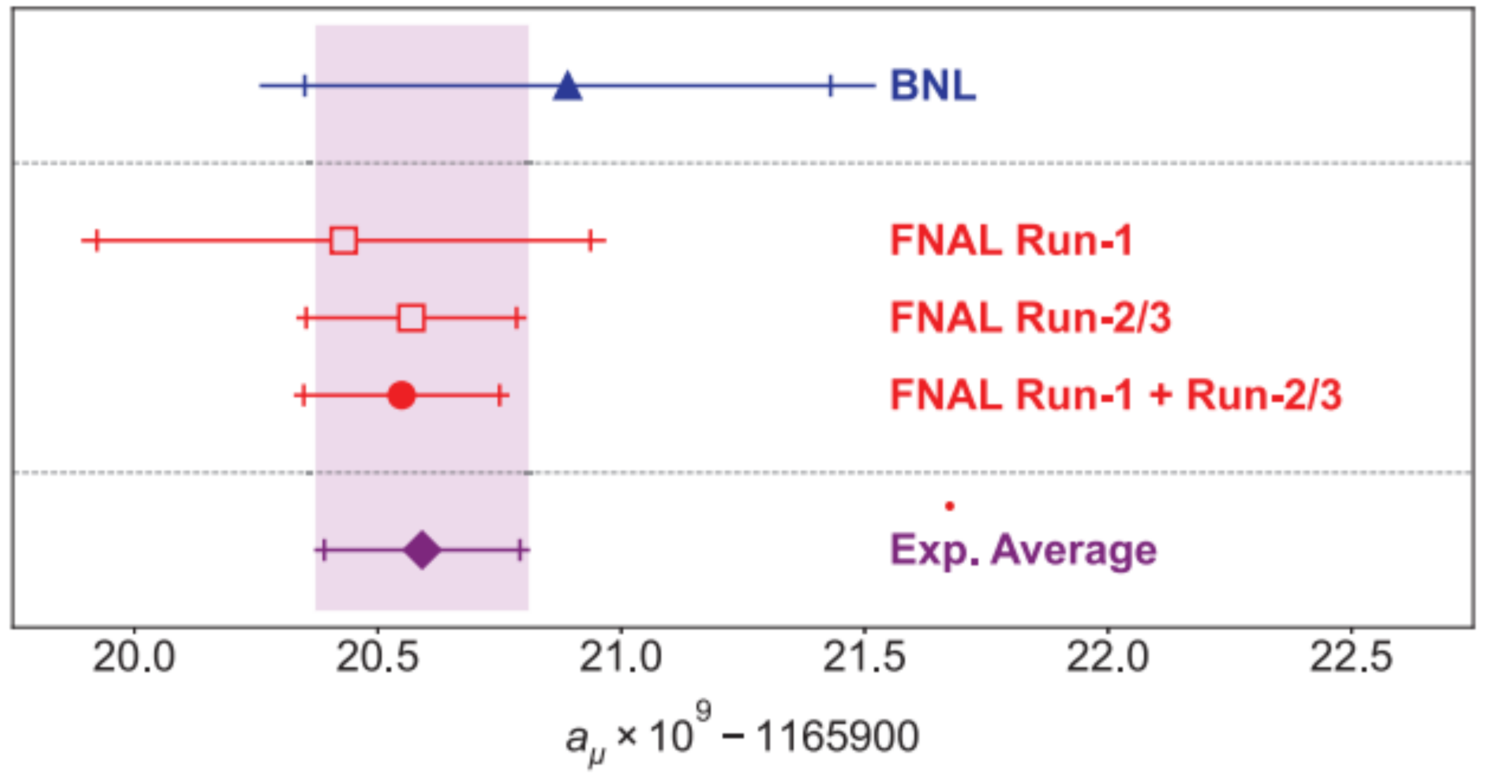
$$H = \frac{\vec{p}^2}{2m} + V(r) + \frac{e}{2m}\vec{B} \cdot (\vec{L} + g\vec{S})$$

- In the tree-level, the Dirac equation implies $g = 2$

- By the late 1940s there were experimental data that could be partially explained by the electron having an anomalous magnetic moment, $a_\mu = \frac{g-2}{2}$



Muon G-2 experiment. UCL g-2. (n.d.). https://www.hep.ucl.ac.uk/muons/g-2/



(Aguillard et al., Measurement of the positive muon anomalous magnetic moment to 0.20 PPM, 2023)

→ New physics beyond the Standard Model?

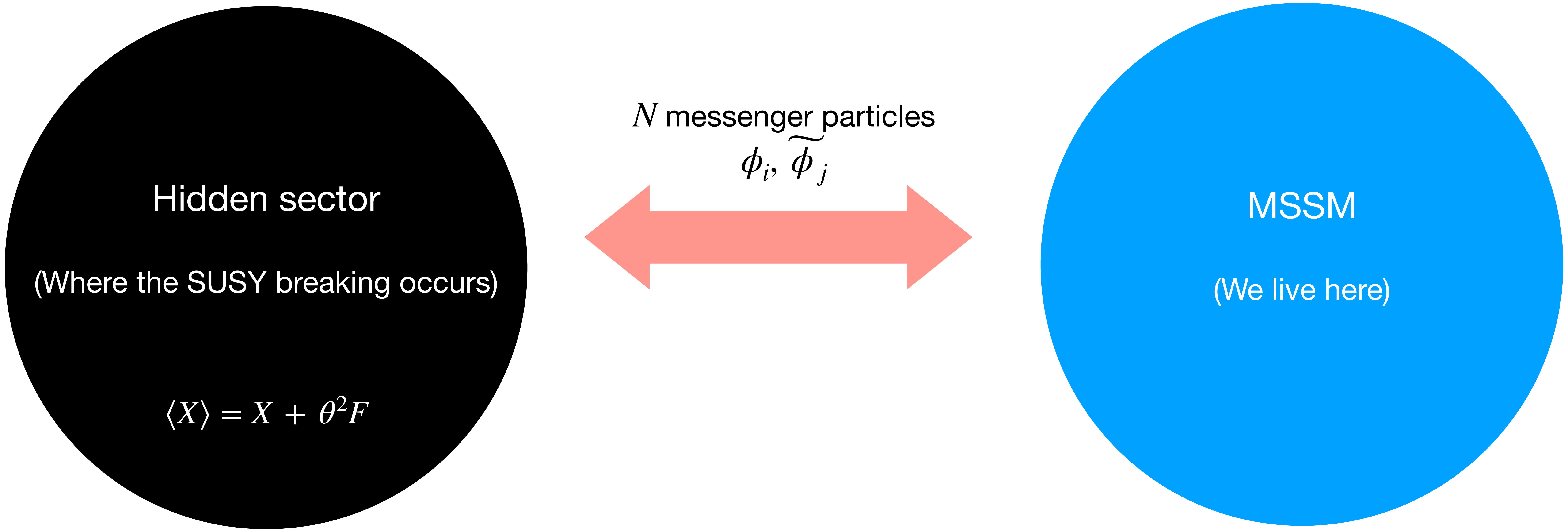
SUSY & its contribution to the muon g-2

- We extend the Standard Model by introducing supersymmetry.
- In this scenario, we have supersymmetric pairs for each Standard Model particles.
- New interactions resulting from new symmetry can contribute to the muon g-2.

- To address the current muon g-2 anomaly, we need to constrain the parameter space of the sparticle masses. In particular, we need light slepton and gaugino.

SUSY breaking - (Ordinary)Gauge mediation

- The hidden sector is parameterized by a singlet field X which is a spurioun for SUSY breaking
- N pairs of messenger fields $\phi_i, \tilde{\phi}_j$
- The messengers interact with X via Yukawa-like couplings, $W = \lambda_{ij} X \phi_i \tilde{\phi}_j$

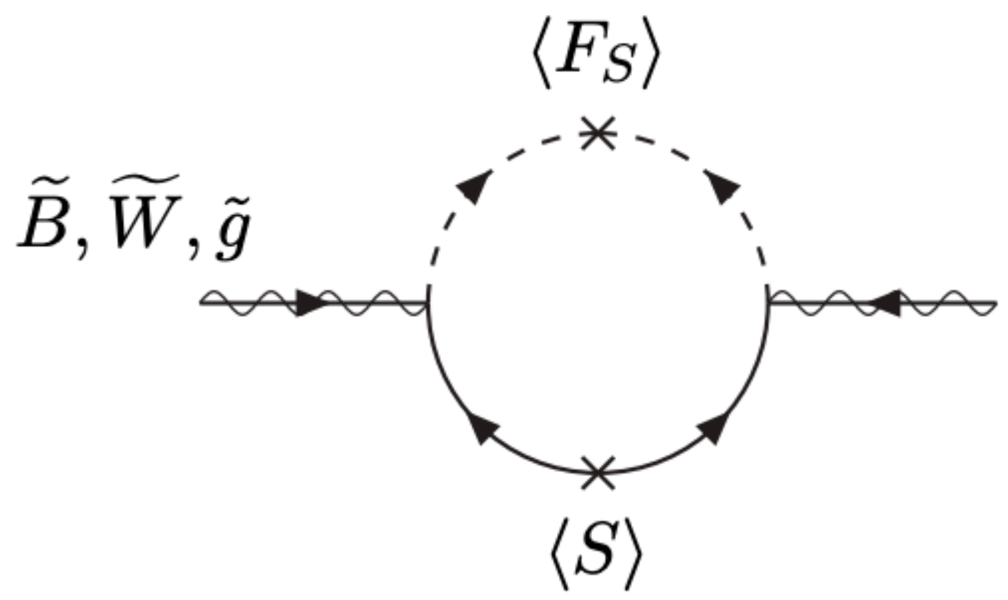


SUSY breaking - (Ordinary)Gauge mediation

- The MSSM gauginos obtain masses from the 1-loop diagram

$$M_i = \frac{\alpha_i}{4\pi}\Lambda N, \text{ Where } \Lambda \equiv \langle F \rangle / \langle X \rangle$$

  - The gaugino mass ratios $M_1 : M_2 : M_3 = \alpha_1 : \alpha_2 : \alpha_3 \approx 1 : 2 : 6$

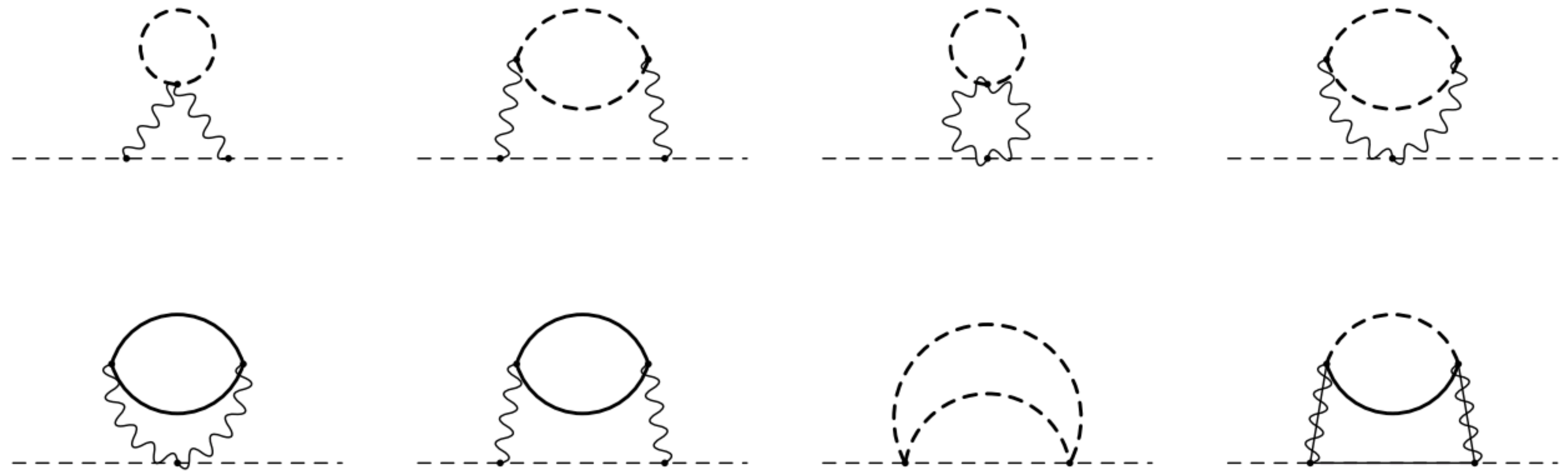


In this diagram, S corresponds to the X

- The scalars of the MSSM gets a squared mass given by

$$m_{\tilde{f}}^2 = 2\sum_{i=1}^{3} C_{\tilde{f}}^i \left(\frac{\alpha_i}{4\pi}\right)^2 \Lambda^2 N$$

  - C^i is the corresponding quadratic Casimir invariants.



We can split the masses of sparticles!

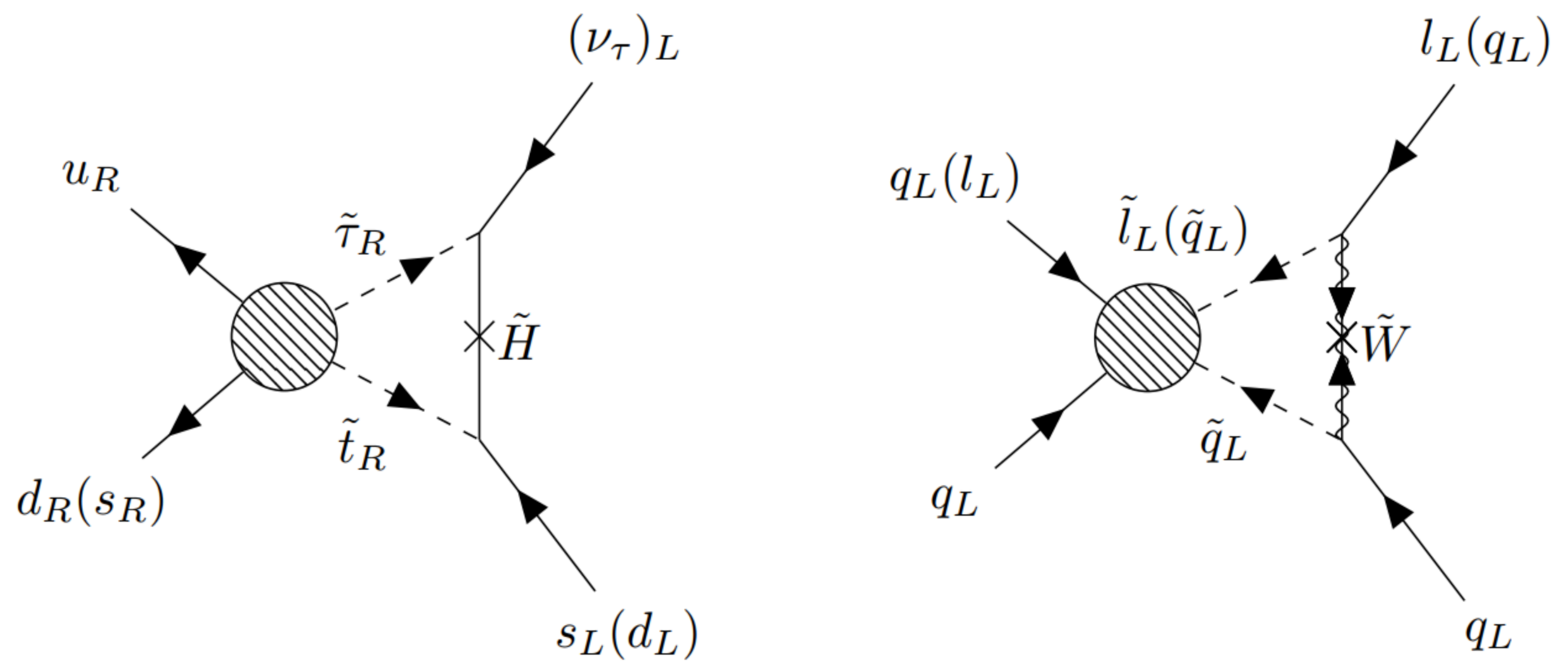
Proton decay

- From the Baryon and Lepton number violating superpotential, proton decay can be expected in the SUSY SU(5) scenario.

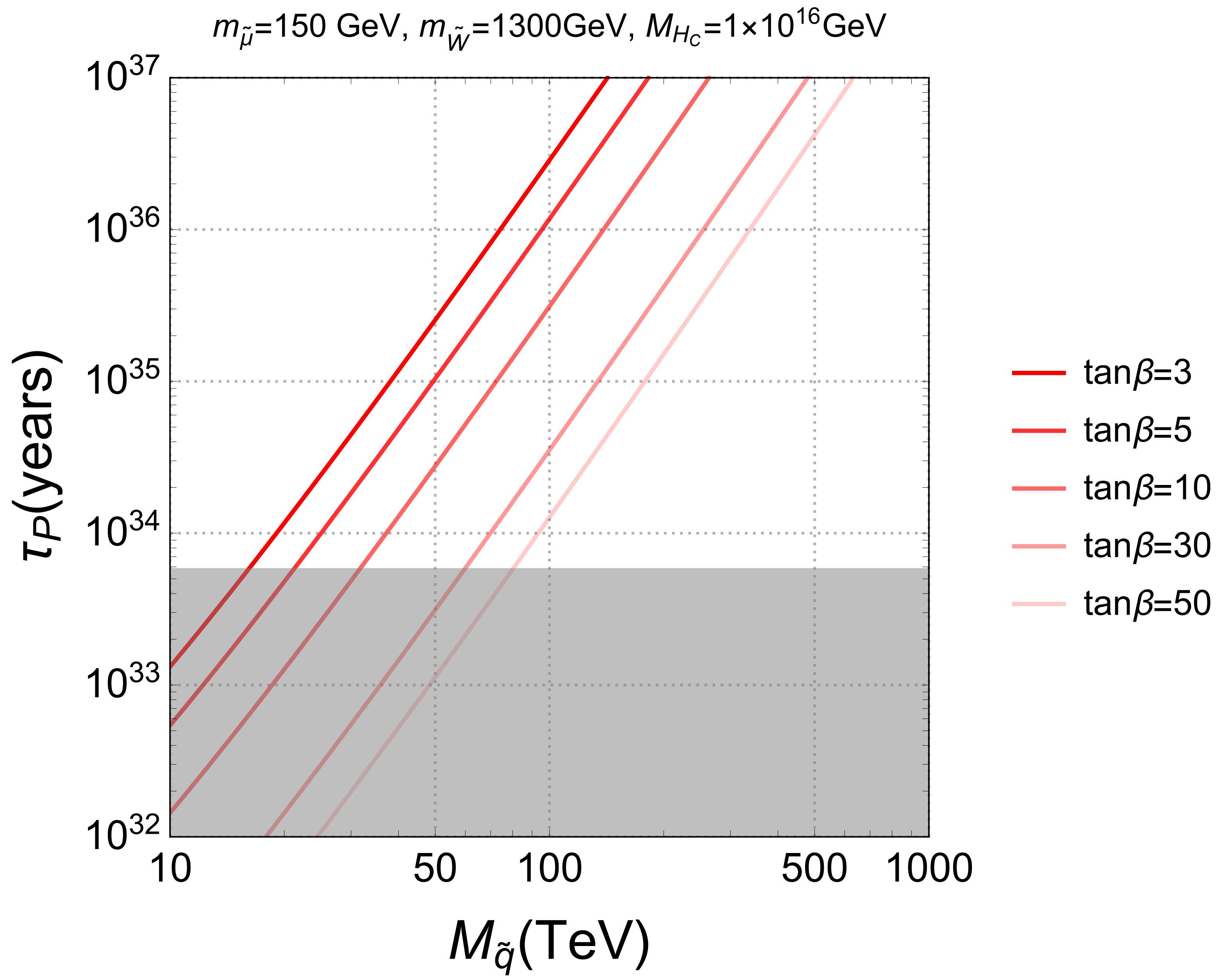
$$W_5 = \frac{1}{2M_{H_C}} QQQL + \frac{1}{M_{H_C}} u^c e^c u^c d^c$$

- Proton decay lifetime can be obtained by

$$\tau_p \simeq 10^{35} \times \sin^4 2\beta \times \left(\frac{F(\mu_H, M_{\tilde{q}}^2, m_{\tilde{l}}^2)^{-1}}{10^2 TeV} \right)^2 \left(\frac{M_{H_C}}{10^{16} GeV} \right)$$



Feynman diagrams for $p \to K^+ \bar{\nu}$

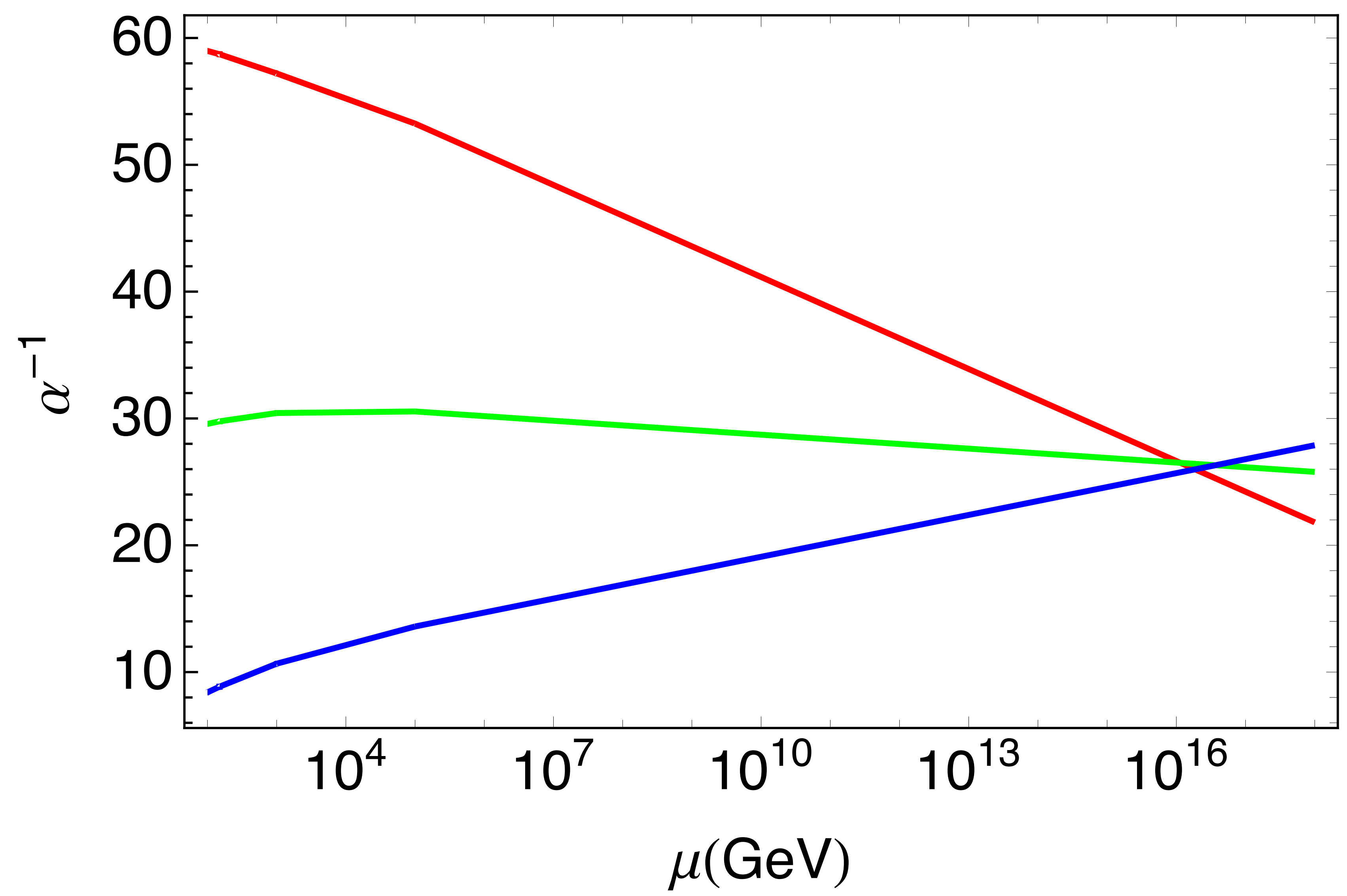


Gauge coupling unification

- Under GUT scenario, we expect gauge couplings to be unified at GUT scale($\sim 10^{16}\ GeV$)
- The running of gauge coupling in on loop is given by $\alpha_i^{-1}(\mu_2) = \alpha_i^{-1}(\mu_1) - \frac{b_i}{4\pi}\ln\left(\frac{\mu_2^2}{\mu_1^2}\right)$, where $\mu_2 > \mu_1$
- b_i's are the 1-loop beta function coefficients which are derived from group theory. According to the mass spectrum we have, they are as follows:

$$b_i^{SM} = \left(\frac{41}{10}, \frac{19}{6}, -7\right),\ b_i' = \left(5, -\frac{13}{6}, -6\right),\ b_i'' = \left(\frac{27}{5}, -\frac{1}{6}, -4\right),\ b_i^{MSSM} = \left(\frac{33}{5}, 1, -3\right)$$

SM particle | Bino, slepton | Wino, higgsino, and gluino | Squarks



Summary

- There exist an anomalous magnetic moment which can't be explained by a standard model
- Supersymmetry is a leading candidate for the extension of standard model
- With the split mass spectrum, SUSY SU(5) can address the muon g-2 anomaly
- With this model, expected proton decay lifetime is compatible with the experimental bound, and it can unify the gauge couplings well.

Non-Thermal Leptogenesis in Peccei-Quinn Inflation

Theoretical High Energy Physics Group
Jun Ho Song
CAU Hep Workshop

What is leptogenesis?

- Leptogenesis is a model that can explain baryon asymmetry of the current universe through the seesaw model

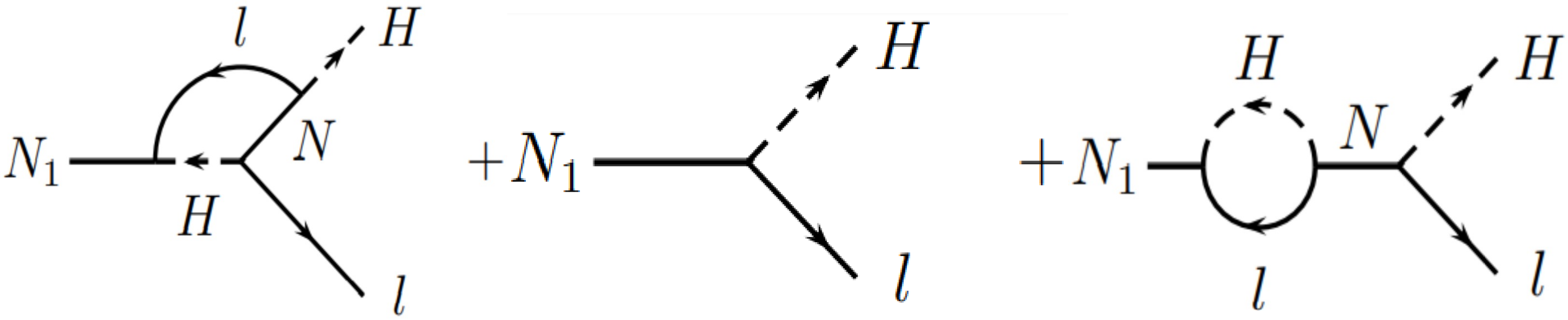
What is different about using the PQ inflation model?

- Non zero Initial number density of RHN!

Leptogenesis

$$\mathcal{L} \supset i\overline{N}_R\gamma^\mu\partial_\mu N_R + h\bar{l}_L H N_R + M_N \overline{N}^c_R N_R + h.c$$

Seesaw Type-1

$\Gamma(N_1 \to \bar{l}_L H) =$ 

$$i\mathcal{M} = h_{1i} + h^*_{ij}h_{kj}h_{ki}F_N$$

$$\epsilon_1 = \frac{\Gamma(N_1 \to \bar{l}_L H) - \Gamma(N_1 \to l_L H^*)}{\Gamma(N_1 \to \bar{l}_L H) + \Gamma(N_1 \to l_L H^*)} \simeq -\frac{3}{16\pi}\frac{1}{(hh^\dagger)_{11}}\sum_{j=2,3}\mathrm{Im}[(hh^\dagger)^2_{1j}]\frac{M_{N_1}}{M_{N_j}}$$

$$N_1(t_e < t < t_{RH}) \cong 0,$$

t_e: end of inflation

Initial number density of RHN

$$N_1(t_e < t < t_{RH}) \cong 0,$$

t_e: end of inflation

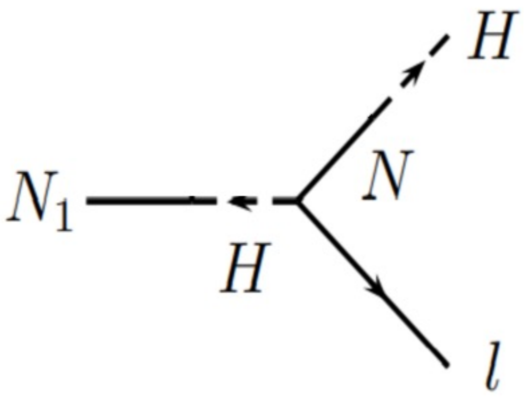
Thermal leptogenesis scenario

$$N_1(t_e < t < t_{RH}) \neq 0,$$

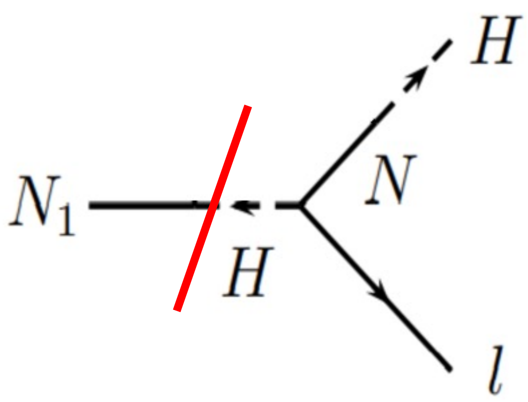
t_e: end of inflation

Non-Thermal leptogenesis scenario
(Our assumption)

Decay



Inverse Decay



$$M_{RHN} \gg T_{RH}$$

Initial number density of RHN

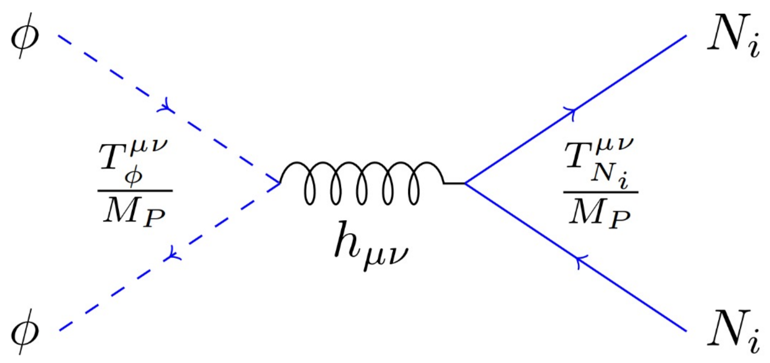
$N_1(t_e < t < t_{RH}) \neq 0,$

t_e: end of inflation

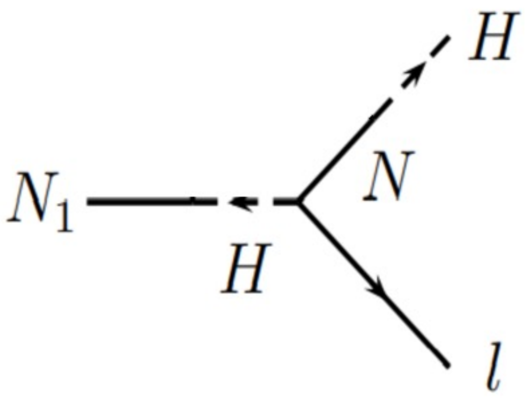
Non-Thermal leptogenesis scenario
(Our assumption)

$$M_{RHN} \gg T_{RH}$$

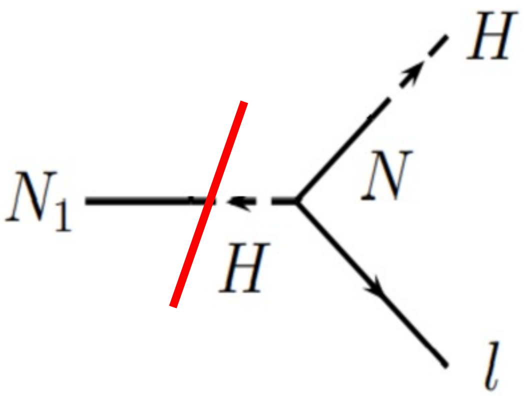
Inflaton scattering



RHN Decay



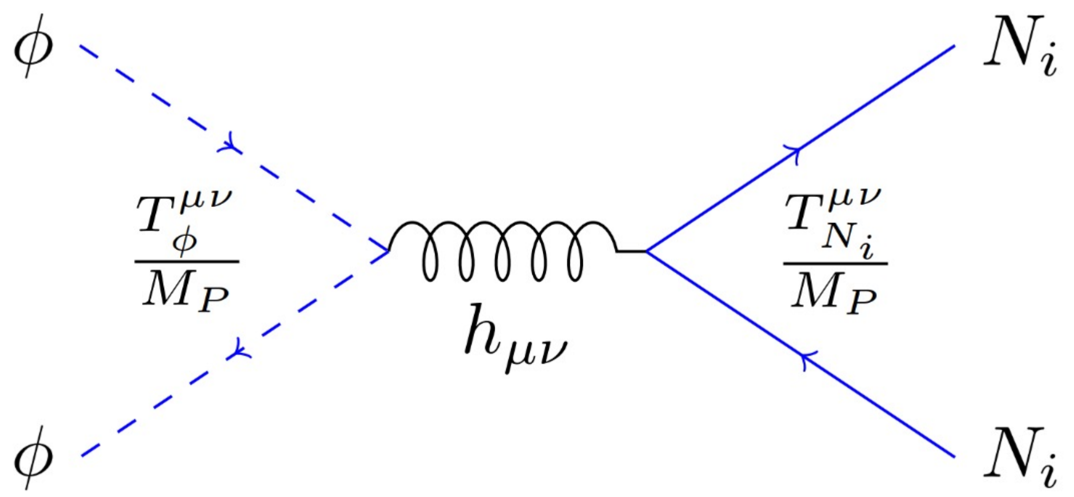
Inverse Decay



Gravitational Production of RHN

$$\sqrt{-g}L^1_{int} = \frac{1}{2M_P} h_{\mu\nu}\left(T^{\mu\nu}_{SM} + T^{\mu\nu}_{\phi} + T^{\mu\nu}_{N}\right).$$

$$g_{\mu\nu} \cong \eta_{\mu\nu} + \frac{h_{\mu\nu}}{M_P}$$

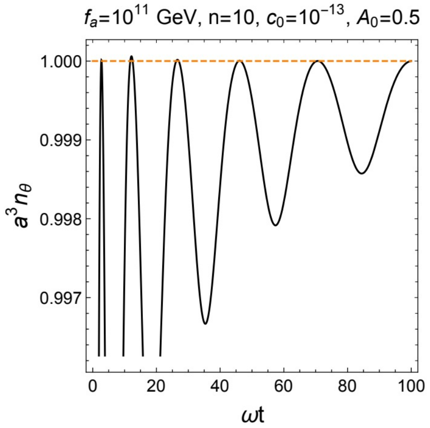


$$|\mathcal{M}^{\phi^k}_n|^2 = \frac{2\rho_\phi^2}{M_P^4}\frac{m_N^2}{s}\left[1 - \frac{4m_N^2}{s}\right]|(\mathcal{P}^k)_n|^2$$

$$\frac{dY^{\phi^k}_{N_1}}{da} = \frac{\sqrt{3}M_P}{\sqrt{\rho_{RH}}} a^2 \left(\frac{a}{a_{RH}}\right)^{\frac{3k}{k+2}} R^{\phi^k}_{N_1}(a)$$

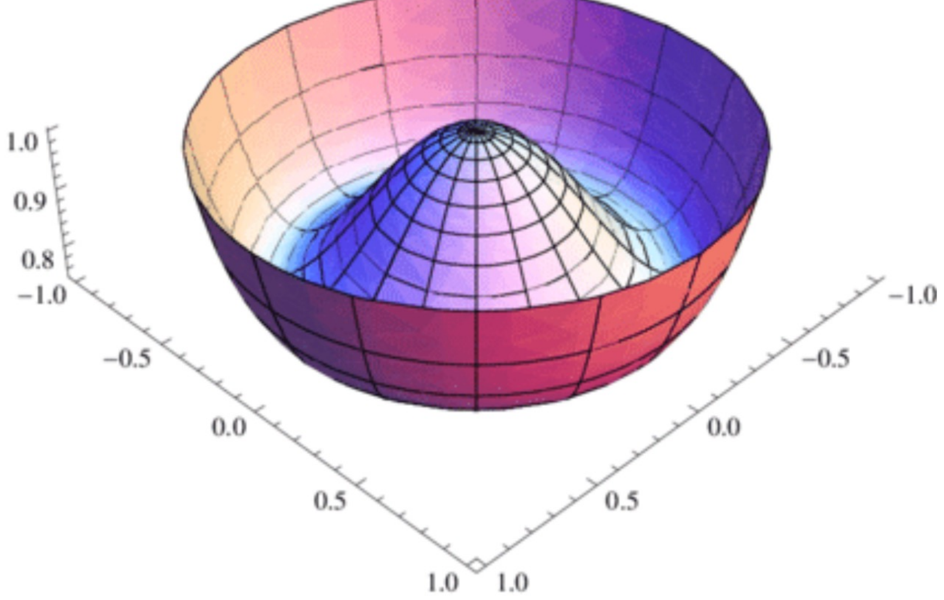
Peccei-Quinn inflation $U(1)_{PQ}$

$$\Phi = \frac{1}{\sqrt{2}}\,\rho\, e^{i\theta}$$



$$\frac{\mathcal{L}_E}{\sqrt{-g_E}} = -\frac{1}{2}M_P^2 R + \frac{1}{2}(\partial_\mu\phi)^2 + 3M_P^2\sinh^2\left(\frac{\phi}{\sqrt{6}M_P}\right)(\partial_\mu\theta)^2 - V_E(\phi,\theta)$$

$$V_E(\phi,\theta) \;=\; V_{\rm PQ}(\phi) + V_{\rm PQV}(\rho,\theta)$$



$$V_{\rm PQ}(\phi) \;=\; V_0 + \frac{1}{4}\lambda_\Phi\left(6M_P^2\tanh^2\left(\frac{\phi}{\sqrt{6}M_P}\right) - f_a^2\right)^2$$

$$V_{\rm PQV}(\rho,\theta) \;=\; 3^{n/2}M_P^4\tanh^n\left(\frac{\phi}{\sqrt{6}M_P}\right)\sum_{k=0}^{[n/2]}|c_k|\cos\Big((n-2k)\theta + A_k\Big)$$

Interaction

Sub-dominant about Reheating

$$\mathcal{L}_{Q,\mathrm{int}} = -y_Q \Phi \bar{Q}_R Q_L + \mathrm{h.c.}$$

$$\mathcal{L}_{\mathrm{gluons}} = \frac{g_s^2}{32\pi^2}\left(\bar{\theta} + \xi \frac{a}{f_a}\right) G^a_{\mu\nu} \tilde{G}^{a\mu\nu}$$

Due to Strong CP problem!

dominant about Reheating

$$\Delta V_E = \lambda_{H\Phi} |\Phi|^2 |H|^2$$

$$\mathcal{L}_{\mathrm{int}} = -\frac{1}{2}\lambda_\Phi \phi^2 a^2$$

$$\frac{\Gamma_{\phi\phi\to aa}}{\Gamma_{\phi\phi\to HH}} \simeq \frac{\lambda_\Phi^2}{2\lambda_{H\Phi}^2}$$

$$\lambda_{H\Phi} \gtrsim \frac{1}{\sqrt{2}}\lambda_\Phi$$ Reheating condition

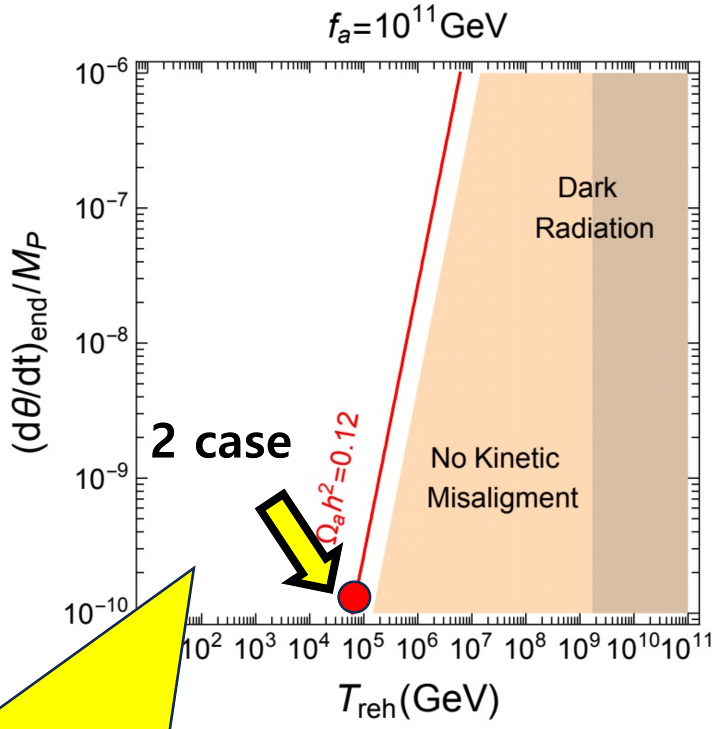
Matching

Constraints

1. $M_{N_1} = 10^{13} GeV$ 2. $\epsilon_1 = 10^{-6}$ 3. $f_a = 10^{11} GeV$

$$Y_B = \frac{n_B}{s} = \frac{28}{79}\epsilon_1 \frac{n_{N_1}(T_{RH})}{s} \quad (s = \frac{2\pi^2 g_{RH} T_{RH}^3}{45}, \text{entropy density})$$

$Y_B^{Planck} \cong 8.7\times10^{-11}$.



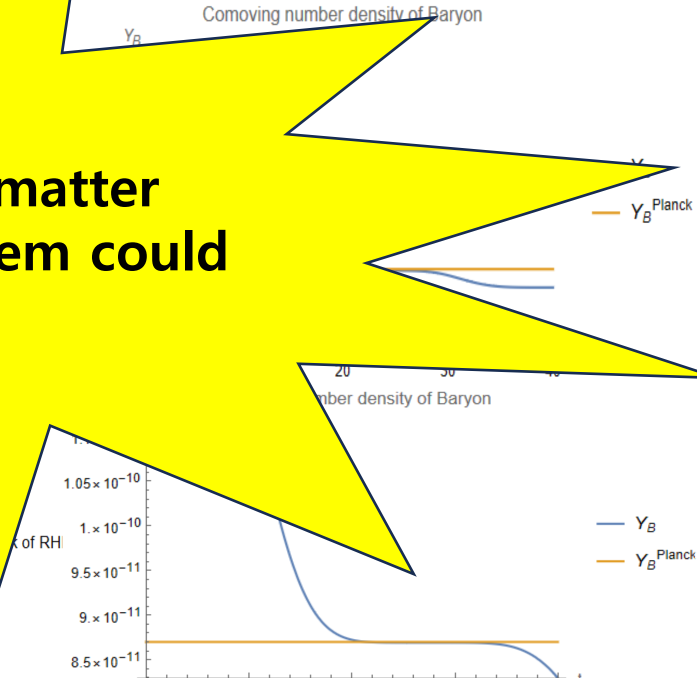
1 case :

$\lambda_{H\Phi} = 10^{-10.57}$
T_{reh}: $6.0 * 10^4 GeV$
initial axial velo...
:$10^{-6} M_p$

2 case :

$\lambda_{H\Phi} = 10^{-10.6}$
T_{reh}: $5.5 * 10^4 GeV$
initial axial velocity
$10^{-10} M_p$

baryon asymmetry, dark matter abundance, strong CP problem could be solved!

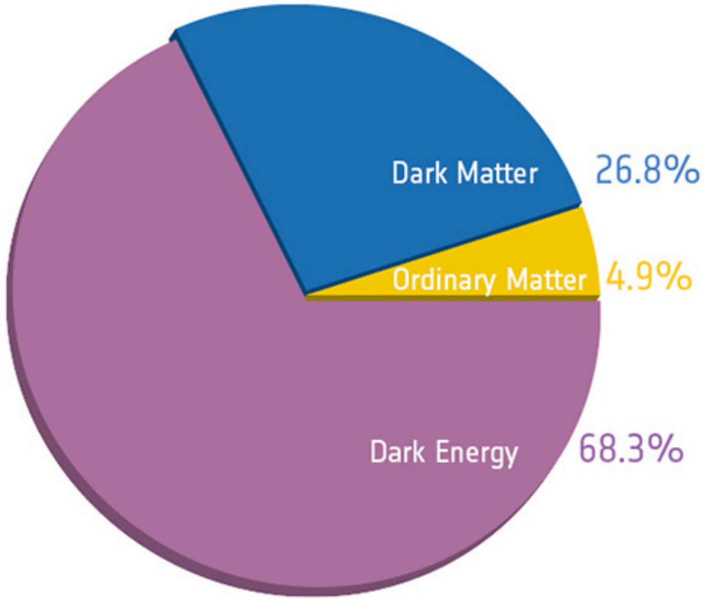


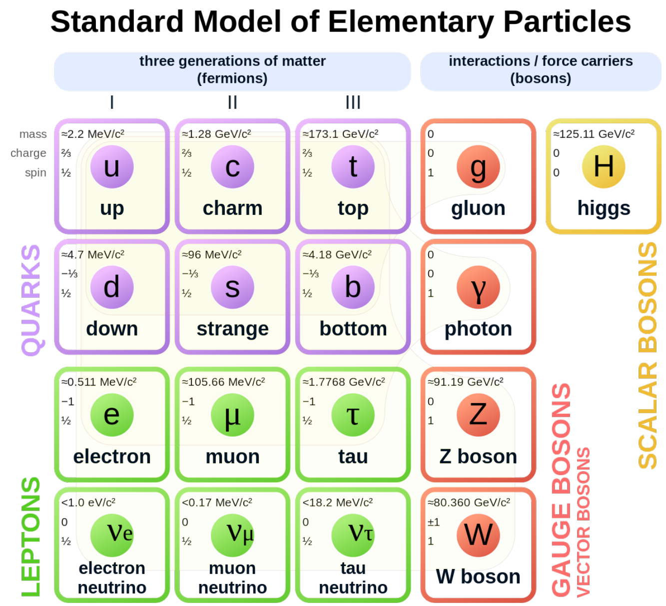
Thank you

Positivity Bounds on Higgs-Portal DM
Freeze-out vs. Freeze-in

SeongSik Kim[1], Hyun Min Lee[1], and Kimiko Yamashita[2]
(Chung-Ang University[1], Ibaraki University[2])
JHEP 11 (2023) 119 & *JHEP* 06 (2023) 124

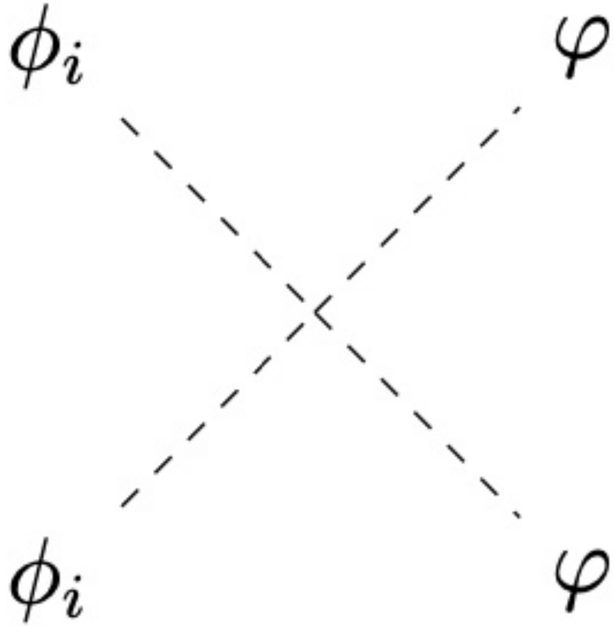
Dark Matter and Higgs





- Most of the nature of Dark Matter (DM) is currently unknown, their origin and interactions especially.
- Higgs is the last particle discovered in the Standard Model (SM). Within our current understanding, Higgs is the most probable particle in the SM sector interacting with DM.

Dark Matter and Higgs



dim-8 operators

$$\mathscr{L}_{\text{dim}-8} \supset \frac{4}{6\Lambda^4} d_2' \lambda_H |H|^4 (\partial_\mu \varphi)^2 + \frac{2}{6\Lambda^4} d_4' \lambda_H |H|^4 |\partial_\mu H|^2$$

$$\begin{aligned}\mathcal{L}_2 \; &= \; \frac{C^{(1)}_{H^2\varphi^2}}{\Lambda^4} O^{(1)}_{H^2\varphi^2} + \frac{C^{(2)}_{H^2\varphi^2}}{\Lambda^4} O^{(2)}_{H^2\varphi^2} \\ &+ \frac{C_{\varphi^4}}{\Lambda^4} O_{\varphi^4} + \frac{C^{(1)}_{H^4}}{\Lambda^4} O^{(1)}_{H^4} + \frac{C^{(2)}_{H^4}}{\Lambda^4} O^{(2)}_{H^4} + \frac{C^{(3)}_{H^4}}{\Lambda^4} O^{(3)}_{H^4}\end{aligned}$$

- Effective Field Theory (EFT) allows us to investigate the theory of DM without knowing their identity exactly, up to the cutoff scale.
- Thus, it is natural to think EFT of Higgs interacts with DM.
- This possibility is called Higgs-portal.
- In EFT, dim-4, dim-6, dim-8, … operators contribute the process.

Positivity Bounds

$$\mathscr{L}_{\mathrm{dim-8}} \supset \frac{4}{6\Lambda^4} d_2' \lambda_H |H|^4 (\partial_\mu \varphi)^2 + \frac{2}{6\Lambda^4} d_4' \lambda_H |H|^4 |\partial_\mu H|^2$$

(2-derivative operators)

$$\mathcal{L}_2 = \frac{C^{(1)}_{H^2\varphi^2}}{\Lambda^4} O^{(1)}_{H^2\varphi^2} + \frac{C^{(2)}_{H^2\varphi^2}}{\Lambda^4} O^{(2)}_{H^2\varphi^2} + \frac{C_{\varphi^4}}{\Lambda^4} O_{\varphi^4} + \frac{C^{(1)}_{H^4}}{\Lambda^4} O^{(1)}_{H^4} + \frac{C^{(2)}_{H^4}}{\Lambda^4} O^{(2)}_{H^4} + \frac{C^{(3)}_{H^4}}{\Lambda^4} O^{(3)}_{H^4}$$

(4-derivative operators)

- Axioms of Quantum Field Theory may restrict the form of EFT.
- One of the known restrictions is Positivity Bounds, which restricts the coefficient of interaction terms.

Especially coefficients of dim-8 operator are restricted

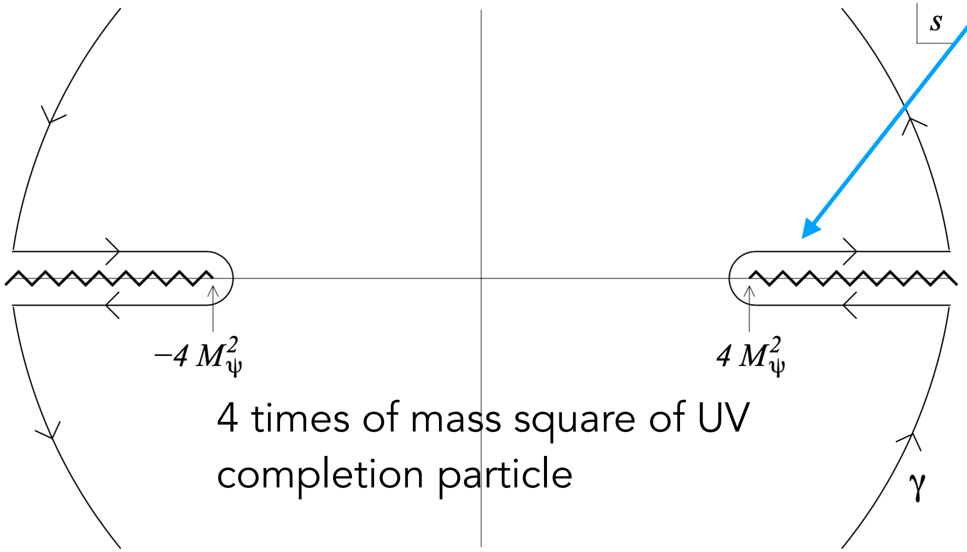
Positivity Bounds

From the Forward ($t=0$) Scattering Cross-section

Consider Effective Lagrangian $\mathcal{L} = \partial^\mu \pi \partial_\mu \pi + \frac{c_3}{\Lambda^4}(\partial_\mu \pi \partial^\mu \pi)^2 + \ldots$

Total cross-section must be $\sigma \geq 0$ & Optical Theorem suggests $\sigma = \frac{\mathrm{Im}\mathcal{M}(s, t=0)}{2E_{CM}p_{CM}}$

Evaluation of $\mathrm{Im}\mathcal{M}(s,t=0)$: $\lim_{\varepsilon \to 0^+} \mathcal{M}(s+i\varepsilon) - \mathcal{M}(s-i\varepsilon) = 2i\mathrm{Im}\mathcal{M}(s)$



Same mathematical quantity arise from contour integral with branch cut

$$\sum \text{Poles} = \frac{3c_3}{\Lambda^4} = \int_\gamma \frac{ds}{2\pi i}\frac{\mathcal{M}(s)}{s^3} \simeq 2\int_{4M_\Psi^2}^{\Lambda^2} \frac{ds}{\pi}\frac{\mathrm{Im}\mathcal{M}(s)}{s^3}$$

$$\boxed{\frac{c_3}{\Lambda^4} = \frac{4}{3\pi}\int_{4M_\Psi^2}^{\Lambda^2} ds \frac{E_{CM}p_{CM}}{s^3}\sigma(s) \geq 0}$$

Ref : Nima Arkani-Hamed et al, arXiv:hep-th/0602178v2

Positive property of cross-section argues positive coefficient.

Positivity Analysis for Higgs-DM

Positivity Constraints are obtained from 2→2 scattering amplitude

$$u_i v_j u_k^* v_l^* \frac{d^2}{ds^2} M(ij \to kl)(s, t=0)\Big|_{s\to 0} \geq 0 \quad \left(u_i, v_i : \begin{array}{c} |a\rangle = \sum_{i=1}^{5} u_i |i\rangle,\ |b\rangle = \sum_{i=1}^{5} v_i |i\rangle \\ |H\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} |1\rangle + i|2\rangle \\ |3\rangle + i|4\rangle \end{pmatrix} \quad |\varphi(\mathrm{DM})\rangle = |5\rangle \end{array} \right)$$

Valid coefficient space for Higgs-Singlet DM model

$$C^{(1)}_{H^4} + C^{(2)}_{H^4} \geq 0, \qquad C^{(1)}_{H^2\varphi^2} \geq 0,$$

$$C^{(1)}_{H^4} + C^{(2)}_{H^4} + C^{(3)}_{H^4} \geq 0, \qquad C_{\varphi^4} \geq 0,$$

$$C^{(2)}_{H^4} \geq 0, \qquad 4\sqrt{(C^{(1)}_{H^4} + C^{(2)}_{H^4} + C^{(3)}_{H^4}) C_{\varphi^4}} \geq \left| C^{(1)}_{H^2\varphi^2} + 2C^{(2)}_{H^2\varphi^2} \right| - C^{(1)}_{H^2\varphi^2}.$$

$$\mathcal{L}_2 = \frac{C^{(1)}_{H^2\varphi^2}}{\Lambda^4} O^{(1)}_{H^2\varphi^2} + \frac{C^{(2)}_{H^2\varphi^2}}{\Lambda^4} O^{(2)}_{H^2\varphi^2} + \frac{C_{\varphi^4}}{\Lambda^4} O_{\varphi^4} + \frac{C^{(1)}_{H^4}}{\Lambda^4} O^{(1)}_{H^4} + \frac{C^{(2)}_{H^4}}{\Lambda^4} O^{(2)}_{H^4} + \frac{C^{(3)}_{H^4}}{\Lambda^4} O^{(3)}_{H^4}$$

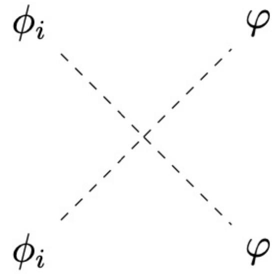
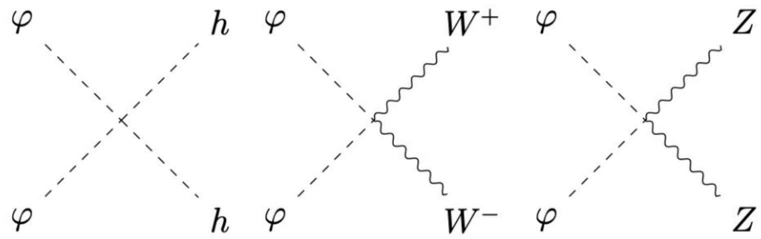
$$O^{(1)}_{H^2\varphi^2} = (D_\mu H^\dagger D_\nu H)(\partial^\mu \varphi \partial^\nu \varphi) \qquad O^{(2)}_{H^2\varphi^2} = (D_\mu H^\dagger D^\mu H)(\partial_\nu \varphi \partial^\nu \varphi)$$

$$O_{\varphi^4} = \partial_\mu \varphi \partial^\mu \varphi \partial_\nu \varphi \partial^\nu \varphi$$

$$O^{(1)}_{H^4} = (D_\mu H^\dagger D_\nu H)(D^\nu H^\dagger D^\mu H) \qquad O^{(2)}_{H^4} = (D_\mu H^\dagger D_\nu H)(D^\mu H^\dagger D^\nu H)$$

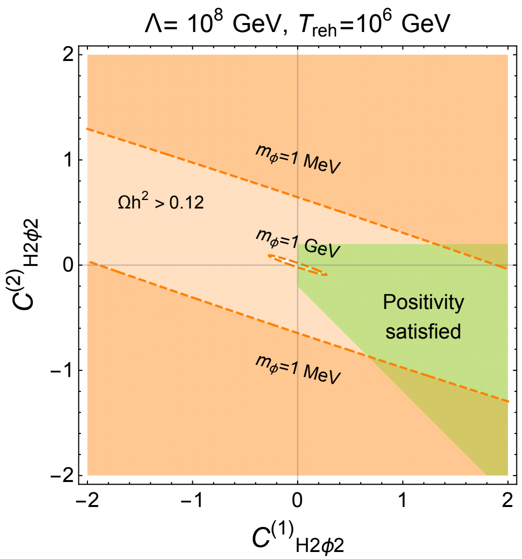
$$O^{(3)}_{H^4} = (D_\mu H^\dagger D^\mu H)(D_\nu H^\dagger D^\nu H)$$

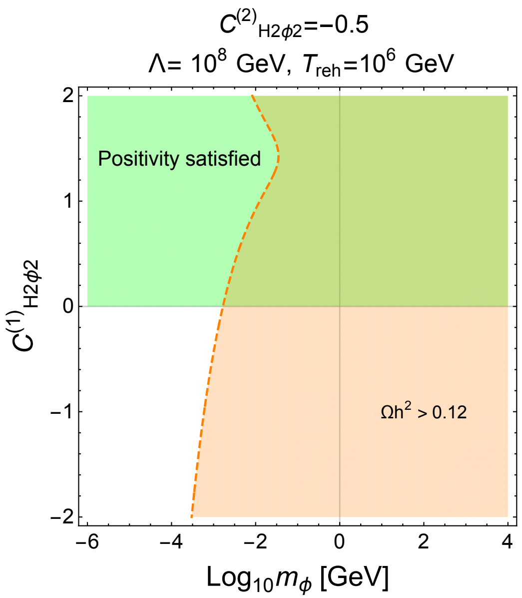
DM Analysis : Relic Abundance

	Freeze-In (FIMP)	Freeze-Out (WIMP)
DM Production/ Decay Process		
Requirements, Assumptions, Consequences	• DM was never thermally equilibrated with thermal plasma. It requires 3 conditions. $c_3 m_\varphi^2/\Lambda^4 \leq 10^{-6}$, $d_3 m_\varphi^2/\Lambda^4, d_4/\Lambda^4 \leq (T_{\text{reh}}^3 M_{pl})^{-1/2}$, $C_{H^2\varphi^2}^{(1,2)}/\Lambda^4 \leq (T_{reh}^7 M_{pl})^{-1/2}$ • We focus on the case of third condition is most important. • Cutoff scale is higher than reheating temperature T_{reh} • DM are produced from SM. Their production is mainly done around the T_{reh}	• DM was thermally equilibrated with SM until it decouples. • DM-fermion interaction is strongly restricted by phenomenological constraint. • Relativity Low Cutoff Allowed, for instances, $\mathcal{O} \sim$ 1TeV • DM are annihilate to SM particles.

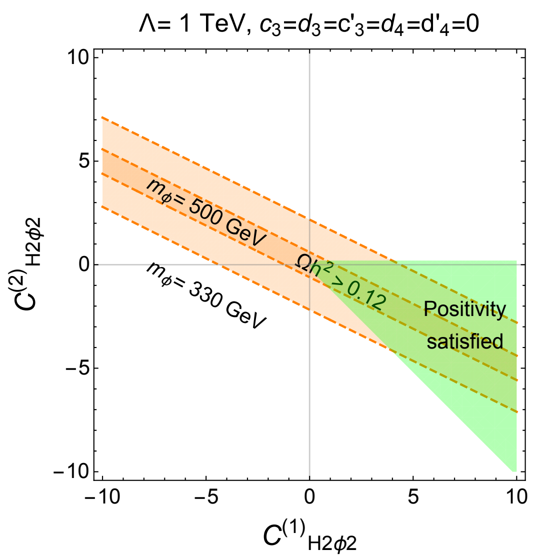
DM Analysis : Relic and Positivity

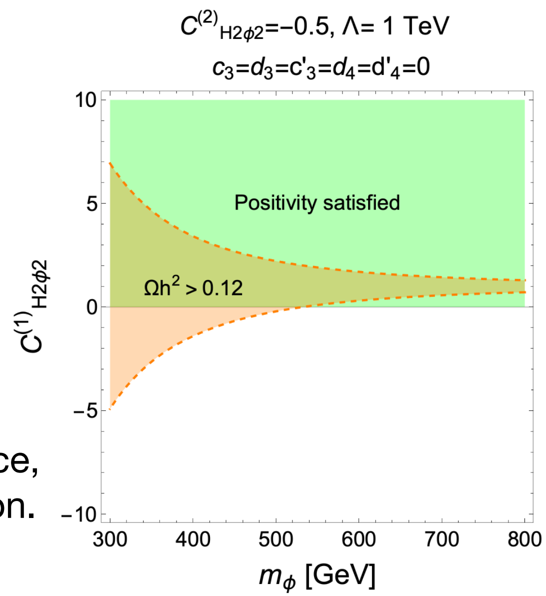
Freeze-in (FIMP)





Freeze-out (WIMP)





- In the FIMP scenario, Higher m_φ leads to restrictive parameter space and limited interaction with Higgs.
- In the WIMP scenario, in contrast, Higher m_φ leads to more free parameter space and allows more interaction with Higgs.
- Positivity forbids half or greater coefficient spaces.

Colored region stands for overabundance, which cannot explain current observation.

Conclusion of this work

- We investigated positivity bound for general Higgs portal scalar DM model. And we combine it to phenomenological bounds.
- We discussed two scenarios, Freeze-in and Freeze-out.
- Both scenario shows different DM mass preference.
  - Parameter spaces are more open to low DM mass for FIMP, and high DM mass for WIMP.
- In FIMP scenario, DM mainly produced at the reheating epoch.
- And positivity forbids almost half (or more) of coefficient spaces.