Chung-Ang University High Energy Physics Center Workshop, Gwangju, Dec. 28, 2023

Probing Dark Matter with Small Scale Astrophysical Structures

Jeong Han Kim

(Chungbuk Natl. U.)

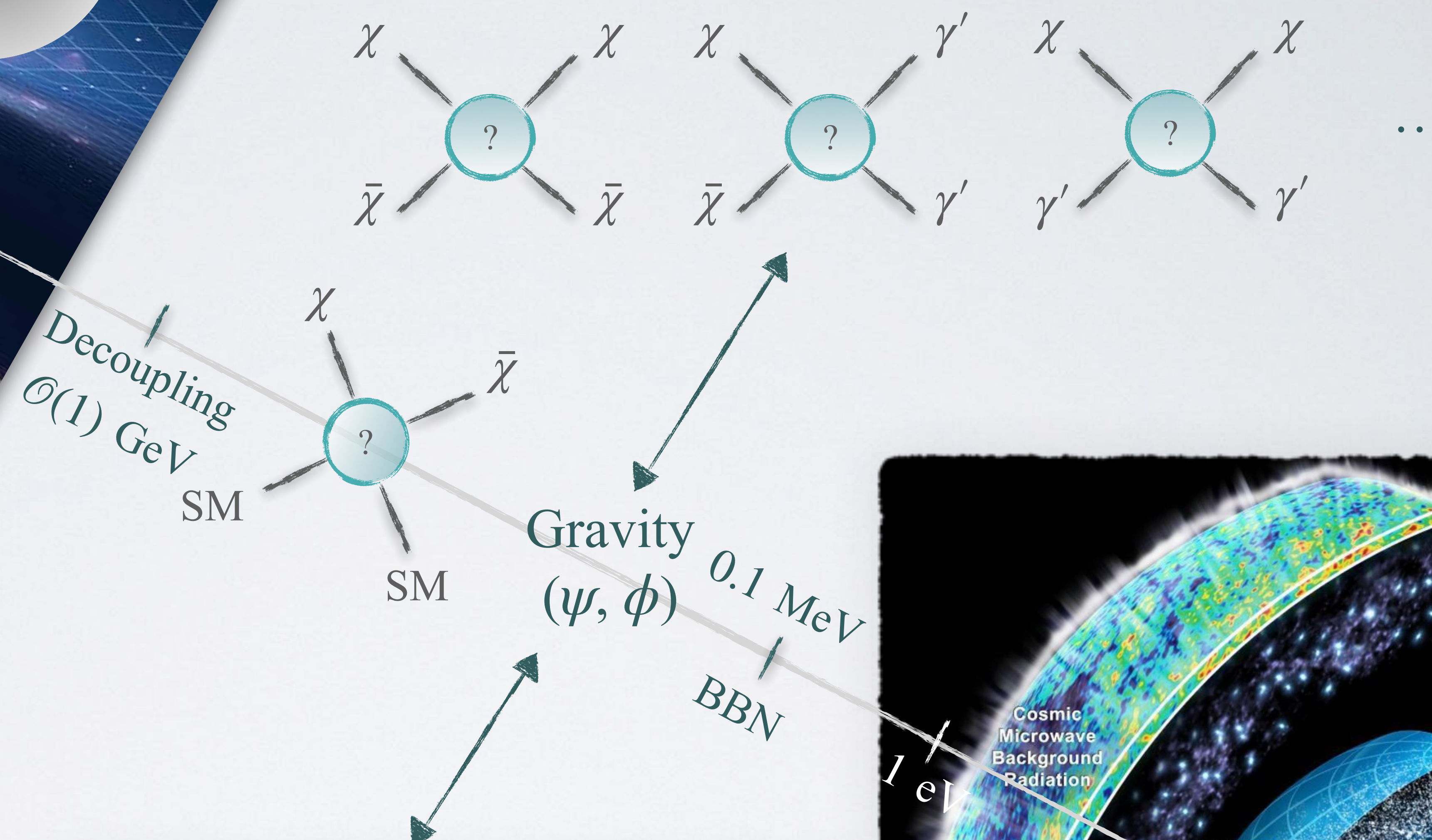
Based on :

Sehwan Lim, **Jeong Han Kim**, Kyoungchul Kong, Jong Chul Park - [arXiv:2312.07660]

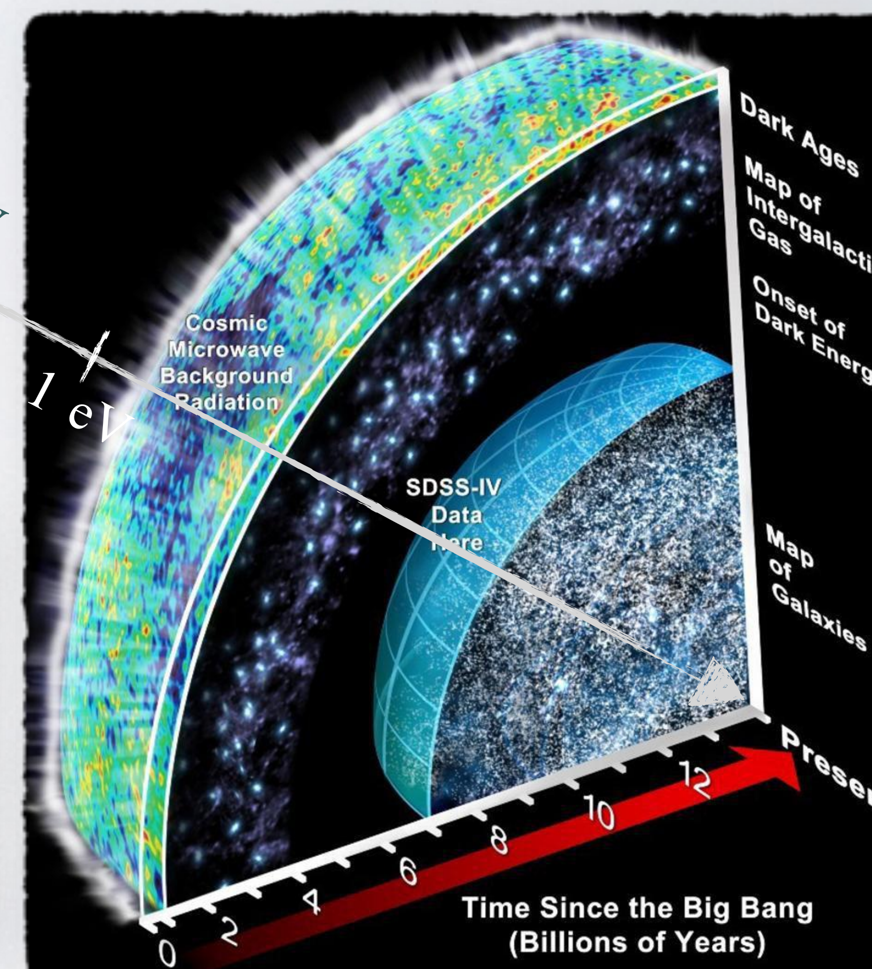
Kenji Kadota, **Jeong Han Kim**, Pyungwon Ko, and Xing-Yu Yang - [arXiv:2306.10828]

Saurabh Bansal, **Jeong Han Kim**, Christopher Kolda, Matthew Low, and Yuhsin Tsai - [JHEP 05 (2022) 050]

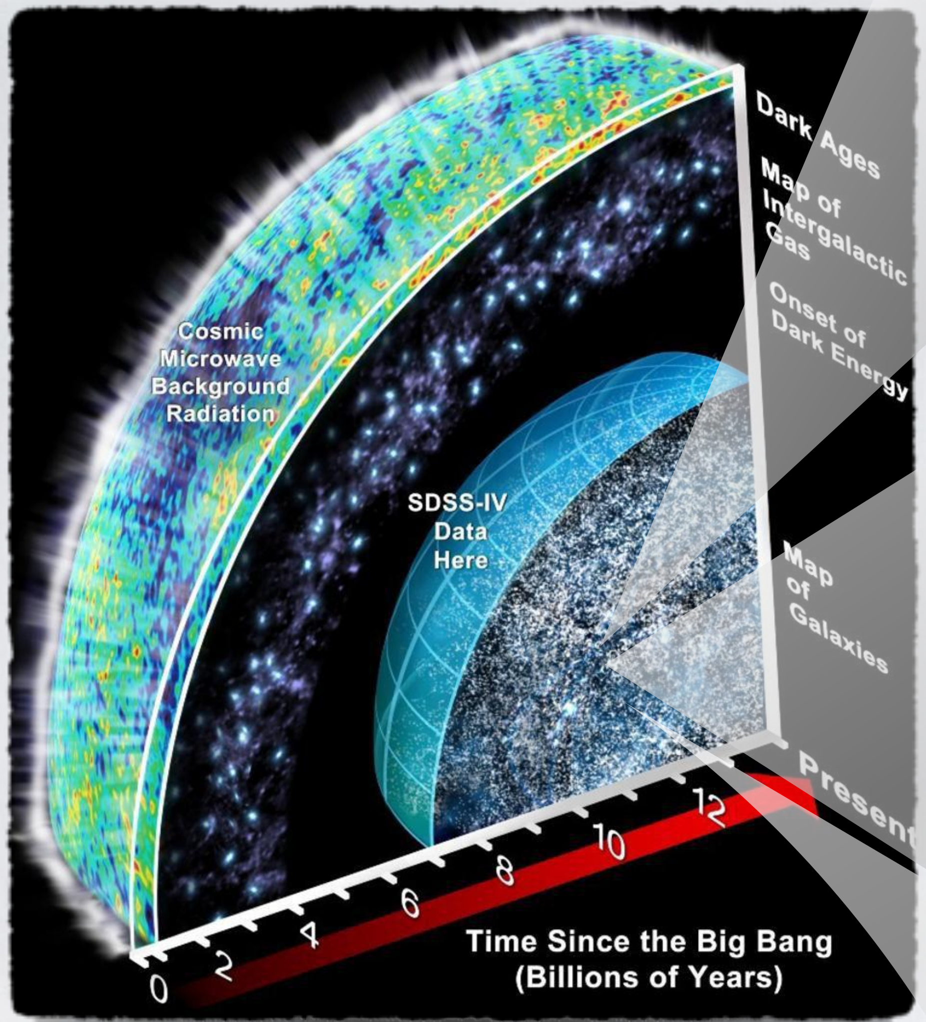
Cosmic Probes of Dark Sector



- What is a hidden dynamics of a dark sector?
- What are useful cosmological data to illuminate them?
- Use the gravitational interaction as a main source to probe the dark sector.

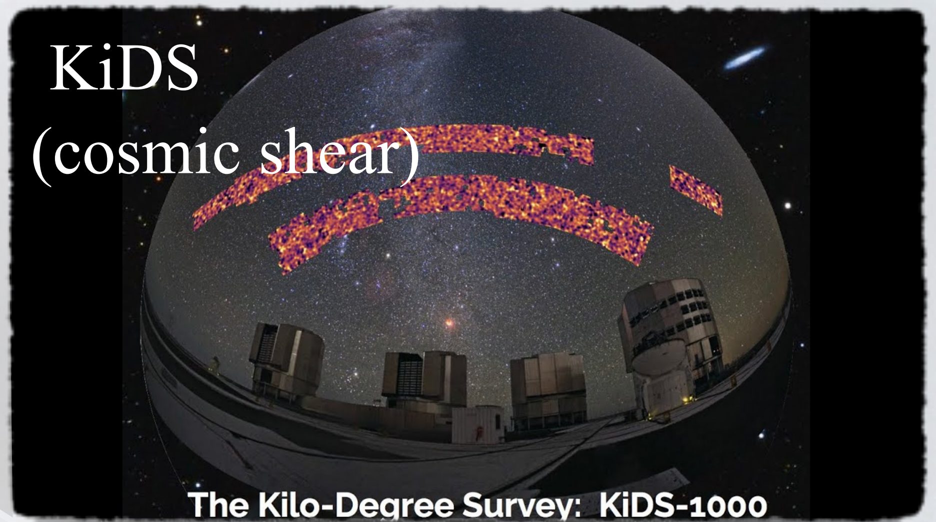


Structure Formation of the Universe



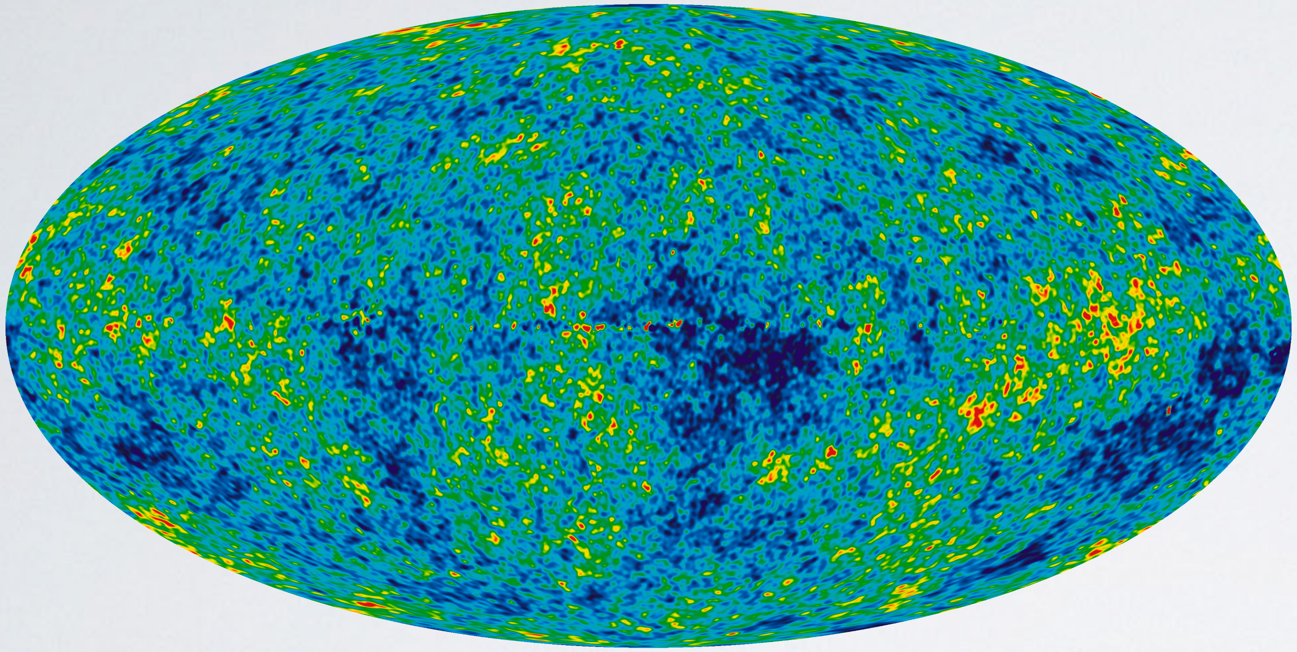
redshift $z \simeq [0,\ 2]$

redshift $z \simeq [0,\ 1]$



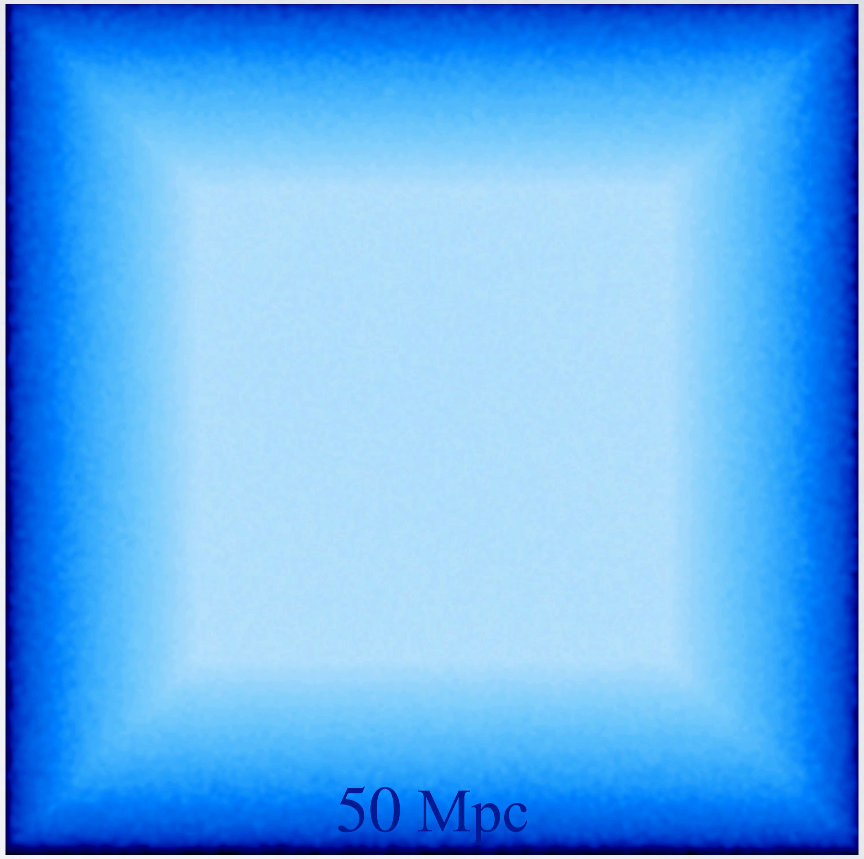
- LSS provides a wide range of opportunities to probe gravitational interactions of DM.
- It has a larger amount of Fourier modes (3D data).
- It enables us to probe much smaller scales where new physics may be lurking around.

Beyond CMB Measurements



vs

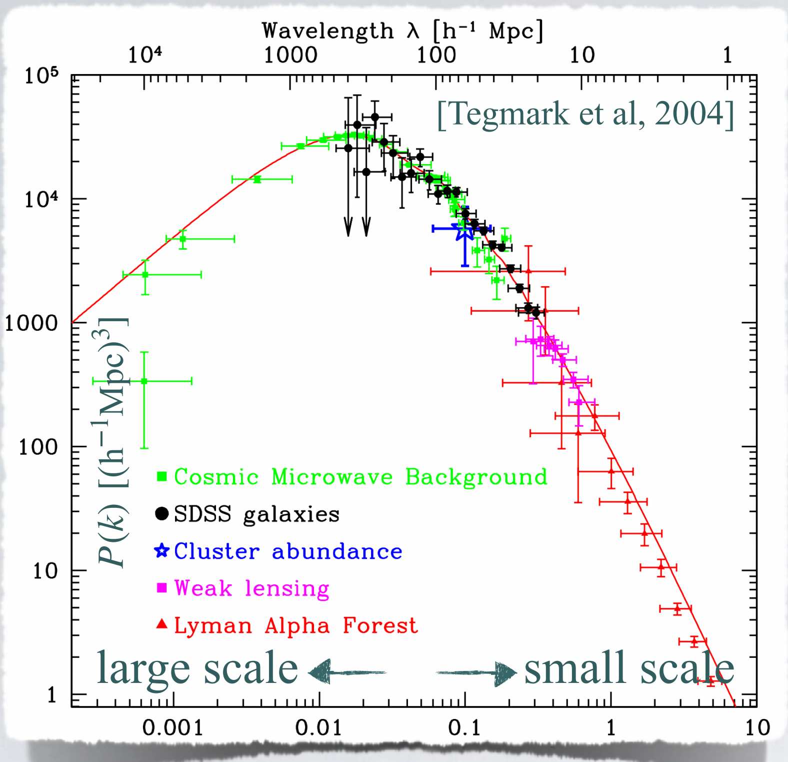
redshift $z \simeq [0,\ 2]$



- CMB contains many constraints that make it hard to get a robust measurement.
- CMB measurements are largely constrained due to a cosmic variance.
- CMB is a 2D surface which limits the amount of Fourier modes that we can measure.

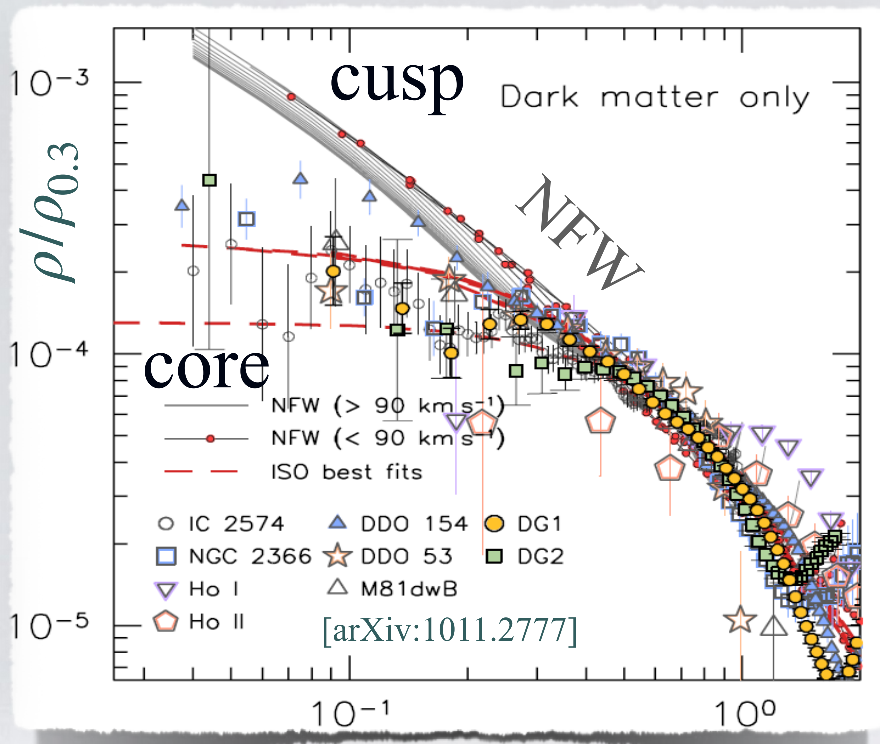
Abundant Observational Data

Matter Power Spectrum



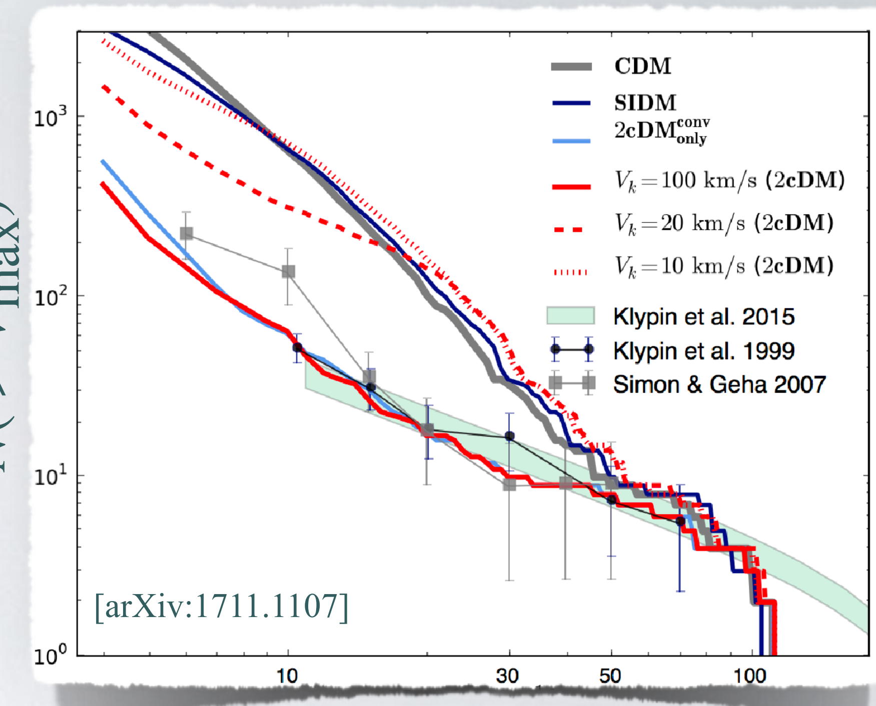
k [h/Mpc^{-1}]

Density Profile of Halos



$r/r_{0.3}$

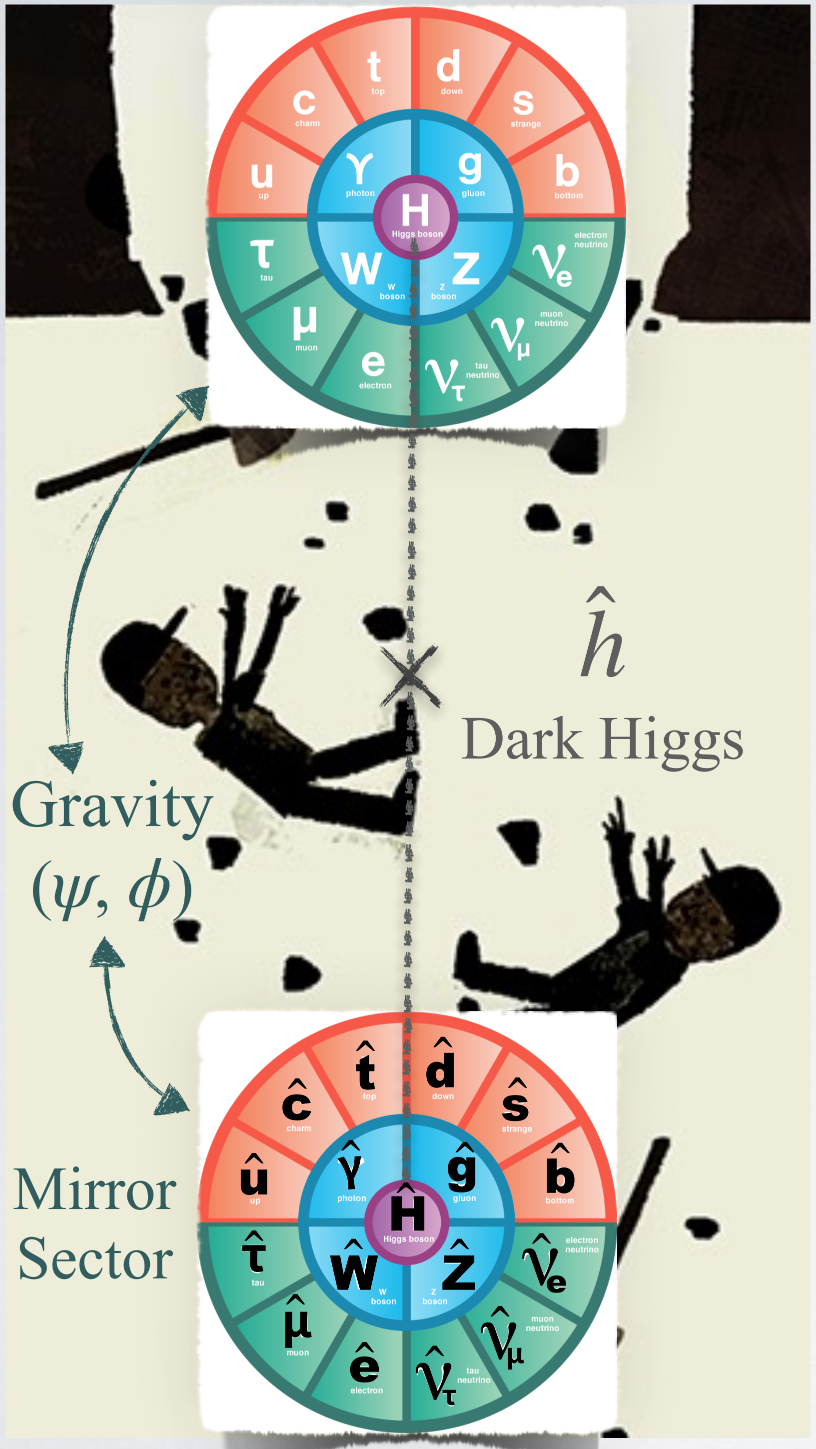
Maximum Circular Velocity Distribution



V_{max}[km/s−1] ~ mass of subhalos

- The power spectrum or N-point correlation functions to study density perturbations.
- At much smaller scales, we can study the density profiles of subhalos.
- We can study statistical distributions of subhalo masses.
- Weak lensing data, peak statistics, … and so on

Benchmark Dark Matter Models

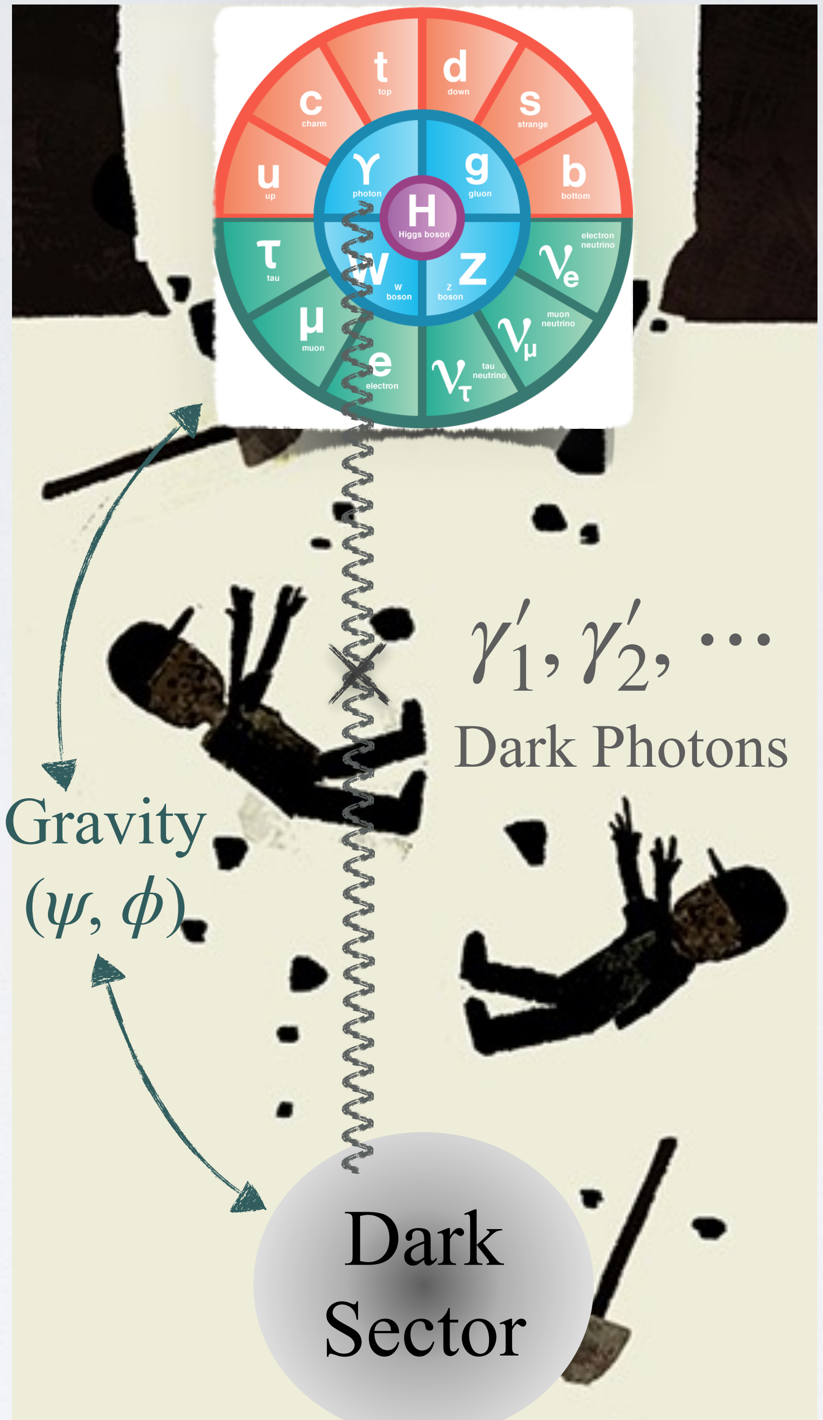


Z. Chacko, H. Goh, R. Harnik [2005]

Z. Chacko, D. Curtin, M. Geller, Y. Tsai [2018]

…

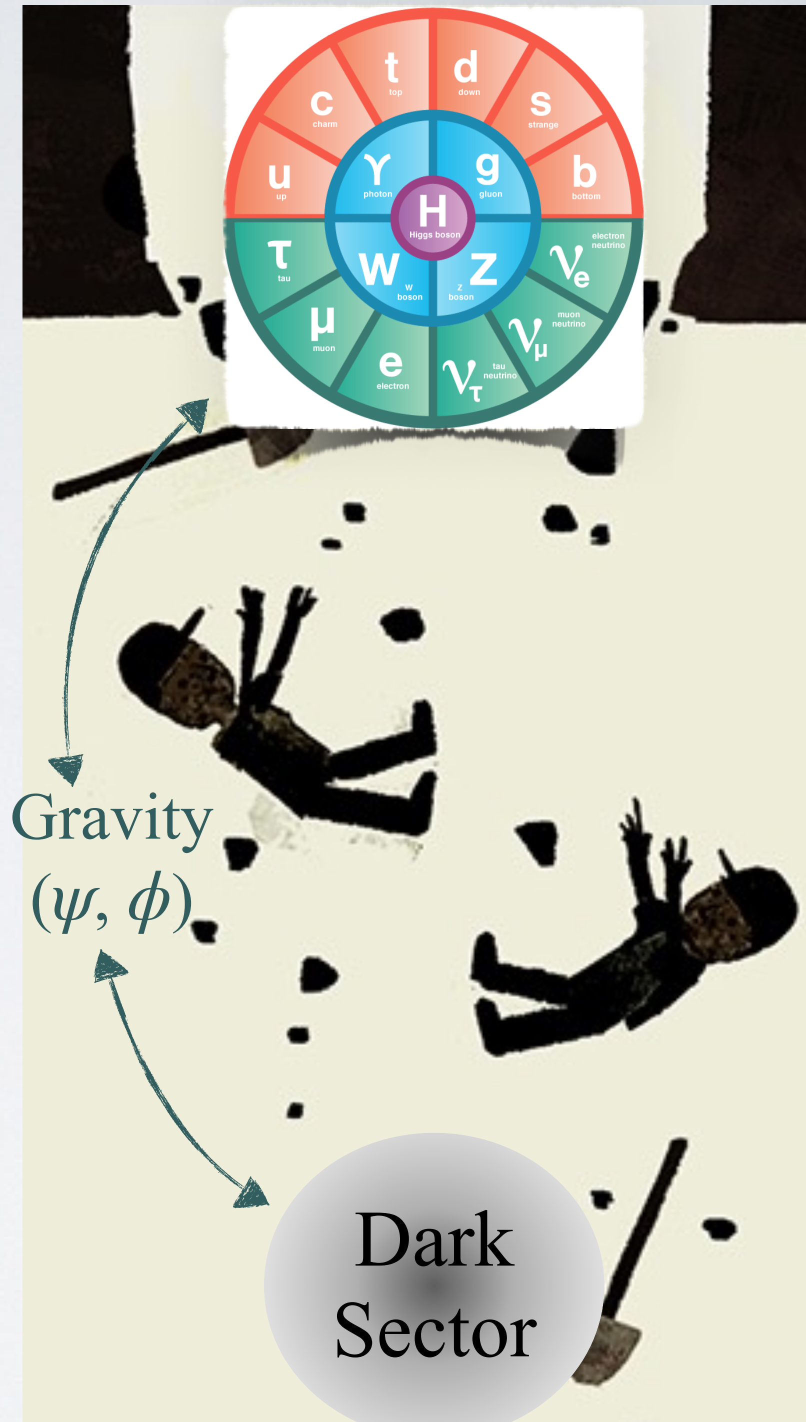
S. Bansal, **J.H. Kim**, C. Kolda, M. Low, Y. Tsai [2022]



$\chi_1, \chi_2, \cdots$

Multi-component Model

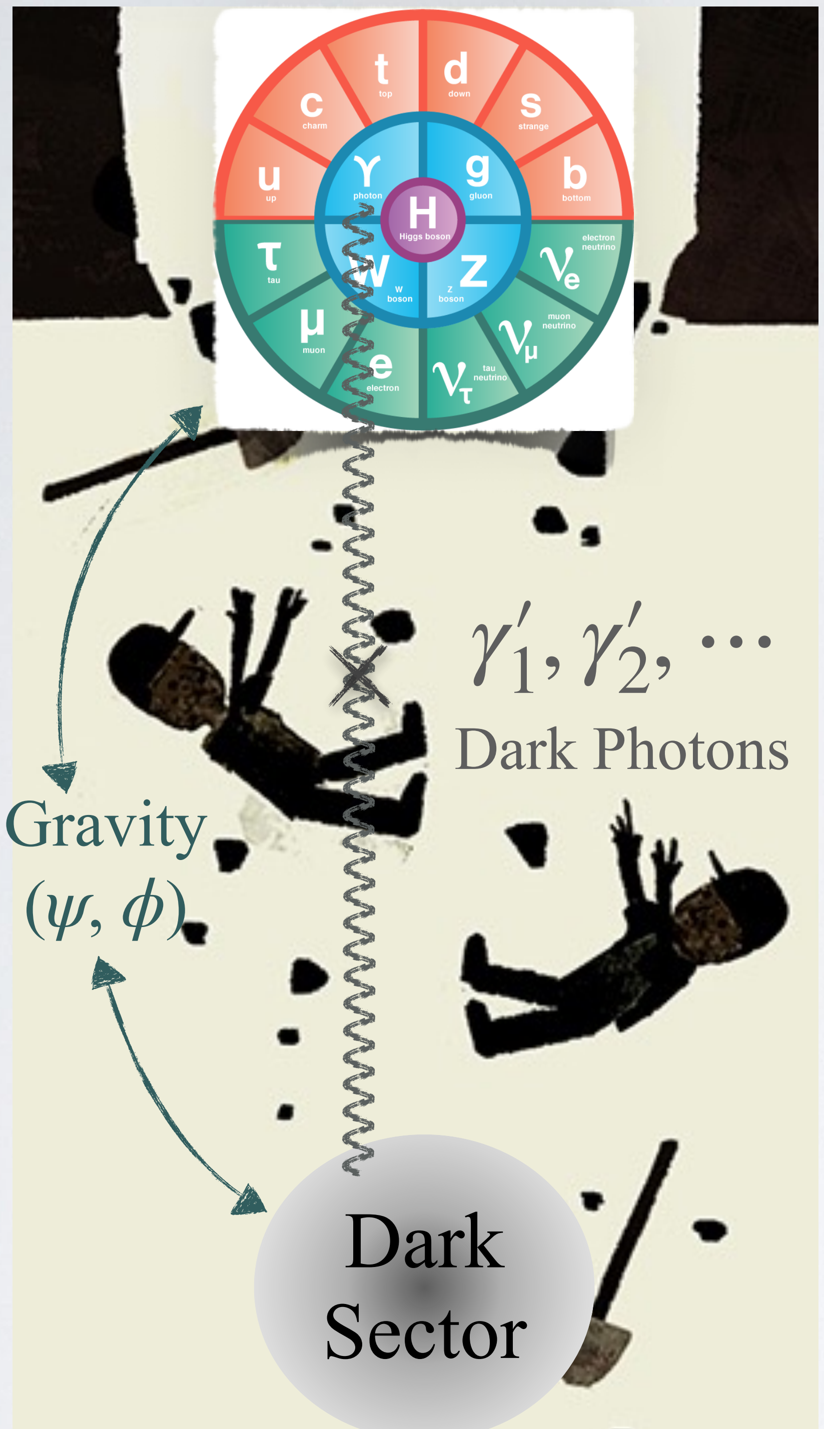
[See the talk by Prof. Seodong Shin]



χ_1

Ultra-light Self-Interacting Dark Matter

Benchmark Dark Matter Models



$\chi_1, \chi_2, \cdots$

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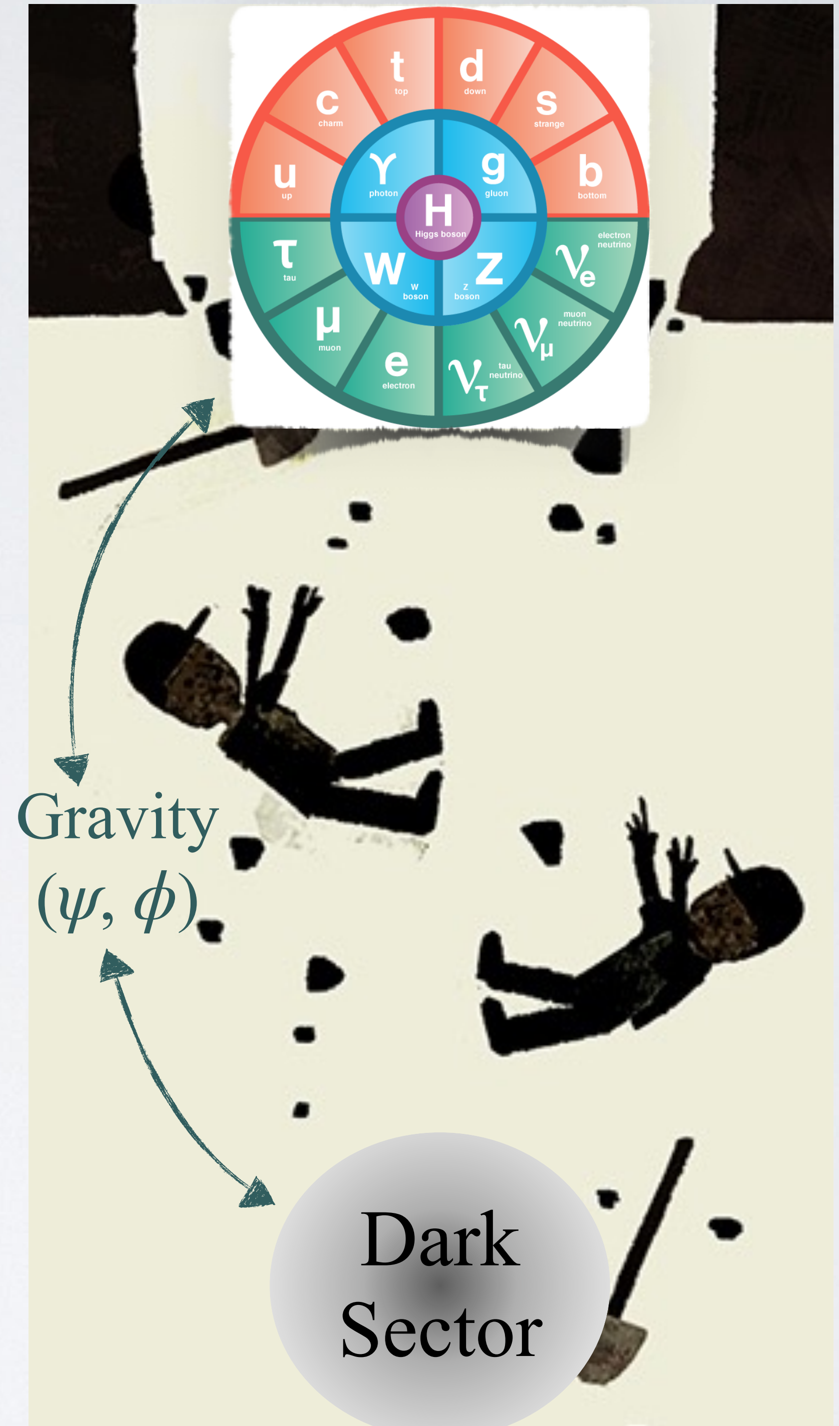
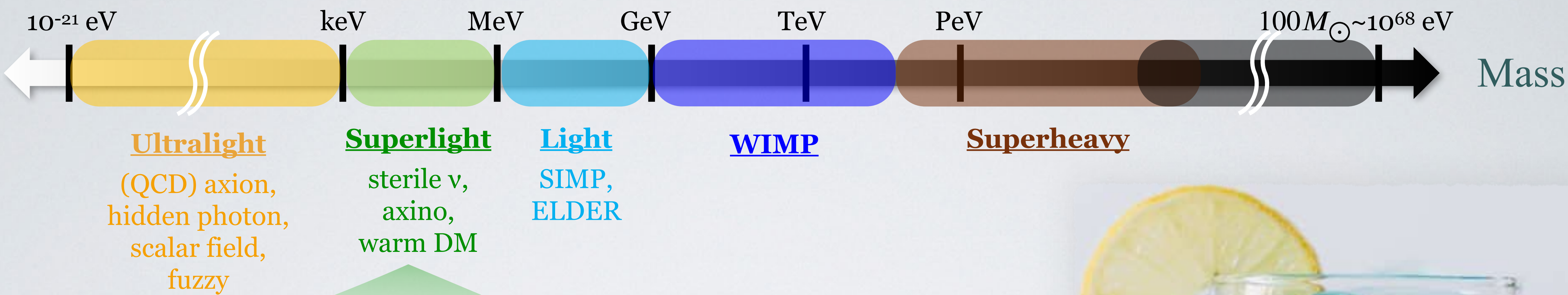


χ_1

Ultra-light Self-Interacting Dark Matter

1. Two-Component Dark Matter

Simple Extension of ΛCDM



$m \sim \mathcal{O}(\text{keV})$

Warm Dark Matter (WDM)

WDM

+

Cold Dark Matter (CDM)

CDM

Mixed Cold and Warm Dark Matter (CWDM) (~27 %)

Boyarsky et al. [0812.0010]

Anderhalden et al. [1212.2967]

Maccio et al. [1202.2858]

...

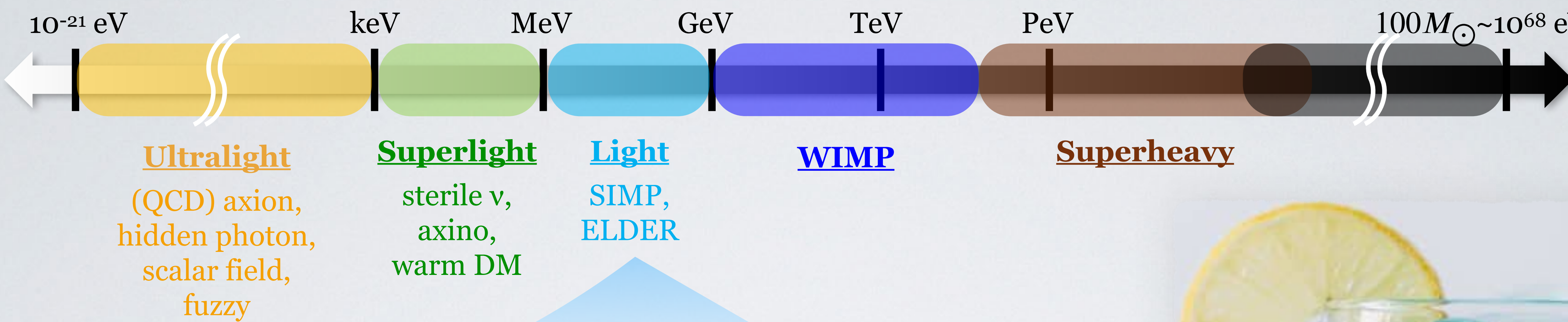
Dark Energy (~68 %)

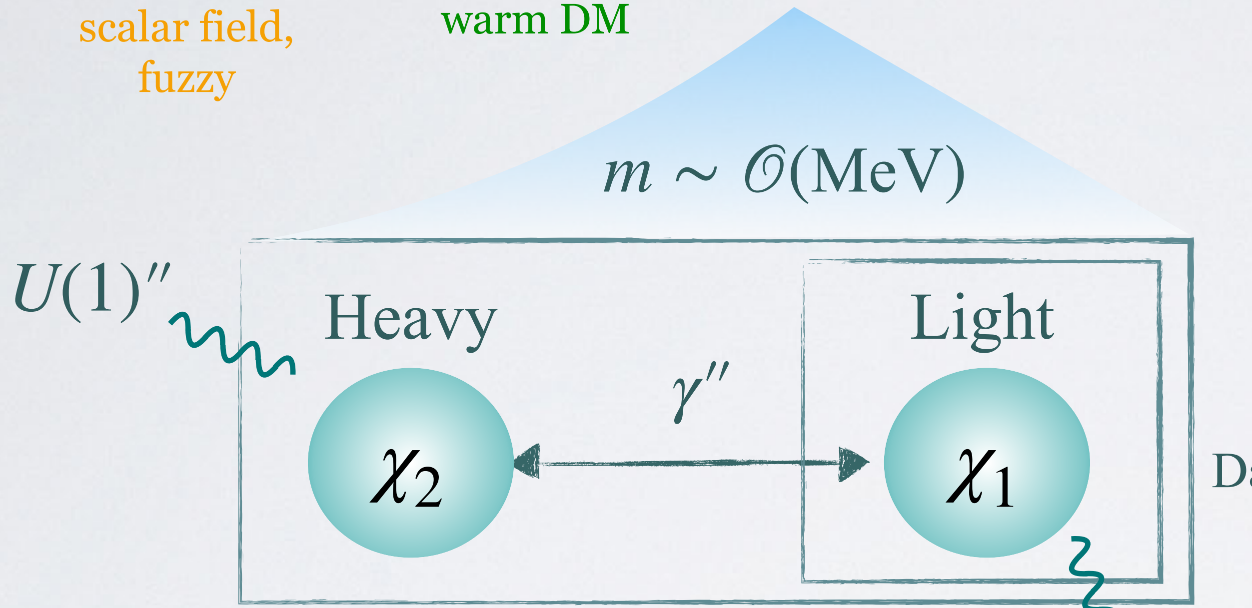
Ordinary Matter (~5 %)

SM

- The WDM is able to free-stream and dampens density perturbations at small scales.
- To have a significant impact on astrophysical data, $m \sim \mathcal{O}(\text{keV})$.

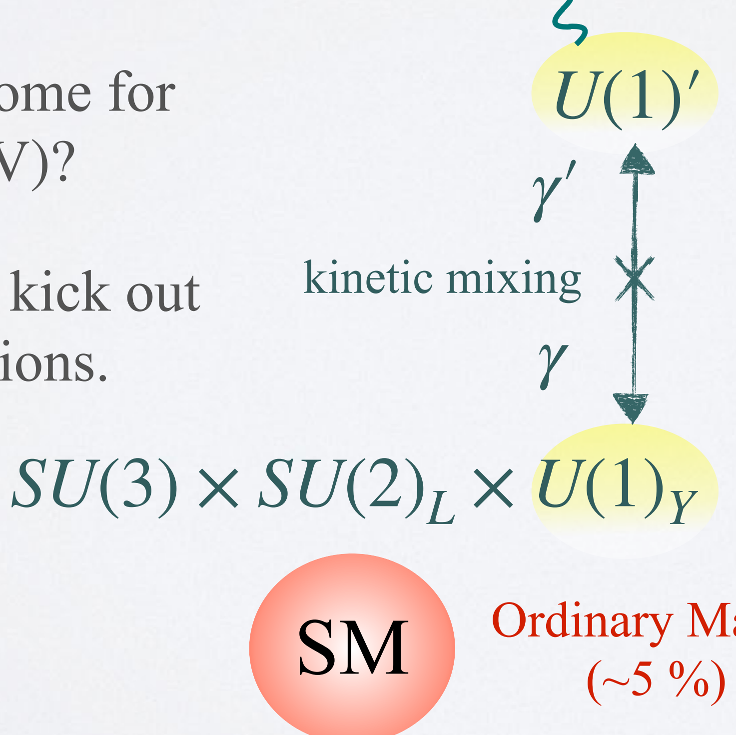
Simplified Two-Component DM





- How to achieve a similar outcome for DM masses above $m \gg \mathcal{O}(\text{keV})$?

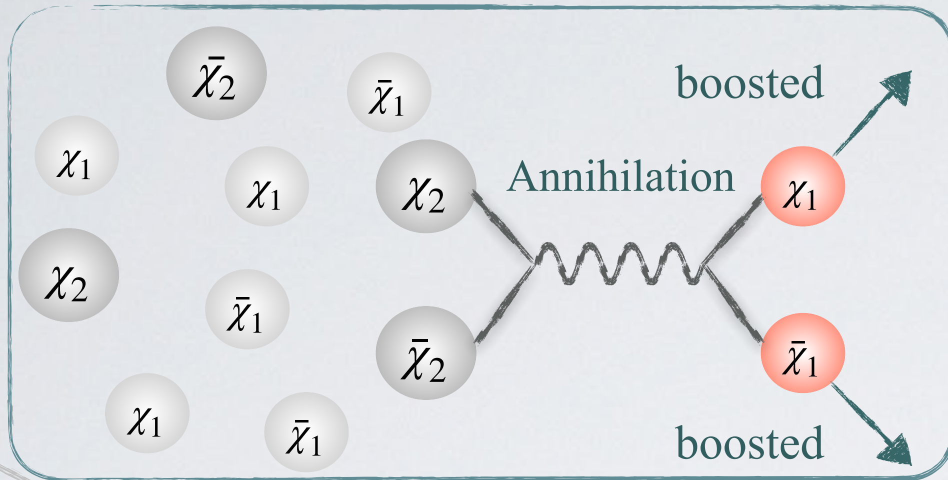
- Introduce the mass gap Δm to kick out light species through annihilations.



Belanger, J. Park, [2012]
Agashe, Cui, Necib, Thaler [2014]
...
A. Kamada, H. Kim, J. Park, S. Shin [2021]

"Self-Heating Effects"

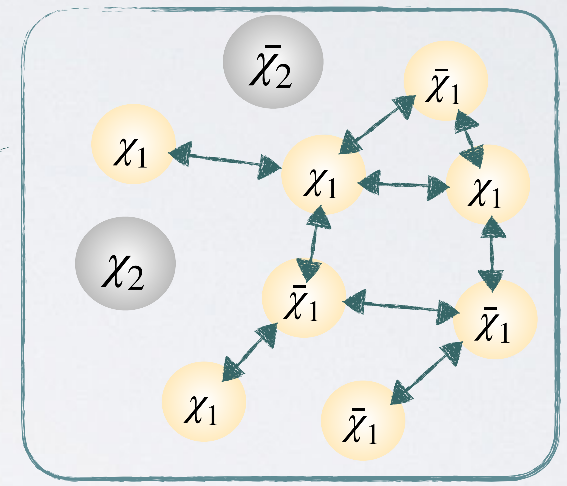
1. The heavy χ_2 annihilates to light χ_1 which becomes boosted.



...

χ_2 Decoupling

2. Sharing energies through self-interaction $\sigma_{11\to 11}/m_{\chi_1}$ which increases the χ_1 temperature.



(with $\Gamma_{11\to 11} > H$)

$$\gamma_{\rm heat} = \frac{2n_{\chi_2}^2 \langle \sigma v \rangle_{22\to 11}}{3n_{\chi_1} T}(m_{\chi_2} - m_{\chi_1})$$

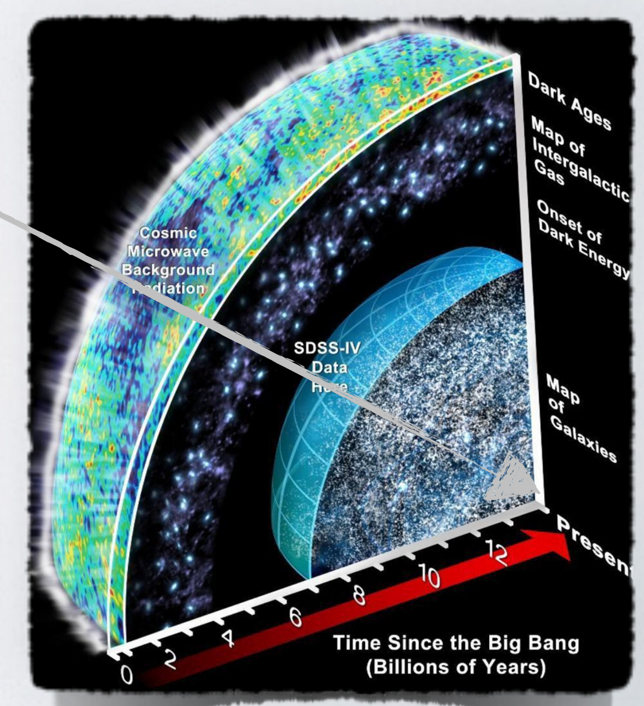
$$\dot{T}_{\chi_1} + 2HT_{\chi_1} \simeq \gamma_{\rm heat} T - 2\gamma_{\chi_1,SM}\left(T_{\chi_1} - T\right)$$

Cooling due the Hubble expansion

Kinetic scattering of χ_1 with a thermal bath

χ_1 Decoupling

...



A. Kamada, H. Kim, J. Park, S. Shin [2021]

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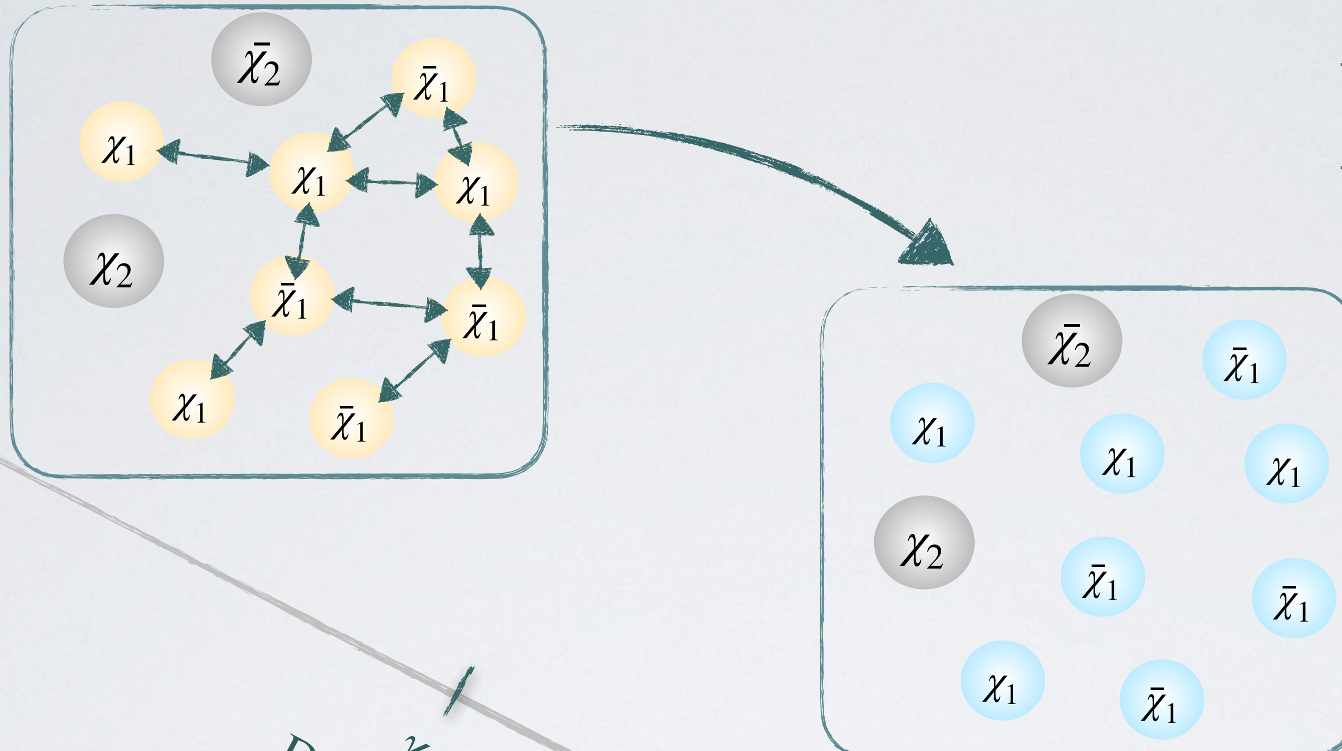
"Self-Heating Effects"

3. When the self-interaction rate drops below the Hubble scale, it starts to cool down.



…

χ_2 Decoupling

χ_1 Decoupling

$$\dot{T}_{\chi_1} + 2HT_{\chi_1} \simeq 0$$

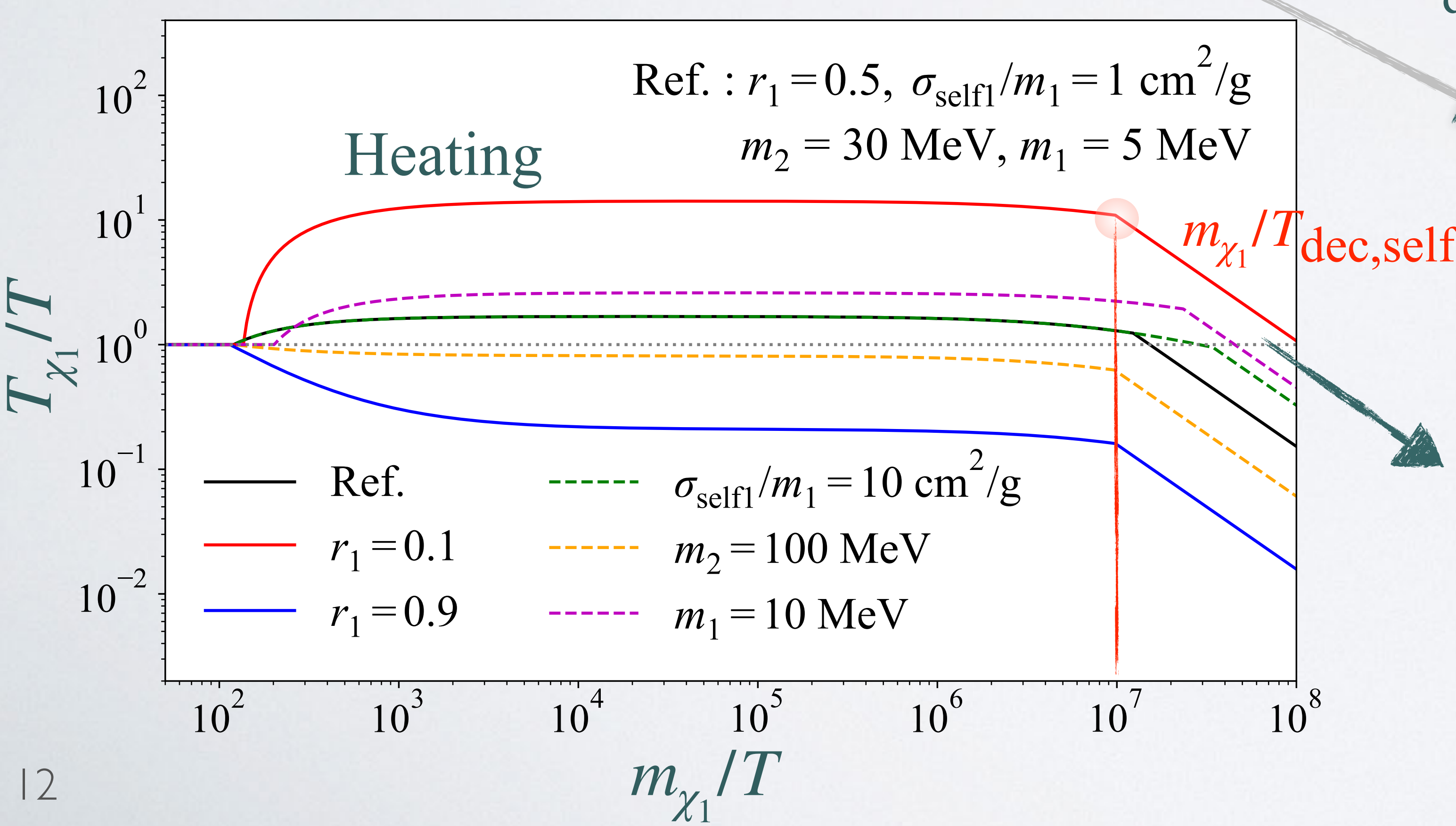
(when $\Gamma_{11\to 11} < H$)

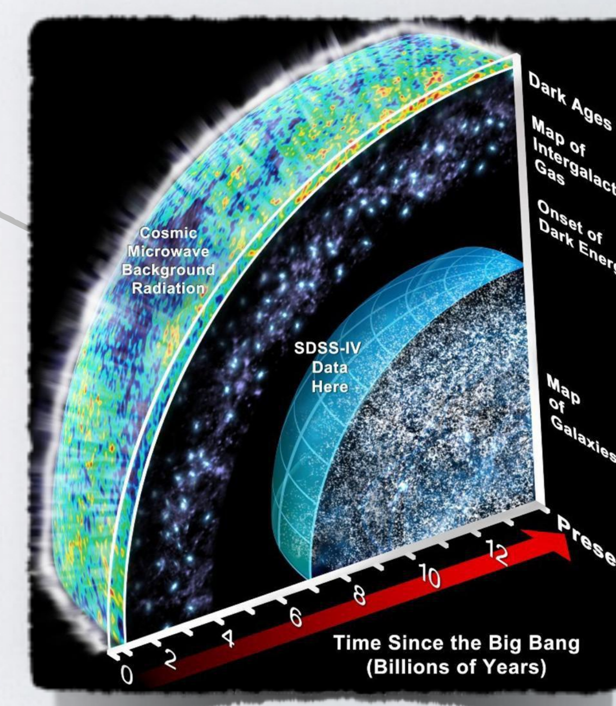
Decoupling temperature of the self-interaction

$$T_{\rm dec,self} \sim \left(\frac{0.3}{r_1}\right)^{2/3} \left(\frac{m_{\chi_1}}{100\ {\rm MeV}}\right)^{1/3} \left(\frac{1 {\rm cm}^2/{\rm g}}{\sigma_{11\to 11}/m_{\chi_1}}\right)^{2/3}$$

Ratio of relics :

$$r_1 = \Omega_{\chi_1}/(\Omega_{\chi_1} + \Omega_{\chi_2})$$



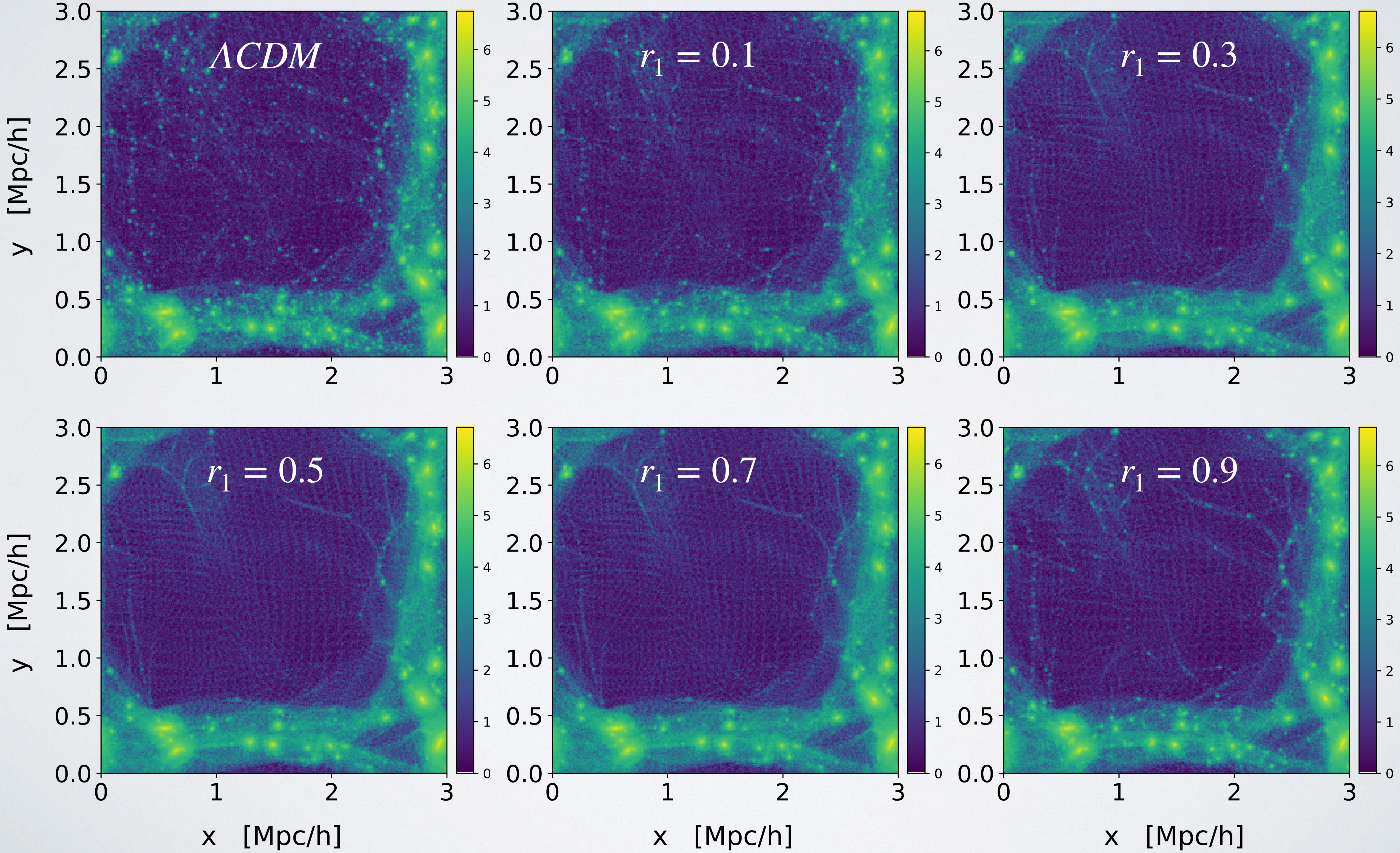


How Does the Structure Formation Change?

Sehwan Lim, **J. H. Kim**, K.C. Kong, J. Park [2023]

- There seem to be fewer subhalos in the two-component Universe.

(For fixed $\sigma_{11\to11}/m_{\chi_1} = 1\text{cm}^2/\text{g}$, $m_{\chi_2} = 30$ MeV, $m_{\chi_1} = 5$ MeV)



Perturbed Boltzmann Equations

Sehwan Lim, **J. H. Kim**, K.C. Kong, J. Park [2023]

- Use the FRW metric with the following convention

$$ds^2 = -(1+2\Psi)dt^2 + (1-2\Phi)a(t)^2\delta_{ij}dx^i dx^j$$

- Density contrasts δ_{χ_i} dictate amount of matter perturbations.

$$\rho_{\chi_i} = \bar{\rho}_{\chi_i}\left(1+\delta_{\chi_i}\right) \quad (\text{with } i = 1,\, 2)$$

- Perturbed velocities $\vec{v}_{\chi_i}$ of dark matters.

$$\theta_{\chi_i} = \nabla \cdot \vec{v}_{\chi_i}$$

- Perturbation equations for χ_2 .   See also the lecture by Lam Hui

(number density)

$$n_{\chi_i,\mathrm{eq}} \simeq g_{\chi_i}\, e^{-m_{\chi_i}/T}\left(\frac{m_{\chi_i}\,T}{2\pi}\right)^{3/2}$$

(energy density)

$$\rho_{\chi_i,\mathrm{eq}} \simeq m_{\chi_i}\, n_{\chi_i,\mathrm{eq}}$$

(perturbation for $\rho_{\chi_i,\mathrm{eq}}$)

$$\delta_{\chi_i,\mathrm{eq}} = \frac{n_{\chi_i,\mathrm{eq}}}{\bar{n}_{\chi_i,\mathrm{eq}}} - 1$$

$$\frac{d\delta_{\chi_2}}{dt} + \frac{\theta_{\chi_2}}{a} - 3\frac{d\Phi}{dt} = \frac{\langle\sigma v\rangle_{22\to11}}{m_{\chi_2}\bar{\rho}_{\chi_2}}\left(-\Psi\left(\bar{\rho}^2_{\chi_2} - \frac{\bar{\rho}^2_{\chi_2,\mathrm{eq}}}{\bar{\rho}^2_{\chi_1,\mathrm{eq}}}\bar{\rho}^2_{\chi_1}\right) - \bar{\rho}^2_{\chi_2}\delta_{\chi_2} + \frac{\bar{\rho}^2_{\chi_2,\mathrm{eq}}}{\bar{\rho}^2_{\chi_1,\mathrm{eq}}}\bar{\rho}^2_{\chi_1}\left(2\delta_{\chi_2,\mathrm{eq}} - \delta_{\chi_2} - 2\delta_{\chi_1,\mathrm{eq}} + 2\delta_{\chi_1}\right)\right)$$

$$\frac{d\theta_{\chi_2}}{dt} + H\theta_{\chi_2} + \frac{\nabla^2\Psi}{a} = \frac{\langle\sigma v\rangle_{22\to11}}{m_{\chi_2}\bar{\rho}_{\chi_2}}\frac{\bar{\rho}^2_{\chi_2,\mathrm{eq}}}{\bar{\rho}^2_{\chi_1,\mathrm{eq}}}\bar{\rho}^2_{\chi_1}\left(\theta_{\chi_1} - \theta_{\chi_2}\right)$$

$$c^2_{s,\chi_2}\frac{\nabla^2\delta_{\chi_2}}{a} \to 0$$

We neglect the sound speed of χ_2

$T_{\chi_2} \simeq 0$ (same as CDM)

- And two independent Einstein equations.

Perturbed Boltzmann Equations

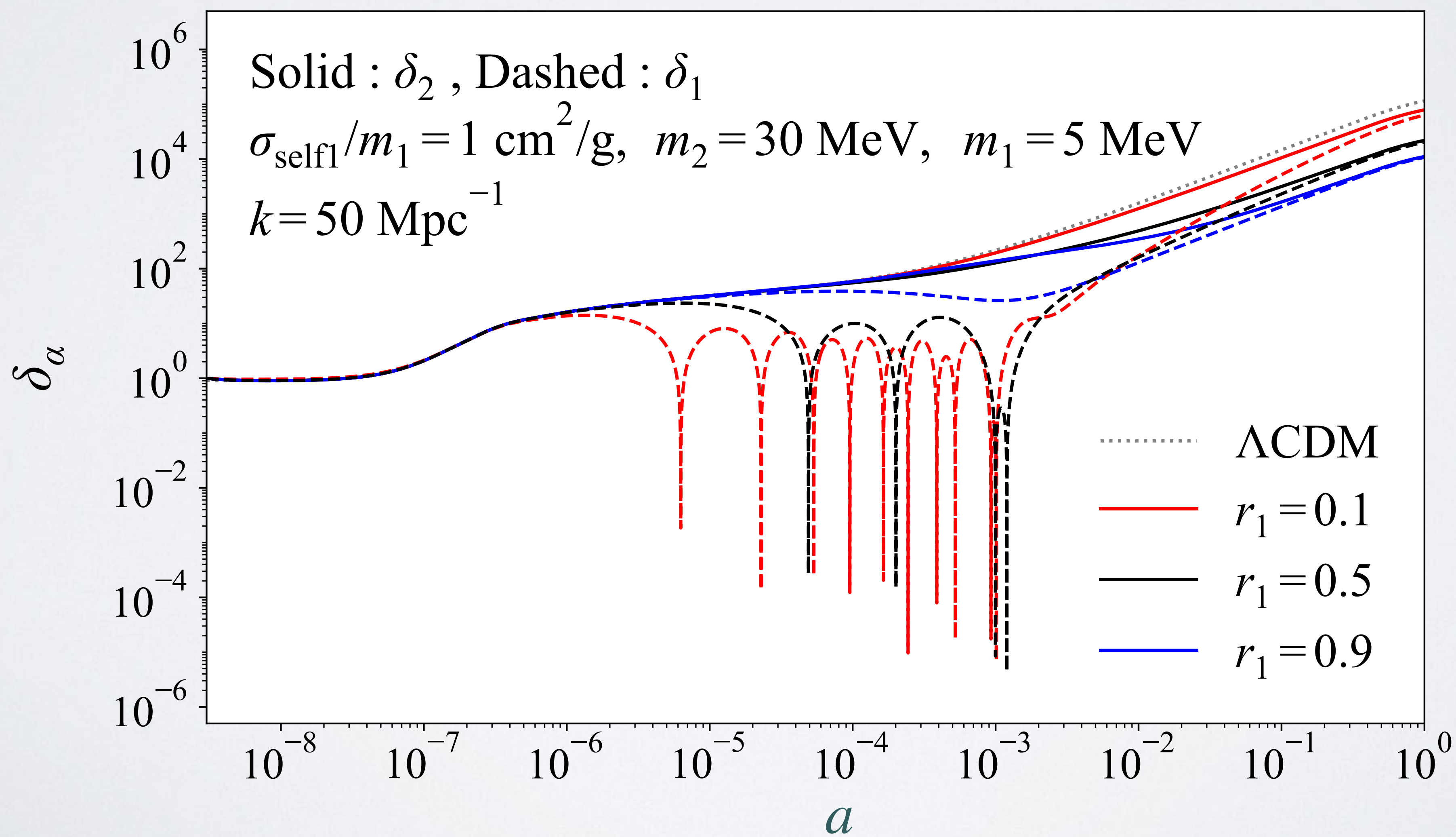
Sehwan Lim, **J. H. Kim**, K.C. Kong, J. Park [2023]

- When $T \ll m_{\chi_i}$ (at around matter-dominated era)

$$\frac{d^2\delta_2}{dt^2} + \left(2H + \frac{\langle \sigma v \rangle_{22\to 11}}{m_2}\bar{\rho}_2\right)\frac{d\delta_2}{dt} - \left(\frac{\langle \sigma v \rangle_{22\to 11}}{m_2}H + 4\pi G\right)\bar{\rho}_2\delta_2 = \left(\text{terms of gravity}\right) + \left(\text{coupled terms with } \delta_1\right)$$

Friction caused by χ_2 annihilation

Negative (δ_{χ_2} grows)



Perturbed Boltzmann Equations

- When $T \ll m_{\chi_i}$ (at around matter-dominated era)

Sehwan Lim, **J. H. Kim**, K.C. Kong, J. Park [2023]

Friction caused by χ_1 annihilation

$$\frac{d^2\delta_1}{dt^2} + \left(2H + 2\frac{\langle\sigma v\rangle_{22\to 11}}{m_2}\frac{\bar\rho_2^2}{\bar\rho_1} + \frac{\langle\sigma v\rangle_{11\to \mathrm{SMSM}}}{m_1}\bar\rho_1\right)\frac{d\delta_1}{dt} - \left(\frac{\langle\sigma v\rangle_{22\to 11}}{m_2}\frac{\bar\rho_2^2}{\bar\rho_1}H + \frac{\langle\sigma v\rangle_{11\to \mathrm{SMSM}}}{m_1}\bar\rho_1 H + 4\pi G\bar\rho_1 - c_{s,1}^2\frac{k^2}{a^2}\right)\delta_1$$

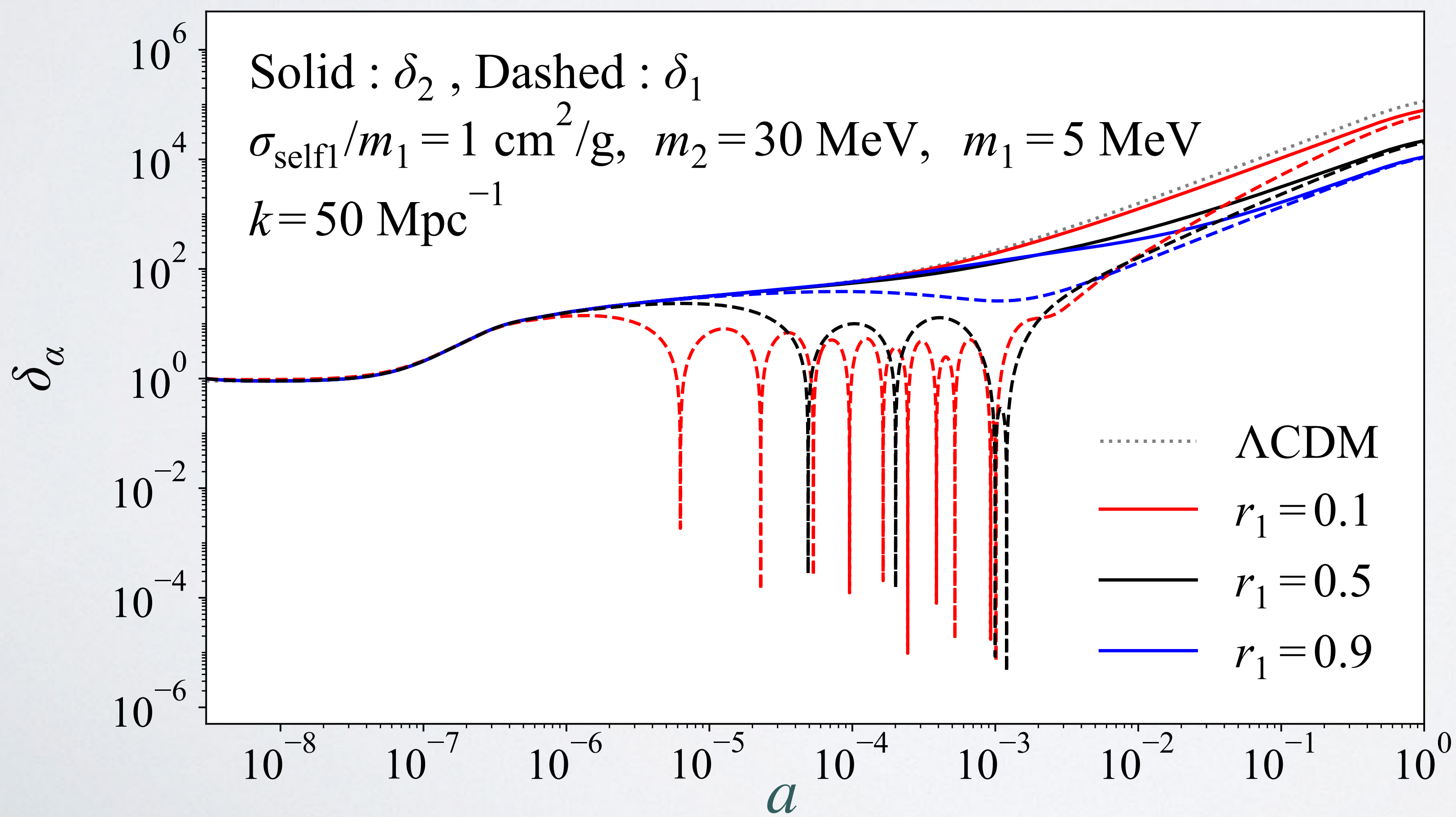
$$= \left(\text{terms of gravity}\right) + \left(\text{coupled terms with } \delta_2\right)$$

Negative: δ_{χ_1} grows

Positive: δ_{χ_1} oscillates

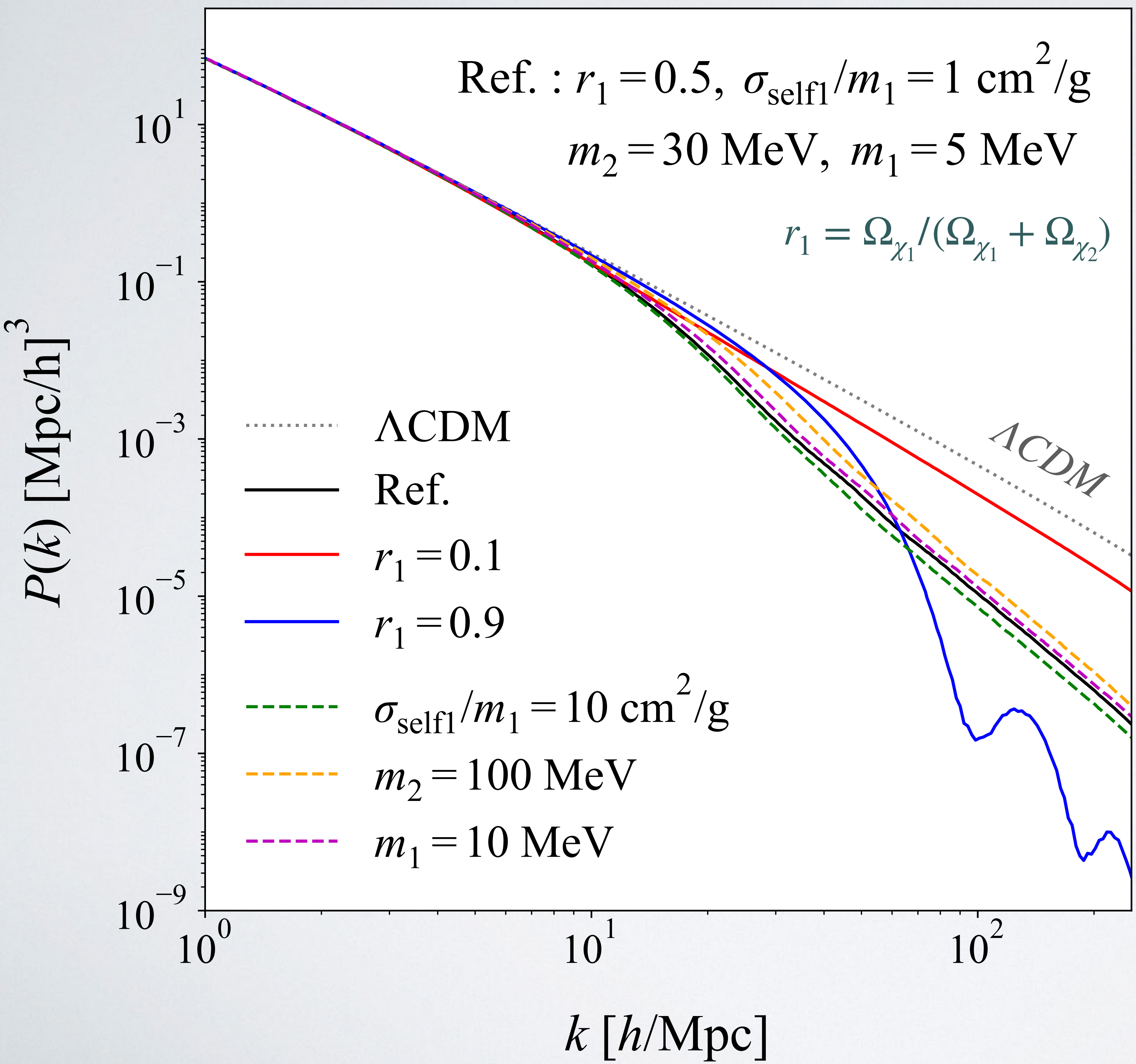
Sound speed of χ_1 resists against the gravity

$$c_{s,\chi_1}^2 = \frac{T_{\chi_1}}{m_{\chi_1}}\left(1 - \frac{1}{3}\frac{\partial \ln T_{\chi_1}}{\partial \ln a}\right)$$



Linear Matter Power Spectrum

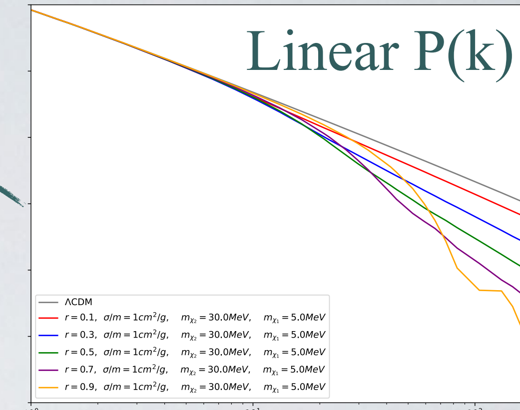
Sehwan Lim, **J. H. Kim**, K.C. Kong, J. Park [2023]



Including Non-Linear Effects

Solve linear Einstein-Boltzmann equations until today

$z = 0$



The input of the simulation is the linear P(k) at $z = 0$.

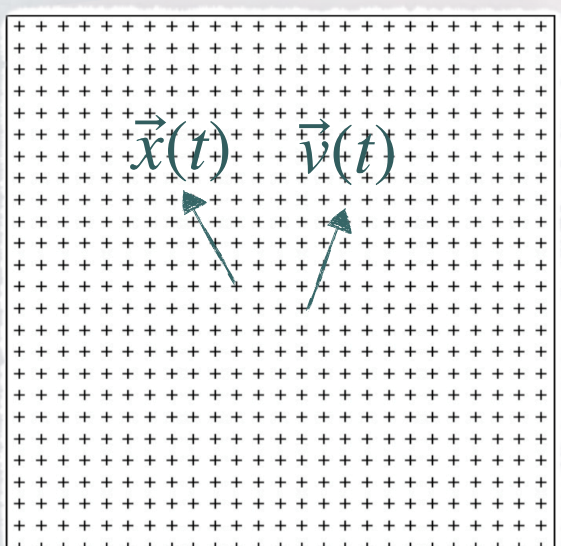
$z \sim 100$

Back-scaling

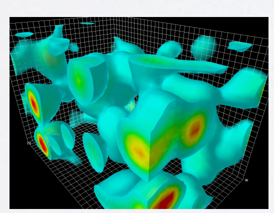
Newtonian linear growth factor
(Including only background quantities)

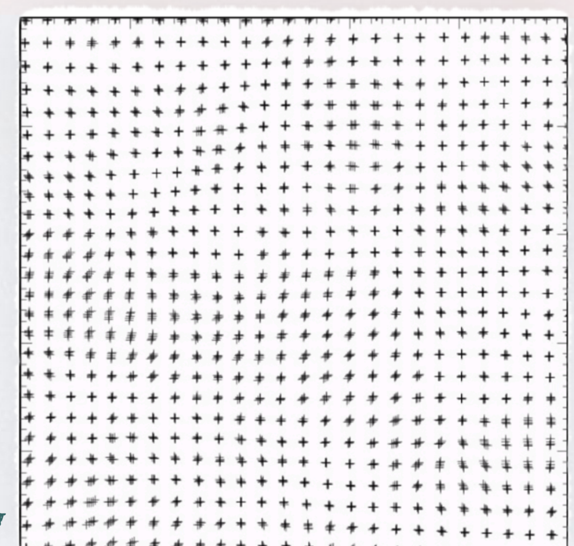
$$\delta_m(z_{\text{start}}) = \delta_m(z=0)\frac{D(z=start)}{D(z=0)}$$

N-body simulations

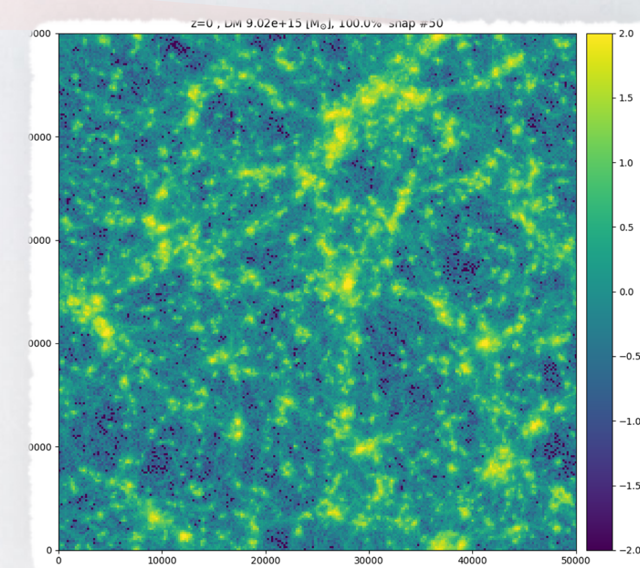


Applying the gaussian initial condition





Simulation starts

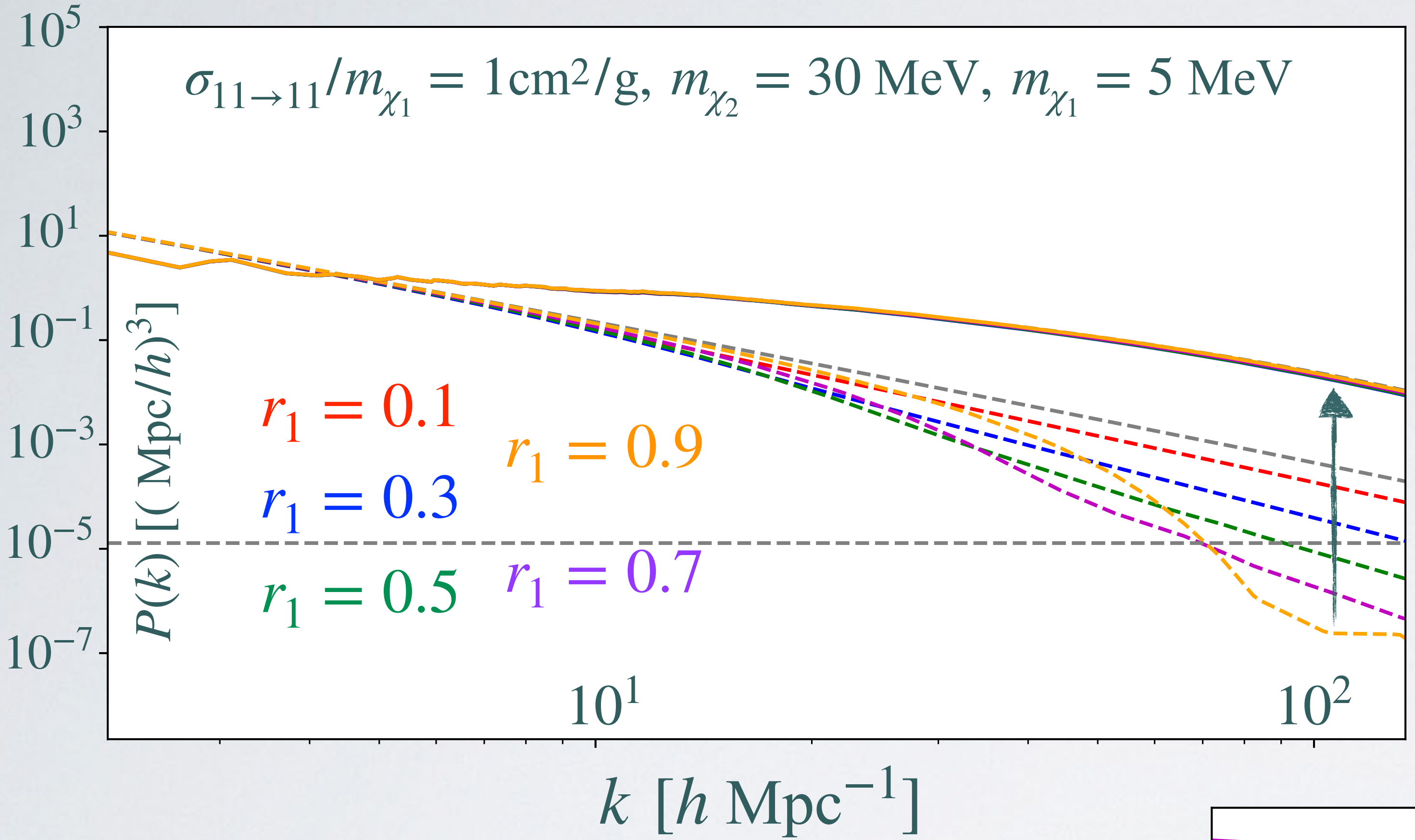


Extracting various small scales observables

1. $\vec{x}(t) = \vec{q} + \vec{\psi}^{(1)}(t, \vec{q})$
2. $\vec{v}(t) = \dot{x}(t)$

Including Non-Linear Effects

Sehwan Lim, **J. H. Kim**, K.C. Kong, J. Park [2023]



- We performed N-body simulations to include non-linear effects.

  Size of a box $= (3\mathrm{Mpc}/h)^3$

  Number of DM particles $= 128^3$

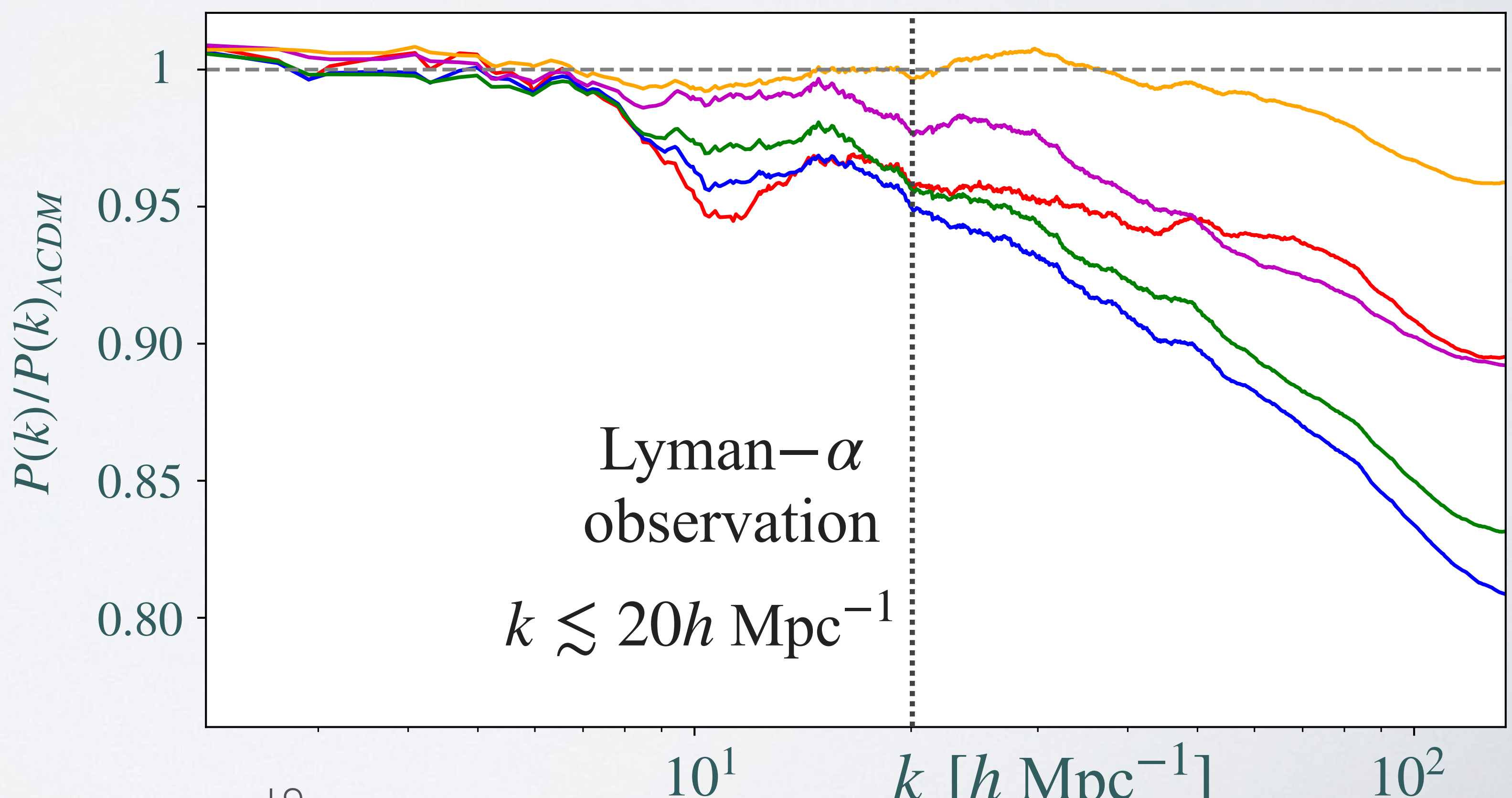
  Starting redshift $z = 100$

  Input $=$ Linear $P(k)$ at $z = 100$

- Non-linear effects can significantly wash out the linear features.

- Nonetheless, there are $5 \sim 20\,\%$ deviations for $k \gtrsim 10h\ \mathrm{Mpc}^{-1}$.

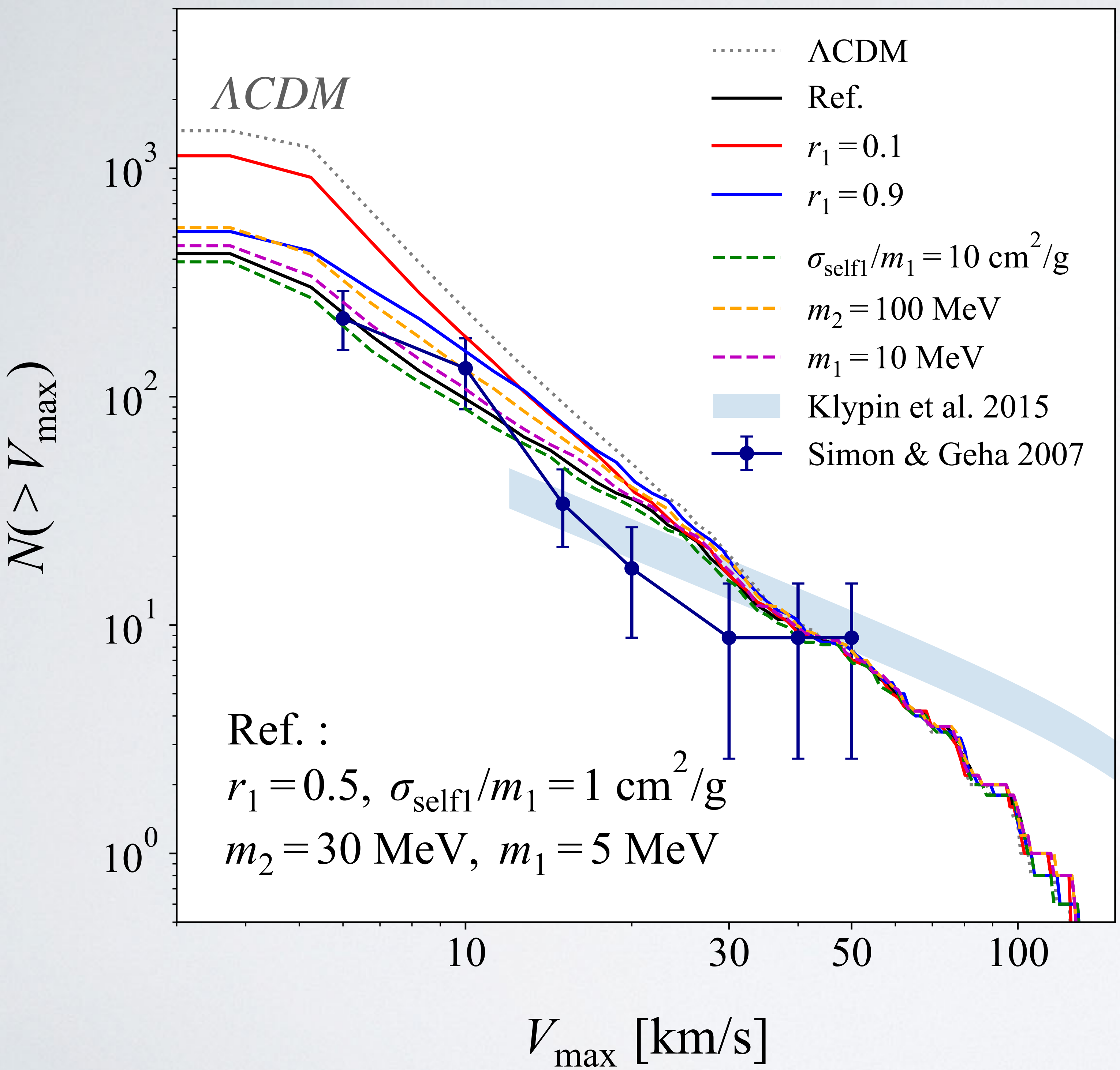
- Lyman$-\alpha$ data can put constraint in the region of $0.5 < k < 20h\ \mathrm{Mpc}^{-1}$.



Observational Constraints

Sehwan Lim, **J. H. Kim**, K.C. Kong, J. Park [2023]

Maximum Circular Velocity Distribution



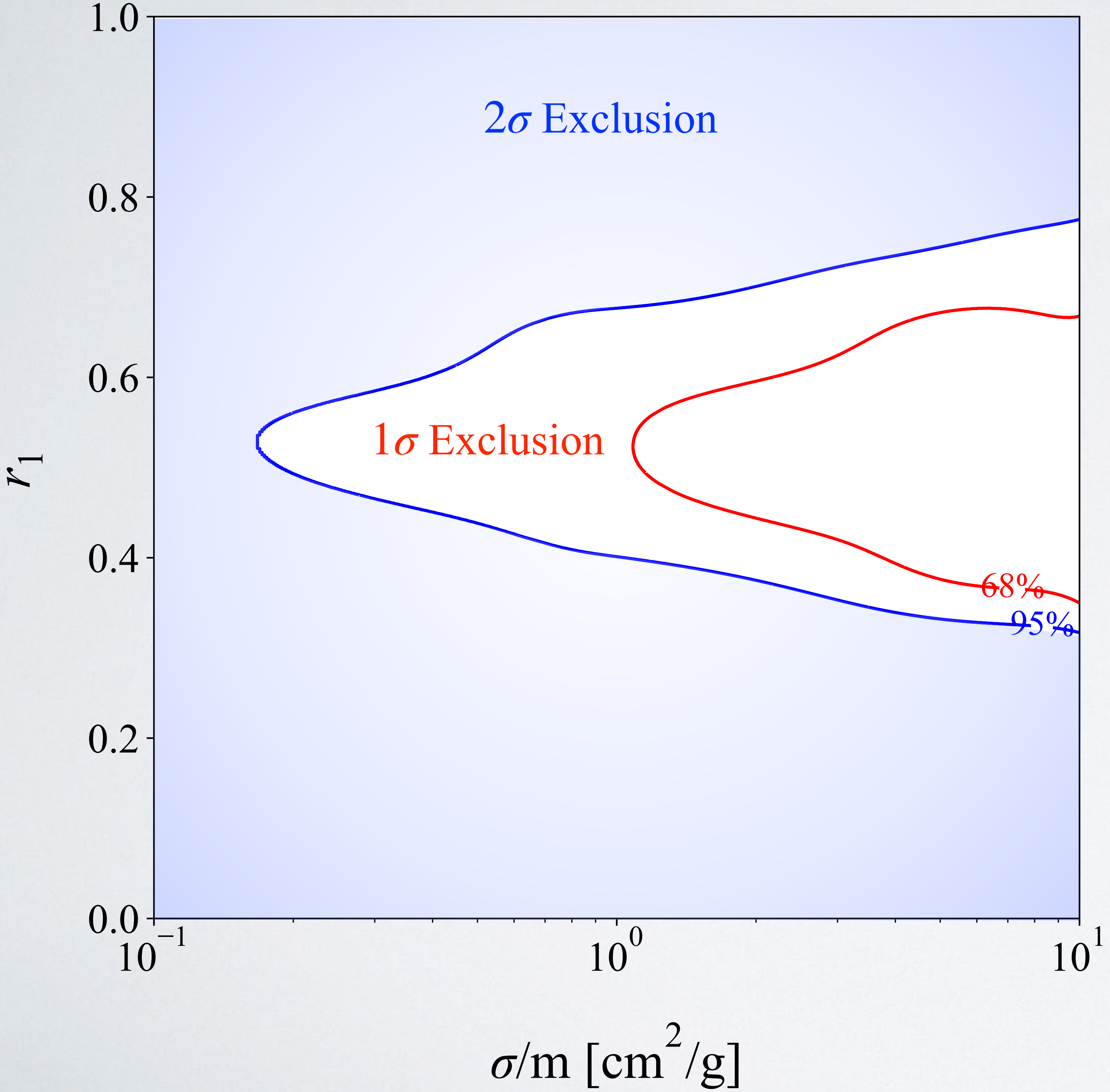
~ mass of subhalos

- The data prefers the Universe with mixed two-component DM.
- The data disfavors large masses m_{χ_1} and m_{χ_2} .
- The data prefers a larger $\sigma_{11\to11}/m_{\chi_1}$.
- *ΛCDM* model is strongly disfavored.

Observational Constraints

Sehwan Lim, **J. H. Kim**, K.C. Kong, J. Park [In Progress]

$m_{\chi_2} = 30$ MeV, $m_{\chi_1} = 5$ MeV

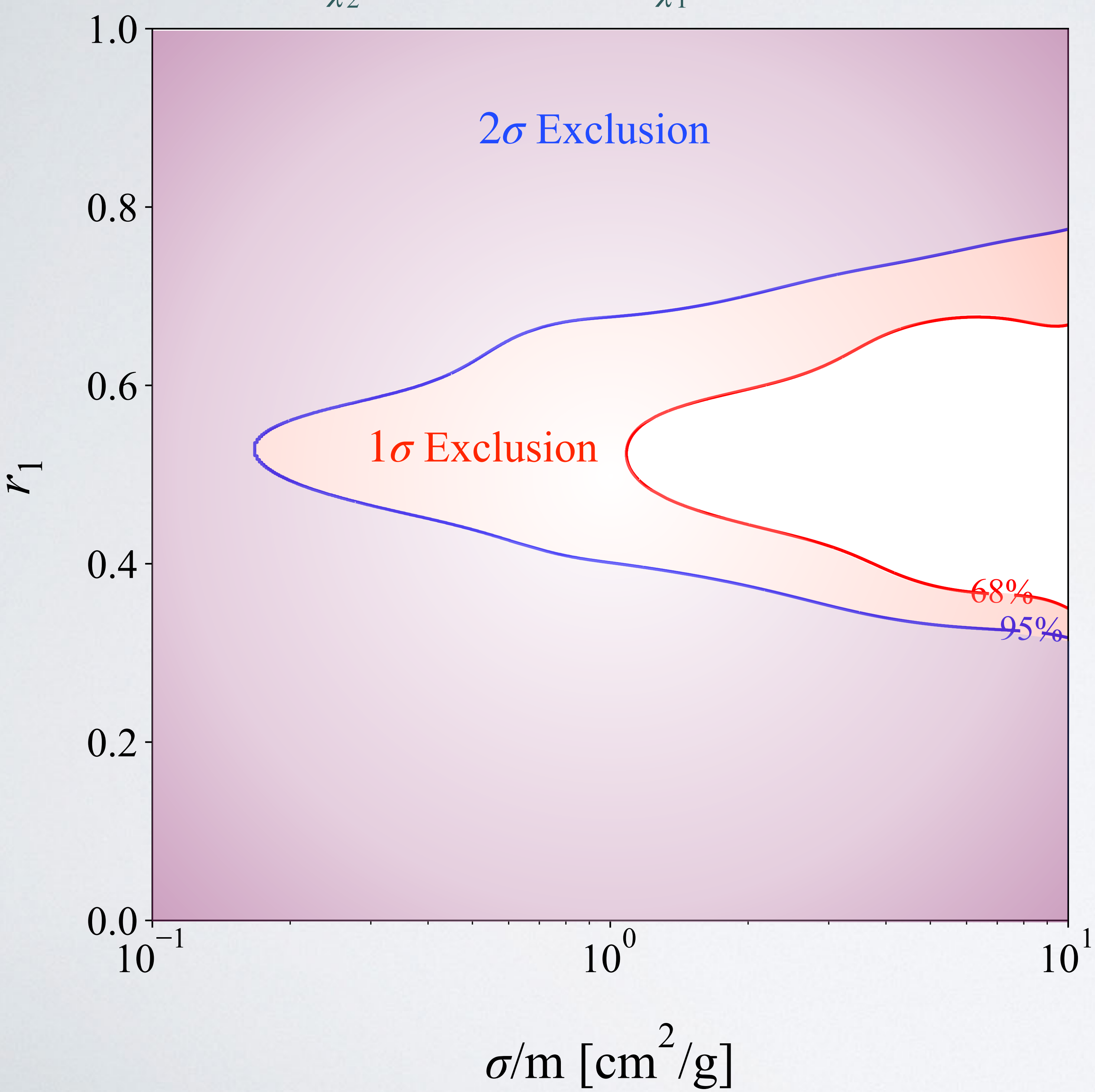


- We perform a chi-square test using the maximum circular velocity distribution
- Single-component limits ($r_1 \sim 1$ or $r_1 \sim 0$) are excluded.
- The data prefers a larger $\sigma_{11\to11}/m_{\chi_1}$.

Observational Constraints

Sehwan Lim, **J. H. Kim**, K.C. Kong, J. Park [2023]

$m_{\chi_2} = 30$ MeV, $m_{\chi_1} = 5$ MeV

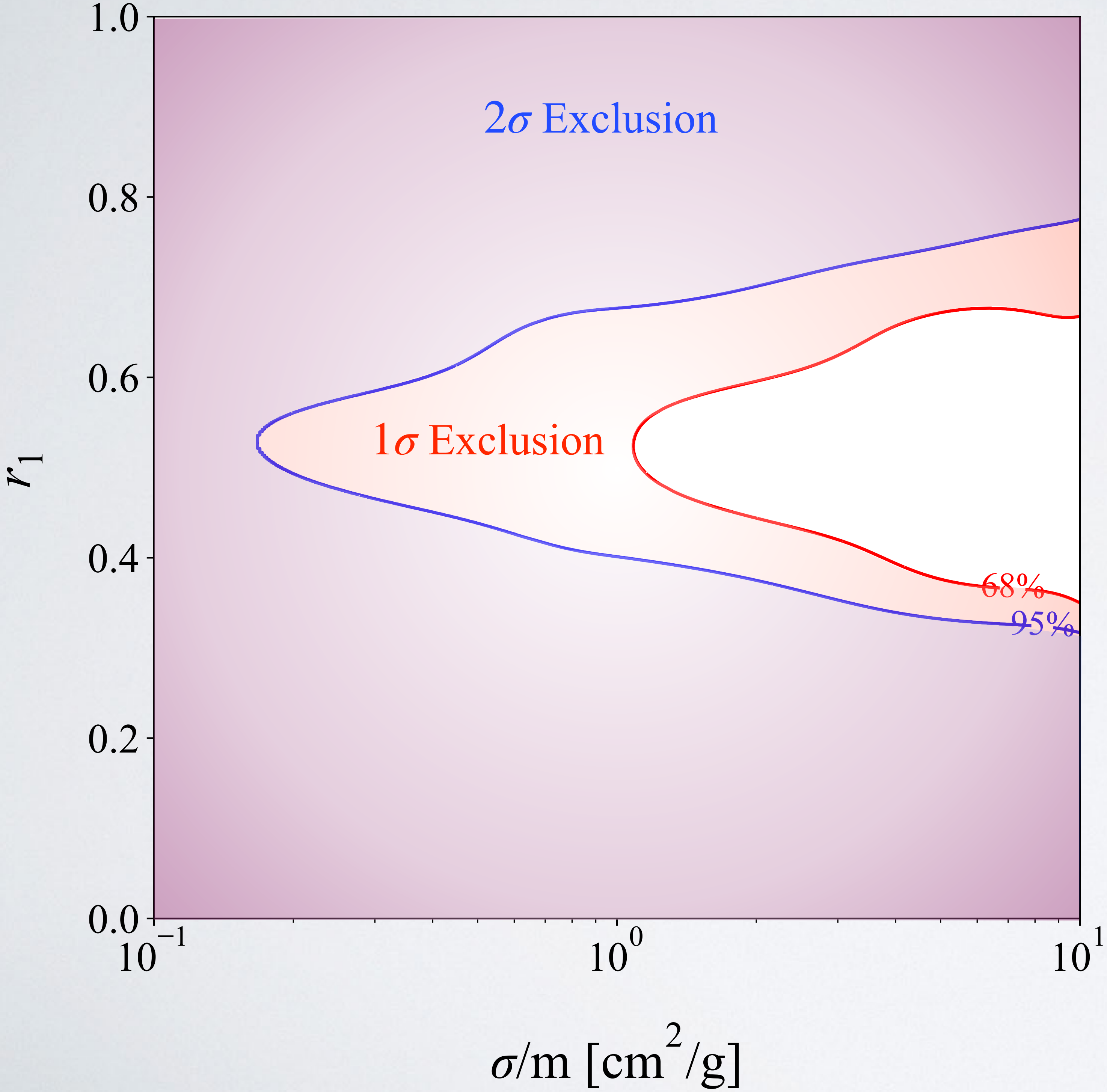


- We perform a chi-square test using the maximum circular velocity distribution
- Single-component limits ($r_1 \sim 1$ or $r_1 \sim 0$) are excluded.
- The data prefers a larger $\sigma_{11\to11}/m_{\chi_1}$.

Future Studies

Sehwan Lim, **J. H. Kim**, K.C. Kong, J. Park [In Progress]

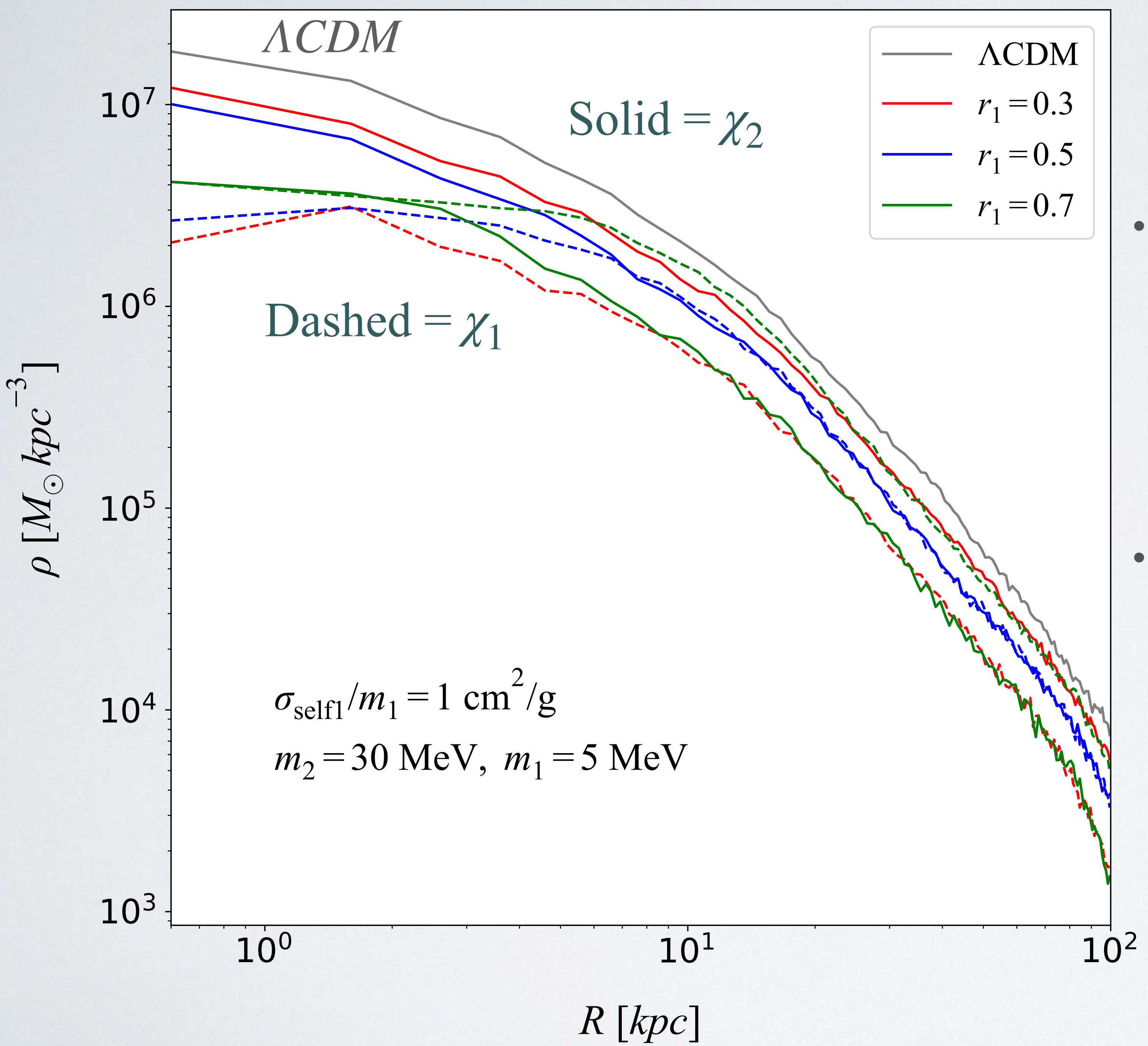
$m_{\chi_2} = 30$ MeV, $m_{\chi_1} = 5$ MeV



- How does the bound change for different masses, m_{χ_1} and m_{χ_2} ?
- How does the bound change if we include the self-interaction of χ_2 ?
- How does the bound change if we include baryons in the simulation ?
- Is the bound compatible with direct detection experiments?
- What are other observables in the small scale structure?

Density Profiles of Halos

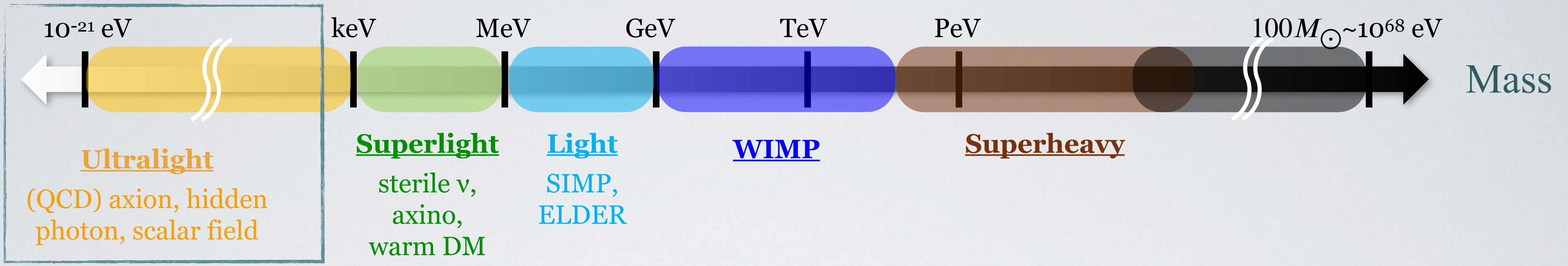
Sehwan Lim, **J. H. Kim**, K.C. Kong, J. Park [In Progress]



- Heavy χ_2 displays a cusp shape of halo.
- Light χ_1 displays a core shape of halo.

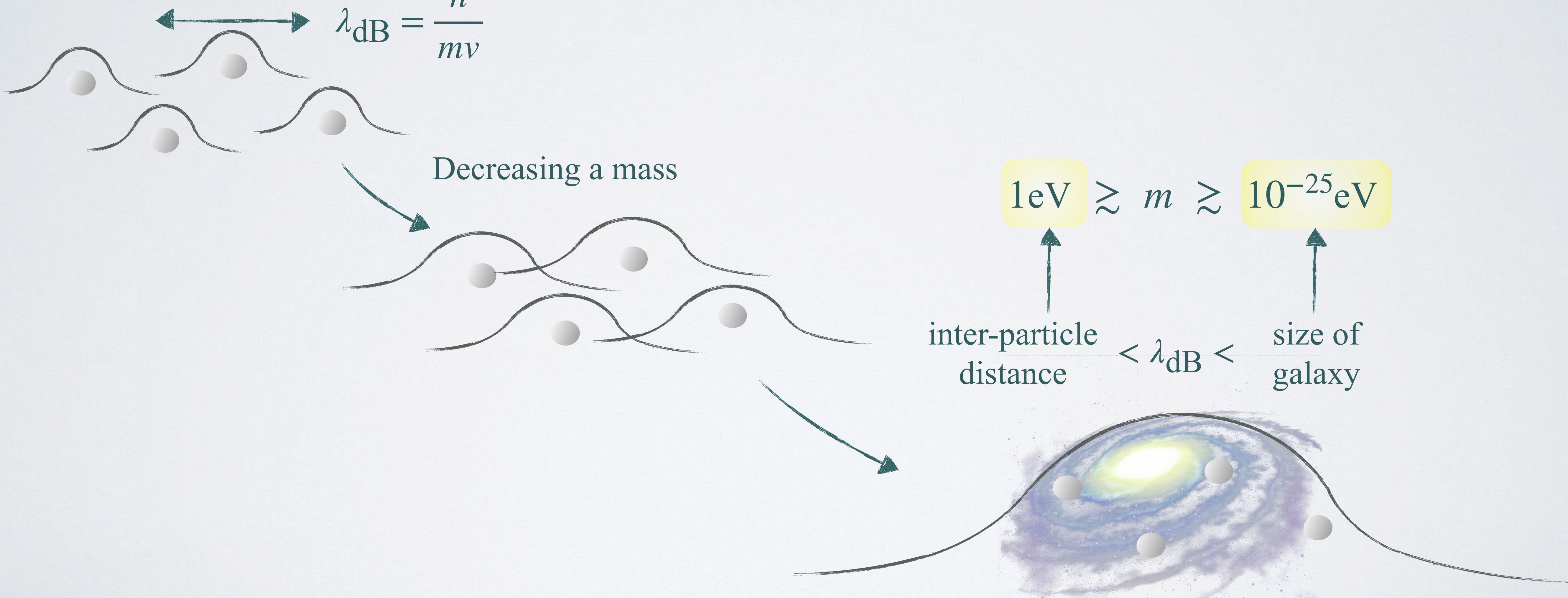
2. Ultra Light Self-Interacting Dark Matter

Ultra Light DM



Consider bosonic DM

$$\lambda_{\rm dB} = \frac{h}{mv}$$



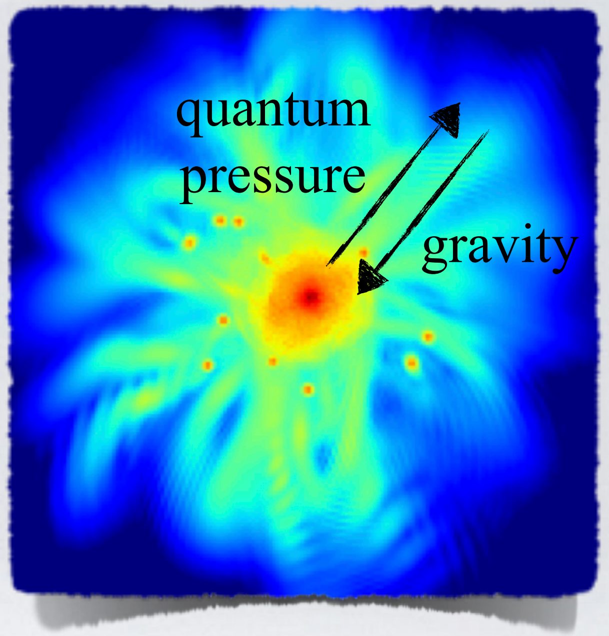
The whole system can be described by a single wave function ψ (cf. superfluid)

Ultra-Light Scalar DM

- Let's consider a scalar ultra-light DM :

$$\mathscr{L} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{m^2}{2}\phi^2$$

Fuzzy DM (= Wave DM)



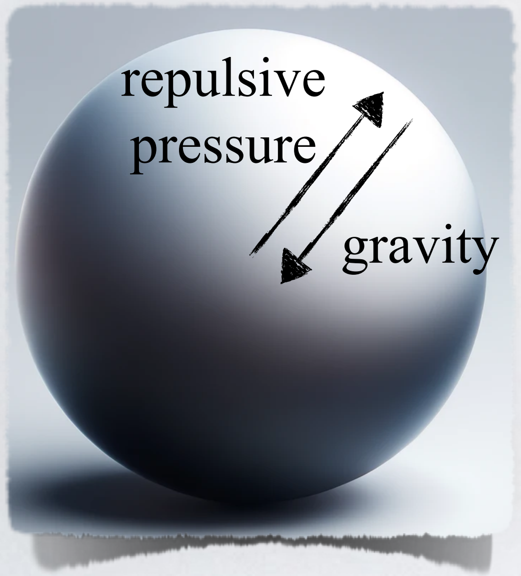
S. Park, D. Bak, J. Lee, I. Park [2022]

- Including a self-interaction :

$$\mathscr{L} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4}\phi^4$$

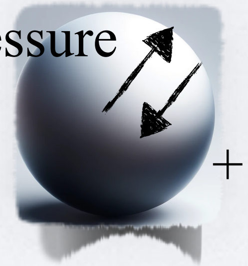
Ultra-light Self-interacting DM (SIDM)

$\lambda > 0$ (repulsive)



Larger soliton

$\lambda < 0$ (attractive) ⟵ (cf. axion)



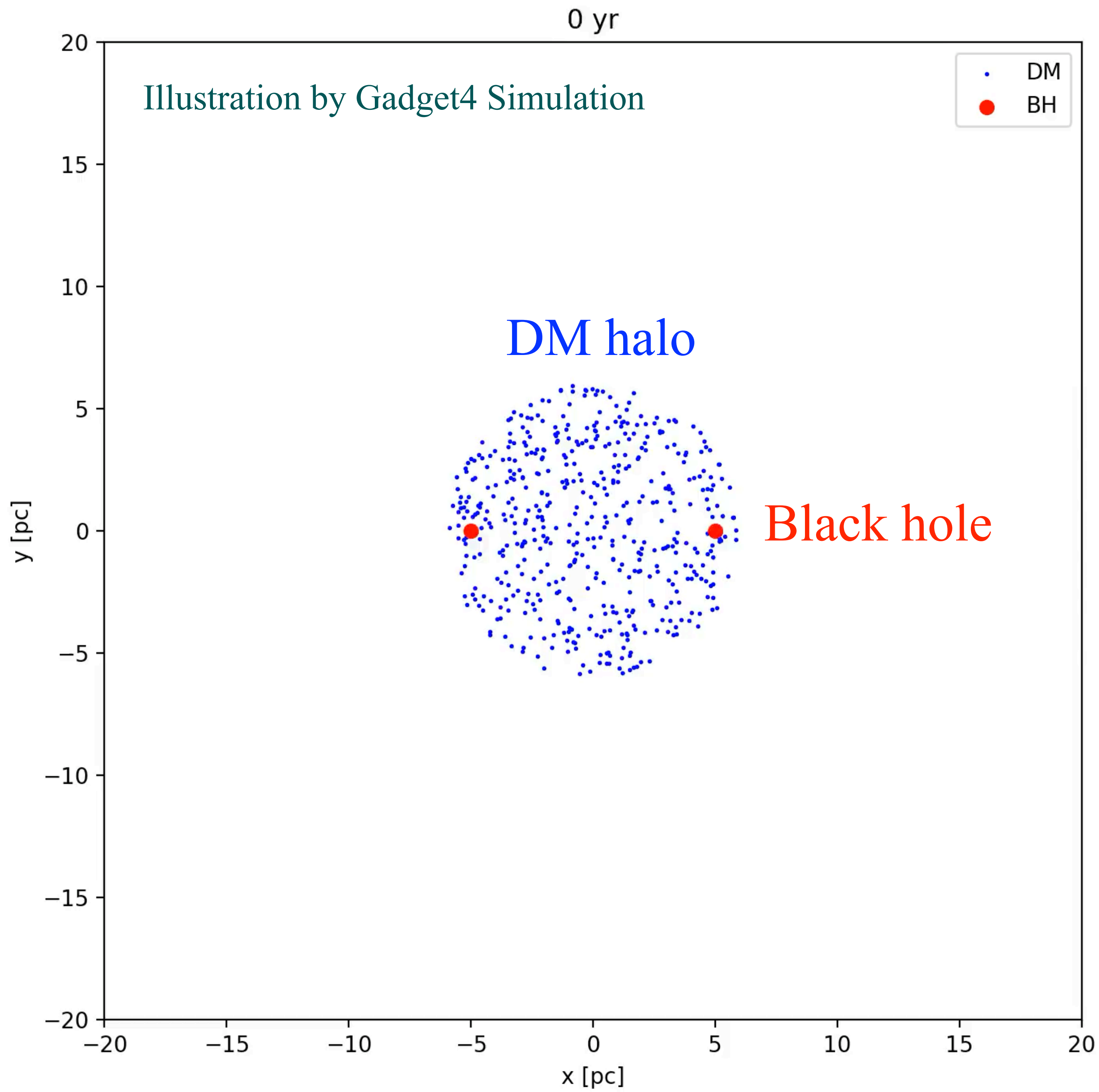
Smaller soliton

$$V(\phi) = \Lambda^4(1 - \cos\frac{\phi}{f})$$

$$m \approx \frac{\Lambda^2}{f} \quad \lambda \approx \frac{\Lambda^4}{6f^4}$$

Gravitational Wave Probes on SIDM

K. Kadota, **J. H. Kim**, Pyungwon Ko, Xing-yu Yang [2306.10828]

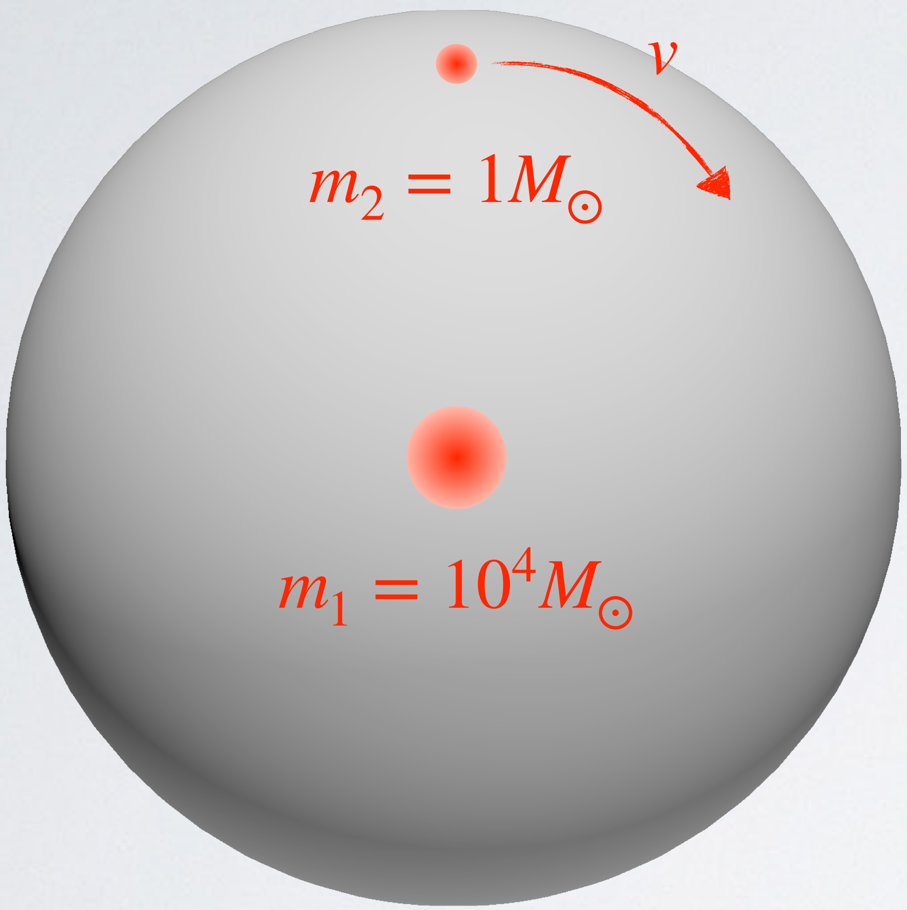


- The shape of DM overdensities can influence the evolution of a binary system.
- The dense region of DM can lead to the dephasing of GWs which can be detected by a future observation by LISA.
- The dynamical friction and accretion of the black hole due to DM should be carefully taken into account.

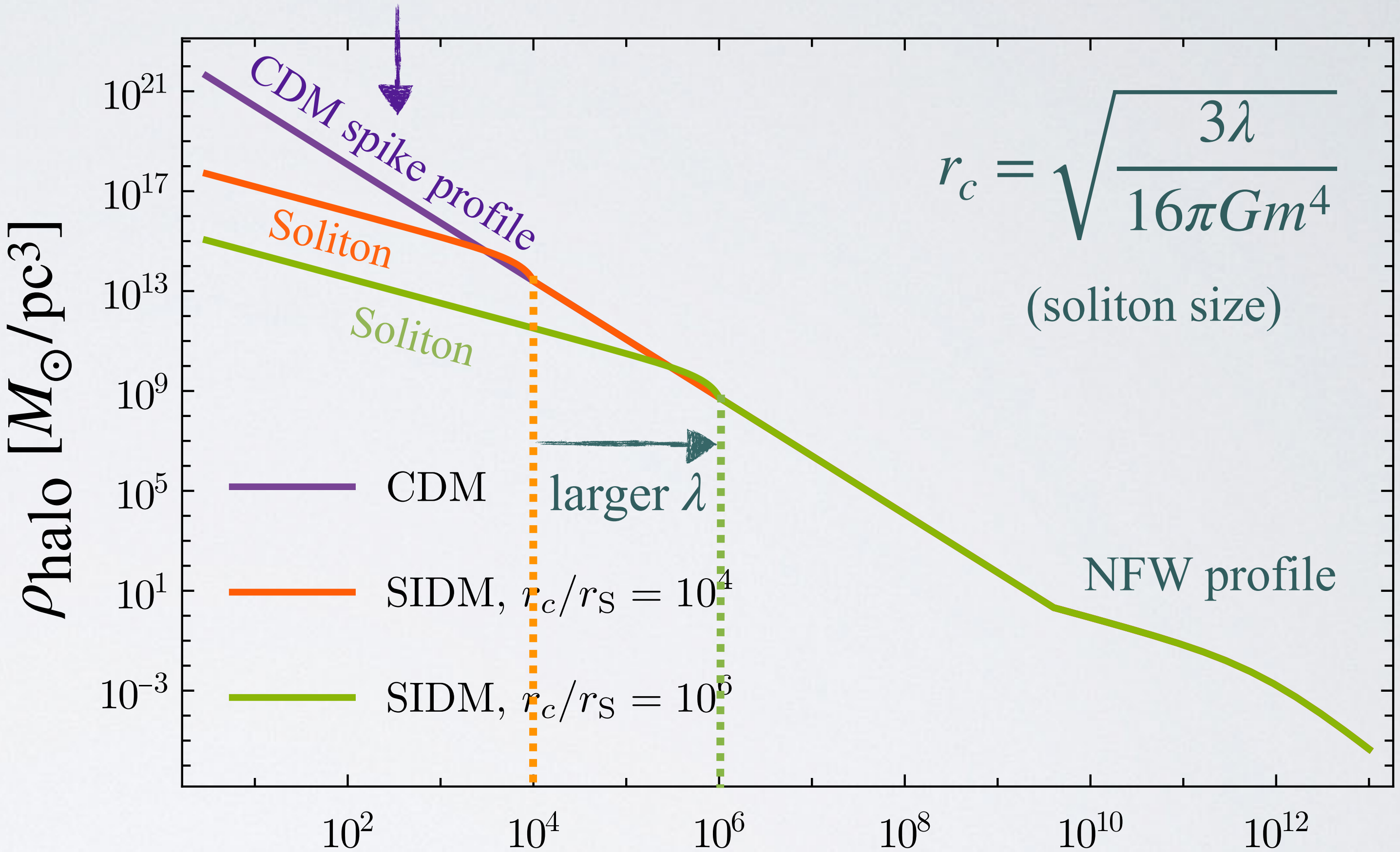
Gravitational Wave Probes on SIDM

K. Kadota, **J. H. Kim**, Pyungwon Ko, Xing-yu Yang [2306.10828]

- To simplify the analysis, we consider a large mass ratio limit $m_2/m_1 \ll 1$.



In the presence of BH



$$r_c = \sqrt{\frac{3\lambda}{16\pi G m^4}}$$

(soliton size)

r/r_s (in a unit of $r_s = 2GM_{\rm BH}$)

Orbital energy of a binary

$$E_{\rm orb} = -\left(\frac{G^2 M_c^5 w_{\rm GW}^2}{32}\right)^{1/3}$$

$$-\frac{dE_{\rm orb}}{dt} = P_{\rm GW} + P_{\rm DF} + P_{\rm Ac}$$

$$w_{\rm GW} = 2\pi f$$

$$w_s = \sqrt{\frac{GW}{r^3}}$$

$$w_{\rm GW} = 2w_s$$

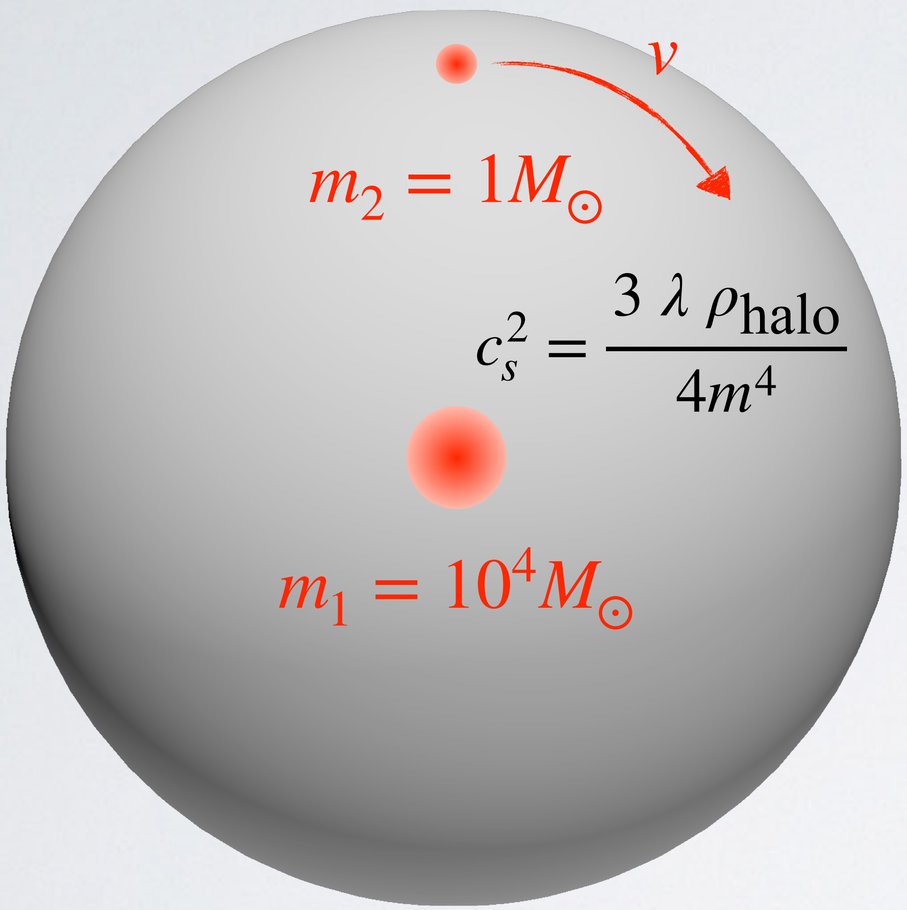
$$P_{\rm GW} = \frac{32}{5}\frac{c^5}{G}\left(\frac{GM_c w_{\rm GW}}{2c^3}\right)^{10/3}$$

Power of energy loss due to GW emission

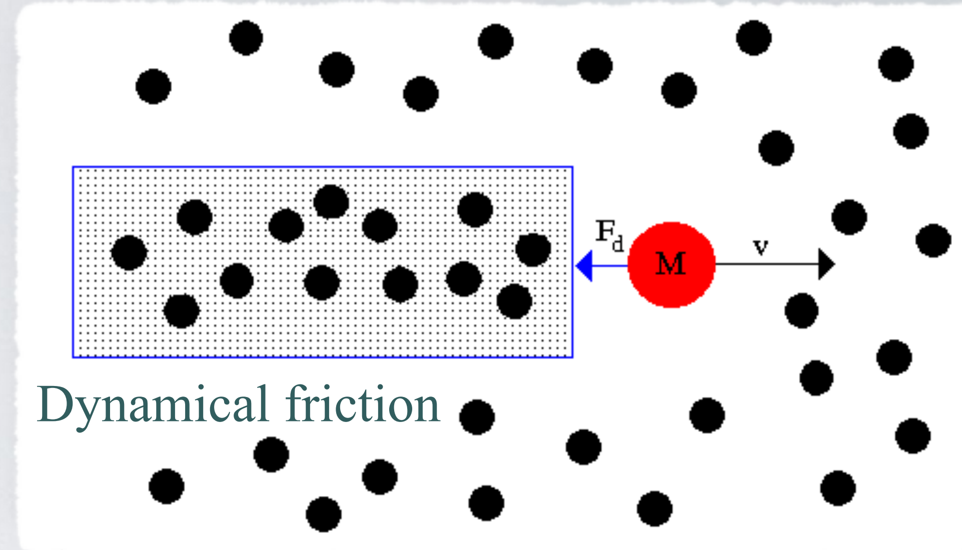
Gravitational Wave Probes on SIDM

K. Kadota, **J. H. Kim**, Pyungwon Ko, Xing-yu Yang [2306.10828]

- Power of energy loss due to a dynamical friction.

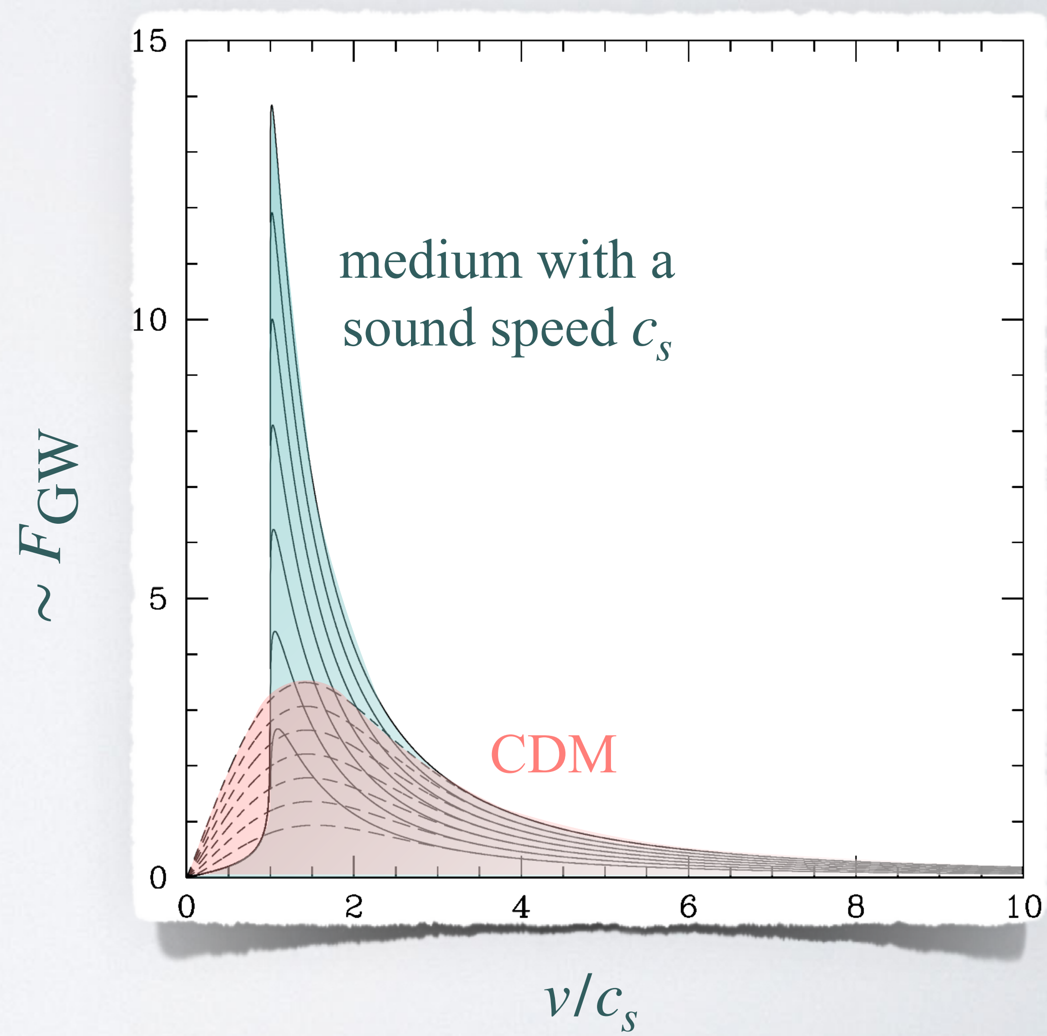


$$c_s^2 = \frac{3\,\lambda\,\rho_{\rm halo}}{4m^4}$$ (sound speed of SIDM medium)



$$-\frac{dE_{\rm orb}}{dt} = P_{\rm GW} + P_{\rm DF} + P_{\rm Ac}$$

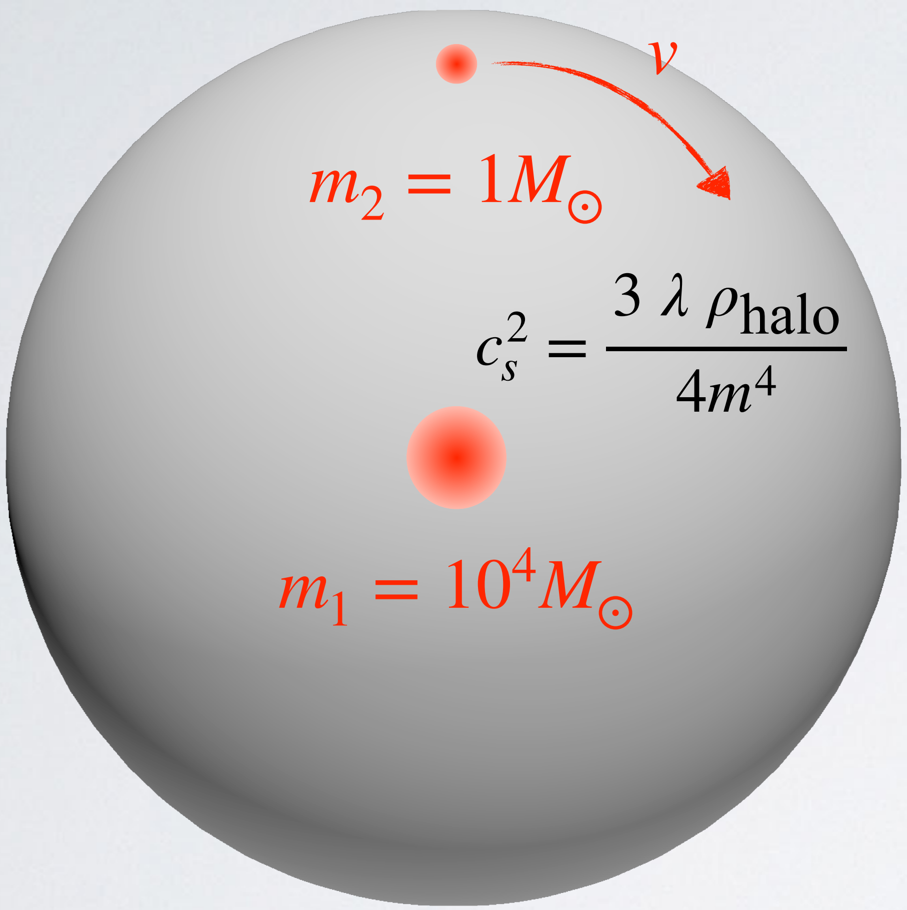
$$P_{\rm DF} = v\,F_{\rm DF} = \frac{4\pi(Gm_2)^2\rho_{\rm halo}}{v} I(\mathcal{M}, \Lambda)$$



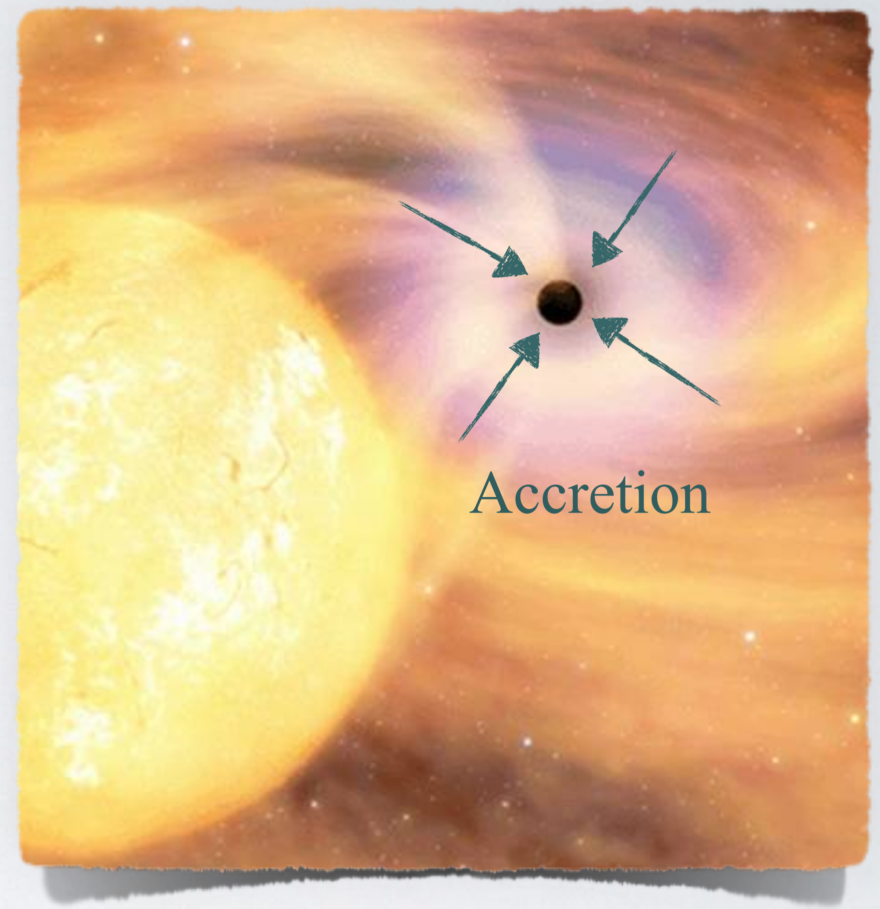
Gravitational Wave Probes on SIDM

K. Kadota, **J. H. Kim**, Pyungwon Ko, Xing-yu Yang [2306.10828]

- Power of energy loss due to an accretion.



(sound speed of SIDM medium)



$$-\frac{dE_{\rm orb}}{dt} = P_{\rm GW} + P_{\rm DF} + P_{\rm Ac}$$

$$P_{\rm Ac} = v\,F_{\rm Ac} = \frac{4\pi (G m_2)^2 \rho_{\rm halo}}{(c_s^2 + v^2)^{3/2}} v^2 \left(1 + \frac{m_2}{m_1}\right)^{-2}$$

- The accretion rate of SIDM is much larger than the CDM case.

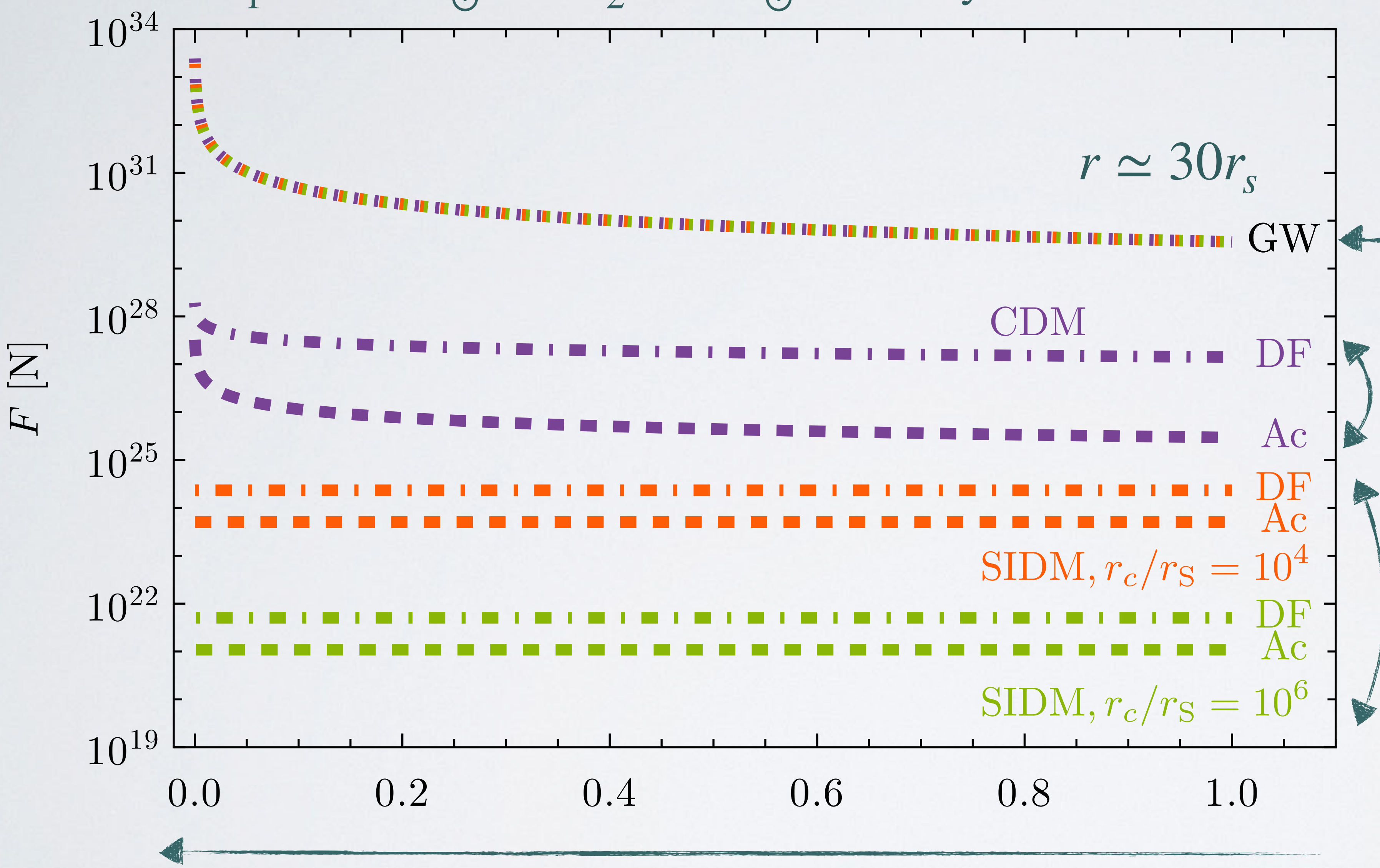
$$\frac{P_{\rm Ac}({\rm CDM})}{P_{\rm Ac}({\rm SIDM})} \sim \frac{v^2}{c^2} \ll 1$$

Dephasing of GWs

K. Kadota, **J. H. Kim**, Pyungwon Ko, Xing-yu Yang [2306.10828]

- The evolution of the forces contributed by GWs, dynamical friction, and accretion.

$m_1 = 10^4 M_\odot$ $m_2 = 1 M_\odot$ One year of observation



τ [yr] (remaining time to merge)

GWs contribution dominates

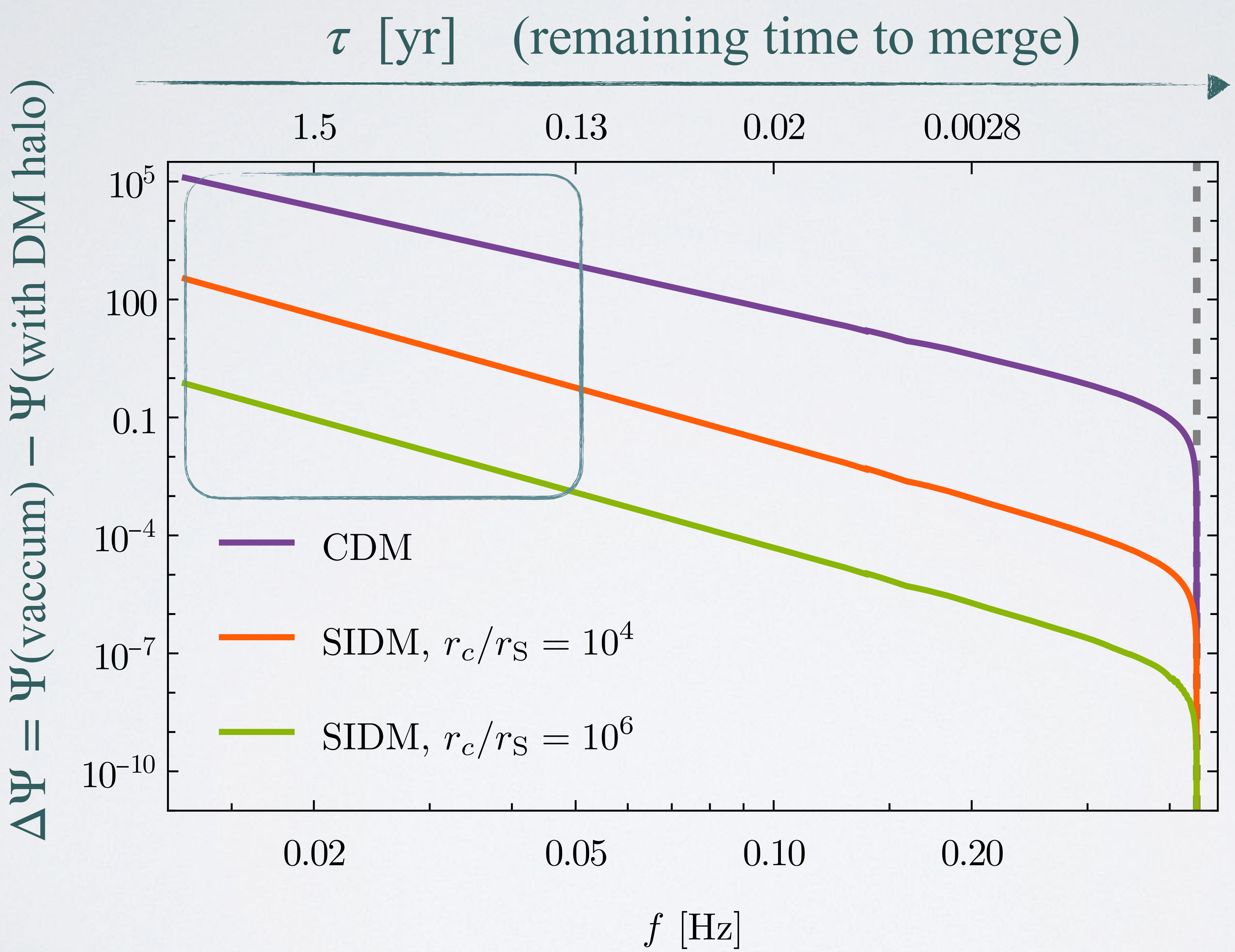
Dynamical friction is larger than the accretion

SIDM has smaller DF and AC compared with CDM

Dephasing of GWs

K. Kadota, **J. H. Kim**, Pyungwon Ko, Xing-yu Yang [2306.10828]

- Dephasing of the gravitational waveform in the presence of DM halo.



$$r_c = \sqrt{\frac{3\lambda}{16\pi G m^4}}$$

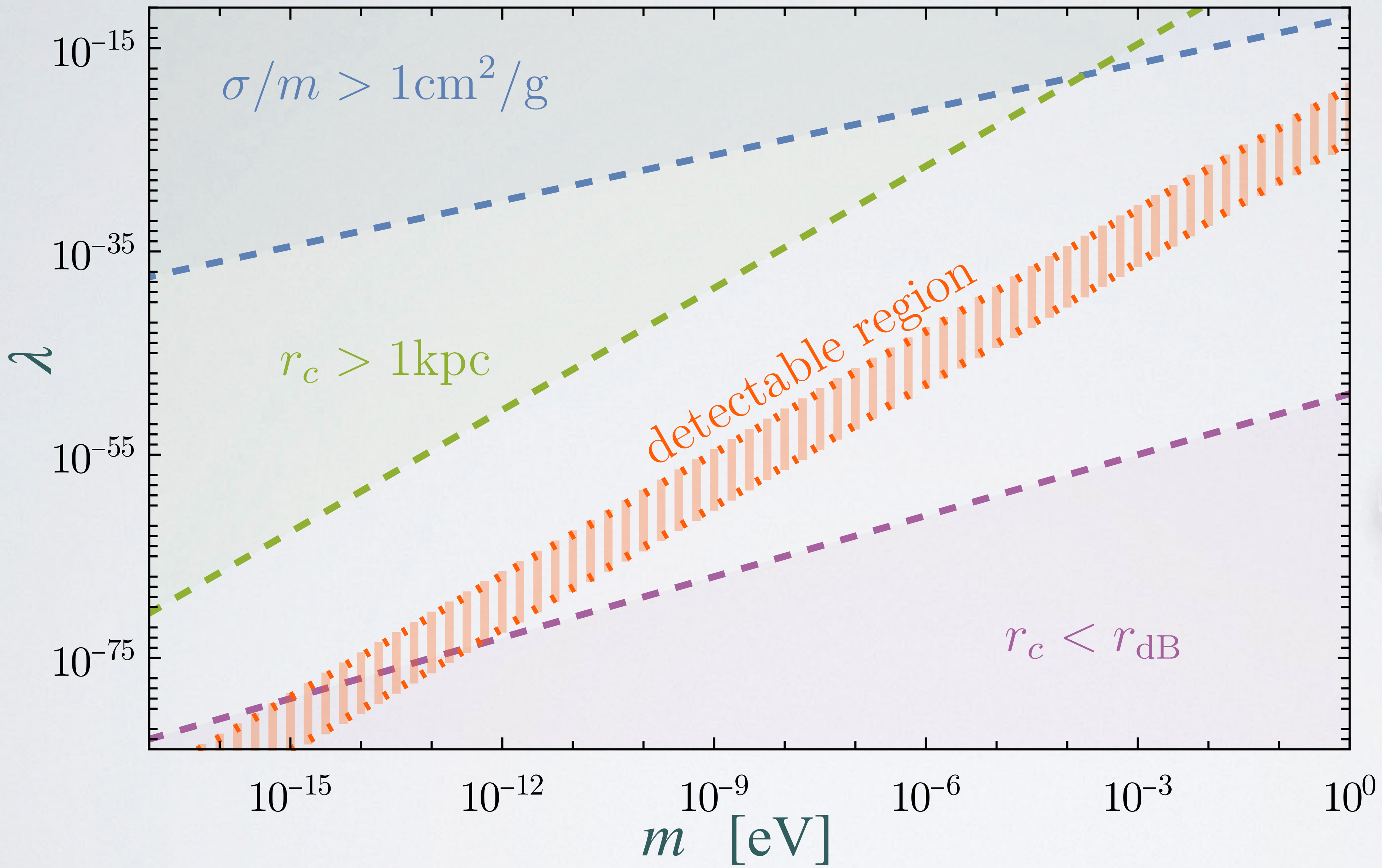
(soliton size)

- The dephasing effect is maximal when the distance is farther away from the r_s.

Gravitational Wave Probes on SIDM

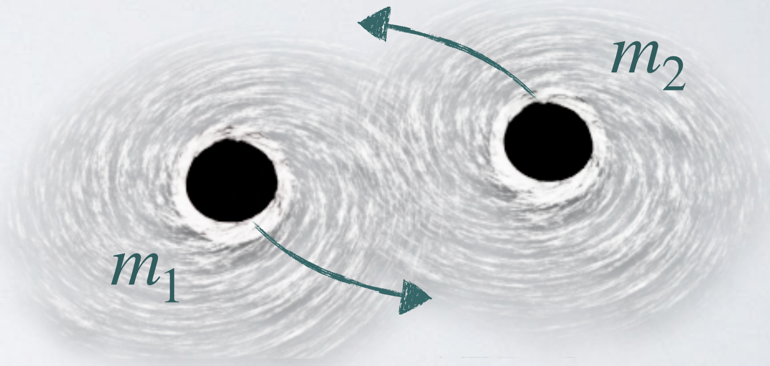
K. Kadota, **J. H. Kim**, Pyungwon Ko, Xing-yu Yang [2306.10828]

$m_1 = 10^4 M_\odot$ $m_2 = M_\odot$



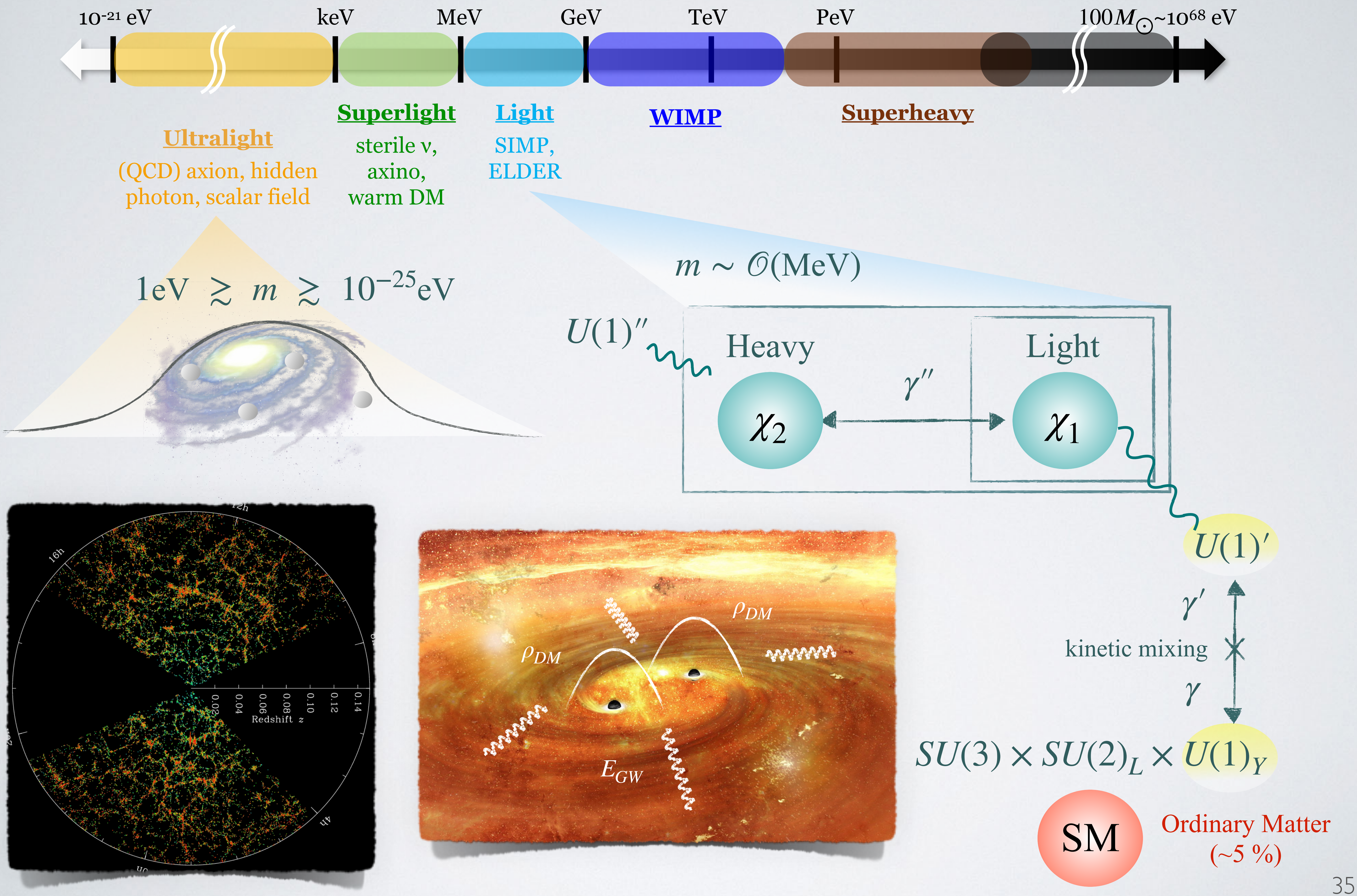
Half of year observation of LISA

$r \simeq [3r_s,\ 15r_s]$



- GW probes on the DM model will be able to shed light on the uncharted parameter space.
- Distinguishing different DM models from GWs will be interesting future works.
- Another complementary handle to probe the dark sector.

Summary



10⁻²¹ eV
keV
MeV
GeV
TeV
PeV
$100M_\odot \sim 10^{68}$ eV
Ultralight
(QCD) axion, hidden photon, scalar field
Superlight
sterile ν, axino, warm DM
Light
SIMP, ELDER
WIMP
Superheavy
$1\text{eV} \gtrsim m \gtrsim 10^{-25}\text{eV}$
$m \sim \mathcal{O}(\text{MeV})$
$U(1)''$
Heavy
Light
γ''
χ_2
χ_1
$U(1)'$
γ'
kinetic mixing
γ
$SU(3) \times SU(2)_L \times U(1)_Y$
SM
Ordinary Matter (~5 %)
Redshift z
0.02
0.04
0.06
0.08
0.10
0.12
0.14
ρ_{DM}
ρ_{DM}
E_{GW}

Back-up

Coupled Background Boltzmann Equations

A. Kamada, H. Kim, J. Park, S. Shin [2021]

- Cosmological background evolutions are governed by coupled Boltzmann equations for χ_1 and χ_2 .

$\chi_2\bar{\chi}_2 \to \chi_1\bar{\chi}_1$

$$1. \quad \frac{d\rho_{\chi_2}}{dt} + 3H\rho_{\chi_2} = -\frac{\langle \sigma v \rangle_{22\to 11}}{m_{\chi_2}}\left(\rho_{\chi_2}^2 - \frac{\rho_{\chi_2,\mathrm{eq}}^2}{\rho_{\chi_1,\mathrm{eq}}^2}\rho_{\chi_1}^2\right)$$

(where $\langle \sigma v \rangle_{22\to 11} \simeq 0.2 \left(\frac{5\times 10^{-26}\mathrm{cm}^3/\mathrm{s}}{\Omega_{\chi_2}}\right)$)

Hubble friction

collision terms

χ_2 relic abundance

$\chi_1\bar{\chi}_1 \to$ SM SM

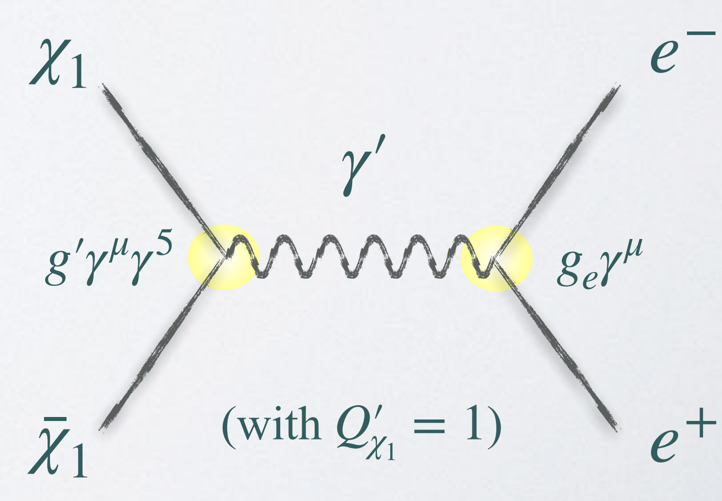
$$2. \quad \frac{d\rho_{\chi_1}}{dt} + 3H\rho_{\chi_1} = -\frac{\langle \sigma v \rangle_{11\to \mathrm{SM\,SM}}}{m_{\chi_1}}\left(\rho_{\chi_1}^2 - \rho_{\chi_1,\mathrm{eq}}^2\right) + \frac{m_{\chi_1}}{m_{\chi_2}}\frac{\langle \sigma v \rangle_{22\to 11}}{m_{\chi_2}}\left(\rho_{\chi_2}^2 - \frac{\rho_{\chi_2,\mathrm{eq}}^2}{\rho_{\chi_1,\mathrm{eq}}^2}\rho_{\chi_1}^2\right)$$

- Here, SM = $e^-, e^+, \gamma, \cdots$ denotes relativistic particles.

- We consider the p-wave cross section $\chi_1\bar{\chi}_1 \to$ SM SM (not to screw CMB, BAO, …).

$$\langle \sigma v \rangle_{11\to \mathrm{SM\,SM}} = \frac{g'^2 g_e^2 \left(2m_{\chi_1}^2 + m_e^2\right)\sqrt{m_{\chi_1}^2 - m_e^2}}{6m_{\chi_1}\left(m_{\gamma'}^2 - 4m_{\chi_1}^2\right)^2\pi}\, v^2 + \mathcal{O}(v^3)$$

Dark photon mass



Coupled Background Boltzmann Equations

A. Kamada, H. Kim, J. Park, S. Shin [2021]

- Large $\langle \sigma v \rangle_{11\to \text{SM SM}}$ can significantly affect the CMB at the 0^{th}-order.

If SM particles are relativistic

$\chi_1\bar{\chi}_1 \to$ SM SM

Neglected in this work

$$3. \quad \frac{d\rho_{\text{SM}}}{dt} + 4H\rho_{\text{SM}} = \frac{\langle \sigma v \rangle_{11\to \text{SM SM}}}{m_{\chi_1}}\left(\rho_{\chi_1}^2 - \rho_{\chi_1,\text{eq}}^2\right)$$

(with SM $= e^-, e^+, \gamma, \cdots$)

- The energy injection to the SM plasma can change the ionization history, Compton scattering, …

D. Green, P.D. Meerburg, J. Meyers [2018]
N. Padmanabhan, D.P. Finkbeiner [2005]
…

- With the p-wave cross section $\langle \sigma v \rangle_{11\to \text{SM SM}}$, we can evade this constraint.

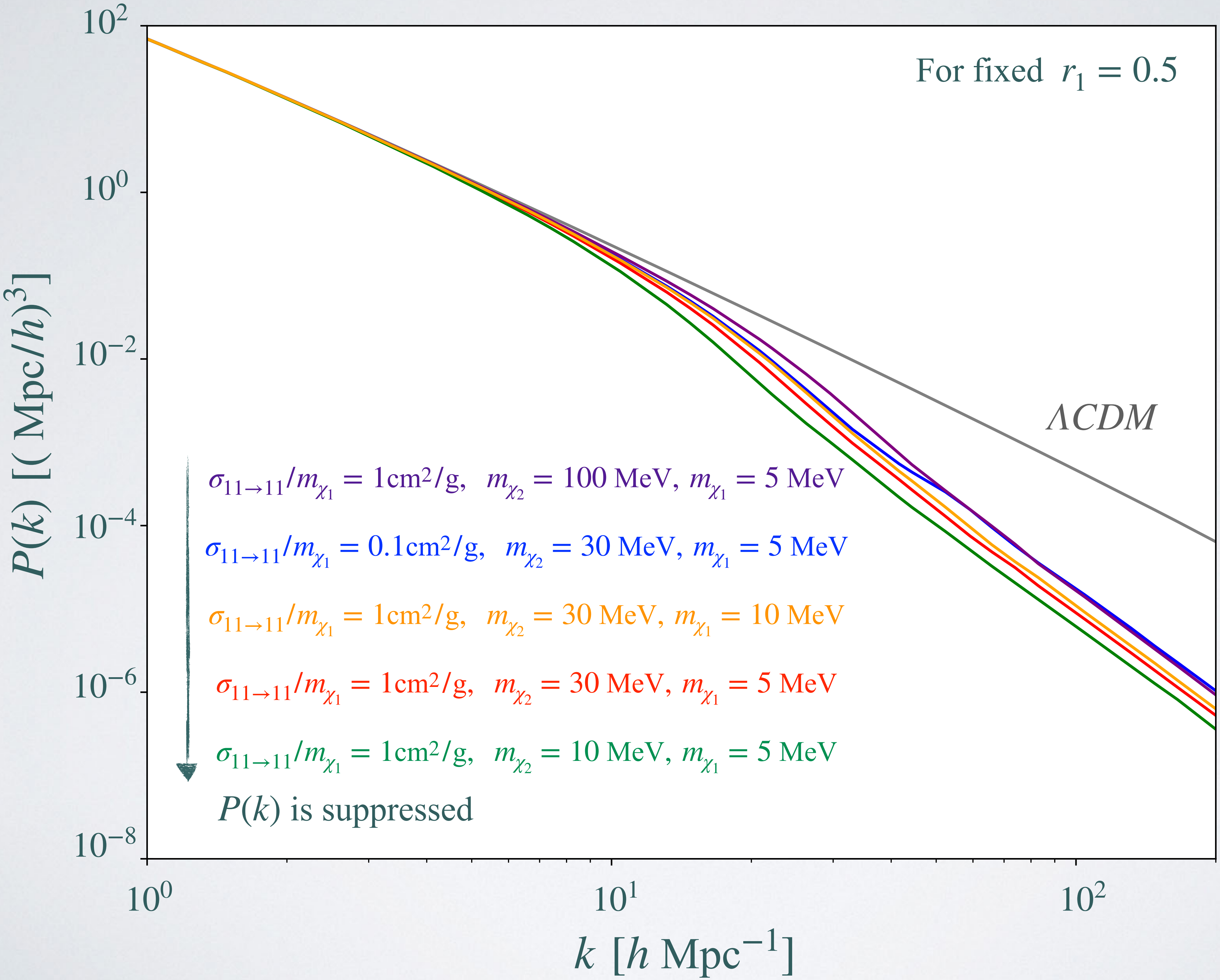
See also other way around, P.J. Fitzpatrick, H. Liu, T.R. Slatyer, Y.D. Tsai [2011]

- In this work, we focus on the evolution of DM matter densities, and neglect the effect of “3” in the structure formation of the Universe.

(future study)

Matter Power Spectrum

Sehwan Lim, **J. H. Kim**, K.C. Kong, J. Park [In Progress]



Initial Conditions

◎ Initial Conditions

Recall For adiabatic perturbations, the fluctuations in all components are related by

$$\delta_\gamma = \delta_\nu = \frac{4}{3}\delta_{CDM} = \frac{4}{3}\delta_b = -2\Phi_i \quad (\text{with } \Phi_i \simeq \Psi_i)$$

where Φ_i is the primordial potential which is given by

$$\Phi_i = \frac{2}{3} R_i$$

Where R_i is the gauge-invariant curvature perturbation. It connects between the era of end of inflation and a deep radiation-dominated era.

Remark From the above initial conditions, we are able to write down the photon and matter fluctuations as (δ_m denotes a matter density contrast)

$$\therefore \delta_\gamma = \frac{4}{3}\delta_m = -2\Phi_i = -\frac{4}{3}R_i$$

Recall The curvature perturbation R_i is determined by

$$\Delta_R^2(k) \equiv \frac{k^3}{2\pi^2}|R_i(k)|^2 = A_s\left(\frac{k}{k_*}\right)^{n_s-1}$$

$\therefore$ scalar amplitude $A_s = \dfrac{1}{8\pi^2}\dfrac{1}{\varepsilon_*}\dfrac{H_*^2}{M_{Pl}^2}$

$\therefore$ spectral index $n_s = 1 - 2\varepsilon_* - \kappa_*$

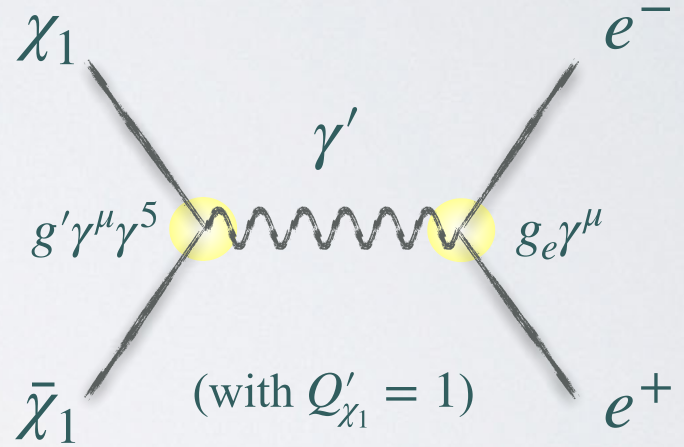
All these quantities are computed at the time of horizon exit

$$\left\langle \sigma v \right\rangle_{\chi_1,X} = \frac{c_a^2 e_v^2 m_{\chi_1} m_e (3m_{\chi_1}^2 + 2m_{\chi_1} m_e + m_e^2)}{2(m_{\chi_1} + m_e)^2 m_{\gamma'}^4 \pi}$$

$$\gamma_{\chi_1 \mathrm{sm}} = \frac{\delta E}{T} n_{\mathrm{sm}} \left\langle \sigma v \right\rangle_{\chi_1,sm}$$

p-wave annihilation

$$\left\langle \sigma v \right\rangle_{11\to\mathrm{SM\ SM}} = \frac{g'^2 g_e^2 \left(2m_{\chi_1}^2 + m_e^2\right)\sqrt{m_{\chi_1}^2 - m_e^2}}{6m_{\chi_1}\left(m_{\gamma'}^2 - 4m_{\chi_1}^2\right)^2 \pi} v^2 + \mathcal{O}(v^3)$$



$$\left\langle \sigma v \right\rangle_{\chi_1,\mathrm{SM}\to\chi_1,\mathrm{SM}} = \frac{3g'^2 g_e^2 m_{\chi_1}^2 m_e^2}{\pi m_{\gamma'}^4 \left(m_{\chi_1} + m_e\right)^2 \pi} v + \mathcal{O}(v^3)$$